# Quaternion EKF

严远斌

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## 文章导航

# 1 Quaternion Fusion

1. Initializes the Quaternion:

$$q0 = 1, q1 = 0, q2 = 0, q3 = 0 \tag{1}$$

2. Calculate direction of gravity indicated by algorithm:

$$\begin{aligned} Gravityx &= 2.f * (q1 * q3 - q0 * q2) && (2)\\ Gravityy &= 2.f * (q0 * q1 + q2 * q3) && (3)\\ Gravityz &= q0 * q0 - q1 * q1 - q2 * q2 + q3 * q3 && (4) \end{aligned}$$

3. Calculate direction of gravity indicated by measurement:

$$\begin{aligned} axNorm &= ax/InverseSqrt(ax * ax + ay * ay + az * az) && (5)\\ ayNorm &= ay/InverseSqrt(ax * ax + ay * ay + az * az) && (6)\\ azNorm &= az/InverseSqrt(ax * ax + ay * ay + az * az) && (7) \end{aligned}$$

4. Calculate accelerometer feedback scaled by 0.5:

$$\begin{aligned} deviategx &= 0.5f * (ayNorm * Gravityz - azNorm * Gravityy) && (8)\\ deviategy &= 0.5f * (azNorm * Gravityx - axNorm * Gravityz) && (9)\\ deviategz &= 0.5f * (axNorm * Gravityy - ayNorm * Gravityx) && (10) \end{aligned}$$

5. Apply accelerometer feedback to gyroscope,Convert gyroscope to radians per second scaled by 0.5:

$$halfgxdt = 0.5f * (gx - deviategx) * dt \tag{11}$$

$$halfgydt = 0.5f * (gy - deviategy) * dt \tag{12}$$

$$halfgzdt = 0.5f * (gz - deviategz) * dt \tag{13}$$

6. Integrate rate of change of quaternion:

$$\begin{aligned} q0 = &\quad q0 - halfgxdt * q1 - halfgydt * q2 - halfgzdt * q3 && (14)\\ q1 = &\quad q1 + halfgxdt * q0 + halfgzdt * q2 - halfgydt * q3 && (15)\\ q2 = &\quad q2 + halfgydt * q0 - halfgzdt * q1 + halfgxdt * q3 && (16)\\ q3 = &\quad q3 + halfgzdt * q0 + halfgydt * q1 - halfgxdt * q2 && (17)\\ & && (18) \end{aligned}$$

7. Normalise quaternion:

$$q0* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \tag{19}$$

$$q1* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \tag{20}$$

$$q2* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \tag{21}$$

$$q3* = InverseSqrt(q0 * q0 + q1 * q1 + q2 * q2 + q3 * q3) \tag{22}$$

8. Converts a quaternion to Euler angles in radians:

$$Euler[0] = atan2f(2.f * (q0 * q3 + q1 * q2), 2.f * (q0 * q0 + q1 * q1) - 1.f); \tag{23}$$

$$Euler[1] = asinf(-2.f * (q1 * q3 - q0 * q2)); \tag{24}$$

$$Euler[2] = atan2f(2.f * (q0 * q1 + q2 * q3), 2.f * (q0 * q0 + q3 * q3) - 1.f); \tag{25}$$

# 2 Extended Kalman Filter

$$\begin{aligned} x_k &= f(x_{k-1}, u_{k-1}, w_{k-1}) & (26) \\ z_k &= h(x_k, v_k) & (27) \end{aligned}$$

$$x = \begin{bmatrix} q0 \\ q1 \\ q2 \\ q3 \\ deviategx \\ deviategy \end{bmatrix} z = \begin{bmatrix} axNorm \\ ayNorm \\ azNorm \end{bmatrix}$$

1. $A = \frac{\partial f}{\partial x}$:

$$A = \begin{bmatrix} 1, & -halfgxdt, & -halfgydt, & -halfgzdt, & -0.5f*q1*dt, & -0.5f*q2*dt \\ halfgxdt, & 1, & halfgzdt, & -halfgydt, & 0.5f*q0*dt, & -0.5f*q3*dt \\ halfgydt, & -halfgzdt, & 1, & halfgxdt, & 0.5f*q3*dt, & 0.5f*q0*dt \\ halfgzdt, & halfgydt, & -halfgxdt, & 1, & -0.5f*q2*dt, & 0.5f*q1*dt \\ 0, & 0, & 0, & 0, & 1, & 0 \\ 0, & 0, & 0, & 0, & 0, & 1 \end{bmatrix}$$

2. $H = \frac{\partial h}{\partial x}$:

$$H = \begin{bmatrix} -2.f*q2, & 2.f*q3, & -2.f*q0, & 2.f*q1, & 0, & 0 \\ 2.f*q1, & 2.f*q0, & 2.f*q3, & 2.f*q2, & 0, & 0 \\ q0, & -q1, & -q2, & q3, & 0, & 0 \end{bmatrix}$$

3. Extended kalman filter

$$\begin{aligned} \hat{x_k^-} &= f(\hat{x_{k-1}}, u_{k-1}, 0) & (28) \\ P_k^- &= AP_{k-1}A\top + WQW\top & (29) \\ K_k &= \frac{P_k^- H\top}{HP_k^- H\top + VRV\top} & (30) \\ \hat{x_k} &= \hat{x_k^-} + K_k(z_k - h(\hat{x_k^-}, 0)) & (31) \\ P_k &= (I - K_k H)P_k^- & (32) \end{aligned}$$