June 2014 Paper II

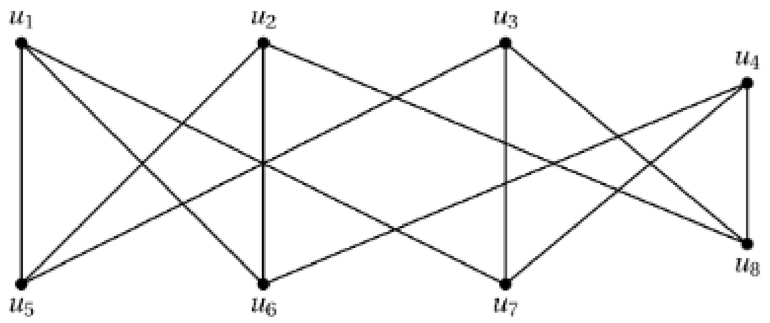
Consider a complete bipartite graph km,n. For which values of m and n does this, complete graph have a Hamilton circuit

(A) m=3, n=2
(B) m=2, n=3
(C) m=n≥2
(D) m=n≥3

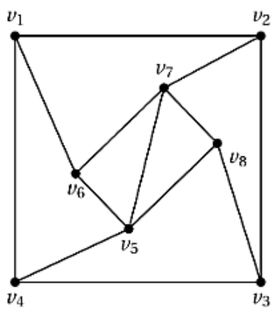
(C) m=n≥2

Hamiltonian circuit: In this circuit, every vertex is visited only once. It must start and end at the same vertex. In complete bipartite graph Km, n, when m = n, then in that case, it has a Hamiltonian circuit.

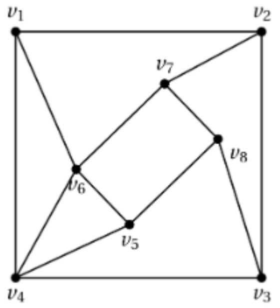
Consider the graph given below as :



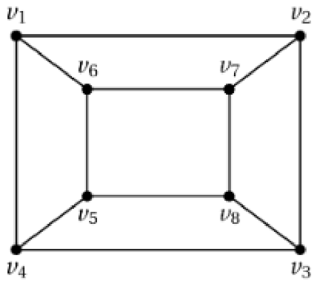
Which one of the following graph is isomorphic to the above graph?



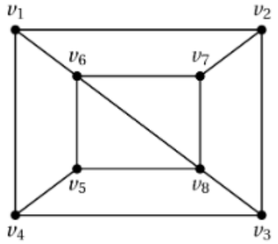
A.



B.



C.



D.

Answer: C

From given figure we can say that, every vertex has degree = 3,

in all options but C, graph has at least one vertex with degree = 4

Paper II December 2014

A certain tree has two vertices of degree 4, one vertex of degree 3 and one vertex of degree 2. If the other vertices have degree 1, how many vertices are there in the graph?

(A) 5
(B) n – 3
(C) 20
(D) 11

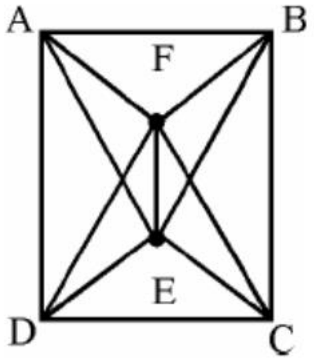
(D) 11

There are 2 vertices of degree 4, 1 vcertex of degree 3, 1 vcertex of degree 2 and vertex of degree one is unknown.

Let's assume k be the nof vertex of degree one.
Total vertex = 2 + 1 + 1 + k = k + 4.
Number of edges = vertex - 1 i.e. k + 4 - 1 = k + 3.
Now apply handshaking lemma(For more information on handshaking
2 * 4 + 1 * 3 + 1 * 2 + 1 * K = 2 * (No of edges)
i.e. 13 + k = 2 * (k + 3) k = 7.
Total vertex = 7 + 4 = 11.

Paper II December 2014

Consider the Graph shown below :



This graph is a ....

(A) Complete Graph
(B) Bipartite Graph
(C) Hamiltonian Graph
(D) All of the above

(C) Hamiltonian Graph

A. In complete graph, every vertex should have an edge to all other vertices.
In given graph, there is no edge between D and B, A and C.
Graph is not complete

B. If nodes in graph can be colored with just two colors, it is bipartitie.
Suppose we colored A with red and all neighbours B, D, F with blue.
But neighbours B, F and D,F are connected. So they cant have same color.
It is not 2 colorable
It is not bipartite

C. According to Dirac's theorem, in a graph of n nodes, if each node has degree greater than n/2, graph is Hamiltonian
In given graph n =6
All nodes have degree =4
Hence graph is Hamiltonian
Sample Hamiltonian path is ABCDEF

Paper II June 2015

Consider a Hamiltonian Graph (G) with no loops and parallel edges. Which of the following is true with respect to this Graph (G) ?

(a) $deg(v) \geq n/2$ for each vertex of G
(b) $|E(G)| \geq 1/2\ (n-1)(n-2)+2$ edges
(c) $deg(v) + deg(w) \geq n$ for every v and w not connected by an edge

(A) (a) and (b)
(B) (b) and (c)
(C) (a) and (c)
(D) (a), (b) and (c)

(C) (a) and (c)

In an Hamiltonian Graph (G) with no loops and parallel edges:
According to Dirac's theorem in a n vertex graph, deg (v) ≥ n / 2 for each vertex of G.
According to Ore's theorem deg (v) + deg (w) ≥ n for every n and v not connected by an edge is sufficient condition for a graph to be Hamiltonian.
If |E(G)| ≥ 1 / 2 * [(n - 1) (n - 2)] then graph is connected but it doesn't guaranteed to be Hamiltonian Graph.
(a) and (c) is correct regarding to Hamiltonian Graph.

Paper II December 2015

Which of the following statement(s) is/are false?

(a) A connected multigraph has an Euler Circuit if and only if each of its vertices has even degree.
(b) A connected multigraph has an Euler Path but not an Euler Circuit if and only if it has exactly two vertices of odd degree.
(c) A complete graph (Kn) has a Hamilton Circuit whenever n≥3
(d) A cycle over six vertices (C6) is not a bipartite graph but a complete graph over 3 vertices is bipartite.

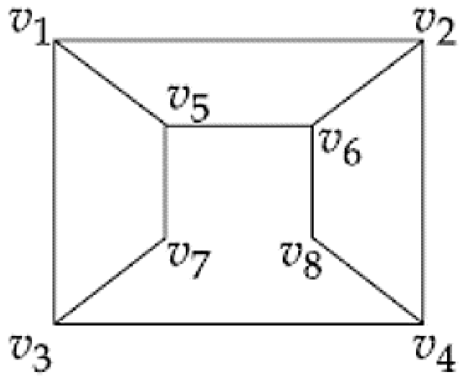
Codes:
(A) (a) only
(B) (b) and (c)
(C) (c) only
(D) (d) only

(D) (d) only

- **An Euler circuit of a graph G is a simple circuit that contains every edge of G.**
- **A connected multigraph has an Euler circuit if and only if each of its vertices has even degree.**
- **A connected multigraph has an Euler path but not an Euler circuit if and only if it has exactly two vertices of odd degree.**
- **A complete graph Kn has a Hamilton circuit for n≥3.**
- **Cycle graphs with an even number of vertices are bipartite.**
- **Thus C 8 also can be bipartite.**

Paper II December 2015

Consider the graph given below:



The two distinct sets of vertices, which make the graph bipartite are:

(A) (v1, v4, v6); (v2, v3, v5, v7, v8)
(B) (v1, v7, v8); (v2, v3, v5, v6)
(C) (v1, v4, v6, v7); (v2, v3, v5, v8)
(D) (v1, v4, v6, v7, v8); (v2, v3, v5)

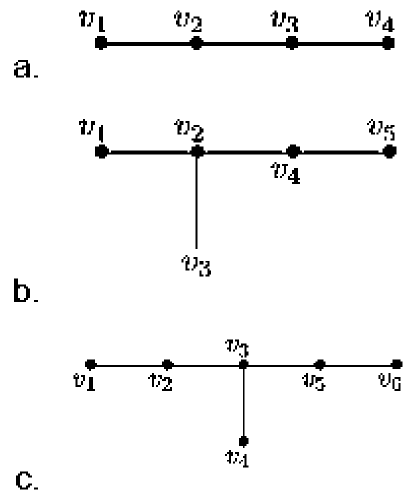
(C) (v1, v4, v6, v7); (v2, v3, v5, v8)

A simple graph G=(V,E) is called bipartite if its vertex set can be partitioned into two disjoint subsets $V=V1 \cup V2$, such that every edge has the form e=(a,b) where $a \in V1$ and $b \in V2$.

Bipartite graphs are equivalent to two-colorable graphs.

1. **Assign Red color to the source vertex (putting into set V1).**
2. **Color all the neighbours with Black color (putting into set V2).**
3. **Color all neighbour's neighbour with Red color (putting into set V1).**
4. **This way, assign color to all vertices such that it satisfies all the constraints of m way coloring problem where m = 2.**
5. **While assigning colors, if we find a neighbour which is colored with same color as current vertex, then the graph cannot be colored with 2 colors (ie., graph is not Bipartite).**

A tree with n vertices is called graceful, if its vertices can be labelled with integers 1, 2, ...,n such that the absolute value of the difference of the labels of adjacent vertices are all different. Which of the following trees are graceful?

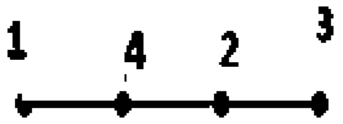


Codes:

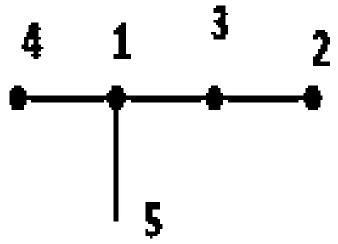
(A) (a) and (b)
(B) (b) and (c)
(C) (a) and (c)
(D) (a), (b) and (c)

(D) (a), (b) and (c)

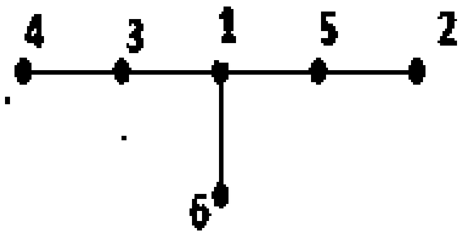
(a)



(b)



(c)



Paper II December 2015

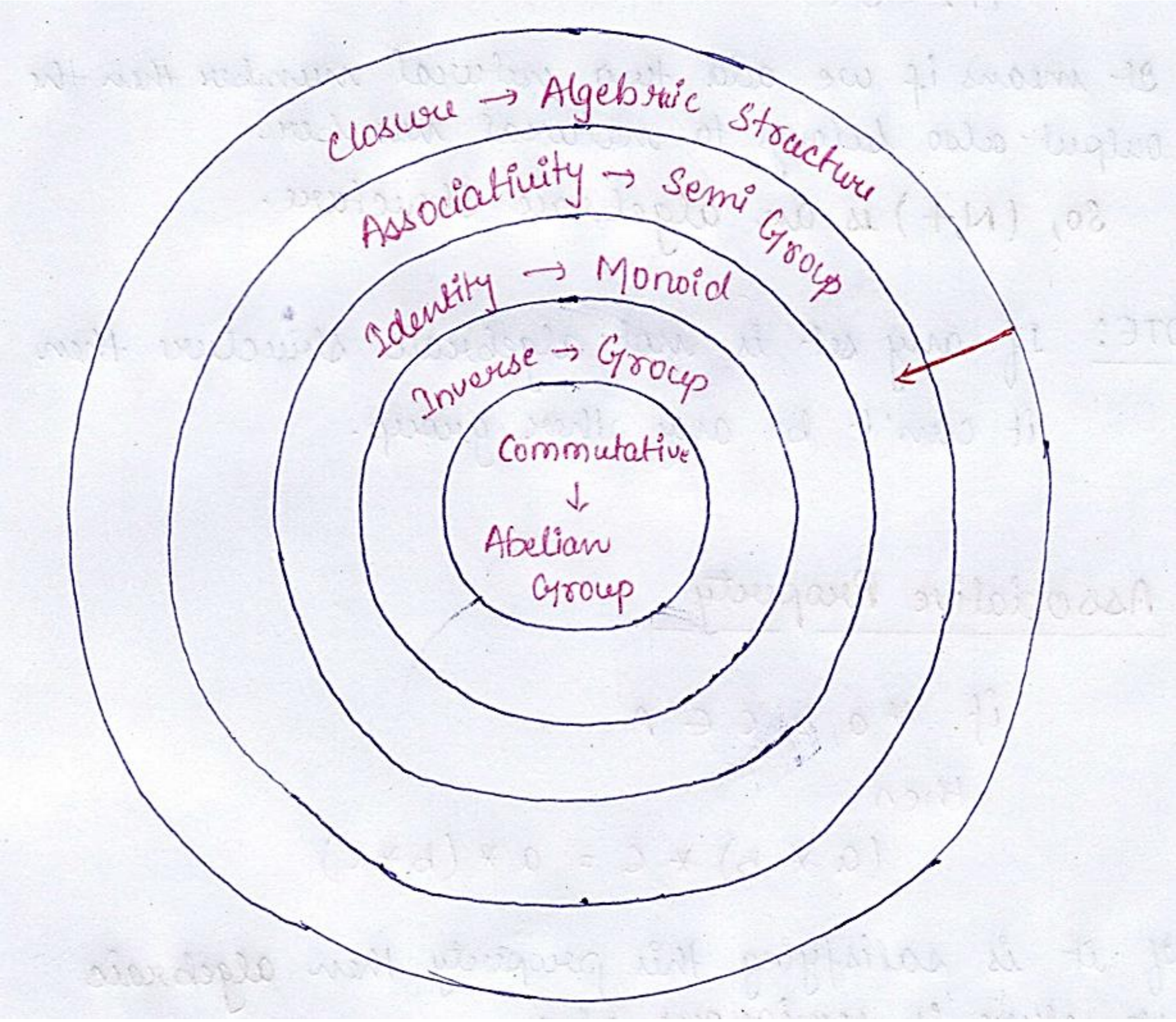
Which of the following propertyies a Group G must hold, in order to be an Abelian group?

(a) The distributive property
(b) The commutative property
(c) The symmetric property

Codes:

(A) (a) and (b)
(B) (b) and (c)
(C) (a) only
(D) (b) only

(D) (b) only


Closure → Algebric Structure
Associativity → Semi Group
Identity → Monoid
Inverse → Group
Commutative
↓
Abelian Group

Paper II July 2016

The number of different spanning trees in complete graph, K4 and bipartite graph K2,2 have .... and .... respectively.

(A) 14, 14
(B) 16, 14
(C) 16, 4
(D) 14, 4

(C) 16, 4

For any complete graph K_n with n nodes, different spanning trees possible is $n^{(n-2)}$

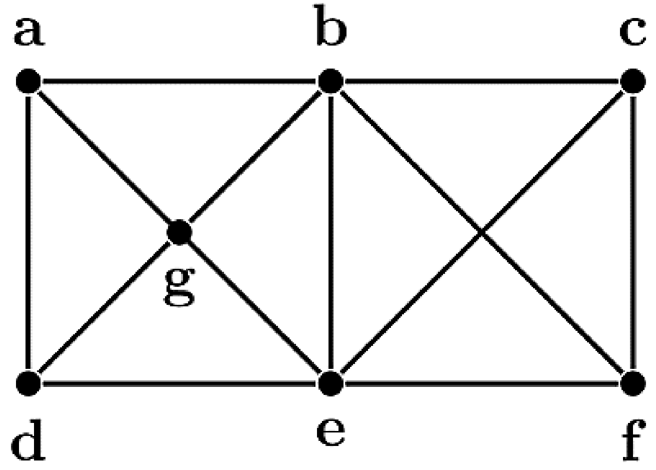
So, for K_4, its $4^{(4-2)} = 16$

For any Bipartite graph $K_{m,n}$ with m and n nodes, different spanning trees possible is $m^{(n-1)}.n^{(m-1)}$

So, for $K_{2,2}$ its $2^{(2-1)}.2^{(2-1)} = 2.2 = 4$

Paper II July 2016

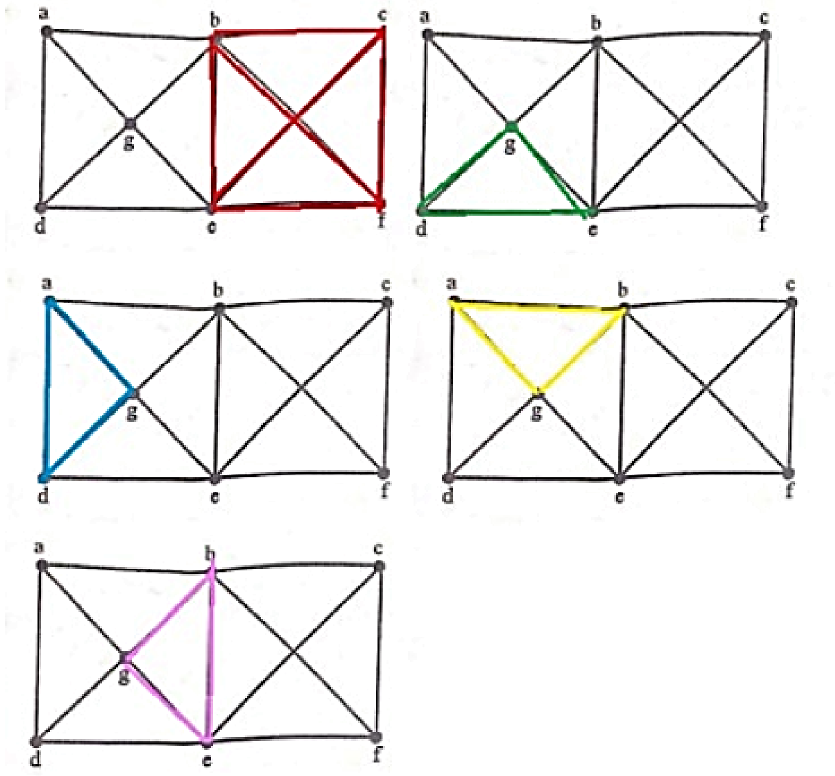
A clique in a simple undirected graph is a complete subgraph that is not contained in any larger complete subgraph. How many cliques are there in the graph shown below?



(A) 2
(B) 4
(C) 5
(D) 6

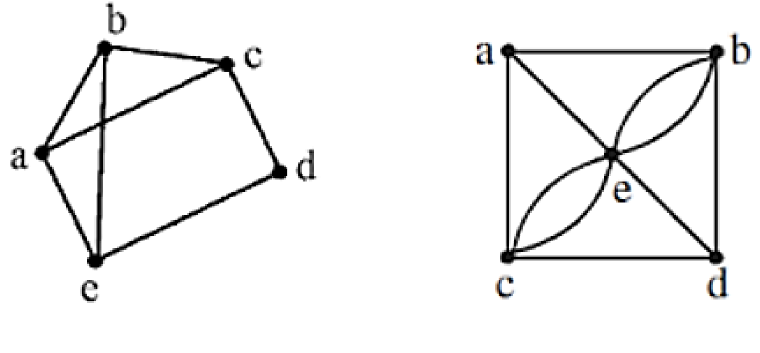
(C) 5

Complete subgraph means each vertex should be connected with all other vretices in the subgraph



Paper II August 2016 (Re-test)

Given the following graphs :



(G_1) (G_2)

Which of the following is correct?

(A) G1 contains Euler circuit and G2 does not contain Euler circuit.
(B) G1 does not contain Euler circuit and G2 contains Euler circuit.
(C) Both G1 and G2 do not contain Euler circuit.
(D) Both G1 and G2 contain Euler circuit.

(C) Both G1 and G2 do not contain Euler circuit.

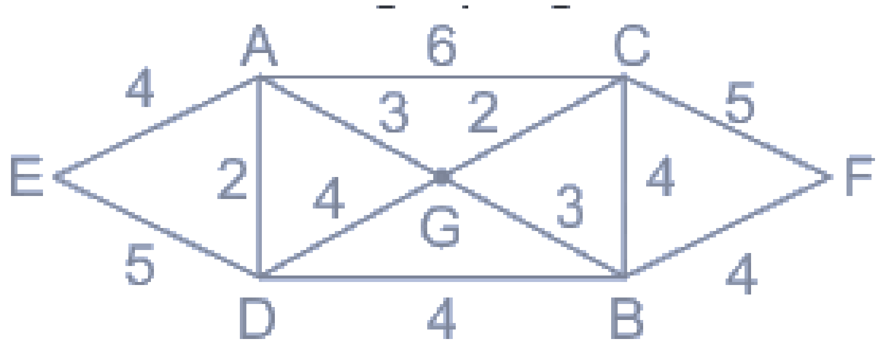
- **A graph has an Euler circuit if and only if the degree of every vertex is even.**
- **A graph has an Euler path if and only if there are at most two vertices with odd degree.**

For a graph to be euler graph every vertex in the graph should have have even degree in graph G1 all the vertices have odd degree so no euler circuit possible therfore g1 is not euler graph.

Graph G2 has vertex a ,d,e all these vertex have odd degree as 3 ,5,3 so here no euler ciruit possible .no this graph is also not euler graph .

Paper II November 2017

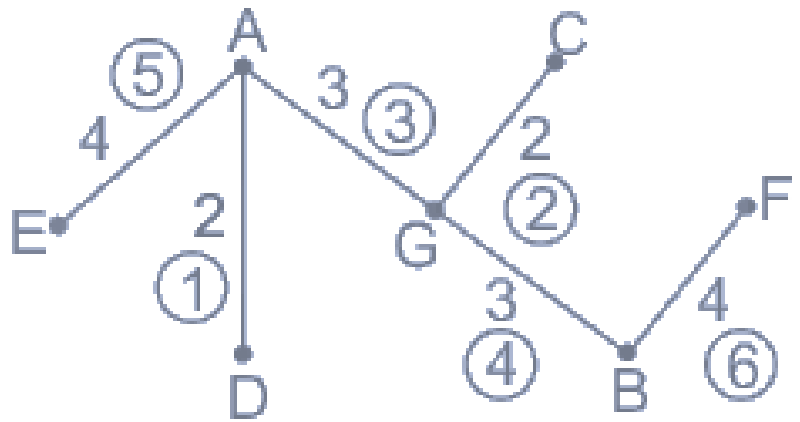
Consider the graph given below :



Use Kruskal's algorithm to find a minimal spanning tree for the graph. The List of the edges of the tree in the order in which they are chosen is?

(1) AD, AE, AG, GC, GB, BF
(2) GC, GB, BF, GA, AD, AE
(3) GC, AD, GB, GA, BF, AE
(4) AD, AG, GC, AE, GB, BF

(3) GC, AD, GB, GA, BF, AE



Paper II December 2018

If a graph (G) has no loops or parallel edges, and if the number of vertices (n) in the graph is $n \geq 3$, then graph G is Hamiltonian if

(i) $deg(v) \geq n/3$ for each vertex v
(ii) $deg(v) + deg(w) \geq n$ whenever v and w are not connected by an edge
(iii) $E(G) \geq 1/3(n-1)(n-2)+2$

Choose the correct answer from the code given below:

(1) (i) and (iii) only
(2) (ii) only
(3) (ii) and (iii) only
(4) (iii) only

(2) (ii) only

A Hamiltonian graph is one which contain a Hamiltonian cycle. A Hamiltonian cycle is a cycle in which each vertex is visited exactly once. Properties of Hamiltonian graph:

1) A graph has a Hamiltonian circuit if each vertex has degree >=3
2) If G= (V, E) has n>=3 vertices and every vertex has degree >=n/2, then G has a Hamilton circuit
3) If G is a graph with n vertices and n>=3, also deg(u) + deg(v) >=n, if u and v are not connected by an edge, then G has a Hamiltonian circuit.
4) $E(G) = \frac{1}{2}(n - 1)(n - 2) + 2$

Paper II June 2019

For which values of m and n does the complete bipartite graph $K_{m,n}$ have a Hamilton circuit?

(a) $m \neq n$, $m,n \geq 2$
(b) $m \neq n$, $m,n \geq 3$
(c) $m = n$, $m,n \geq 2$
(d) $m = n$, $m,n \geq 3$

Paper II June 2019

For which values of m and n does the complete bipartite graph Km, n have a Hamilton circuit?

(a) $m \neq n,\ m,n \geq 2$
(b) $m \neq n,\ m,n \geq 3$
(c) $m = n,\ m,n \geq 2$
(d) $m = n,\ m,n \geq 3$

Paper II December 2019

Consider the following statements:

S1: If a group (G,*) is of order n, and a ∈ G is such that am = e for some integer m≤n, then m must divide n.

S2: If a group (G,*) is of even order, then there must be an element and a ∈ G is such that a ≠ e and a * a = e.

Which of the statements is (are) correct?

(1) Only S1
(2) Only S2
(3) Both S1 and S2
(4) Neither S1 nor S2

(3) Both S1 and S2

S_1: If a group (G, *) is of order n, and a ∈ G is such that $a^m = e$ for some integer $m \leq n$, then m must divide n.

Given statement is correct. As a ∈ G , is such that $a^m = e$ for some integer m<=n, then it means it is a subgroup of G and m is the order of a. where order of given group is n. By the property of subgroup or Lagrange's theorem, a order of subgroup divides the order of a group. So, here m must divide n is true.

S_2: If a group (G, *) is of even order, then there must be an element a ∈ G such that a ≠ e and a * a = e

This statement is correct. Consider an example for this : Consider G is of order 2n. There exists a ∈ G such that a^p = e and p divides 2n. Let n = pq. So, $(a^n)^2 = (a^{pq})^2 = ((a^p)^q)^2 = (e^q)^2 = e$. It means a^n is an element which satisfy the condition and a =! e. Here, a is non trivial subgroup of G.

Paper II November 2020

Let G be a directed graph whose vertex set is the set of numbers from 1 to 100. There is an edge from a vertex i to a vertex j if and only if either j=i+1 or j = 3i. The minimum number of edges in a path in G from vertex 1 to vertex 100 is ____

a) 23
b) 99
c) 4
d) 7

d) 7

Edge set consists of edges from i to j, using either two conditions are j = i + 1 or j = 3i
Second choice helps us to move from 1 to 100. The trick to slot this is to think the other way around.
Try to find a 100 to 1 trail, instead of having a 1 trail to 100.
So, the edge sequence with the minimum number of edges is 1 → 3 → 9 → 10 → 11 → 33 → 99 → 100 which consists of 7 edges.

Paper II November 2020

Consider the following statements:

(A) Any tree is 2-colorable
(B) A graph G has no cycles of even length if it is bipartite.
(C) A graph G is 2-colorable if is bipartite
(D) A graph G can be colored with d+1 colors if d is the maximum degree of any vertex in the graph G.
(E) A graph G can be colored with O(log) colors if it has O(v) edges.

Choose the correct answer from the options given below

a) (C) and (E) are incorrect
b) (B) and (C) are incorrect
c) (B) and (E) are incorrect
d) (A) and (D) are incorrect

c) (B) and (E) are incorrect

Any tree is 2-colourable. True every tree is no loops and a bipartite graph with chromatic number 2. Odd levels coloured with one colour and even levels coloured with another colour.

A graph G has no cycles of even length if it is bipartite. False

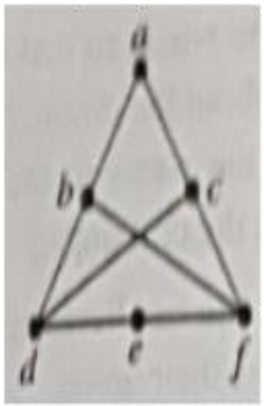
A graph G is 2-colourable if is bipartite True. A graph K(2,3), set of V1 vertices coloured with one colour and set of V2 vertices coloured with another colour.

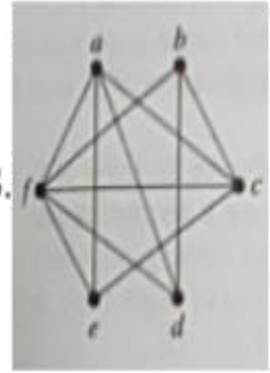
A graph G can be coloured with d+1 colours if d is the maximum degree of any vertex in the graph G.True as the vertex of degree d means it is adjacent to d vertices hence need d+1 different colours required.

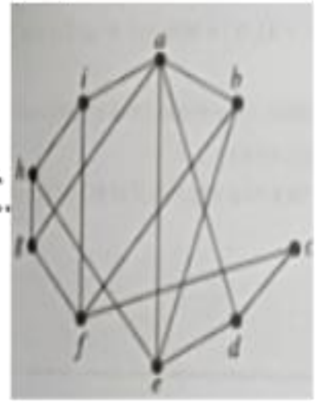
A graph G can be coloured with O(log|v|) colours if it has O(|v|) degrees False as Kn has O(n^2) edges with chromatic number n so it needs O (√ V) colours

Paper II November 2021

Which of the following graphs is/are planar?

A.


B.


C.


Choose the correct answers from the option given below:

a) **A and B only**
b) **A only**
c) **B and C only**
d) **B only**

a) A and B only

Figure A can be redraw as:

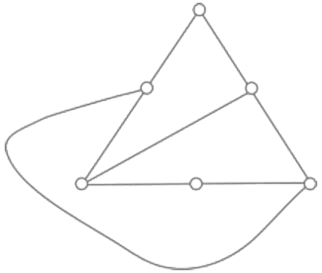
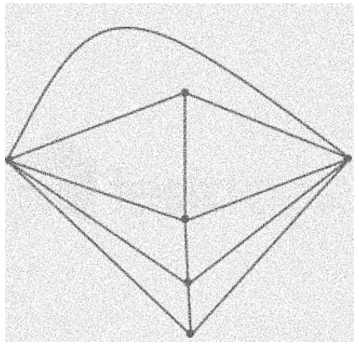


Figure B can be redraw as :



But, Figure C cant not be redraw in such a way that no edge intersect each other. Therefore ,Figure A and B both are planar graph.

Paper II November 2021

Given below are two statements

Statement I: In an undirected graph, number of odd degree vertices is even.

Statement II: In an undirected graph, sum of degrees of all vertices is even.

In light of the above statements, choose the correct answer from the options given below.

a) Both Statement I and Statement II are false
b) Both Statement I and Statement II are true
c) Statement I is false but Statement II is true
d) Statement I is true but Statement II is false

b) Both Statement I and Statement II are true

In an undirected graph, number of odd degree vertices is even as per handshaking lemma.

In an undirected graph, sum of degrees of all vertices is even, as sum of degree of the vertices = 2 * number of edges .

Paper II November 2021

Let (X, *) be a semigroup. Furthermore, for every a and b in X, if $a \neq b$, then $a*b \neq b*a$. Based on the defined semigroup, choose the correct equalities from the options given below:

A. For every a in X, a*a = a
B. For every a, b in X, a*b *a= a
C. For every a, b, c in X, a*b *c= a*c

a) A and B only
b) A and C only
c) A, B and C
d) B and C only

c) A, B and C

"For every a in X, a*a = a" - TRUE

Proof - Let a*a = b.

⇒ (a * a) * a = b * a

⇒ => Since (A, *) is a semigroup, * is closed and associative.

⇒ => So, (a * a) * a = a * (a * a)

⇒ => a * b = b * a, which is possible only if a = b.

⇒ => Hence, a * a = a.

For every a, b in X, a*b*a= a - TRUE

Proof - Let (a * b) * a = c

⇒ ((a * b) * a) * a = c * a

⇒ => (a * b) * (a * a) = c * a

⇒ => (a * b) * a = c * a

⇒ => c * a = a ----- (1) Similarly, a * (a + b + a) = a * c

⇒ => a * (a * (b * a)) = a * c

⇒ => (a * a) * (b * a) = a * c

⇒ => a * (b * a) = a * c

⇒ => (a * b) * a = a * c => c = a * c ----- (2)

⇒ From (1) and (2), we can get c * a = a * c => c = a.

For every a, b, c in X, a*b*c= a*c - TRUE

Proof - Let (a * b) * c = d =

(a * b) * c * c = d * c

a * b * c = d * c

=> d * c = d. Similarly, a * (a * b * c) = a * d

=> a * a * (b * c) = a * d

=> a * (b * c) = a * d => a * d = d. Thus, d * c = a * d = d.

=> Now, c * d * c = c * a * d = c * d

=> c = c * a * d = c * d and d * c * a = a * d * a = d * a

=> d * c * a = a = d * a Hence, a * c = (d * a) * (c * d)

=> d * (a * c) * d = d. Therefore, a * b * c = a * c.

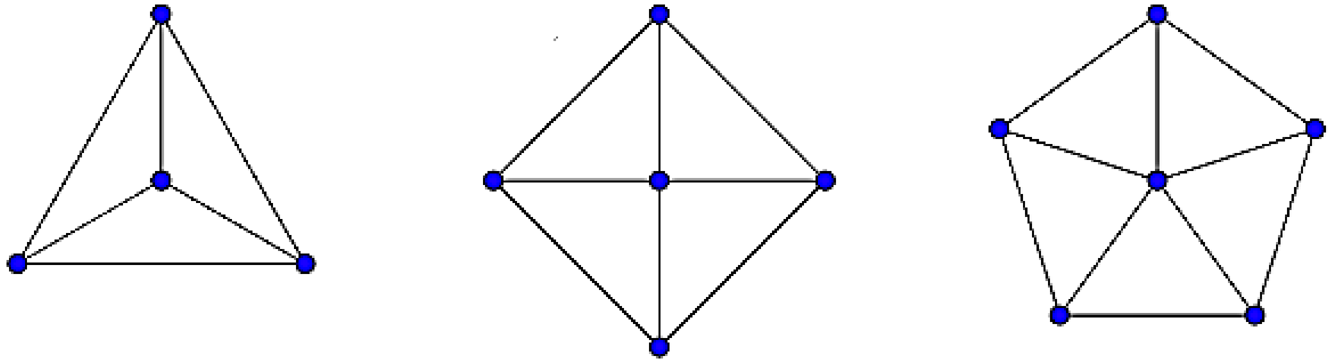
Paper II November 2021

For which value of n is Wheel graph W_n regular?

a) 2
b) 3
c) 4
d) 5

b) & c)

Wheel Graph: Wn is a wheel graph. A wheel graph is a graph formed by connecting a single universal vertex to all vertices of a cycle. Following are the examples of wheel graph:



We can see in the above diagrams that the polygon is connected to a vertex at the centre resulting in a wheel like formation. It is regular for n = 3 or degree = 3.

Paper II October 2022

Let ({a,b}, *) be a semigroup, where a*a=b.

(A) a*b = b*a
(B) b* b = b

Choose the most appropriate answer from the options given below:

a) (A) Only true
b) (B) Only true
c) Both (A) and (B) true
d) Neither (A) nor (B)

d) Neither (A) nor (B) true.

Commutative Property which is a * b = b * a is not true for semigroups.
The equation b * b = b is not true for all semigroups. A semigroup is a set S equipped with a binary operation * that is associative, which means (a * b) * c = a * (b * c) for all a, b, c in S. For a given semigroup, the equation b * b = b may or may not hold true. In fact, there are many semigroups where this equation is not satisfied.

For example, consider the set of non-negative integers under addition modulo 2. That is, we define a binary operation * on this set by a * b = (a + b) mod 2. This forms a semigroup, as * is associative. However, the equation b * b = b is not generally true in this semigroup. For instance, if b = 1, then 1 * 1 = 0, which is not equal to 1. Therefore, the equation b * b = b is not generally true for all semigroups.

Paper II March 2023

Consider the following statements:

P. There exists no simple, undirected and connected graph with 80 vertices and 77 edges.
Q: All vertices of Euler graph are of even degree.
R. Every simple, undirected, connected and acyclic graph with 50 vertices has at least two vertices of degree one.
S. There exits a bipartite graph with more than ten vertices which is 2-colorable.

What is the number of correct statements among the above statements.

1. 1
2. 2
3. 3
4. 4

4. 4

P. According to Euler's formula for planar graphs, a connected graph with V vertices and E edges satisfies V - E + F = 2, where F is the number of faces. In this case, V = 80 and E = 77, so we have 80 - 77 + F = 2, which implies F = -1. There exists no simple connected graph.

Q. This property is a consequence of the handshaking lemma, which states that the sum of the degrees of all vertices in a graph is twice the number of edges.

R. In a simple, undirected, connected, and acyclic graph, which is a tree, there must always be at least two vertices of degree one. This is because a tree has exactly V - 1 edges, where V is the number of vertices. If there were no vertices of degree one, then every vertex would have a degree of at least two, resulting in at least 2V degrees, which contradicts the fact that the sum of the degrees is equal to twice the number of edges.

S. In a bipartite graph, it is always possible to 2-color the vertices because the two sets can be assigned different colors. As long as the graph has more than ten vertices, there will exist a bipartite graph that satisfies this condition.

Paper II March 2023

Match List I with List II

LIST I		LIST II	
A.	Planar Graph	I.	Probabilistic Model
B.	Bipartite Graph	II.	Deterministic Model
C.	PERT	III.	4-Colorable
D.	CPM	IV.	2-Colorable

Choose the correct answer from the options given below:

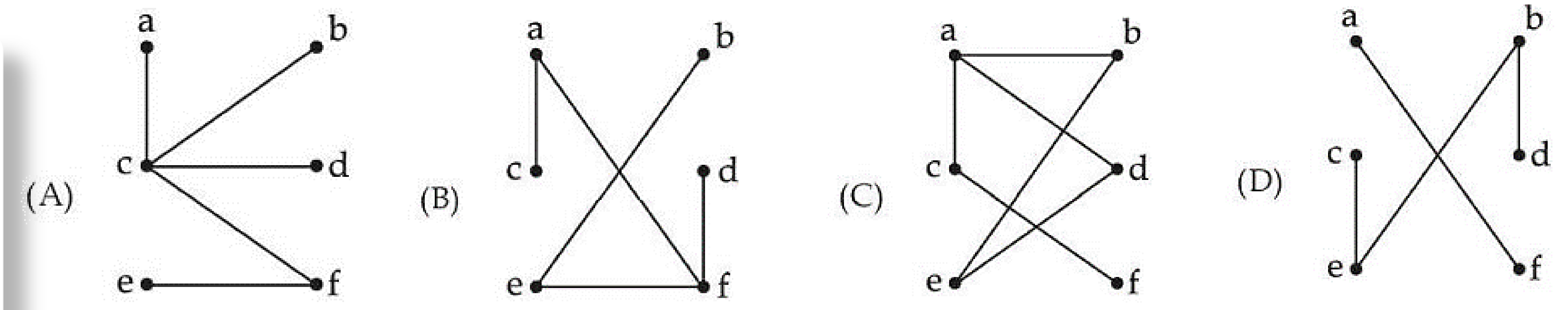
1. A-IV, B-III, C-I, D-II
2. A-III, B-IV, C-II, D-I
3. A-II, B-IV, C-I, D-III
4. A-III, B-IV, C-I, D-II

4. A-III, B-IV, C-I, D-II

BASIS FOR COMPARISON	PERT	CPM
Meaning	PERT is a project management technique, used to manage uncertain activities of a project.	CPM is a statistical technique of project management that manages well defined activities of a project.
What is it?	A technique of planning and control of time.	A method to control cost and time.
Orientation	Event-oriented	Activity-oriented
Evolution	Evolved as Research & Development project	Evolved as Construction project
Model	Probabilistic Model	Deterministic Model
Focuses on	Time	Time-cost trade-off
Estimates	Three time estimates	One time estimate
Appropriate for	High precision time estimate	Reasonable time estimate
Management of	Unpredictable Activities	Predictable activities
Nature of jobs	Non-repetitive nature	Repetitive nature
Critical and Non-critical activities	No differentiation	Differentiated
Suitable for	Research and Development Project	Non-research projects like civil construction, ship building etc.

Paper II Dec 2023

Which of the following graphs are trees ?



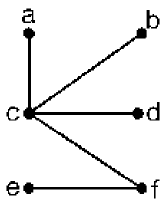
Choose the correct answer from the options given below :

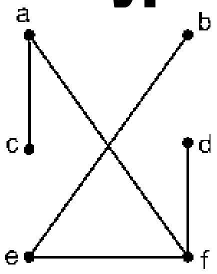
(1) (A) and (B) Only
(2) (A), (B) and (D) Only
(3) (A) and (D) Only
(4) (A), (B), (C) and (D) Only

(1) (A) and (B) Only

Tree:

- **A tree is a type of graph that is acyclic, meaning there are no cycles or loops. It consists of nodes connected by edges in a hierarchical structure.**
- **Each node, except the root, has exactly one parent, forming a parent-child relationship.**
- **There are no loops or cycles in a tree structure.**
- **A tree is a connected graph, meaning there is a unique path between any pair of nodes. Each node in a tree (except the root) has exactly one parent.**
- **There is a unique topmost node called the root from which all other nodes are descendants.**
- **Trees are usually directed, meaning edges have a specific direction (from parent to child).**
- **Trees are commonly used for hierarchical data representation, such as file systems, organizational charts, and expression trees.**
- **Binary Search Trees (BSTs) are a specific type of tree used for efficient searching and sorting.**





- **A graph is a more general structure that can be cyclic or acyclic. It consists of nodes (vertices) and edges connecting these nodes.**
- **There are no strict hierarchical relationships between nodes in a graph.**
- **A graph may or may not be connected. There can be isolated components within a graph.**
- **Nodes in a graph can have multiple incoming edges, meaning they can have multiple parents.**
- **There is no concept of a root node in a general graph. Graphs can be directed or undirected.**
- **In directed graphs, edges have a direction, while in undirected graphs, edges have no direction.**
- **Graphs are more general-purpose and can represent a wide range of relationships between entities.**
- **They are used in applications like social networks, road networks, dependency graphs, and more.**