Chitraarth Singh
A20387080

HOMEWORK ASSIGNMENT - 5.

1. Given: Potential Function proportional to the sum of the depths of the nodes in the heap.

$D_i \leftarrow$ Heap after operation.

$\therefore \Phi(D_i) = \sum_{i=1}^{n} \log n.$ $\left\{ n \leftarrow \text{number of elements.} \right\}$

We know,

$$\boxed{\text{Amortized Cost} = \text{Actual Cost} + \Phi(\text{after}) - \Phi(\text{before})}$$

Amortized cost for Insertion when heap is non empty.

$$\hat{C}_i = C_i + \Phi(D_i) - \Phi(D_{i-1})$$

$$= \log n + \log n - \log(n-1)$$

$$= 2\log n - \log(n-1)$$

$$\approx O(\log n)$$

$\therefore$ Amortized cost of Insertion operation is $O(\log n)$

Amortized cost for EXTRACT-MIN when heap is non-empty

$\hat{c}_i = c_i + \Phi(D_i) - \Phi(D_{i-1})$

When we execute this operation, the root element would be extracted.

$\therefore$ Actual cost $= \log n + 1$.

$\therefore \hat{c}_i = \log(n) + 1 + 0 - \cancel{\log(n)} \log(n)$

$= 0 + 1 = 1 \in O(1)$.

$\therefore$ Amortized cost of EXTRACT-MIN is $O(1)$.

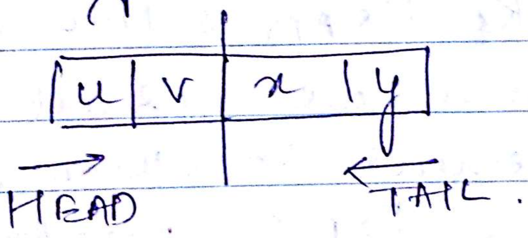
Scanned by CamScanner

2.

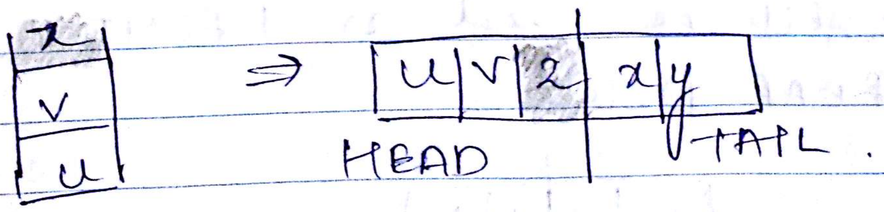
(a) The deque has 2 stacks HEAD & TAIL and it places both the stacks back-to-back so that operations are fast at both the end.

Insert operation

Inserting element at the Head.

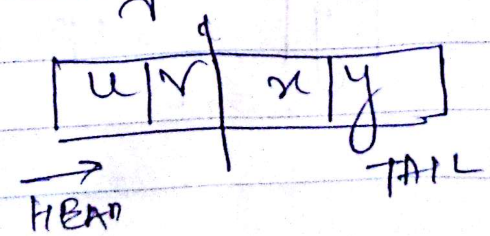


when we insert a new element z

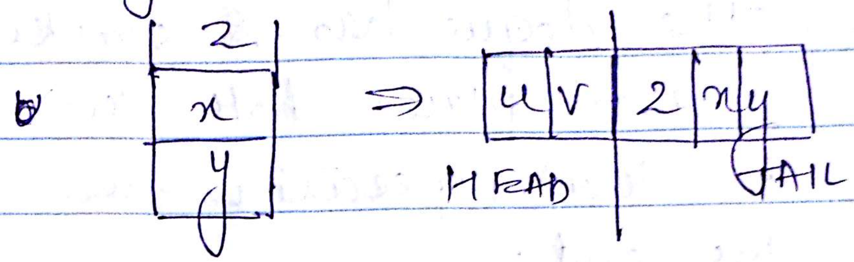


This operation will take time O(1).

Inserting element at the TAIL.

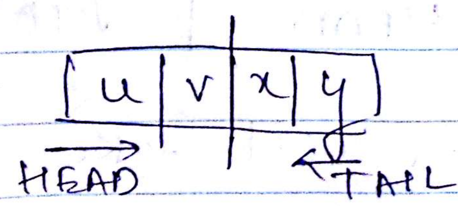


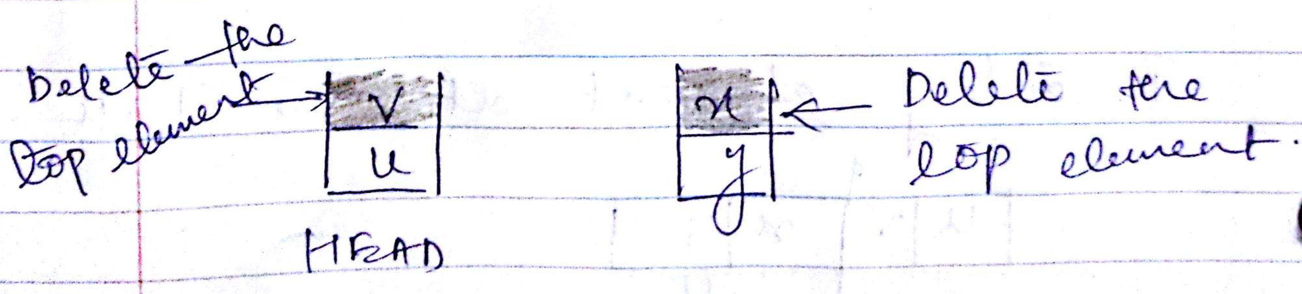
When we insert element 2 at the TAIL we get



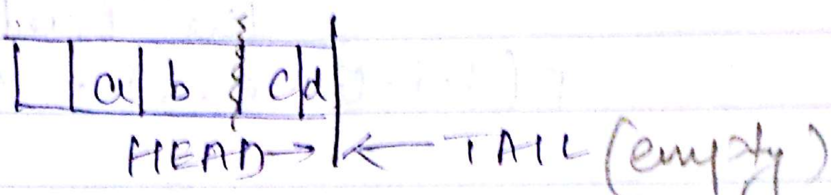
∴ This operation will take time of O(1)

(b) If stacks HEAD & TAIL are not empty, then we extract the topmost element in other words we perform Pop() operation on the HEAD or TAIL stack which will be used as FRONT Delete and REAR Delete.

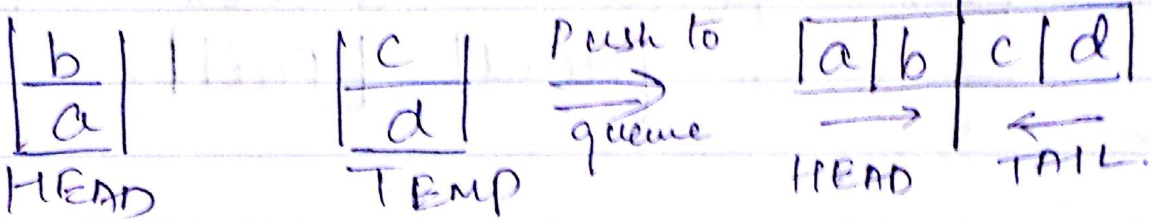




If either a stack HEAD or TAIL stack is empty, then we split the non empty stack using TEMP stack and then afterwards push the elements back to Queue.



use Temp stack and split HEAD



(C). Worst case cost for four operations:-

1. Insert Front : Push element in the Head stack.

   $\therefore$ T(insert front) = O(1).

2. Insert Rear :- Push element in the Tail stack.

   $\therefore$ T(insert rear) = O(1).

3. Delete Front :- Pop element from the Head stack.

$T(\text{deletefront}) = O(1)$

4. Delete Rear :- Pop element from the Tail stack.

$T(\text{delete rear}) = O(1)$.

(d) Potential Function is proportional to $|Head| - |Tail|$.

(1.) Amortized cost for Insert Front

$$\hat{c} = \text{Actual cost} + \Phi_i - \Phi_{i-1}$$

$$= (Head + Tail) + |Head + 1 - Tail| - |Head - Tail|$$

$$= Head + Tail + 1$$

$$\therefore \hat{c} = O(1)$$

2. Amortized cost for Insert Rear

$$\hat{c} = (Head + Tail) + |Head - (Tail + 1)| - |Head - Tail|$$

$= (Head + Tail) + 1$

$\therefore \hat{C} = O(1)$

3. Amortized cost for Delete Front

$\hat{c} = (Head + Tail) + |(Head-1) - Tail| - |Head - Tail|$

$\therefore \hat{c} = O(1)$

4. Amortized cost for Delete Rear

$\hat{c} = (Head + Tail) + |Head - (Tail-1)| - |Head - Tail|$

$\therefore \hat{c} = O(1)$