Quaternion EKF

严远斌

2023/04/27

文章导航

1 Quaternion Fusion

1. Initializes the Quaternion:
$$q0 = 1, q1 = 0, q2 = 0, q3 = 0 \tag{1}$$

2. Calculate direction of gravity indicated by algorithm:
$$Gravityx = 2.f * (q1 * q3 - q0 * q2) \tag{2}$$
$$Gravityy = 2.f * (q0 * q1 + q2 * q3) \tag{3}$$
$$Gravityz = q0 * q0 - q1 * q1 - q2 * q2 + q3 * q3 \tag{4}$$

3. Calculate direction of gravity indicated by measurement:
$$axNorm = ax/InverseSqrt(ax * ax + ay * ay + az * az) \tag{5}$$
$$ayNorm = ay/InverseSqrt(ax * ax + ay * ay + az * az) \tag{6}$$
$$azNorm = az/InverseSqrt(ax * ax + ay * ay + az * az) \tag{7}$$

4. Calculate accelerometer feedback scaled by 0.5:
$$deviategx = 0.5f * (ayNorm * Gravityz - azNorm * Gravityy) \tag{8}$$
$$deviategy = 0.5f * (azNorm * Gravityx - axNorm * Gravityz) \tag{9}$$
$$deviategz = 0.5f * (axNorm * Gravityy - ayNorm * Gravityx) \tag{10}$$

5. Apply accelerometer feedback to gyroscope,Convert gyroscope to radians per second scaled by 0.5:
$$halfgxdt = 0.5f * (gx - deviategx) * dt \tag{11}$$
$$halfgydt = 0.5f * (gy - deviategy) * dt \tag{12}$$
$$halfgzdt = 0.5f * (gz - deviategz) * dt \tag{13}$$

6. Integrate rate of change of quaternion:
$$q0 = q0 - halfgxdt * q1 - halfgydt * q2 - halfgzdt * q3 \tag{14}$$
$$q1 = q1 + halfgxdt * q0 + halfgzdt * q2 - halfgydt * q3 \tag{15}$$
$$q2 = q2 + halfgydt * q0 - halfgzdt * q1 + halfgxdt * q3 \tag{16}$$
$$q3 = q3 + halfgzdt * q0 + halfgydt * q1 - halfgxdt * q2 \tag{17}$$
$$\tag{18}$$

7. Normalise quaternion:
$$q0* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \tag{19}$$
$$q1* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \tag{20}$$
$$q2* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \tag{21}$$
$$q3* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \tag{22}$$

8. Converts a quaternion to Euler angles in radians:

$$Euler[0] = atan2f(2.f * (q0 * q3 + q1 * q2), 2.f * (q0 * q0 + q1 * q1) - 1.f); \tag{23}$$

$$Euler[1] = asinf(-2.f * (q1 * q3 - q0 * q2)); \tag{24}$$

$$Euler[2] = atan2f(2.f * (q0 * q1 + q2 * q3), 2.f * (q0 * q0 + q3 * q3) - 1.f); \tag{25}$$

2 Extended Kalman Filter

$$
\begin{aligned}
x_k &= f(x_{k-1}, u_{k-1}, w_{k-1}) &(26)\\
z_k &= h(x_k, v_k) &(27)
\end{aligned}
$$

$$
x = \begin{bmatrix} q0 \\ q1 \\ q2 \\ q3 \\ deviategx \\ deviategy \end{bmatrix} z = \begin{bmatrix} axNorm \\ ayNorm \\ azNorm \end{bmatrix}
$$

1. $A = \frac{\partial f}{\partial x}$:

$$
A = \begin{bmatrix}
1, & -halfgxdt, & -halfgydt, & -halfgzdt, & 0.5f*q1*dt, & 0.5f*q2*dt \\
halfgxdt, & 1, & halfgzdt, & -halfgydt, & -0.5f*q0*dt, & 0.5f*q3*dt \\
halfgydt, & -halfgzdt, & 1, & halfgxdt, & -0.5f*q3*dt, & -0.5f*q0*dt \\
halfgzdt, & halfgydt, & -halfgxdt, & 1, & 0.5f*q2*dt, & -0.5f*q1*dt \\
0, & 0, & 0, & 0, & 1, & 0 \\
0, & 0, & 0, & 0, & 0, & 1
\end{bmatrix}
$$

2. $H = \frac{\partial h}{\partial x}$:

$$
H = \begin{bmatrix}
-2.f*q2, & 2.f*q3, & -2.f*q0, & 2.f*q1, & 0, & 0 \\
2.f*q1, & 2.f*q0, & 2.f*q3, & 2.f*q2, & 0, & 0 \\
2.f*q0, & -2.f*q1, & -2.f*q2, & 2.f*q3, & 0, & 0
\end{bmatrix}
$$

3. Extended kalman filter

$$
\begin{aligned}
\hat{x_k^-} &= f(\hat{x_{k-1}}, u_{k-1}, 0) &(28)\\
P_k^- &= AP_{k-1}A\top + WQW\top &(29)\\
K_k &= \frac{P_k^- H\top}{HP_k^- H\top + VRV\top} &(30)\\
\hat{x_k} &= \hat{x_k^-} + K_k(z_k - h(\hat{x_k^-}, 0)) &(31)\\
P_k &= (I - K_k H)P_k^- &(32)
\end{aligned}
$$