Quiz II

Algorithms Analysis and Design
IIIT Hyderabad, Monsoon 2025

October 27, 2025

There are 3 questions 10 marks each. Maximum Marks: 30. Time: 45 min

---

1.
   - Suppose that in addition to edge capacities, a flow network has vertex capacities. That is each vertex v has a limit $\ell(v)$ on how much flow can pass through v. Show how to transform a flow network $G = (V, E)$ with vertex capacities into an equivalent flow network $G' = (V', E')$ without vertex capacities, such that a maximum flow in G' has the same value as a maximum flow in G.
   - The vertex-connectivity (analogous to edge connectivity) of an undirected graph is defined as the minimum number k of vertices that must be removed to disconnect the graph. Assuming that edge connectivity can be computed using maximum flow, can you also compute the vertex-connectivity (say, using vertex capacities etc.)? $7 + 3 = 10$ marks

2.
   - Give an efficient greedy algorithm that finds an optimal vertex cover for a tree. Can you find a linear time algorithm for the same?
   - Argue for (or against) how a max-flow algorithm can be used to design a 2-approximate algorithm for minimum vertex cover problem. $6 + 4 = 10$ marks

3.
   - Is the class **P** closed under intersection? What about the class **NP**? Prove your answers (make suitable assumptions, if required). $1 + 2 = 3$ marks
   - Let $\text{CLIQUE}_i = \{\langle G \rangle \mid G \text{ has a clique of size } i\}$. Prove that $\text{CLIQUE}_{1000} \in \mathbf{P}$. Assuming that 3-SAT is **NP**-Complete, prove that $\text{CLIQUE}_{\lceil \frac{n}{2} \rceil}$ is **NP**-Complete where n is the number of vertices in graph G. Show that for any i: $\text{CLIQUE}_{i+1} \leq_p \text{CLIQUE}_i$. Spot the error in the following proof of $\mathbf{P} = \mathbf{NP}$ and *suggest* suitable modifications to the author to correct the error: Consider induction on the clique size k. The base case is $k = 1$, and clearly $\text{CLIQUE}_1 \in \mathbf{P}$. Also, you just proved that if $\text{CLIQUE}_i \in \mathbf{P}$ then $\text{CLIQUE}_{i+1} \in \mathbf{P}$. From induction, we know: $\forall k \in \mathbf{N}\ \text{CLIQUE}_k \in \mathbf{P}$. Thus $\mathbf{P} = \mathbf{NP}$. $1 + 3 + 1 + 2 = 7$ marks

ALL THE BEST!