Solution to Homework Assignment 8(CS 430)

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1 Question 1. Problem 34.4-3 on page 1085

Answer -

Lets say 3-CNF-SAT formula have x variables , then, the truth table corresponding to this formula would have 2^x combinations. The reduction step needs to consider every possible values to all the variables. This then means that the reduction process will need to iterate over all the rows of the Truth table , so complexity will be O(2^x) . Hence, we can conclude that this strategy does not yield a polynomial-time reduction.

2 Question 2. Problem 34.4-7 on page 1086

Answer -

- We will create a Graph where vertexes are variable v and $\neg v$
- We will draw an Edge (α, β) iff there exists a clause in the 2-CNF-SAT as $(\neg\alpha \vee \beta)$
- So, If there's an edge (α, β) then there must be an edge $(\neg\alpha, \neg\beta)$
- The 2-CNF formula will be unsatisfiable if and only if there exists a variable v, such that there is a path from (v and $\neg v$) and ($\neg v$ and v)

  Proof: Suppose there are path(s) (v and $\neg v$) and ($\neg v$ and v) and there exists a satisfying assignment for the 2-CNF-SAT

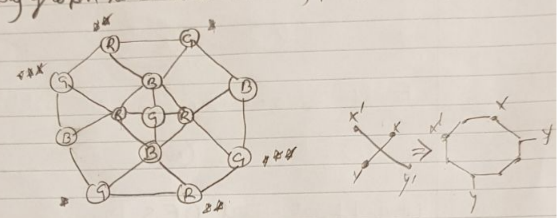
  Case1: Let v =TRUE. Let there be a path in the graph be $v.... \rightarrow \alpha \rightarrow \beta \rightarrow .... \rightarrow \neg v$. The way we constructed the graph, there is an edge between (M,N) iff there is a clause $(\neg M \vee N)$. An edge from M to N represents that if M is TRUE, then N must be TRUE (for the clause to be TRUE). Now since v is true, all literals in path from (v to α)must be TRUE. And all literals in the path from ($\beta to \neg v$) must be FALSE. This results in an edge between (α ,β) where α = TRUE and β = FALSE. So $(\neg\alpha \vee \beta)$ becomes FALSE, contradicting our assumption that there exists a satisfying assignment. Case2: Let v = FALSE. similar as above

- So if there is no such edge , here is we can assign boolean value -
  a) pick an unassigned vertex and assign it T
  b) Assign T to all reachable vertices
  c) Assign F to their negations
  d) Repeat until all vertices are assigned

- So following algo can determine if $2 - CNF - SAT \in P$
  a) For each variable v find if there is a path from v to $\neg v$ and vice-versa.
  b) Reject if any of these tests succeeded.
  c) Accept otherwise
  Complexity of this Algo is O(n) so $2 - CNF - SAT \in P$

3 Question 3.a Prove that in any legal 3-coloring of the crossover gadget, the opposite corners are forced to have the same color 3.b Prove that any assignment of colors to the corners such that opposite corners have the same color extends to a legal 3-coloring of the entire crossover gadget . 3.c Use the following idea to prove that 3-colorability of planar graphs is NP-hard: Replace each point at which another edge crosses edge (u, v) with a copy of the crossover gadget G.

(3) a&b We can use this gadget to reduce the 3-colorability of arbitary graph to 3-colorability for planer graphs



The given gadget is 3-Colorable.

The above graph is symmetric shown by *. So If there is a cross edge we can replace it by connecting opposite corner with this gadget. Complexity will be $O(E^2)$. And resulting graph is planner.

Thus resulting graph will be planner & symmetric property implies that connected vertices of a cross over edge can't be of same color.

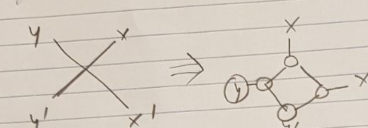
So using this gadget we can reduce the 3-Colorability of any arbitory graph to 3-Colorability of a planner graph.

We can verify 3-Color solution in $O(E)$ time and 3-Color of graph is np-hard problem so 3-Coloring is a NP-Complete problem

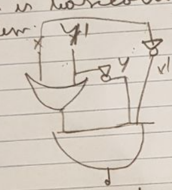
It's a symmetric graph and that's why we are able to replace cross over edges with the gadget. Symetric property ensure ~~if we~~ "opposite corner forced to have same color".

(3)(b)

3(a) Lets say we have following graph



which is basically a circuit satis[fiability] problem.



~~which~~ which is a NP-hard problem so 3-Co[loring] is a NP-hard problem.

(3)(b) We Can maintain symmetry of the g[raph] only iff we color opposite vertex [same] color.

So that will ensure 3-Color legal.

References

[1] CLSR Solution

texttthttp://sites.math.rutgers.edu/ ajl213/CLRS/Ch34.pdf

[2] 2-SAT Solution

texttthttp://www.dei.unipd.it/ geppo/AA/DOCS/2SAT.pdf

[3] 2-SAT IIT Solution

textttShttps://www.iitg.ac.in/deepkesh/CS301/assignment-2/2sat.pdf

[4] 3 color

texttthttps://courses.cs.washington.edu/courses/cse431/14sp/scribes/lec15.pdf