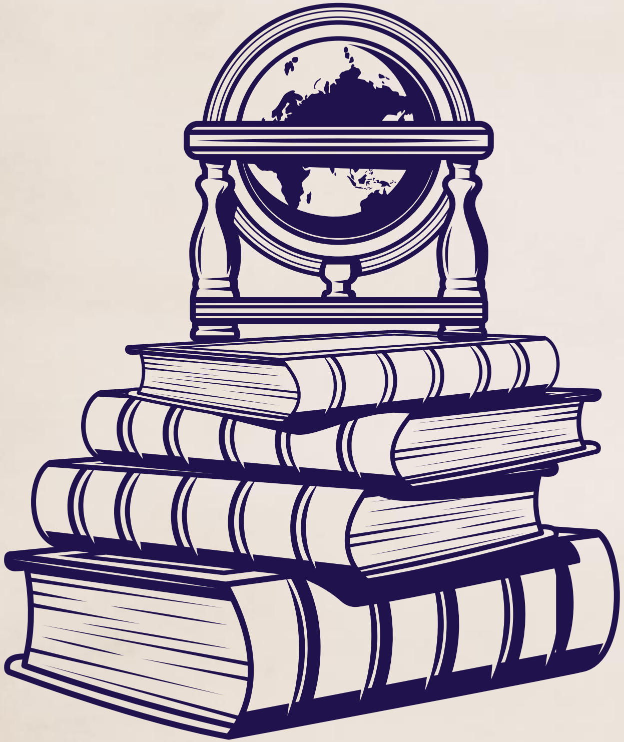

Propositional Logic

By Aditi mam

June 2014 Paper II

The notation $\exists!xp(x)$ denotes the proposition "there exists a unique x such that P(x) is true".

Give the truth values of the following statements :

I. $\exists!xP(x) \rightarrow \exists xP(x)$
II. $\exists!x \neg P(x) \rightarrow \neg\forall xp(x)$

(A) Both I and II are true
(B) Both I and II are false
(C) I-false, II-true
(D) I-true, II-false

(A) Both I and II are true

I. $\exists!xP(x) \rightarrow \exists xP(x)$

This statement asserts that if there exists a unique x such that P(x) is true, then there exists at least one x such that P(x) is true.

If there exists a unique x such that P(x) is true, then obviously there exists at least one x such that P(x) is true. So, this statement is true.

II. $\exists!x \neg P(x) \rightarrow \neg\forall xP(x)$

This statement asserts that if there exists a unique x such that ¬P(x) (not P(x)) is true, then it is not the case that for all x, P(x) is true.

If there exists a unique x such that ¬P(x) is true, then obviously it is not the case that for all x, P(x) is true. So, this statement is also true.

June 2014 Paper II

Give a compound proposition involving propositions p, q and r that is true when exactly two of p, q and r are true and is false otherwise.

(A) $(p \vee q \wedge \neg r) \wedge (p \wedge \neg q \wedge r) \wedge (\neg p \wedge q \wedge r)$
(B) $(p \wedge q \wedge \neg r) \wedge (p \vee q \wedge \neg r) \wedge (\neg p \wedge q \wedge r)$
(C) $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \wedge (\neg p \wedge q \wedge r)$
(D) $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r)$

June 2014 Paper II

Give a compound proposition involving propositions p, q and r that is true when exactly two of p, q and r are true and is false otherwise.

(A) $(p \vee q \wedge \neg r) \wedge (p \wedge \neg q \wedge r) \wedge (\neg p \wedge q \wedge r)$
(B) $(p \wedge q \wedge \neg r) \wedge (p \vee q \wedge \neg r) \wedge (\neg p \wedge q \wedge r)$
(C) $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \wedge (\neg p \wedge q \wedge r)$
(D) $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r)$

Paper III December 2014

The resolvent of the set of clauses

$$(A \vee B, \sim A \vee D, C \vee \sim B)$$

is

A. $A \vee B$

B. $C \vee D$

C. $A \vee C$

D. $A \vee D$

Answer: B

$$A \vee B, \ \sim A \vee D, \ C \vee \sim B$$

A and $\sim A$ will cancel out and so will B and $\sim B$

Paper II June 2015

"If my computations are correct and I pay the electric bill, then I will run out of money. If I don't pay the electric bill, the power will be turned off. Therefore, if I don't run out of money and the power is still on, then my computations are incorrect."

Convert this argument into logical notations using the variables c, b, r, p for propositions of computations, electric bills, out of money and the power respectively. (Where ¬ means NOT)

(A) if $(c \wedge b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(\neg r \wedge p) \rightarrow \neg c$
(B) if $(c \vee b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(r \wedge p) \rightarrow c$
(C) if $(c \wedge b) \rightarrow r$ and $\neg p \rightarrow \neg b$, then $(\neg r \vee p) \rightarrow \neg c$
(D) if $(c \vee b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(\neg r \wedge p) \rightarrow \neg c$

(A) if $(c \wedge b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(\neg r \wedge p) \rightarrow \neg c$

Let's break down the argument into logical propositions:

"If my computations are correct and I pay the electric bill, then I will run out of money."
This can be represented as: $(c \wedge b) \rightarrow r$
"If I don't pay the electric bill, the power will be turned off."
This can be represented as: $\neg b \rightarrow \neg p$
"Therefore, if I don't run out of money and the power is still on, then my computations are incorrect."
This can be represented as: $(\neg r \wedge p) \rightarrow \neg c$

A. if $(c \wedge b) \rightarrow r$ and $\neg b \rightarrow \neg p$, then $(\neg r \wedge p) \rightarrow \neg c$ This matches the given propositions and their logical notation.

Paper II June 2015

Match the following:

List - I

(a) $(p \rightarrow q) \Leftrightarrow (\neg q \rightarrow \neg p)$

(b) $[(p \wedge q) \rightarrow r] \Leftrightarrow [p \rightarrow (q \rightarrow r)]$

(c) $(p \rightarrow q) \Leftrightarrow [(p \wedge \neg q) \rightarrow o]$

(d) $(p \Leftrightarrow q) \Leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow p)]$

List - II

(i) Contrapositive

(ii) Exportation law

(iii) Reductio ad absurdum

(iv) Equivalence

Codes:

	(a)	(b)	(c)	(d)
(A)	(i)	(ii)	(iii)	(iv)
(B)	(ii)	(iii)	(i)	(iv)
(C)	(iii)	(ii)	(iv)	(i)
(D)	(iv)	(ii)	(iii)	(i)

(A) (i) (ii) (iii) (iv)

(a)The contrapositive of a conditional statement is formed by negating both the hypothesis and the conclusion, and then interchanging the resulting negations. In other words, the contrapositive negates and switches the parts of the sentence. It does BOTH the jobs of the INVERSE and the CONVERSE.

Ex:- $(p \rightarrow q) \Leftrightarrow (\neg q \rightarrow \neg p)$

(b) Exportation is a valid rule of replacement in propositional logic. The rule allows conditional statements having conjunctive antecedents to be replaced by statements having conditional consequents and vice versa inlogical proofs. It is the rule that:

$((P \wedge Q) \rightarrow R) \Leftrightarrow (p \rightarrow (Q \rightarrow R))$

(C)

(d) Logical equality (also known as biconditional) is an operation on two logical values, typically the values of two propositions, that produces a value of true if and only if both operands are false or both operands are true.It is logically equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$.

Paper II June 2015

Consider a proposition given as:

"$x \geq 6$, if $x2 \geq 25$ and its proof as:
If $x \geq 6$, then $x2 = x.x = 6.6 = 36 \geq 25$

Which of the following is correct w.r.to the given proposition and its proof ?

(a) The proof shows the converse of what is to be proved.
(b) The proof starts by assuming what is to be shown.
(c) The proof is correct and there is nothing wrong.

(A) (a) only
(B) (c) only
(C) (a) and (b)
(D) (b) only

(C) (a) and (b)

let p: x>=6 q: x^2=25 then given statement can be written as q->p

and its given proof may be written as p->q which is converse of q->p so

a) is correct

we have to prove p and we are assuming it in our proof so b is also true

so ans is C) a and b

Paper III June 2015

The clausal form of the disjunctive normal form $\neg A \vee \neg B \vee \neg C \vee D$ is:

(A) $A \wedge B \wedge C \Rightarrow D$
(B) $A \vee B \vee C \vee D \Rightarrow$ true
(C) $A \wedge B \wedge C \wedge D \Rightarrow$ true
(D) $A \wedge B \wedge C \wedge D \Rightarrow$ false

(A) $A \wedge B \wedge C \Rightarrow D$

We know that $P \Rightarrow Q = \neg P \vee Q$
Similarly $A \wedge B \wedge C \Rightarrow D = \neg(A \wedge B \wedge C) \vee D = \neg A \vee \neg B \vee \neg C \vee D$

Paper III June 2015

In propositional logic $P \leftrightarrow Q$ is equivalent to (Where ~ denotes NOT):

(A) $\sim(P \vee Q) \wedge \sim(Q \vee P)$
(B) $(\sim P \vee Q) \wedge (\sim Q \vee P)$
(C) $(P \vee Q) \wedge (Q \vee P)$
(D) $\sim(P \vee Q) \rightarrow \sim(Q \vee P)$

(B) $(\sim P \vee Q) \wedge (\sim Q \vee P)$

$P \leftrightarrow Q$

$(P \rightarrow Q) \wedge (Q \rightarrow P)$

$(\sim P \vee Q) \wedge (\sim Q \vee P)$

Paper II December 2015

Which of the following arguments are not valid?

(a) “If Gora gets the job and works hard, then he will be promoted. If Gora gets promotion, then he will be happy. He will not be happy, therefore, either he will not get the job or he will not work hard”.

(b) “Either Puneet is not guilty or Pankaj is telling the truth. Pankaj is not telling the truth, therefore, Puneet is not guilty”.

(c) If n is a real number such that n>1, then n2>1. Suppose that n2>1, then n>1.

Codes:

(A) (a) and (c)
(B) (b) and (c)
(C) (a), (b) and (c)
(D) (a) and (b)

Answer: Marks to all

Option A let

- p: Gora get the job
- q: he works hard
- r: he will be promoted
- s: he will be happy

So we have $p \wedge q \rightarrow r$, $r \rightarrow s$ which will give us $\neg s \rightarrow \neg r$ (by contrapositive law) and $\neg r \rightarrow \neg(p \wedge q)$. Now $\neg s$ is given so it implies $\neg r$ and so we have $\neg p \vee \neg q$ i.e either he does not get the job or he does not work hard. So it is valid

Option B valid as **EXACTLY** one of two statements must be true. Given one statement is not true so other must be true

Option C not valid. Here we have $a \rightarrow b$. But this does not always mean $b \rightarrow a$.

Hence A and B are valid C is no valid so no option is correct .

Paper II December 2015

Let P(m,n) be the statement "m divides n" where the Universe of discourse for both the variables is the set of positive integers. Determine the truth values of the following propositions.

(a) $\exists m \, \forall n \, P(m,n)$ (b) $\forall n \, P(1,n)$ (c) $\forall m \, \forall n \, P(m,n)$

(A) (a)-True; (b)-True; (c)-False
(B) (a)-True; (b)-False; (c)-False
(C) (a)-False; (b)-False; (c)-False
(D) (a)-True; (b)-True; (c)-True

(A) (a)-True; (b)-True; (c)-False

$\forall m\ \forall n\ P(m, n)$ says that every number divides every other number and result should be a postive integer.
Clearly it is a false proposition.
Eg: if m=10 n=3 10 divides 3 does not follow the proposition
a is false

$\forall n\ P(1, n)$ says that any positive integer is divisible by 1 and result will be a +ve integer.
That is correct
b is true

$\exists m \forall n P(m,n)$ says that there are some +ve integers which divides any other +ve integer.
The proposition is correct
example: 1
c is true

Paper II December 2015

Match the following terms:

List - I	List - II
(a) Vacuous proof	(i) A proof that the implication $p \rightarrow q$ is true based on the fact that p is false
(b) Trivial proof	(ii) A proof that the implication $p \rightarrow q$ is true based on the fact that q is true
(c) Direct proof	(iii) A proof that the implication $p \rightarrow q$ is true that proceeds by showing that q must be true when p is true.
(d) Indirect proof	(iv) A proof that the implication $p \rightarrow q$ is true that proceeds by showing that p must be false when q is false.

Codes:

	(a)	(b)	(c)	(d)
(A)	(i)	(ii)	(iii)	(iv)
(B)	(ii)	(iii)	(i)	(iv)
(C)	(iii)	(ii)	(iv)	(i)
(D)	(iv)	(iii)	(ii)	(i)

Paper II December 2015

Match the following terms:

List - I	List - II
(a) Vacuous proof	(i) A proof that the implication $p \rightarrow q$ is true based on the fact that p is false
(b) Trivial proof	(ii) A proof that the implication $p \rightarrow q$ is true based on the fact that q is true
(c) Direct proof	(iii) A proof that the implication $p \rightarrow q$ is true that proceeds by showing that q must be true when p is true.
(d) Indirect proof	(iv) A proof that the implication $p \rightarrow q$ is true that proceeds by showing that p must be false when q is false.

Codes:

	(a)	(b)	(c)	(d)
(A)	(i)	(ii)	(iii)	(iv)
(B)	(ii)	(iii)	(i)	(iv)
(C)	(iii)	(ii)	(iv)	(i)
(D)	(iv)	(iii)	(ii)	(i)

Paper II December 2015

Consider the compound propositions given below as:

(a) $p \vee \sim(p \wedge q)$ (b) $(p \wedge \sim q) \vee \sim(p \wedge q)$ (c) $p \wedge (q \vee r)$

Which of the above propositions are tautologies?

(A) (a) and (c)
(B) (b) and (c)
(C) (a) and (b)
(D) only (a)

(D) only (a)

$p \vee \sim(p \wedge q) = p + (pq)` = p + p` + q` = 1 + q` = 1$. This is a tautology.
$(p \wedge \sim q) \vee \sim(p \wedge q) = pq` + (pq)` = pq` + p` + q` = p` + q`$. This is not a tautology.
$p \wedge (q \vee r) = pq + pr$. This is not a tautology.

Paper II January 2017

Match the following:

List-I	List-II
a. Absurd	i. Clearly impossible being contrary to some evident truth.
b. Ambiguous	ii. Capable of more than one interpretation or meaning.
c. Axiom	iii. An assertion that is accepted and used without a proof.
d. Conjecture	iv. An opinion Preferably based on some experience or wisdom.

Codes:

	a	b	c	d
(1)	i	ii	iii	iv
(2)	i	iii	iv	ii
(3)	ii	iii	iv	i
(4)	ii	i	iii	iv

Paper II January 2017

Match the following:

List-I	List-II
a. Absurd	i. Clearly impossible being contrary to some evident truth.
b. Ambiguous	ii. Capable of more than one interpretation or meaning.
c. Axiom	iii. An assertion that is accepted and used without a proof.
d. Conjecture	iv. An opinion Preferably based on some experience or wisdom.

Codes:

	a	b	c	d
(1)	i	ii	iii	iv
(2)	i	iii	iv	ii
(3)	ii	iii	iv	i
(4)	ii	i	iii	iv

Paper II January 2017

In propositional logic if $(P \rightarrow Q) \wedge (R \rightarrow S)$ and $(P \vee R)$ are two premises such that

$$\begin{array}{c} (P \rightarrow Q) \wedge (R \rightarrow S) \\ P \vee R \\ \hline Y \\ \hline \end{array}$$

Y is the premise:

(1) $P \vee R$
(2) $P \vee S$
(3) $Q \vee R$
(4) $Q \vee S$

Paper II January 2017

In propositional logic if $(P \rightarrow Q) \wedge (R \rightarrow S)$ and $(P \vee R)$ are two premises such that

$$\begin{array}{c} (P \rightarrow Q) \wedge (R \rightarrow S) \\ P \vee R \\ \hline Y \\ \hline \end{array}$$

Y is the premise:

(1) $P \vee R$
(2) $P \vee S$
(3) $Q \vee R$
(4) $Q \vee S$

Paper III January 2017

Which of the following statements is true?

(1) The sentence S is a logical consequence of S1,..., Sn if and only if S1^S2^........^Sn→S is satisfiable.
(2) The sentence S is a logical consequence of S1,..., Sn if and only if S1^S2^........^Sn→S is valid.
(3) The sentence S is a logical consequence of S1,..., Sn if and only if S1^S2^........^Sn^¬S is consistent.
(4) The sentence S is a logical consequence of S1,..., Sn if and only if S1^S2^........^Sn^S is inconsistent.

(2) The sentence S is a logical consequence of S1,..., Sn if and only if S1^S2^........^Sn→S is valid.

$S_1, S_2, \ldots, S_n$ can be considered as premises and S is conclusion. Argument $\mathit{premises} \implies \mathit{conclusion}$ is said to be valid(TRUR) if $S_1, S_2, S_3, \ldots, S_n \rightarrow S$ is valid. (False $\rightarrow$ or True $\rightarrow$ True.)

Paper III January 2017

The first order logic (FOL) statement $((R \vee Q) \wedge (P \vee \neg Q))$ is equivalent to which of the following?

(1) $((R \vee \neg Q) \wedge (P \vee \neg Q) \wedge (R \vee P))$
(2) $((R \vee Q) \wedge (P \vee \neg Q) \wedge (R \vee P))$
(3) $((R \vee Q) \wedge (P \vee \neg Q) \wedge (R \vee \neg P))$
(4) $((R \vee Q) \wedge (P \vee \neg Q) \wedge (\neg R \vee P))$

(2) ((RvQ)∧(Pv¬Q)∧(RvP))

(R+Q) (P+~Q)

=RP+R~Q+PQ

now option (b)

(R+Q) (P+~Q) (R+P)

(PR+~QR+PQ)(R+P)

PR+~QR+PQR+PR+P~QR+PQ

(~QR+P~QR)+(PQR+PQ)+PR

~QR+PQ+PR

Paper II November 2017

Let P and Q be two propositions, $\neg(P \leftrightarrow Q)$ is equivalent to:

(1) $P \leftrightarrow \neg Q$
(2) $\neg P \leftrightarrow Q$
(3) $\neg P \leftrightarrow \neg Q$
(4) $Q \rightarrow P$

Answer: 1, 2

P	Q	$\sim p$	$\sim Q$	$Q \rightarrow P$	$\sim (P \leftrightarrow Q)$	$P \leftrightarrow \sim Q$	$\sim P \leftrightarrow Q$	$\sim P \leftrightarrow \sim Q$
0	0	1	1	1	0	0	0	1
0	1	1	0	0	1	1	1	0
1	0	0	1	1	1	1	1	0
1	1	0	0	1	0	0	0	1

Paper III November 2017

Negation of the proposition $\exists$ x H(x) is

(1) $\exists$ x $\neg$H(x)
(2) $\forall$ x $\neg$H(x)
(3) $\forall$ x H(x)
(4) $\neg$ x H(x)

(2) $\forall x \neg H(x)$

$\exists x\ H(x)$: There exists some x for which H(x) is true

$\neg\exists x\ H(x)$: The negation for $\exists x\ H(x)$ will be for all x H(x) is not true.

$\therefore \neg\exists x\ H(x) = \forall x\ \neg H(x)$

Paper III November 2017

Let P, Q, R and S be Propositions. Assume that the equivalences $P \Leftrightarrow (Q \vee \neg Q)$ and $Q \Leftrightarrow R$ hold. Then the truth value of the formula $(P \wedge Q) \Rightarrow ((P \wedge R) \vee S)$ is always :

(1) True
(2) False
(3) Same as truth table of Q
(4) Same as truth table of S

(1) True

Equivalence condition (Both RHS and LHS should be True)= $P \Leftrightarrow (Q \vee \neg Q)$
Holding condition (Both RHS and LHS may True/False)=$Q \Leftrightarrow R$

Step 1: $(P \wedge Q) \Rightarrow ((P \wedge R) \vee S)$[Note: Q, R, and S is True]

P	Q	R	S	$(P \wedge Q)$	$((P \wedge R) \vee S)$	$(P \wedge Q) \Rightarrow ((P \wedge R) \vee S)$
T	T	T	T	T	T	T

Note: The given condition is True

Step-2:$(P \wedge Q) \Rightarrow ((P \wedge R) \vee S)$ [Note: Q, R, and S is False]

P	Q	R	S	$(P \wedge Q)$	$((P \wedge R) \vee S)$	$(P \wedge Q) \Rightarrow ((P \wedge R) \vee S)$
T	F	F	F	F	F	T

Paper III November 2017

"If X, then Y unless Z" is represented by which of the following formulae in propositional logic?

(1) $(X \wedge Y) \rightarrow \neg Z$
(2) $(X \wedge \neg Z) \rightarrow Y$
(3) $X \rightarrow (Y \wedge \neg Z)$
(4) $Y \rightarrow (X \wedge \neg Z)$

(2) $(X \wedge \neg Z) \rightarrow Y$

"If X, then Y unless Z" means $\neg Z \rightarrow (X \rightarrow Y)$

$Z \vee \neg X \vee Y \neg X \vee Z \vee Y$

$A). (X \wedge Y) \rightarrow \neg Z \neg(X \wedge Y) \vee \neg Z \neg X \vee \neg Y \vee \neg Z$	$B). (X \wedge \neg Z) \rightarrow Y \neg(X \wedge \neg Z) \vee Y \neg X \vee Z \vee Y$	$C. X \rightarrow (Y \wedge \neg Z) \neg X \vee (Y \wedge \neg Z) \neg X \vee Y \wedge \neg Z$	$D. Y \rightarrow (X \wedge \neg Z) Y \vee (X \wedge \neg Z) Y \vee X \wedge \neg Z$

Paper III November 2017

Consider the following two well-formed formulas in prepositional logic.

F1 : $P \Rightarrow \neg P$
F2 : $(P \Rightarrow \neg P) \vee (\neg P \Rightarrow P)$

Which of the following statements is correct?

(1) F1 is Satisfiable, F2 is valid
(2) F1 is unsatisfiable, F2 is Satisfiable
(3) F1 is unsatisfiable, F2 is valid
(4) F1 and F2 both are Satisfiable

(1) F1 is Satisfiable, F2 is valid

$$F1 : P \to \neg P$$

$$= \neg P \vee \neg P$$

$$= \neg P.$$ can be true when P is false (Atleast one T hence satisfiable)

$$F2 : (P \to \neg P) \vee (\neg P \to P)$$

$$= \neg P \vee (P \vee P)$$

$$= \neg P \vee P$$

$$= T.$$

VALID

Paper II July 2018

Consider the following English sentence:
“Agra and Gwalior are both in India”.

A student has written a logical sentence for the above English sentence in First-Order Logic using predicate In(x, y), which means x is in y, as follows:

In(Agra, India) V In(Gwalior, India)

Which one of the following is correct with respect to the above logical sentence?

(1) It is syntactically valid but does not express the meaning of the English sentence.
(2) It is syntactically valid and expresses the meaning of the English sentence also.
(3) It is syntactically invalid but expresses the meaning of the English sentence.
(4) It is syntactically invalid and does not express the meaning of the English sentence.

(1) It is syntactically valid but does not express the meaning of the English sentence.

Given that "Agra and Gwalior are both in India"
representation :- In(x,y) means x is in India
In(Agra,India) ----(1)
In(Gwalior,India) ---(2)

According to the english statement, In(Agra,India) is should be true, In(Gwalior,India) is should be true

therefore use Conjunction as Connector ===> In(Agra,India) ^ In(Gwalior,India).

but given that In(Agra,India) V In(Gwalior,India) ====> syntactically correct but not represent the given enlish statement.

Paper II July 2018

The equivalence of

$\neg \exists x\, Q(x)$ is:

(1) $\exists\, x \neg Q(x)$
(2) $\forall x \neg Q(x)$
(3) $\neg \exists x \neg Q(x)$
(4) $\forall x\, Q(x)$

(2) $\forall x \neg Q(x)$

Given statement is: $\neg \exists x\ Q(x)$ is : This negation $\neg$ will change the quantifier and also it negates the element with the quantifier. So, it becomes. $\forall x \neg Q(x)$

Paper II December 2018

In mathematical logic, which of the following are statements?

(i) There will be snow in January.
(ii) What is the time now?
(iii) Today is Sunday.
(iv) You must study Discrete mathematics

Choose the correct answer from the code given below:

(1) i and iii
(2) i and ii
(3) ii and iv
(4) iii and iv

(1) i and iii

Statement (i): There will be snow in January It is a mathematical statement because either there will be snow in January or there will not be snow in January. It has only two possible values either true or false.

Statement (ii): What is the time now? In this case, there is no meaning of true or false. It is only asking the current time and you can answer that but not in true or false.

Statement (iii): Today is Sunday It is a statement. As, it is true of Sunday and false on any other day. So, there are two possibilities of either true or false.

Statement (iv): You must study Discrete Mathematics. It is not a statement. It is not giving meaning in true or false sense.

Paper II December 2018

Match the List-I with List-II and choose the correct answer from the code given below:

List I	List II
(a) Equivalence	(i) $p \Rightarrow q$
(b) Contrapositive	(ii) $p \Rightarrow q : q \Rightarrow p$
(c) Converse	(iii) $p \Rightarrow q : \sim q \Rightarrow \sim p$
(d) Implication	(iv) $p \Leftrightarrow q$

Code:

(1) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
(2) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)
(3) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)
(4) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

Paper II December 2018

Match the List-I with List-II and choose the correct answer from the code given below:

List I	List II
(a) Equivalence	(i) $p \Rightarrow q$
(b) Contrapositive	(ii) $p \Rightarrow q : q \Rightarrow p$
(c) Converse	(iii) $p \Rightarrow q : \sim q \Rightarrow \sim p$
(d) Implication	(iv) $p \Leftrightarrow q$

Code:

(1) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
(2) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)
(3) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)
(4) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

Paper II December 2018

Consider a vocabulary with only four propositions A, B, C and D. How many models are there for the following sentence?

$\neg A \lor \neg B \lor \neg C \lor \neg D$

(1) 7
(2) 8
(3) 15
(4) 16

(3) 15

Number of models is nothing but number of TRUE in final statement.

Paper II December 2018

Consider the sentence below:
“There is a country that borders both India and Nepal”

Which of the following represents the above sentence correctly?

(1) $\exists c$ Country(c) $\wedge$ Border(c,India) $\wedge$ Border(c,Nepal)
(2) $\exists c$ Country(c) $\Rightarrow$ [Border(c,India) $\wedge$ Border(c,Nepal)]
(3) [$\exists c$ Country(c)] $\Rightarrow$ [Border(c,India) $\wedge$ Border(c,Nepal)]
(4) $\exists c$ Border(Country(c),India) $\wedge$ Nepal)

(1) $\exists c$ Country(c) ∧ Border(c,India) ∧ Border(c,Nepal)

c- It represents country
$\exists c$ – it means there exists a country
Border(c, India) – it means border between India and c
Border(c, Nepal) – it means border between Nepal and c

Variables are two: India and Nepal Given statement: "There is a country that borders both India and Nepal."
AND is true only when Border(c, India) and Border(c, Nepal) is 1.
Country must have border with both India and Nepal. So, and operation supports it.
AND is denoted by ∧ in proposition logic. Correct option is: $\exists c$ Country(c) ∧ Border(c, India) ∧ Border(c, Nepal)
It means there exists a country c and it borders both India and Nepal.

Paper II June 2019

Which of the following is principal conjunctive normal form for $[(p \vee q) \wedge \neg p \rightarrow \neg q]$?

(a) $p \vee \neg q$
(b) $p \vee q$
(c) $\neg p \vee q$
(d) $\neg p \vee \neg q$

Paper II June 2019

Which of the following is principal conjunctive normal form for $[(p \vee q) \wedge \neg p \rightarrow \neg q]$?

(a) $p \vee \neg q$
(b) $p \vee q$
(c) $\neg p \vee q$
(d) $\neg p \vee \neg q$

Paper II June 2019

Match List-I with List-II:

List-I	List-II
A. $p \rightarrow q$	1. $\neg(q \rightarrow \neg p)$
B. $p \vee q$	2. $p \wedge \neg q$
C. $p \wedge q$	3. $\neg p \rightarrow q$
D. $\neg(p \rightarrow q)$	4. $\neg p \vee q$

Choose the correct option from those given below:

(a) A-2, B-3, C-1, D-4
(b) A-2, B-1, C-3, D-4
(c) A-4, B-1, C-3, D-2
(d) A-4, B-3, C-1, D-2

Paper II June 2019

Match List-I with List-II:

List-I	List-II
A. $p \rightarrow q$	1. $\neg(q \rightarrow \neg p)$
B. $p \vee q$	2. $p \wedge \neg q$
C. $p \wedge q$	3. $\neg p \rightarrow q$
D. $\neg(p \rightarrow q)$	4. $\neg p \vee q$

Choose the correct option from those given below:

(a) A-2, B-3, C-1, D-4
(b) A-2, B-1, C-3, D-4
(c) A-4, B-1, C-3, D-2
(d) A-4, B-3, C-1, D-2

Paper II November 2020
Consider the statement below.

A person who is radical (R) is electable (E) if he/she is conservative (C), but otherwise is not electable. Few probable logical assertions of the above sentence are given below:

(A) $(R \wedge E) \Longleftrightarrow C$

(B) $R \Rightarrow (E \Leftrightarrow C)$

(C) $R \Rightarrow ((C \Rightarrow E) V \neg E)$

(D) $(\neg R \vee \neg E \vee C) \wedge (\neg R \vee \neg C \vee E)$

Which of the above logical assertions are true? Choose the correct answer from the options given below:

a) (B) Only
b) (C) only
c) (A) and (C) only
d) (B) and (D) only

d) (B) and (D) only

1) $(R \wedge E) \Leftrightarrow C$ says that all (and only) conservatives are radical and electable. So, this assertion is not true.

2) $R \Rightarrow (E \Leftrightarrow C)$ says that same as the given assertion. This is a correct assertion.

3) $R \Rightarrow ((C \Rightarrow E) \vee \neg E) = \neg R \vee (\neg C \vee E \vee \neg E)$ which is true for all interpretations. This is not a correct assertion.

4) $(\neg R \vee \neg E \vee C) \wedge (\neg R \vee \neg C \vee E) = (\neg R \vee (E \Rightarrow C)) \wedge (\neg R \vee (C \Rightarrow E)) = R \Rightarrow (E \Leftrightarrow C)$ which is equivalent to assertion B. This is also true.

Paper II November 2020

Which of the following pairs of propositions are not logically equivalent?

a) $((p \rightarrow r) \wedge (q \rightarrow r))$ and $((p \vee q) \rightarrow r)$
b) $p \leftrightarrow q$ and $(\neg p \leftrightarrow \neg q)$
c) $(p \rightarrow q) \wedge (q \rightarrow p)$ and $p \leftrightarrow q$
d) $((p \wedge q) \rightarrow r)$ and $((p \rightarrow r) \wedge (q \rightarrow r))$

d) 4

P	Q	R	¬P	¬Q	P⇒R	Q⇒R	**(P⇒R)∧(Q⇒R)**	P∧Q	**P∧Q⇒R**
T	T	T	F	F	T	T	T	T	T
T	T	F	F	F	F	F	F	T	F
T	F	T	F	T	T	T	T	F	T
T	F	F	F	T	F	T	F	F	T
F	T	T	T	F	T	T	T	F	T
F	T	F	T	F	T	F	F	F	T
F	F	T	T	T	T	T	T	F	T
F	F	F	T	T	T	T	T	F	T

Consider the following argument with premise for all x (P(x) V Q(x)) and conclusion (for all x P(x))∧(for all x Q(x))

(A)	$\forall_x (P(x) \vee Q(x))$	Premise
(B)	$P(c) \vee Q(c)$	Universal instantiation from (A)
(C)	$P(c)$	Simplification from (B)
(D)	$\forall_x P(x)$	Universal Generalization of (C)
(E)	$Q(c)$	Simplification from (B)
(F)	$\forall_x Q(x)$	Universal Generalization of (E)
(G)	$(\forall_x P(x)) \wedge (\forall_x Q(x))$	Conjunction of (D) and (F)

a) This is a valid argument
b) Steps (C) and (E) are not correct inferences
c) Steps (D) and (F) are not correct inferences
d) Step (G) is not a correct inference

b) Steps (C) and (E) are not correct inferences

For C and E to be true, the assertion should be $P(c) \wedge Q(c)$. Hence, this is not a correct inference

Paper II November 2021

Which of the following are logically equivalent?

A. $\neg p \rightarrow (q \rightarrow r)$ *and* $q \rightarrow (p \vee r)$

B. $(p \rightarrow q) \rightarrow r$ *and* $p \rightarrow (q \rightarrow r)$

C. $(p \rightarrow q) \rightarrow (r \rightarrow s)$ *and* $(p \rightarrow r) \rightarrow (q \rightarrow s)$

Choose the correct answers from the option given below:

a) A and B only
b) A and C only
c) A only
d) B and C only

C) A only

A. p V q’ V r and q’V p V r logically equivalent

B. (p’ + q)’ + r => p.q’ + r

p’ + (q’ + r)

C. (p->q)’ + (r->s) => p.q’ + r’ + s

(p->r) + (q’ + s) => p’ + r + q’ + s

Paper II October 2022

The logic expression $(P' \wedge Q) \vee (P \wedge Q') \vee (P \wedge Q)$ is equivalent to

a) $P' \vee Q$
b) $P \vee Q'$
c) $P \vee Q$
d) $P' \vee Q'$

c) P V Q

$P'Q + PQ' + PQ$
$Q(P + P') + PQ'$
$Q + PQ'$
$(Q + P)(Q + Q')$
$Q + P$
$P \vee Q$

Paper II October 2022

Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R):

Assertion (A): p'

Reason (R): $(r \rightarrow q', r \vee s, s \rightarrow q', p \rightarrow q)$

In the light of the above statements, choose the correct answer from the options given below:

a) Both (A) and (R) are true and (R) is the correct explanation of (A)
b) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)
c) (A) is true but (R) is false
d) (A) is false but (R) is true

a) Both (A) and (R) are true and (R) is the correct explanation of (A)

$r' \vee q'$
$r \vee s$
$s' \vee \bar{q}$
$p' \vee q$
here complement of element is cancel by its original element like r' and r cancel after solving like that answer is p'

Paper II October 2022

Consider α, β, γ as logical variables. Identify which of the following represents correct logical equivalence :

A. $(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$
B. $(\alpha \vee \beta) \equiv \sim \alpha \vee \beta$
C. $(\alpha \rightarrow \beta) \equiv (\sim \beta \rightarrow \sim\alpha)$
D. $(\sim(\alpha \vee \beta)) \equiv (\sim\alpha \rightarrow \sim\beta)$

Choose the correct answer from the options given below:

a) (A) and (D) only
b) (B) and (C) only
c) (A) and (C) only
d) (B) and (D) only

c) (A) and (C) only

A. $(\alpha \wedge (\beta \vee \gamma)) = (\alpha.(\beta+\gamma)) = (\alpha.\beta) + (\alpha\ .\gamma) = (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$ =RHS

B. $(\alpha \vee \beta) \neq \sim \alpha \vee \beta$

C. LHS: $\alpha \rightarrow \beta = \sim \alpha \vee \beta$
RHS: $\sim \beta \rightarrow \sim\alpha = \beta \vee \sim \alpha$
LHS = RHS

D. LHS: $\sim(\alpha \vee \beta) = \sim\alpha \wedge \sim \beta$
RHS: $(\sim\alpha \rightarrow \sim\beta\ ) = \alpha \vee \sim\beta$
LHS ≠ RHS

Paper II March 2023

The negation of "Some students like hockey" is:

1. Some students dislike hockey
2. Every student dislike hockey
3. Every student like hockey
4. All students like hockey

2. Every student dislike hockey

~(Some students like hockey) => Every student dislike hockey

Paper II DEC 2023

Match List - I with List - II

	List - I		List - II
	Propositions		Disjunctive Normal Form (DNF)
(A)	$P\wedge(P\rightarrow Q)$	(I)	$P\vee Q$
(B)	$\neg(P\vee Q)\rightarrow(P\wedge Q)$	(II)	$(P\wedge\neg P)\vee(P\wedge Q)$
(C)	$P\rightarrow Q$	(III)	$(\neg P)\vee Q$
(D)	$P\vee(Q\wedge R)$	(IV)	$(P\wedge P)\vee(P\wedge Q)\vee(P\wedge R)\vee(Q\wedge R)$

Choose the correct answer from the options given below :

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(2) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
(3) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
(4) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

Paper II DEC 2023

Match List - I with List - II

	List - I Propositions		List - II Disjunctive Normal Form (DNF)
(A)	$P \wedge (P \rightarrow Q)$	(I)	$P \vee Q$
(B)	$\neg(P \vee Q) \rightarrow (P \wedge Q)$	(II)	$(P \wedge \neg P) \vee (P \wedge Q)$
(C)	$P \rightarrow Q$	(III)	$(\neg P) \vee Q$
(D)	$P \vee (Q \wedge R)$	(IV)	$(P \wedge P) \vee (P \wedge Q) \vee (P \wedge R) \vee (Q \wedge R)$

Choose the correct answer from the options given below :

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(2) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
(3) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
(4) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

Paper II Dec 2023

If universe of disclosure are all real numbers, then which of the following are true?

(A) $\exists x \ \forall y \ (x + y = y)$

(B) $\forall x \ \forall y(((x \geq 0) \wedge (y < 0)) \rightarrow (x - y > 0))$

(C) $\exists x \ \exists y(((x \leq 0) \wedge (y \leq 0)) \wedge (x - y > 0))$

(D) $\forall x \ \forall y((x \neq 0) \wedge (y \neq 0) \leftrightarrow (xy \neq 0))$

Choose the correct answer from the options given below:

(1) (A) and (B) Only
(2) (A), (C) and (D) Only
(3) (A), (B) and (D) Only
(4) (A), (B), (C) and (D) Only

Paper II Dec 2023

If universe of disclosure are all real numbers, then which of the following are true?

(A) $\exists x\ \forall y\ (x+y=y)$

(B) $\forall x\ \forall y(((x \geqslant 0) \wedge (y<0)) \rightarrow (x-y>0))$

(C) $\exists x\ \exists y(((x \leqslant 0) \wedge (y \leqslant 0)) \wedge (x-y>0))$

(D) $\forall x\ \forall y((x \neq 0) \wedge (y \neq 0) \leftrightarrow (xy \neq 0))$

Choose the correct answer from the options given below:

(1) (A) and (B) Only
(2) (A), (C) and (D) Only
(3) (A), (B) and (D) Only
(4) (A), (B), (C) and (D) Only