Structure	Definition	Properties	Mathematical Structure
Semigroup	A set equipped with an associative binary operation.	Associativity	Closed under operation *
Monoid	A semigroup with an identity element.	Associativity, Identity element	Semigroup with identity element
Group	A monoid in which every element has an inverse.	Associativity, Identity element, Inverses for every element	Monoid with inverses for every element
Abelian Group	A group in which the binary operation is commutative.	Associativity, Identity element, Inverses for every element, Commutativity	Group with commutative operation
Ring	A set equipped with two binary operations (usually addition and multiplication); these operations make the set an abelian group under addition and a semigroup under multiplication.	Associativity for both operations, Identity for addition, Additive inverses, Distributive property of multiplication over addition	Abelian group under +, Semigroup under â‹… â‹…
Ring with Unity (Unital Ring)	A ring that has a multiplicative identity (1).	Associativity for both operations, Identity for both addition and multiplication, Additive inverses, Distributive property of multiplication over addition	Ring with identity for â‹…â‹…
Commutative Ring with Unity	A ring with unity where the multiplication is commutative.	Associativity for both operations, Identity for both addition and multiplication, Additive inverses, Commutativity of multiplication, Distributive property of multiplication over addition	Ring with unity and commutative multiplication
Integral Domain	A commutative ring with unity, having no zero divisors (except zero itself).	Associativity for both operations, Identity for both addition and multiplication, Additive inverses, No zero divisors, Commutativity of multiplication, Distributive property of multiplication over addition	Commutative ring with unity and no zero divisors
Field	An integral domain in which every non-zero element has a multiplicative inverse.	Associativity for both operations, Identity for both addition and multiplication, Additive inverses, Multiplicative inverses for non-zero elements, Commutativity of multiplication, No zero divisors, Distributive property	Integral domain with inverses for every non-zero element

Vector Space	A set of vectors with two operations: vector addition and scalar multiplication, where scalars are elements of a field.	Associativity, Commutativity, Identity and inverse for addition, Distributivity of scalar multiplication over vector addition and field addition, Identity element of scalar multiplication	Field, plus additional structure to handle linear combinations of vectors
Module	Similar to vector spaces, but scalar multiplication is defined over a ring instead of a field, making modules more general than vector spaces.	Properties similar to vector spaces, but the scalars come from a ring, not necessarily a field	Ring, generalization of vector spaces
Algebra	A vector space equipped with a bilinear binary operation âˆ—âˆ— (in addition to the addition and scalar multiplication of the vector space), which is associative and has a multiplicative identity.	All vector space properties, plus associativity and distributivity of the binary operation âˆ—âˆ—, and existence of a multiplicative identity with respect to âˆ—âˆ—	Vector space with an additional associative binary operation
Lie Algebra	An algebra over a field where the binary operation, known as the Lie bracket [,][,], is anticommutative [x , y] = âˆ' [y , x][*x*,*y*]=âˆ'[*y*,*x*] and satisfies the Jacobi identity.	Anticommutativity, Jacobi identity, Linearity in both arguments	Algebra with additional structure satisfying Lie properties
Boolean Algebra	A complemented distributive lattice that captures the essentials of set operations and logic, with operations AND (âˆ§âˆ§), OR (âˆ¨âˆ¨), and NOT (Â¬Â¬).	Associativity, Commutativity, Idempotence, Distributivity, Complements	Special ring-like structure tailored for logical operations
Topological Group	A group G*G* equipped with a topology Ï„*Ï„* such that the group operation and the taking of inverses are continuous functions.	All group properties, Continuity of group operations with respect to topology	Combination of group structure with topological properties