

Solutions to Homework Assignment 9

CS 430 Introduction to Algorithms
Spring Semester, 2017

1. It is easy to show that the gadget itself is 3-colorable as Fig. 1. Furthermore, this gadget has a symmetric

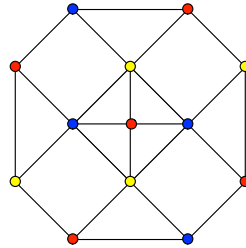


Figure 1: 3-colorable gadget

property, which is the color of the node at the 11:00 clock position could be always the same as the node at the 5:00 clock position, and the color of the node at the 1:00 clock position could be always the same as the node at the 7:00 clock position, and so on. Thus we can always replace a crossed edge (u, v) with u embedded at a corner of the gadget and v connected to the opposite corner with an edge, as Fig. 2, which takes time $O(E^2)$. The resulting graph is planar and the symmetric property of the gadget implies that u and v cannot be assigned the same color. We have shown a reduction of the 3-colorability

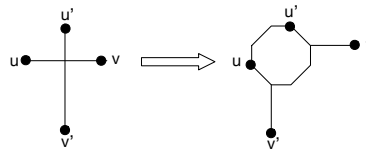


Figure 2: Replacing a cross with the gadget

for an arbitrary graph to the same problem for a planar by using this gadget. Since we can verify a color assignment by checking if the end points of every edge have different colors in $O(E)$ time, 3-coloring a planar graph belongs to the class NP. Because 3-colorable is NP-complete for general graphs, we can prove that 3-colorable is NP-complete for planar graphs.

2. The corresponding decision problem is: given an undirectional graph G and a positive integer k , determine if there is a simple cycle in G of length at least k .

To shown that the longest-simple-cycle problem is NP complete, we will firstly show that the problem is in class NP, and then show that this problem is NP hard.

Let (G, k) be an instance of the longest-simple-cycle problem. Given a certificate c , which is a sequence of vertices, we can go through c to check whether c is a simple cycle and whether the length of c is greater than k in polynomial time. Thus the longest-simple-cycle problem is in class NP.

To show that the longest-simple-cycle problem is NP hard, we will reduce the Hamiltonian cycle problem to the longest-simple-cycle problem. Given an instance of the Hamiltonian cycle problem, which is a graph H defined by edges E and vertices V , the corresponding instance of the longest-simple-cycle problem is $(H, |V|)$. If there exists a simple path of length at least $|V|$, there must exist a Hamiltonian cycle in H . If there exists a Hamiltonian cycle in H , there must exist a simple path of length $|V|$.