Hierarchy of Automata from RE to TM

Automaton Type Example Language	Formal Definition	Recognized Languages	Grammar
Regular Expressions (RE) $\{a, b\}^*$, the set of all strings over $\{a, b\}$	$L = \{w \in \Sigma^* \mid$ w matches a regular expression$\}$	Regular Languages	Regular Grammar
Finite Automaton (FA) $\{a^n b^n \mid n = 1\}$, strings of the form $a^n b^n$	$M = (Q, \Sigma, \delta, q_0, F)$	Regular Languages	Regular Grammar
Pushdown Automaton (PDA) $\{a^n b^n \mid n \geq 0\}$, balanced parentheses or palindromes	$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$	Context-Free Languages (CFL)	Context-Free Grammar (CFG)
Linear Bounded Automaton (LBA) $\{a^n b^n c^n \mid n \geq 1\}$, equal numbers of a, b, and c	$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$	Context-Sensitive Languages (CSL)	Context-Sensitive Grammar (CSG)
Turing Machine (TM) The halting problem: the set of TMs that halt on a given input	$M = (Q, \Sigma, \Gamma, \delta, q_0, , F)$	Recursively Enumerable Languages (REL)	Unrestricted Grammar

Table 1: Comparison of Automata from RE to TM

Hierarchy Summary

Regular Expressions (RE) < Finite Automaton (FA) < Pushdown Automaton (PDA) < Linear Bounded Automaton (LBA) < Turing Machine (TM)

This hierarchy shows that each machine type can recognize all languages recognized by the machines below it and more. The Turing machine is the most powerful, capable of recognizing the broadest class of languages.