Quiz 11 - Solutions

Monday, November 26, 2018 10:01 PM

Note: Your quiz might have different numbers.

Question 1:

Here is an SVM model. The weights are as follows:

w1 = 4

w2 = 3

w3 = -2 (note negative sign)

b = 1

And here is an object:

x1 = 5

x2 = 6

x3 = 30

What is the class of this object with respect to this SVM? (Note that the classes in SVM are represented as +1 or -1).

You should be able to calculate it without a calculator.

Solution. $\text{sign}(4\times5+3\times6-2\times30+1)$

$= \text{sign}(20+18-60+1)$

$= \text{sign}(-21)$

$= -1$

Question 2

We have a linear kernel SVM where weights can be calculated (slides 9 through 17). Here is a dataset and the corresponding α values.

X1	X2	Y	α
1	1	+1	0
2	3	+1	8
2.5	3	-1	8
3	1	-1	0

Calculate w0 and w1. What is w1? To receive credits, your answer has to be exactly correct.
You should be able to calculate it without a calculator

Solution: $\langle w_0, w_1, w_2\rangle = 8\times\langle 2,3\rangle - 8\times\langle 2.5,3\rangle$

$= \langle 16,24\rangle - \langle 20,24\rangle$

$= \langle -4,0\rangle$

Question 3

We have a linear kernel SVM where weights and the bias can be calculated. The data is linearly separable and hence we will use the hard-margin SVM (no slack variables). Here is a dataset and the corresponding α values.

X1	X2	Y	α

1	1	+1	0
2	3	+1	18
7/3	3	-1	18
3	1	-1	0

Solution: $\langle w_1, w_2 \rangle = 18\langle 2,3 \rangle - 18\langle 7/3, 3 \rangle$

Calculate w1, w2, and b's b. What is b? To receive credits, your answer has to be exactly correct.
You should be able to calculate it without a calculator

$= \langle 36, 84 \rangle - \langle 42, 84 \rangle$

$= \langle -6, 0 \rangle$

$\langle -6,0 \rangle \cdot \langle 2,3 \rangle + b = +1$

$-12 + b = 1$

$b = 13$

Alternatively

$\langle -6,0 \rangle \langle 7/3, 3 \rangle + b = -1$

$-14 + b = -1$

$b = 13$

Question 4

We have a hard-margin SVM with a Gaussian kernel (slide 19). We have 4 data points: d1, d2, d3, d4. Here is the corresponding dataset and α values.

Object	Y	α
d1	+1	0
d2	+1	5
d3	-1	5
d4	-1	0

What is the bias value b?
Note: if the above information is insufficient to calculate b, then enter -1.

Solution: There are two support vectors. We know that in hard-margin SVM, support vectors x_s have the following property

$\sum \alpha_i y_i K(x_i, x_s) + b = y_s$

For d_2: $5 \times (+1) \times K(d_2, d_2) + 5 \times (-1) \times K(d_2, d_3) + b = +1$ (1)

For d_3: $5 \times (+1) \times K(d_3, d_2) + 5 \times (-1) \times K(d_3, d_3) + b = -1$ (2)

Note that $K(d_2, d_2) = K(d_3, d_3) = 1$ for Gaussian kernels

Therefore

$5 - 5 \cdot K(d_2, d_3) + b = +1$ (1)

$5 \times K(d_3, d_2) - 5 + b = -1$ (2)

Add (1) & (2): $2b = 0 \Rightarrow b = 0$

Question 5

You need a calculator (or better yet, a python interpreter) to answer this question.
We have a hard-margin SVM with a Gaussian kernel (slide 19). Here is a dataset and corresponding α values.

x1	x2	y	α
1.5	3	+1	0
2	3	+1	10
2.459	3	-1	10
3	3	-1	0

The Gaussian kernel parameter s is 1. What is the functional margin of the following data point?
x1 = 1
x2 = 4
To receive credits, your answer has to be within 0.001 of the correct answer.

Solution: Functional margin of $\langle 1,4 \rangle$ is

$$10 \times (+1) \times K\left(\langle 1,4 \rangle, \langle 2,3 \rangle\right) + 10 \times (-1) \times K\left(\langle 1,4 \rangle, \langle 2.459, 3 \rangle\right)$$

We know from question 4 that for a Gaussian kernel, with two support vectors, th
opposing labels, and equal α values, that $b = 0$.
All we need is to calculate the above equation using a Python interpreter
For this particular set of numbers, the answer is. 1.5865