EE5604: Introduction to Statistical Learning Theory, Fall 2022

Indian Institute of Technology Hyderabad
HW 1, Assigned: Tuesday 18.10.2022. 25 points.
Due: Sunday 23.10.2022 at 11:59 pm.

1. State and prove the Sauer's lemma and the corollary. (5)

2. Now use the Sauer's lemma to derive a probabilistic upper bound on the generalization error of the infinite cardinality of Λ case. (5)

3. Let $(X_n : n \geq 1)$ be the sequence of random variables on the standard unit-interval probability space defined by $X_n(\omega) = \omega^n$. Does this sequence converge to a finite limit almost surely? If yes, find the limit. If not, state why not. (5)

4. Let $(X_n : n \geq 1)$ be the sequence of random variables on the standard unit-interval probability space. Any n can be expressed as $n = 2^k + j$ where $k = \lfloor \ln_2(n) \rfloor$, $0 \leq j < 2^k$. The random variable $X_n(\omega)$ is one in the interval $(j2^{-k}, (j+1)2^{-k}]$ and zero elsewhere. Does this sequence converge almost surely? If yes, find the limit. If not, state why not. (5)

5. Let U be uniformly distributed on the interval $[0, 1]$, and for $n \geq 1$, let $X_n = \frac{(-1)^n U}{n}$. Let X denote the random variable that is 0 for all ω. Does X_n converge to X:

   (a) almost surely, (1)

   (b) in probability, (1)

   (c) in distribution. (3)

   Plot the CDF of X_n for even and odd n.