Exercises 11.1 - 11.4

Introduction:

Exercises feature nonstatistical applications of hypothesis testing. For each, identify the hypotheses, define Type I and Type II errors, and discuss the consequences of each error. In setting up the hypotheses, you must consider where to place the 'burden of proof.'

11.1

In Europe it is the responsibility of the European Medicines Agency (EMA) to judge the safety and effectiveness of new drugs. There are two possible decisions: approve the drug or disapprove the drug.

Where is the burden of proof and set hypothesis?

$$H_0$$

The drug is not safe and effective

$$H_1$$

The drug is safe and effective

11.2

You are contemplating a PhD in business or economics. If you succeed, a life of fame, fortune and happiness awaits you. If you fail, you've wasted 5 years of your life. Should you go for it?

Where is the burden of proof and set hypothesis?

$$H_0$$

I will complete the Ph.D.

$$H_1$$

I will not be able to complete the Ph.D.

11.3

You are faced with two investments. One is very risky, but the potential returns are high. The other is safe, but the potential is quite limited. Pick one.

Where is the burden of proof and set hypothesis?

$$H_0$$

Risky investment is more successful

$$H_1$$

Risky investment is not more successful

11.4

You are the pilot of a jumbo jet. You smell smoke in the cockpit. The nearest airport is less than 5 minutes away. Should you land the plane immediately?

Where is the burden of proof and set hypothesis?

$$H_0$$

?

$$H_1$$

?

11.5

Several years ago in a high-profile case, a defendant was acquitted in a double murder trial but was sub-sequently found responsible for the deaths in a civil trial. In a civil trial the plaintiff (the victims' relatives) are required only to show that the preponderance of evidence points to the guilt of the defendant. Aside from the other issues in the cases, discuss why these results are logical.

$$H_0$$

?

$$H_1$$

?

Exercises 11.6 - 11.11

In Exercises, calculate the value of the **test statistic**, set up the **rejection region**, determine the **p-value**, **interpret the result**, and **draw the sampling distribution**.

11.6

H_0: μ = 1000 (Null Hypothesis)

H_1: μ ≠ 1000 (Alternative Hypothesis)

x̄ = 980 (sample mean)

σ = 200 (standard deviation)

n = 100 (number of observations)

α = 0.01 (the level of significance)

μ = 1000 (population mean)

Step 1: Calculate the Test Statistic

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{980 - 1000}{200/\sqrt{100}} = -1.00$$

Step 2: Set up the Rejection Region

Since we are using a two-tailed test we want to find the critical values for a two-tailed test for our significance level:

$\alpha = 0.01/2 = 0.005$

$z < -z_{0.005} = -2.575$

$z > z_{0.005} = 2.575$

Step 3: Determine the p-value

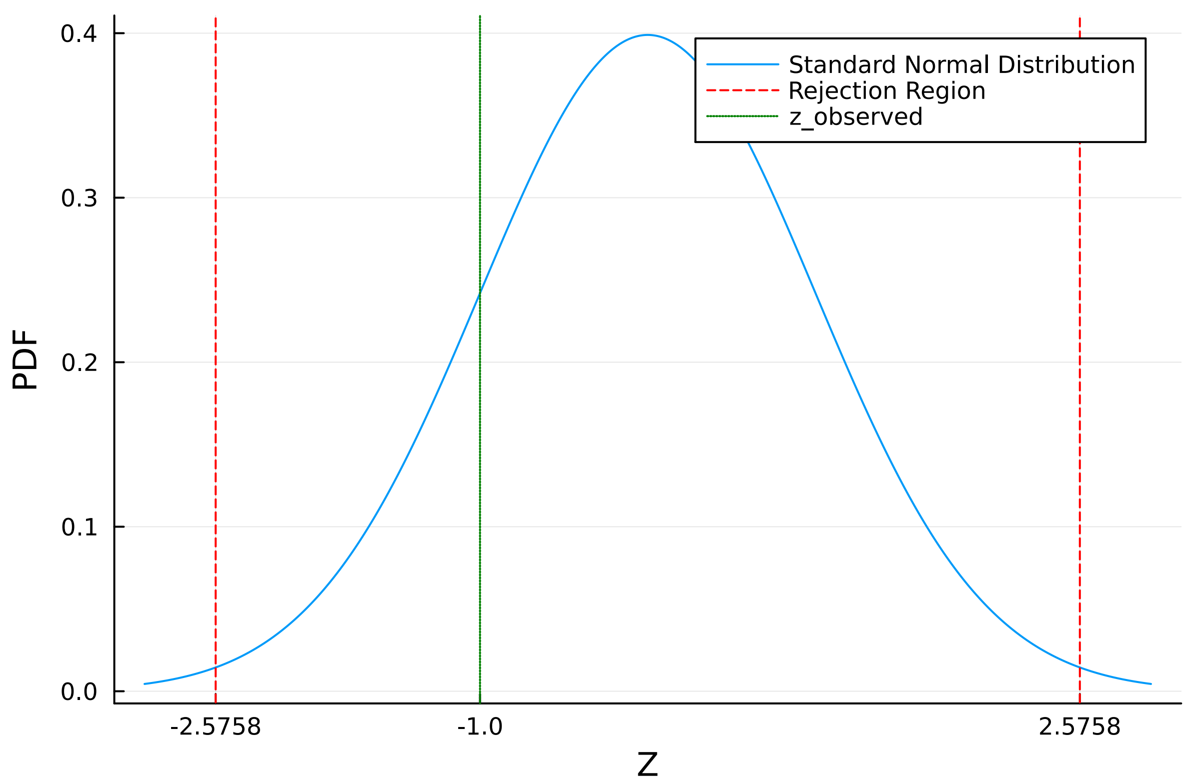
We can use z-score to calculate p-value:

p-value = $2P(Z < -1.00) = 2(0.1587) = 0.3174$

Step 4: Interpretation of the result

There is not enough evidence to infer that $\mu \neq 1000$.

Step 5: Sampling distribution



11.7

H_0: $\mu = 50$

H_1: $\mu > 50$

$\bar{x} = 51$

$\sigma = 5$

$n = 9$

$\alpha = 0.03$

$\mu = 50$

Step 1: Calculate the Test Statistic

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{51 - 50}{5 / \sqrt{9}} = 0.60$$

Step 2: Set up the Rejection Region

Since we are using a one-tailed test we want to find the critical values for a one-tailed test for our significance level:

$z > z_{0.03} = 1.88$

Step 3: Determine the p-value

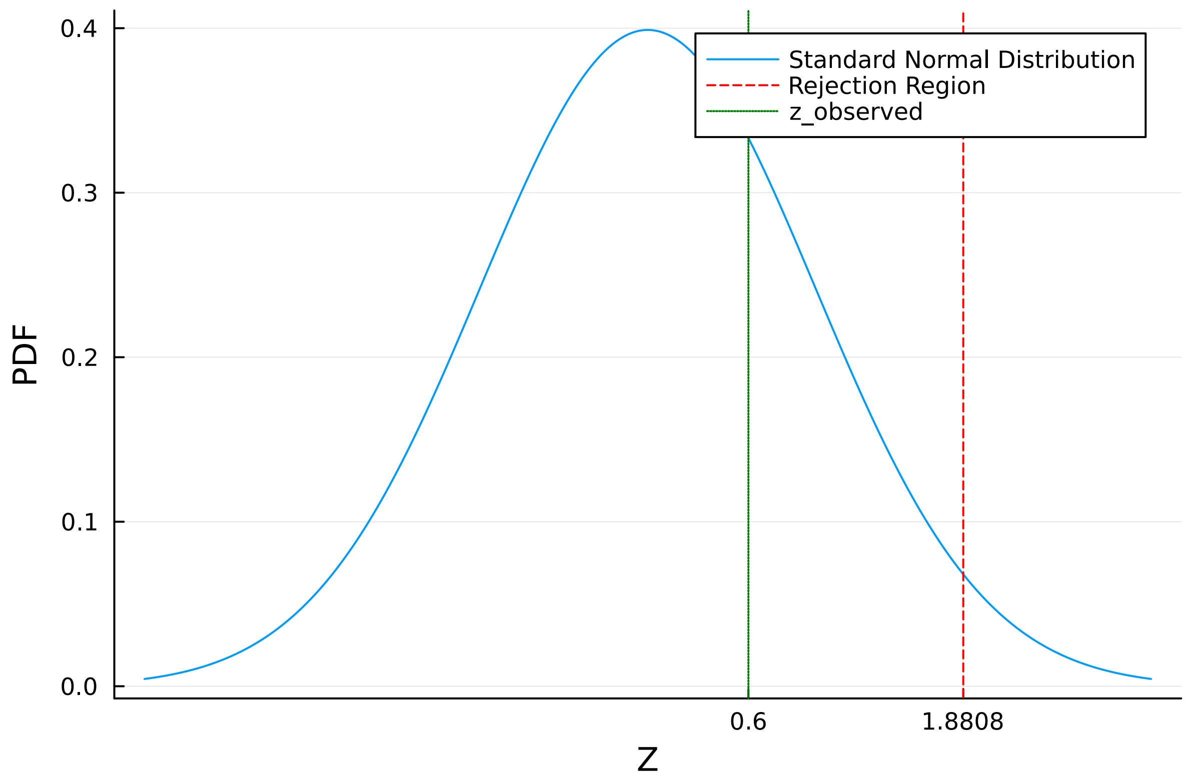
We can use z-score to calculate p-value:

p-value = $P(Z > 0.60) = 1 - 0.7257 = 0.2743$

Step 4: Interpretation of the result

There is not enough evidence to infer that $\mu > 50$.

Step 5: Sampling distribution



EXERCISES

Developing an Understanding of Statistical Concepts

In Exercises 11.6–11.11, calculate the value of the test statistic, set up the rejection region, determine the p-value, interpret the result and draw the sampling distribution.

11.6 H_0: $\mu = 1{,}000$

H_1: $\mu \neq 1{,}000$

$\sigma = 200$, $n = 100$, $\bar{x} = 980$, $\alpha = 0.01$

11.7 H_0: $\mu = 50$

H_1: $\mu > 50$

$\sigma = 5$, $n = 9$, $\bar{x} = 51$, $\alpha = 0.03$

11.8 H_0: $\mu = 15$

H_1: $\mu > 15$

$\sigma = 2$, $n = 25$, $\bar{x} = 14.3$, $\alpha = 0.10$

11.9 H_0: $\mu = 100$

H_1: $\mu \neq 100$

$\sigma = 10$, $n = 100$, $\bar{x} = 100$, $\alpha = 0.05$

11.10 H_0: $\mu = 70$

H_1: $\mu > 70$

$\sigma = 20$, $n = 100$, $\bar{x} = 80$, $\alpha = 0.01$

11.11 H_0: $\mu = 50$

H_1: $\mu < 50$

$\sigma = 15$, $n = 100$, $\bar{x} = 48$, $\alpha = 0.05$

For Exercises 11.12–11.18, calculate the p-value of the test to determine that there is sufficient evidence to infer each research objective.

11.12 Research objective: The population mean is less than 250.

$$\sigma = 40,\ n = 70,\ \bar{x} = 240$$

11.13 Research objective: The population mean is not equal to 1,500.

$$\sigma = 220,\ n = 125,\ \bar{x} = 1{,}525$$

11.14 Research objective: The population mean is greater than 7.5.

$$\sigma = 1.5,\ n = 30,\ \bar{x} = 8.5$$

11.15 Research objective: The population mean is greater than 0.

$$\sigma = 10,\ n = 100,\ \bar{x} = 1.5$$

Exercises 11.12 - 11.18

In Exercises, calculate the **p-value** of the test to determine that there is sufficient evidence to infer each research objective.

11.12

Research objective: The population mean is less than 250.

H_0: $\mu = 250$

H_1: $\mu < 250$

$\bar{x} = 240$

$\sigma = 40$

$n = 70$

Step 1: Calculate the Test Statistic

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{240 - 250}{40 / \sqrt{70}} = -2.09$$

Step 2: Determine the p-value

We can use z-score to calculate p-value:

p-value = $P(Z < -2.09) = 0.0182$

11.13

Research objective: The population mean is not equal to 1,500.

H_0: $\mu = 1500$

H_1: $\mu \neq 1500$

$\bar{x} = 1525$

$\sigma = 220$

$n = 125$

Step 1: Calculate the Test Statistic

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{1525 - 1500}{220 / \sqrt{125}} = 1.27$$

Step 2: Determine the p-value

We can use z-score to calculate p-value:

p-value = $2P(Z > 1.27) = 2(1-P(Z < 1.27) = 2(1 - 0.8980) = 2(0.1020) = 0.2040$

EXERCISES

Developing an Understanding of Statistical Concepts

In Exercises 11.6–11.11, calculate the value of the test statistic, set up the rejection region, determine the p-value, interpret the result and draw the sampling distribution.

11.6 H_0: $\mu = 1{,}000$

H_1: $\mu \neq 1{,}000$

$\sigma = 200$, $n = 100$, $\bar{x} = 980$, $\alpha = 0.01$

11.7 H_0: $\mu = 50$

H_1: $\mu > 50$

$\sigma = 5$, $n = 9$, $\bar{x} = 51$, $\alpha = 0.03$

11.8 H_0: $\mu = 15$

H_1: $\mu > 15$

$\sigma = 2$, $n = 25$, $\bar{x} = 14.3$, $\alpha = 0.10$

11.9 H_0: $\mu = 100$

H_1: $\mu \neq 100$

$\sigma = 10$, $n = 100$, $\bar{x} = 100$, $\alpha = 0.05$

11.10 H_0: $\mu = 70$

H_1: $\mu > 70$

$\sigma = 20$, $n = 100$, $\bar{x} = 80$, $\alpha = 0.01$

11.11 H_0: $\mu = 50$

H_1: $\mu < 50$

$\sigma = 15$, $n = 100$, $\bar{x} = 48$, $\alpha = 0.05$

For Exercises 11.12–11.18, calculate the p-value of the test to determine that there is sufficient evidence to infer each research objective.

11.12 Research objective: The population mean is less than 250.

$$\sigma = 40,\ n = 70,\ \bar{x} = 240$$

11.13 Research objective: The population mean is not equal to 1,500.

$$\sigma = 220,\ n = 125,\ \bar{x} = 1{,}525$$

11.14 Research objective: The population mean is greater than 7.5.

$$\sigma = 1.5,\ n = 30,\ \bar{x} = 8.5$$

11.15 Research objective: The population mean is greater than 0.

$$\sigma = 10,\ n = 100,\ \bar{x} = 1.5$$

Exercises 11.20 - 11.34

The Exercises are **'what if analyses'** designed to determine what happens to the **test statistic** and **p-value** when the sample size, standard deviation, and sample mean change. These problems can be solved manually or by using an Excel spreadsheet.

11.20

a. Compute the p-value in order to test the following hypotheses given that:

$\bar{x} = 52$

$\sigma = 5$

$n = 9$

H_0: $\mu = 50$

H_1: $\mu > 50$

Test Statistic

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{52 - 50}{5 / \sqrt{9}} = 1.20$$

$$\text{p-value} = P(Z > 1.20) = 1 - 0.8849 = 0.1151$$

b. Repeat part (a) with n = 25:

$\bar{x} = 52$

$\sigma = 5$

n = 25

H_0: $\mu = 50$

H_1: $\mu > 50$

Test Statistic

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{52 - 50}{5 / \sqrt{25}} = 2.00$$

$$\text{p-value} = P(Z > 2.00) = 1 - 0.9772 = 0.0228$$

c. Repeat part (a) with n = 100:

$\bar{x} = 52$

$\sigma = 5$

n = 100

H_0: $\mu = 50$

H_1: $\mu > 50$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{52 - 50}{5 / \sqrt{100}} = 4.00$$

$$\text{p-value} = P(Z > 4.00) = 1 - 0.999 = 0.001$$

d. Could you describe what happens to the value of the test statistic and its p-value when the sample size increases?

Case	n	z	p-value
a	9	1.2	0.1151
b	25	2.0	0.0228
c	100	4.0	0

The value of the test statistic increases and the p-value decreases.

11.16 Research objective: The population mean is less than 0.

$$\sigma = 25,\ n = 400,\ \bar{x} = -2.3$$

11.17 Research objective: The population mean is not equal to 0.

$$\sigma = 50,\ n = 90,\ \bar{x} = -5.5$$

11.18 Research objective: The population mean is not equal to –5.

$$\sigma = 5,\ n = 25,\ \bar{x} = -4.0$$

11.19 You are conducting a test to determine whether there is enough statistical evidence to infer that a population mean is greater than 100. You discover that the sample mean is 95.

a. Is it necessary to do any further calculations? Explain.

b. If you did calculate the p-value would it be smaller or larger than 0.5? Explain.

Exercises 11.20 to 11.34 are 'what if analyses' designed to determine what happens to the test statistic and p-value when the sample size, standard deviation and sample mean change. These problems can be solved manually or by using the Excel spreadsheet.

11.20 a. Compute the p-value in order to test the following hypotheses given that: $\bar{x} = 52$, $n = 9$, and $\sigma = 5$.

$$H_0: \mu = 50$$
$$H_1: \mu > 50$$

b. Repeat part (a) with $n = 25$.

c. Repeat part (a) with $n = 100$.

d. Describe what happens to the value of the test statistic and its p-value when the sample size increases.

11.21 a. A statistics practitioner formulated the following hypotheses:

$$H_0: \mu = 200$$
$$H_1: \mu < 200$$

and learned that $\bar{x} = 190$, $n = 9$ and $\sigma = 50$. Compute the p-value of the test.

b. Repeat part (a) with $\sigma = 30$.

c. Repeat part (a) with $\sigma = 10$.

d. Discuss what happens to the value of the test statistic and its p-value when the standard deviation decreases.

11.22 a. Given the following hypotheses, determine the p-value when $\bar{x} = 21$, $n = 25$ and $\sigma = 5$.

$$H_0: \mu = 20$$
$$H_1: \mu \neq 20$$

b. Repeat part (a) with $\bar{x} = 22$.

c. Repeat part (a) with $\bar{x} = 23$.

d. Describe what happens to the value of the test statistic and its p-value when the value of $\bar{x}$ increases.

11.23 a. Test these hypotheses by calculating the p-value given that $\bar{x} = 99$, $n = 100$ and $\sigma = 8$.

$$H_0: \mu = 100$$
$$H_1: \mu \neq 100$$

b. Repeat part (a) with $n = 50$.

c. Repeat part (a) with $n = 20$.

d. What is the effect on the value of the test statistic and the p-value of the test when the sample size decreases?

11.24 a. Find the p-value of the following test given that $\bar{x} = 990$, $n = 100$ and $\sigma = 25$.

$$H_0: \mu = 1{,}000$$
$$H_1: \mu < 1{,}000$$

b. Repeat part (a) with $\sigma = 50$.

c. Repeat part (a) with $\sigma = 100$.

d. Describe what happens to the value of the test statistic and its p-value when the standard deviation increases.

11.25 a. Calculate the p-value of the test described here.

$$H_0: \mu = 60$$
$$H_1: \mu > 60$$
$$\bar{x} = 72,\ n = 25,\ \sigma = 20$$

b. Repeat part (a) with $\bar{x} = 68$.

c. Repeat part (a) with $\bar{x} = 64$.

d. Describe the effect on the test statistic and the p-value of the test when the value of $\bar{x}$ decreases.

11.26 Repeat Example 11.1 with:

a. $n = 200$

b. $n = 100$

c. Describe the effect on the test statistic and the p-value when n increases.

11.27 Repeat Example 11.1 with:

a. $\sigma = 35$

b. $\sigma = 100$

c. Describe the effect on the test statistic and the p-value when σ increases.

11.28 While conducting a test to determine whether a population mean is less than 900, you find that the sample mean is 1,050.

a. Can you make a decision on this information alone? Explain.

b. If you did calculate the p-value, would it be smaller or larger than 0.5? Explain.