Assignment 6

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HOMEWORK ASSIGNMENT 6

1. Prove that $A_3(j) > \text{tower}(j)$, where

$$\text{tower}(n) = \begin{cases} 2^{\text{tower}(n-1)} & \text{if } n > 0, \\ 1 & \text{if } n = 0. \end{cases}$$

Solution:-

We know that Ackermann's function is defined as:-

$$A_k(j) = \begin{cases} j+1 & \text{if } k = 0 \\ A_{k-1}^{j+1}(j) & \text{if } k \geq 1 \end{cases}$$

For $j = 1$:

$A_k(j) = A_{k-1}^{j+1}(j)$

$\therefore A_3(1) = A_2^2(1)$

$= A_2(A_2(1))$

$= A_2(3) = 2047$

$\therefore \boxed{A_3(1) = 2047}$ — (1)

For $j = 0$

$A_3(0) = A^1{}_2(0)$

$= 1$

$\text{tower}(0) = 1$

$\therefore \boxed{A_3(0) = \text{tower}(0)}$

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Now lets consider lower(j) for j=1.

lower(0) = 1

lower(1) = 2 lower(1-1)
= 2 lower(0)
= 2(1)
= 2

∴ lower(1) = 2 — (2)

Comparing (1) and (2) we conclude that

$A_3(1) > \text{lower}(1)$

For j=2

$A_3(2) = A_2^3(2)$
$= A_2(A_2(A_2(2)))$
$= A_2(A_2(23))$
$= A_2(2^{24} \cdot 24 - 1)$ $\left(\because A_2(j) = 2^{j+1}(j+1)-1\right)$

$A_3(2) = A_2\, 2^{24} \cdot 24 - 1$ — (1)

Now lets consider lower j=2.

lower(0) = 1.
lower(1) = 2.
lower(2) = 2 lower(2-1)
$= 2^2$
lower(2) = 4 — (2).

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Comparing (1) and (2) we conclude that:

$$A_3(2) > \text{tower}(2)$$

From the above conclusions we can say that

$$A_3(j) > \text{tower}(j)$$

As the value of j increases the function $A_3(j)$ also grows so rapidly that it becomes nearly non computable after certain value of j.

Therefore $A_3(j)$ will always be greater than $\text{tower}(j)$ no matter what the value of j is, in a certain computation. except when the value of $j = 0$

Hence we have proved that:-

$$A_3(j) \geq \text{tower}(j)$$

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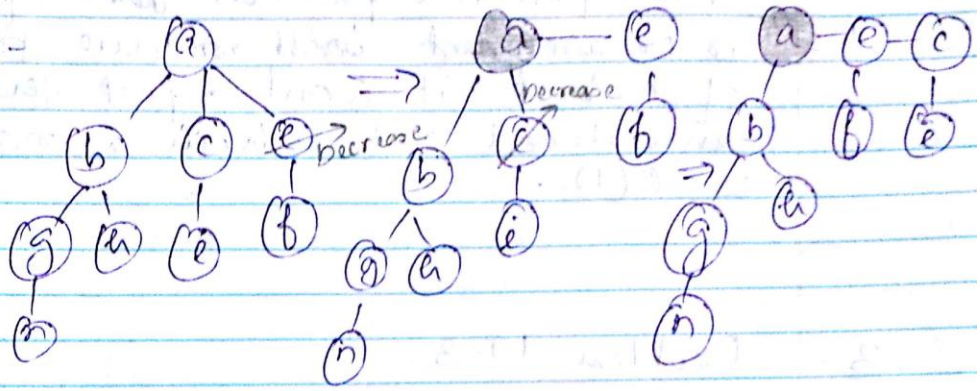
2. 19.3-1

Solution :-



A root x in Fibonacci heap is marked because it would have lost a child at some point whose key was decreased and made lesser than x.
It doesn't matter to the analysis that x is marked because in the CASCADING-CUT algorithm the condition that y.mark == FALSE is only run on nodes whose parent is ≠ NIL i.e. z ≠ NIL.
Since x is root and has no parent - therefore CASCADING-CUT will never run on the root node x.

As discussed in the lectures:

Amortized cost = $O(c) + [\ell(H) + c - 1 + 2[m(H) - (c-1)+1]] - [\ell(H) + 2m(H)]$

$= O(c) - c + 3$

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$= O(1)$

The potential function after will increase due to marking of x but it won't affect the total amortized cost which remains same as $O(1)$.

3. Problem 19-3.

(a). Operation FIB-HEAP-CHANGE-KEY(H, x, k)

(1) When k is less than x.key
If $k < x.key$ the procedure will be same as FIB-HEAP-DECREASE-KEY and hence amortized cost will be $O(1)$

(2) When k is equal to x.key
If $k = x.key$ then the operation will simply take $O(1)$ amortized time.

(3) When k is greater than x.key
If $k > x.key$, we delete the current value of x which will take time equal to $O(\lg n)$ and then insert the new key, which will take time proportional to $O(1)$. Hence the total time will be $O(\lg n) + O(1) = O(\lg n)$

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(b) : FIB-HEAP-PRUNE(H, r)

Solution :-

We introduce an additional cost to the potential function which is proportional to the size of the structure. Now the cost could only increase by a constant if we do an insertion.

$$\Phi(D_i) = t(H) + 2m(H)$$

Given that $q = \min(r, H.n)$

Now after modifying the potential function we modify the heap by connecting the leaf nodes with the help of doubly linked list.
This representation of leaf nodes with doubly linked list will help in pruning the heap more efficiently.

We proceed with pruning in the following the manner :-

- Pick any leaf node and remove it from its parent child list.
- Remove the leaf node from the list containing the leaves.

The above pruning procedure is repeated

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q times. This will make potential to drop by factor 'r'. The deletion from a deque linked list will take constant time for each operation. Therefore Amortized cost is constant.

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REFERENCE:

http://www.math.rutgers.edu/~ajl213/CLRS/Ch19.pdf