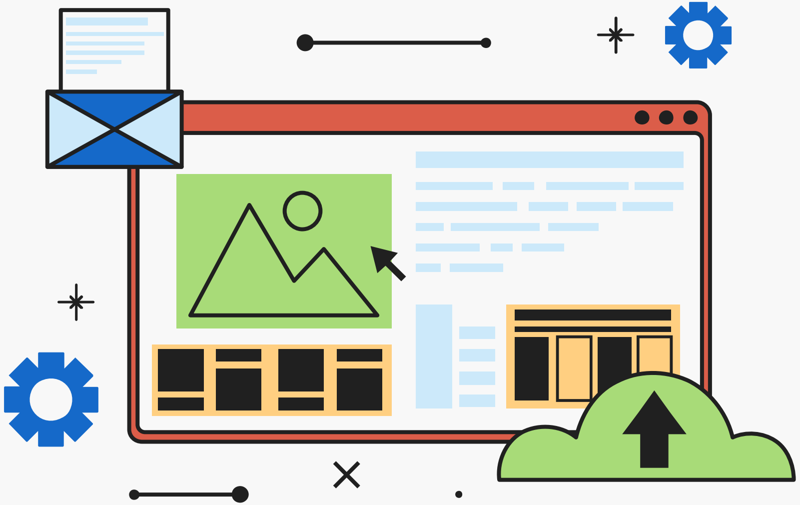
DM Miscellanoues



June 2014 Paper II

How many cards must be chosen from a deck to guarantee that atleast

i. two aces of two kinds are chosen.
ii. two aces are chosen.
iii. two cards of the same kind are chosen.
iv. two cards of two different kinds are chosen

(A) 50, 50, 14, 5
(B) 51, 51, 15, 7
(C) 52, 52, 14, 5
(D) 51, 51, 14, 5

(A) 50, 50, 14, 5

since we have to be sure (guarantee) consider the worst cases for all

i) two aces of same kind are chosen (first 48 cards without ace 49th will surely be an ace and 50 th will be of another ace) Here kind word is incorrectly given

ii) two aces are chosen first 48 cards can be without ace then 49th and 50th will definitely be ace

iii)two cards of same kind that is same number so first 13 can be of different numbers but 14 th will definitely match with someone

iv)2 cards of 2 different kinds let first 4 are of same number say all ace or all 2 etc now 5th one will be definitely different number

So ans is A 50 50 14 5

Paper II December 2014

Consider a set A = {1, 2, 3, ..., 1000}. How many members of A shall be divisible by 3 or by 5 or by both 3 and 5 ?

(A) 533
(B) 599
(C) 467
(D) 66

(C) 467

(A U B) = (A) + (B) - (A ∩ B)

A=1000/3 = 333   [No's divisible by 3]

B=1000/5=200   [No's divisible by 5]

A ∩ B= 1000/15=66   [No's divisible by both 3 and 5]

A U B = 333+200-66=533-66=467

Paper II December 2014

A computer program selects an integer in the set $\{k : 1 \leq k \leq 10,00,000\}$ at random and prints out the result. This process is repeated 1 million times. What is the probability that the value $k = 1$ appears in the printout atleast once ?

(A) 0.5
(B) 0.704
(C) 0.632121
(D) 0.68

(C) 0.632121

Probability of k=1 is 1/1000000= 10^{-6}

Probability of k!= 1 is 1- 10^{-6} = 0.999

Probability that k=1 is never printed in all 10^6 print outs = 0.999*0.999*.......0.9999 (10^{6} times)

= 0.999^{10^6}

Probability that 1 is printed at least once = 1- probability that 1 is never printed

= $1-0.999^{10^6}$

=0.6321

Paper II June 2015

How many strings of 5 digits have the property that the sum of their digits is 7 ?

(A) 66
(B) 330
(C) 495
(D) 99

(B) 330

Let five digit A, B, C, D, E then A + B + C + D + E = 7

given n = 7, r = 5 we know that

$^{n+r-1}C_{r-1} = {}^{7+5-1}C_{5-1} = {}^{11}C_{4} = 11x\ 10x9x8/1x2x3x4 = 330$

Paper II June 2015

Consider an experiment of tossing two fair dice, one black and one red. What is the probability that the number on the black die divides the number on red die ?

(A) 22 / 36
(B) 12 / 36
(C) 14 / 36
(D) 6 / 36

(C) 14 / 36

If number on the black die divides the number on red die, respective numbers on the black die and red die must be any of the following

(1,1),(1,2),(1,3),(1,4),(1,5),(1,6), (2,2),(2,4),(2,6), (3,3),(3,6), (4,4), (5,5), (6,6)

Total number of possible outcomes = 14

The probability that the number of the black die divides the number of red die = 14/36

Paper II June 2015

In how many ways can 15 indistinguishable fish be placed into 5 different ponds, so that each pond contains at least one fish ?

(A) 1001
(B) 3876
(C) 775
(D) 200

(A) 1001

Number of ways it can be done = number of positive integer solutions of

=> x1+x2+x3+x4+x5=15x1+x2+x3+x4+x5=15

=> (15-1)C4 = 14C4 = 1001

Paper II December 2015

How many committees of five people can be chosen from 20 men and 12 women such that each committee contains at least three women?

(A) 75240
(B) 52492
(C) 41800
(D) 9900

(B) 52492

Total 5 people need to be chosen with condition that at least 3 must be women and no restriction on men

For that, we have 20 men and 12 women

Case 1: Take 2 men from 20 men and 3 women from 12 women

$20C2 * 12C3$

Case 2: Take 1 man from 20 men and 4 women from 12 women

$20C1 * 12C4$

Case 3: Take 0 man from 20 men and 5 women from 12 women

$20C0 * 12C5$

All these 3 cases are mutually exclusive:

ANS= $20C2 * 12C3 + 20C1 * 12C4 + 20C0 * 12C5$

ANS=52492

Paper II July 2016

There are three cards in a box. Both sides of one card are black, both sides of one card are red, and the third card has one black side and one red side. We pick a card at random and observe only one side.

What is the probability that the opposite side is the same colour as the one side we observed?

(A) 3/4
(B) 2/3
(C) 1/2
(D) 1/3

(B) 2/3

There are BB RR and BR, total outcome is 3, probability that the opposite side is the same color as the one side we observed:

It is clear that BB and RR will show the same color on opposite side, so favorable outcome will be 2.

Probability will be favorable outcome / total outcome
i.e. 2 / 3.

Paper II August 2016 (Re-test)

What is the probability that a randomly selected bit string of length 10 is a palindrome?

(A) 1/64
(B) 1/32
(C) 1/8
(D) 1/4

(B) 1/32

- Total number of bit-strings of length 10 possible = 2^{10}
- To make a string palindrome, we only care about first half digits and rest of them are just replicated .
- Hence, here we care of first 5 digits only in 2^5 ways and rest 5 are replicated in only 1 way .

Required Probability = $\frac{2^5}{2^{10}} = \frac{1}{2^5}$

Paper II January 2017

How many multiples of 6 are there between the following pairs of numbers?

0 and 100 and -6 and 34

(1) 16 and 6
(2) 17 and 6
(3) 17 and 7
(4) 16 and 7

(3) 17 and 7

Between 0 and 100 multiple of 6 are: 0,6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96 ie. 17 multiple. Between -6 and 34 multiple of 6 are: -6, 0, 6, 12, 18, 24, 30.

Paper II November 2017

If the time is now 4 O'clock, what will be the time after 101 hours from now?

(1) 9 O'clock
(2) 8 O'clock
(3) 5 O'clock
(4) 4 O'clock

(1) 9 O'clock

In a day, 24 hours are there Therefore, after multiple of 24 hours' time will be 4 O' clock (101 mod 24) = 5

Time after 101 hours = 4 O' clock + 5 hours = 9 O' clock

Paper II November 2017

How many distinguishable permutations of the letters in the word BANANA are there?

(1) 720
(2) 120
(3) 60
(4) 360

(3) 60

In BANANA we have six letters in total but here we have some duplicate letters too so we have to deal with it and have to remove those duplicate case.

B - 1 A - 3 N - 2

So total no of words possible is factorial(6) ie 6!

but we must remove duplicate words: ie- (6!/(2!*3!)) which gives 60

Paper II July 2018

Digital data received from a sensor can fill up 0 to 32 buffers. Let the sample space be S={0, 1, 2, ..., 32} where the sample j denote that j of the buffers are full and p(i) =1/561 (33 - i). Let A denote the event that the even number of buffers are full. Then p(A) is:

(1) 0.515
(2) 0.785
(3) 0.758
(4) 0.485

(1) 0.515

Buffers are from 0 to 32

Sample space S = {0, 1, 2............32}

Probability of i^{th} buffer getting full is denotes by p(i) which is given as : $p(i) = \frac{1}{561}(33 - i)$

Probability of even number of buffers are full is denote by p(A).

We have to consider only the even numbers from the sample space i.e. 0, 2, 4, ...32

For this, find the probability for p(0),p(2), p(4), p(6),..........p(32)

$$p(0) = \frac{1}{561}(33 - 0) = \frac{1}{561}(33)$$

$$p(2) = \frac{1}{561}(33 - 2) = \frac{1}{561}(31)$$

$$p(4) = \frac{1}{561}(33 - 4) = \frac{1}{561}(29)$$

$$p(6) = \frac{1}{561}(33 - 6) = \frac{1}{561}(27)$$

$$p\,(32) = \frac{1}{561}\,(33 - 32) = \ \frac{1}{561}\,(1)$$

So, every time bracketed value is decreasing by 2 while outside value remains same.

So, p(A) = p(0) + p(2) + p(4) +...............+ p(32)

P(A) = (1/561) [33 + 31 + 29 + 27 + 25 + 23 + 21 + ...1]

p(A) = (1/561) × 289

p(A) = 0.515

Paper II December 2018

A box contains six red balls and four green balls. Four balls are selected at random from the box. What is the probability that two of the selected balls will be red and two will be green?

(1) 1/14
(2) 3/7
(3) 1/35
(4) 1/9

(2) 3/7

Total balls = 6 + 4 = 10

4 balls from 10 balls can be selected in $^{10}C_4$

2 red balls can be selected in $^{6}C_2$ ways.

2 green balls from 4 green balls can be selected in $^{4}C_2$ ways.

Probability that two selected balls will be red and two will be green = $\frac{^{6}C_2 \times {^{4}C_2}}{^{10}C_4} = \frac{15 \times 6}{210} = \frac{3}{7}$

Paper II December 2018

A full joint distribution for the Toothache, Cavity and Catch is given in the table below.

	Toothache		¬ Toothache	
	Catch	¬ Catch	Catch	¬ Catch
Cavity	0·108	0·012	0·072	0·008
¬ Cavity	0·016	0·064	0·144	0·576

What is the probability of Cavity, given evidence of Toothache?

(1) <0.2,0.8>
(2) <0.4,0.8>
(3) <0.6,0.8>
(4) <0.6,0.4>

(4) <0.6,0.4>

Probability of cavity given evidence of toothache is denoted here as:

Case1:

$$P\left(\frac{Cavity}{Toothache}\right) = \frac{P(cavity \cap Toothache)}{P(Toothache)}$$

P(Toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2

P(cavity ∩ Toothache) = 0.108 + 0.012 = 0.12

$$P\left(\frac{Cavity}{Toothache}\right) = \frac{0.12}{0.2} = 0.6$$

Case2:

$$P\left(\frac{\neg Cavity}{Toothache}\right) = \frac{P(\neg cavity \cap Toothache)}{P(Toothache)}$$

P(¬cavity ∩ Toothache) = 0.016 + 0.064 = 0.08

$$P\left(\frac{\neg Cavity}{Toothache}\right) = \frac{0.08}{0.2} = 0.4$$

Paper II June 2019

How many cards must be selected from a standard deck of 52 cards to guarantee that at least three hearts are present among them?

(a) 9
(b) 13
(c) 17
(d) 42

(d) 42

The worst case, we may selects all the clubs, diamonds, and spades (39 cards) before any hearts.

So, to guarantee that at least three hearts are selected, 39+3=42 cards should be selected.

Paper II December 2019

How many cards must be selected from a standard deck of 52 cards to guarantee that at least three hearts are present among them?

(a) 9
(b) 13
(c) 17
(d) 42

Paper II December 2019

How many cards must be selected from a standard deck of 52 cards to guarantee that at least three hearts are present among them?

(a) 9
(b) 13
(c) 17
(d) 42

Paper II November 2020

How many ways are there to pack six copies of the same book into four identical boxes, where a box can contain as many as six books?

a) 4
b) 6
c) 7
d) 9

d) 9

Here, there are six copies of the same book into four identical boxes(same box). We will enumerate all ways to pack the books. For each way to pack the books, we will list the number of books in the box with the largest number of books, followed by the numbers of books in each box containing at least one book, in order of decreasing the number of books in a box

6,0,0,0
5,1,0,0
4,2,0,0
4,1,1,0
3,3,0,0
3,2,1,0
3,1,1,1
2,2,2,0
2,2,1,1.

So the total number of ways is 9.

Paper II November 2020

The number of positive integers not exceeding 100 that are either odd or the square of an

a) 63
b) 30
c) 55
d) 60

c) 55

By using inclusion exclusion principle, $|A \cup B| = |A|+|B|-|A \cap B|$

Number of odd numbers in the range of (1-100)=100 ÷ 2=50 (1,3,5,7....97,99)=$|A|$

Number of squares=10 (1,4,9,16,25,36,49,64,81,100)=$|B|$

Number of odd numbers and squares=5 (1,9,25,49,81)=$|A \cap B|$

Number of positive integers not exceeding 100 that are either odd or the square=$|A \cup B|$= 50+10-5=55

Paper II November 2021

How many ways are there to assign 5 different jobs to 4 different employees if every employee is assigned at least 1 job?

a) 1024
b) 20
c) 240
d) 625

c) 240

	E1	E2	E3	E4
Case 1	2 jobs	1 job	1 job	1job
Case 2	1 job	2 jobs	1 job	1 job
Case 3	1 job	1 job	2 jobs	1 job
Case 4	1 job	1 job	1 job	2 jobs

For Case 1,Number of ways : 5C2 * 3C1 * 2C1 * 1C1 =10*3*2*1 = 60ways
For Case 2,Number of ways : 5C1 * 4C2 * 2C1 * 1C1 =5*6*2*1= 60ways
For Case 3,Number of ways : 5C1 * 4C1 * 3C2 * 1C1 =5*4*3*1 = 60ways
For Case 4,Number of ways : 5C1 * 4C1 * 3C1 * 2C2 =5*4*3*1 = 60ways
Total number of possible ways is = (60+60+60+60)=240

Paper II November 2021

A company stores products in a warehouse. Storage bins in this warehouse are specified by their aisle, location in the aisle, and self. There are 50 aisles, 85 horizontal locations in each aisle, and 5 shelves throughout the warehouse. What is the least number of products the company can have so that at least two products must be stored in the same bin?

a) 21251
b) 251
c) 4251
d) 426

a) 21251

In the warehouse , No. of aisles = 50
Horizontal locations in each aisle = 85
No. of shelves =5
Therefore, total number of bins = 50 * 85 * 5 = 21250

According to the pigeonhole principle, if there are N+1 pigeon then there must be N pigeonholes, such that at least two pigeons are in same pigeonholes.
Here, N = 21250,therefore number of products = 21250+1 =21251 so that at least two products are in the same bin.

Paper II November 2021

Let us assume a person climbing the stairs can take one stair or two stairs at a time. How many ways can this person climb a flight of eight stairs?

a) 21
b) 24
c) 31
d) 34

d) 34

Let us consider 1 as one step and 2 as two step. Consider the following conditions to reach the flight with 1 or 2.

1, 1, 1, 1, 1, 1, 1, 1 - This can be done in 8C0 ways = 1.

1, 1, 1, 1, 1, 1, 2 - This can be done in 7C1 ways = 7.

1, 1, 1, 1, 2, 2 - This can be done in 6C2 ways = 15.

1, 1, 2, 2, 2 - This can be done in 5C3 ways = 10.

2, 2, 2, 2 - This can be done in 4C0 ways = 1

Total number of ways = 1 + 7 + 15 + 10 + 1 = 34

Paper II June 2023

Find the sum of all four digit numbers formed using the digits 1,2,4 and 6 .

1. 86,658
2. 88,8858
3. 91,958
4. 93,358

1) 86,658.

There are total $4! = 4*3*2*1 = 24$ ways to permute 4 different digits in 4 places.
The sum of all 4-digit numbers can be found by calculating the sum for each of the 4 positions (Thousands, Hundreds, Tens, and Units), then summing those results.
For each position:
Each of the 4 numbers (1, 2, 4, 6) will appear in each position 1/4th of the time in the total permutations, so $24 / 4 = 6$ times for each.
The sum of the digits is $1 + 2 + 4 + 6 = 13$. So, the contribution for each position will be $13 * 6 = 78$.
Now, we calculate the total sum taking into account the place value:
The Thousands place contributes $78 * 1000 = 78,000$.
The Hundreds place contribute $78 * 100 = 7,800$. The Tens place gives $78 * 10 = 780$.
The Units place contributes $78 * 1 = 78$.
Adding those up, the total sum of all 4-digit numbers that can be made with the digits 1, 2, 4, and 6 is $78,000 + 7,800 + 780 + 78 = 86,658$. So the answer is option 1) 86,658.

Paper II Dec 2023

What is the probability that a positive integer selected at random from the set of positive integer not exceeding 100 is divisible by either 2 or 5?

(1) 10/5
(2) 3/5
(3) 2/5
(4) 1/5

(2) 3/5

We need to find the probability that a positive integer, selected randomly from 1 to 100 is divisible by either 2 or 5.
First, find the total number of positive integers from 1 to 100, which is 100.
Divisible by 2: Every second number is divisible by 2, so there are 100 / 2 = 50 such numbers.
Divisible by 5: Every fifth number is divisible by 5, so there are 100 / 5 = 20 such numbers. There is an intersection between these two sets of numbers, namely, those numbers which are divisible by both 2 and 5 (that is, numbers divisible by 10). To avoid counting these twice, we need to subtract these from the total.
Divisible by 10 (both 2 and 5): Every tenth number is divisible by 10, so there are 100 / 10 = 10 such numbers.
So, numbers that are divisible by either 2 or 5 are 50 (divisible by 2) + 20 (divisible by 5) - 10 (divisible by both) = 60.
Now, the probability of a number randomly picked from 1 to 100 being divisible by either 2 or 5 is 60/100 = 3/5.