## C O M P U T E R A S S I G M E N T

## EXERCISE 1

A set of n lamps illuminates an area that we divide into m regions (pixels). We let  $l_i$  denotes the lighting level in region i, so the m-vector l gives the illumination levels across all regions. We let  $p_i$  denote the power at which lamp i operates, so the n-vector p gives the set of lamp powers. The vector of illumination levels is a linear function of the lamp powers, so we have l = Ap for some  $m \times n$  matrix A. The jth column of A gives the illumination pattern for lamp j, i.e., the illumination when lamp j has power 1 and all other lamps are off. We will assume that A has linearly independent columns (and therefore is tall). The *i*th row of A gives the sensitivity of pixel i to the n lamp powers. Your mission is to find lamp powers that result in a desired illumination pattern  $l^{\text{des}}$ . such as  $l^{\text{des}} = \alpha \cdot \mathbf{1}$ , (1 the vector with all ones) which is uniform illumination with value across the area In other words, we seek p so that  $Ap \approx l^{\text{des}}$ . We can use least squares to find  $\hat{p}$  that minimizes the sum square deviation from the desired illumination,  $||Ap-l^{\text{des}}||_2^2$ . As an example use n=10 lamps, the area being an  $25 \times 25$  grid with m = 625 pixels, each  $1m^2$ . The (x, y) positions of lamps and their height above floor are

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\{(4.1, 20.4, 4), (14.1, 21.3, 3.5), (22.6, 17.1, 6), (5.5, 12.3, 4), (12.2, 9.7, 4)\}
\{(15.3, 13.8, 6), (21.3, 10.5, 5.5), (3.9, 3.3, 5), (13.1, 4.3, 5), (20.3, 4.2, 4.5)\}
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The illumination decays with an inverse square law, so  $A_{ij}$  is proportional to  $d_{ij}^{-2}$ , where  $d_{ij}$  is the (3-D) distance between the center of the pixel and the lamp position. The matrix A is scaled so that when all lamps have power one, the average illumination level is one. The desired illumination pattern is  $\mathbb{F}$ , i.e., uniform with value 1.

- 1. Create two graphs to show the illumination of the two patterns: The first with all lamps set to 1 and the other that minimize the sum square deviation with a desired uniform illumination. Which are the RMS errors in both cases?
- 2. Create the histogram of patch illumination values for all lamp powers one, and for lamp powers found by LS.

## EXERCISE 2

Your aim is to predict whether a given banknote is authentic given a number of measures. In the file you can find the training and test set with the corresponding outputs.

1. Using the least square procedure create a classifier (see chapter 14 from vmls book). Then, create the confusion matrix for the training set. If the logical assumption: "If the prediction value > 0.5 then classify the item as 1 otherwise as 0." leads to serious unbalancing number of negative false and positive false responses, then following the technique proposed in 14.2.3, try to find -heuristically- a value that balance the two numbers. For this threshold, run your model for the test set. Give the confusion matrix for the test set, and the error rates for the 2 sets.

Bonus Following techniques proposed in paragraph 13.3 try to increase the accuracy of the model