Artificial Intelligence
CS-401

Chapter # 03

Solving Problems by Searching

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Chapter's Outline

- Problem Solving Agents
- Well Defined Problems and Solutions
- Example Problems
- Searching for Solutions
  - Infrastructure of Search Algorithms
  - Performance Metric
- **Uninformed Search Strategies**
- Informed (Heuristic) Search Strategies

Search Algorithms

- Uninformed Blind search
  1. Breadth-first
  2. depth-first
  3. uniform cost
  4. Iterative deepening depth-first
  5. Bidirectional
- Informed Heuristic search
  - Greedy search, A*, Hill climbing, Local Searches
- Important concepts:
  1. Completeness
  2. Time complexity
  3. Space complexity
  4. Quality of solution

Search Strategies

- A search strategy is defined by picking the **order of node expansion**

- Strategies are **evaluated** along the following four dimensions:
  1. **Completeness**: does it always find a solution if one exists?
  2. **Optimality**: does it always find a least-cost solution?
  3. **Time complexity**: number of nodes generated
  4. **Space complexity**: maximum number of nodes in memory

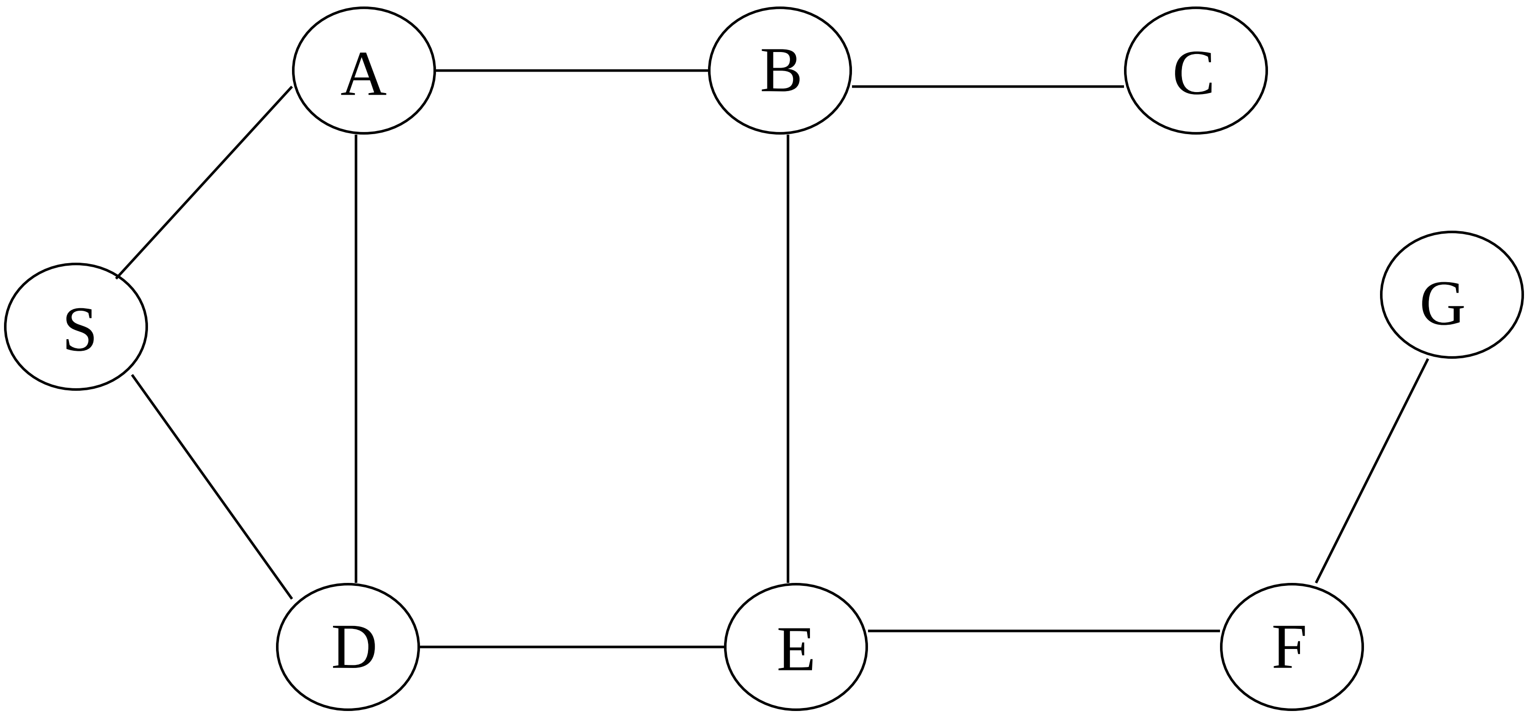
- Time and space complexity are measured in terms of
  - ***b:*** maximum branching factor of the search tree
  - ***d:*** depth of the least-cost solution
  - ***m*:** maximum depth of the state space (may be ∞)

Breadth-First Search (BFS)

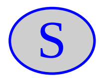
- Expand shallowest unexpanded node
- **Fringe**: nodes waiting in a queue to be explored, also called OPEN
- Implementation:
  - For BFS, *fringe* is a first-in-first-out (FIFO) queue
  - new successors go at end of the queue
- Repeated states?
  - Simple strategy: do not add parent of a node as a leaf

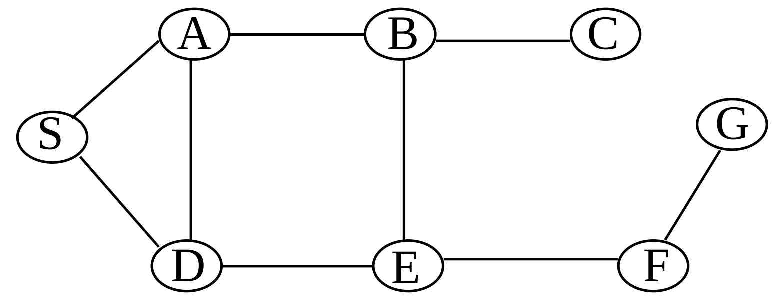
Example: Map Navigation

State Space:
S = start,   G = goal, other nodes = intermediate states, links = legal transitions



BFS Search Tree





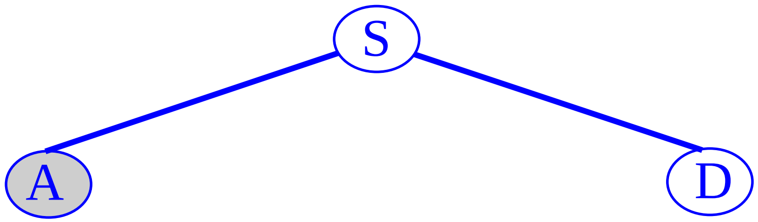
Queue = {S}

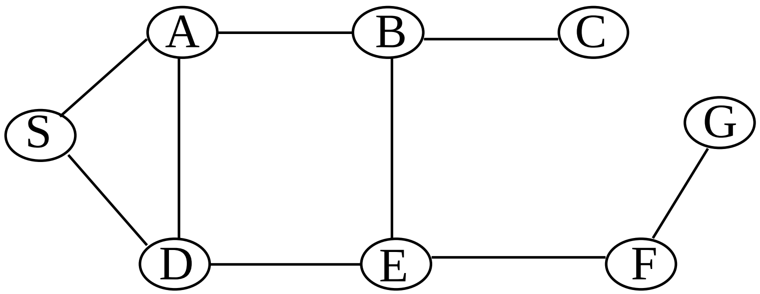
Select S

Goal(S) = true?

If not, Expand(S)

BFS Search Tree





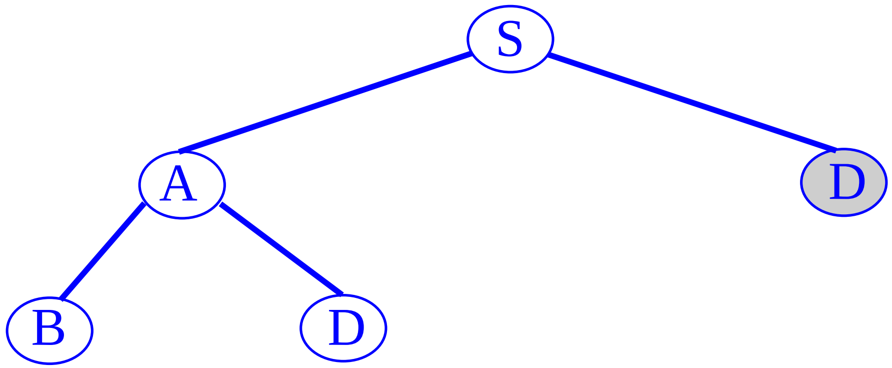
Queue = {A, D}

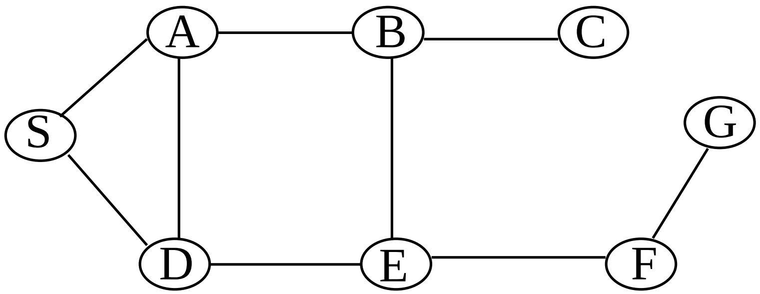
Select A

Goal(A) = true?

If not, Expand(A)

BFS Search Tree





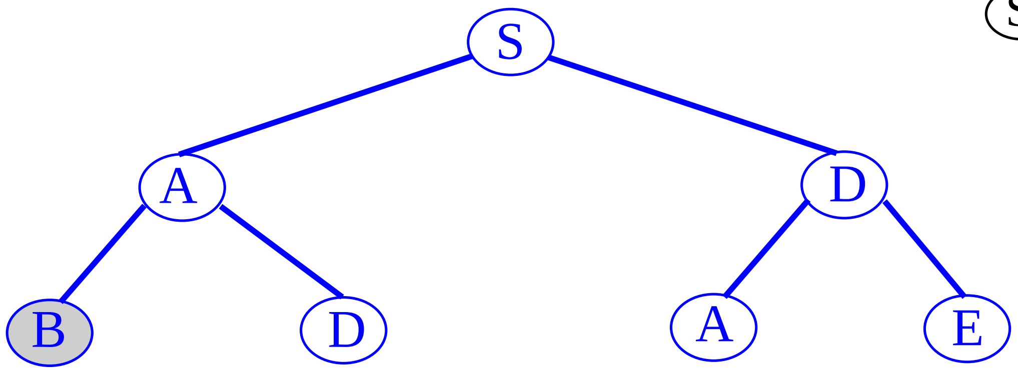
Queue = {D, B, D}

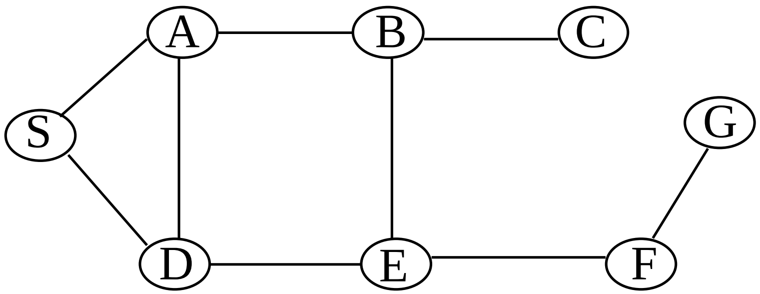
Select D

Goal(D) = true?

If not, expand(D)

BFS Search Tree

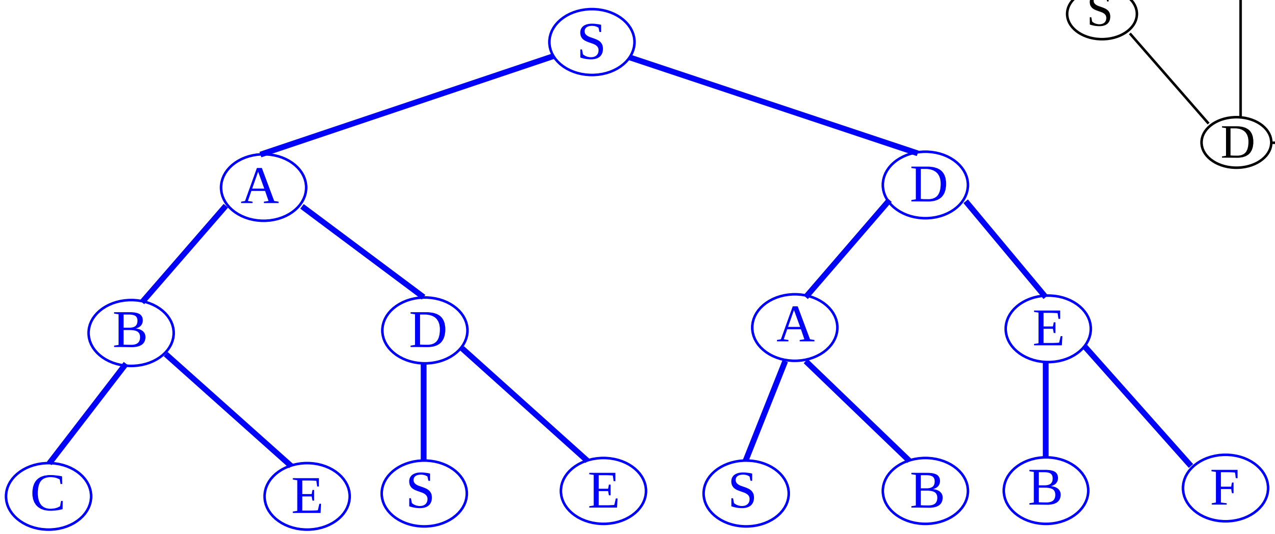


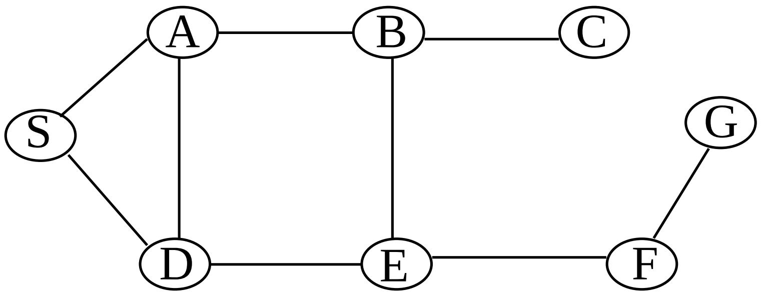


Queue = {B, D, A, E}

Select B
etc.

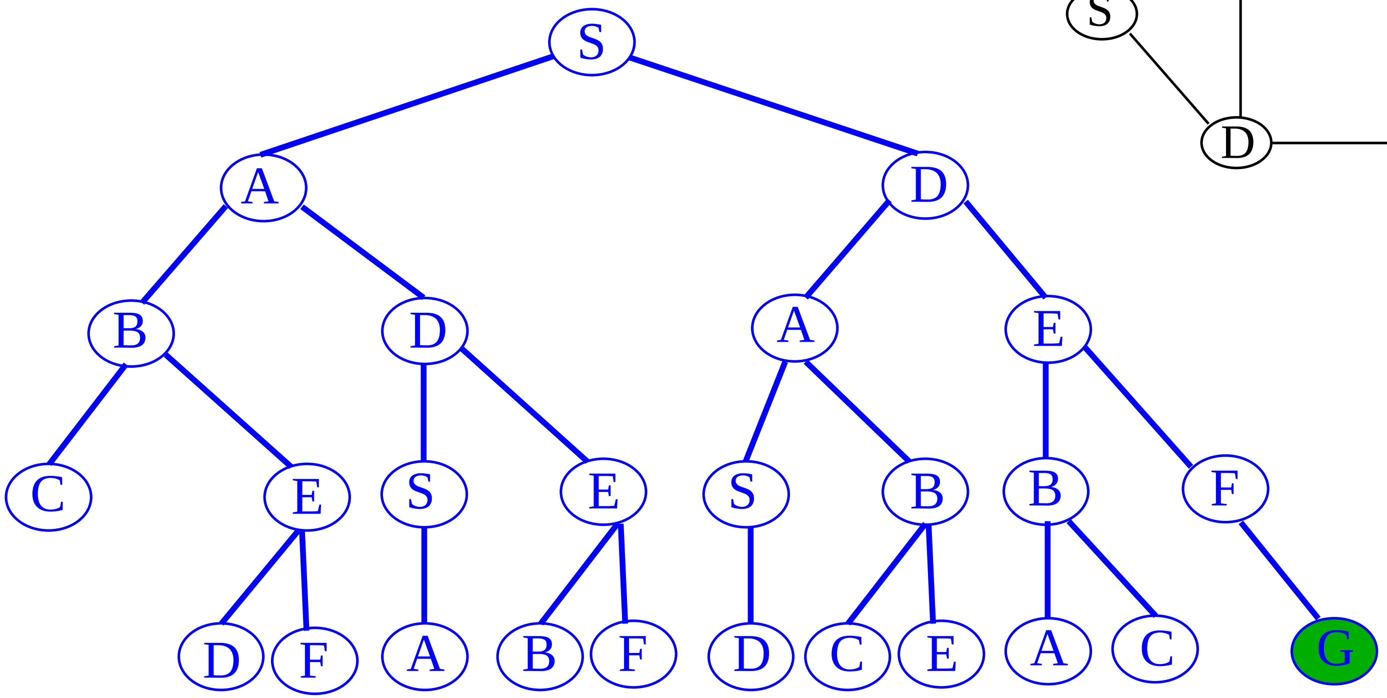
BFS Search Tree

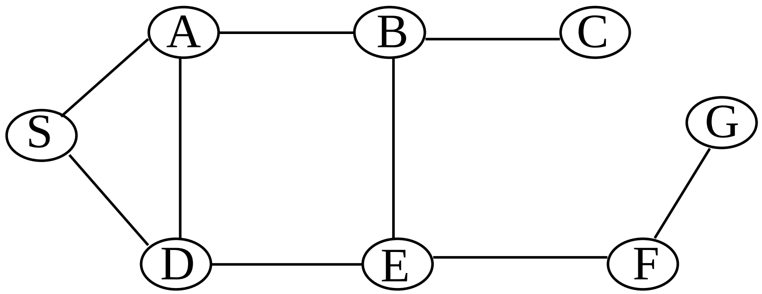




Level 3
Queue = {C, E, S, E, S, B, B, F}

BFS Search Tree





Level 4
Expand queue until G is at front
Select G
Goal(G) = true

Breadth-First Search (BFS) Properties

- Complete?
  Yes

- Optimal?
  Only if path-cost = **non-decreasing function of depth**

- Time complexity
  $O(b^d)$

- Space complexity
  $O(b^d)$

- Main practical drawback?
  exponential time and **space** complexity

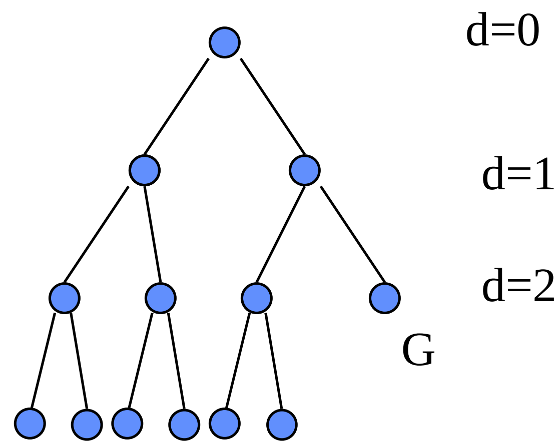
Complexity of Breadth-First Search

- **Time Complexity**
  - assume (worst case) that there is **1 goal** leaf at the RHS at **depth d**
  - so BFS will generate

    $= b + b^2 + \;.....\; + b^d + b^{d+1} - b$

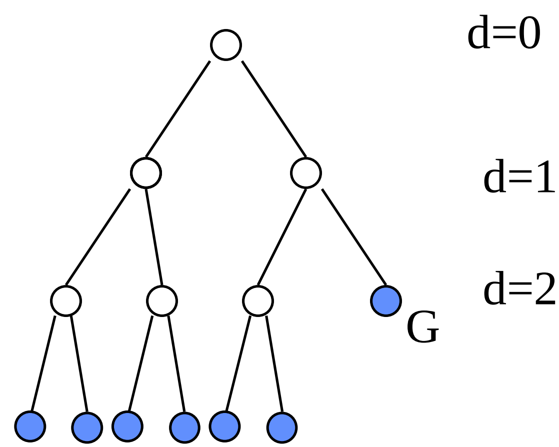
    $= \mathbf{O\ (b^{d+1})}$
  - If goal test is applied **before expansion** then complexity will be:

    $= \mathbf{O\ (b^d)}$



- **Space Complexity**
  - how many nodes can be in the queue (worst-case)?
  - at depth **d** there are b^{d+1} unexpanded nodes in the queue.

  $= \mathbf{O\ (b^{d+1})}$



Examples of Time and Memory Requirements for Breadth-First Search

Assuming b=10, 10,000 nodes/sec, 1kbyte/node

Depth of Solution	Nodes Generated	Time	Memory
2	1100	0.11 seconds	1 MB
4	111,100	11 seconds	106 MB
8	10^9	31 hours	1 TB
12	10^{13}	35 years	10 PB

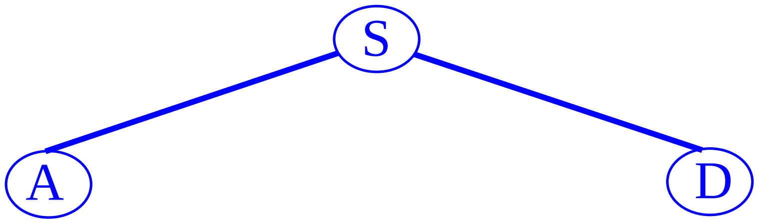
Depth-First Search (DFS)

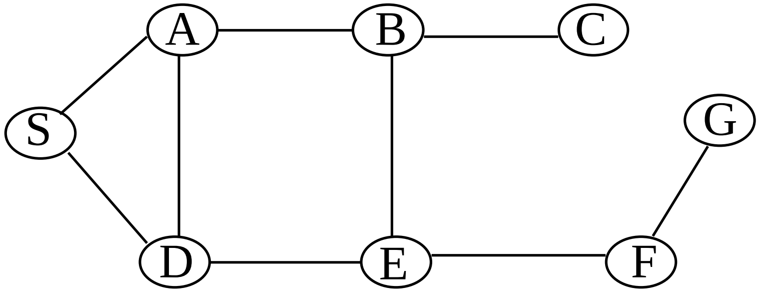
- Expand **deepest** unexpanded node

- Implementation:
  - For DFS, *fringe* is a **Last-in-first-out (LIFO)** queue
  - new successors go at beginning of the queue

- **Repeated nodes**?
  - Simple strategy: Do not add a state in the fringe if that state is already on the path from the root to the current node

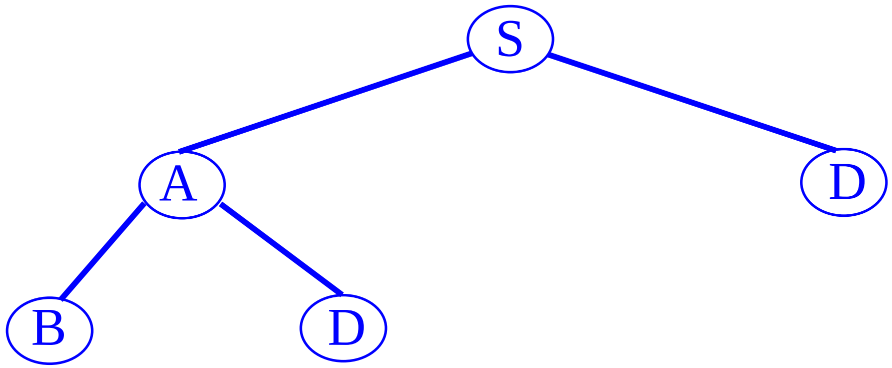
DFS Search Tree

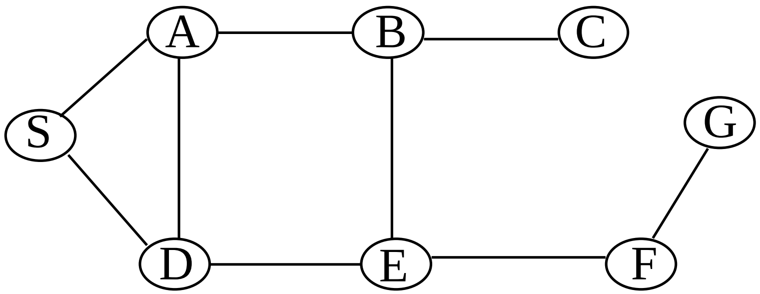




Queue = {A,D}

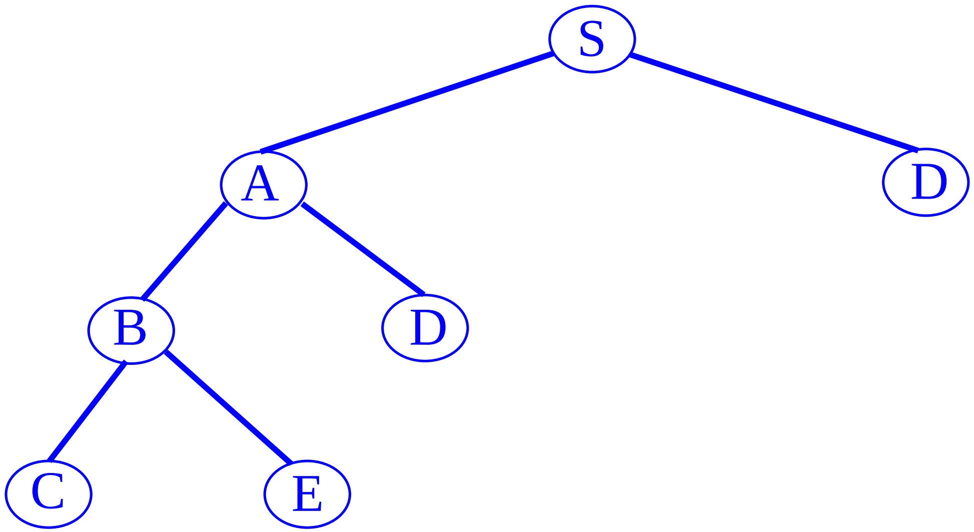
DFS Search Tree

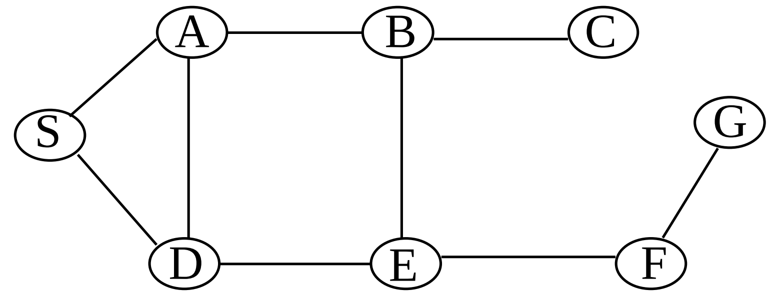




Queue = {B,D,D}

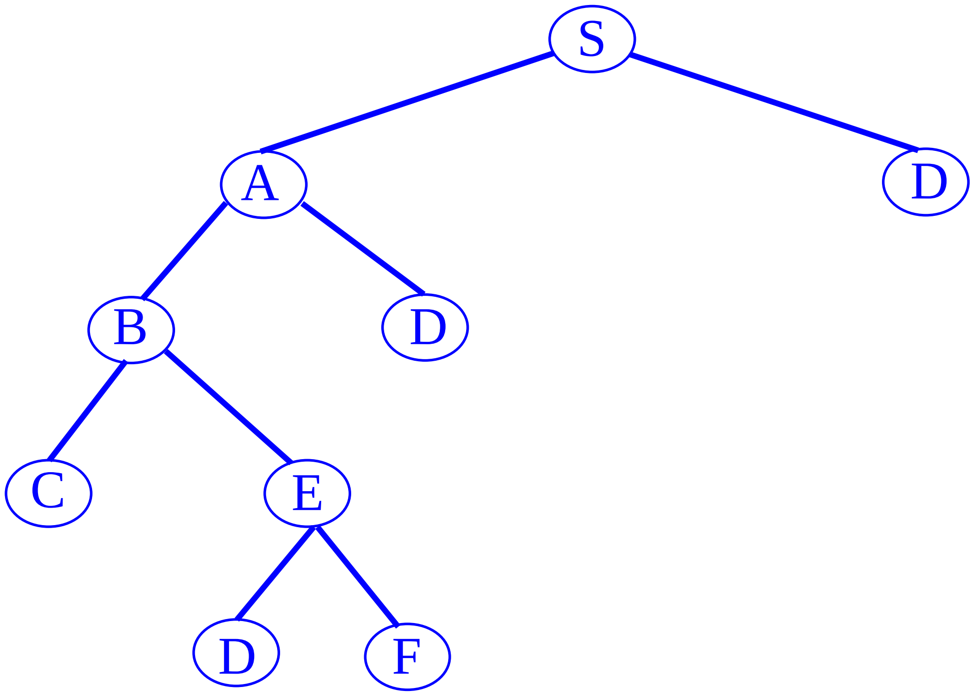
DFS Search Tree

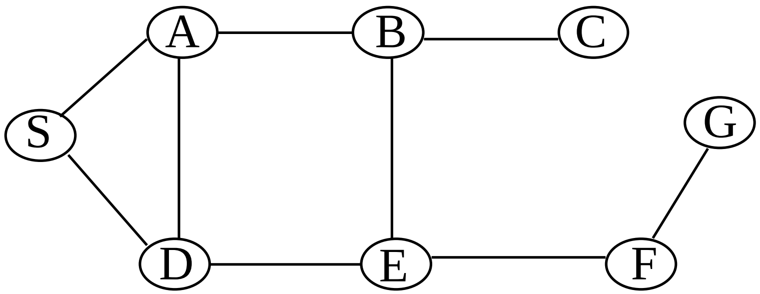




Queue = {C,E,D,D}

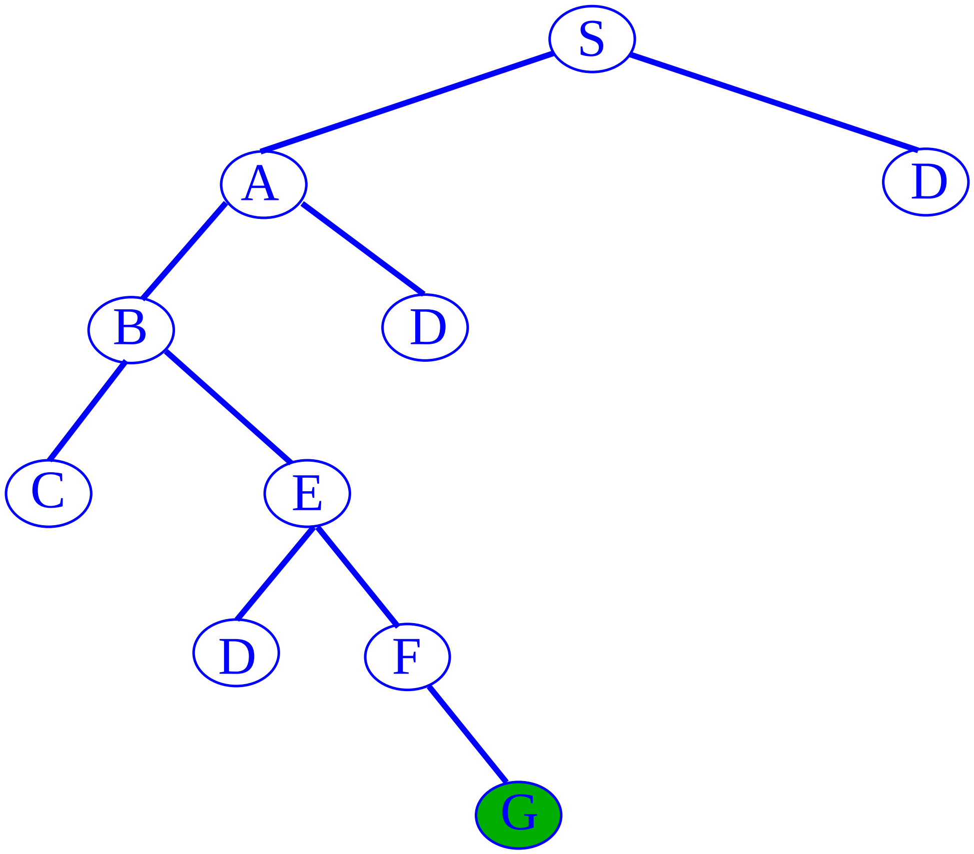
DFS Search Tree

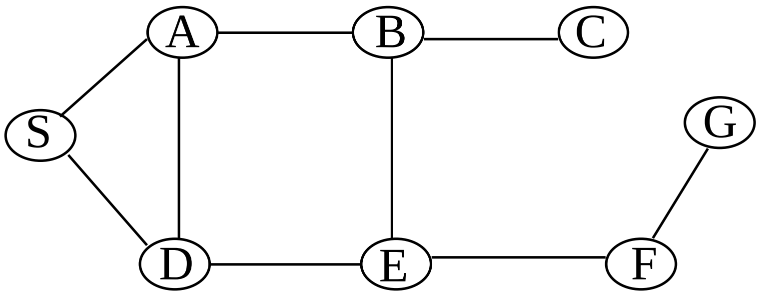




Queue = {D,F,D,D}

DFS Search Tree

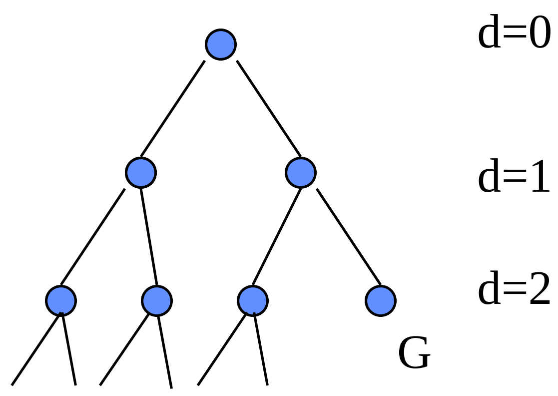




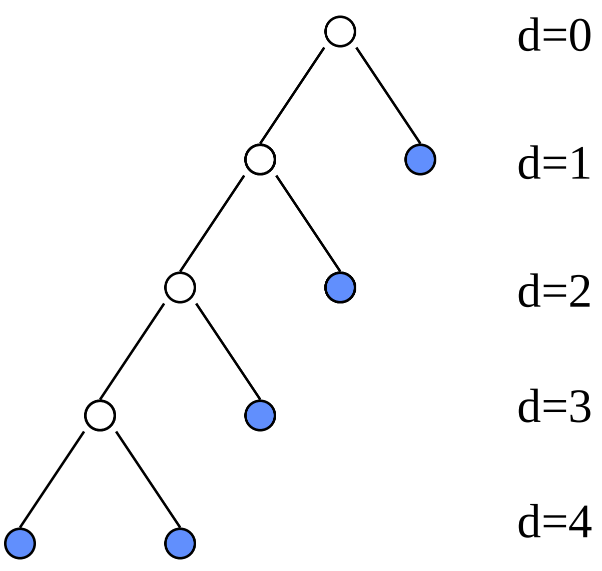
Queue = {G,D,D}

What is the Complexity of Depth-First Search?

- Time Complexity
  - maximum tree depth = m
  - assume (worst case) that there is 1 goal leaf at the RHS at depth d
  - so DFS will generate **O (b^m)**



- Space Complexity
  - how many nodes can be in the queue (worst-case)?
  - at depth m we have b nodes
  - and b-1 nodes at earlier depths
  - total = b + (m-1)*(b-1) = **O(bm)**



Examples of Time and Memory Requirements for Depth-First Search

Assuming b=10, m = 12, 10,000 nodes/sec, 1kbyte/node

Depth of Solution	Nodes Generated	Time	Memory
2	10^{12}	3 years	120kb
4	10^{12}	3 years	120kb
8	10^{12}	3 years	120kb
12	10^{12}	3 years	120kb

Depth-First Search (DFS) Properties

- Complete?
  - Not complete **if tree has unbounded depth**
- Optimal?
  - **No**
- Time complexity?
  - **Exponential**
- Space complexity?
  - **Linear ☺**

Comparing DFS and BFS

- Time complexity: same, but
  - In the worst-case BFS is always better than DFS
  - Sometime, on the average DFS is better if:
    - ✓ many goals, no loops and no infinite paths

- BFS is much worse memory-wise
  - DFS is linear space
  - BFS may store the whole search space.

- In general
  - BFS is better if goal is not deep, if infinite paths, if many loops, if small search space
  - DFS is better if many goals, not many loops,
  - **DFS is much better in terms of memory**

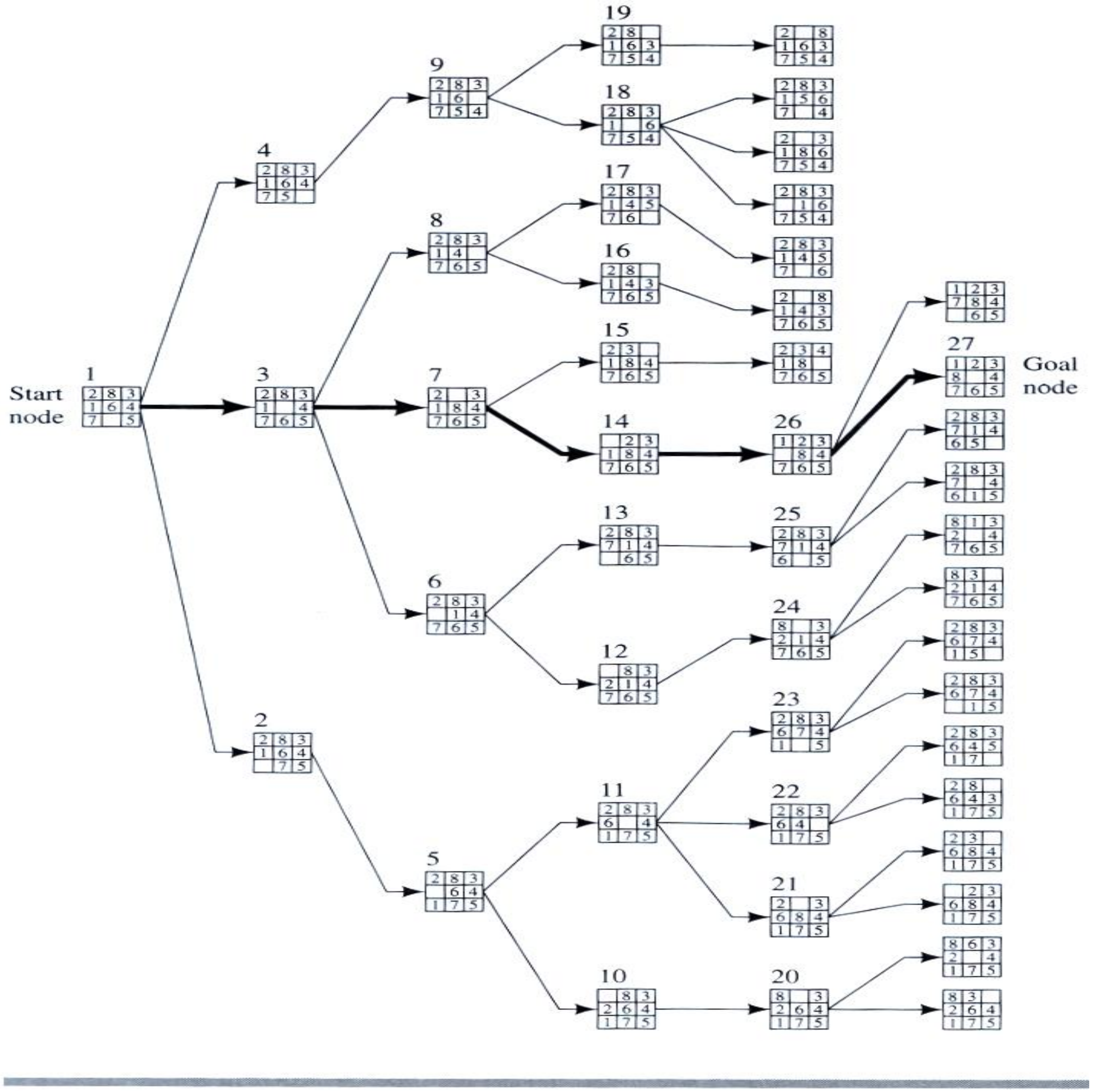
DFS with a depth-limit L

- Standard DFS, but tree is not explored below some depth-limit L

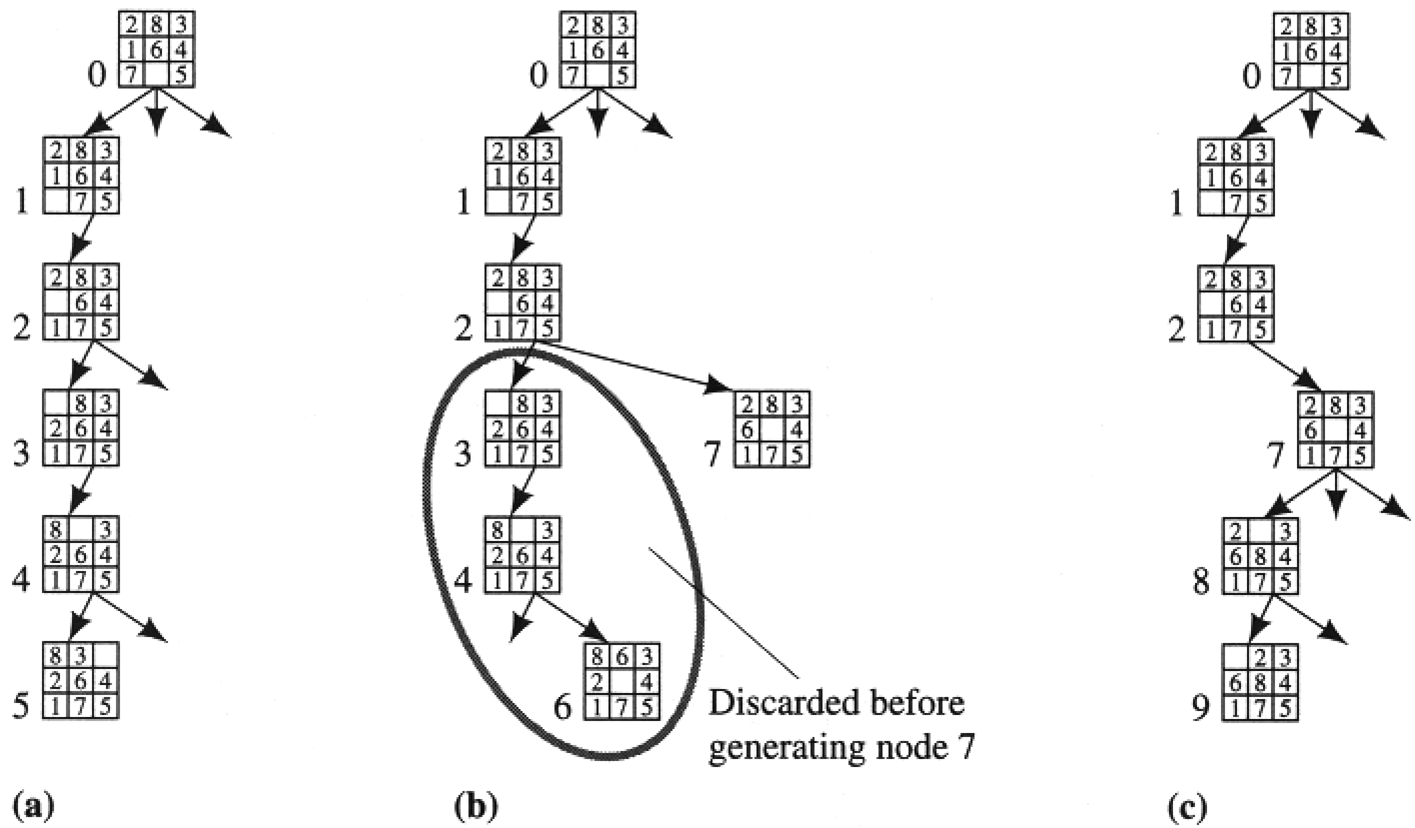
- Solves problem of infinitely deep paths with no solutions
  - But **will be incomplete** if solution is below depth-limit

- Depth-limit **L** can be selected based on **problem knowledge**
  - E.g., diameter of state-space:
    - E.g., max number of steps between 2 cities
  - But typically not known ahead of time in practice ☹

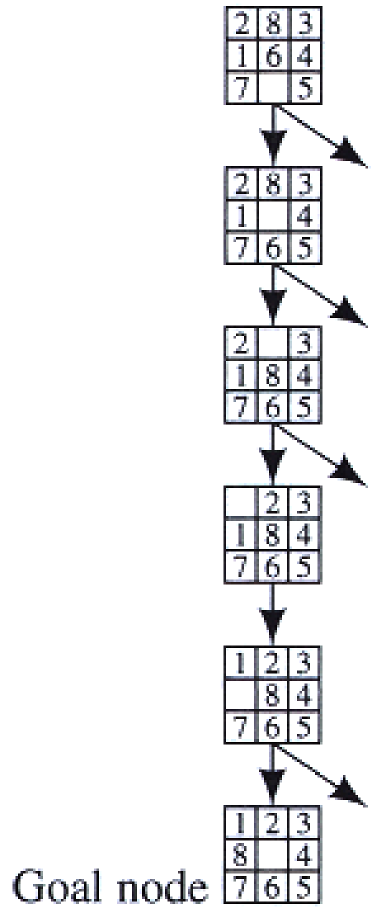
Depth-First Search with a depth-limit, L = 5



Depth-First Search with a depth-limit



Generation of the First Few Nodes in a Depth-First Search



The Graph When the Goal Is Reached in Depth–First Search

Iterative Deepening Search (IDS)

- Run multiple DFS searches with increasing depth-limits

- Iterative deepening search
  - L = 1
  - While no solution, do

    DFS from initial state S_0 with cutoff L

    If found goal,

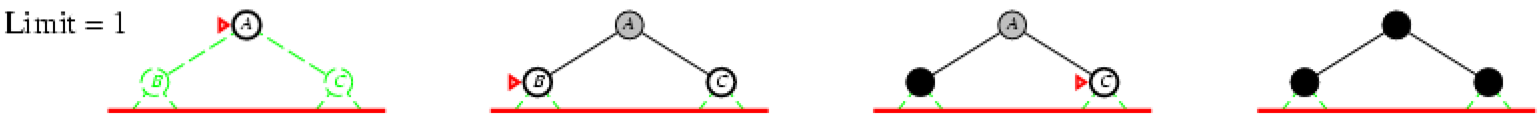
    stop and return solution,

    else, increment depth limit L

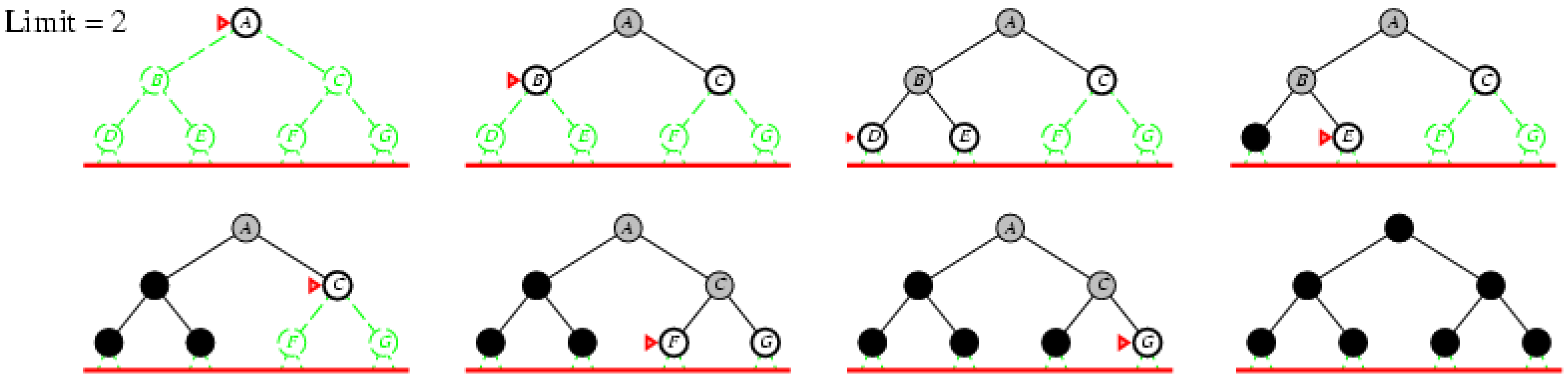
Iterative deepening search *L*=0

Limit = 0

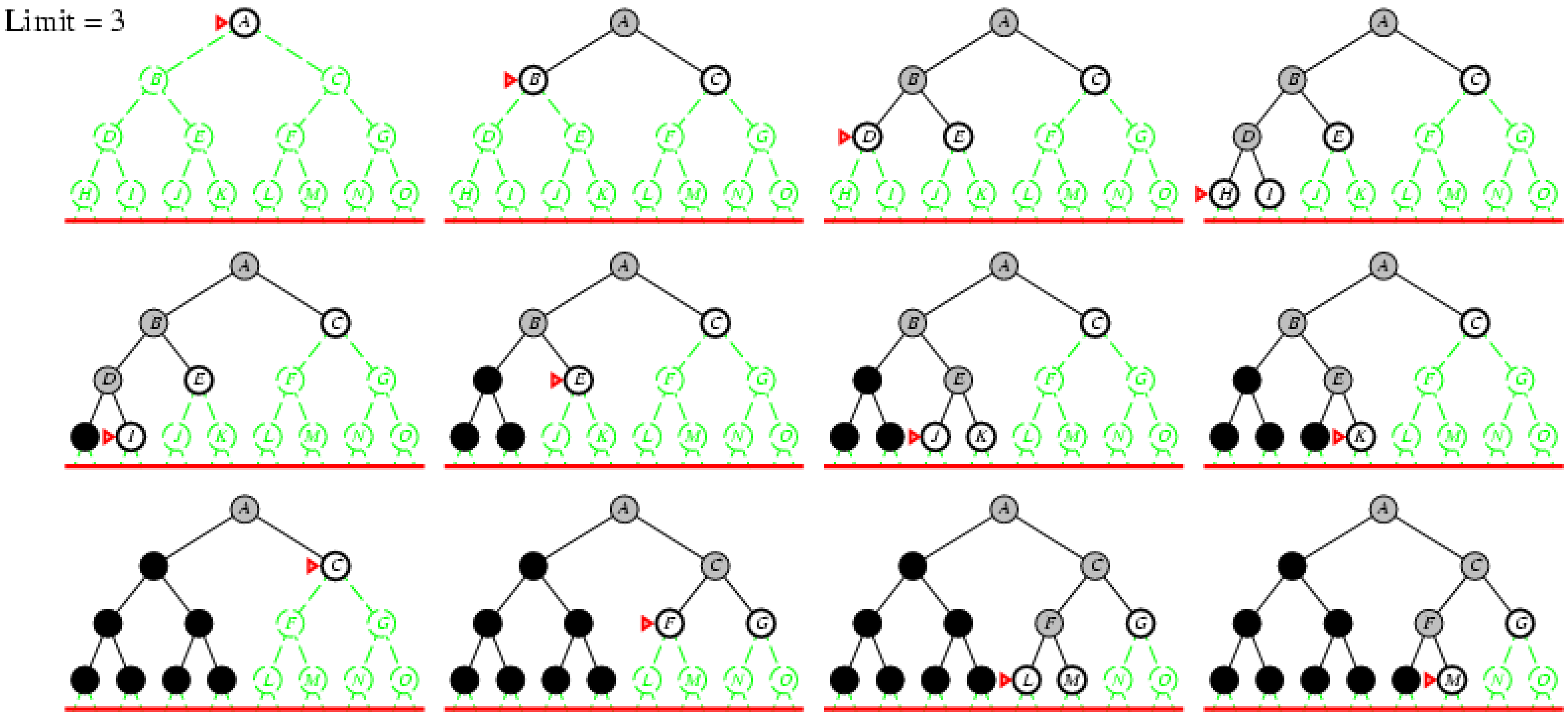
Iterative deepening search *L*=1



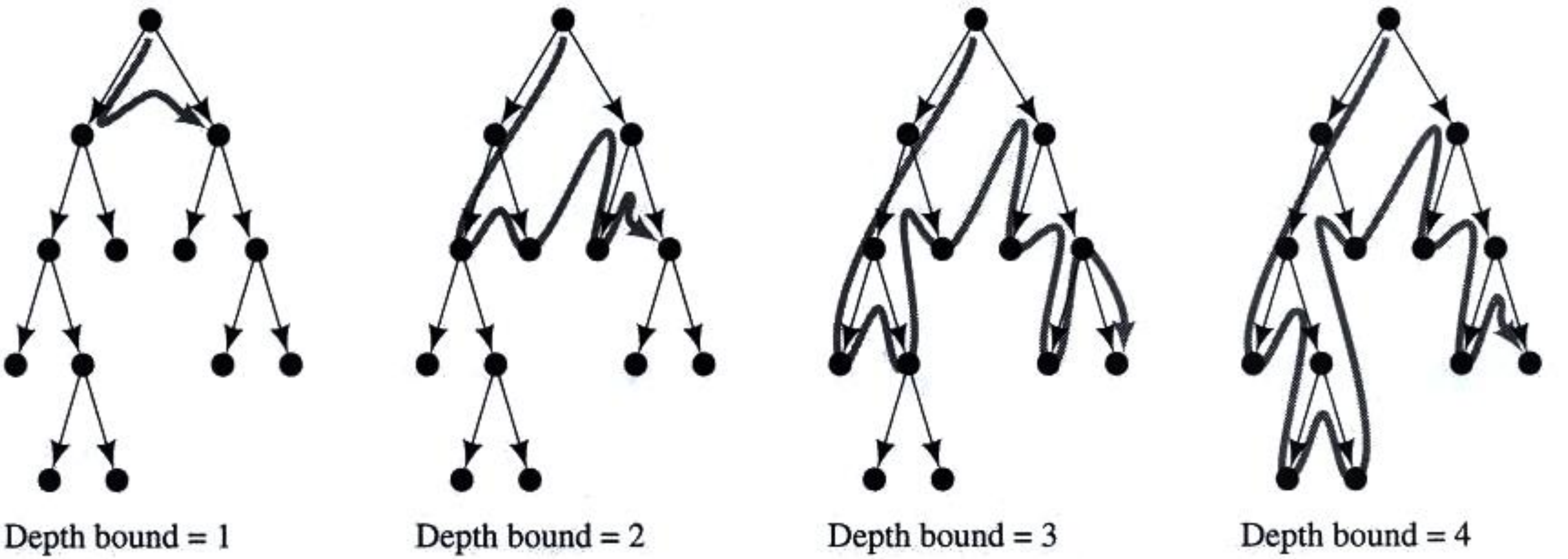
Iterative deepening search L=2



Iterative Deepening Search L=3



Iterative deepening search



Stages in Iterative-Deepening Search

Properties of Iterative Deepening Search

- Space complexity = **O(bd)**
  - (since its like depth first search run different times, with maximum depth limit d)
- Time Complexity
  - **$(d)\ b + (d-1)\ b^2 + \ldots. + (1)\ b^d = O(b^d)$**
    (i.e., asymptotically the same as BFS or DFS to limited depth d in the worst case)
- **Complete**?
  - **Yes**
- **Optimal**
  - **Only if path cost is a non-decreasing function of depth**
- IDS combines the small memory footprint of DFS, and has the completeness guarantee of BFS

IDS in Practice

- Isn't IDS wasteful?
  - Repeated searches on different iterations

  - **Compare IDS and BFS:**
    - **E.g., $b = 10$ and $d = 5$**

    - **$N(IDS) \sim db + (d-1)b^2 + \ldots\ldots b^d = 123{,}450$**

    - **$N(BFS) \sim b + b^2 + \ldots\ldots b^d = 111{,}110$**

    - **Difference is only about 10%**
      - Most of the time is spent at depth d, which is the same amount of time in both algorithms

- In practice, IDS is the **preferred uniform search method** with a **large search space** and **unknown solution depth**

Uniform Cost Search

- Optimality: path found = lowest cost
  - Algorithms so far are only optimal under restricted circumstances
- Let **g(n)** = cost from start state S to node n
- Uniform Cost Search:
  - **Always expand the node on the fringe with minimum cost g(n)**
  - **Note that if costs are equal (or almost equal) will behave similarly to BFS**

Uniform Cost Search

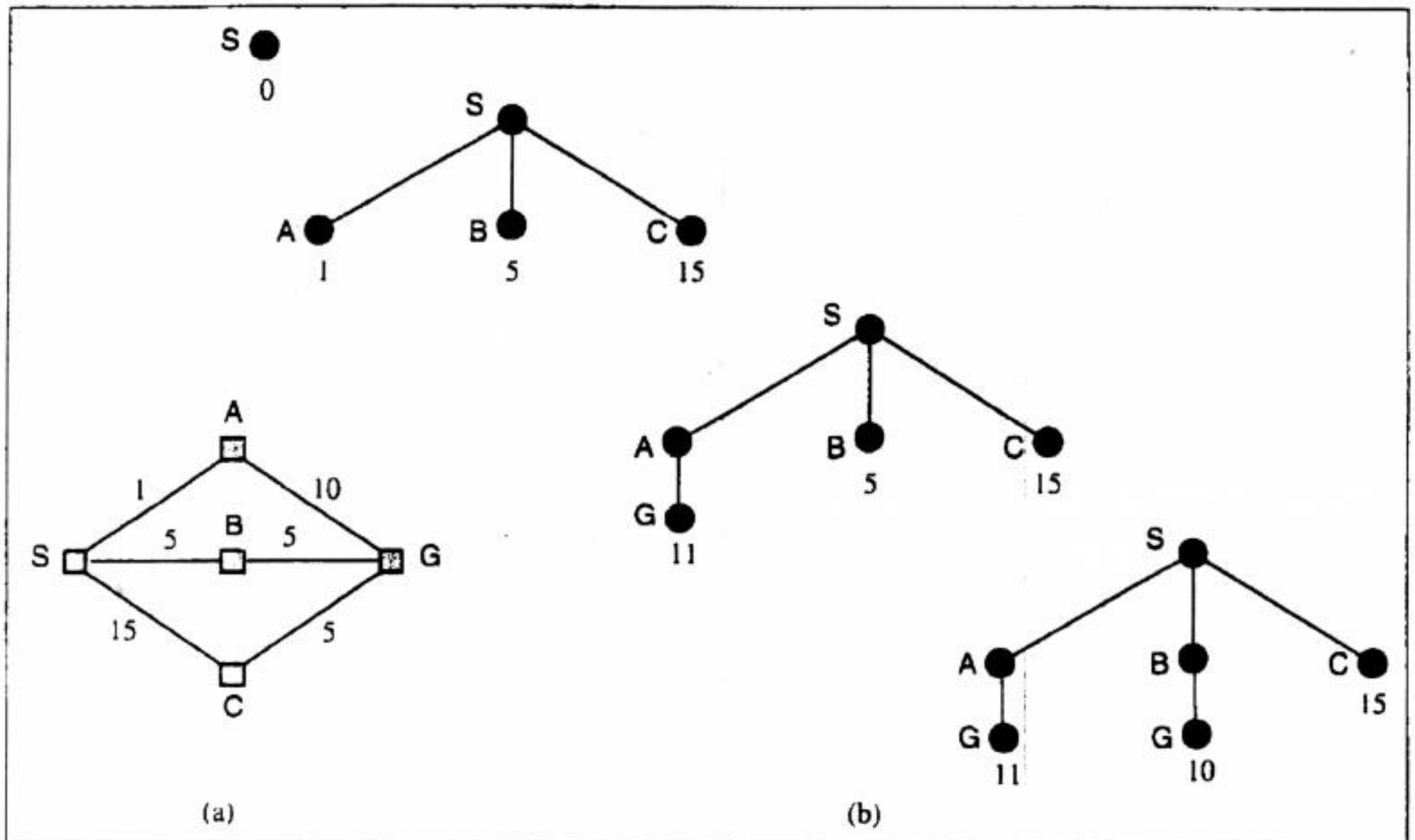
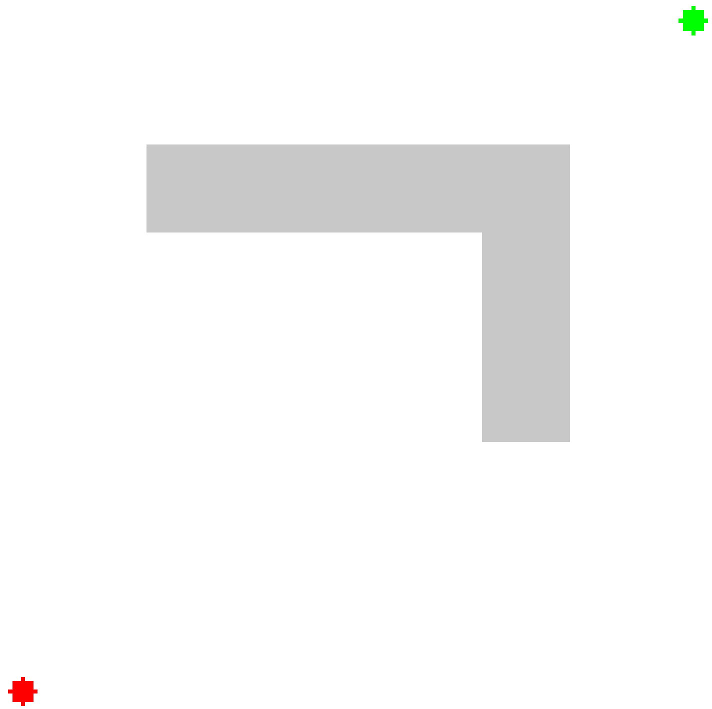


Figure 3.13 A route-finding problem. (a) The state space, showing the cost for each operator. (b) Progression of the search. Each node is labelled with $g(n)$. At the next step, the goal node with $g = 10$ will be selected.

Another example of uniform-cost search



Source: Wikipedia

Optimality of Uniform Cost Search?

- Assume that every step costs at least $\varepsilon > 0$
- **Proof of Completeness**:

  Given that every step will cost more than 0, and assuming a **finite branching factor**, there is a **finite number of expansions** required before the **total path cost** is equal to the **path cost of the goal state**. Hence, we will reach it in a finite number of steps.
- **Proof of Optimality given Completeness**:
  - Assume UCS is not optimal.
  - Then there must be a goal state with path cost smaller than the goal state which was found (invoking completeness)
  - However, this is impossible because UCS would have expanded that node first by definition.
  - Contradiction.

Optimality of Uniform Cost Search?

- Assume that every step costs at least $\varepsilon > 0$
- **Proof of Completeness**:

  Given that every step will cost more than 0, and assuming a **finite branching factor**, there is a **finite number of expansions** required before the **total path cost** is equal to the **path cost of the goal state**. Hence, we will reach it in a finite number of steps.

- **Proof of Optimality given Completeness**:
  - OR Alternatively:
  - Suppose UCS terminates at goal state n with path cost **g(n)** but there exists another goal state **n'** with **g(n') < g(n)**
  - By the graph separation property, there must exist a node **n"** on the frontier that is on the optimal path to **n'** But because **g(n") ≤ g(n') < g(n)**, **n"** should have been expanded first!

Complexity of Uniform Cost

- Let C* be the cost of the optimal solution

- Assume that every step costs at least $\varepsilon > 0$

- Worst-case **time** and **space** complexity is:

$$O\left(b^{[1 + \mathrm{floor}(C^*/\varepsilon)]} \right)$$

Why?

floor(C*/ε) ~ depth of solution if all costs are approximately equal

- This **can be greater** than $\mathbf{O(b^d)}$: the search can explore long paths consisting of small steps before exploring shorter paths consisting of larger steps

Bi-Directional Search

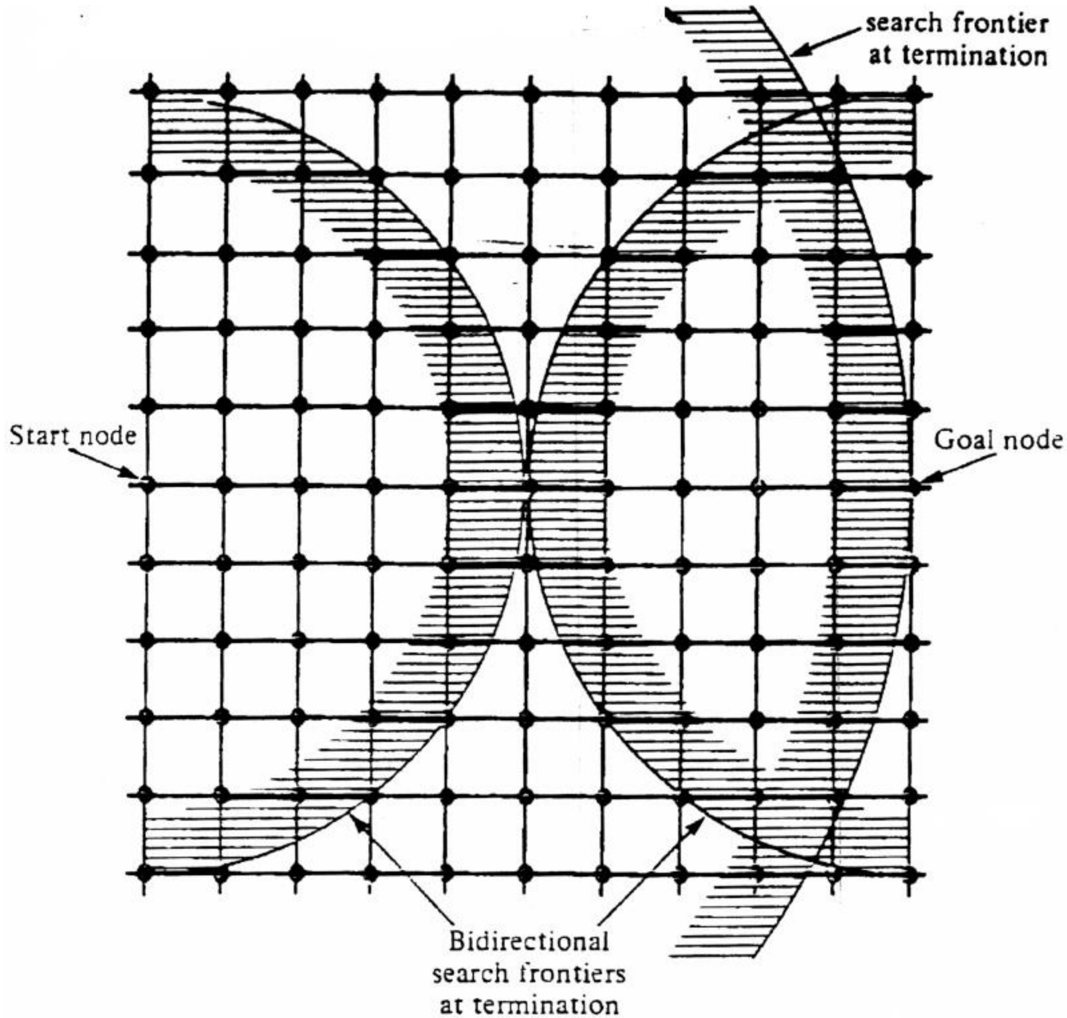


Fig. 2.10 Bidirectional and unidirectional breadth-first searches.

Bidirectional Search

- Idea
  - simultaneously search forward from S and backwards from G
  - stop when both “meet in the middle”
  - need to keep track of the intersection of 2 open sets of nodes

- What does searching backwards from G mean
  - need a way to specify the predecessors of G
    - this can be difficult,
    - e.g., predecessors of checkmate in chess?
  - **what if there are multiple goal states?**
  - **what if there is only a goal test, no explicit list? E.g. Traveling Salesman Problem**

- Complexity
  - time complexity at best is: $O(2\ b^{(d/2)}) = O(b^{(d/2)})$
  - memory complexity is the same

Comparison of Uninformed Search Algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

Summary

- A review of search
  - a search space consists of states and operators: it is a graph
  - a search tree represents a particular exploration of search space
- There are various strategies for “uninformed search”
  - breadth-first
  - depth-first
  - iterative deepening
  - bidirectional search
  - Uniform cost search
- Various trade-offs among these algorithms
  - “best” algorithm will depend on the nature of the search problem
- Next up – heuristic search methods