

3. Mathematically derive the average runtime complexity of the non-random pivot version of quicksort.

First, picking a pivot and computing it with each element of an array which takes time complexity of $\Theta(n)$.

The pivot partition of the array to subarray of sizes k and $n-k-1$. we assume that the pivot divides the array into somewhere equal halves making the recurrence relation to

$$T(n) = T(k) + T(n-k-1) + \Theta(n)$$

for average case we expect the value of k roughly $n/2$,

$$T(n) = T(n/2) + T(n/2) + \Theta(n)$$

$$T(n) = 2T(n/2) + \Theta(n)$$

Using Master Theorem,

$$T(n) = aT(n/b) + \Theta(n^d)$$

$$a=2 \quad b=2 \quad d=1$$

Since, this is the case of $a = b^d$
we have,

$$\Theta(n^d \lg(n))$$

So,

$$T(n) = \Theta(n^1 \lg(n))$$

$$T(n) = \Theta(n \log n)$$