

1. Paper - II December - 2004

Q1. If $f(x) = x+1$ and $g(x) = x+3$ then $f \circ f \circ f \circ f$ is:

- (A) g (B) $g+1$
(C) g^4 (D) None of the above

Ans: B

Explanation:

If $f(x) = x+1$ and $g(x) = x+3$, then $f \circ g(x) = x+4$ and $g \circ f(x) = x+4$
 $f \circ f(x) = x+2$, $f \circ f \circ f(x) = x+3$ and $f \circ f \circ f \circ f(x) = x+4 = x+3+1 = g+1$

Q2. The following lists are the degrees of all the vertices of a graph:

- (i) 1, 2, 3, 4, 5 (ii) 3, 4, 5, 6, 7
(iii) 1, 4, 5, 8, 6 (iv) 3, 4, 5, 6

then, which of the above sequences are graphic?

- (A) (i) and (ii) (B) (iii) and (iv)
(C) (iii) and (ii) (D) (ii) and (iv)

Ans: B

Explanation:

Rest can't be graphs as the number of vertices with odd degree in a graph should be even.

In list (i), number of vertices with odd degree is 3.

In list (ii), number of vertices with odd degree is 3.

In list (iii), number of vertices with odd degree is 2.

In list (iv), number of vertices with odd degree is 2.

Q3 If I_m denotes the set of integers modulo m , then the following are fields with respect to the operations of addition modulo m and multiplication modulo m :

- (i) Z_{23} (ii) Z_{29}
(iii) Z_{31} (iv) Z_{33}

Then

- (A) (i) only (B) (i) and (ii)
only

(C) (i), (ii) and (iii) only
(iii) and (iv)

(D) (i), (ii),

Ans: C

Explanation:

Basically, a field is a thing where you can add, subtract, multiply and divide. It is a bit tricky to see that the first three examples (Z_{23} , Z_{29} , Z_{31}) are indeed fields. In fact, $Z_p Z_p$ happens to be a field always when p is prime, and this result follows from Fermat's little theorem.

But let us look at the fourth example. Assume you can divide the elements by 11, then you have

$$\begin{aligned} 3 &= (11/11) * 3 \\ &= (11 * 3) / 11 \\ &= 33 / 11 = 0 \end{aligned}$$

a contradiction. (The latter equality holds because $33 \equiv 033 \equiv 0$ modulo 3333.) A similar argument shows you that $Z_q Z_q$ cannot be a field if Q is any composite number.

Q4. Weighted graph:

- (A) Is a bi-directional graph
- (B) Is directed graph
- (C) Is graph in which number associated with arc
- (D) Eliminates table method

Ans: C

Explanation:

Weighted Graphs: A graph in which each carries a value is said to be a weighted graph. Weighted graphs are used to represent applications in which the value of connection between the vertices is important, not just existence of the connection. In graph theory which one of these two will be called a weighted graph ? A graph where vertices have some weights or values . A graph where edges have some weights or values . A graph where both edges and vertices have some weights or values A graph where neither edges nor vertices have any weights or values.

2. Paper-II June-2005

Q5. The transitive closure of a relation R on set A whose relation matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

is :-

(A) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

Ans: B

Q6. Consider the relation on the set of non-negative integers defined by $X \equiv Y$ if and only if:

(A) $x \bmod 3 = 3 \bmod y$

(B) $3 \bmod x \equiv 3$

$\bmod y$

(C) $x \bmod 3 = y \bmod 3$

(D) None of the above

Ans: C

Q7. Minimum number of individual shoes to be picked up from a dark room (containing 10 pair of shoes) if we have to get at least one proper pair:

(A) 2

(B) 20

(C) 11

(D) None of these

Ans: C

Explanation:

10 pair of shoes = 20 individual shoes Number of unpaired individual shoes = 10 so, on picking up 11th shoe, they will get at least one proper pair Hence, required Number = 11

11 individual shoes, for if you picked up only 10, you could possibly pick 10

left shoes and no right shoes, or vice-versa. But if so, the 11th one has to

match one of the other 10.

For your information:

If you pick then the probability of this many shoes, getting one matched pair is

1	0
2	$1/19 = .053$
3	$3/19 = .158$
4	$99/323 = .307$
5	$155/323 = .480$
6	$211/323 = .653$
7	$259/323 = .802$
8	$3815/4199 = .909$
9	$4071/4199 = .970$
10	$45933/46189 = .994$
11	1

If you pick 6 shoes out of the 20 you are more likely than not to have picked a matching pair.

Q8. Which of the following statement is false ?

- (A) Every tree is a bipartite graph (B) A tree contains a cycle
(C) A tree with n nodes contains n-1 edges (D) A tree is a connected graph

Ans: B

Explanation:

Because A tree cannot contain cycle.

- 1) In a tree every node except the root has exactly one parent.
- 2) A tree with n-nodes has exactly n-1 branches.
- 3) A tree is connected graph

3. Paper - II December - 2005

Q10. Consider the graph, which of the following is a valid topological sorting?

(A) ABCD

(B) BACD

(C) BADC

(D) ABDC

Ans: D

4. Paper - II June - 2006

Q11. Let $A = \{x \mid -1 < x < 1\} = B$. The function $f(x) = x/2$ from A to B is:

(A) injective

(B) subjective

(C) both injective and subjective

(D) neither injective nor subjective

Ans: C

5. Paper - II December - 2006

Q12. The number of edges in a complete graph with N vertices is equal to:

(A) $N(N-1)$

(B) $2N-1$

(C) $N-1$

(D) $N(N$

$-1)/2$

Ans: D

Explanation:

Number of edge in a complete graph = Number of ways of selecting two vertices out of n = $n(n-1)/2$.

Q13. If $(a^2 - b^2)$ is a prime number where a and $b \in \mathbb{N}$, then:

(A) $a^2 - b^2 = 3$

(B) $a^2 - b^2 = a$

$-b$

(C) $a^2 - b^2 = a + b$

(D) $a^2 - b^2 = 5$

Ans: C

Explanation:

Here, $a^2 - b^2 = \text{prime}$

$$\implies (a+b)(a-b) = \text{some_prime} \times 1$$

$$\implies a+b = \text{some_prime}; a-b \text{ must be } = 1$$

6. Paper - II December - 2007

Q14. A box contains six red balls and four green balls. Four balls are selected at random from the box. What is the probability that two of the selected balls are red and two are green?

(A) $3/7$

(B) $4/7$

(C) $5/7$

(D) $6/7$

Ans: A

Explanation:

$$\frac{{}^6C_2 {}^4C_2}{{}^{10}C_4}$$

4 balls are selected from 10 balls.

So, total number of ways to select 4 balls = ${}^{10}C_4$.

Number of ways of selecting 2 Red balls out of 6 red balls = 6C_2 .

Number of ways of selecting 2 Green balls out of 4 Green balls = 4C_2 .

Hence, required probability = ${}^6C_2 \times {}^4C_2 / {}^{10}C_4$.

$$= 15 \times 6 / 210$$

$$= 60 / 210.$$

$$= 3/7$$

Q15. The number of edges in a complete graph with ' n ' vertices is

equal to:

(A) $n(n-1)$

(B) $n(n-1)/2$

(C) n^2

(D) $2n-1$

Ans: B

Explanation:

Number of edge in a complete graph = Number of ways of selecting two vertices out of $n = n(n-1)/2$.

7. Paper - II June - 2008

Q16. The set of positive integers under the operation of ordinary multiplication is:

(A) not a monoid

(B) not a group

(C) a group

(D) an

Abelian group

Ans: D

Explanation:

A group is said to be abelian if it satisfies the following additional condition: (AS) Commutative: $a * b = b * a$ for all a, b , in G . The set of integers (positive, negative, and 0) under addition is an abelian group. The set of real numbers under multiplication is an abelian group.

Q17. In a set of 8 positive integers, there always exists a pair of numbers having the same remainder when divided by:

(A) 7

(B)

11

(C) 13

(D) 15

Ans: A

Explanation:

For example, the integers 2, 9, 16, all leave the same remainder when divided by 7. The special relationship between the numbers 2, 9, 16 with respect to the number 7 is indicated by saying these numbers are congruent to each other modulo 7, and writing. $16 \equiv 9 \equiv 2 \pmod{7}$
According to pigeonhole principle 7 can have 7 remainder values i.e.

0,1,2,3,4,5,6

so in 8 integers at least 1 remainder value will occur twice

8. Paper - II December - 2008

Q18. The graph $K_{3,4}$ has:

- (A) 3 edges (B) 4 edges
(C) 7 edges (D) 12 edges

Ans: D

Explanation:

A bipartite graph is a complete bipartite graph if every vertex in U is connected to every vertex in V. If U has n elements and V has m, then the resulting complete bipartite graph can be denoted by $K_{n,m}$ and the number of edges is given by $n*m$. The number of edges = $K_{3,4} = 3 * 4 = 12$

Q19. A relation R in $\{1,2,3,4,5,6\}$ is given by $\{(1,2),(2,3),(3,4),(4,4),(4,5)\}$. The relation is : (D-2008)

(A) Reflexive
(B) symmetric
(C) Transitive
(D) not reflexive, not symmetric and not transitive.

Ans: D

Explanation

Reflexive Relation : Let P is a binary relation on a set A. The relation P is said to be reflexive if $(a,a) \in P \implies (b,a) \in P$ i.e. $pb \implies bpa$.

Transitive Relation : Let P be a binary relation on a set A.

The relation P is said to be Transitive if for any 3 elements $a,b,c \in A$, $(a,b) \in P$ and $(b,c) \in P \implies (a,c) \in P$ i.e. apb and $bpc \implies apc$.

Hence, given relation R is not reflexive, not symmetric and not transitive.

9. Paper - II December - 2009

Q20. If she is my friend and you are her friend, then we are friends. Given this, the friend relationship in this context is

- (i) Commutative (ii) Transitive
- (iii) Implicative (iv) Equivalence

(A) (i) and (ii)

(B) (iii)

(C) (i), (ii), (iii) and (iv)

(D) None

of these

Ans: D

10. Paper - II June - 2010

Q21. R is a robot of M' means R can perform some of the tasks that otherwise M would do and R is unable to do anything else. Which of the following is the most appropriate representation to model this situation?

(A)

(B)

(C)

(D) None of these

Answer: (B)

Explanation:

M is the big set and R is the small set as 'R is a robot of M'

For example, 'Pigeon is a bird' where bird is big set and pigeon is the small set

As R cannot perform any other tasks compared to M, R set should be inside the M set

11. Paper - II December - 2010

Q22. The number of integers between 1 and 250 that are divisible by 2, 5 and 7 is

- (A) 2 (B) 3
(C) 5 (D) 8

Ans: B

Explanation:

LCM of 2, 5 and 7 = 70

Number of integers between 1 and 250 that are divisible by 2, 5 and 7 =

There are only 3 numbers (70, 140 and 210) which are divisible by 2, 5

Hence, Option (B) 3.

L.C.M of 2, 5, and 7 = $2 * 5 * 7 = 70$ Number divisible by 70 and lies between 1 to 250 are 70, 140, 210. Hence, number of such number's = 3

Q23. An undirected graph possesses an eulerian circuit if and only if it is connected and its vertices are

- (A) all of even degree (B) all of odd degree
(C) of any degree (D) even in number

Ans: A

Explanation:

An undirected graph possesses an eulerian circuit if and only if it is

connected and its vertices are All of even degree .

Q24. A partially ordered set is said to be a lattice if every two elements in the set have

(A) a unique least upper bound (B) a unique greatest lower bound

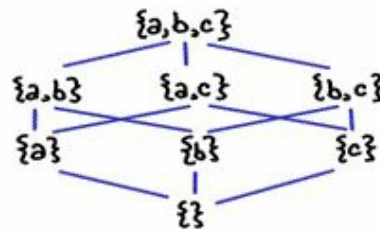
(C) both (A) and (B) (D) none of the above

Ans: C

Explanation:

A partially ordered set is said to be a lattice if every two elements in the set have

1. A unique least upper bound
2. A unique greatest lower bound



Q25. The minimum number of edges in a connected graph with 'n' vertices is equal to

(A) $n(n-1)$

(B) $n(n-1)/2$

(C) n^2

(D) $n-1$

Ans: D

Explanation:

The minimum number of edges in a connected graph with n vertex is n-1 i.e. Tree.

Q26. Consider the problem of connecting 19 lamps to a single electric outlet by using extension cords each of which has four outlets. The number of extension cords required is

(A) 4

(B) 5

(C) 6

(D) 7

Ans: C

Explanation:

first extension chord is connected to 4 ext. chords (which can connect 15 lamps +one slot free for another ext chord which can further connect 4 more lamps hence $15+4=19$ lamps)

4 chords can connect max 13 lamps

5 chords can connect max 16 lamps

12. Paper - II June - 2011

Q27. Any integer composed of 3^n identical digits divisible by

(A) 2^n (B) 3^n

(C) 5^n (D) 7^n

Ans: B

Explanation:

Let's take $n=1$

Now $3^1=3$ identical digits i.e.

111 which is not divisible by 2,5,7

And divisible by 3.

All other 3 digit numbers like 222,333,444 are multiple of 111 and hence of 3.

Now, for $n=2$, we get $3^2=9$. 11111111 is a multiple of 9. and similarly any 3^n digit number composed of only 1, is divisible by 3^n , and composed of any other number is also divisible by 3^n .

13. Paper - II December- 2011

Q28. Domain and Range of the function

$Y = -\sqrt{-2x + 3}$ is

(A) $x \geq 3/2, y \geq 0$

(B) $x > 3/2,$

$$y \leq 0$$

$$(C) \ x \geq 3/2, y \leq 0$$

$$(D) \ x \leq 3/2,$$

$$y \leq 0$$

Ans: D

Explanation:

$-2x+3 \geq 0$ (As Square Root cannot have negative value)

$-2x \geq -3$ implies $x \leq 3/2$

for $x=3/2$, $y=0$

for any value of x , $y < 0$

The possible values are $x \leq 3/2$, $y \leq 0$

Q29. Maximum number of edges in a n-Node undirected graph without self loop is

$$(A) \ n^2$$

$$(B) \ n(n$$

- 1)

$$(C) \ n(n + 1)$$

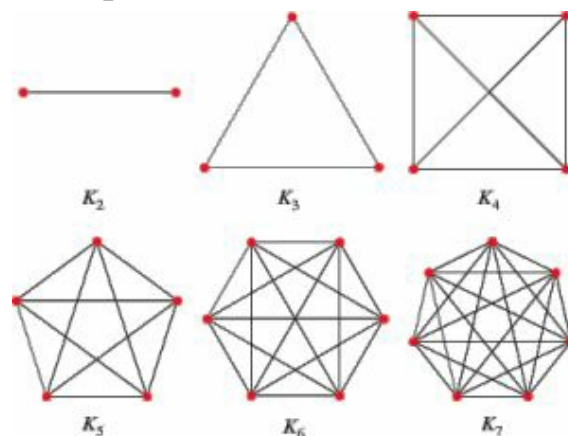
$$(D) \ n(n - 1)/2$$

Ans: D

Total no of pairs = Total no of edges (max) = $\frac{n(n-1)}{2}$

-1)2

Examples:



Explanation:-

if $n=2$ then no. of edges is 1

if $n=3$ then no. of edges are 3

if $n= 5$ then no. of edges are 10

all are true in option d

Q30. What is the probability of choosing correctly an unknown integer between 0 and 9 with 3 chances?

- (A) 963/1000 (B) 973/1000
(C) 983/1000 (D) 953/1000

Ans None of these

Here we can get right digit in 1st attempt, 2nd attempt or 3rd attempt

Right number - R

Wrong number - W

So, attempt could be $(R, _, _)$, $(W, R, _)$, (W, W, R)

So, probability is $1/10 + 9/100 + 81/1000 = 271/1000$

14. Paper - II June - 2012

Q31. The number of colours required to properly colour the vertices of every planer graph is

- (A) 2 (B) 3
(C) 4 (D) 5

Ans: D

According to the 4-color theorem states that the vertices of every planar graph can be coloured with at most 4 colours so that no two adjacent vertices receive the same colour.

15. Paper - III June - 2012

Q32. Let $Q(x, y)$ denote “ $x + y = 0$ ” and let there be two quantifications given as

(i) $\exists y \forall x Q(x, y)$

(ii) $\exists x \forall y Q(x, y)$

where x & y are real numbers. Then which of the following is valid?

(A) (i) is true & (ii) is false. (B) (i) is false & (ii) is true.

(C) (i) is false & (ii) is also false. (D) both (i) & (ii) are true.

Ans: A

Explanation:

If then for all x there exists some y $x+y=0$ (e.g take any number as x then some $y=-x$ will always be there)

In symbolic form it is written as $\exists y \forall x Q(x,y)$ so first is true

Second quantification is a typo here in the actual question it was

Now for some y all x are not here to hold $x+y=0$ for some y some x are there so second is not true

Hence Answer is A

Q33. How many relations are there on a set with n elements that are symmetric and a set with n elements that are reflexive and symmetric?

(A) $2^{n(n+1)/2}$ and $2^n \cdot 3^{n(n-1)/2}$ (B) $3^{n(n-1)/2}$ and $2^{n(n-1)}$

(C) $2^{n(n+1)/2}$ and $3^{n(n-1)/2}$ (D) $2^{n(n+1)/2}$ and $2^{n(n-1)/2}$

Ans: D

Q34. Let $a * H$ and $b * H$ be two cosets of H .

(i) Either $a * H$ and $b * H$ are disjoint

(ii) $a * H$ and $b * H$ are identical

Then,

(A) only (i) is true (B) only (ii) is true

(C) (i) or (ii) is true (D) (i) and (ii) is false

Ans: C

Explanation:

Let's Take a Example

$A = \{1, 2, 3\}$

$A \times A = \{ (1,1)(2,2)(3,3)(1,2)(2,1)(1,3)(3,1)(2,3)(3,2) \}$

Symmetric Relation:- A relation 'R' on set A is said to be symmetric if (xRy) then $(yRx) \forall x, y \in A$

--

$(1,1)(2,2)(3,3)$	$(1,2)(2,1)(1,3)(3,1)(2,3)(3,2)$
n	n^2-n

For n diagonal elements $(1,1)(2,2)(3,3)$ there are 2 choices for each. Either it can include in relation or it can't include in relation.

For remaining n^2-n elements according to definition of symmetric relation we can form pairs of $(1,2)(2,1)$ and $(1,3)(3,1)$ and $(2,3)(3,2)$. For each pair there are 2 choices. Either it can include in relation or it can't not include in relation.

Hence, Total Number of Symmetric Relation =

$$2^n \cdot 2^{(n^2-n)/2} = 2^{n + (n^2-n)/2} = 2^{(n^2+n)/2}$$

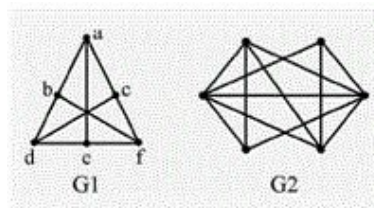
Now For Reflexive and Symmetric relation there are only one choices for diagonal elements $(1,1)(2,2)(3,3)$ and

for remaining. we can form pairs of $(1,2)(2,1)$ and $(1,3)(3,1)$ and $(2,3)(3,2)$. For each pair there are 2 choices. Either it can include in relation or it can't not include in relation.

Hence, Total Number of Reflexive and Symmetric Relation = $2^{(n^2+n)/2}$

Hence, Option **(D)** $2^{(n^2+n)/2}$ and $2^{(n^2-n)/2}$ is the correct choice.

Q35.



G1 and G2 are two graphs as shown:

(A) Both G1 and G2 are planar graphs.

(B) Both G1 and G2

are not planar graphs.

(C) G1 is planar and G2 is not planar graph.

(D) G1 is not

planar and G2 is planar graph.

Ans: D

Explanation:

Planar Graph: - A **planar graph** is a **graph** that can be embedded in the

plane, i.e., it can be drawn on the plane in such a way that its edges intersect only at their endpoints. In other words, it can be drawn in such a way that no edges cross each other.

Here only G_2 is planar graph.



Hence, Option (D) G_1 is not planar and G_2 is planar.

16. Paper - II December - 2012

Q36. The power set of the set $\{\phi\}$ is

(A) $\{\phi\}$

(B) $\{\phi,$

$\{\phi\}\}$

(C) $\{0\}$

(D) $\{0, \phi,$

$\{\phi\}\}$

Ans: B

Explanation:

If A is a finite set then set of all subset of A is called power set A denoted by $P(A)$

Here, $A = \{\Phi\}$

$P(A) = \{\Phi, \{\Phi\}\}$

Hence, Option(B) $\{\Phi, \{\Phi\}\}$ is the correct choice.

Q37. Suppose that someone starts with a chain letter. Each person who receives the letter is asked to send it on to 4 other people. Some people do this, while some do not send any letter. How many people have seen the letter, including the first person, if no one receives more than one letter and if the chain letter ends after there have been 100 people who read it but did not send it out ? Also find how many people

sent out the letter?

(A) 122 & 22

(B) 111 & 11

(C) 133 & 33

(D) 144 &

44

Ans: C

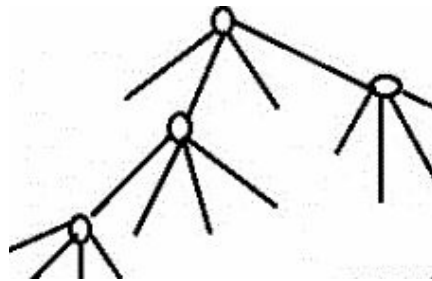
Explanation:

Either a person sends to 44 people or he doesn't send it. So, let x be the number of persons who sends to 44 people. 100 people didn't send any letter. Except 1 (the initial person) all others $(x+100)$ must have a sender. So, we can write

No. of receivers = $4 \times$ No. of senders + 1 (Initial person doesn't have a sender)
No. of receivers = $4 \times$ No. of senders + 1 (Initial person doesn't have a sender)

$$x+100=4x+1 \quad 3x=99 \quad x=33 \quad x+100=4x+1 \quad \square \quad 3x=99 \quad x=33$$

Let n represent the no of people who sent the letter out. At each stage a person sends 4 letters.



Let circles presents the people who sent the letter out and let the same be represented by n . So here 4 people sent the letter. So $n=4$ now count the branches that did not sent the letter out. Here those are 13. A simple reasoning shows it will be $3n+1$ people who did not send the letter out.

Trying the above branching diagram a couple of times tells us it is $3n+1$ people who read(received) the letter but did not send out one,

Hence,

$$3n + 1 = 100$$

$$n = 33.$$

Thus 33 people have sent out the letter and $100+33 = 133$ have seen the letter.

17. Paper - III December - 2012

Q38. Two graphs A and B are shown below:
Which one of the following statement is true ?

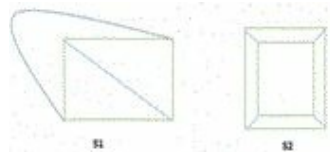
- (A) Both A and B are planar. (B) Neither A nor B is planar.
(C) A is planar and B is not. (D) B is planar and A is not.

Ans: A

Explanation:

Planar Graph:-A graph G is planar if it can be drawn in the plane in such a way that no two edges meet each other except at a vertex to which they are incident.

Here Both S1 and S2 are Planar graph .Here no edges cross each other.



Hence, Option (A) **Both A and B Planar.**

Q39. The number of distinct bracelets of five beads made up of red, blue, and green beads (two bracelets are indistinguishable if the rotation of one yield another) is,

- (A) 243 (B) 81
(C) 51 (D) 47

Ans: C

Q40. 58 lamps are to be connected to a single electric outlet by using an extension board each of which has four outlets. The number of extension boards needed to connect all the light is

(A) 29

(B) 28

(C) 20

(D) 19

Ans: D

Explanation:

$\text{Ceil}(58/4) = 15$

$\text{Ceil}(15/4) = 3$

$\text{Ceil}(4/4) = 1$

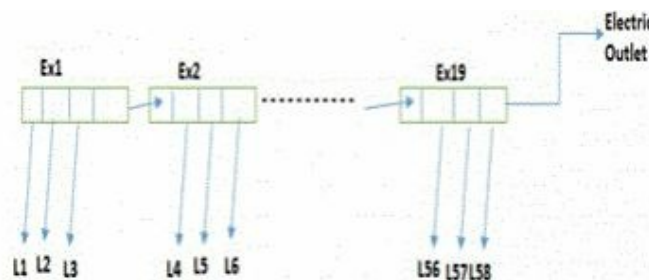
Here I'm getting 19

Or

Connect First 4 Lamps(L1 , L2 , L3 , L4) to extension 1 and take power from Extension 2 . In Extension 2 we can connect only 3 lamps because out of 4 one port is busy to supply power to extension 1.

Similarly, For other extensions we can connect only 3 lamps because one port is busy to supply power to other extension.

Connect Last Extension (Ex 19) to Electric outlet.



We can connect 4 Lamps to first extension

Number of Extension required for Remaining 54 lamps = $54/3 = 18$
Extension

Total Extension required to connect all 58 Lamps = $18 + 1 = 19$

Q41. The power set of $A \cup B$, where $A = \{2, 3, 5, 7\}$ and $B = \{2, 5, 8, 9\}$ is

(A) 256

(B) 64

(C) 16

(D) 4

Ans: B

Explanation:

$$A = \{2,3,5,7\} \quad B = \{2,5,8,9\}$$

$$A \cup B = \{2,3,5,7,8,9\}$$

Power set going to have 2^6 elements which is 64

Q42. The no. of ways to distribute n distinguishable objects into k distinguishable boxes, so that n_i objects are placed into box i , $i = 1, 2, \dots, k$ equals which of the following ?

(A)
$$\frac{n!}{n_1! + n_2! + \dots + n_k!}$$

(B)
$$\frac{n_1! + n_2! + \dots + n_k!}{n_1! n_2! n_3! \dots n_k!}$$

(C)
$$\frac{n!}{n_1! n_2! n_3! \dots n_k!}$$

(D)
$$\frac{n_1! n_2! \dots n_k!}{n_1! - n_2! - n_3! \dots - n_k!}$$

Ans: C

Explanation:

The number of ways to distribute n distinguishable objects into k distinct boxes so that n_i objects are placed in box i , $i=1, \dots, k$, and $n_1 + \dots + n_k = n$, is

$$\frac{n!}{n_1! n_2! n_3! \dots n_k!}$$

18. Paper - II June - 2013

Q43. A test contains 100 true/false questions. How many different ways can a student Answer the questions on the test, if the Answer may be left blank also.

(A) $^{100}P_2$

(B) $^{100}C_2$

(C) 2^{100}

(D) 3^{100}

Ans: D

if there are one 1 question it can be answered in 3 ways T F or blank(-) (i.e 3^1)

if there are one 2 questions it can be answered in 9 ways TT, TF ,FT ,FF, T- F- -F -T - - (i.e 3^2)

if there are one 100 questions it can be answered in 3^{100} WAYS

or

For every question we can leave it blank or Answer TRUE or Answer FALSE. So, for each question we have 3 options.

So, total ways of Answering the test is $3 \times 3 \times 3 \times \dots \times 100 \text{ times} = 3^{100}$

Q44. Which of the following connected simple graph has exactly one spanning tree?

(A) Complete graph

(B) Hamiltonian graph

(C) Euler graph

(D) None of the

above

Ans: D

Explanation:

When a graph has cycle, then it may or may not have the unique spanning tree.

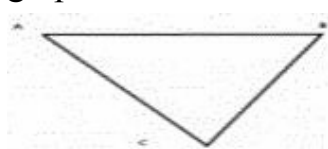
Each and every graph, which has been given in the list are cyclic graph.

Every connected graph has at least one spanning tree.

Let G be a connected graph.

If G has no cycles, then it is its own spanning tree.

If G has cycles, then on deleting one edge from each of the cycles, the graph remains connected and cycle free containing all the vertices of G.



The answer to first 3 options is false considering above graph.

Q45. The relation "divides" on a set of positive integers is.....

(A) Symmetric and transitive

(B) Anti symmetric and transitive

(C) Symmetric only

(D) Transitive only

Ans: B

Explanation:

The 'divide' operation is ant-symmetric because if a divides b does not necessarily implies that b divides a. If a divides b and b divides c then a divides c. So, it is transitive as well.

19. Paper - III June - 2013

Q46. A vertex cover of an undirected graph $G(V, E)$ is a subset $V_1 \subseteq V$ vertices such that

(A) Each pair of vertices in V_1 is connected by an edge

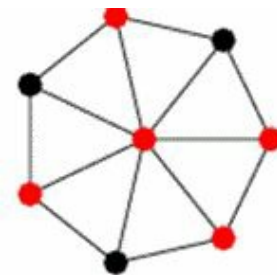
(B) If $(u, v) \in E$ then $u \in V_1$ and $v \in V_1$

(C) If $(u, v) \in E$ then $u \in V_1$ or $v \in V_1$

(D) All pairs of vertices in V_1 are not connected by an edge

Ans: C

A set of vertices such that each edge of the graph is incident to at least one vertex of the set.



- Set consists of rest of nodes is called **independent set**.
- Every planar graph has a vertex-cover of size at most $3n/4$

20. Paper - II June - 2013 (Re test)

Q47. Find the number of ways to paint 12 offices so that 3 of them will be green, 2 of them pink, 2 of them yellow and the rest ones white.

(A) 55,440 (B)

1,66,320

(C) 4.790E+08 (D)

39,91,680

Ans: B

Total number of ways 12 offices can be painted = $12!$

But 3 of them will be green, 2 of them pink, 2 of them yellow and 5 of them white.

Answer = $12! / (3! * 2! * 2! * 5!) = 166320$

Q48. Consider the following statements:

(i) A graph in which there is a unique path between every pair of vertices is a tree.

(ii) A connected graph with $e = v - 1$ is a tree.

(iii) A graph with $e = v - 1$ that has no circuit is a tree.

Which of the above statements is/are true?

(A) (i) & (iii) (B) (ii) &

(iii)

(C) (i) & (ii) (D) All of

the above

Ans: D

Explanation:

Let's go one by one,

(i) A graph in which there is a unique path between every pair of vertices is a tree.

\Rightarrow **This statement is true**, Because graph can have unique path only when it does not have cycle. And according to the definition of tree, its a graph without cycle. Hence this is a valid statement.

(ii) A connected graph with $e = v - 1$ is a tree.

\Rightarrow **This statement is true**. Not every graph with $e = v - 1$, will be a tree. But if the graph is connected hence its true.

(iii) A graph with $e = v - 1$ that has no circuit is a tree.

\Rightarrow **This statement is also true**, here he has not mentioned the connected thing, but mentioned that it has no circuit. It means that it's

connected.

21. Paper - III June - 2013 (Re test)

Q49. How many people must there be in a room before there is a 50% chance that two of them were born on the same day of the year?

(A) At least 23

(B) At least

183

(C) At least 366

(D) At least 730

Ans: A

Explanation:

At least 23 People

This is standard problem known as Birthday problem.

the probability, $P(1)$, that Person 1 does not share his/her birthday with previously analyzed people is 1, or 100%. Ignoring leap years for this analysis, the probability of 1 can also be written as $365/365$, for reasons that will become clear below.

For Event 2, the only previously analyzed people are Person 1. Assuming that birthdays are equally likely to happen on each of the 365 days of the year, the probability, $P(2)$, that Person 2 has a different birthday than Person 1 is $364/365$. This is because, if Person 2 was born on any of the other 364 days of the year, Persons 1 and 2 will not share the same birthday.

Similarly, if Person 3 is born on any of the 363 days of the year other than the birthdays of Persons 1 and 2, Person 3 will not share their birthday. This makes the probability $P(3) = 363/365$.

This analysis continues until Person 23 is reached, whose probability of not sharing his/her birthday with people analyzed before, $P(23)$, is $343/365$.

$P(A')$ is equal to the product of these individual probabilities:

--	--	--

$P(A') = \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365}$	
------------------------------------------------------------------------------------------------	--

The terms of equation (1) can be collected to arrive at:

$$P(A') = \left(\frac{1}{365}\right)^{23} \times (365 \times 364 \times 363 \times \cdots \times 343)$$

Evaluating equation (2) gives $P(A') \approx 0.492703$

22. Paper - II December - 2013

Q50. Let f and g be the functions from the set of integers to the set integers defined by

$$f(x) = 2x + 3 \text{ and } g(x) = 3x + 2$$

Then the composition of f and g and g and f is given as

- (A) $6x + 7, 6x + 11$ (B) $6x + 11, 6x + 7$
 (C) $5x + 5, 5x + 5$ (D) None of the above

Ans: A

Explanation:

$$f \circ g(x) = f(g(x)) = f(3x+2) = 2(3x+2) + 3 = 6x+7$$

$$g \circ f(x) = g(f(x)) = g(2x+3) = 3(2x+3) + 2 = 6x+11$$

Q51. If n and r are non-negative integers and $n \geq r$, then $p(n+1, r)$ equals to

- (A) $P(n,r)(n+1) / (n+1-r)$ (B) $P(n,r)(n+1) / (n-1+r)$
 (C) $p(n,r)(n-1) / (n+1-r)$ (D) $p(n,r)(n+1) / (n+1+r)$

Ans: A

Explanation:

$$p(n, r) = n! / (n-r)!$$

$$p(n+1, r) = (n+1)! / (n+1-r)!$$

$$= (n+1) n! / (n+1-r) (n-r)!$$

$$= P(n, r)(n+1) / (n+1-r)$$

Q52. A graph is non-planar if and only if it contains a sub-graph homeomorphic to

- (A) $K_{3,2}$ or K_5 (B) $K_{3,3}$ and K_6
(C) $K_{3,3}$ or K_5 (D) $K_{2,3}$ and K_5

Ans: C

Explanation:

Kuratowski's Theorem: A graph is non-planar if and only if it contains a sub-graph that is homeomorphic to either K_5 or $K_{3,3}$.

23. Paper - III December - 2013

Q53. A..... complete sub-graph and a subset of vertices of a graph $G=(V,E)$ are a clique and a vertex cover respectively.

- (A) minimal, maximal (B) minimal, minimal
(C) maximal, maximal (D) maximal, minimal

Ans: D

Explanation:

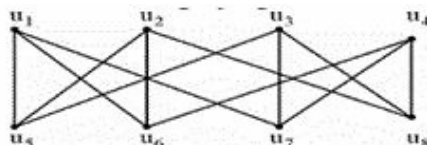
With choice should be different may be maximal ,maximal

In the [mathematical](#) area of [graph theory](#), a **clique** ([/'kli:k/](#) or [/'klɪk/](#)) is a subset of vertices of an [undirected graph](#) such that its [induced sub-graph](#) is [complete](#); that is, every two distinct vertices in the clique are adjacent.

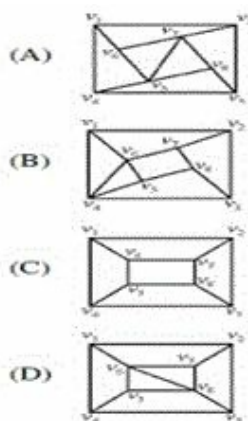
In the mathematical discipline of graph theory, a **vertex cover** (sometimes node **cover**) of a graph is a set of **vertices** such that each edge of the graph is incident to at least one **vertex** of the set.

24. Paper - II June - 2014

Q54. Consider the graph given below as :



Which one of the following graph is isomorphic to the above graph?



Ans: C

Explanation:

From given figure we can say that, every vertex has degree = 3, in all options but C, graph has at least one vertex with degree = 4 so I think option C would be the correct answer Because it has 12 edges which is equal to above graph edges equality of number edges is one of the criterion for showing isomorphism, also we have to take into account degree sequence

Q55. How many cards must be chosen from a deck to guarantee that at least

- two aces of two kinds are chosen.
- two aces are chosen.
- two cards of the same kind are chosen.
- two cards of two different kinds are chosen

(A) 50, 50, 14, 5
15, 7

(B) 51, 51,

(C) 52, 52, 14, 5
14, 5

(D) 51, 51,

Ans: A

Explanation:

Since we have to be sure (guarantee) consider the worst cases for all

i) two aces of same kind are chosen (first 48 cards without ace 49th will surely be one kind of an ace and 50th will be of another kind of ace

ii) two aces are chosen first 48 cards without ace then 49th and 50th will definitely be ace

iii) two cards of same kind (same number) all first 13 will be different i.e. of one colour either spade or heart or club or diamond now 14th will surely be of same number of any one of these 13 cards

iv) 2 cards of 2 different kinds let first 4 are of same kind say all ace or all 2 etc now 5th one will be definitely different kind

So answer is A 50 50 14 5

Q56. Consider a complete bipartite graph $K_{m,n}$. For which values of m and n does this, complete graph have a Hamilton circuit

(A) $m=3, n=2$
 $n=3$

(B) $m=2,$

(C) $m=n \geq 2$

(D) $m=n \geq 3$

Ans: C

Explanation:

$K_{m,n}$ has a Hamilton circuit if and only if $m=n \geq 2$

Circuit is path similar to a cycle that starts and ends at the same vertex. $K_{2,2}$ also contains Hamiltonian cut

Q57. A text is made up of the characters $\alpha, \beta, \gamma, \delta$ and σ with the probability 0.12, 0.40, 0.15, 0.08 and 0.25 respectively. The optimal coding technique will have the average length of

(A) 1.7

(B) 2.15

(C) 3.4

(D) 3.8

Ans: B

Explanation:

Alpha=.12 Beta=.40 Gamma=.15 Delta=.08 Sigma=.25

(All are given in question)

Path length with 4= Alpha, Delta. (.48+.32)

Path length with 3= gamma.(.45).

Path length with 2=sigma..(.50).

Path length with 1= beta(.40).

Average path length=2.15 .

25. Paper - III June - 2014

Q58. Given the following statements :

S1: The sub-graph isomorphism problem takes two graphs G1 and G2 and asks whether G1 is a sub- graph of G2.

S2: The set-partition problem takes as input a set S of numbers and asks whether the numbers can be partitioned into two sets A

$$\text{and } \bar{A} = S - A \text{ such that } \begin{matrix} \blacksquare & x = & \blacksquare & x \\ x \in A & & x \in \bar{A} \end{matrix}$$

Which of the following is true ?

- (A) S1 is NP problem and S2 is P problem.
- (B) S1 is NP problem and S2 is NP problem.
- (C) S1 is P problem and S2 is P problem.
- (D) S1 is P problem and S2 is NP problem.

Ans: B

26. Paper - II December - 2014

Q59. Consider a set $A = \{1, 2, 3, \dots, 1000\}$. How many members of A shall be divisible by 3 or by 5 or by both 3 and 5 ?

- (A) 533
- (B) 599

(C) 467 (D) 66

Ans: C

Explanation:

- Number of members divisible by 3 = $\{3, 6, 9, \dots, 999\}$ // Total 333 terms : easy way $= 999/3 = 333$
- Number of members divisible by 5 = $\{5, 10, \dots, 995, 1000\}$ // Total 200 terms: easy way $= 1000/5 = 200$
- Number of members divisible by 15 = $\{15, 30, \dots, 990\}$ // total 66
thus total number of members in set = $333 + 200 - 66 = 467$

Q60. A certain tree has two vertices of degree 4, one vertex of degree 3 and one vertex of degree 2. If the other vertices have degree 1, how many vertices are there in the graph?

- (A) 5 (B) $n - 3$
(C) 20 (D) 11

Ans: D

there are two vertices with degree 4, one vertex with degree 3, one vertex with degree 2 and let x vertex with degree 1.....therefore total number of vertices is $(x+4)$

Now by handshaking lemma we know..... sum of degrees of all vertices $= 2e$ (where e is number of edges)

therefore $8+3+2+x=2e$

$2e-x=13$(i)

Now in a tree number of edges is one less than number of vertices..

$e-x=3$(ii)

Solving (i) and (ii) we get..... $e=10$ and $x=7$

Now total number of vertices is $x+4$ which is 11...

Q61. Consider the Graph shown below :

This graph is a

(A) Complete Graph

(B) Bipartite

Graph

(C) Hamiltonian Graph

(D) All of the above

Ans: C

Explanation:

A. In complete graph, every vertex should have an edge to all other vertices.

- In given graph, there is no edge between D and B, A and C.
- Graph is not complete

B. If nodes in graph can be coloured with just two colours, it is bipartite.

- Suppose we coloured A with red and all neighbours B, D, F with blue.
- But neighbours B, F and D, F are connected. So they can't have same colour.
- It is not 2 colourable
- It is not bipartite

C. According to Dirac's theorem, in a graph of n nodes, if each node has degree greater than $n/2$, graph is Hamiltonian

- In given graph $n=6$
- All nodes have degree $=4$
- Hence graph is Hamiltonian
- Sample Hamiltonian path is ABCDEF

A is false because for a complete graph with n vertices, should be $n(n-1)/2$ edges. But here for 6 vertices, no. of edges $=13 \neq [6(6-1)/2] = 15$ for bipartite graph at least two partition should be there with (m,n) vertices, but there is no partition possible. so option B is false

Q62. A computer program selects an integer in the set $\{k : 1 \leq k \leq 10,00,000\}$ at random and prints out the result. This process is repeated 1 million times. What is the probability that the value $k = 1$ appears in the printout at least once ?

(A) 0.5

(B) 0.704

(C) 0.632121

(D) 0.68

Ans: C

Explanation:

Probability of $k=1$ is $1/1000000 = 10^{-6}$

Probability of $k \neq 1$ is $1 - 10^{-6} = 0.999$

Probability that $k=1$ is never printed in all 10^6 print outs =
 $0.999 \times 0.999 \times \dots \times 0.999$ (10^6 times)

$$= 0.999^{10^6}$$

Probability that 1 is printed at least once = $1 - \text{probability that 1 is never printed}$

$$= 1 - 0.999^{10^6}$$

$$= 0.6321$$

or

$$n=1000000, p=1/1000000$$

Then Mean:

$$m=np$$

$$m=1000000 \times (1/1000000)$$

$$m=1$$

As $m=1$, hence this question can be solved by **Poisson Distribution** method.

Therefore, required probability:

$$P = (e^{-m} m^r) / r!$$

$$P = 1 - P(0)$$

$$P = 1 - (e^{-1} 1^0) / 0!$$

$$P = 0.6321$$

Q63. If we define the functions f , g and h that map R into R by :
 $f(x) = x^4$, $g(x) = \sqrt{(x^2+1)}$, $h(x) = x^2+72$, then the value of the composite functions $ho(gof)$ and $(hog)of$ are given as

(A) $x^8 - 71$ and $x^8 - 71$ (B) $x^8 - 73$ and $x^8 - 73$

(C) $x^8 + 71$ and $x^8 + 71$ (D) $x^8 + 73$ and $x^8 + 73$

Ans: D

Explanation:

$$f = x^4$$

$$gof = \sqrt{(x^8 + 1)}$$

So $\text{gof} = x^8 + 1 + 72 = x^8 + 73$ //only option D holds this!

27. Paper - II June - 2015

Q64. How many strings of 5 digits have the property that the sum of their digits is 7 ?

- (A) 66 (B) 330
(C) 495 (D) 99

Ans: B

Let five digit A, B, C, D, E then $A + B + C + D + E = 7$

given $n = 7$, $r = 5$ we know that

$${}^{n+r-1}C_{r-1} = {}^{7+5-1}C_{5-1} = {}^{11}C_4 = 11 \times 10 \times 9 \times 8 / 1 \times 2 \times 3 \times 4 = 330$$

Explanation:

Let $n=7$ and $r=5$.

$${}^{n+r-1}C_{r-1} = 330.$$

Q65. Consider an experiment of tossing two fair dice, one black and one red. What is the probability that the number on the black die divides the number on red die ?

- (A) $22 / 36$ (B) $12 / 36$
(C) $14 / 36$ (D) $6 / 36$

Ans: C

Explanation:

If number on the black die divides the number on red die, respective numbers on the black die and red die must be any of the following

(1,1),(1,2),(1,3),(1,4),(1,5),(1,6), (2,2),(2,4),(2,6), (3,3),(3,6), (4,4), (5,5), (6,6)

Total number of possible outcomes = 14

The probability that the number of the black die divides the number of red die = $\frac{14}{36}$

Hence, Option(c) 14361436.

Or

1st die (1,2,3,4,5,6): 6 ways

2nd die (1,2,3,4,5,6): 6 ways

Thus On tossing two dice, total number of possible outcomes = $6 \times 6 = 36$

- If 1 comes in black dice, number on red dice divisible by 1 = $\{1,2,3,4,5,6\}$ // 6 success
- If 2 comes in black dice, number on red dice divisible by 2 = $\{2,4,6\}$ // 3 success
- If 3 comes in black dice, number on red dice divisible by 3 = $\{3,6\}$ // 2 success
- If 4 comes in black dice, number on red dice divisible by 2 = $\{4\}$ // 1 success
- If 5 comes in black dice, number on red dice divisible by 2 = $\{5\}$ // 1 success
- If 6 comes in black dice, number on red dice divisible by 2 = $\{6\}$ // 1 success

Total number of favourable outcomes = 14

Required probability = $14/36$

Q66. In how many ways can 15 indistinguishable fish be placed into 5 different ponds, so that each pond contains at least one fish ?

(A) 1001

(B) 3876

(C) 775

(D) 200

Ans: A

This given question is equivalent to distribution of 15 identical objects into 5 distinct boxes where each box contains at least one object.

Number of ways it can be done = number of positive integer solutions of

$$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 15 \quad x_1 + x_2 + x_3 + x_4 + x_5 = 15$$

$$\Rightarrow (15-1)C_4 = 14C_4 = 1001$$

Or

Given that distribution of 15 identical fish into 5 distinct ponds where

each box contains at least one fish.

Number of ways fishes placed in ponds

$$= (n-1r-1)(n-1r-1) = (144)(144) = 1001$$

28. Paper - II December - 2015

Q67. How many committees of five people can be chosen from 20 men and 12 women such that each committee contains at least three women?

(A) 75240

(B) 52492

(C) 41800

(D) 9900

Ans: B

Explanation:

We must choose at least 3 women, so we calculate the case of 3 women, 4 women and 5 women and by addition rule add the results.

$$\begin{aligned} {}^{12}C_3 \times {}^{20}C_2 + {}^{12}C_4 \times {}^{20}C_1 + {}^{12}C_5 \times {}^{20}C_0 &= \left(\frac{{}^{12}C_3 \times {}^{20}C_2}{3 \times 2 \times 1} \right) \times \left(\frac{{}^{20}C_1}{2 \times 1} \right) \\ &\quad + \left(\frac{{}^{12}C_4 \times {}^{20}C_1}{4 \times 3 \times 2 \times 1} \right) \times 20 \\ &\quad + \left(\frac{{}^{12}C_5 \times {}^{20}C_0}{5 \times 4 \times 3 \times 2 \times 1} \right) \times 1 \\ &= 220 \times 190 + 495 \times 20 + 792 \\ &= 52492 \end{aligned}$$

Q68. Which of the following statement(s) is/are false?

(a) A connected multi-graph has an Euler Circuit if and only if each of its vertices has even degree.

(b) A connected multi-graph has an Euler Path but not an Euler Circuit if and only if it has exactly two vertices of odd degree.

(c) A complete graph (K_n) has a Hamilton Circuit whenever $n \geq 3$

(d) A cycle over six vertices (C_6) is not a bipartite graph but a complete graph over 3 vertices is bipartite.

Codes:

(A) (a) only

(B) (b) and (c)

(C) (c) only

(D) (d) only

Ans: D

Explanation:

[Euler Circuits](#)

[Hamilton Circuit](#)

Bipartite Graph

Thus C & D also can be bipartite.

D is false

Q69. Which of the following is/are not true?

(a) The set of negative integers is countable.

(b) The set of integers that are multiples of 7 is countable.

(c) The set of even integers is countable.

(d) The set of real numbers between 0 and $\frac{1}{2}$ is countable.

(A) (a) and (c)

(B) (b) and

(d)

(C) (b) only

(D) (d) only

Ans: D

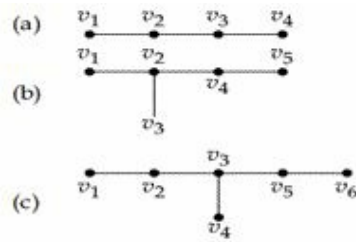
Explanation:

a set is countable if each element can be associated with a natural number e.g set of whole numbers like 0, 1, 2, 3,

now such mapping is not possible to set of all real numbers between 0 to $\frac{1}{2}$ hence it is not countable

for rest options such mapping is possible

Q70. A tree with n vertices is called graceful, if its vertices can be labelled with integers 1, 2, ..., n such that the absolute value of the difference of the labels of adjacent vertices are all different. Which of the following trees are graceful?



Codes:

(A) (a) and (b)

(B) (b) and

(c)

(C) (a) and (c)

(D) (a), (b)

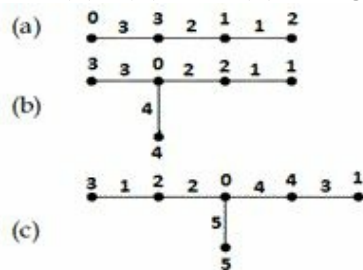
and (c)

Ans: D

Caterpillar tree: In graph theory, a caterpillar is a tree in which all the vertices are within distance 1 of a central path.

Theorem: All caterpillars are graceful.

So, (a), (b) and (c) are graceful.



Q71. Which of the following property/ies a Group G must hold, in order to be an Abelian group?

(a) The distributive property

(b) The commutative property

(c) The symmetric property

Codes:

(A) (a) and (b)

(B) (b) and

(c)

(C) (a) only

(D) (b) only

Ans: D

Explanation:

The commutative property must hold, in order to be an Abelian group. since group is given so all other property is inclusive.

Q72. A data cube C, has n dimensions, and each dimension has exactly p distinct values in the base cuboids. Assume that there are no concept hierarchies associated with the dimensions. What is the maximum number of cells possible in the data cube, C?

- (A) p^n (B) p
(C) $(2^n-1)p+1$ (D) $(p+1)^n$

Ans: D

Explanation:

(a) What is the maximum number of cells possible in the base cuboids?
 p^n .

This is the maximum number of distinct tuples that you can form with p distinct values per dimensions.

(b) What is the minimum number of cells possible in the base cuboids?
p.

You need at least p tuples to contain p distinct values per dimension. In this case no tuple shares any value on any dimension.

(c) What is the minimum number of cells possible in the data cube, C?
 $(2^n-1) \times p + 1$.

The minimum number of cells is when each cuboids contains only p cells, except for the apex, which contains a single cell.

(d) What is the maximum number of cells possible (including both base cells and aggregate cells) in the data cube, C?

$(p+1)^n$.

The argument is similar to that of part (a), but now we have p+1 because in addition to the p distinct values of each dimension we can also choose *.

Q73. Suppose that from given statistics, it is known that meningitis causes stiff neck 50% of the time, that the proportion of persons having meningitis is 1/50000, and that the proportion of people having stiff neck is 1/20. Then the percentage of people who had meningitis and complain about stiff neck is:

- (A) 0.01% (B) 0.02%
(C) 0.04% (D) 0.05%

Ans: B

Explanation:

The computation is based on the simplified Bayes' formula.

$$P\{B|A\} = (P\{A|B\} \cdot P\{B\}) / P\{A\}.$$

$P\{M|S\}$ = probability that a person had meningitis, conditioned by the existence of stiff neck.

$P\{S|M\}$ = probability that a person complains about stiff neck, conditioned by the existence of meningitis. = 50%=1/2

$P\{S\}$ = proportion of people who complain about stiff neck. = 1/20

$P\{M\}$ = proportion of people who had meningitis. = 1/50,000

Then:

$$P\{M|S\} = (P\{S|M\} \cdot P\{M\}) / P\{S\} = (1/2 \times 1/50,000) / 1/20 = 0.0002 = 0.02\%$$

Q74. Consider the graph given below:

The two distinct sets of vertices, which make the graph bipartite are:

(A) $(v_1, v_4, v_6); (v_2, v_3, v_5, v_7, v_8)$ (B) $(v_1, v_7, v_8);$

(v_2, v_3, v_5, v_6)

(C) $(v_1, v_4, v_6, v_7); (v_2, v_3, v_5, v_8)$ (D) $(v_1, v_4, v_6,$

$v_7, v_8); (v_2, v_3, v_5)$

Ans: C

Explanation:

A simple graph $G=(V,E)$ is called bipartite if its vertex set can be partitioned into two disjoint subsets $V=V_1 \cup V_2$, such that every edge has the form $e=(a,b)$ where $a \in V_1$ and $b \in V_2$.

Bipartite graphs are equivalent to two-colorable graphs.

1. Assign Red colour to the source vertex (putting into set V_1).
2. Colour all the neighbours with Black colour (putting into set V_2).
3. Colour all neighbour's neighbour with Red colour (putting into set V_1).

4. This way, assign colour to all vertices such that it satisfies all the constraints of m way colouring problem where $m = 2$.

5. While assigning colors, if we find a neighbour which is coloured with same colour as current vertex, then the graph cannot be coloured with 2 colors (i.e., graph is not Bipartite).

So Answer is option (C).

(v_1, v_4, v_6, v_7) put these vertex with red colour

(v_2, v_3, v_5, v_8) put these vertex with blue colour

et Between any two vertex there is direct no edge.

or

as we know a graph is bipartite if it can be divided into 2 set of vertices such that each edge in graph joins vertex of one part to vertex of another

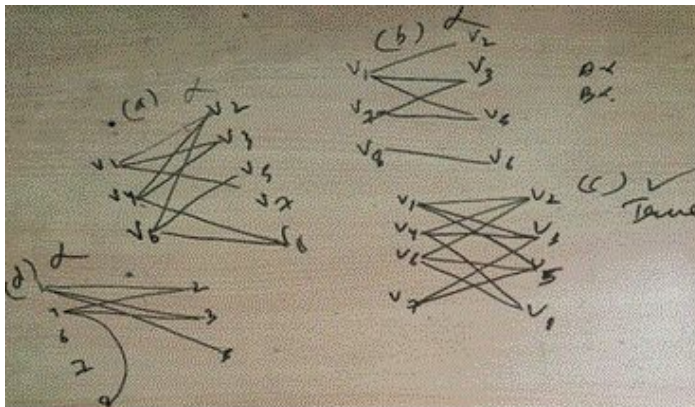
let $v_1v_2=a$, $v_1v_3=b$, $v_1v_5=c$, $v_2v_4=d$, $v_2v_6=e$, $v_3v_4=f$, $v_3v_7=g$, $v_4v_8=h$, $v_5v_6=i$, $v_5v_7=j$, $v_6v_8=k$

option C is the answer since

edge $a, b, c, d, e, f, g, h, i, j, k$ will join vertex of left hand side to r.h.s

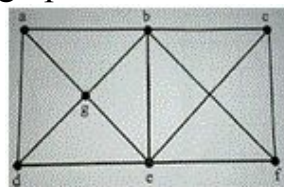
$v_1, v_4, v_6, v_7, v_2, v_3, v_5, v_8$

Or



29. Paper - II July - 2016

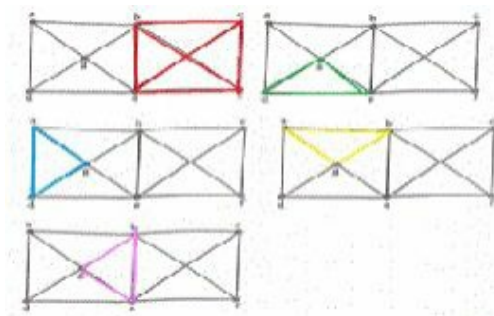
Q75. A clique in a simple undirected graph is a complete sub-graph that is not contained in any larger complete sub-graph. How many cliques are there in the graph shown below?



- (A) 2 (B) 4
(C) 5 (D) 6

Answer: C

Complete sub-graph means each vertex should be connected with all other vertices in the sub-graph



1. (a, d, g)

2. (b, d, g)
3. (d, e, g)
4. (b, e, g)
5. (b, c, e, f)

Q76. How many different equivalence relations with exactly three different equivalence classes are there on a set with five elements?

- (A) 10 (B) 15
(C) 25 (D) 30

Answer: C

Explanation:

$${}^5P_{(3,1,1)} = \frac{{}^5C_3 \times {}^2C_1 \times {}^1C_1}{2!} = \frac{10 \times 2 \times 1}{2} = 10$$

$${}^5P_{(2,2,1)} = \frac{{}^5C_2 \times {}^3C_2 \times {}^1C_1}{2!} = \frac{10 \times 3 \times 1}{2} = 15$$

$$= 10 + 15 = 25$$

Q77. The number of different spanning trees in complete graph, K_4 and bipartite graph $K_{2,2}$ have and respectively.

- (A) 14, 14 (B) 16, 14
(C) 16, 4 (D) 14, 4

Answer: C

The number of different spanning trees in complete graph with n vertices

$$= n(n-2)n(n-2) = \text{here } n=4 \text{ so } 16$$

and for $K_{2,2}$ 4 edges are there so 4 spanning tree are there.

Explanation:

$K_4 = n^{n-2} = 4^{4-2} = 4^2 = 16$	$K_{2,2} = 4$
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- Q78. Suppose that R_1 and R_2 are reflexive relations on a set A . Which of the following statements is correct?
- (A) $R_1 \cap R_2$ is reflexive and $R_1 \cup R_2$ is irreflexive.
 - (B) $R_1 \cap R_2$ is irreflexive and $R_1 \cup R_2$ is reflexive.
 - (C) Both $R_1 \cap R_2$ and $R_1 \cup R_2$ are reflexive.

(D) Both $R_1 \cap R_2$ and $R_1 \cup R_2$ are irreflexive.

Answer: C

Explanation:

A **relation** R on a set A is called **reflexive** if and only if $\langle a, a \rangle \in R$ for every element a of A

because R_1 and R_2 are reflexive relations So Both $R_1 \cap R_2$ and $R_1 \cup R_2$ are reflexive.

Q79. There are three cards in a box. Both sides of one card are black, both sides of one card are red, and the third card has one black side and one red side. We pick a card at random and observe only one side.

What is the probability that the opposite side is the same colour as the one side we observed?

(A) $3/4$

(B) $2/3$

(C) $1/2$

(D) $1/3$

Answer: B

3 cards BB RR BR

BB RR will have same colour on both sides

so favourable cases = 2 total cases = 3

answer is $2/3$

30. Paper-II Jan-2017

Q80. Consider a sequence F_0 defined as:

$$F_0(0) = 1, \quad F_0(1) = 1$$

$$F_0(n) = \frac{10 * F_0(n-1) + 100}{F_0(n-2)} \text{ for } n \geq 2$$

Then what shall be the set of values of the sequence F_0 ?

(A) (1, 110, 1200)

(B) (1, 110, 600, 1200)

(C) (1, 2, 55, 110, 600, 1200)

(D) (1, 55, 110, 600, 1200)

Ans A

$$F_{00}(2) = (10 \cdot F_{00}(1) + 100) / F_{00}(0) = (10 \cdot 1 + 100) / 1 = 110$$

$$F_{00}(3) = 1200$$

Q81. Match the following:

List-I

List-II

- a. Absurd truth.
b. Ambiguous
c. Axiom proof.
d. Conjecture wisdom.
- i. Clearly impossible being contrary to some evident truth.
ii. Capable of more than one interpretation or meaning.
iii. An assertion that is accepted and used without a proof.
iv. An opinion Preferably based on some experience or wisdom.

Codes:

a b c d

(A) i ii iii iv

(B) i iii iv ii

(C) ii iii iv i

(D) ii i iii iv

Ans A

- Absurd i. Clearly impossible being contrary to some evident truth.
Ambiguous ii. Capable of more than one interpretation or meaning.
Axiom iii. An assertion that is accepted and used without a proof.
Conjecture iv. An opinion Preferably based on some experience or wisdom.

Q82. The functions mapping R into R are defined as:

$$f(x) = x^3 - 4x, g(x) = 1/(x^2 + 1) \text{ and } h(x) = x^4$$

Then find the value of the following composite functions:

hog(x) and hogof(x)

(A) $(x^2 + 1)^4$ and $[(x^3 - 4x)^2 + 1]^4$

(B) $(x^2 + 1)^4$ and $[(x^3 - 4x)^2 + 1]^{-4}$

(C) $(x^2 + 1)^{-4}$ and $[(x^3 - 4x)^2 + 1]^4$

(D) $(x^2 + 1)^{-4}$ and $[(x^3 - 4x)^2 + 1]^{-4}$

Ans D

$$h(g(x)) = h(1/(x^2 + 1)) = (x^2 + 1)^{-4}$$

$$h(g(f(x))) = h(g(x^3 - 4x)) = [(x^3 - 4x)^2 + 1]^2 - 4$$

Q83. How many multiples of 6 are there between the following pairs of numbers?

0 and 100 and -6 and 34

- (A) 16 and 6
- (B) 17 and 6
- (C) 17 and 7
- (D) 16 and 7

Ans C

multiples of 6 are

$$\text{between 0 and 100} = 100/6 - 1/6 = 16 - (-1) = 17$$

$$\text{between -6 and 34} = 34/6 - -7/6 = 5 - (-2) = 7$$

Q84. Consider a Hamiltonian Graph G with no loops or parallel edges and with $|V(G)| = n \geq 3$. Then which of the following is true?

- (A) $\deg(v) \geq n/2$ for each vertex v.
- (B) $|E(G)| \geq 1/2(n-1)(n-2) + 2$
- (C) $\deg(v) + \deg(w) \geq n$ whenever v and w are not connected by an edge.
- (D) All of the above

Ans D

(A) is true: If $G = (V, E)$ has $n \geq 3$ vertices and every vertex has degree $\geq n/2$ then G has a Hamilton circuit. it is a proved theorem. (B) is also true.