Mid Semester Examination

Algorithms Analysis and Design
IIIT Hyderabad, Monsoon 2025

September 25, 2025

There are 10 questions 10 marks each. Maximum Marks: 100. Time: 90 min

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1. Fill in the following blanks. $1 \times 10 = 10$ marks

    1. The FFT of $(1, 0, 1, -1)$ is __________.
    2. The worst-case complexity of deterministic quick-sort is __________.
    3. The `g.c.d.` of two consecutive Fibonacci numbers is __________.
    4. Solving the recurrence $T(n) = O(n) + \sum_{i=2}^{\lfloor\sqrt{n}\rfloor} T(\frac{n}{i^2})$, $T(\leq 1) = 1$, gives __________.
    5. A Huffman code for `aababcabcdabcdeabcdef` is __________.
    6. It is possible to find the median of n numbers in $O($__________$)$ time.
    7. The edit-distance between SUNNY and SNOWY is __________.
    8. An optimal parenthesization of a matrix-chain product with sequence of dimensions $\{5, 10, 3, 12, 5, 50, 6\}$ is __________.
    9. The cut-property states that __________.
    10. A language $A \subset \{0,1\}^*$ is said to be mapping-reduced to language $B \subset \{0,1\}^*$ if there exists a computable function f such that for all inputs in $\{0,1\}^*$, __________.

2. Give an efficient algorithm to find the longest palindrome that is a subsequence of a given input string. Analyze and prove your answer. 10 marks

3. Design and analyze with proof an algorithm to compute the length of the longest increasing back-and-forth subsequence (alternatively increasing and decreasing) in an array. 10 marks

4. Given positive integers n and k, along with $p_1, \ldots, p_n \in [0, 1]$, you wish to determine the probability of obtaining exactly k heads when n biased coins are tossed independently at random, where p_i is the probability that the i^{th} coin comes up heads. Give an efficient algorithm for this task. Suppose you wish to simulate an unbiased coin using all/some of the above coin(s), how would you go about it? $7 + 3 = 10$ marks

5. Design an algorithm to make change (if possible) for n rupees using at most k coins of denominations in the set $\{x_1, x_2, \ldots, x_n\}$ assuming infinite supply of each coin. If $x_i = c^i$, for some $c > 1$, show a greedy algorithm that yields an optimal solution. $6 + 4 = 10$ marks

6. Design an algorithm to count the number of ways to construct sum n by throwing a dice one or more times. Each throw produces an outcome between 1 and 6. Modify the problem by adding a *cost* (associated with every solution) that you wish to optimize on such that (a) greedy strategy fails and (b) greedy works. Prove your answers, $4 + 3 + 3 = 10$ marks

7. Define *matroids* and use a graphic matroid to solve the problem of minimum/maximum spanning tree using a greedy strategy. Give full details. 10 marks

8. You are given two multisets of integers, (a) A set of apples $A = \{a_1, a_2, \ldots, a_n\}$ and (b) A set of bananas $B = \{b_1, b_2, \ldots, b_m\}$, where each element lies in the range $[1, k]$. Define the pair sum frequency function $f : \{2, 3, \ldots, 2k\} \to \mathbb{N}$ as $f(w) = |\{(a, b) \in A \times B \mid a + b = w\}|$. That is, $f(w)$ counts the number of pairs consisting of one apple and one banana whose weights sum to exactly w. Design and analyze an efficient algorithm (using FFT that computes discrete convolutions efficiently) to compute $f(w)$ for all $w \in \{2, \ldots, 2k\}$. 10 marks

9. Prove the following: $2\frac{1}{2} \times 4 = 10$ marks

   1. the set of all subsets of $\mathbb{N}$ is uncountable.
   2. A_{TM} is undecidable.
   3. $\overline{A_{TM}}$ is unrecognizable.
   4. A_{DFA} is decidable.

10. Design and analyze (both efficacy/correctness/optimality and efficiency/complexity) an algorithm that computes the longest common subsequence (LCS) of two sequences. Execute your algorithm on some example of your choice. $5 + 2 + 1 + 2 = 10$ marks.

ALL THE BEST!