

Combinational circuit is a circuit which consist of logic gate whose outputs at any instant of time are determined directly from the present combination of input without regard to previous input.

- ~~There will be two combination of inputs~~
→ The combinational circuit does not use any memory.

The procedure involves the following steps:

- The problem is stated.
- The number of available input variables and required output variable is determined.
- The input and output variable are assigned their letter symbol.
- The truth table that defines the required relationship between the input and the output is derived.
- The simplified Boolean function for each output is obtained.
- The logic diagram is drawn.

Adder:

Adder are the combinational circuit which is used to add two or more than two bit at a time.

Type of adder

- Half adder.
- full adder.

1. Half Adder

A combinational circuit that performs the addition of bit is called half adder. The circuit needs two binary inputs and two binary outputs. The input variables designate the augend ('A') and addend ('B') bits; the output variables produce the sum ('S') and carry ('C').

Truth table to identify the function of half adder:

INPUT OUTPUT

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

K-map:

for sum

A\B	0	1
0	0	1
1	1	0

for carry

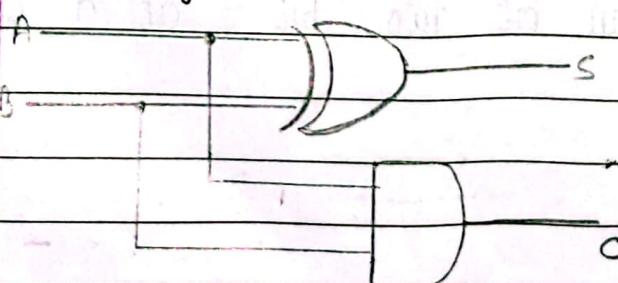
A\B	0	1
0	0	0
1	0	1

from K-map the logic expression from sum & carry:

$$C = AB$$

$$S = AB' + A'B = A \oplus B$$

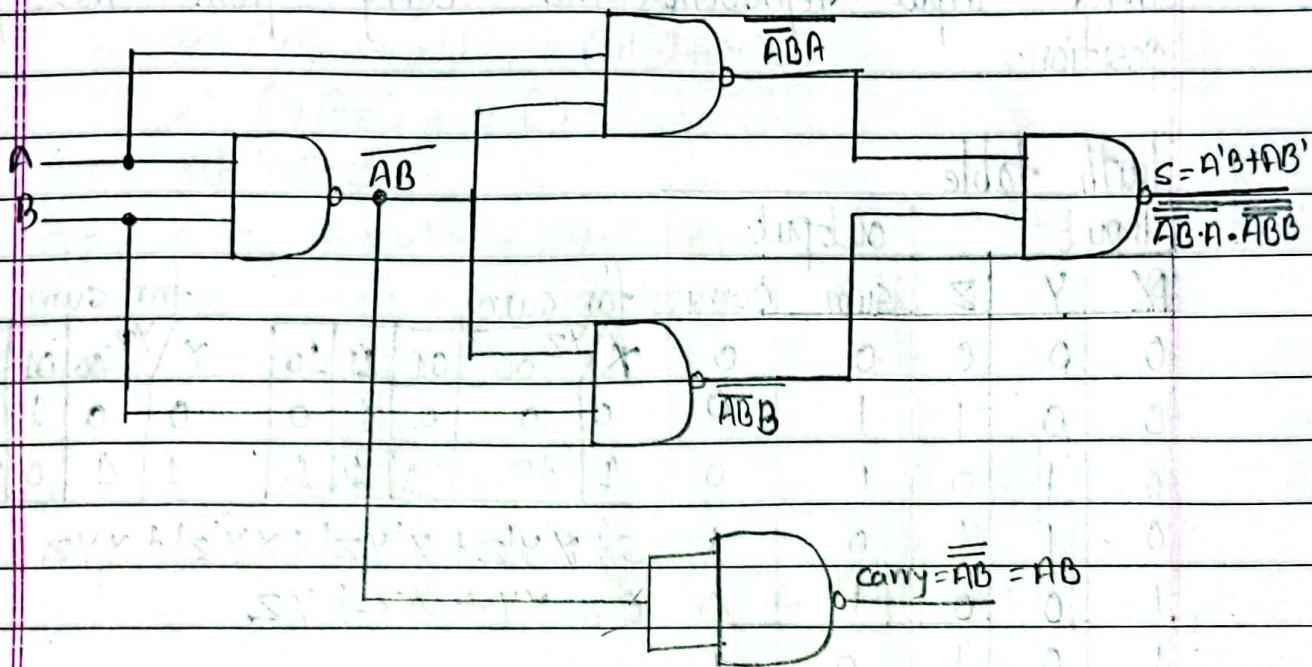
logic diagram



Design a half adder using only NAND gate.

$$S = A'B + AB'$$

$$C = AB$$



Design a half adder using NOR gate.

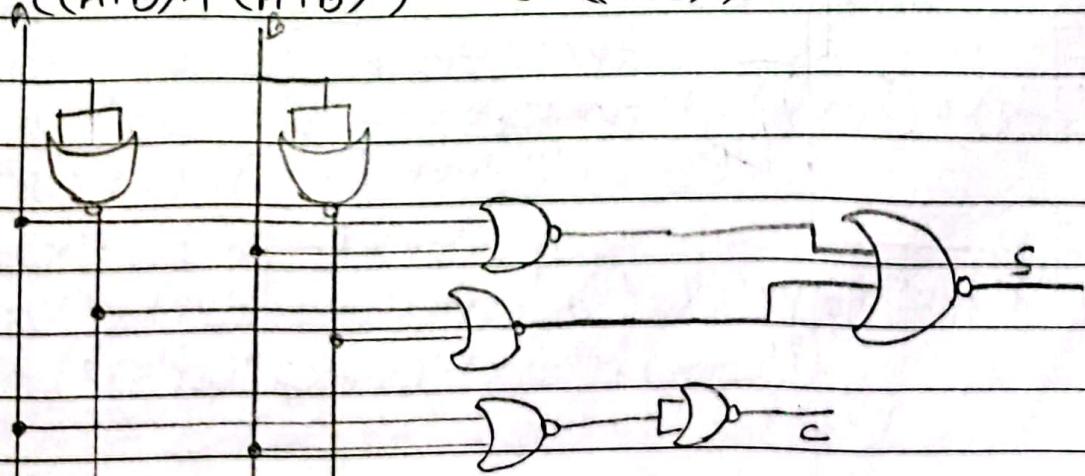
$$S = (A+B) \cdot (A'+B')$$

$$C = \overline{AB} \cdot \overline{A+B}$$

$$S = (A+B) \cdot (A'+B')$$

$$S' = ((A+B) \cdot (A'+B'))' = ((A+B)' + (A'+B')')'$$

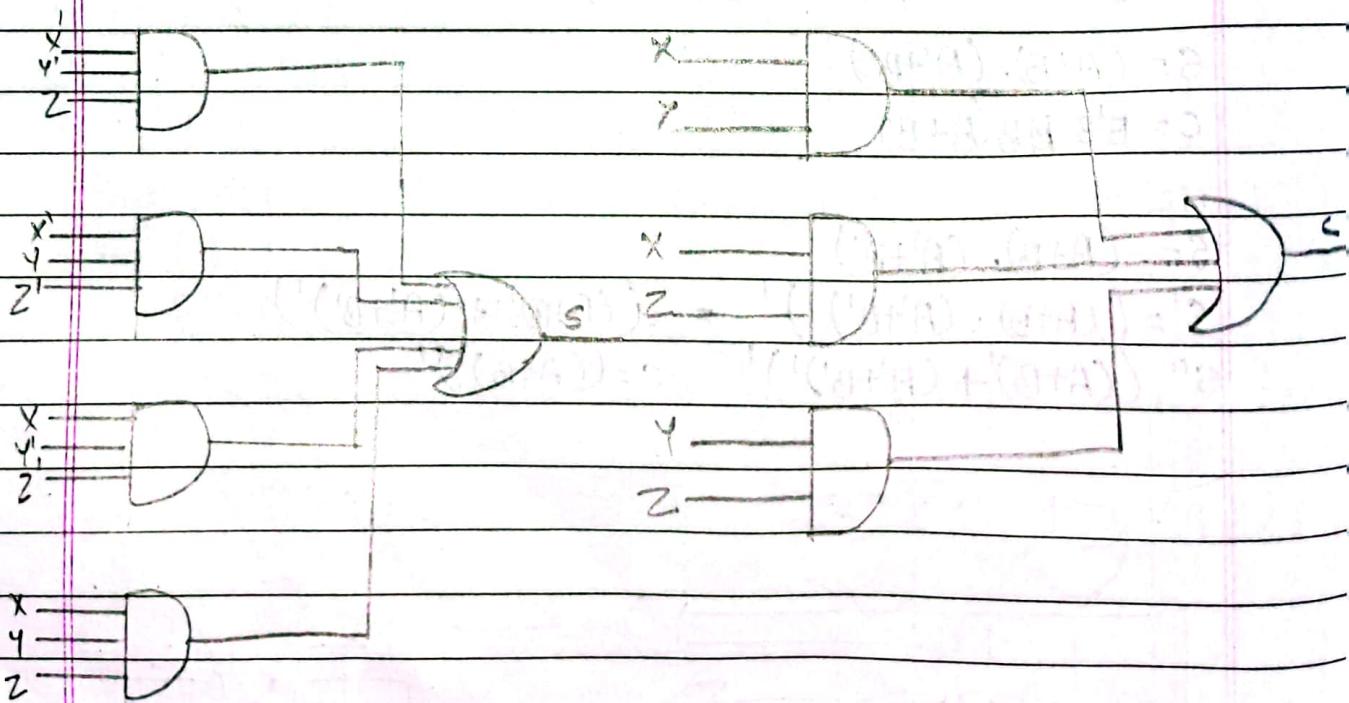
$$S'' = ((A+B)' + (A'+B')')' = C = ((A+B)')'$$



* Full Adder: A combinational circuit that performs the addition of three bit at a time is called full adder. It consists of three inputs and two outputs, two inputs are the bits to be added, the third input represents the carry from the previous position.

Truth table

Input			Output		for carry					for sum					
X	Y	Z	Sum	Carry	YZ	00	01	11	10	X	YZ	00	01	11	10
0	0	0	0	0	0	00	01	11	10	0	00	01	11	10	
0	0	1	1	0	0	0	0	0	1	0	0	0	1	0	
0	1	0	1	0	1	0	1	1	1	1	1	1	0	1	
0	1	1	0	1	1	Y'Z + X'YZ' + XY'Z' + XYZ									
1	0	0	1	0	0					C = XY + XZ + YZ					
1	0	1	0	1											
1	1	0	0	1											
1	1	1	1	1											



⊕ Design full adder with two half adder and an OR gate.

$$\bar{x}\bar{y}z + S = \bar{x}\bar{y}z + \bar{x}yz' + xy'z' + xyz$$

$$C = \bar{x}y + xz + yz (\bar{x}\bar{y}z + \bar{x}\bar{y}z + xy'z' + xyz)$$

$$S = \bar{x}\bar{y}z + \bar{x}yz' + xy'z' + xyz$$

$$= x' (y'z + yz') + x (y'z + yz)$$

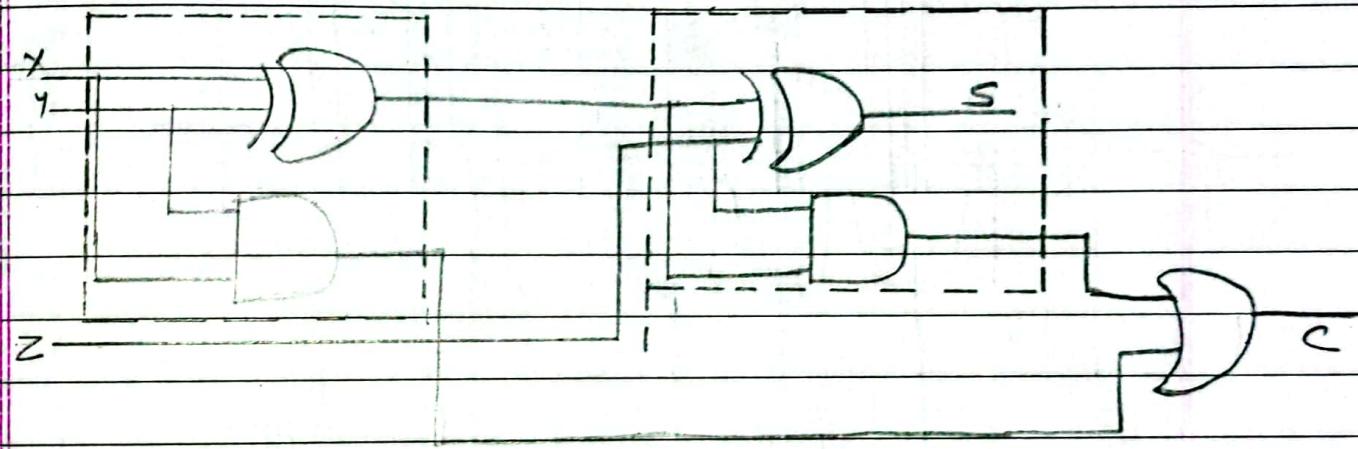
$$= x' (y \oplus z) + x (y \oplus z)$$

$$= x \oplus y \oplus z$$

$$C = \bar{x}\bar{y}z + \bar{x}yz' + xy'z' + xyz$$

$$= yz (x + x') + x (y'z + yz')$$

$$= yz + x (y \oplus z)$$



⊕ Design full adder using $NAND^2$ gate:

$$Sum = \bar{x}\bar{y}z + \bar{x}yz' + xy'z' + xyz$$

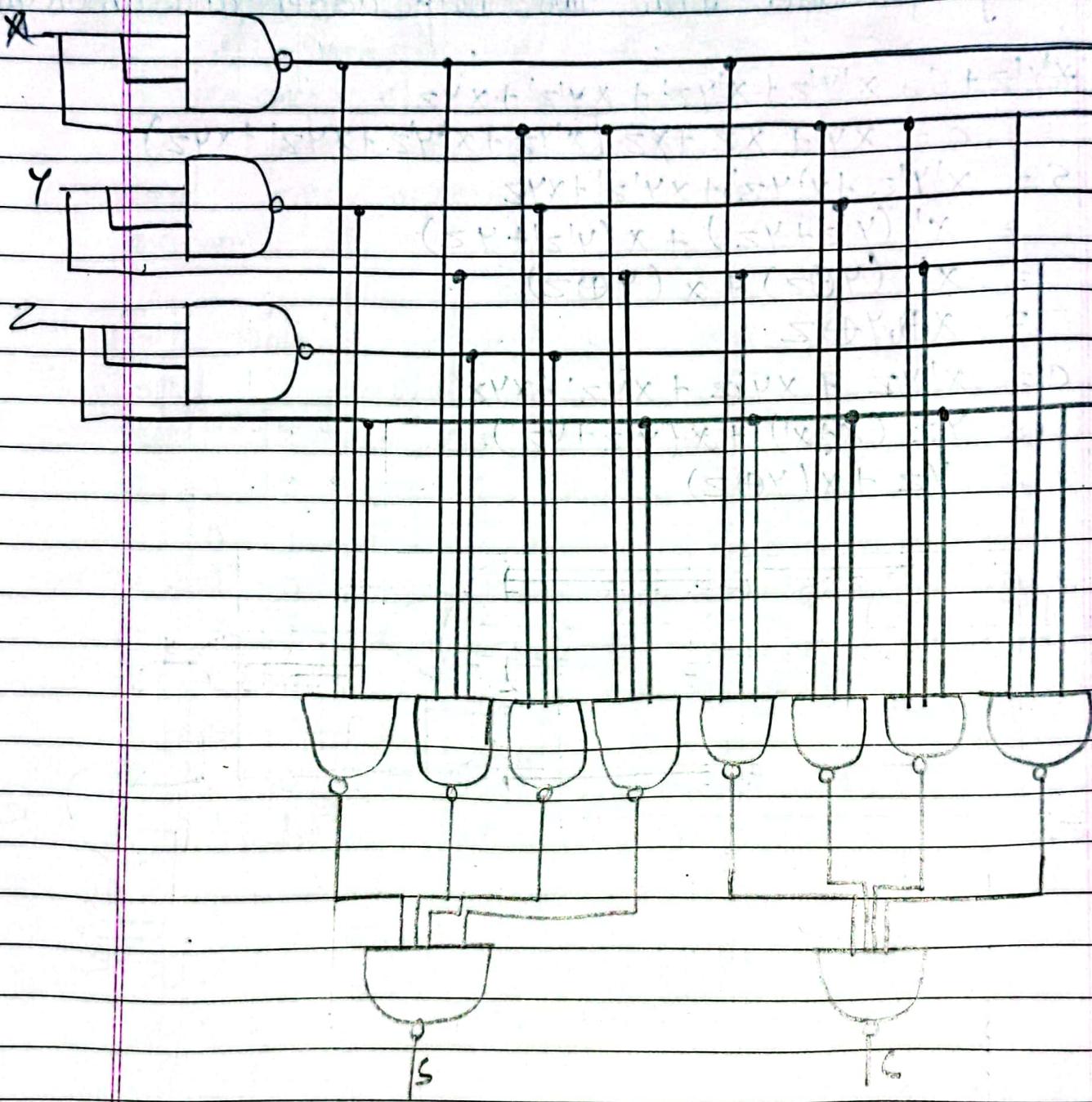
$$S' = (\bar{x}\bar{y}z + \bar{x}yz' + xy'z' + xyz)' = ((\bar{x}\bar{y}z)' + (\bar{x}yz')' + (xy'z')' + (xyz)')$$

$$S'' = ((\bar{x}\bar{y}z)'.(\bar{x}yz')'.(xy'z')'.(xyz)')$$

$$C = \bar{x}\bar{y}z + \bar{x}yz' + xy'z' + xyz$$

$$C' = (\bar{x}\bar{y}z + \bar{x}yz' + xy'z' + xyz)' = ((\bar{x}\bar{y}z)' + (\bar{x}yz)' + (xy'z)' + (xyz)')$$

$$C'' = (((\bar{x}\bar{y}z)'.(\bar{x}yz)'.(xy'z)'.(xyz)'))'$$



Design full adder using NOR gate

$$S = X'Y'Z + X'YZ' + X'Y'Z' + XYZ \quad C = X'YZ + XY'Z + XY'Z' + XYZ$$

$X \backslash Y \backslash Z$	00	01	11	10	$X \backslash Y \backslash Z$	00	01	11	10
0	0		0		0	0	0		
1		0	0	0	1	0			

$$S = (X+Y+Z) \cdot (X'+Y+Z') \cdot (X+Y'+Z') \cdot (X'+Y'+Z) \quad C = (Y+Z) \cdot (X+Y) \cdot (X+Y')$$

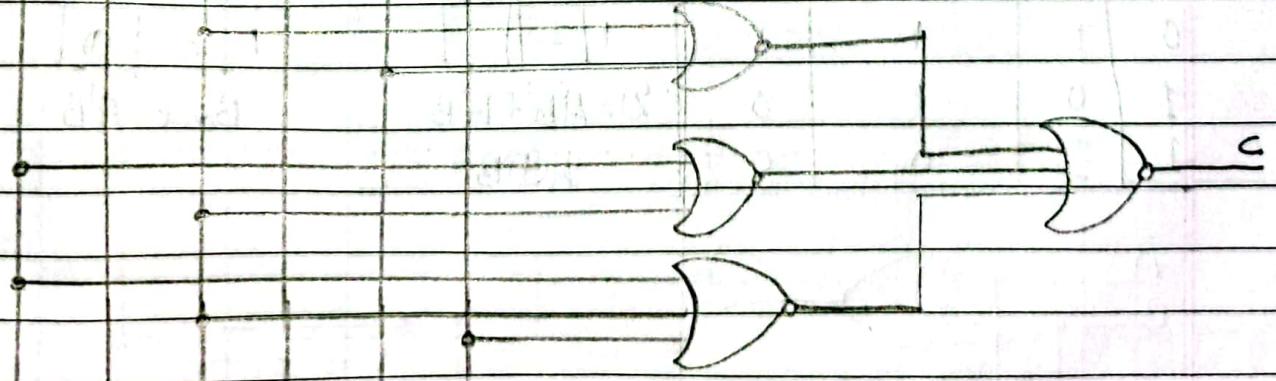
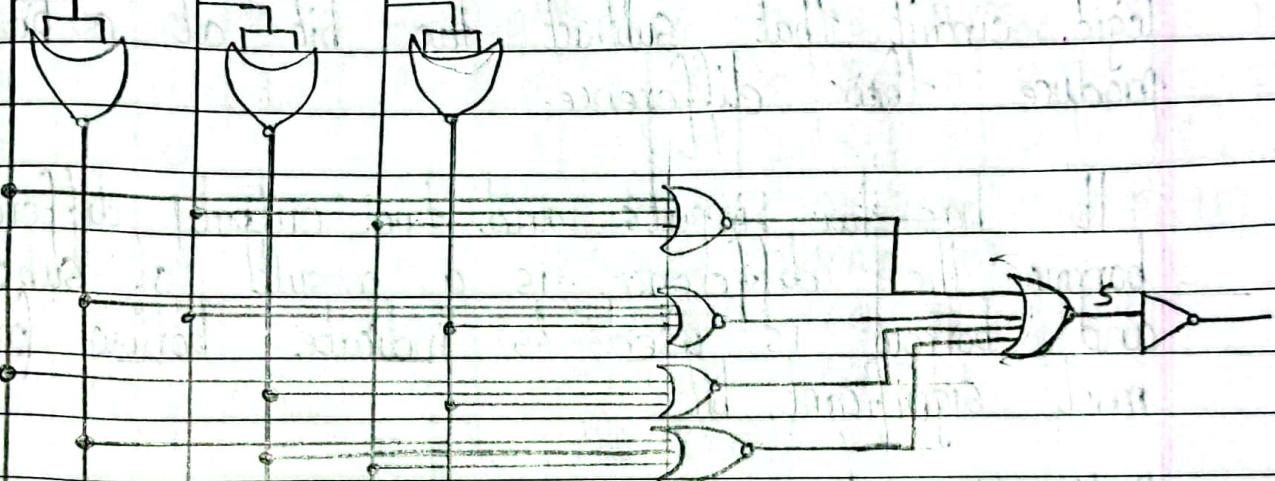
$$\begin{aligned}
 S' &= ((x+y+z) \cdot (x'+y+z') \cdot (x+y'+z') \cdot (x'+y'+z'))' \\
 &= ((x+y+z)' + (x'+y+z')' + (x+y'+z')' + (x'+y'+z')) \\
 S'' &= ((x+y+z)' + (x+y'+z')' + (x+y'+z')' + (x'+y'+z'))'
 \end{aligned}$$

$$C = (y+z) \cdot (x+y) \cdot (x+y'+z)$$

$$C' = (y+z) \cdot (x+y) \cdot (x+y'+z)' = (y+z)' + (x+y)' + (x+y'+z)'$$

$$C'' = (y+z)' + (x+y)' + (x+y'+z)'$$

x y z



Subtractors:

Subtractor is a combinational logic circuit which is used to subtract two or more than two bits at a time and provides difference and borrow as an output.

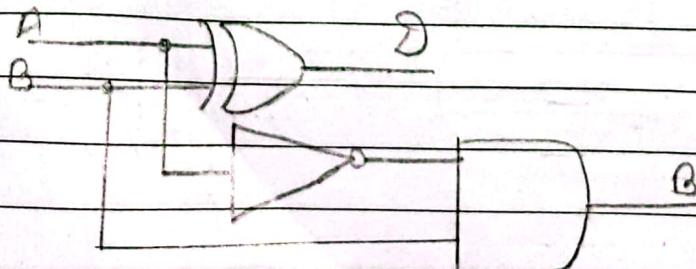
Type of subtractor:

- ④ Half subtractor: A half subtractor is a combinational logic circuit that subtract two bit at a time and produce their difference.

It has two inputs and two output difference and borrow. The difference is a result of subtraction and borrow is used to indicate borrow from next most significant bit.

Truth Table:

Input		Output		for difference	for Borrow
A	B	Difference	Borrow	A'B 0 0 1	A'B 0 0 1
0	0	0	0	0 0 1	0 0 1
0	1	1	1	1 1	1 0 0
1	0	1	0	$D = AB' + A'B$	
1	1	0	0	$= A \oplus B$	

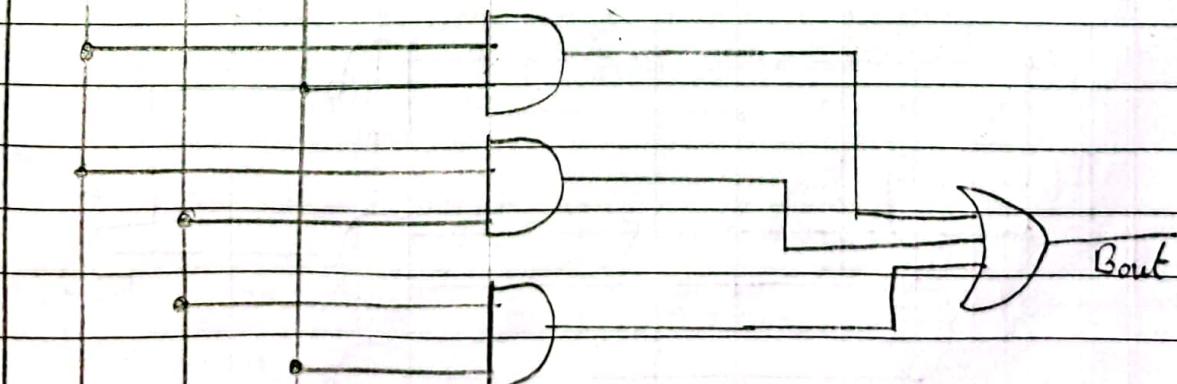
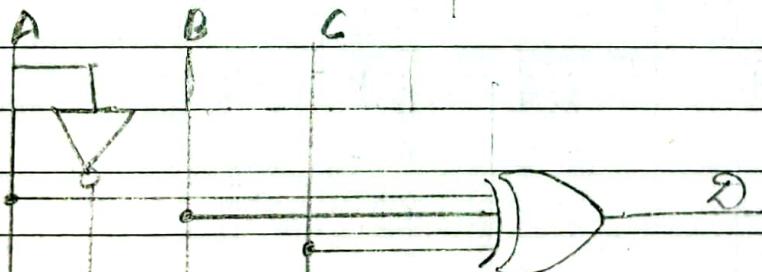


④ Full Subtractor: A combinational logic circuit used to subtract three binary digit at a time is called full subtractor. The circuit has three input and two output. These three input are A, B, C & two output D and Bout respectively.

Truth table for full subtractor:

Input Output

A	B	C	D	Bout	for D	for Bout
0	0	0	0	0	$A \setminus B \setminus C$	$A \setminus B \setminus C$
0	0	1	1	1	$0 \setminus 0 \setminus 1$	$0 \setminus 0 \setminus 1$
0	1	0	1	1	$1 \setminus 0 \setminus 0$	$1 \setminus 0 \setminus 1$
0	1	1	0	1	$D = A'B'C + A'B'C' + ABC' + ABC$	$Bout = A'B + A'B + BC$
1	0	0	1	0	$= C(A'B' + A'B) + C'(A'B + A'B)$	
1	0	1	0	0	$= C(A \oplus B) + C'(A \oplus B)$	
1	1	0	0	0	$= C(A \oplus B)' + C'(A \oplus B)$	
1	1	1	1	1	$= A \oplus B \oplus C$	



Draw and implement full subtractor using NAND gate.

$$D = A'B'C + A'BC' + AB'C' + ABC$$

$$D' = (A'B'C + A'BC' + AB'C' + ABC)'$$

$$' = ((A'B'C)') + (A'BC')' + (AB'C')' + (ABC)'$$

$$D'' = ((A'B'C)') + (A'BC')' + (AB'C')' \cdot (ABC)'$$

$$B_{out} = A'C + A'B + BC$$

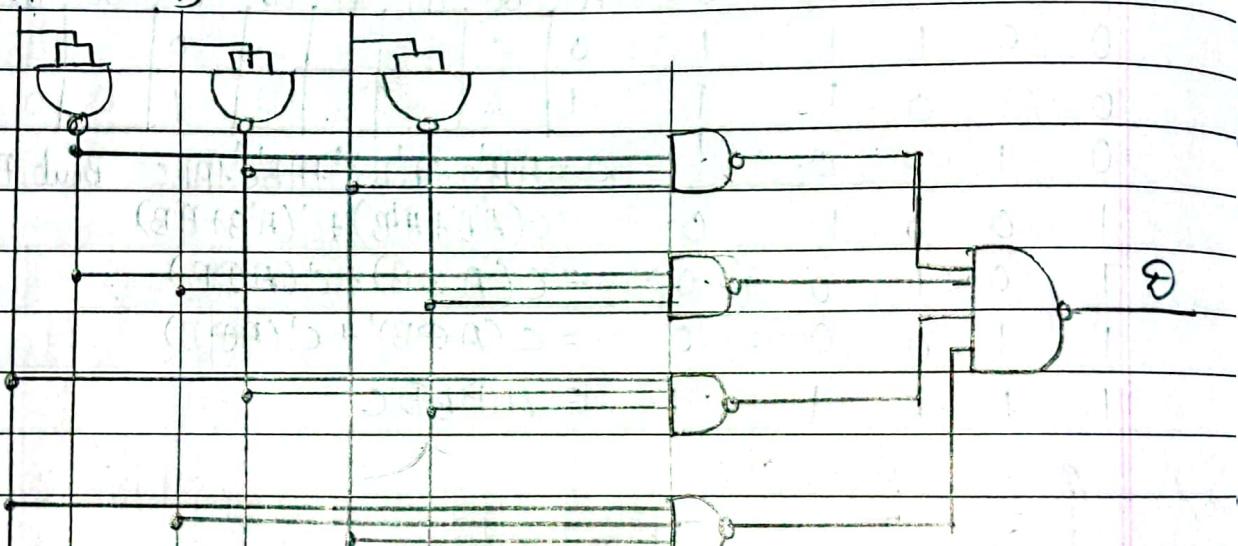
$$B'_{out} = (A'C + A'B + BC)' = ((A'C)' + (A'B)' + (BC)')$$

$$B''_{out} = ((A'C)' \cdot (A'B)' \cdot (BC)')$$

A

B

C



D

D

D

Bout

Draw and implement full subtractor using NOR gate.

$$D = A'B'C + A'B'C' + AB'C' + ABC$$

A	B	C	00	01	11	10
0	0	1	0	1		
1	1	0	1	0		

$$D = (A+B+C)(A+B'+C')(A'+B+C')(A'+B'+C)$$

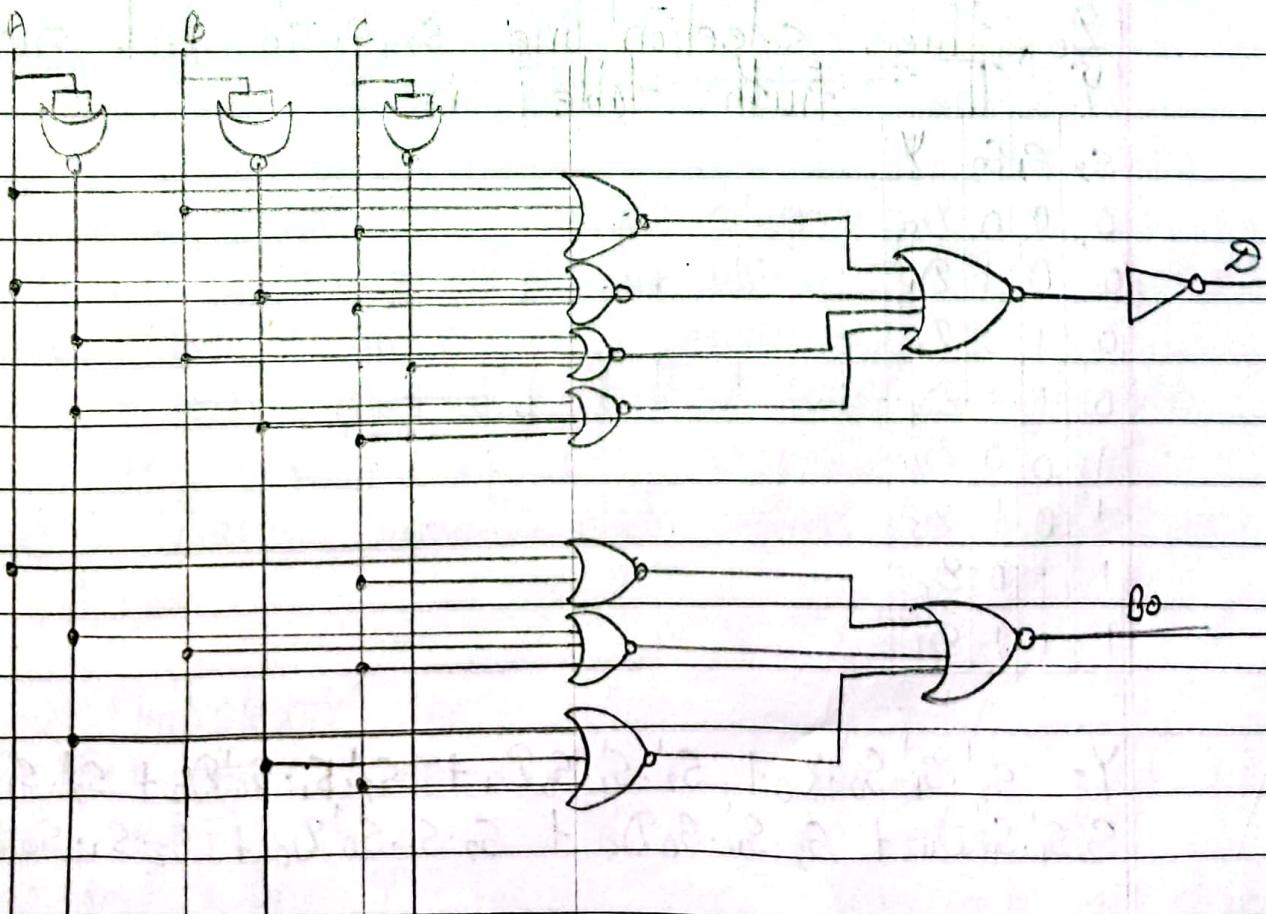
$$D' = ((A+B+C)(A+B'+C')(A'+B+C')(A'+B'+C))'$$

$$= ((A+B+C) + (A+B'+C') + (A'+B+C') + (A'+B'+C))'$$

$$D'' = ((A+B+C)' + (A+B'+C')' + (A'+B+C')' + (A'+B'+C)')$$

$$B_{out} = A'C + A'B + BC$$

A	B	C	00	01	11	10	B _{out}	B'	B''
0	0	1	1	1	1	1	(B+C)(A'+B+C')(A'+B'+C)	((B+C)(A'+B+C')(A'+B'+C))'	((B+C)' + (A'+B+C)' + (A'+B'+C))'
1	0	0	1	0	0	0			



Multiplexer and Demultiplexer

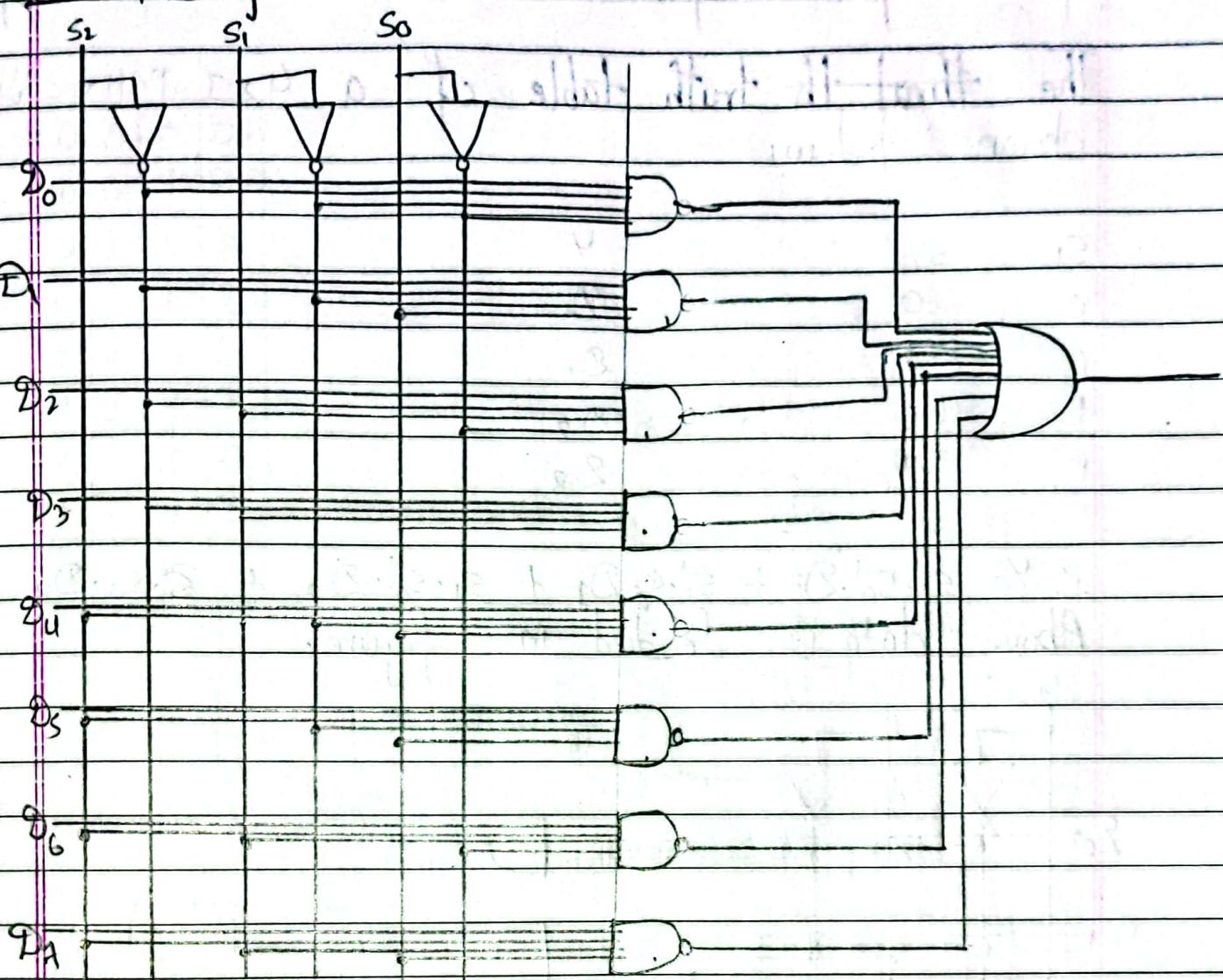
1. ~~Multiplexer~~: ~~Multiplexer~~ means a circuit used to select and route any one of several input signals to a single output. It has maximum 2^n data input, ' n ' selection of line and ~~maximum of 2^n data input, 'n' single selection line and~~.
2. ~~Multiplexer~~: Multiplexer is a combinational circuit that has maximum of 2^n data input, ' n ' single selection line and single output line. There are ' n ' selection line, there will be 2^n possible combinations of zeros and ones. It is also called as Mux.

8x1 Multiplexer

let the 8x1 Multiplexer has eight data input D_0 to D_7 , three selection line S_2, S_1, S_0 and one output Y . The truth table is

S_2	S_1	S_0	Y
0	0	0	D_0
0	0	1	D_1
0	1	0	D_2
0	1	1	D_3
1	0	0	D_4
1	0	1	D_5
1	1	0	D_6
1	1	1	D_7

$$Y = S_2' \cdot S_1' \cdot S_0' \cdot D_0 + S_2' \cdot S_1' \cdot S_0 D_1 + S_2' \cdot S_1 \cdot S_0' D_2 + S_2' \cdot S_1 \cdot S_0 D_3 + S_2 \cdot S_1' \cdot S_0' D_4 + S_2 \cdot S_1' \cdot S_0 D_5 + S_2 \cdot S_1 \cdot S_0' D_6 + S_2 \cdot S_1 \cdot S_0 D_7$$

Circuit diagram

In above figure the logical diagram of 8 to 1 mux is implemented by using 8 AND gate, three NOT gate for complement of Input and one OR gate. When enable pin 1 is set mux will disable & if it is zero then input data pass through output.

4x1 Multiplexer

A 4x1 mux consist four data input as D_0 to D_3 , two select line as S_0 and S_1 and single output line y . The Select line S_0 & S_1 one

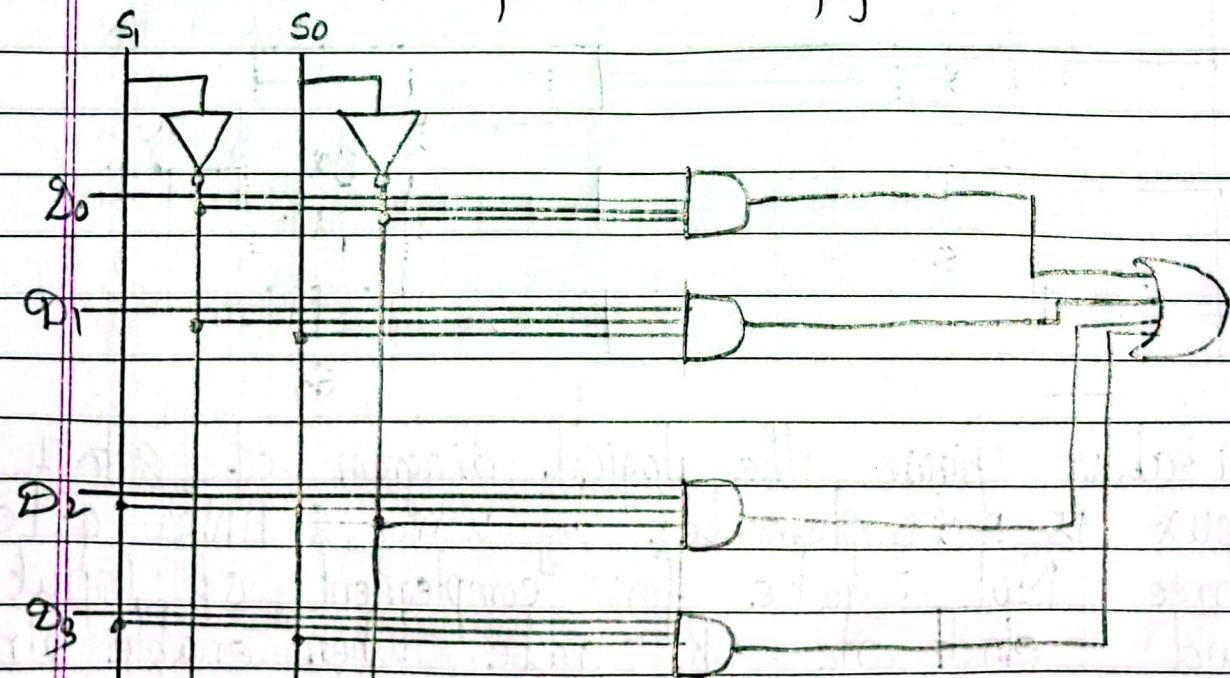
of the four input line to connect the output line.

The truth table of a 4×1 MUX is shown below:

S_1	S_0	Y
0	0	D_0
0	1	D_1
1	0	D_2
1	1	D_3

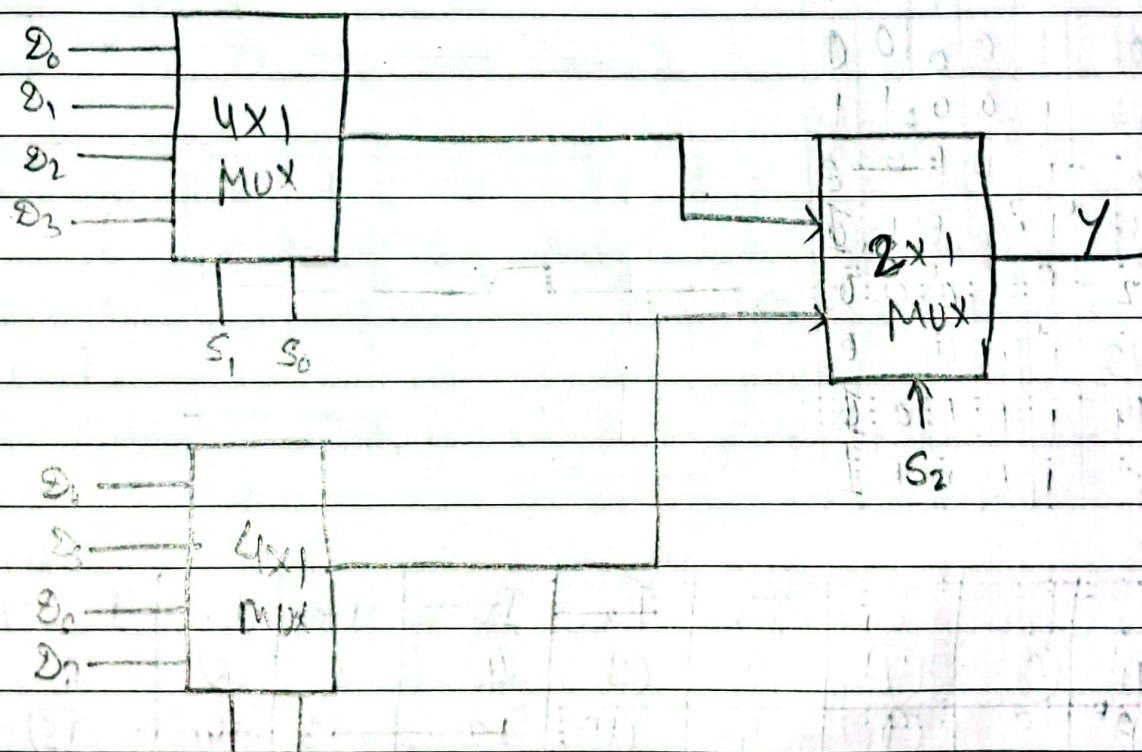
$$\therefore Y = S_1' \cdot S_0' \cdot D_0 + S_1' \cdot S_0 \cdot D_1 + S_1 \cdot S_0' \cdot D_2 + S_1 \cdot S_0 \cdot D_3$$

Above data is feeded in figure.



Design 8x1 mux using lower mux

S_2	S_1	S_0	Y
0	0	0	D_0
0	0	1	D_1
0	1	0	D_2
0	1	1	D_3
1	0	0	D_4
1	0	1	D_5
1	1	0	D_6
1	1	1	D_7



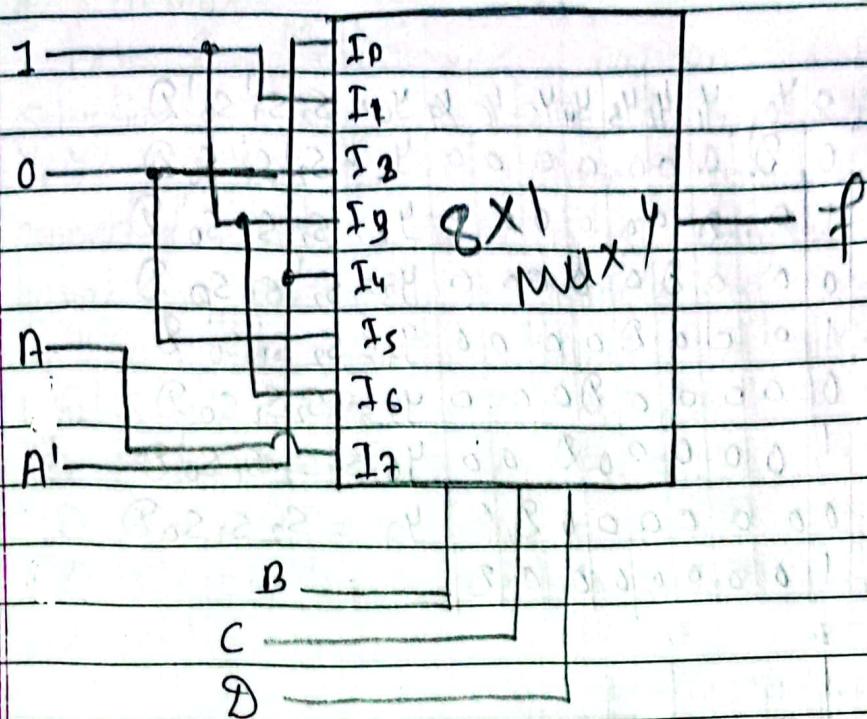
→ In figure selection line S_1, S_0 are applied in 4×1 Multiplexers. The data input in upper MUX from D_0 to D_3 and data input in lower MUX from D_4 to D_7 . Each 4×1 Multiplexer produce an output based on value of selection line. S_2 is applied to 2×1 Mux.

Q Design Multiplexer for give boolean function
 $f = AB'C'D'$

SOL

Min	A	B	C	D	f
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	0
3	0	0	1	1	1
4	0	0	0	0	1
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	1
15	1	1	1	1	1

	I ₀	I ₁	I ₂	I ₃	I ₄	I ₅	I ₆	I ₇
A'	①	②	2	③	④	5	⑥	7
A'	8	9	10	11	12	13	14	15
A'	1	0	1	A'	0	1	1	A



Demultiplexer:

A combinational circuit that performs the reverse operation of Multiplexer. It has single input, 'n' selection line and maximum of 2^n outputs. The address input determine which data output is going to have the same value as the data input. The other data outputs will have the value 0.

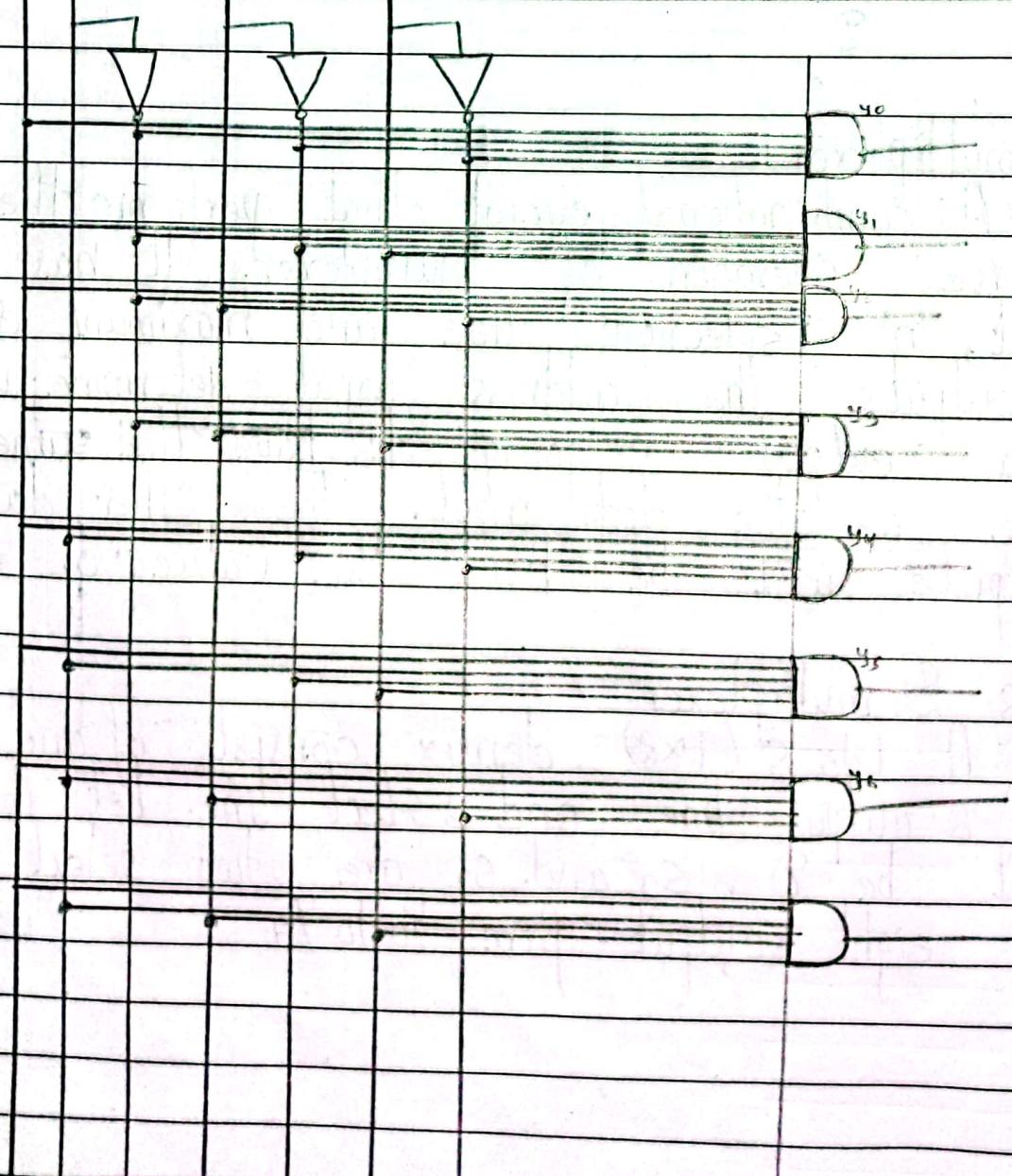
1x8 Demultiplexer

A 1 to 8 (1x8) demux consist of one input and 8 output line and 3 select line. Let the input be D, S₁ and S₂ are two select line and eight output from Y₀ to Y₇

Truth table

Input	S_2	S_1	S_0	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	$y_0 = S_2'S_1S_0'D$
0	0	0	0	0	0	0	0	0	0	0	0	$y_1 = S_2'S_1.S_0'D$
0	0	0	1	0	0	0	0	0	0	0	0	$y_2 = S_2'S_1.S_0'D$
0	0	1	0	0	0	0	0	0	0	0	0	$y_3 = S_2'S_1.S_0'D$
0	0	1	1	0	0	0	0	0	0	0	0	$y_4 = S_2S_1S_0'D$
0	1	0	0	0	0	0	0	0	0	0	0	$y_5 = S_2S_1S_0'D$
0	1	0	1	0	0	0	0	0	0	0	0	$y_6 = S_2S_1S_0'D$
0	1	1	0	0	0	0	0	0	0	0	0	$y_7 = S_2S_1S_0'D$
0	1	1	1	0	0	0	0	0	0	0	0	

Q S_2 S_1 S_0

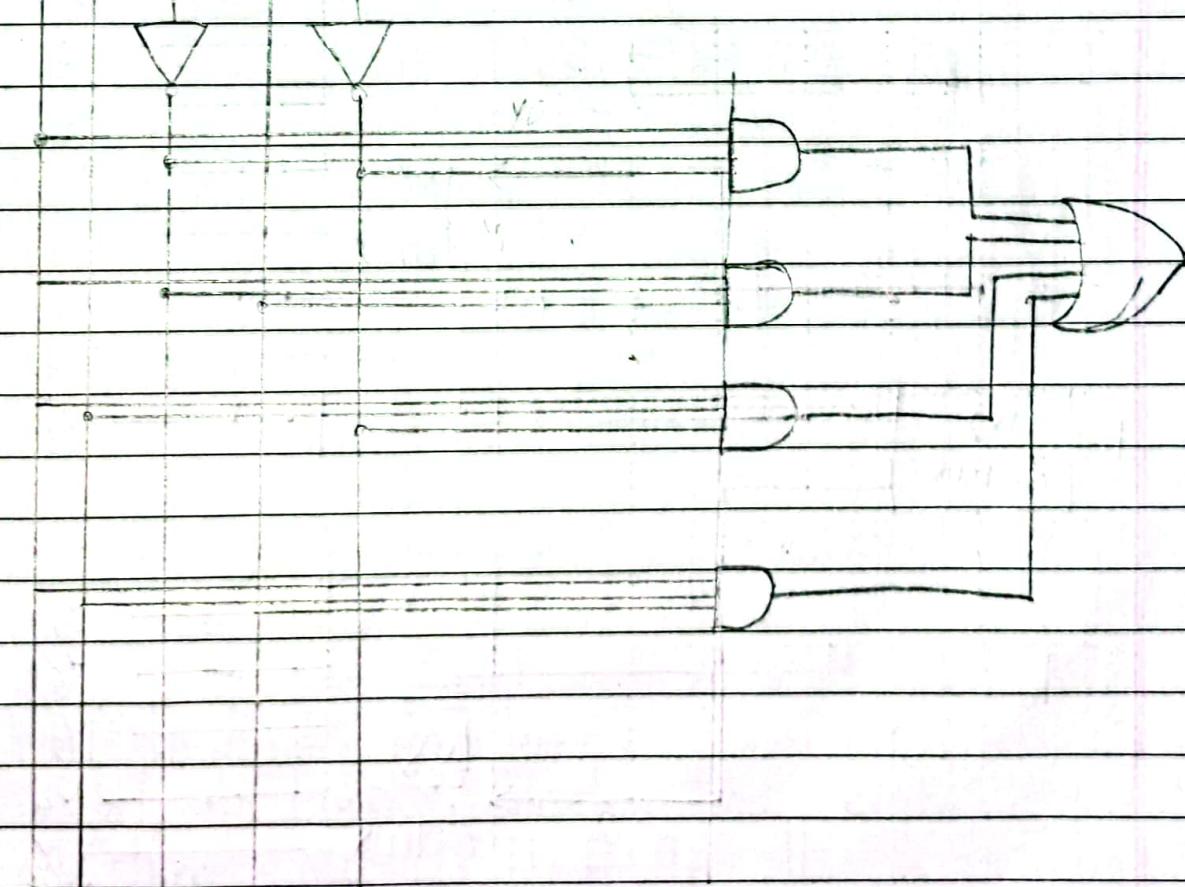


4x4 demux :

1x4 De-Multiplexer has one input D , two selection lines S_1 & S_0 and four output y_3 , y_2 , y_1 & y_0 . The single input D will be connected to one of four output, y_3 to y_0 based on the value of Selection line S_1 & S_0 . The truth table of 1x4 DeMUX is:

Input $S_1 S_0$	y_0	y_1	y_2	y_3
$0 0$	D	0	0	0
$0 1$	0	D	0	0
$1 0$	0	0	D	0
$1 1$	0	0	0	D

④ S_1 S_0

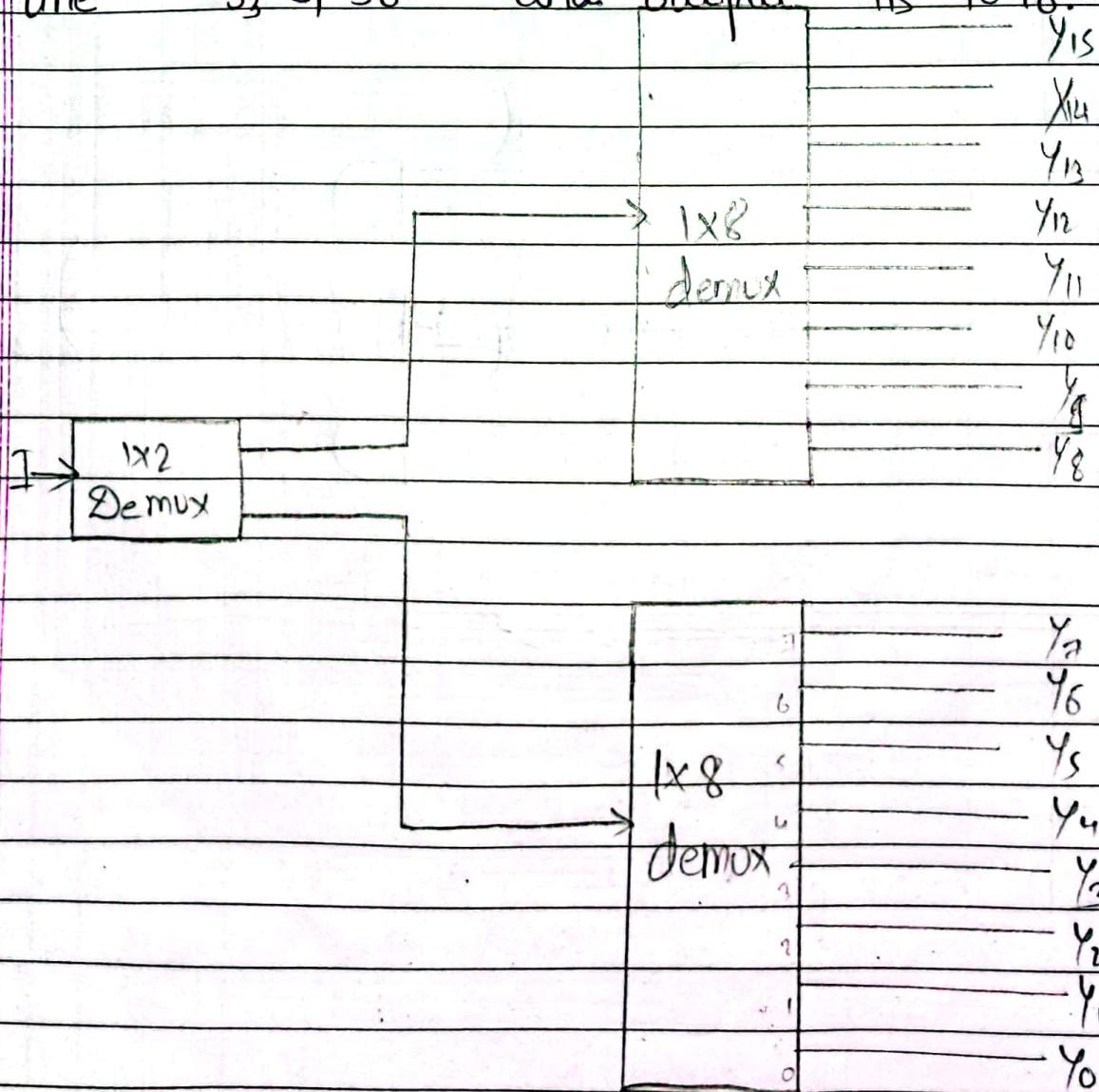


1x16

In this section let us implement 1×16 Demux using 1×8 Demux and 1×2 Demux.

~~There are three~~ In 1×16 de Mux has single input, four selection line and sixteen output. for 1×8 demux in second stage in order to get the final sixteen output. The number of input in second stage is two. We require 1×2 demux in first stage so that the output of first stage will be input of second stage.

Let 1×16 De-mux has one input, four selection line s_3, s_2, s_1, s_0 and output y_{15} to y_0 .



- Encoder

An Encoder is combinational circuit that performs the reverse operation of decoder. It has maximum of 2^n input lines and 'n' output lines. It will produce a binary code which active high.

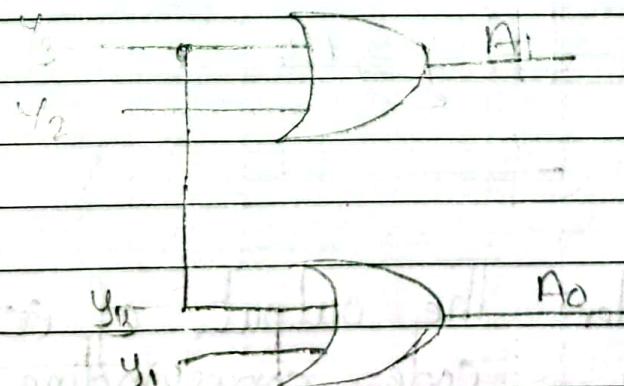
4 to 2 encoder

y_3	y_2	y_1	y_0	A_1	A_0
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	0	1	0
1	0	0	0	1	1

from Above table

$$A_1 = y_3 + y_2 \quad A_0 = y_3 + y_1$$

The circuit diagram is



Octal to Binary Encoder

Octal to Binary Encoder has eight input, y_7 to y_0 and three output A_2 , A_1 , A_0 . It is nothing but 8 to 3 encoder. Only one of these eight input can be '1' in order to get the respective binary code.

y_7	y_6	y_5	y_4	y_3	y_2	y_1	y_0	A_2	A_1	A_0
0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	1	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	1	1	0	
1	0	0	0	0	0	0	1	1	1	1

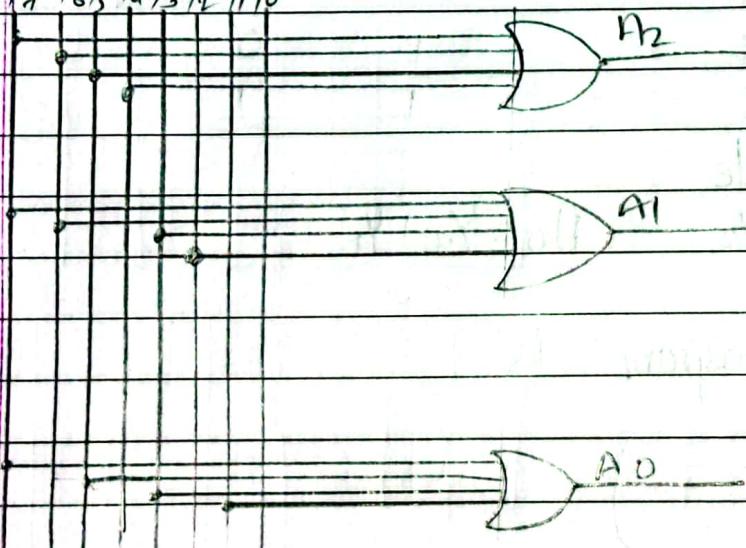
$$A_2 = y_7 + y_6 + y_5 + y_4$$

$$A_1 = y_7 + y_6 + y_3 + y_2$$

$$A_0 = y_7 + y_5 + y_3 + y_1$$

We can implement it in circuit diagram

$y_7 \ y_6 \ y_5 \ y_4 \ y_3 \ y_2 \ y_1 \ y_0$



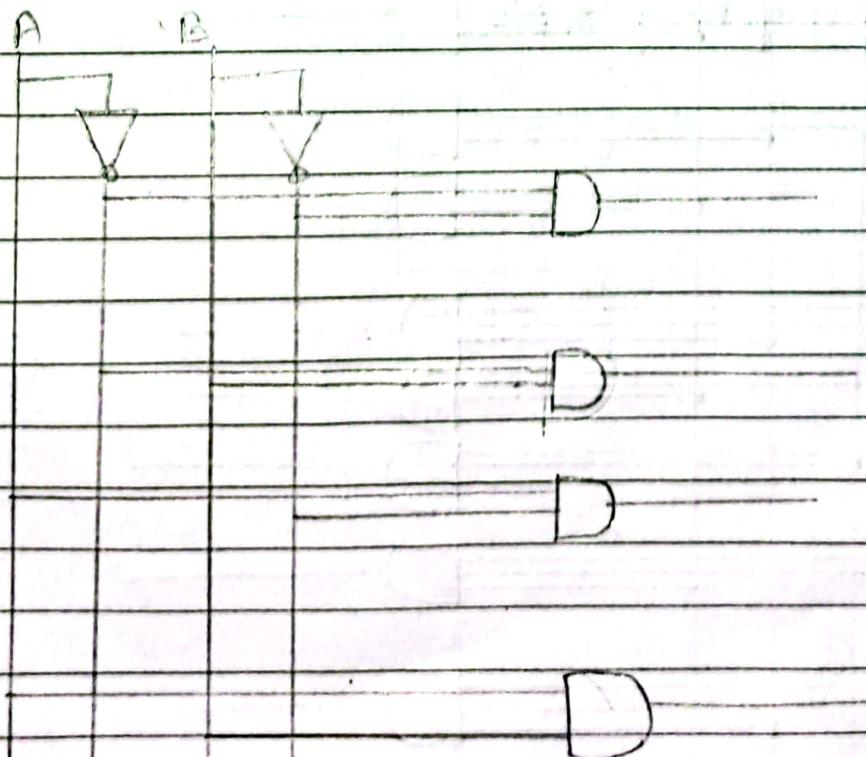
Priority encoder: The output of encoder will be the binary code corresponding to active high input and which has higher priority is called as priority encoder.

Decoder: Decoder is a combinational circuit that has ' n ' input line and maximum of 2^n output line. One of these outputs will be active high based on the combination of inputs present. When the decoder is enabled. That means decoder detects a particular code.

* **2 to 4 decoder:** Let 2 to 4 decoder has two input A_1 and A_0 and four output P_3 , P_2 , P_1 & P_0 . One of these four output will be '1' for each combination of input when enable E is 1.

Truth table

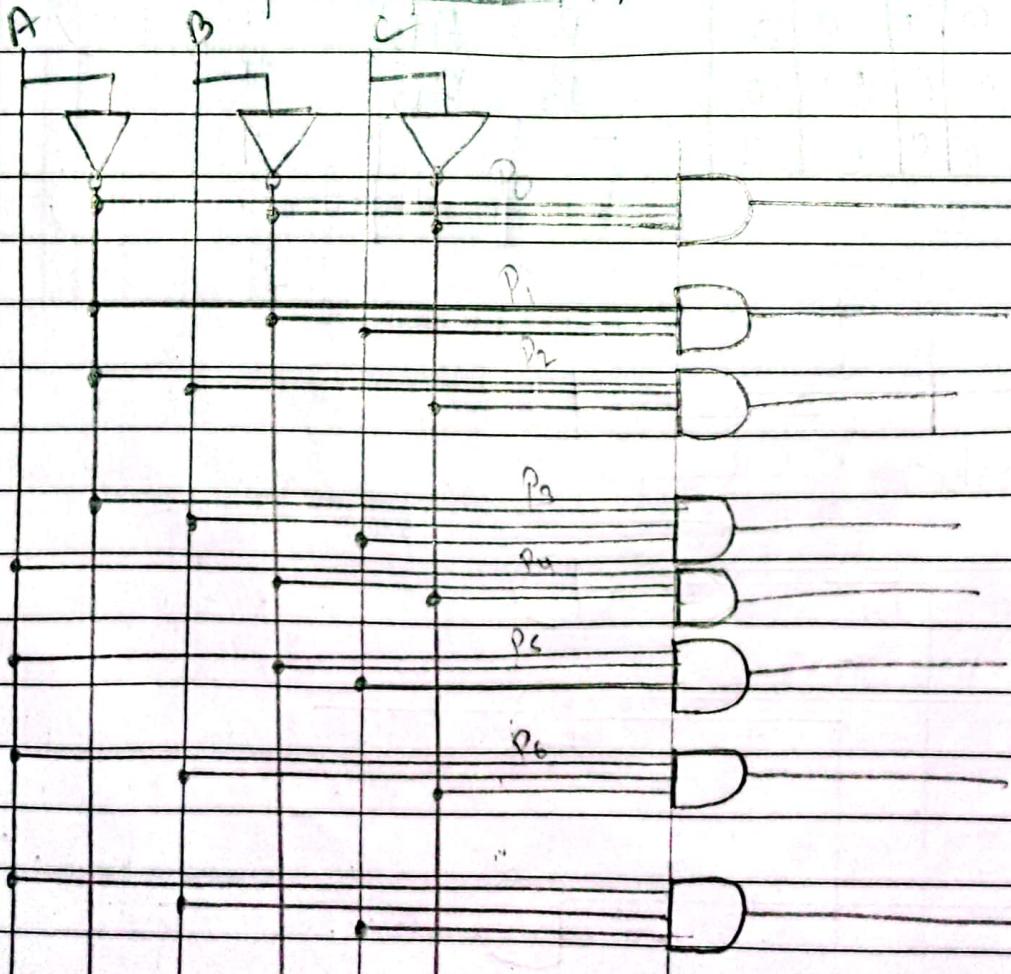
A	B	P_0	P_1	P_2	P_3	$P_0 = A'B'$
0	0	1	0	0	0	$P_1 = A'B$
0	1	0	1	0	0	$P_2 = AB'$
1	0	0	0	1	0	$P_3 = AB$
1	1	0	0	0	1	



- 3-8 line decoder:

let 3 to 8 line decoder has three input A, B, C, and eight output put P_0 to P_7 . one of these eight output will be 1. Therefore the above Truth table is structured as :

A	B	C	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
0	0	0	1	0	0	0	0	0	0	$P_0 = A'B'C'$
0	0	1	0	1	0	0	0	0	0	$P_1 = A'B'C$
0	1	0	0	0	1	0	0	0	0	$P_2 = A'BC'$
0	1	1	0	0	0	1	0	0	0	$P_3 = A'BC$
1	0	0	0	0	0	0	1	0	0	$P_4 = AB'C'$
1	0	1	0	0	0	0	0	1	0	$P_5 = AB'C$
1	1	0	0	0	0	0	0	0	1	$P_6 = ABC'$
1	1	1	0	0	0	0	0	0	1	$P_7 = ABC$



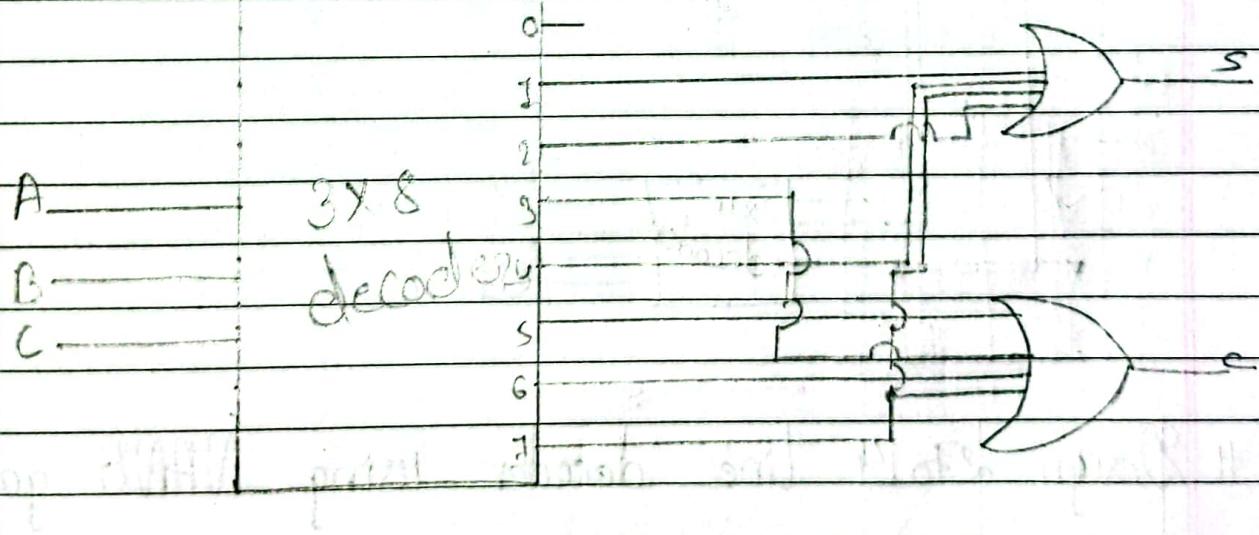
* Design a full adder using decoder and two OR gate

A	B	D	S	Cout
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

from truth table $s(ABD) = \Sigma (1, 2, 4, 7)$

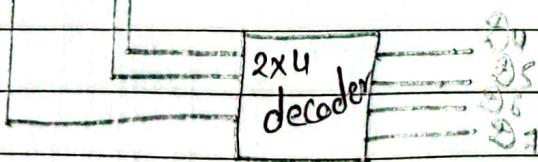
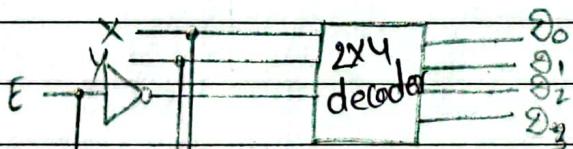
$c(ABD) = \Sigma (3, 5, 6, 7)$

Since, there are three input and total of eight minterms. So, we need 3 to 8 line decoder. The decoder generate the eight minterms for AB & C. The OR gate for output sum (S) & carry is (C).



④ Design 3 to 8 line decoder using lower decoder

X	Y	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7
0	0	0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	0
0	1	0	0	1	0	0	0	0	0
0	1	1	0	0	1	0	0	0	0
1	0	0	0	1	0	0	0	0	0
1	0	1	0	0	0	1	0	0	0
1	0	0	0	0	1	0	0	0	0
1	1	0	0	0	0	1	0	0	0
1	1	1	0	0	0	0	1	0	0
1	1	0	0	0	0	0	1	0	0
1	1	1	0	0	0	0	0	1	0

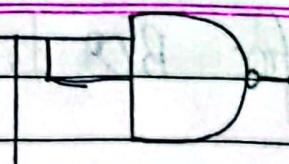


Design 2 to 4 line decoder using NAND gate

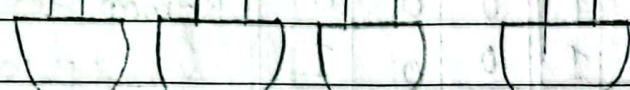
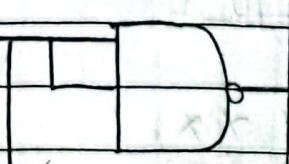
Truth table

X	Y	D_0	D_1	D_2	D_3	D_0	$x'y'$
0	0	1	0	0	0	D_0	$x'y'$
0	1	0	1	0	0	D_1	$x'y$
1	0	0	0	1	0	D_2	$x'y'$
1	1	0	0	0	1	D_3	$x'y$

X



Y

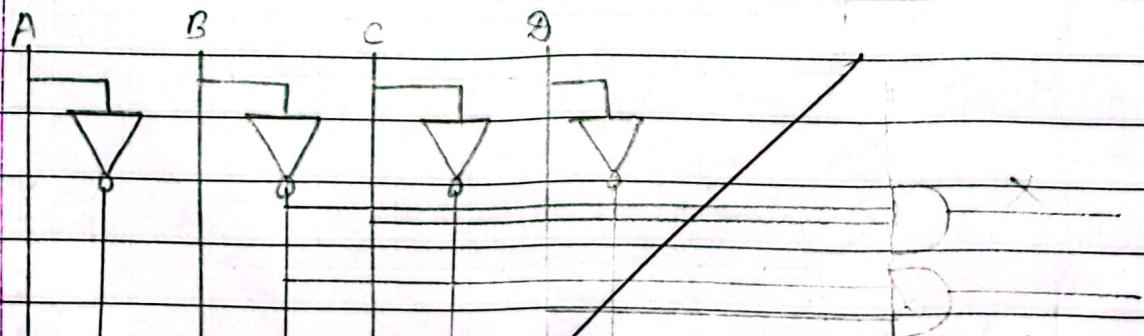
 D_0 D_1 D_2 D_3

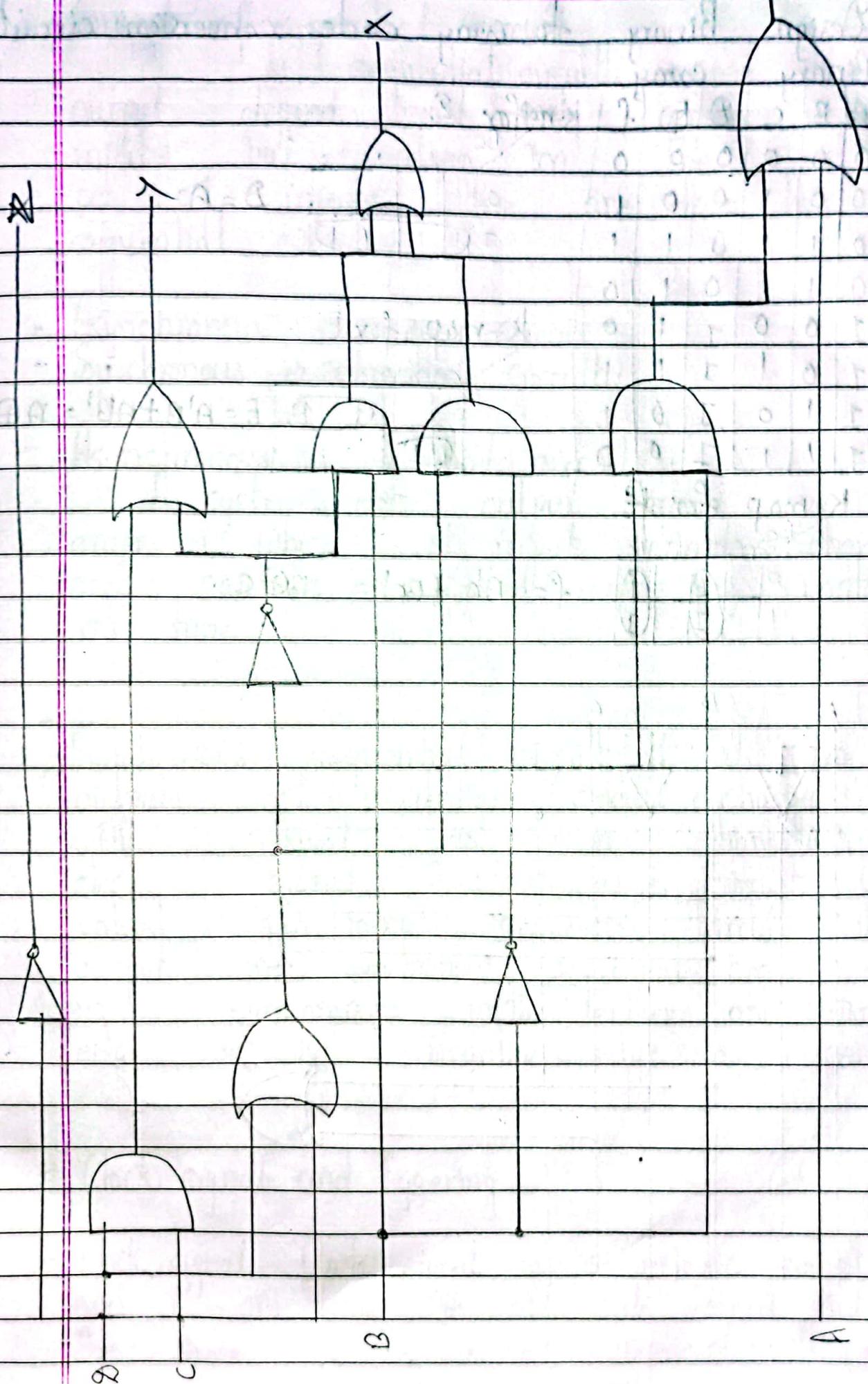
Design a circuit diagram for BCD to excess-3 code converter.

Input BCD	Output excess-3 code			K-map for -x				
A B C D	W	X	Y	Z	A B C D	W	X	
0 0 0 0	0	0	1	1	00	1	1	
1 0 0 1	0	1	0	0	01	1		
2 0 0 1 0	0	1	0	1	11	x	x	
3 0 0 1 1	0	1	1	0	10	1	x	
4 0 1 0 0	0	1	1	1	$x = B'C + B'D + BC'D' = B'(C+D) + BC'D'$			
5 0 1 0 1	1	0	0	0	K-map for Y			
6 0 1 1 0	1	0	0	1	AB C D	01	11	10
7 0 1 1 1	1	0	1	0	00	1	1	
8 1 1 1 1	1	0	1	1	01	1	1	
9 1 0 0 1	1	1	0	0	11	x	x	x
					10	1	x	x

for k-map z

AB	C D	00	01	11	10
00	1	1			
01	1		1		
11	x	x	x		
10	x		x		





Design Binary to Gray code conversion circuit

Binary Gray

A B C D E F k-map for D

0	0	0	0	0	0
---	---	---	---	---	---

0	0	1	0	0	1
---	---	---	---	---	---

0	1	0	0	1	1
---	---	---	---	---	---

0	1	1	0	1	0
---	---	---	---	---	---

1	0	0	1	1	0
---	---	---	---	---	---

1	0	1	1	1	1
---	---	---	---	---	---

1	1	0	1	0	1
---	---	---	---	---	---

1	1	1	1	0	0
---	---	---	---	---	---

k-map for E

00	01	11	10
----	----	----	----

01	11	11	10
----	----	----	----

11	11	11	10
----	----	----	----

11	11	11	10
----	----	----	----

$$D = A$$

$$E = A'B + AB' = A \oplus B$$

K-map for F

A	Bc
---	----

00	01	11	10
----	----	----	----

01	11	11	10
----	----	----	----

11	11	11	10
----	----	----	----

$$F = B'C + BC' = B \oplus C$$

