Chapter 6 - Exotic options

Modelling Derivatives in C++

1 Barrier options

Barrier options are standard options that can 'disappear' and 'reappear' if asset prices cross a pre-determined set of dates. There are 8 types of barrier options classified by the option payoff, whether the asset price is above or below the barrier price, and whether the option disappears or appears when the barrier is crossed.

Barrier options are cheaper than standard options due to disappearance.

We can price barrier options analytically, an example payoff for a down-and-out call option would be:

$$\begin{aligned} f(S_T, T) &= max(S_T - X, 0) \\ f(B, t^*) &= 0 \text{ for } t \in [t, T] \end{aligned}$$

The analytical solution is provided as:

$$C_B^{DAO}(S,t) = C(S,t) - \left(\frac{B}{S}\right)^{\gamma} C\left(\frac{B^2}{S}, t\right)$$

Where $C(S,t)$ is the formula for a conventional call option.

There are two inaccuracies related to pricing barrier options with a binomial lattice:

1. Quantisation error - asset price is allowed to take only the values of the points in the lattice
2. Specification error - binomial trees are ineffective in capturing the terms of the option

2 Barrier options implementation

The Python code provided will show an implementation of the DAO option through a binomial lattice.

3 Asian options

Asian options are like vanilla options - but the option depends on the average price of the underlying security during the term of the option.

4 Geometric averaging

Should the price depends on the geometric or arithmetic average - the geometric average price and the arithmetic geometric prices are different, and take on different (long) numeric formulae.

5 Arithmetic averaging

As above.

6 Seasoned Asian options

To simplify some processes we can take averages at specific time intervals, and then take weighted averages of the averages to get the approximate price of the Asian option using Monte Carlo methods.

7 Lookback options

A lookback option is like an Asian option - but rather than using the average, the payoff depends on the maximum or minimum price of the option. Fixed lookback options depend on a specified strike price, whereas floating lookback options depend on the price of the underlying at a specific end date.

Analytical formulae exists for the call and put option versions of the lookback options.

8 Implementation of floating lookback options

This is done in the provided Python code.

9 Implementation of fixed lookback options

This is also done in the provided Python code.