

# Quaternion EKF

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2023/04/27

## 文章导航

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## 1 Quaternion Fusion

1. Initializes the Quaternion:

$$q0 = 1, q1 = 0, q2 = 0, q3 = 0 \quad (1)$$

2. Calculate direction of gravity indicated by algorithm:

$$halfvx = q1 * q3 - q0 * q2 \quad (2)$$

$$halfvy = q0 * q1 + q2 * q3 \quad (3)$$

$$halfvz = q0 * q0 + q3 * q3 - 0.5f \quad (4)$$

3. Calculate accelerometer feedback scaled by 0.5:

$$axNorm = ax / InverseSqrt(ax * ax + ay * ay + az * az) \quad (5)$$

$$ayNorm = ay / InverseSqrt(ax * ax + ay * ay + az * az) \quad (6)$$

$$azNorm = az / InverseSqrt(ax * ax + ay * ay + az * az) \quad (7)$$

$$deviategx = 1.f * (ayNorm * halfvz - azNorm * halfvy) \quad (8)$$

$$deviategy = 1.f * (azNorm * halfvx - axNorm * halfvz) \quad (9)$$

$$deviategz = 1.f * (axNorm * halfvy - ayNorm * halfvx) \quad (10)$$

4. Convert gyroscope to radians per second scaled by 0.5:

$$halfgxdt = 0.5f * gx * dt \quad (11)$$

$$halfgydt = 0.5f * gy * dt \quad (12)$$

$$halfgzdt = 0.5f * gz * dt \quad (13)$$

$$q0 = q0 - halfgxd t * q1 - halfgyd t * q2 - halfgzd t * q3 - 0.5f * q1 * dt * deviategx - 0.5f * q2 * dt * deviategy - 0.5f * q3 * dt * deviategz \quad (14)$$

$$q1 = q1 + halfgxd t * q0 + halfgzd t * q2 - halfgyd t * q3 + 0.5f * q0 * dt * deviategx - 0.5f * q3 * dt * deviategy + 0.5f * q2 * dt * deviategz \quad (15)$$

$$q2 = q2 + halfgyd t * q0 - halfgzd t * q1 + halfgxd t * q3 + 0.5f * q3 * dt * deviategx + 0.5f * q0 * dt * deviategy - 0.5f * q1 * dt * deviategz \quad (16)$$

$$q3 = q3 + halfgzd t * q0 + halfgyd t * q1 - halfgxd t * q2 - 0.5f * q2 * dt * deviategx + 0.5f * q1 * dt * deviategy + 0.5f * q0 * dt * deviategz \quad (17)$$

5. Normalise quaternion:

$$q0* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \quad (18)$$

$$q1* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \quad (19)$$

$$q2* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \quad (20)$$

$$q3* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \quad (21)$$

## 2 Extended Kalman Filter

$$x(k) = f(x(k-1), u(k-1), w(k-1)) \quad (22)$$

$$z(k) = h(x(k), v(k)) \quad (23)$$

$$x = \begin{bmatrix} q0 \\ q1 \\ q2 \\ q3 \\ deviategx \\ deviategy \end{bmatrix} \quad z = \begin{bmatrix} accelxNorm \\ accelyNorm \\ accelzNorm \end{bmatrix}$$

1.  $A = \frac{\partial f}{\partial x}$ :

$$A = \begin{bmatrix} 1, -halfgxd t, -halfgyd t, -halfgzd t, -0.5f * q1 * dt, -0.5f * q2 * dt \\ halfgxd t, 1, halfgzd t, -halfgyd t, 0.5f * q0 * dt, -0.5f * q3 * dt \\ halfgyd t, -halfgzd t, 1, halfgxd t, 0.5f * q3 * dt, 0.5f * q0 * dt \\ halfgzd t, halfgyd t, -halfgxd t, 1, -0.5f * q2 * dt, 0.5f * q1 * dt \\ 0, 0, 0, 0, 1, 0 \\ 0, 0, 0, 0, 0, 1 \end{bmatrix}$$

2.  $H = \frac{\partial h}{\partial x}$ :

$$H = \begin{bmatrix} \frac{0.5f*q2*dt}{halfvz}, \frac{0.5f*q3*dt}{halfvz}, -\frac{0.5f*q0*dt}{halfvz}, -\frac{0.5f*q1*dt}{halfvz}, 0, -\frac{1}{halfvz}, \\ \frac{0.5f*q1*dt}{halfvz}, -\frac{0.5f*q0*dt}{halfvz}, -\frac{0.5f*q3*dt}{halfvz}, \frac{0.5f*q2*dt}{halfvz}, -\frac{1}{halfvz}, 0, \\ \frac{0.5*q1*dt}{halfvy} - \frac{0.5*q2*dt}{halfvx}, -\frac{0.5*q0*dt}{halfvy} - \frac{0.5*q3*dt}{halfvx}, -\frac{0.5*q3*dt}{halfvy} + \frac{0.5*q0*dt}{halfvx}, \frac{0.5*q2*dt}{halfvy} + \frac{0.5*q1*dt}{halfvx}, -\frac{1}{halfvy}, \frac{1}{halfvx} \end{bmatrix}$$