INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY

QUANTUM MECHANICS

Monsoon 2025 – CND Core – Credit 4

Instructor: Subhadip Mitra | **Date:** SEPTEMBER 22, 2025

Time: 1 H 30 M

Mid-Semester Examination

Total Marks: 50

Instructions:

- **Class notes or books are not permitted. But you may bring one A4 sheet of handwritten material (not photo-copied/printed). Calculators are allowed.**
- You may skip 'trivial' steps. However, unless the logic is clear, you will not get any credit for a problem.
- Illegible answers will not be graded. No 'benefit of doubt' because of bad notation/illegible hand-writing etc.

Q 1. Show that

(a) For any two observables represented by two operators, A and B,

$$\sigma_A^2\sigma_B^2 \geq \left(\frac{1}{2i}\langle[\hat{A},\hat{B}]\rangle\right)^2$$

(where σ denotes the standard deviation) and that for $\hat{A}=\hat{x}$ and $\hat{B}=\hat{p}$, the above equation reduces to the Heisenberg's uncertainty principle.

(b) If $\hat{A}$ and $\hat{B}$ have a complete set of common eigenstates (which then can form a basis), then $[\hat{A},\hat{B}]\,|\psi\rangle = 0$ for any $|\psi\rangle$ in the Hilbert space.

(c) Eigenvalues of Hermitian operators are real, and the eigenstates corresponding to different eigenvalues of a Hermitian operators are orthogonal. [5+2+3 = 10 **CO:** 1,2,5]

Q 2. A particle of mass m is constrained to move on the surface of a sphere of radius a.

(a) Write down the time-independent Schrödinger equation for the particle.

(b) Using the method of separation of variables, find the eigenfunctions (up to a proportionality factor) and identify the appropriate quantum numbers.

(c) Determine the allowed energy levels of the particle and their degeneracies.

(d) Sketch (qualitatively) the polar plots for the ground state and the first excited states on the surface of the sphere.

(e) Apply the L^2 operator on the wave functions and find its eigenvalues. [2+6+2+4+6 = 20 **CO:** 1,2,3,5]

Q 3. For a simple harmonic oscillator about the point $x=0$,

(a) Obtain the wave function for the first excited state ($n=1$) up to an overall multiplicative constant. What is its energy? Sketch $|\psi_1(x)|^2$ as a function of x.

(b) Let $\psi_n(x)$ denote the normalized (stationary) wavefunction of the n^{th} energy state. Express $\psi_n(x)$ in terms of $\psi_0(x)$.

(c) For the first excited state $\psi_1(x)$, calculate the expectation values $\langle x\rangle$ and $\langle x^2\rangle$.

(d) Suppose you measured the energy of the oscillator and found that the particle did not have sufficient energy to be in the second excited state or above. However, because of limited resolution of your measurement, you could not measure its energy more precisely. What would be the wave function $\psi(x)$ of the particle? Will $|\psi(x)|^2$ differ from that of the ground state? If so, how?

(e) Consider the *half* harmonic oscillator, defined by:

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2x^2, & x>0, \\ \infty, & x<0. \end{cases}$$

Find the allowed energy levels for this system. [5+4+4+3+4 = 20 **CO:** 1,2,3,5]