Paper II December 2015

Which of the following is/are not true?

(a) The set of negative integers is countable.
(b) The set of integers that are multiples of 7 is countable.
(c) The set of even integers is countable.
(d) The set of real numbers between 0 and 1/2 is countable.

(A) (a) and (c)
(B) (b) and (d)
(C) (b) only
(D) (d) only

(D) (d) only

a set is countable if it each element can be associated with a natural number e.g set of whole numbers like 0,1,2,3,....c

now such mapping is not possible to set of all real numbers between 0 to 1/2 hence it is not countable

Paper II July 2016

How many different equivalence relations with exactly three different equivalence classes are there on a set with five elements?

(A) 10
(B) 15
(C) 25
(D) 30

(C) 25

Step-1: Given number of equivalence classes with 5 elements with three elements in each class will be 1,2,2 (or) 2,1,2 (or) 2,2,1 and 3,1,1.

Step-2: The number of combinations for three equivalence classes are

2,2,1 chosen in $({}^5C_2 * {}^3C_2 * {}^1C_1)/2! = 15$

3,1,1 chosen in$({}^5C_2 * {}^3C_2 * {}^1C_1)/2! = 10$

Step-3: Total differential classes are 15+10

=25.

Paper II July 2016

Suppose that R1 and R2 are reflexive relations on a set A. Which of the following statements is correct?

(A) R1∩R2 is reflexive and R1UR2 is irreflexive.
(B) R1∩R2 is irreflexive and R1UR2 is reflexive.
(C) Both R1∩R2 and R1UR2 are reflexive.
(D) Both R1∩R2 and R1UR2 are irreflexive.

(C) Both R1∩R2 and R1UR2 are reflexive.

suppose we have A={a,b,c}

so reflexive relation must have R1={(a,a),(b,b),(c,c)} all diagonal elements+ any thing

similarly R2= {(a,a),(b,b),(c,c)} all diagonal elements+ any thing

so R1 intersection R2 must have {(a,a),(b,b),(c,c)}==>reflexive

and R1 union R2 must have {(a,a),(b,b),(c,c)}==>reflexive

Paper II August 2016 (Re-test)

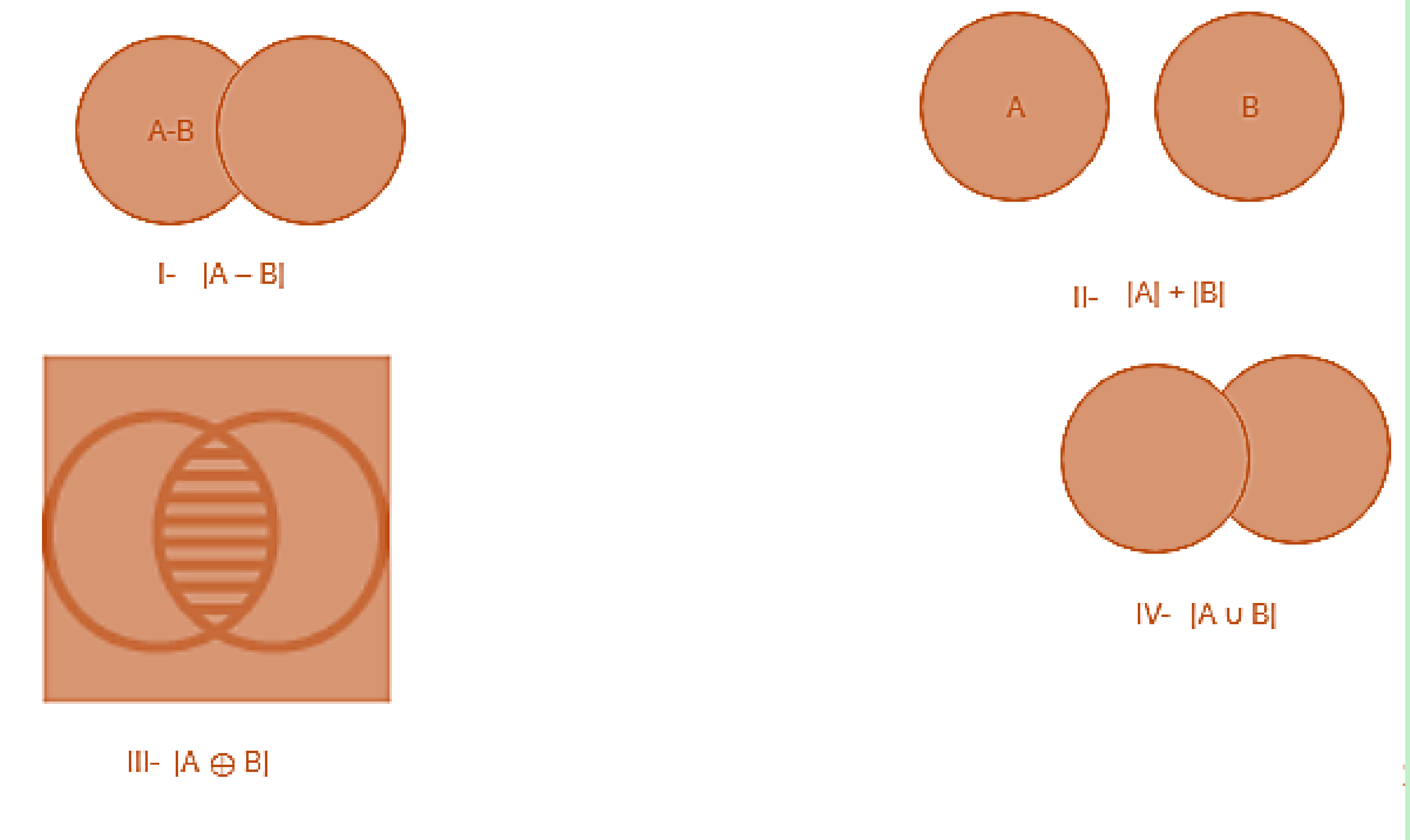
Let A and B be sets in a finite universal set U. Given the following:

$|A - B|, |A \oplus B|, |A| + |B|$ and $|A \cup B|$

Which of the following is in order of increasing size ?

A. $|A - B| < |A \oplus B| < |A| + |B| < |A \cup B|$
B. $|A \oplus B| < |A - B| < |A \cup B| < |A| + |B|$
C. $|A \oplus B| < |A| + |B| < |A - B| < |A \cup B|$
D. $|A - B| < |A \oplus B| < |A \cup B| < |A| + |B|$

Answer: D



Paper II January 2017

The functions mapping R into R are defined as:

$f(x) = x^3 - 4x, g(x) = \frac{1}{x^2+1}$ and $h(x) = x^4$.

Then find the value of the following composite functions: hog(x) and hogof(x)

A. $(x^2+1)^4$ and $\left[(x^3-4x)^2+1\right]^4$
B. $(x^2+1)^4$ and $\left[(x^3-4x)^2+1\right]^{-4}$
C. $(x^2+1)^{-4}$ and $\left[(x^2-4x)^2+1\right]^4$
D. $(x^2+1)^{-4}$ and $\left[(x^3-4x)^2+1\right]^{-4}$

Answer: 4

h(g(x))=h(1/(x^2 +1)) = (x^2 +1)^ -4

h(g(f(x)))= h(g(x^3 -4x)) = [(x^3-4x)^2+1]^
– 4

Paper II July 2018

If Ai = {−i, … −2,−1, 0, 1, 2, ... i}

then $\bigcup_{i=1}^{\infty} A_i$ is :

(1) Z
(2) Q
(3) R
(4) C

(1) Z

In this question, we have to define which type of numbers are given in the set Ai = {-i, …….-2, -1, 0, 1, 2, ……i}

Various type of number with their representation symbol are:

- **Rational numbers: A rational number is any real number that can be written as a fraction or in p/q form where p and q are integers. Rational numbers are denoted by Q.**
- **Complex number: Complex numbers are represented in the form of a + bi where a and b are real numbers and i stands for iota which has only two values either -1 or 1. Complex numbers are denoted by C.**
- **Real numbers: Real numbers include both rational and irrational numbers. These are denoted by R.**
- **Integers: Integers are like whole numbers, but they also include negative numbers. The range is from – infinity to + infinity. They are denoted by Z. So, set Ai = {−i, … −2, −1, 0, 1, 2, ….. i} Is the set of all integers which are denoted by Z.**

Paper II July 2018

Match the following in List - I and List - II, for a function f:

List– I	List- II
(a) $\forall x \forall y (f(x)=f(y) \rightarrow x=y)$	(i) Constant
(b) $\forall y \exists x (f(x)=y)$	(ii) Injective
(c) $\forall x\, f(x)=k$	(iii) Surjective

Code:

	(a)	(b)	(c)
(1)	(i)	(ii)	(iii)
(2)	(iii)	(ii)	(i)
(3)	(ii)	(i)	(iii)
(4)	(ii)	(iii)	(i)

(4) (ii) (iii) (i)

$\forall x \forall y \, (f(x) = f(y) \rightarrow x = y)$, that means if two functions maps same value then input of the functions should be same. This is definition of injective (or one-to-one) function. An injective function or injection or one-to-one function is a function that preserves distinctness: it never maps distinct elements of its domain to the same element of its codomain.

$\forall y \exists x \, (f(x) = y)$, that means for all y, there is a mapping function from x. This is definition of surjective (or onto) function. A function f from a set X to a set Y is surjective (or onto), or a surjection, if for every element y in the codomain Y of f there is at least one element x in the domain X of f such that f(x) = y.

$\forall x \, f(x) = k$, that means for all x, the output or mapping is only k and never changes. This is definition of constant function. A constant function is a function whose (output) value is the same for every input value. For example, the function is a constant function because the value of is 4 regardless of the input value.

Paper II July 2018

Which of the relations on {0, 1, 2, 3} is an equivalence relation?

(1) { (0, 0) (0, 2) (2, 0) (2, 2) (2, 3) (3, 2) (3, 3) }
(2) { (0, 0) (1, 1) (2, 2) (3, 3) }
(3) { (0, 0) (0, 1) (0, 2) (1, 0) (1, 1) (1, 2) (2, 0) }
(4) { (0, 0) (0, 2) (2, 3) (1, 1) (2, 2) }

(2) { (0, 0) (1, 1) (2, 2) (3, 3) }

Given set = {0,1,2, 3}

(1){(0, 0) (0, 2) (2, 0) (2, 2) (2, 3) (3, 2) (3, 3)} It is not equivalence relation. Here, element 1 is not matched to itself. It is not reflexive.

(2){(0, 0) (1, 1) (2, 2) (3, 3)} It is equivalence relation. As, it is satisfying property of all three: reflexive, symmetric and transitive.

(3){(0, 0) (0, 1) (0, 2) (1, 0) (1, 1) (1, 2) (2, 0)} This relation is neither reflexive nor symmetric so, it cannot be equivalence relation.

(4){(0, 0) (0, 2) (2, 3) (1, 1) (2, 2)} This is neither reflexive nor symmetric. So, it cannot be equivalence relation.

Paper II July 2018

Which of the following is an equivalence relation on the set of all functions from Z to Z?

(1) { (f, g) | f(x)−g(x)=1 ∀x e Z }
(2) { (f, g) | f(0)=g(0) or f (1)=g(1) }
(3) { (f, g) | f(0)=g(1) and f (1)=g(0) }
(4) { (f, g) | f(x)−g(x)=k for some k ∈ Z }

(4) { (f, g) | f(x)−g(x)=k for some k ∈ Z }

(1){ (f, g) | f (x) − g (x) = 1 x ∈ Z } It is not reflexive. As f(x) – f(x) = 0 it is not 1. It is also not transitive. So, it cannot be an equivalence relation.

(2){(f, g) | f (0) = g (0) or f (1) = g (1) } This relation is not transitive. Suppose f(x) = 0, g(x) = x and h(x) = 1. Here f is not related to h. Only we have a relation given between f and g, g and h. but not between f and h. So, it cannot be an equivalence relation.

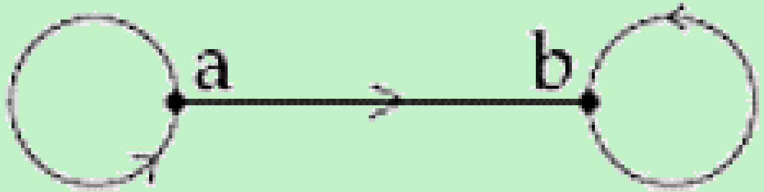
(3){(f, g) | f (0) = g (1) and f (1) = g (0) } It is not always true f(0) = f(1) for reflexive case. So, it is not reflexive. Hence, no equivalence relation.

(4){ (f, g) | f (x) − g (x) = k for some k ∈ Z } It is reflexive relation, consider constant as 0. It is also symmetric because the difference will be equal to a constant value. It is also transitive. So, it is an equivalence relation.

Paper II July 2018

Which of the following statements is true?

(1) (Z, ≤) is not totally ordered
(2) The set inclusion relation ⊆ is a partial ordering on the power set of a set S
(3) (Z, ≠) is a poset
(4) The directed graph is not a partial order



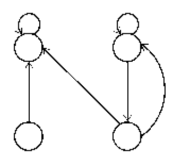
(2) The set inclusion relation ⊆ is a partial ordering on the power set of a set S

Option 1: (Z, ≤) is not totally ordered
This is false. As, ≤ is totally ordered. A partial ordered set with comparison is known as totally ordered set. Consider the set contains elements {1, 2, 3, 4}. This represents as totally ordered set. As 1<= 2, 2<=3, 3< = 4.

Option 2: The set inclusion relation ⊆ is a partial ordering on the power set of a set S If we consider the set S = {a, b} . So possibilities with this set are {Ø, a, b , ab}. As, subset relation is reflexive, antisymmetric and transitive. So, it is partially ordered. Because if a is subset of b, then b is superset of a.

Option 3: (Z, ≠) is a poset It is not partially ordered set. As, it follows symmetric property. Example if a is not equal to b , then b is also not equal to a. It can not be poset. Given statement is incorrect.

Option 4: The directed graph is not a partial order This statement is incorrect. This graph represents partial order. It can be represented as:



It is satisfying the property of partially ordered set. So, given directed graph is poset. It is reflexive, antisymmetric and transitive.

Paper II December 2018

A survey has been conducted on methods of commuter travel. Each respondent was asked to check Bus, Train and Automobile as a major method of travelling to work. More than one answer was permitted. The results reported were as follows:

Bus 30 people; Train 35 people; Automobile 100 people; Bus and Train 15 people; Bus and Automobile 15 people; Train and Automobile 20 people; and all the three methods 5 people. How many people completed the survey form?

(1) 120
(2) 165
(3) 160
(4) 115

(1) 120

A=Bus
B=Train
C=Automobile

$$n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(C\cap A)+n(A\cap B\cap C)$$
$$n(A\cup B\cup C)=30+35+100-15-20-15+5$$
$$n(A\cup B\cup C)=120$$

Paper II December 2018

The relation ≤ and > on a boolean algebra are defined as:
$x \leq y$ if and only if $x \vee y = y$
$x < y$ means $x \leq y$ but $x \neq y$
$x \geq y$ means $y \leq x$ and
$x > y$ means $y < x$

Considering the above definitions, which of the following is not true in the boolean algebra?
(i) If $x \leq y$ and $y \leq z$, then $x \leq z$
(ii) If $x \leq y$ and $y \leq x$, then $x = y$
(iii) If $x < y$ and $y < z$, then $x \leq y$
(iv) If $x < y$ and $y < z$, then $x < y$

Choose the correct answer from the code given below:
(1) (i) and (ii) only
(2) (ii) and (iii) only
(3) (iii) only
(4) (iv) only

(3) (iii) only

Consider all the options one by one:

1) If $x \leq y$ and $y \leq z$, then $x \leq z$ This is true by transitive property. As $x \leq y$ and $y \leq z$, then x should be less than or equal to z.

2) If $x \leq y$ and $y \leq x$, then x=y As x<=y, it means $x \vee y = y$ //given in question
Y<=x, means $x \vee y = x$
Here, $x \vee y = y = x$
So, this is true.

3) If $x < y$ and $y < z$, then $x \leq y$ In this, it says that x< y which means
x< =y where, x should not be equal to y.
But in this only first condition is given, second is not present.
So, it is false.

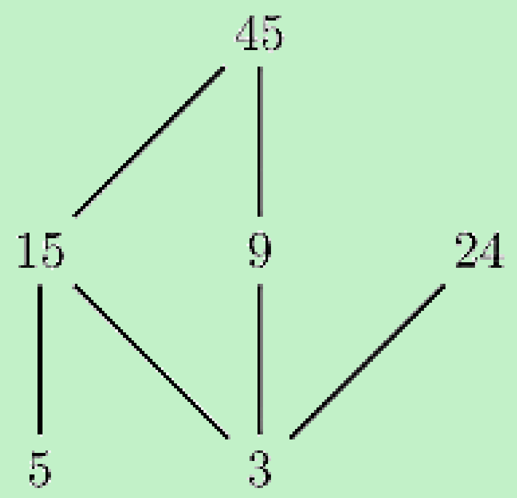
4) If $x < y$ and $y < z$, then $x < y$ This statement is true.
As x< y, then $x < y$ which is same in both the cases.

Paper II June 2019

Consider the poset ({3, 5, 9, 15, 24, 45}, |). Which of the following is correct for the given poset?

(a) There exists a greatest element and a least element.
(b) There exists a greatest element but not a least element.
(c) There exists a least element but not a greatest element.
(d) There does not exist a greatest element and a least element.

(d) There does not exist a greatest element and a least element.



There are two maximal elements 24 and 45.

There are two minimal elements 5 and 3.

So there is no greatest and least element.

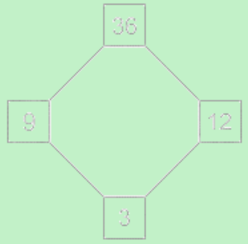
Paper II December 2019

What are the greatest lower bound (GLB) and the least upper bound (LUB) of the sets A={3, 9, 12} and B={1, 2, 4, 5, 10} if they exist in poset (z*,/)?

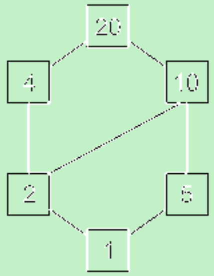
(1) A(GLB – 3, LUB – 36); B(GLB – 1, LUB – 20)
(2) A(GLB – 3, LUB – 12); B(GLB –1, LUB – 10)
(3) A(GLB –1, LUB – 36); B(GLB – 2, LUB – 20)
(4) A(GLB – 1, LUB – 12); B (GLB – 2, LUB – 10)

(1) A(GLB – 3, LUB – 36); B(GLB – 1, LUB – 20)

In poset (z+, /), / is the division relation. Hence, Hasse diagram for the given POSET
A = {3, 9, 12, 36 (added)}
A with GLB = 3 and LUB = 36



B = {1, 2, 4, 5, 10, 20 (added)}
B with GLB = 1 and LUB = 20



Paper II November 2020

Consider the following properties:

A Reflexive
B Antisymmetric
C Symmetric

Let A={a, b, c, d, e, f, g} and R={(a, a), (b, b), (c, d), (c, g), (d, g), (e, e) (f, f), (g,g)} be a relation on A. Which of the following property (properties) is (are) satisfied by the relation R

a) Only A
b) Only C
c) Both A and B
d) B and not A

d) B and not A

If a binary relation R over a set X relates every element of X to itself, it is said to be reflexive. Since (c,c) and (d,d) is not given in the given question, it is not reflexive.

A binary relation is antisymmetric if there is no pair of distinct elements of X each of which is related by R to the other. The given relation R is antisymmetric because for (c,d) (d,c) is not present in R. Similarly for (c,g) and (d,g).

A binary relation is a type of binary relation. An example is the relation "is equal to", because if a=b is true then b=a is also true. Here since (c,d) is pair in a given relation R for which (d,c) is not present in it. So. it violates the Symmetric property of the relation. Hence it is not symmetric.

Paper II November 2020

Match List I with List II

Let R1 = {(1, 1), (2, 2), (3, 3)} and R2 = {(1, 1), (1, 2), (1, 3), (1, 4)}

List I	List II
A. R1 ∪ R2	I. {(1, 1), (1,2), (1,3), (1,4), (2,2), (3,3)}
B. R1 - R2	II. {(1,1)}
C. R1 ∩ R2	III. {(1, 2), (1, 3), (1, 4)}
D. R2 – R1	IV. {(2, 2), (3,3)}

Choose the correct answer from the options given below

a) A-I B-II C-IV D-III
b) A-I B-IV C-III D-II
c) A-I B-III C-IV D-II
d) A-I B-IV C-II D-III

Paper II November 2020

Match List I with List II

Let R1 = {(1, 1), (2, 2), (3, 3)} and R2 = {(1, 1), (1, 2), (1, 3), (1, 4)}

List I	List II
A. R1 ∪ R2	I. {(1, 1), (1,2), (1,3), (1,4), (2,2), (3,3)}
B. R1 - R2	II. {(1,1)}
C. R1 ∩ R2	III. {(1, 2), (1, 3), (1, 4)}
D. R2 – R1	IV. {(2, 2), (3,3)}

Choose the correct answer from the options given below

a) A-I B-II C-IV D-III
b) A-I B-IV C-III D-II
c) A-I B-III C-IV D-II
d) A-I B-IV C-II D-III

Paper II October 2022

Let ϵ =0.0005, and Let R_ϵ be the relation {(x,y) belongs to R: |x-y|< ϵ). R_ϵ could be interpreted as the relation approximately equal. R_ϵ is

(A) Reflexive
(B) Symmetric
(C) transitive

Choose the correct answer from the options given below

a) (A) and (B) only true
b) (B) and (C) only true
c) (A) and (C) only true
d) (A), (B) and (C) true

a) (A) and (B) only true

A relation is said to be reflexive if every element in the set is related to itself. In this case, for any element x in R2, |x - x| = 0, which is less than ∈, meaning that (x, x) is in the relation R_ϵ . So R_ϵ is reflexive.

A relation is said to be symmetric if, for any ordered pair (x, y) in the relation, the ordered pair (y, x) is also in the relation. In this case, if (x, y) is in R_ϵ , meaning |x - y| < ∈, then |y - x| = |x - y| < ∈, meaning (y, x) is also in R_ϵ . So R_ϵ is symmetric.

However, Re is not transitive. Consider the following counterexample: let x = 0, y = 0.0004, and z = 0.0009. Then |x - y| = |0 - 0.0004| = 0.0004 < ∈, |y - z| = |0.0004 - 0.0009| = 0.0005 < ∈, but |x - z| = |0 - 0.0009| = 0.0009 ≠ |x - y| + |y - z|, meaning (x, z) is not in R_ϵ . Therefore, R_ϵ is reflexive and symmetric but not transitive.

Paper II March 2023

A relation "R" is defined on ordered pairs of integers as: (x,y) R (u,v) if x < u and y > v. Then R is

1. **Neither a partial order nor an equivalence relation**
2. **A partial order but not a total order**
3. **A total order**
4. **An equivalence relation**

1. Neither a partial order nor an equivalence relation

Because the relation is not reflexive x < u and y > v here no equal sign present which is a necessary condition for both partial order and equivalence relation, so R is neither a partial order nor an equivalence relation.

Paper II March 2023

Consider the following statements:

A. There exists a Boolean algebra with '5' elements.
B. Every element of Boolean algebra has unique complement.
C. If a Lattice 'L' is a Boolean algebra then 'L' is not relatively complemented.
D. The direct product of two Boolean Algebras is also a Boolean algebra.

Choose the correct answer about the four statements given above.

1. Only A and D are correct
2. Only B and D are correct
3 All statements are NOT correct
4. All statements are correct

2. Only B and D are correct

A. This statement is incorrect. The number of elements in a Boolean algebra must be a power of 2. Therefore, there is no Boolean algebra with exactly 5 elements.

B. This statement is correct. In a Boolean algebra, every element has a unique complement. The complement of an element is another element in the algebra that, when combined with the original element using the Boolean operations (AND, OR, NOT), yields the identity element (0 or 1, depending on the algebra).

C. This statement is incorrect. In a Boolean algebra, every element has a complement, and therefore, the lattice is relatively complemented.

D. This statement is correct. The direct product of two Boolean algebras is defined as the Cartesian product of their underlying sets equipped with component-wise Boolean operations. This construction preserves the algebraic properties of Boolean algebras, such as closure under the Boolean operations and the existence of a complement for each element. Therefore, the direct product of two Boolean algebras is also a Boolean algebra.

Paper II June 2023

Let N denote the set of all natural numbers and R be the relation on NXN defined by (a,b)R(c,d), if ad(b+c) = bc(a+d). Then R is

1. **Symmetric only**
2. **Reflexive only**
3. **Transitive only**
4. **An equivalence relation**

4. An equivalence relation

Symmetric: To check symmetry, we need to see if (c, d)R(a, b) holds as well:

$$\begin{aligned} & ad(b+c) = bc(a+d) \\ \Rightarrow\ & bc(a+d) = ad(b+c) \\ \Rightarrow\ & cb(d+a) = da(c+b) \\ \Rightarrow\ & \quad (c,d)R(a,b) \end{aligned}$$

Reflexive: To check reflexivity, we need to see if (a, b)R(a, b) for every (a, b) in N × N: ab(b+a) = ba(a+b)

Transitive: To check transitivity, assume that (a, b)R(c, d) and (c, d)R(e, f). This means:

$$\begin{aligned} & ad(b+c) = bc(a+d) \\ \Rightarrow\ & adb + adc = abc + bcd \\ \Rightarrow\ & abd - abc = bcd - acd \\ \Rightarrow\ & ab(d-c) = cd(b-a) \\ \Rightarrow\ & \frac{ab}{b-a} = \frac{cd}{d-c} \qquad (1) \end{aligned}$$

And let $(c,d)R(e,f)$.

Therefore,

$$\begin{aligned} & cf(d+e) = de(c+f) \\ \Rightarrow\ & cfd + cef = ced + edf \\ \Rightarrow\ & cfd - ced = edf - cef \\ \Rightarrow\ & cd(f-e) = ef(d-c) \\ \Rightarrow\ & \frac{cd}{d-c} = \frac{ef}{f-e} \qquad (2) \end{aligned}$$

From equations (1) and (2) we get,

$$\begin{aligned} & \frac{ab}{b-a} = \frac{ef}{f-e} \\ \Rightarrow\ & abf - abe = efb - efa \\ \Rightarrow\ & abf + efa = efb + abe \\ \Rightarrow\ & af(b+e) = be(a+f) \\ \Rightarrow\ & \quad (a,b)R(e,f) \end{aligned}$$

Paper II June 2023

If A= (4n + 2 | n is a natural number) and B={3n | n is a natural number). Which of the following is correct for A ∩ B?

1. **($12n^2$ + 6n | n is a natural number}**
2. **(24n - 12 | n is a natural number)**
3. **(60n + 30 | n is a natural number}**
4. **(12n - 6 | n is a natural number}**

4. (12n-6 | n is a natural number}

Given A = 4n + 2 | n is a natural number = 6,10,14,18,...
B = 3n| n is a natural number = 3,6,9,12,....
$(A \cap B)$=6,18,30,...
$= 6 + (n - 1)12, n \in N$
$= 12n - 6, n \in N$

Paper II June 2023

Let R = {x : x ∈ to N, x is multiple of 3 and x ≤100) and {x : x ∈ to N, x is multiple of 5 and x < 100). What is the number of elements in (R ∩ S)×(S ∩ R)?

1. 36
2. 33
3. 20
4. 6

1.36

R= {x| x ∈ N, x is a multiple of 3 and x ≤ 100} = {3, 6, 9, 12, ….., 99}
⇒ n (R) = 33.
S = {y| y ∈ N, x is a multiple of 5 and y ≤ 100} = {5, 10, 15, …., 100}
⇒ n (S) = 20.

∵ R ∩ S = {15, 30, 45, 60, 75, 90} = S ∩ R
⇒ (R ∩ S) × (R ∩ S) = {15, 30, 45, 60, 75, 90} × {15, 30, 45, 60, 75, 90}
⟹ n ((R ∩ S) × (R ∩ S)) = 36.

Paper II December 2023

Match List - I with List - II

	List - I		List - II
(A)	$\phi \cap \{\phi\} =$	(I)	ϕ
(B)	$\{\phi\} \cap \{\phi\} =$	(II)	$\{\phi\}$
(C)	$\{\phi, \{\phi\}\} - \phi =$	(III)	$\{\{\phi\}\}$
(D)	$\phi \cup \{\{\phi\}\} =$	(IV)	$\{\phi, \{\phi\}\}$

Choose the correct answer from the options given below :

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(2) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
(3) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
(4) (A)-(I), (B)-(II), (C)-(IV), (D)-(III)

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Match List - I with List - II

List - I		List - II	
(A)	$\phi \cap \{\phi\} =$	(I)	ϕ
(B)	$\{\phi\} \cap \{\phi\} =$	(II)	$\{\phi\}$
(C)	$\{\phi, \{\phi\}\} - \phi =$	(III)	$\{\{\phi\}\}$
(D)	$\phi \cup \{\{\phi\}\} =$	(IV)	$\{\phi, \{\phi\}\}$

Choose the correct answer from the options given below :

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(2) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
(3) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
(4) (A)-(I), (B)-(II), (C)-(IV), (D)-(III)