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Assignment:- Parameter estimation.

1.) $x_1, x_2 \cdots x_n \rightarrow$ sample of size $n$

mean = $\theta_1$

variance = $\theta_2$.

$$f(x_i \mid \theta_1, \theta_2) = (1/\sqrt{2\pi\theta_2})\, e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

$$L(\theta_1, \theta_2 \mid x_1 \cdots x_n) = \prod (1/\sqrt{2\pi\theta_2})\, e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

Taking log

$$\ln(L(\theta_1, \theta_2 \mid x_1 \cdots x_n)) = -n \ln(\sqrt{2\pi\theta_2}) - \Sigma((x_i - \theta_1)^2/(2\theta_2))$$

Taking partial derivatives.

$$\partial \ln(L)/\partial\theta_1 = (1/\theta_2)^* \Sigma(x_i - \theta_1)$$

equating to zero

$\Sigma(x_i - \theta_2) = 0$

$\theta_1 = (\Sigma x_i)/n \rightarrow$ MLE of Mean

$$\partial \ln(L)/\partial\theta_2 = -n/2(1/\theta_2) + (1/2\theta_2^2))\Sigma(x_i - \theta_1)^2$$

$$\theta_2^2 = \Sigma(x_i - \theta_1)^2/(n-1)$$

Mean $(\theta_1) = (\Sigma x_i)/n$

Variance $(\theta_2^2) = \Sigma(x_i - \theta_1)^2/(n-1)$

2.) $x_1, x_2, \dots x_n$

B(m, $\theta$) distribution

$\theta \in \Theta = (0,1)$

$f(x_i \mid \theta) = \theta^{x_i} (1-\theta)^{(1-x_i)}$

$L(\theta \mid x_1 \dots x_n) = \prod f(x_i \mid \theta)$

$L(\theta \mid x_1 \dots x_n) = \prod (\theta^{x_i} (1-\theta)^{(1-x_i)})$

$\ln(L(\theta \mid x_1 \dots x_n)) = \sum (x_i \ln(\theta) + (1-x_i)\ln(1-\theta))$

$d \ln(L) / d\theta = \sum((x_i/\theta) - ((1-x_i)/(1-\theta))) = 0$

$\sum(x_i) - \theta \sum(1) = \theta \sum(1/\theta) - \sum(1/1-\theta))$

$\sum(x_i) - n\theta = n - \theta$

$\theta = \sum(x_i)/n$

MLE of $\theta = (\sum x_i)/n$