

Not so surprise Quiz 2: Probability and Statistics (25 mks)

Each question: 5 marks

1. (Memory test:)

- Define convergence in probability and convergence in r^{th} mean. (3 marks)
- When do we say that an estimator $\hat{\theta}$ is strongly consistent. Give an example. (2 mks)

2. Find the stationary distribution π for the Markov chain with transition matrix

$$P = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.5 & 0.5 \end{bmatrix}.$$

(5 marks)

3. Give a method to simulate n successive states of a Markovian coin with initial distribution $\mu = [0.5, 0.5]$ and

$$P = \begin{bmatrix} 0.1 & 0.9 \\ 0.3 & 0.7 \end{bmatrix}.$$

You are given access to only samples from a Uniform $[0, 1]$ random variable.

Each question 10mks

Let X_1, X_2, \dots, X_n be i.i.d. random variables with $E[X_i] = \theta$ and $Var(X_i) = \sigma^2$. Let $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$ and c denote a known constant. Derive the Mean square error (MSE) for the following estimators for θ :

- $\hat{\theta}_1 = 2\bar{X}_n - X_1$ (5mks). (Hint: \bar{X}_n and X_1 are not independent, so careful with covariance)
- $\hat{\theta}_2 = \frac{1}{3}(X_1 + X_3)$. (2.5mks)
- $\hat{\theta}_3 = c\bar{X}_n$. (2.5mks)