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### Assignment:- Parameter estimation.

1)  $x_1, x_2, \dots, x_n \rightarrow$  sample of size  $n$

mean =  $\theta_1$

variance =  $\theta_2$ .

$$f(x_i | \theta_1, \theta_2) = (1/\sqrt{2\pi\theta_2}) e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

$$L(\theta_1, \theta_2 | x_1, \dots, x_n) = \prod (1/\sqrt{2\pi\theta_2}) e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

Taking log

$$\ln(L(\theta_1, \theta_2 | x_1, \dots, x_n)) = -n \ln(\sqrt{2\pi\theta_2}) - \frac{\sum (x_i - \theta_1)^2}{2\theta_2}$$

Taking partial derivatives.

$$\partial \ln(L) / \partial \theta_1 = (1/\theta_2) \sum (x_i - \theta_1)$$

Equating to zero

$$\sum (x_i - \theta_1) = 0$$

$$\theta_1 = (\sum x_i) / n \rightarrow \text{MLE of Mean}$$

$$\partial \ln(L) / \partial \theta_2 = -n/2 \left( 1/\theta_2 + (1/2\theta_2^2) \sum (x_i - \theta_1)^2 \right)$$

$$\theta_2^2 = \sum (x_i - \theta_1)^2 / (n-1)$$

$$\text{Mean}(\theta_1) = (\sum x_i) / n$$

$$\text{Variance}(\theta_2^2) = \sum (x_i - \theta_1)^2 / (n-1)$$

2.)

$x_1, x_2, \dots, x_n$

$B(m, \theta)$  distribution

$\theta \in \Theta = (0, 1)$

$$f(x_i | \theta) = \theta^{x_i} (1-\theta)^{(1-x_i)}$$

$$L(\theta | x_1, \dots, x_n) = \prod f(x_i | \theta)$$

$$L(\theta | x_1, \dots, x_n) = \prod (\theta^{x_i} (1-\theta)^{(1-x_i)})$$

$$\ln[L(\theta | x_1, \dots, x_n)] = \sum (x_i \ln(\theta) + (1-x_i) \ln(1-\theta))$$

$$d \ln(L) / d\theta = \sum ((x_i / \theta) - ((1-x_i) / (1-\theta))) = 0$$

$$\sum (x_i) = \theta \sum (1) = \theta \sum (1/\theta) - \sum (1/(1-\theta))$$

$$\sum (x_i) = n\theta = n - \theta$$

$$\theta = \sum (x_i) / n$$

$$\text{MLE of } \theta = (\sum x_i) / n$$