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Multiobjective Optimisation of Bevel Gear Pair design using NSGA-II

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Abstract

Spiral bevel gears mesh with additional overlapping tooth action compared to straight bevels. Because of this, the transmission of motion will be smoother than that by straight bevel gears or zero bevel gears. As a result they have reduced noise and vibration, particularly occuring at high speeds. Weight reduction with simultaneous increase in transmission efficiency, by optimization method, is very much essential for gear applications which transmit very high mechanical power, with a significant reduction in speed. In many instances, in order to satisfy technical, economic and security requirements, it is essential to undertake a long development process. This research paper aims to optimize the design of a pair of spiral bevel gears, using NSGA-II, a non-dominated sorting genetic algorithm for optimization of multiple objective functions.

Keywords: spiral bevel gear; NSGA-II; multi-objective optimization; evolutionary algorithms; pareto optimal solutions;

1. Introduction

Spiral bevel gears have curved teeth which provide gradual and smooth engagement and transmit motion between concurrent axes. The spiral bevel transmission has merits, such as the big overlap ratio, the high loading capacity, the high transmission efficiency, less noises, and they are widely used in automotive vehicle, planes, machine tools and all kinds of machines [1]. The level of noise, vibration and the endurance are generally due contact pattern and transmission errors [2].

Emmanuel Mermoz, et. Al [3] proposed an optimization procedure of the spiral bevel gear by finite element method. Xiaoqin Zhang, et al [4] developed optimization design software for straight bevel gear to achieve the automatic optimization by using VB and MATLAB, in which the genetic algorithm is selected and the augmented penalty function and integer serial number encoding is used, so the global optimal solution can be obtained.

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Nomenclature

P	Power transmitted in kW
i	Gear (or) transmission ratio
m_t	Transverse module in mm
m_{av}	Average module in mm
z_1, z_2	Number of teeth in pinion, gear
d_1, d_2	PCD of large end of bevel pinion, Gear in mm
f	Average coefficient of friction
R_c	Outside radius of large end of bevel gear in mm
R	Pitch radius of large end of bevel gear in mm
r_o	Pitch radius of large end of bevel gear in mm
r	Outside radius of large end of bevel gear in mm
R_c	Cone distance in mm
b	Face width of gear and pinion in mm
ρ	Density of the material in kg/mm^2
E	Young's modulus in N/ mm^2
σ_b	Induced bending stress in N/ mm^2
$[\sigma_b]$	Design bending stress in N/ mm^2
σ_c	Induced crushing stress in N/ mm^2
$[\sigma_c]$	Design crushing stress in N/ mm^2
$[M_t]$	Design twisting moment in N mm
H_s	Specific sliding velocity at start of approach action
H_t	Specific sliding velocity at end of recess action
θ	Pitch cone angle of bevel gear
γ	Pitch cone angle of bevel pinion
$\varnothing_n$	Normal pressure angle
P_L	Percent power loss
η	Efficiency
y	Form factor

For optimization of bevel gear pair design, Padmanabhan.S, et.al. [5] developed a combined objective function by using RCGA and LINGO. Tetsu Nagata, et.al [6] developed tooth contact analysis technique that calculates a meshing condition, tooth flank form measurement technique applicable to large spiral bevel gears. G. González Rey Rey [7] proposed a computation program in Visual Basic 4.0, for optimization of straight bevel gear. A comprehensive analysis of recent developments in the research on bevel gears is provided in a research paper by G.Chandrasekaran [8]. This paper attempts to use NSGA-II, a proven genetic algorithm technique, to optimize the design of spiral bevel gear pair to maximise efficiency and minimize weight and cone distance, considering several constraints.

2. Multi objective evolutionary algorithms

Multi objective evolutionary algorithms simulate the evolution of individual structures via the processes of selection and reproduction.

2.1. The multi objective genetic algorithm

Multi-objective optimization is the task of finding one or more optimum solutions for all the objective functions. [9], [10]. The aim is to find a set of multiple optimal solutions, commonly known as the Pareto-optimal solutions [9]. Obtaining the Pareto front of a multi objective problem is the main goal of multi-objective optimization. The Pareto front is obtained by automatically extracting non dominated solutions from a set of multi objective and multidimensional solutions. Theoretically, a Pareto front could contain a large number of points (even infinite)[11].

There are plenty of methods to solve multi objective problems [12]. Among them, the multi-objective genetic algorithm has attracted remarkable attention [13]. The variants of multi-objective genetic algorithm are vector evaluated genetic algorithms (VEGA)[14], niched Pareto genetic algorithms (NPGA)[15], strength Pareto evolutionary algorithms (SPEA)[13], SPEA2[16], and non-dominated sorting genetic algorithms (NSGA)[17].

2.2. NSGA-II Software functionality

NSGA-II represents the state of the art of multi objective evolutionary optimization, proposed by K. Deb[18]. NSGA-II is an overall good MOEA with respect to several different classes of problems [19]. For multi-objective genetic algorithms, they face two difficult challenges [20]: one is to find the Pareto optimal set; the other is to avoid premature convergence by maintaining population diversity and make the solution set have good distribution.

2.3. Concept of NSGA-II

NSGA-II uses (i) a faster non-dominated sorting approach, (ii) an elitist strategy, and (iii) no niching parameter. Diversity is preserved by the use of crowded comparison criterion in the tournament selection and in the phase of population reduction [9].

The procedure of NSGA-II algorithm is simple and straightforward and explained elsewhere in detail[18],[19].

3. Design optimization of spiral bevel gear pair

In this section an existing production spiral bevel gear pair is considered for minimization of weight and cone distance and maximization of efficiency. Efficiency depends on the power loss of the gear pair. Oil churning losses, bearing churning losses, seal losses and windage losses are dependent on the speed of the gear pair. Bearing losses, sliding friction losses and rolling friction losses are dependent on the load on the gear pair. To measure air losses (windage losses) researchers in the past conducted experiments on test rigs. Diab, et. al [21] conducted experiments in a test rig and obtained an empirical equation for windage power loss by considering dimensional analysis and fluid flow analysis.

3.1. Establishing a mathematical model

Establishing a mathematical model is the first step in spiral bevel gear design optimization. The design variables, objective functions and necessary constraints have to be developed.

The most general form of the constrained multi objective optimization problem is described by Equations 1-4.

$$\text{minx}\left[f_1(x), f_2(x), ., .fi(x), \ldots .fn(x)\right] \quad (1)$$

$$\text{Subject to: } g(x) \geq 0, \quad (2)$$

$$h(x) = 0, \quad (3)$$

$$x_l \leq x \leq x_u \quad (4)$$

where f_i is the i^{th} objective, n is the number of objectives, g is a vector of inequality constraints, h is a vector of equality constraints, x is the vector of design variables x_l is the lower limit for x and x_u is the upper limit for x.

3.2. Objective functions for the spiral bevel gear pair

The objective functions considered for the spiral bevel gear pair are as given below:

1. Minimisation of weight of the spiral bevel gear pair:
$$f_1 = Total\ Weight\ W = W_1 + W_2 \tag{5}$$
Where, $W_1 = Weight\ of\ pinion = 42.438\,\rho m_t^3 z_1$
$W_2 = Weight\ of\ gear = 68.52\,\rho\, m_t^3 z_2$

2. Maximisation of efficiency of gear pair:
$$f_2 = Efficiency\,\eta = 100 - P_L \tag{6}$$
Where P_L = Power Loss, which is given by the following equation:
$$50 f \left\{ \frac{\cos + \cos}{\cos\phi_n} \right\} \cos^2\beta\, \frac{\left(H_s^2 + H_t^2\right)}{\left(H_s + H_t\right)} \tag{7}$$
To calculate H_s and H_t, following equations (8) and (9) are used
$$H_S = (i+1)\left\{\left[\sqrt{\left(\frac{R_o}{R}\right)^2 - \cos^2\varnothing_n}\right] - \sin\varnothing_n\right\} \tag{8}$$
$$H_t = \left(\frac{i+1}{i}\right)\left\{\left[\sqrt{\left(\frac{r_o}{r}\right)^2 - \cos^2\varnothing_n}\right] - \sin\ \varnothing_n\right\} \tag{9}$$
$R_o = R + one\ addendum$
One addendum for 20° full depth involute system = $One\ average\ Module = m_{av}$
Where,
$$m_{av} = m_t\left(\frac{\Psi_y - 0.5}{\Psi_y}\right)$$
Also,
$r_o = r + m_{av}$; $R_o = R + m_{av}$; $r = d_1/2$ & $R = d_1/2$
Where,
d_1 = Pitch diameter of the large end of bevel pinion in mm = $m_t z_1$
d_2 = Pitch diameter of the large end of bevel gear in mm = $m_t z_2$

3. Minimisation of pitch cone distance of gear pair:
Eqn. 10 represents this objective function.
$$f3 = Rc = 0.5 m_t z_1 \sqrt{i^2 + 1} \tag{10}$$
Since weight and pitch cone distance should be minimized and efficiency should be maximized, the following notation is denoted in this model, since NSGA II is essentially a minimization software:
$f_1 = W$; $f_2 = -\eta$; $f_3 = R_c$

3.3. Design variables

The independent design parameters of spiral bevel gear drive include transverse module m_t and number of tooth of pinion z_1. So the design variables are,
$$X = [m_t, z_1]_T = [x_1, x_2]_T \tag{11}$$

3.4. Constraints for Spiral Bevel Gear

The constraints considered are bending stress (12), crushing stress (13), gear ratio (14) and average module(15).

(a) Bending stress

$$\sigma_b \le [\sigma_b]$$

$$\sigma_b = \left(\frac{0.7R\sqrt{(i^2+1)}[M_t]}{(R-0.5b)^2\, b m_n y_v} \right) \tag{12}$$

(b) Crushing stress

$$\sigma_c \le [\sigma_c]$$

$$\sigma_c = \frac{0.72}{(R-0.5b)}\sqrt{\frac{(i^2 \pm 1)^3}{ib} E\ [M_t]} \tag{13}$$

(c) Cone distance

$$R_{\min} \le R$$

$$\frac{41.4885}{(0.357Z_1 - 0.5)^{\frac{2}{3}}} \le R \tag{14}$$

(d) Average Module

$$m_{av} \ge 1.15 cos\beta_{av} \sqrt[3]{\frac{[M_t]}{y_{v[\sigma_b]\psi_m Z_1}}} \tag{15}$$

Additionally gear ratio is kept constant.

(e) Gear ratio

$i = 4.778$

4. Design data for optimization

This section describes the problem formulation for the existing production spiral gear pair for optimization of weight, efficiency and cone distance.

4.1. The input parameters

Following are the parameters considered for the spiral bevel gear design based on existing production gears:

Material of pinion and gear= Steel 20 Mn 5 Cr 5 IS:4432-1988

Alternate material for pinion and gear = Steel 15 Ni 4 Cr 4 Mo1 IS:4432-1988

Density of the material ρ = $7.86 \times 10^{-6} Kg / mm^3$

Design bending stress $[\sigma_b] = 430 N / mm^2$

Design crushing stress $[\sigma_c] = 1100 N / mm^2$

Input speed N= 500 rpm

Power transmitted P= 50 HP

Young's modulus of the material (E) = $2.15 \times 10^5 N / mm^2$

Pressure angle $\varnothing_n$ = 20°

Co-efficient of friction f = 0.08

Ratio between cone distance and face width $\psi_y = 0.357Z_1$

Ratio between face width and average module $\psi_m = 8.0$

4.2. Spiral bevel gear design optimisation formulation

The complete spiral bevel gear design problem is as formulated below:

Minimise $f_1 = W = 380.8\ x\ \rho\ x\ m_t^3 x\ z_1$

Maximise $f_2 = \eta = 100 - P_L$

Where, $P_L = 3.8781\dfrac{\left(H_S^2 + H_t^2\right)}{\left(H_S + H_t\right)}$

$$H_S = 5.778\left\{\left[\left(\frac{4.778Z_1}{4.778Z_1 - 1.6888}\right)^2 - 0.883\right]^{1/2} - 0.342\right\}$$

$$H_t = 1.2093\left\{\left[\left(\frac{Z_1}{Z_1 - 1.3467}\right)^2 - 0.883\right]^{1/2} - 0.342\right\}$$

$f_3 = R_c = 2.44m_t\, z_1$

Subject to,

$$\sigma_b = \frac{974286.17Z_1^2}{m_t^3\left(5.938Z_1^2 - 16.643Z_1 + 11.66\right)\left(0.154Z_1 - 0.603\right)} \leq 430$$

$$\sigma_c = \frac{179401.3}{\left(2.4368m_t^{3/2}Z_1 - 3.415m_t^{3/2}\right)} \leq 1100$$

$$i = 4.778$$

$$m_{av} \geq \frac{6.304}{\left(0.154Z_1 - 0.603\right)^{1/3}}$$

5. Methodology of optimization of spiral bevel gear pair using NSGA-II

In this section it is described how the NSGA-II software is used to solve the optimization problem of spiral bevel gear pair.

5.1. Variable bounds

Transverse module (m_t) and no. of pinion teeth (Z_1) are considered as variables (x_1,x_2) and their lower and upper bounds considered are as follows:

No. of pinion teeth (Z_1) : 5 to 12

Transverse module (m_t) : 5 to 10 mm

5.2. Input parameters

The objective functions and design constraints formulated in section IV.A and IV.B are entered in the software coding, making necessary modifications as required for the optimization. The probability of cross over and mutation are taken as per the recommendations given in the software. The input parameters as read by the program are as given below:

Population size = 100

Number of generations = 100

Number of objective functions = 3

Number of constraints = 4
Number of real variables = 2
Lower limit of real variable 1 = 5.0
Upper limit of real variable 1 = 10
Lower limit of real variable 2 = 5.0
Upper limit of real variable 2 = 12
Probability of crossover of real variable = 0.8
Probability of mutation of real variable = 0.5
Distribution index for crossover = 10
Distribution index for mutation = 20
Number of binary variables = 0
Seed for random number generator = 0.123
Number of crossover of real variable = 3915
Number of mutation of real variable = 9734

6. Results and discussions

In this section, the results obtained by NSGA-II are discussed and compared with the design of existing production gear pair.

In Table I, a sample of NSGA-II output is shown. Since NSGA-II software accepts constraints of the form of ≥ only, all the columns corresponding to the four constraints shown in the below table I indicate results obtained by modifying the constraint equations accordingly.

Table 1. Sample of NSGA-II output

No. of objectives = 3			**No. of constraints = 4**				**No. of real_vars = 2**	
Weight (kg)	Efficiency	Cone Distance (mm)	Constraint 1 (Equation 12)	Constraint 2 (Equation 13)	Constraint 3 (Equation 14)	Constraint 4 (Equation 15)	Transverse module	no. of pinion teeth
18.41154	97.74524	220.9782	1.112549	775.4411	20.2504	1.064271	8.241486	10.9889
24.54296	97.07801	202.2817	0.4036546	761.4918	13.31362	1.108052	9.945356	8.335782
17.58065	97.87989	227.3236	4.022834	781.5242	23.41929	1.066596	7.940181	11.73341
22.11699	97.32315	207.94	3.696449	765.7091	15.3011	1.113999	9.311705	9.152063
22.11874	97.32334	207.9564	3.767405	765.7402	15.31428	1.114423	9.311705	9.152786
18.09362	97.79036	222.8746	1.334874	777.1123	21.11947	1.061128	8.135188	11.22802
20.32261	97.51073	212.8715	2.160672	768.9739	16.94883	1.090753	8.821981	9.889211
17.59139	97.85791	225.7121	0.631713	779.3893	22.34959	1.050433	7.97091	11.60532
18.92338	97.68034	218.5578	1.665979	773.52	19.24639	1.07343	8.401396	10.66166
19.10668	97.64848	217.1469	0.2371318	771.9934	18.50551	1.068139	8.469367	10.50783
19.43179	97.61183	215.9854	1.023509	771.2167	18.09499	1.075794	8.564053	10.33606
19.60074	97.58959	215.1875	0.794508	770.5024	17.74286	1.076349	8.617135	10.23445
19.923	97.54605	213.6253	0.0099179	769.0331	17.03203	1.075345	8.719394	10.04099

In the Table II, a comparison has been made between the results that are obtained with NSGA-II and the design of existing production gear pair for best performance. No. of teeth of pinion were selected as 10 and 11 from the best populations of NSGA-II results for comparison, as suggested by AGMA recommendations (ref. chart 1 [22]). The values of variables and objective functions are indicated in the Table II for comparison. From the NSGA-II results, nearest integer values of no. of teeth of pinion Z_1 of 10 and 11 were taken for comparison with the existing production gear pair. The total weight of pinion and gear in Kgs, the efficiency and the cone distance in mm were taken for comparison, as shown in the table II.

The comparison shows that the results obtained from NSGA-II software are superior by optimizing the three objective functions without any constraint violation. The results were obtained with minimum computational effort and time.

Chart 1. Recommendations of AGMA for no. of teeth pinion

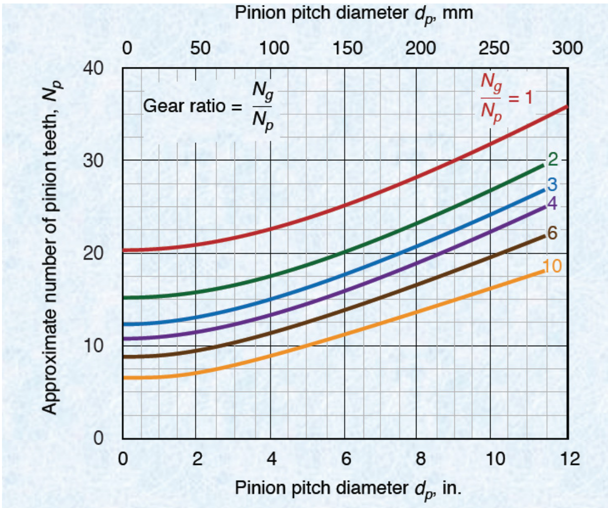

Table 2. Optimised design variables and objective functions of bevel gear pair using NSGA-II

	Variables		Objective Functions		
	Transverse Module m_t(mm)	No. of Teeth (Pinion) Z_1	Total Weight W(Kgs)	Efficiency η	Cone Distance R_c(mm)
Existing production gear pair design	9.418	9	22.522	97.281	206.889
NSGA-II design	8.719	10	19.923	97.546	213.625
	8.241	11	18.411	97.745	220.978

The total weight of existing production spiral bevel gear pair was 22.522 Kg. In the case of NSGA-II, selecting no. of teeth in pinion as 10, the weight is reduced to 19.923 Kg. Hence, there is a reduction of 11.54 % in weight by applying NSGA-II software compared to the existing production gears. Similarly the efficiency of existing production gear pair was 97.2818%. In the case of NSGA-II, there is an increase in efficiency to 97.546%. For cone distance, there is a marginal increase from 206.889 to 213.625, compared to the existing production gear pair. Similarly, selecting no. of teeth in pinion as 11 from the NSGA-II results, the weight is reduced to 18.411 Kg. Hence, there is a reduction of 18.25 % in weight by applying NSGA-II software compared to the existing production gears. Similarly the efficiency of existing production gear pair was 97.2818%. In the case of NSGA-II, there is an increase in efficiency to 97.745%. For cone distance, there is a marginal increase from 206.889 to 220.978.

Fig 1 shows the 3D plot of objective functions, viz., Total weight, efficiency and cone distance of the best population generated by NSGA-II. Here all generations were having no constraint violation. The Pareto front is formed for the population of best solutions.

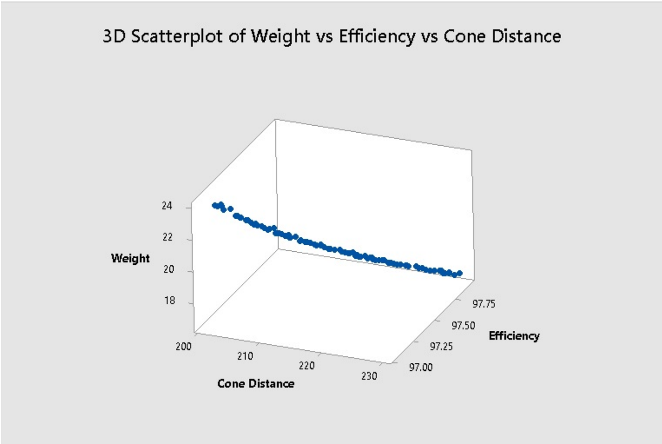

FIG. 1. Best population of the three objective functions

Fig 2 shows the comparison of objective functions for existing spiral gear pair design and the optimized design based on NSGA-II results. The design with number of teeth in pinion as 11 from NSGA-II optimizes best with reduction of 18.2 % in weight with improved efficiency compared to the existing production gear set. There is a marginal increase in the cone distance of 14.09 mm only.

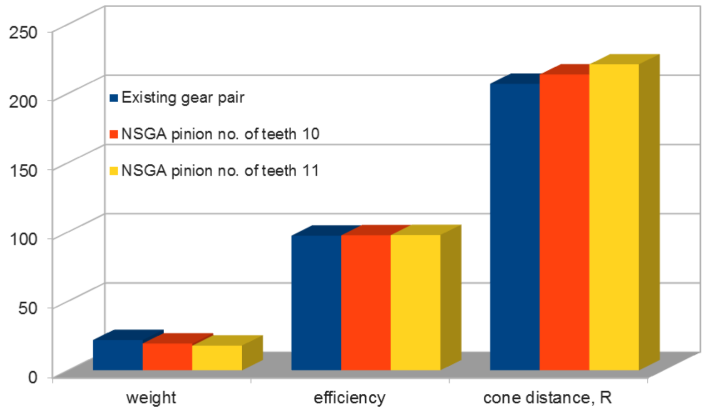

FIG. 2. Comparison of existing and optimised NSGA-II design

7. Conclusions

Optimisation of spiral bevel pair involves many complex objective functions and constraints and require tremendous amount of time for computation.

Multi-objective Optimization procedure is developed in this work for the design of spiral bevel gear which is based on the concept of Non-dominated Sorting Genetic Algorithm. In order to improve the performance of the bevel gears used in various industrial applications, three objective functions are considered; Minimization of weight, cone distance and maximization of transmission efficiency. Global Pareto front is obtained by implementing the procedure using the software developed with C-language. Results are compared with the existing bevel gear design used in industries and it is found to be superior in terms of cost, efficiency and compactness of the gear. Optimization results are obtained with minimal computational effort. It is planned to extend this multi-objective optimization procedure for designing the complete gear box consisting of spiral bevel pair and planetary gears used in trucks.

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