Value Iteration with Guessing

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Objective

Compute the value of Markov Decision Processes,
using only local updates,
for reachability (and shortest stochastic path).

$$v \in \mathbb{R}^S \mapsto \mathrm{Update}(v)(s) \coloneqq \max_{a \in A} \left\{ \sum_{s' \in S} p(s' \mid s, a)\ v(s') \right\}$$

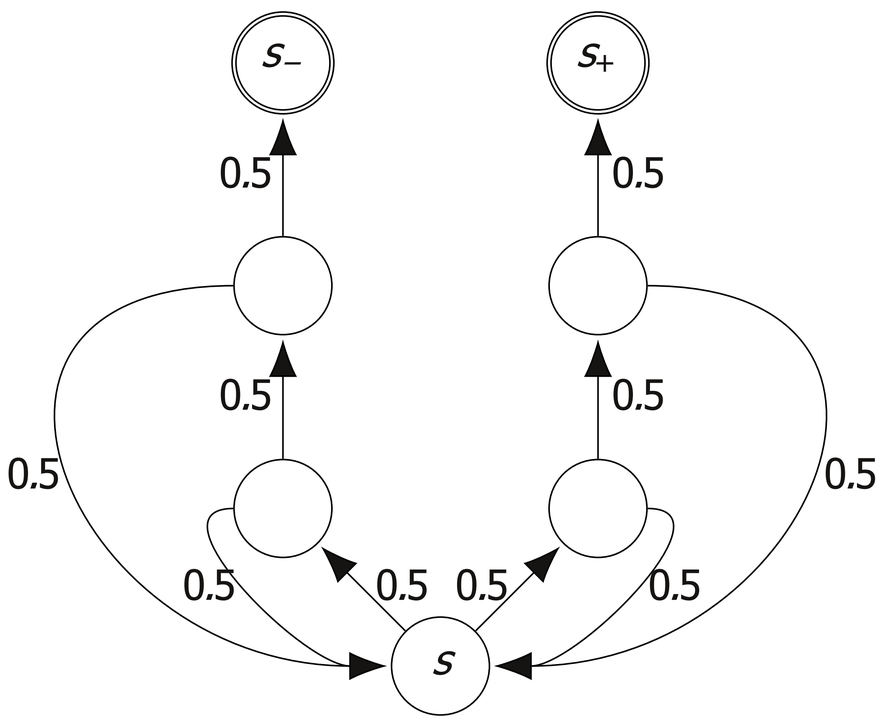
Value Iteration

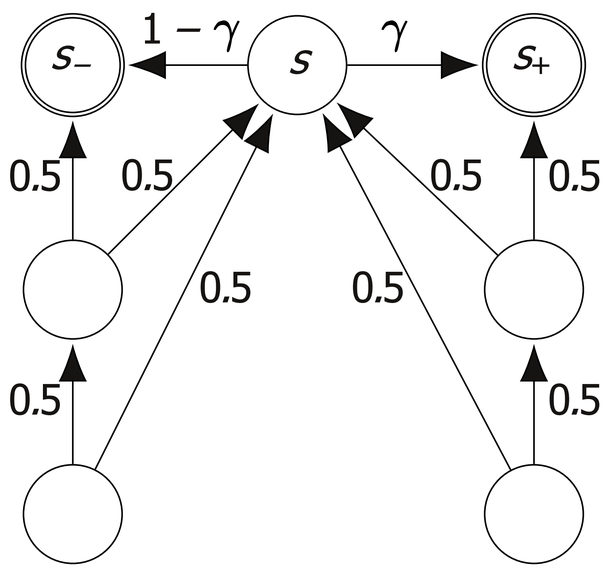
$$v_0(s) \coloneqq 1[s \text{ is target}]$$
$$v_{i+1} \coloneqq \mathrm{Update}(v_i)$$

$$\| v_{|S|i} - \mathrm{val} \|_\infty \leq \left(1 - p_{\min}^{|S|}\right)^i \| v_0 - \mathrm{val} \|_\infty$$

Number of updates required for approximation: $(-\log(\varepsilon)|S|/p_{\min})^{\Omega(|S|)}$

Guessing to simplify structure





Inequalities

Gueessed values can be verified with the recursive solution.

$$\mathrm{Update}(\gamma, \mathrm{val}[s \coloneqq \gamma])(s) > \gamma \ \Leftrightarrow\ \mathrm{val}(s) > \gamma$$

Approximate verification is possible.

$$\mathrm{Update}(\gamma, \mathrm{val}[s \coloneqq \gamma])(s) > \gamma - \varepsilon p_{\min}^{|S|} \ \Rightarrow\ \mathrm{val}(s) > \gamma - \varepsilon$$

Markov Chains

We can precompute which updates to ask for using linear space and almost linear time
to obtain an approximation of the value using few updates.

Precomputation

$$\mathcal{O}((|S| + |E|) \log |S|)$$

Numer of updates

$$(-\log(\varepsilon)|S|/p_{\min})^{\mathcal{O}(\sqrt{|S|})}$$

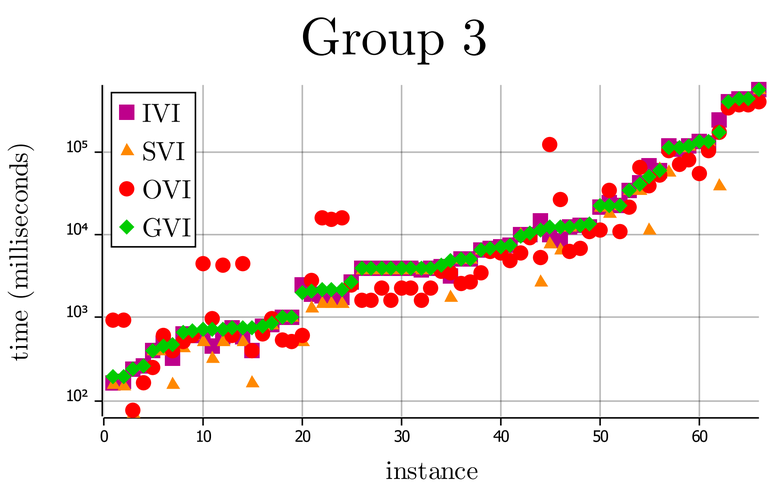
Markov Decision Processes

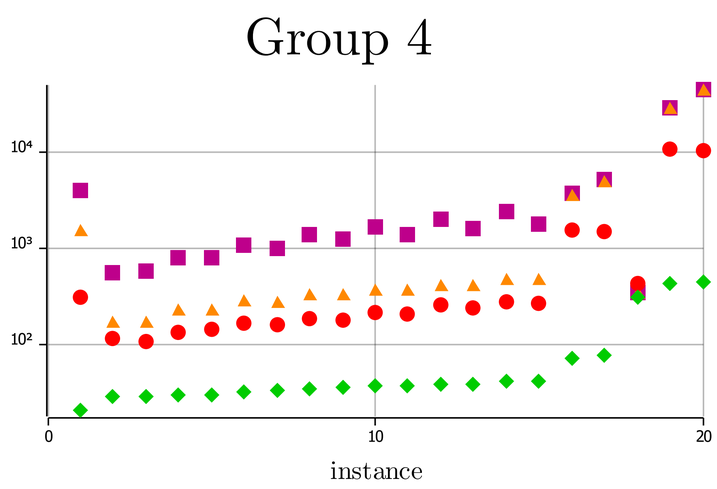
Group 1: 170 instances
All algorithms take less than 0.1 secs

Group 2: 135 instance
Fastest and slowest algorithms are only at most 1.10 times of each other

Group 3: 139 instances
Guessing VI is not the fastest

Group 4: 30 instances
The rest, our improvement





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