Mid-Semester Examination

(MA6.102) Probability and Random Processes, Monsoon 2025

24 September, 2025

Max. Duration: 90 Minutes

Question 1 (4 marks). Consider a two-coin toss experiment with sample space $\Omega = \{H, T\}^2$. For $i \in \{1, 2\}$, define

$$A_i = \{\omega = (\omega_1, \omega_2) \in \Omega : \omega_j = H \text{ for some } j \in [1:i]\}.$$

Construct the smallest σ-field $\mathcal{F}$ that contains the events A_1 and A_2. Further, determine whether the event $\{HH, TH\}$ belongs to $\mathcal{F}$.

Question 2 (3 marks). For n events $A_1, A_2, \ldots, A_n$, show that

$$P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1).$$

[*Hint: Analyze* $P\left(\bigcup_{i=1}^{n} A_i^c\right)$.]

Question 3 (5 Marks). Let $X \sim$ Poisson(λ) and $Y \sim$ Poisson(μ) be independent random variables. Determine the conditional distribution of X given that $X + Y = n$, i.e., $P_{X|\{X+Y=n\}}$. Is this distribution Poisson or Binomial? Specify its parameter(s).

Question 4 (3 marks). Let X and Y be independent Bernoulli($\frac{1}{2}$) random variables. Define $Z = X + Y$ and $W = |X - Y|$. Determine whether Z and W are uncorrelated, and whether they are independent.

Question 5 (5 Marks). Let $X \sim$ Exponential(λ), i.e., $f_X(x) = \lambda e^{-\lambda x}$, $x \geq 0$. Define

$$Y = \lfloor X \rfloor \quad \text{(the integer part of } X\text{)}, \qquad R = X - \lfloor X \rfloor \quad \text{(the fractional part of } X\text{)}.$$

(a) Find the PMF of Y.
(b) Find the CDF and PDF of R.