

Logic Gate & Boolean Algebra with Logics

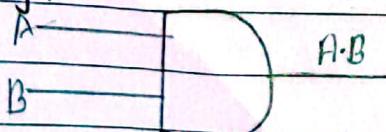
A logic gate is a basic building block of a digital circuit that has two inputs and one output. The relationship between the input and the output is based on a certain logic. These gates are implemented using electronic switches like transistors, diodes.

The basic logic gate are categorized into seven: AND, OR, XOR, NAND, NOR, XNOR and NOT. These logic gates with their logic gate symbol and truth table are explained below:

Basic logical Gate:

- **AND Gate:** The AND gate is a digital gate with inputs and output, which perform logical Conjunction based on the combination of its input. The output of this gate is high only when all the inputs are high otherwise output will be low. The symbol and truth table of an AND gate with two input is shown:

Figure

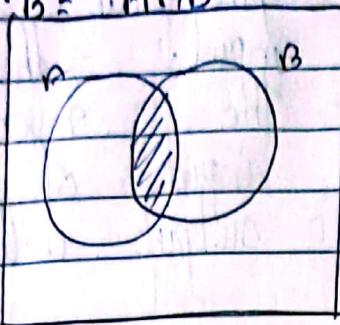


Truth table

Input		Output
A	B	f
0	0	0
0	1	0
1	0	0
1	1	1

Set diagram:

$$A \cdot B = A \cap B$$



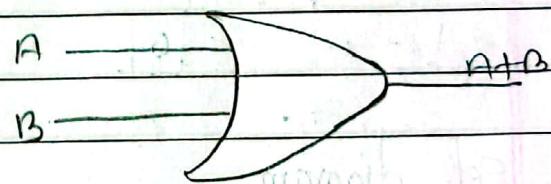
OR Gate: The OR gate is a digital logic gate with one input and one output, that performs a logical conjunction based on the combination of its inputs. The output of the OR gate is high only when one or more inputs are true high. Otherwise output will be low.

Algebraic expression: $f = A + B$

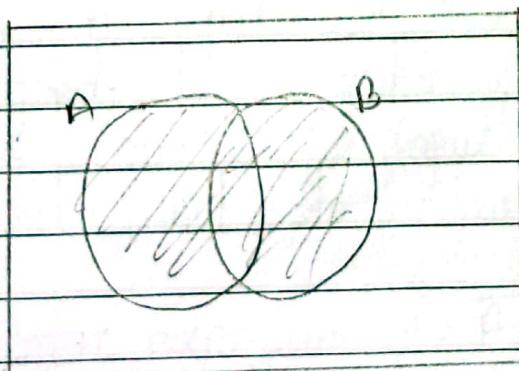
Truth table

Input		Output
A	B	f
0	0	0
0	1	1
1	0	1
1	1	1

figure:



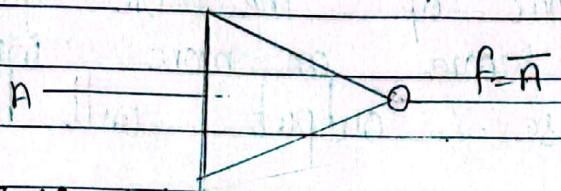
$$\text{Set: } A \cup B = A + B$$



Not Gate: The Not gate is a digital logic gate with one input and one output that an inverter operation of the input. The output of Not gate is the reverse of the input. When the input of the Not gate is high then output will be low and viceversa.

Algebraic expression: $f = A'$

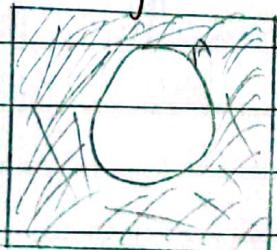
Symbol



Truth table

Input	Output
A	$f = A'$
0	1
1	0

Set diagram

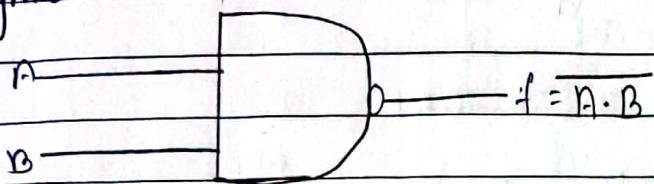


Universal Gate:

- **NAND Gate:** The NAND Gate is a digital logic gate with 'n' input and one output. This performs the operation of the AND gate followed by the operation of NOT gate. NAND gate is designed by combining AND and NOT gates. If the input of NAND gate is high, then the output of - gate will be low.

Algebraic expression: $f = \overline{A \cdot B}$

Symbol :



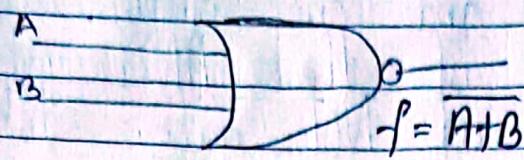
Truth table

Input		Output	
A	B	$A \cdot B$	$f = \overline{A \cdot B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

- **NOR Gate:** The NOR gate is a digital logic gate with n input and one output. NOR gate is designed by combining the OR and NOT gate. When one of input of NOR gate is high the output will be low.

Algebraic expression $f = \overline{A + B}$

Symbol



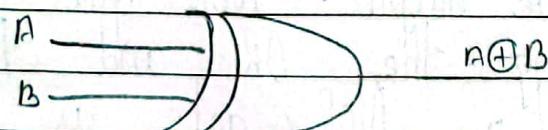
Truth table

Input	Output	Output	
A	B	$A+B$	$f = A+B$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

Other logic Gates:

- Exclusive-OR Gate: The Exclusive-OR Gate is digital logic gate with two inputs and one output. The short form of this gate is Ex-OR. It performs based on the operation of OR-gate. If any one of the inputs of the gate is high, then the output of Ex-OR gate will be high.

Algebraic expression: $A'B + AB' (A \oplus B)$
Symbol



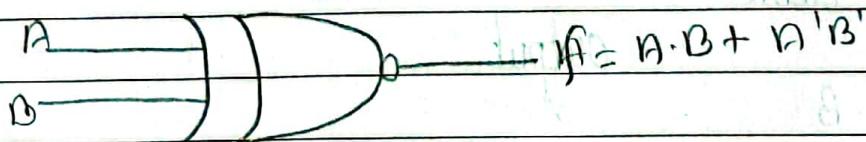
Truth-table:

Input						output $A'B + AB'$	
A	B	A'	B'	$A'B$	AB'		
0	0	1	1	0	0	0	0
0	1	1	0	1	0	1	1
1	0	0	1	0	1	1	1
1	1	0	0	0	0	0	0

- Exclusive-NOR Gates: The Exclusive-NOR gate is a digital logic gate with two input and one output. It is also known as Ex-NOR. When both the inputs of this gate are high, then the output will be high. But if any one of the input is high, the output will be low.

Algebraic expression: $F = A \cdot B + A'B' / A \odot B$

Symbol.



Truth table

Input

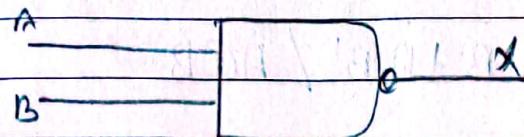
A'	B	A'	B'	$A'B'$	$A \cdot B$	$f = A \cdot B + A'B'$
0	0	1	1	1	0	1
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	1	0	0	0	1	1

Universal Gate: Universal gate are the logic gate which are capable of implementing any Boolean function without requiring any other type of gate. Universal gate can realize all the binary operation and all the basic logic gate can be derived from them, that is why they are called Universal gate.

NAND gate is a universal gate.

NAND gate can be connected to form any other logic gate. NAND gate can be connected to form Inverter, And and OR gates.

Logic symbol

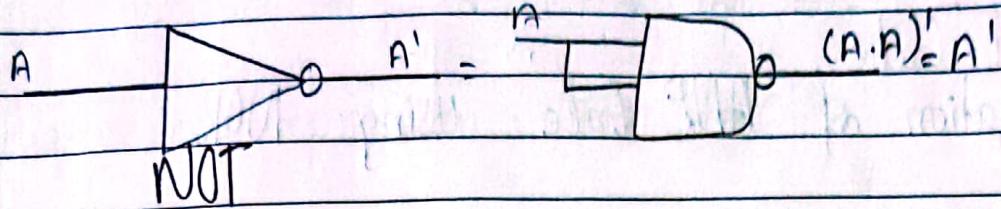


Truth Table

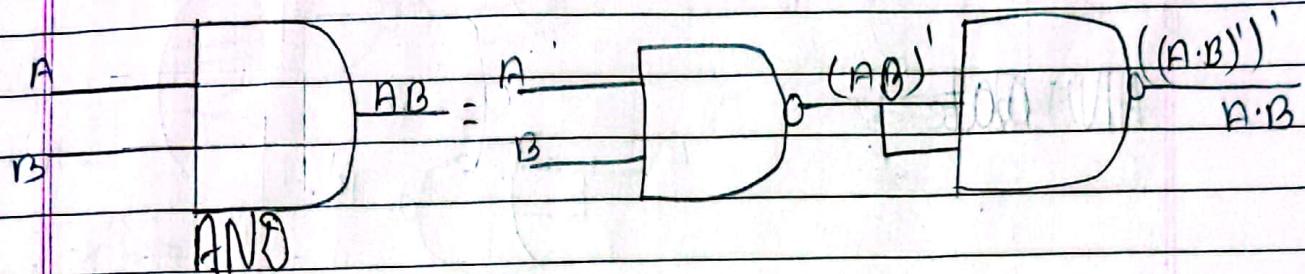
Input	Output	
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Implementing
Releas

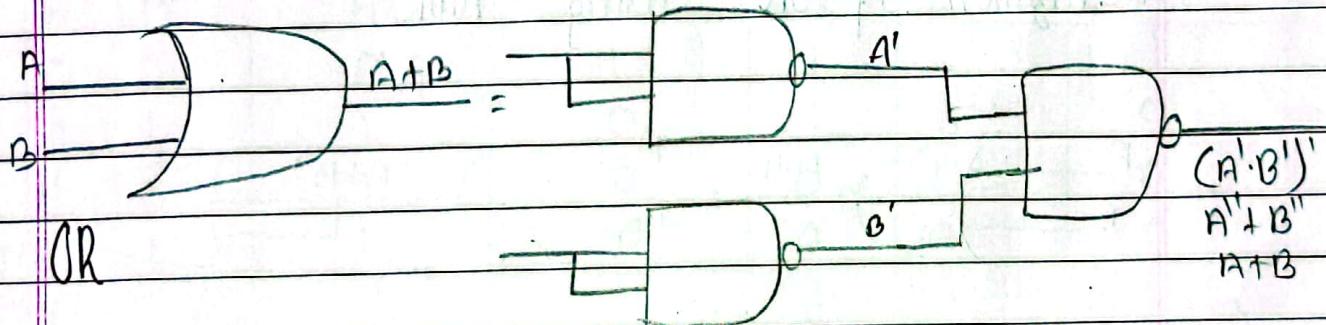
- Realization of Not gate from NAND



- Realization of AND gate from NAND



Realization of OR gate using NAND

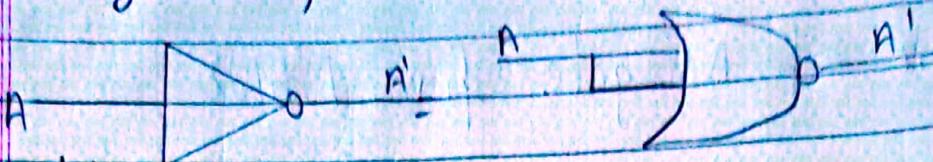


NOR Gate:

NOR Gate is constructed by connecting a NOT Gate at the output terminal of the OR Gate.

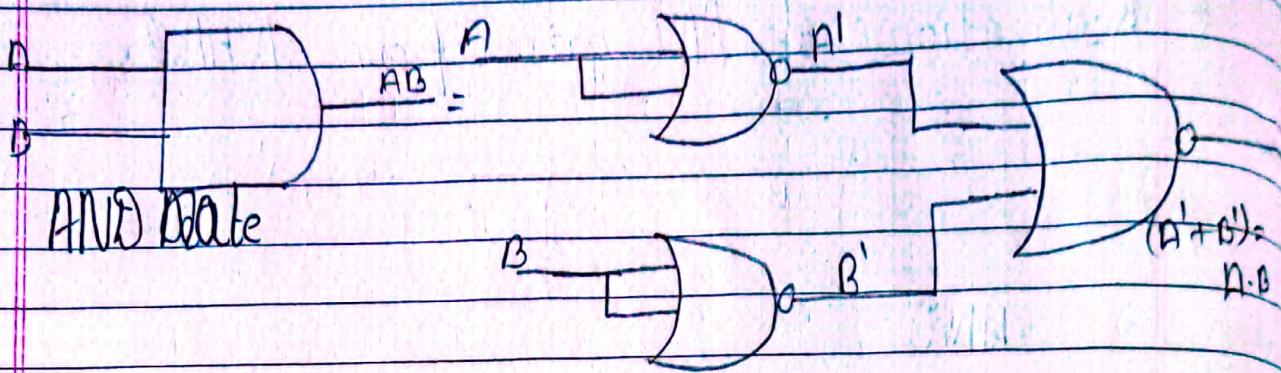
NOR Gate \Leftrightarrow OR Gate + NOT Gate.

- Realization of NOT Using NOR

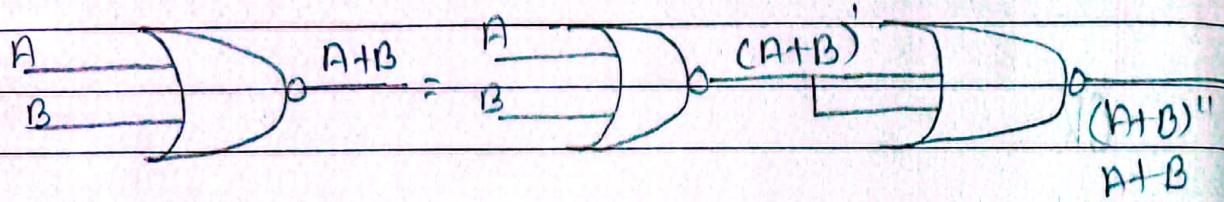


NOT Gate

- Realization of AND Gate Using NOR



Realization of OR using NOR



From above analysis all three basic gate can be independently prepared by NAND and NOR gate. So they any gate can be derived from NAND or NOR gate.

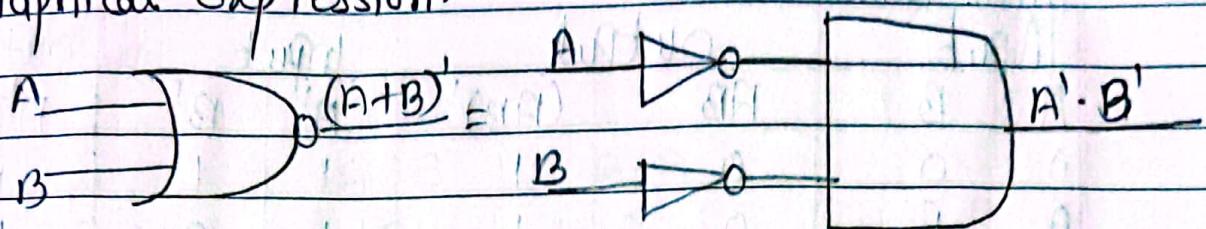
De-Morgan's law

- first law:

The complement of a sum is equal to the product of the component.

Algebraic expression: $(A+B)' = A' \cdot B'$

Graphical expression:



Truth table:

Input		Output	Input		Output	
A	B	$A+B$	$(A+B)'$	A'	B'	$A' \cdot B'$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Conclusion:

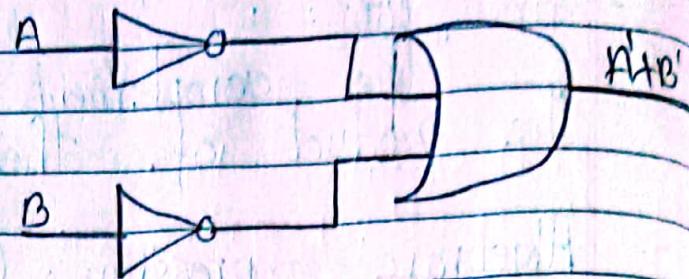
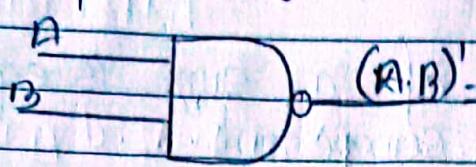
Comparing the value of $(A+B)'$ and $A' \cdot B'$ from truth table, both are equal, hence proved.

+ Second law

Statement: "The component of product is equal to the sum of complement"

Algebraic expression: $(A \cdot B)' = A' + B'$

Graphical symbol:



Truth Table

Input		Output		Input		Output	
A	B	AB	$(AB)'$	A'	B'	$A' + B'$	
0	0	0	1	1	1	1	
0	1	0	1	1	0	1	
1	0	0	1	0	1	1	
1	1	1	0	0	0	0	

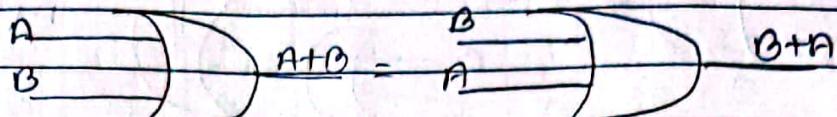
Conclusion: Comparing the value of $(AB)'$ and $A' + B'$ from truth table both are equal, hence proved.

Laws of Boolean Algebra

I. Commutative law:

a. $(A+B) = (B+A)$

(graphical symbol)



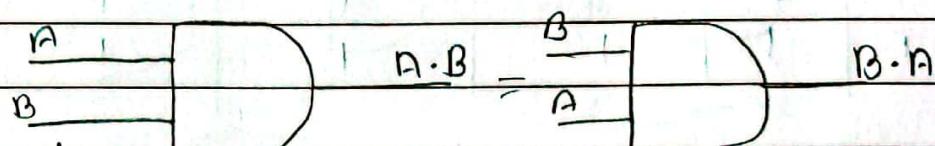
Truth table

Input		Output	
A	B	$A+B$	$B+A$

0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

b. $(A \cdot B) = (B \cdot A)$

(graphical symbol):



Truth table:

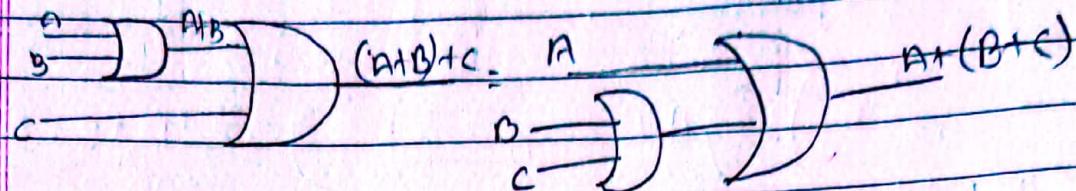
Input		Output	
A	B	$A \cdot B$	$B \cdot A$

0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1

2. Associative law

Algebraic expression: $(A+B)+C = A+(B+C)$

Graphical symbol:



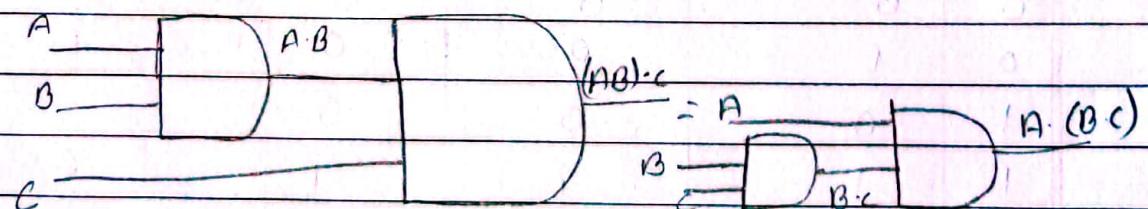
Truth table

Input

A	B	C	$A+B$	$(A+B)+C$	$B+C$	$A+(B+C)$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

Graphical symbol



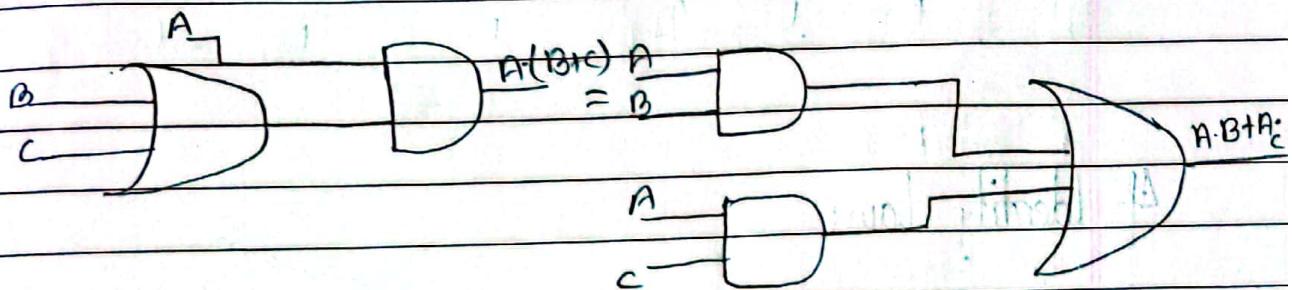
Trillable

A	B	C	$A \cdot B$	$(A \cdot B) \cdot C$	$B \cdot C$	$A \cdot (B \cdot C)$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	1	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

3. Distributive law

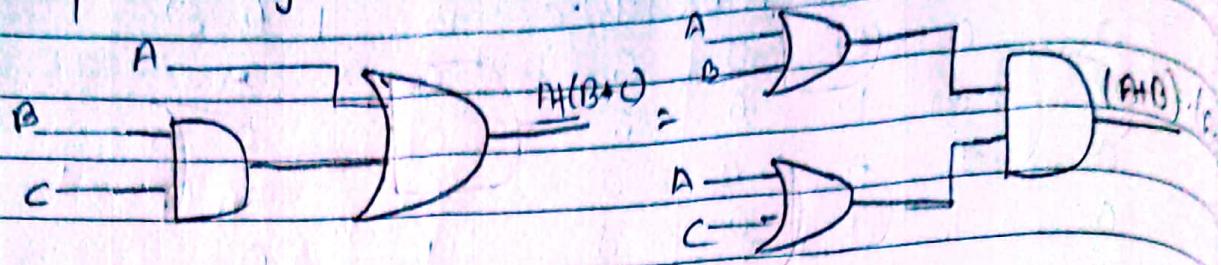
Graphical Symbol:

$$A \cdot (B + C) = A \cdot B + A \cdot C$$



Truth table

$\text{ii} \quad A + (B \cdot C) = (A + B) \cdot (A + C)$
 Graphical symbol



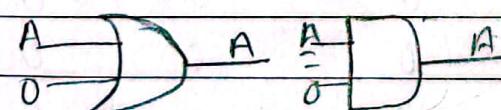
Truth table

A	B	C	B.C	$A + (B \cdot C)$	$A + B$	$A + C$	$(A + B) \cdot (A + C)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	1	0
0	1	1	0	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

4 Identity law:

$\text{ii} \quad A + 0 = A \quad [\because A = IA]$

Graphical symbol

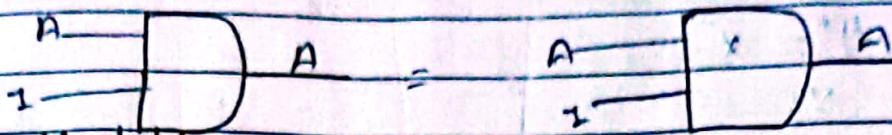


Truth table

A	$A + 0$
0	0
1	1

$$\text{H} \quad A \cdot I = A$$

Graphical symbol:



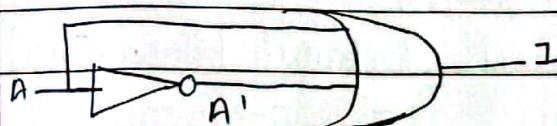
Truth table

A	I	$A \cdot I$
0	1	0
1	1	1

$$5. \text{ Complement Law}$$

$$\text{H} \quad A + A' = I$$

Graphical symbol

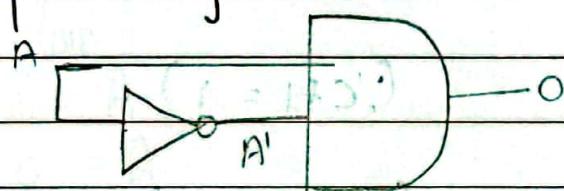


Truth table

A	A'	$A + A'$
0	1	1
1	0	1

$$+ \quad A \cdot A' = 0$$

Graphical symbol:



A	A'	$A \cdot A'$
0	1	0
1	0	0

Simplification Theorem

- * $XY + X'Y' = X$
- * $(X+Y)(X+Y)' = X$
- * $X + XX' = X$
- * $X(X+Y) = X$
- * $(X+Y')Y = XY$
- * $XY' + Y = X+Y$

Consensus Theorem

$$XY + X'Z' + YZ = XY + X'Z$$

$$(X+Y) + (X'+Z)(Y+Z) = (X+Y)(X'+Z)$$

Duality

~~Ex.~~ $(X+Y+Z) = XYZ$

$$(A+B)(A+C) = A+BC$$

$$(A+B)(A+C) =$$

$$= AA + AC + AB + BC \quad (\text{distributive law})$$

$$= A + AC + AB + BC \quad (\because AA = A)$$

$$A(1+C) + B(A+C) \quad (\because C+1 = 1)$$

$$A + AB + BC \quad (\because \text{factoring})$$

$$A(1+B) + BC \quad (\because 1+B = 1)$$

$$A + BC \quad (A \cdot 1 = A)$$

$$H) A + \bar{A}B = A + B$$

$$= (A + AB) + \bar{A}B \quad (\because A = A + AB)$$

$$= (A \cdot A + AB) + \bar{A}B \quad (\because A = A \cdot A)$$

$$= (A + AB) + \bar{A}B \quad (\because A \cdot A = 0)$$

$$= (A + A') A(A + B) + \bar{A}(A + B) \quad (\because A(A + B) = AA + AB)$$

$$(A + A')(A + B)$$

$$1 \cdot (A + B)$$

Other law of boolean algebra

1. Complement law:

A complement = A' i.e input 1 then output 0.

2 AND law

$$- A \cdot A = A$$

$$- A \cdot 0 = 0$$

$$- A \cdot 1 = A$$

$$- A \cdot A' = 0$$

3 OR law

$$i) A + A = A$$

$$ii) A + 0 = A$$

$$iii) A + 1 = 1$$

$$A + A' = 1$$

4. Distributive law

$$A \cdot (B+c) = AB + Ac$$

$$A+(B \cdot c) = (A+B)(A+C)$$

Example

$$A+A'B$$

$$(A+A') \cdot (A+B)$$

$$I = (A+B)$$

$$\text{let } (A+B) = X$$

$$I \cdot X$$

$$X$$

$$A+B$$

$$\# BA'c' + B'A'c' + Bc'$$

$$= Ac'(B+B') + Bc' \quad (B+B=1)$$

$$= Ac' + Bc'$$

$$= c' (A+B)$$

$$\# AB + AB'c + AB'c'$$

$$= AB + AB' (c+c')$$

$$= A (B+B')$$

A

$$3. (A+B+C) \cdot (A+B'C) \cdot (A+B+C')$$

$$\text{let } A+B = x$$

$$= (x+c) (A+B'+C) (x+c')$$

$$= (x+c) (x+c') (A+B'+C)$$

$$= (\cancel{x+c}) (x+c'c) (A+B'+C)$$

$$= x (A+B'+C)$$

$$(A+B) \cdot (A+B'+C)$$

$$= A \cdot A + AB' + AC + A \cdot B (B'+C)$$

$$= A + B \cdot B' + BC$$

$$= A + BC$$

$$4. (A+B) (A+B') (A'+B) (A'+B')$$

SOLY

$$= (A+B \cdot B') (A'+B \cdot B')$$

$$= (A+0) \cdot (A'+0)$$

$$= A \cdot A'$$

$$= 0$$

Redundancy Theorem:

Condition

- i) Three variable
- ii) Each variable is repeated twice
- iii) One variable is complemented

Solution

- iv) Take complimented variable

$$5. AB + A'c + Bc \\ = AB + A'c$$

$$6. XY + XZ + XZ' \\ = XZ + XZ'$$

5. Commutative law

$$A \oplus B = B \oplus A$$

$$A \cdot B = B \cdot A$$

6. Associative law

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A + B) + C = A + (B + C)$$

7. Demorgan's theorem

$$(A \oplus B)' = A' \oplus B'$$

$$(A \cdot B)' = A' + B'$$

8. Priority:

- i) Braces ii) NOT iii) AND iv) OR

A	B	C	$B+C$	$A \cdot (B+C)$	AB	AC	$AB+AC$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	0	0	1
1	1	1	1	1	1	1	1

Boolean algebra it representation

- i) SOP
- ii) POS

- SOP (Sum of product):

If only consider the minterms which have high value.

SOP is symbolized by Σ and minterms is represented by small m.

Example

Truth table

Minterms	Digit	A	B	C	f
$A'B'C$	0	0	0	0	0
$A'B'C'$	1	0	0	1	0
$A'B'C'$	2 (m ₂)	0	1	0	1
$A'B'C$	3	0	1	1	0
$A'B'C'$	4 (m ₄)	1	0	0	0
$A'B'C$	5 (m ₅)	1	0	1	1
$A'B'C'$	6 (m ₆)	1	1	0	1
$A'B'C$	7 (m ₇)	1	1	1	1

$$F = m_2 + m_4 + m_5 + m_6 + m_7$$

Standard form contains all the variable in its minterms.

$$\begin{aligned}
 & A'B'C' + AB'C' + AB'C + ABC' + ABC \\
 & = A'B'C' + AB'(C' + c) + AB(c' + c) \\
 & = A'B'C' + AB' \\
 & = A'B'C' + A(B' + B) \\
 & = A'B'C' + A
 \end{aligned}$$

Let $Bc' = x \quad = A + A'B = A + B$

$$A + A'Bc'$$

$$A + A'x$$

$$A + x$$

$A + Bc'$ (Minimal sop format does not contain all the variable in its minterms)

$$* f(A, B, C) = m_2 + m_4 + m_5 + m_6 + m_7$$

Example.

$$f(A, B) = \Sigma m(0, 2, 3)$$

Digital	A	B	f
0	0	0	1
1	0	1	0
2	1	0	1
3	1	1	1

$$\begin{aligned}
 f &= A'B' + AB' + AB \\
 &= B'(A' + A) + AB \\
 &= B' + AB \quad (\because A + A'B = A + B)
 \end{aligned}$$

$A' + B'$

SOP form : having all the variable

1st type $F = \sum m(0, 1, 3)$

2nd type $f = A'B' + AB + A'B$
 $m_0 \quad 0 \quad 1 \quad 0 \quad 0$
 $m_1 \quad 1 \quad 0 \quad 1 \quad 0$
 $m_2 \quad 0 \quad 1 \quad 0 \quad 0$
 $m_3 \quad 1 \quad 1 \quad 1 \quad 0$

3rd type

Minterm	A	B	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

$$B(A' + A) + A'B'$$

$$B \cdot 1 + A'B'$$

$$B + A'B' \quad (A + A'B = A + B)$$

$$B + B'A'$$

$$B + A'$$

$$F(A, B) \in m(0, 2, 3)$$

$$m_0 + m_2 + m_3$$

$$A'B' + AB' + A'B$$

Rough

$$0 = 00 \quad A'B'$$

$$2 = 10 \quad A'B$$

$$3 = 11 = AB$$

Example $\Sigma m(1,3)$ Convert the above expression into minimal SOP form

$$F = m_1 + m_3$$

$$= A'B + AB$$

$$= B$$

$$\begin{matrix} 0 \\ A'B \end{matrix} \quad \begin{matrix} 1 \\ AB \end{matrix}$$

Example $f(A,B,C) = \Sigma m(0,1,4,5,7)$

$$m_0 + m_1, m_4 + m_5 + m_7$$

$$0 = 000 \quad A'B'C'$$

$$1 = 001 \quad A'B'C$$

$$4 = 100 = AB'C'$$

$$5 = 101 = AB'C$$

$$7 = 111 = ABC$$

$$A'B'C' + A'B'C + AB'C' + AB'C + ABC$$

$$= A'B' + AB' + ABC$$

$$= B'(A+A') + AB'C$$

$$= B' + AB'C$$

$$= B' + B(AC)$$

$$= B' + AC$$

POS (Product of sum):

It only consider the maxterms which have low value
POS is symbolized by Π and maxterm is represented
by Capital M

Truth table

Maxterm	Digit	A	B	C	f
$A'B'C$	0	0	0	0	0
$A'B'C'$	1	0	0	1	0
$A+B'C$	2	0	1	0	1
$A+B'+C'$	3	0	1	1	0
$A+B+C$	4	1	0	0	1
$A'B+A'B'C$	5	1	0	1	1
$A'B'C'+C$	6	1	1	0	1
$A+B'+C'$	7	1	1	1	1

info

$$f(A, B, C) = M_0, M_1, M_3$$

$$f(A, B, C) = \prod M(0, 1, 3)$$

$$= (A+B+C)(A+B'+C')(A+B''+C'')$$

$$= \text{let } A+B = X$$

$$= (x+c)(x+c')(A+B'+c'')$$

$$= (x+c \cdot c') (A+B'+c'')$$

$$\text{let } B'+c' = Y$$

$$(A+B)(A+Y)$$

$$A+B Y$$

$$A+B(B'+c'')$$

$$A+B'C'$$

for Pos

$$F(A, B, C) = \Sigma m(2, 4, 5, 6, 7)$$

$$010 = A'B'C'$$

$$100 = A'B'C'$$

$$101 = AB'C'$$

$$110 = AB'C'$$

$$111 = ABC$$

$$F(A, B, C) = A'B'C' + AB'C + AB'C' + ABC$$

$$f(A, B) = \Sigma m(0, 2, 3)$$

$$= A'B' + AB' + AB$$

$$= B'(A+A') + AB$$

$$= B' + BA \quad (\because A+A' = A+A)$$

$$= B' + A$$

Q1. Simplify the following in Boolean algebra.

a. $A'c + Bc' + A'c' + C'c'$

Sol:

$$c' (A + B + A' + B')$$

$$c' (A + A' + B + B')$$

$$c'$$

$$(A + B) \cdot (A' + B')$$

b. $c(B+c)(A+B+c)$

$$c(B+c)(A+B+c)$$

let $B+c = x$

$$= c(x+A \cdot 0)(A+B+x)$$

$$= c(x+A \cdot 0)$$

$$= c(B+c)$$

$$\therefore Bc + c^2$$

$$\therefore Bc + c$$

$$= c(B+1)$$

$$c$$

b. $AB'C + A'B'C + AB'C' + A'B'C'$

Sol:

$$\begin{aligned}
 & AB'C + A'B'C + AB'C' + A'B'C' \\
 & = AB'(C+C') + A'B'(C+C') \\
 & = AB' + A'B \\
 & = B'(A+A') \\
 & = B'
 \end{aligned}$$

d. $A+B(A+B) + A(A'+B)$

$$\begin{aligned}
 & = A+AB+BB+A \cdot A'+AB \\
 & = A+AB+B+A \\
 & = A+B
 \end{aligned}$$

e. $(AB'C' + AB'C + ABC + ABC') (A+B)$

$$[(AB'(C'+C) + AB(C+C'))] (A+B)$$

$$[(AB'+AB)] (A+B)$$

$$[A(B'+B)] (A+B)$$

$$= A + (A+B)$$

$$= A$$

$$\Rightarrow F = (A \cdot B \cdot C)' + A'B'C + (AC)'$$

$$= A' + B' + C' + A'B'C + A' + C'$$

$$A' \cdot 1 + B' \cdot P' + A'B'C$$

$$A' (1 + BC) + B' + C'$$

$$A' \cdot 1 + B' + C'$$

$$(ABC)'$$

* Comparing SOP & POS

Digit	A	B	C	f
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

SOP

$$Y(A, B, C) = \sum m(0, 2, 3, 6, 7)$$

$$m_0 + m_2 + m_3 + m_6 + m_7$$

$$= A'B'C' + A'B'C + A'BC + AB'C + ABC$$

$$= A'B'C' + A'B(C' + C) + AB(C' + C)$$

$$= A'B'C' + A'B + AB$$

$$= A'B'C' + B(A' + A)$$

$$= A'B'C' + B$$

$$\text{let } A'C' = x$$

$$= xB' + B \quad (A + A'B = A + B)$$

$$= B + A'C'$$

POS

$$Y = (A, B, C) = \prod M(1, 4, 5)$$

$$M_1, M_4, M_5$$

$$(A + B + C') \cdot (A' + B + C) \cdot (A' + B + C') \quad | \quad B + \bar{A} = \bar{C}$$

$$(A + B + C') \cdot (A' + B + C \cdot \bar{C})$$

$$(\bar{A} + B) \cdot (B + \bar{C})$$

$$(A + B + C') \cdot (A' + B)$$

$$[B + (A + \bar{C}) \cdot \bar{A}]$$

$$B + (A \cdot \bar{A} + \bar{A} \cdot \bar{C})$$

K-MAP (Karnaugh map)

mapping structure to arrange boolean algebra
mpsL easiest form to express boolean alge
into minimal form.
groups are formed and while forming groups

- i) try to make large group as possible
- ii) Size of group is give 2^n to the power n

SOP form in K-map

$$f(A, B) = (m_0, m_2, m_3)$$

$$\begin{aligned} &= A'B' + AB' + AB \\ &= B'(A'+A) + AB \\ &= B' + AB \\ &= B' + A \end{aligned}$$

$$\begin{array}{c|cc} A \backslash B & B' & B \\ \hline n' & 00 & 01 \\ n & 10 & 11 \end{array}$$

$$f(A, B) = B' + AB$$

Using k-map

a. $f(w, x, y, z) = \Sigma(7, 13, 14, 15)$

$wx \setminus yz$	00	01	11	10	11	10
00	0	0	1	3	1	2
01	0	4	5	7	7	6
11	12	13	13	15	15	14
10	8	9	9	11	11	10

1st part = wxz

2nd part = xyz

3rd part = wxy

$f = wxz + xyz + wxy$

b. $f(w, x, y, z) = \Sigma(1, 3, 4, 6, 9, 11, 14, 15)$

$wx \setminus yz$	00	01	11	10	11	10
00	0	1	1	3	2	2
01	7	4	5	7	6	6
11	12	13	13	15	15	14
10	8	9	9	11	11	10

1st part = $w'x'z$

4th part = $w'yz$

2nd part = $w'xz'$

$f = w'x'z + w'xz' + wxy + wyz$

3rd part = wxy

d. $f(WX, Y, Z) = \Sigma(1, 3, 4, 5, 7, 8, 9, 13, 15)$

SOL

WX/YZ	00	01	11	10	1	2
00	0	1 1	1 2	1	2	3
01	1 4	1 5	1 7	1 6	1 4	1 6
11	1 2	1 3	1 5	1 4	1 6	1 8
10	1 8	1 9	"	1 10	1 10	1 10

1st part = $X'YZ$

2nd part = $Y'Z$

3rd part = $W'XY'$

4th part = $WX'Y'$

$$f = XZ + Y'Z + W'XY' + WX'Y'$$

X	$AB'CD$	00	01	11	10	1
00	1 0	1	1	1 2	1	2
01	1 4	1 5	1 7	1 6	1 4	1 6
11	1 2	1 3	1 5	1 4	1 6	1 8
10	1 8	1 9	"	1 10	1 10	1 10

1st group $A'B'$

2nd group $B'D'$

3rd group $ABCD$

4th group ACD'

$$F = A'B' + B'D' + ABCD + ACD'$$

$A \setminus BC$	00	01	11	10	
00	1 0	1 1	1 2		2
01	1 4	1 5		7	6
11	1 2	1 4		15	14
10	8	1 9 1 1			10

$$f = \Sigma m(0, 1, 3, 5, 13, 9, 11, 14, 12)$$

1st group : Bc'

2nd group = $B'D$

3rd group = $A'c'$

$$F = Bc' + B'D + A'c'$$

$$f = \Sigma m(0, 1, 2, 4, 5, 6)$$

$A \setminus BC$	00	01	11	10	
0	1 0	1 1	1 3	1 2	
1	1 4	1 5		1 6	

1st part = B'

2nd part = c'

$$f = B' + c'$$

$$\times f(w, x, y, z) = \pi (1, 4, 5, 6, 11, 12, 13, 14, 15)$$

$wx \setminus yz$	0+0	0+1	1+1	1+0	
0+0	0	(0)	3	2	
0+1	0	(0)	7	(0)	6
1+1	0	13	(0)	15	14
1+0	8	9	(0)	11	10

1st part $x' + y$

2nd part $w + y + z'$

3rd part $w' + y' + z'$

4th part $x' + y + z'$

final part $(x' + y)(w + y + z')(w' + y' + z')(x' + y + z')$

$$\times f(w, x, y, z) = \pi (0, 4, 5, 7, 8, 9, 13, 15)$$

$w+x \setminus y+z$	0+0	0+1	1+1	1+0	
0+0	0	1	3	2	
0+1	0	5	0	6	
1+1	12	13	0	15	14
1+0	8	9	11	10	

1st part $= x' + z'$

2nd part $= w + y + z$

3rd part $= w + x + y$

$$f = (x' + z') (w + y + z) (w + x + y)$$

ii. Simplify the following Boolean expression

$$a. W'Z + XZ + X'Y + WX'Z$$

$W'X'Y'$	$W'X'Z$	$WX'Z$	$X'Y$	WZ	XZ	$W'Z$
00	0	0	0	1	0	1
01	0	0	1	0	1	0
11	1	1	1	1	1	0
10	1	0	0	0	0	1

Imp Short Answer

1. Why alphabets are used for representing number 9 in hexadecimal number system?

→ Alphabets are used for representing number above 9 in hexadecimal number system to make the problem easier to solve and reduce the complexity of digit.

2. Why NAND gate is said to be universal gate?

→ NAND gate are called universal gate because any logic function can be done using them and thus can produce a inverter.

3. What are the three different method that are used for representing the negative number in decimal form into binary?

→ The three different method that are used for representing the negative number in decimal form into binary are:

- Sign magnitude representation
- One's complement form
- Two's complement form

4. What is the use of error detection code?

→ The use of error detection code is to transmit binary information from one place to another by electric source.

5. What is Boolean algebra?

→ Boolean algebra of logic which deals with study of binary variable and logic operation

7. What is the advantage of representing a data in hexa-decimal form?

→ The advantage of representing a data in hexa-decimal form is that it is very compact and by using a base 16 means that the number is usually less than in binary or decimal.

8. Differentiate between Pos and SOP expression.

- SOP: It is used more number of AND gate. It takes OR function of product term.

POS: It is used more number of OR gate. It takes AND function of sum term.

9. Why do we need don't care in k-map?

→ Don't care conditions allow us to replace the empty cells of a k-map to form a larger group of the variables. While forming a group of cells, we can simply ignore the cell.

10 Convert a binary number 1101 into equivalent gray code.

$$\begin{array}{r} 1101 \\ \downarrow \\ = 1011 \end{array}$$

15. What is the importance of k-map?

→ K-maps are used to simplify real-world logic requirements so that they can be implemented using a minimum number of physical logic gates. It is also used for

12. Why gray code are used in K-map instead of binary codes?

- Gray code actually follows Adjacency property i.e. between two successive code. There is only one bit change and in the binary code, there will be multiple value change. Because of Adjacency Property when one plots on K-map, literals can be grouped keeping other literal constant.

13. Mention advantage of parity method of error detection.

- Error on a noisy line can be caught quickly
- Only the errant word has to be retransmitted

14. Is gray code weight or un-weighted code?

→ It is the non weighted code and it is not arithmetic codes. That means there are no specific weights assigned to the bit position. It has a very special feature that, only one bit will change each time the decimal number is incremented.

15. Decimal number are weighted number justify.

- Yes because each digit position is being assigned with specific weight.

16. What is advantage of BCD code.

- It is fast and efficient system to convert the decimal number into binary number as compare to pure binary system.

17. What are error detection code? Give example?

→ Error detection is the process of detecting the error that are present in the data transmitted from transmitter to receiver, in a communication system. We use some redundancy code to detect these errors, by adding to the data while it is transmitted from source. These codes are called error detecting code.

18. Why NAND and NOR gate are called universal gate.

- The NAND and NOR gate are known as universal gate, since logic function can be implemented by NAND or NOR gates.

Circuits using NAND and NOR are popular as they are easier to design and therefore cheap. Function of other gate can easily be implemented using NAND and NOR gate; for this reason they are called universal gate.

19. State De-morgan's theorem mathematically