

**EE5605: Kernel Methods, Fall 2022 (56)**

Indian Institute of Technology Hyderabad

HW 0, 30 points. Assigned: Thursday 03.11.2022.

**Due: Wednesday 09.11.2022 at 11:59 pm.**

1. Show that the set of real numbers  $\mathbb{R}$  with the distance measure  $d(x, y) = |x - y|$  is a metric space. (5)
2. Let  $X$  be the set of  $n$ -letter words in a  $k$ -character alphabet  $A = \{a_1, a_2, \dots, a_k\}$ , meaning that  $X = \{(x_1, x_2, \dots, x_n) | x_i \in A\}$ . The distance  $d(x, y)$  between two words  $x, y \in X$  is defined to be the number of places in which the words have different letters. Show that  $(X, d)$  is a metric space. (5)
3. Suppose that  $(X, \|\cdot\|)$  is a normed linear space. Show that  $d(x, y) = \|x - y\|$  is a metric on  $X$ . (5)
4. Suppose that  $\mathcal{H}_1$  and  $\mathcal{H}_2$  are two Hilbert spaces. We define  $\mathcal{H}_1 \oplus \mathcal{H}_2 = \{(x_1, x_2) | x_1 \in \mathcal{H}_1, x_2 \in \mathcal{H}_2\}$  with the inner product  $\langle (x_1, x_2), (y_1, y_2) \rangle_{\mathcal{H}_1 \oplus \mathcal{H}_2} = \langle x_1, y_1 \rangle_{\mathcal{H}_1} + \langle x_2, y_2 \rangle_{\mathcal{H}_2}$ . Prove that  $\mathcal{H}_1 \oplus \mathcal{H}_2$  is a Hilbert space. (5)
5. Following the definition of a kernel from class, show that the sums of kernels are kernels. (5)
6. Let  $x, x' \in \mathbb{R}^d$  for  $d \geq 1$ , and let  $m \geq 1$  be an integer and  $c \geq 0$  be a positive real. Show that  $k(x, x') := (\langle x, x' \rangle + c)^m$  is a valid kernel. (5)