

Quiz 10 - Solutions

Tuesday, November 27, 2018 9:15 AM

Note: Your quiz might have different numbers.

Question 1

The input has 3 dimensions: x_1 , x_2 , x_3 .

Here is a neuron that is connected to the input through following weights:

$$w_0 = 2$$

$$w_1 = 2$$

$$w_2 = 1$$

$$w_3 = -3$$

Here is an input

$$x_1 = 2$$

$$x_2 = -5$$

$$x_3 = 1$$

Assume the neuron's activation function is binary sigmoid, as defined in class (slide 15).

What is the activation output of this neuron for this input?

To receive credits, your answer has to be within 0.001 of the correct answer.

Solution:

$$\begin{aligned} & \text{sigmoid} (2 + 2 \times 2 + 1 \times (-5) + (-3) \times 1) \\ &= \text{sigmoid} (2 + 4 - 5 - 3) \\ &= \text{sigmoid} (-2) \\ &= \frac{1}{1 + e^2} \end{aligned}$$

Question 2

We have a simple multilayer neural network, where input has three dimensions: x_1 , x_2 , x_3 .

The input is connected to a single neuron z whose activation function is tanh (slide 17). For tanh, use tanh function from numpy package.

z is connected to a single output neuron y , whose activation function is binary sigmoid (slide 15).

Here are the weights from x to z :

$$v_0 = 2 \text{ (the bias weight)}$$

$$v_1 = 1$$

$$v_2 = 1$$

$$v_3 = -2$$

Here are the weights from z to y :

$$w_0 = 5 \text{ (the bias weight)}$$

$$w_1 = 1$$

Here is an input:

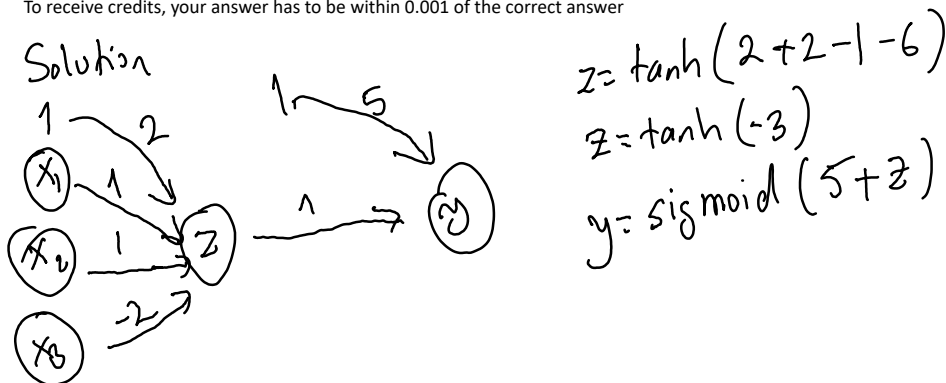
$$x_1 = 2$$

$$x_2 = -1$$

$$x_3 = 3$$

What is the activation value of the output neuron y ?

To receive credits, your answer has to be within 0.001 of the correct answer



Question 3

Here is a neural network:

z's are tanh (slide 17; use numpy's tanh function), y is binary sigmoid (slide 15).

Weights are as follows: v_{ij} is from i th input to j th hidden neuron, and w_i is from i th hidden neuron to output neuron.

Here are the weights from x to z1:

$v_{01} = 2$ (bias weight)

$v_{11} = 1$

$v_{21} = 1$

$v_{31} = -2$

Here are the weights from x to z2:

$v_{02} = -2$ (bias weight)

$v_{12} = 2$

$v_{22} = 4$

$v_{32} = 1$

Here are the weights from z1 and z2 to y:

$w_0 = 3$ (bias weight)

$w_1 = -1$

$w_2 = -1$

Here is an input:

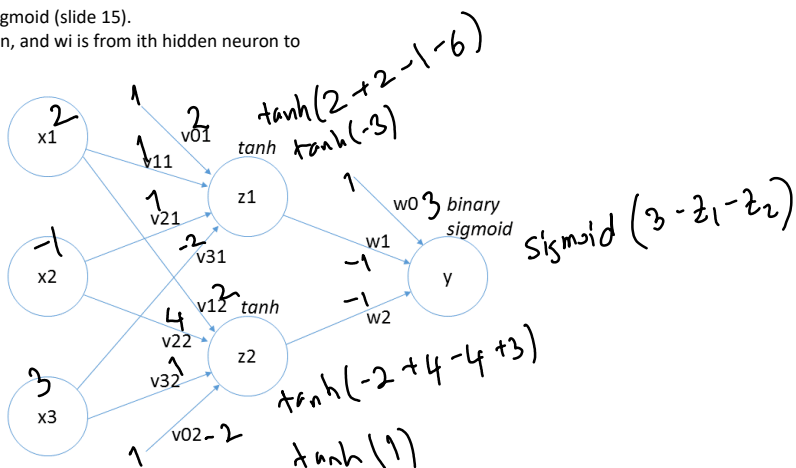
$x_1 = 2$

$x_2 = -1$

$x_3 = 3$

What is the activation value of the output neuron y?

To receive credits, your answer has to be within 0.001 of the correct answer.



Question 4

We have a simple multilayer regression neural network, where x is a single dimensional input, z is the hidden layer neuron with a tanh activation function (slide 17; use numpy's tanh), and y is the output neuron with a linear activation function (identity function). The network is $x \rightarrow z \rightarrow y$. This is a regression problem where the error is defined as: $\frac{1}{2}(t-y)^2$

where t is the true value and y is the predicted value.

Here are the weights:

from x to z :

$v_0 = 3$ (bias)

$v_x = -2$

from z to y :

$w_0 = 2$ (bias)

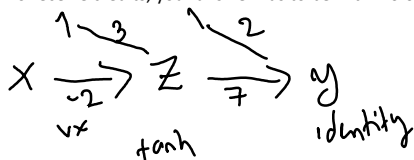
$w_z = 7$

Here is a training instance

$\langle x, t \rangle = \langle 2, 3 \rangle$

What is the value of the gradient of the error with respect to v_x ? (See OneNote)

To receive credits, your answer has to be within 0.001 of the correct answer.



Answer:

First, do a forward pass and calculate z and y .

$z = \text{np.tanh}(3 - 2 * 2) = \text{np.tanh}(-1)$

$y = 2 + 7 * z$

Derivative of E with respect to v_x is the product of the following:

1. Derivative of E with respect to y
2. Derivative of the identity function
3. 7
4. Derivative of the tanh
5. x

$$1) \frac{\partial E}{\partial y} = \frac{\partial}{\partial y} \left(\frac{1}{2} (t-y)^2 \right) = (y-t)$$

$$2) \text{Derivative of the identity function: } 1$$

$$3) 7$$

$$(y-t) \cdot 1 \cdot 7 \cdot (1-z)(1+z) \cdot x$$

$$4) (1-z) \times (1+z)$$

$$5) x$$

Question 5

We have a simple multilayer classification neural network, where x is a single dimensional input, z is the hidden layer neuron with a tanh activation function (slide 17; use numpy's tanh), and y is the output neuron with a binary sigmoid activation function (slide 15). The network is $x \rightarrow z \rightarrow y$. The classification error is defined as:

$$-(1-t) \ln(1-y) - t \ln(y)$$

where t is the true target label and y is the output of the neuron y . t is either 0 or 1.

Here are the weights:

from x to z :

$$v_0 = 3 \text{ (bias)}$$

$$v_x = -2$$

from z to y :

$$w_0 = 2 \text{ (bias)}$$

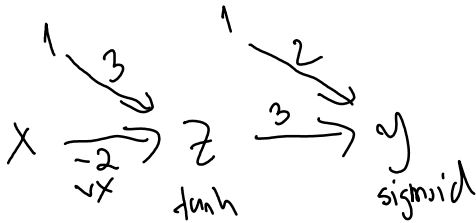
$$w_z = 3$$

Here is a training instance

$$\langle x, t \rangle = \langle 2, 1 \rangle$$

What is the value of the gradient of the error with respect to v_x ? (See OneNote)

To receive credits, your answer has to be within 0.001 of the correct answer.



Answer:

First, do a forward pass and calculate z and y .

$$z = \text{np.tanh}(3 - 2 \times 2) = \text{np.tanh}(-1)$$

$$y = \text{sigmoid}(2 + 3 \times z)$$

Derivative of E with respect to v_x is the product of the following:

1. Derivative of E with respect to y
2. Derivative of the sigmoid function
3. 3
4. Derivative of the tanh
5. x

$$1) \frac{\partial E}{\partial y} = \frac{\partial}{\partial y} \left(-(1-t) \ln(1-y) - t \ln(y) \right) = \frac{(1-t)}{(1-y)} - \frac{t}{y} = \frac{y(1-t) - t(1-y)}{(1-y)y}$$

$$2) \text{Derivative of sigmoid: } y(1-y)$$

$$3) 3$$

$$4) \text{Derivative of tanh } (1-z)(1+z)$$

$$5) x$$

$$= 3 (1-z)(1+z) \cdot x$$

Final answer $y(1-t) - t(1-y)$