Course Code	**:**	**BCS-012**
Course Title	**:**	**Basic Mathematics**
Assignment Number	**:**	**BCA(1)012/Assignment/2022-23**
Maximum Marks	**:**	**100**
Weightage	**:**	**25%**
Last Date of Submission	**:**	**31st October, 2022 (For July Session)** **15th April, 2023 (For January Session)**

Note: This assignment has 15 questions of 80 marks (Q.no.1 to 14 are of 5 marks each, Q15 carries 10 marks). Answer all the questions. Rest 20 marks are for viva voce. You may use illustrations and diagrams to enhance explanations. Please go through the guidelines regarding assignments given in the Programme Guide for the format of presentation.

Q1. Solve the following system of equations by using Matrix Inverse Method.

1. $3x+4y+7z=14$
2. $2x-y+3z=4$
3. $2x+2y-3z=0$

Q2. Use principle of Mathematical Induction to prove that:
$\frac{1}{1*2}+\frac{1}{2*3}+\ldots\ldots\ldots\ldots.+\frac{1}{n(n+1)}=\frac{n}{n+1}$

Q3. How many terms of G.P $\sqrt{3}$, 3 , $3\sqrt{3}$, ……….. Add up to 39

Q4. If $y = a.e^{mx} + b.e^{-mx}$, Prove that $d^2y/dx^2 = m^2\,y$

Q5. For what value of 'k' the points (-k + 1, 2k), (k, 2 - 2k) and (- 4 - k, 6 - 2k) are collinear.

Q6. Evaluate $\int \frac{x\,dx}{[(x+1)\,(2x-1)]}$ and $\int \frac{dx}{(e^x-1)^2}$

Q7. If 1, w, w^2 are Cube Roots of unity show that $(1+w)^2 - (1+w)^3 + w^2 = 0$.

Q8. If α, β are roots of equation $2x^2-3x-5=0$ form a Quadratic equation whose roots are α^2, β^2

Q9. Solve the inequality $\frac{3}{5}(x-2) \leq \frac{5}{3}(2-x)$ and graph the solution set.

Q10. A spherical ballon is being Inflated at the rate of 900 cm^3/sec. How fast is the Radius of the ballon Increasing when the Radius is15 cm.

Q11. Find the area bounded by the curves $x^2 = y$ and $y=x$.

Q12. Determine the values of x for which $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is increasing and for which it is decreasing.

Q13. Using integration, find length of the curve $y = 3 - x$ from (-1, 4) to (3, 0).

Q14. Show that the lines $\frac{X-5}{4}=\frac{y-7}{-4}=\frac{z-3}{-5}$ and $\frac{X-8}{4}=\frac{y-4}{-4}=\frac{z-5}{4}$ Intersect.

Q15. A manufacturer makes two types of furniture, chairs and tables. Both the products are processed on three machines A1, A2 and A3. Machine A1 requires 3 hours for a chair and 3 hours for a table, machine A2 requires 5 hours for a chair and 2 hours for a table and machine A3 requires 2 hours for a chair and 6 hours for a table. The maximum time available on machines A1, A2 and A3 is 36 hours, 50 hours and 60 hours respectively. Profits are \$ 20 per chair and \$ 30 per table. Formulate the above as a linear programming problem to maximize the profit and solve it.

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Q1. Solve the following system of equation by using Matrix Inverse Method

1. $3x+4y+7z=14$
2. $2x-y+3z=4$
3. $2x+2y-3z=0$

Solution:

$$3x+4y+7z=14 \quad \cdots (i)$$
$$2x-y+3z=4 \quad \cdots (ii)$$
$$2x+2y-3z=0 \quad \cdots (iii)$$

We can put the above equations into the single matrix equation $AX=B$ where

$$A=\begin{pmatrix}3 & 4 & 7\\ 2 & -1 & 3\\ 2 & 2 & -3\end{pmatrix}, \quad X=\begin{pmatrix}x\\ y\\ z\end{pmatrix}, \quad B=\begin{pmatrix}14\\ 4\\ 0\end{pmatrix}$$

The cofactors of $|A|$ are,

$$A_{11}=(-1)^{1+1}\begin{vmatrix}-1 & 3\\ 2 & -3\end{vmatrix}=3-6=-3$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix}2 & 3\\ 2 & -3\end{vmatrix}=-1(-6-6)=12$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix}2 & -1\\ 2 & 2\end{vmatrix}=4+2=6$$

As,

$$|A|=a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}$$
$$=3(-3)+4\times12+7\times6$$
$$=-9+48+42=81$$

Since $|A|\neq0$, A is non-singular (invertible) matrix. Its remaing cofactors are.

$$A_{21}=(-1)^{2+1}\begin{vmatrix}4 & 7\\ 2 & -3\end{vmatrix}=-1(-12-14)=26$$

$$A_{22}=(-1)^{2+2}\begin{vmatrix}2 & -3\\ 3 & 7\end{vmatrix}=-9-14=-23$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 4 \\ 2 & 2 \end{vmatrix} = -1(6-8) = 2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 4 & 7 \\ -1 & 3 \end{vmatrix} = 12+7 = 19$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 7 \\ 2 & 3 \end{vmatrix} = -1(9-14) = 5$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -3-8 = -11.$$

Adjoint matrix of A canbe written as,

$$\text{adj } A = \begin{pmatrix} -3 & 26 & 19 \\ 12 & -23 & 5 \\ 6 & 2 & -11 \end{pmatrix}$$

Now,

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{81} \begin{pmatrix} -3 & 26 & 19 \\ 12 & -23 & 5 \\ 6 & 2 & -11 \end{pmatrix}$$

Also,

$$X = A^{-1}B = \frac{1}{81} \begin{pmatrix} -3 & 26 & 19 \\ 12 & -23 & 5 \\ 6 & 2 & -11 \end{pmatrix} \begin{pmatrix} 14 \\ 4 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{81} \begin{pmatrix} -42+104 \\ 168-92 \\ 84+8 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{81} \begin{pmatrix} 62 \\ 76 \\ 92 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{62}{81} \\ 76/81 \\ 92/81 \end{pmatrix}$$

$$\therefore \; x = \frac{62}{81}, \; y = \frac{76}{81}, \; z = 1\frac{11}{81}$$

Q2. Use principle of Mathematical Induction to prove that:

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\cdots\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

Solution:

Let P_n denote the statement.

$$P_n=\frac{1}{1\times2}+\frac{1}{2\times3}+\cdots\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

when $n=1$ LHS RHS

$$P_1=\frac{1}{1\times2}=\frac{1}{1+1}$$

$$\Rightarrow \frac{1}{2}=\frac{1}{2} \quad \text{which is true.}$$

It shows the result (P_n) holds for $n=1$.
Now, Lets assume that the result holds for $n=k$ for some $k\in\mathbb{N}$.

$$P_k=\frac{1}{1\times2}+\frac{1}{2\times3}+\cdots\cdots+\frac{1}{k(k+1)}=\frac{k}{k+1} \quad \cdots (i)$$

when $n=k+1$

$$P_{k+1}=\frac{1}{1\times2}+\frac{1}{2\times3}+\cdots\cdots+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+1+1)}$$

$$=P_k+\frac{1}{(k+1)(k+1+1)}$$

Putting P_k from eq. (i).

$$P_{k+1}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)^{\cancel{2}}}{\cancel{(k+1)}(k+2)}$$

$$=\frac{k+1}{k+1+1}$$

This shows that the result also holds for $n=k+1$ when it holds for $n=k$.

Hence, by mathematical induction principle, the statement is true for each natural number.

Q3. How many terms of GP
$\sqrt{3}, 3, 3\sqrt{3}, \ldots\ldots$ Add upto 39

Solution:

$$GP = \sqrt{3}, 3, 3\sqrt{3}, \ldots\ldots$$

$a = \sqrt{3}$, $ar = 3$ $\therefore$ $r = \frac{3}{a} = \frac{3}{\sqrt{3}} = \sqrt{3}$

In this GP, neither any item would be 39 nor sum upto any item can be 39. The problem has no solution.
Now, assuming the sum to be $39 + 13\sqrt{3}$

$$S_n = 39 + 13\sqrt{3}$$

$$\Rightarrow \frac{a(r^n - 1)}{r - 1} = 39 + 13\sqrt{3}$$

$$\Rightarrow \frac{\sqrt{3}(\sqrt{3}^n - 1)}{\sqrt{3} - 1} = 39 + 13\sqrt{3}$$

$$\Rightarrow 3^{n/2} - 1 = \frac{13\sqrt{3}(\sqrt{3}+1)(\sqrt{3}-1)}{\sqrt{3}}$$

$$\Rightarrow 3^{n/2} - 1 = \frac{13\sqrt{3} \times 2}{\sqrt{3}}$$

$$\Rightarrow 3^{n/2} = 26 + 1 = 27$$

$$\Rightarrow 3^{n/2} = 3^3$$

$\therefore$ $n/2 = 3$ i.e. $n = 6$.

Thus 6 terms of $\sqrt{3}, 3, 3\sqrt{3} \ldots\ldots$ are required to obtain $39 + 13\sqrt{3}$

Q4. If $y = ae^{mx} + be^{-mx}$. Prove that $d^2y/dx = m^2y$.

Solution:

We have $y = ae^{mx} + be^{-mx}$

Differentiating both sides with respect to x, we get.

$$\frac{dy}{dx} = \frac{d(ae^{mx} + be^{-mx})}{dx}$$

$$= ame^{mx} - bme^{-mx}$$

Again differentiating both sides with respet to x.

$$\frac{d^2y}{dx^2} = \frac{d(ame^{mx} - bme^{-mx})}{dx} = am.me^{mx} - bm.(-m)e^{-mx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = am^2e^{mx} + bm^2e^{-mx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = m^2(ae^{mx} + be^{-mx})$$

$$= m^2y.$$

Hence,

$$\frac{d^2y}{dx^2} = m^2y. \quad \text{proved}$$

Q5. For what value of 'k', the points $(-k+1, 2k)$, $(k, 2-2k)$ and $(-4-k, 6-2k)$ are collinear.

Solution:

The three points are collinear if and only if the area of triangle made by the three points is zero (0). Lets find the area of triangle from the given points.

$(-k+1, 2k)$, ~~k-2~~ $(k, 2-2k)$, $(-4-k, 6-2k)$

Area of triangle $\Delta = \frac{1}{2} \left| \begin{vmatrix} -k+1 & 2k & 1 \\ k & 2-2k & 1 \\ -4-k & 6-2k & 1 \end{vmatrix} \right|$

$= \frac{1}{2} \left| \begin{vmatrix} -k+1 & 2k & 1 \\ 2k-1 & 2-4k & 0 \\ -5 & 6-4k & 0 \end{vmatrix} \right|$ Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$= \frac{1}{2} \left| \begin{vmatrix} 2k-1 & 2-4k \\ -5 & 6-4k \end{vmatrix} \right|$

$= \frac{1}{2} \left| \left[(2k-1)(6-4k) + 5(2-4k) \right] \right|$

$= \frac{1}{2} \left| \left[12k - 6 + 4k - 8k^2 + 10 - 20k \right] \right|$

$= \frac{1}{2} \left| (-8k^2 - 4k + 4) \right|$

$= \left| -4k^2 - 4k + 2 \right|$

$= 4k^2 + 4k - 2$

For being collinear, the area of triangle must be zero.

i.e. $4k^2 + 4k - 2 = 0$.

or, $2(2k^2 + k - 1) = 0$

or, $(k+1)(2k-1) = 0$.

Either $k = -1$ or $k = \frac{1}{2}$

Hence

For $k = -1$ and $\frac{1}{2}$, the lines are collinear.

Q6 Evaluate

i) $\displaystyle\int \frac{x\,dx}{(x+1)(2x-1)}$ ii) $\displaystyle\int \frac{dx}{(e^{x}-1)^2}$

i) $\displaystyle\int \frac{x\,dx}{(x+1)(2x-1)}$

Solution:

Lets first resolve the integral into partial fraction

$$\frac{x}{(x+1)(2x-1)} = \frac{A}{x+1} + \frac{B}{2x-1}$$

$$\Rightarrow x = A(2x-1) + B(x+1)$$

Assuming $x = 1/2$ Assuming $x = -1$

$B = 1/3$ $A = 1/3$.

As A and B are constants,

Thus,

$$\int \frac{x\,dx}{(x+1)(2x-1)} = \frac{1}{3}\int \frac{dx}{x+1} + \frac{1}{3}\int \frac{dx}{2x-1}$$

$$= \frac{1}{3}\log|x+1| + \frac{1}{3}\cdot\frac{1}{2}\log|2x-1|$$

$$= \frac{1}{3}\log|x+1| + \frac{1}{6}\log|2x-1|$$

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ii) $\displaystyle\int \frac{dx}{(e^x-1)^2}$

Solution:

$$\text{Let } e^x - 1 = t$$

so that $e^x dx = dt$

Now the problem can be written as,

$$I = \int \frac{dt}{t^2(t+1)}$$

We split $\dfrac{1}{t^2(t+1)}$ into partical fraction.

$$\frac{1}{t^2(t+1)} = \frac{A}{t} + \frac{B}{t^2} + \frac{C}{(t+1)}$$

$$1 = At(t+1) + B(t+1) + Ct^2$$

For ~~to~~ $t = 0$, $\qquad t = -1$

$B(0+1) = 1 \qquad C\cdot(-1)^2 = 1$

$\therefore B = 1 \qquad \therefore C = 1$

Comparing cofficient of t^2, we obtain,

$$0 = A + C$$

$$\Rightarrow A = -C = -1$$

Thus $I = \displaystyle\int \left[-\frac{1}{t} + \frac{1}{t^2} + \frac{1}{t+1}\right] dt.$

$$= -\log|t| - \frac{1}{t} + \log|t+1| + c$$

$$= \log\left|\frac{t+1}{t}\right| - 1/t + c$$

$$= \log\left|\frac{e^x+1}{e^x}\right| - \frac{1}{e^x} + c$$

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Q7. $1, \omega, \omega^2$ are cube roots of unity, show that $(1+\omega)^2 - (1+\omega)^3 + \omega^2 = 0$

Solution:

$1, \omega, \omega^2$ are cube roots of unity.

$\therefore\ 1+\omega+\omega^2 = 0$

i.e.

$1+\omega = -\omega^2 \quad \cdots (i)$

$1+\omega^2 = -\omega \quad \cdots (ii)$

Taking LHS

$$(1+\omega)^2 - (1+\omega)^3 + \omega^2$$
$$= (-\omega^2)^2 - (-\omega^2)^3 + \omega^2$$
$$= \omega^4 + \omega^6 + \omega^2$$
$$= \omega^2(1+\omega^2+\omega^4)$$
$$= \omega^2[1+\omega^2(1+\omega^2)]$$
$$= \omega^2[1+\omega^2(-\omega)]$$
$$= \omega^2(1-\omega^3)$$
$$= \omega^2(1-1)$$
$$= 0$$
$$= \text{RHS.}$$

proved

Q8. If α, β are the roots of equation $2x^2 - 3x - 5 = 0$ Form a quadratic equation whose roots are α^2, β^2.

Solution :

Since α and β are the roots of the equation $2x^2 - 3x - 5 = 0$

$$\alpha + \beta = \frac{-b}{a} = \frac{3}{2} \quad \ldots (i)$$

$$\alpha \beta = \frac{c}{a} = \frac{-5}{2} \quad \ldots (ii)$$

Lets find out $\alpha^2 + \beta^2$

$$= (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{3}{2}\right)^2 - 2\left(-\frac{5}{2}\right)$$

$$= \frac{9}{4} + 5 = \frac{29}{4}$$

$$\alpha^2 \beta^2 = \left(\frac{-5}{2}\right)^2 = \frac{25}{4}.$$

The equation for the roots is

$$x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2 = 0$$

$$\Rightarrow x^2 - \frac{29}{4}x + \frac{25}{4} = 0$$

$$\Rightarrow 4x^2 - 29x + 25 = 0$$

Hence, $4x^2 - 29x + 25$ is the required equation.

Q9. Solve the inequality $\frac{3}{5}(x-2) \leq \frac{5}{3}(2-x)$ and graph the solution set.

Solution:

$$\frac{3}{5}(x-2) \leq \frac{5}{3}(2-x)$$

On solving the inequality, we get.

$$\Rightarrow 9x - 18 \leq 50 - 25x$$

$$\Rightarrow 9x + 25x \leq 50 + 18$$

$$\Rightarrow 34x \leq 68$$

$$\Rightarrow x \leq 2$$

The solution set is $\{x \mid x \leq 2\} = (-\infty, 2]$

Above is the graph of the inequality.

Q10. A spherical ballon is being inflated at the rate of 900 cm^3/sec. How fast is the radius of ballon increasing when the radius is 15cm.

Solution :

Volume of the sphere is given by,

$$V = \frac{4}{3}\pi r^3$$

Now, differentiating the equation with respect to time (t)

$$\frac{dV}{dt} = \frac{d\left(\frac{4}{3}\pi r^3\right)}{dt} \quad \ldots\ldots (i)$$

As per the question,

$$\frac{dV}{dt} = 900\ cm^3/sec \quad \ldots\ldots (ii)$$

Solving (i) & (ii)

$$900 = 4\pi r^2 \frac{dr}{dt}$$

when $r = 15$ cm.

$$900 = 4\pi (15)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{\overset{\cancel{60}\,\cancel{4}}{\cancel{900}}}{\cancel{4}\pi \times \cancel{15} \times \cancel{15}} = \frac{1}{\pi} = \frac{7}{22}\ cm/s$$

The radius of the ballon is increasing by $\frac{7}{22}$ cm/s.

Q11. Find the area bounded by the curves $x^2 = y$ and $y = x$.

Solution:

Two curves are

$$x^2 = y \quad \cdots (i)$$

$$y = x$$

Plotting them in the graph with the table below.

	$x^2 = y$					$y = x$		
X	0	1	2	−2	−1	0	1	2
Y	0	1	4	4	1	0	1	2

From the graph next page to this. the point of contact between two lines (0,0) and (1,1). This implies

Area of the curve

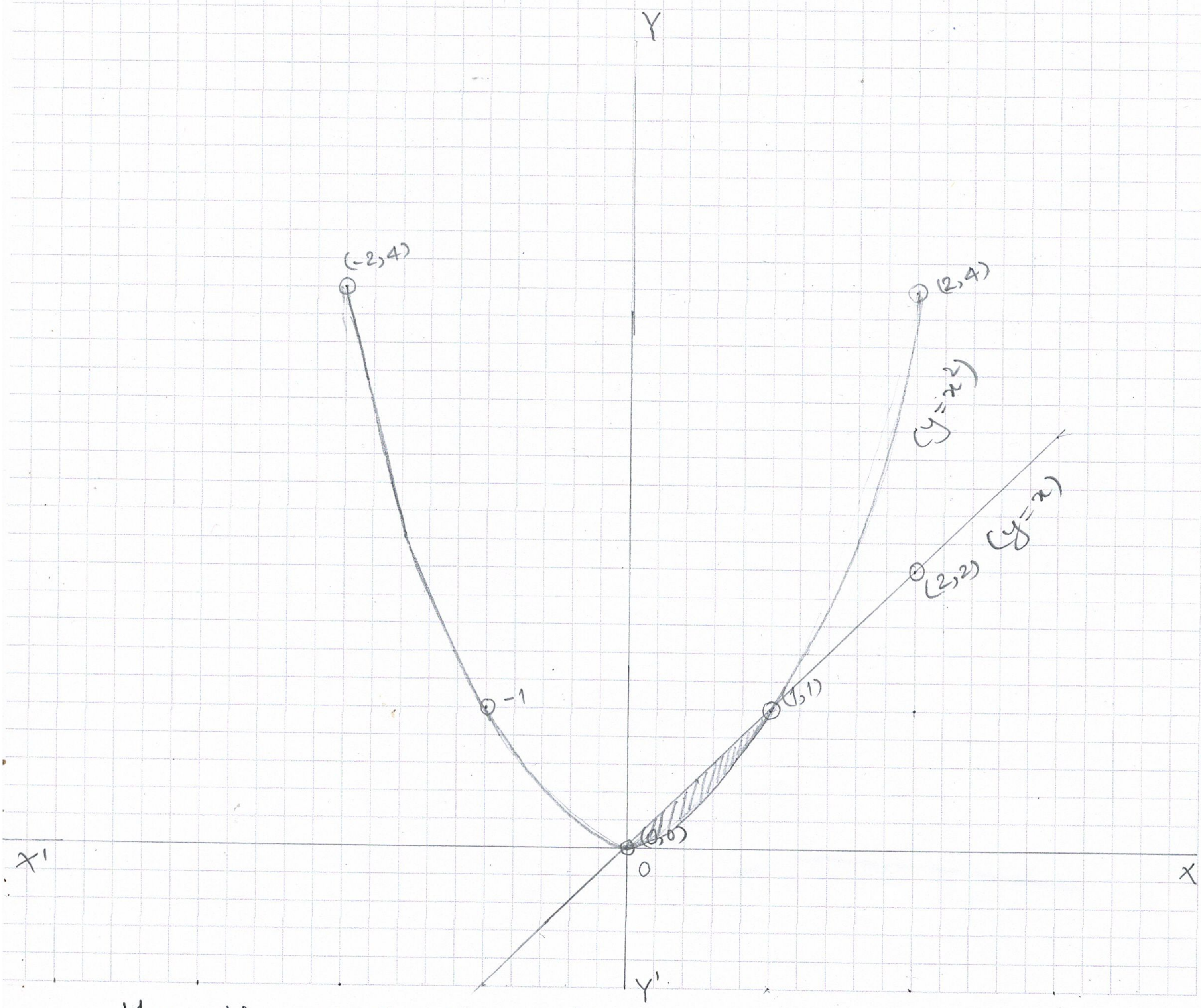
$$\int_0^1 (x - x^2)\, dx$$

$$= \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1$$

$$= \left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{6} \text{ sq. units.}$$

Hence area ~~of~~ between the curves $y = x^2$ and $y = x$ is $1/6$ sq. unit.

Q11.



Here the graph shows the cross section area between the two curves between two points (0,0) and (1,1). This means two curve is bounded from 0, to 1.

Q12. Determine the value of x for which $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is increasing and for which it is decreasing.

Solution:

$$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$$

Differentiating with respect to x.

$$\begin{aligned} f'(x) &= 4x^3 - 24x^2 + 44x - 24 + 0 \\ &= 4(x^3 - 6x^2 + 11x - 6) \\ &= 4(x^3 - x^2 - 5x^2 + 5x + 6x - 6) \\ &= 4(x-1)(x^2 - 5x + 6) \\ &= 4(x-1)(x^2 - 3x - 2x + 6) \\ &= 4(x-1)(x-2)(x-3) \end{aligned}$$

a) for increasing value of x

$$f'(x) > 0$$

Putting $x = 1, 2, 3$ in numberline and applying rule of inequality, we get.

$$x \in (1,2) \cup (3,\infty)$$

b) for decreasing value of x.

$$f'(x) < 0$$

similarly putting $x = 1, 2, 3$ in the number line $x \in (\infty, 1) \cup (2,3)$

Hence, value of $x \in (1,2) \cup (3,\infty)$ for increment while $x \in (\infty, 1) \cup$

Q13. Using integration, find the length of the curve y3-x from (-1,4) to (3,0)

Solution;

Given curve is $y = 3-x$.

Plotting the curve in the graph in the next page, we found the straight line $y = 3-x$ & both the points $(-1,4)$ and $(3,0)$ lie within the equation.

$$y = 3-x$$

$$\frac{dy}{dx} = \frac{d(3)}{dx} - \frac{dx}{dx} \quad \left[\begin{array}{l}\text{Differentiating} \\ \text{wrt } x.\end{array}\right]$$

$$= 0 - 1 = -1$$

$$\frac{dy}{dx} = -1 \quad \dots (1)$$

Length of the curve

$$= \int_{-1}^{3} \sqrt{1 - \left(\frac{dy}{dx}\right)^2}\, dx$$

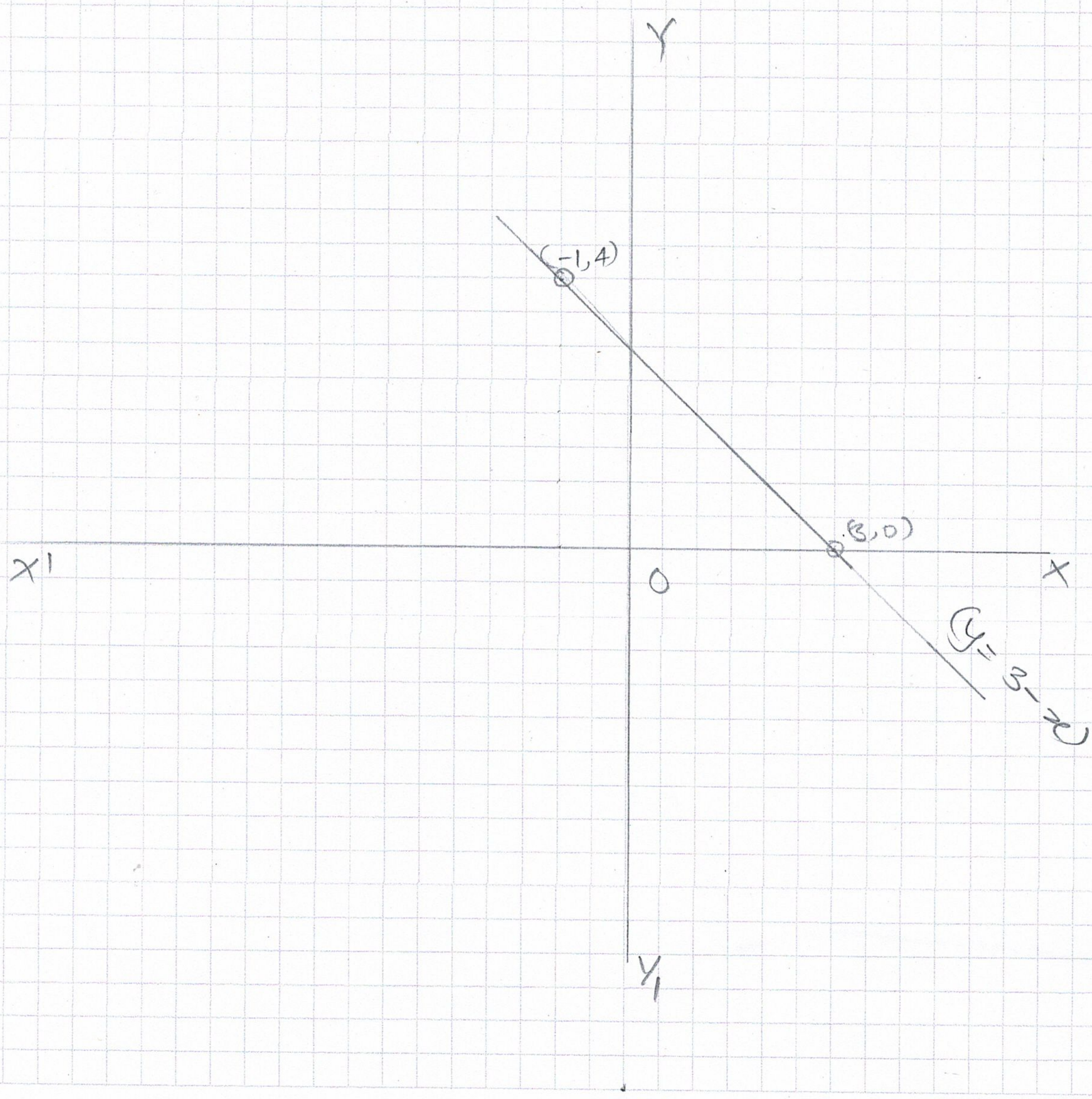
$$= \int_{-1}^{3} \sqrt{1+1}\, dx$$

$$= \sqrt{2} \int_{-1}^{3} dx$$

$$= \sqrt{2}\,[x]_{-1}^{3}$$

$$= \sqrt{2}\,(3+1) = 4\sqrt{2} \text{ units.}$$

Q13.



Length of the line between two points

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(3+1)^2+(4-0)^2}$$

$$= \sqrt{4^2+4^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \text{ units.}$$

Q14. Show that the lines

$$\frac{x-5}{4} = \frac{y-7}{-4} = \frac{z-3}{-5} \quad \text{and} \quad \frac{x-8}{4} = \frac{y-4}{-4} = \frac{z-5}{4}$$

intersect.

Solution:

Given lines are

$$\frac{x-5}{4} = \frac{y-7}{-4} = \frac{z-3}{-5}$$

$$\frac{x-8}{4} = \frac{y-4}{-4} = \frac{z-5}{4}$$

Converting these 3D equations into vectors, we get.

$$\vec{r} = (5\hat{i} + 7\hat{j} + 3\hat{k}) + t(4\hat{i} - 4\hat{j} - 5\hat{k})$$

and

$$\vec{r} = (8\hat{i} + 4\hat{j} + 5\hat{k}) + s(4\hat{i} - 4\hat{j} + 4\hat{k})$$

Comparing the equations with $\vec{r} = \vec{a} + t\vec{b}$ and $\vec{r} = \vec{c} + s\vec{d}$

We get.

$$\vec{a} = 5\hat{i} + 7\hat{j} + 3\hat{k}$$
$$\vec{b} = 4\hat{i} - 4\hat{j} - 5\hat{k}$$
$$\vec{c} = 8\hat{i} + 4\hat{j} + 5\hat{k}$$
$$\vec{d} = 4\hat{i} - 4\hat{j} + 4\hat{k}$$

Now,

$$\vec{c} - \vec{a} = 8\hat{i} - 5\hat{i} + 4\hat{j} - 7\hat{j} + 5\hat{k} - 3\hat{k}$$
$$= 3\hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -4 & -5 \\ 4 & -4 & 4 \end{vmatrix} = (-16-20)\hat{i} + (16+20)\hat{j}$$

$$= -36\hat{i} + 36\hat{j}$$

$$(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d}) = (3\hat{i}-3\hat{j}+2k)(-36\hat{i}+36\hat{j})$$

$$= 108 - 108 = 0$$

Thus the shortest distance between the two lines.

$$\left|\frac{(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})}{|\vec{b}\times\vec{d}|}\right| = 0$$

Hence the two lines intersect.

Q. 15. A manufacturer makes two types of furniture, chairs and tables. Both the products are processed on three machines A_1, A_2 and A_3. Machine A_1 requires 3 hours for a chair and 3 hours for a table, machine A_2 requires 5 hours for a chair and 2 hours for a table and machine A_3 require 2 hours for a chair and 6 hours for a table. The maximum time available for A_1, A_2, A_3 is 36 hours, 50 hours and 60 hours respectively. Profit are \$20 per chair and \$30 per table. Formulate the above as a linear programming problem to maximize the profit and solve it.

Solution:

Lets assume x chairs and y tables are manufactured per week.

Now,

profit per week will be \$$(20x+3y)$

As per the question we need to maximize the profit. So, objective function is

$$P = 20x + 30y$$

For machine A_1.

$$3x + 3y \leq 36 \quad \cdots (i)$$

As it can manufacture a chair in 3 hrs and a table in 3 hrs and has availability of 36 hrs.

Similarly.

For A_2

$$5x + 2y \leq 50 \quad \cdots (ii)$$

For A_3

$$2x + 6y \leq 60 \quad \cdots (iii)$$

Now, the problem can be written as

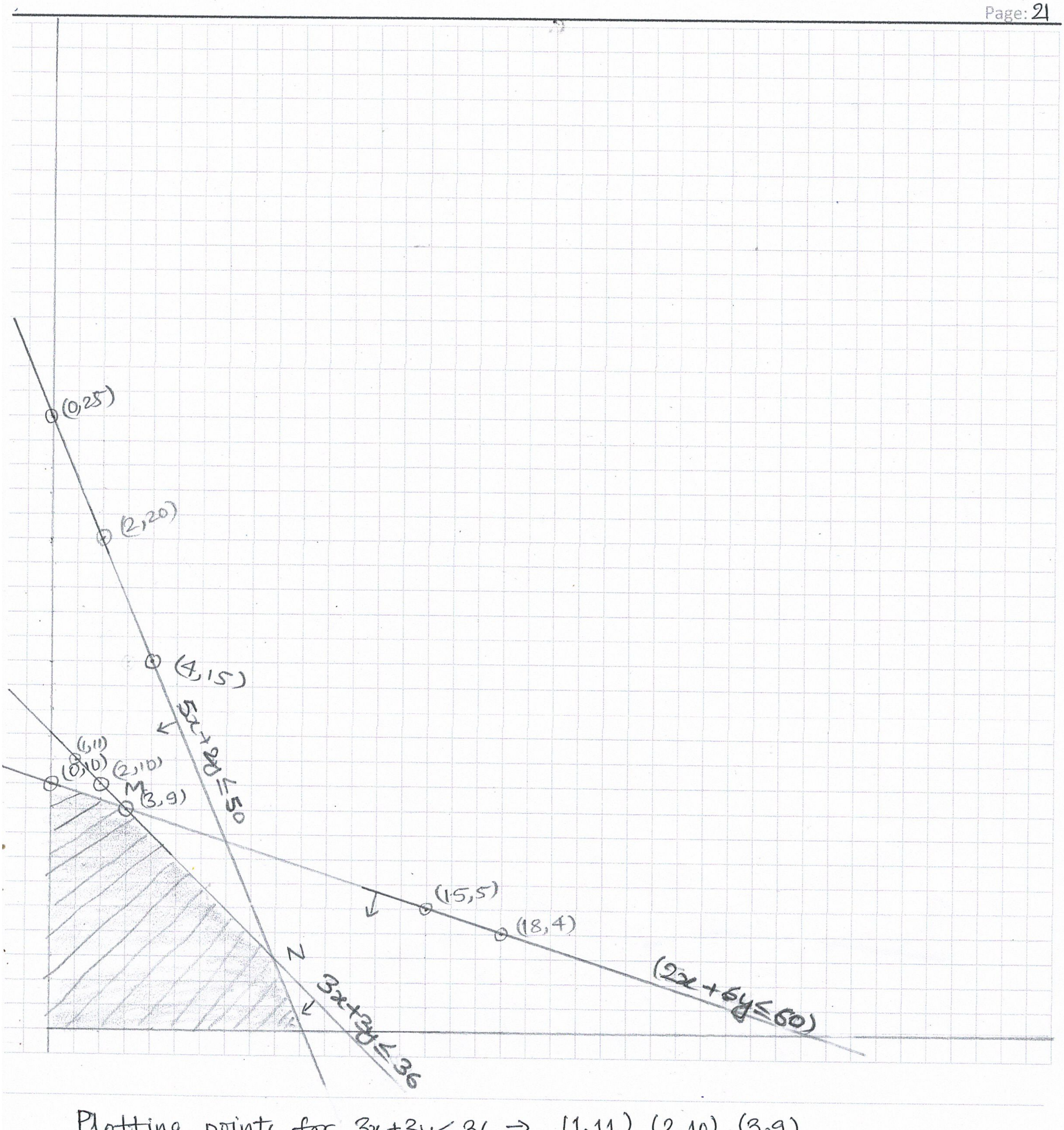
$$P = 20x + 30y$$

subject to

$$3x + 3y \leq 36$$
$$5x + 2y \leq 50$$
$$2x + 6y \leq 60$$

As chairs and tables can't have a negetive or zero value. $x \geq 0$, $y \geq 0$

The inequalities are plotted in graph.



Plotting points for $3x+3y\leq 36 \Rightarrow (1,11),(2,10),(3,9)$

$2x+6y\leq 60$ ~~$5x+2y\leq 50$~~ $\Rightarrow (0,10),(15,5),(18,4)$

$5x+2y\leq 50 \Rightarrow (0,25),(2,20),(4,15)$

Cross section point $M(3,9)$ and $N(\frac{26}{3},\frac{10}{3})$ and shaded area is the possible outcome zone for manufacturing.

Lets find profit at crossection point
M (3,9) and N$\left(\frac{26}{3}, \frac{10}{3}\right)$

$$\text{profit}(M) = 20 \times 3 + 30 \times 9$$
$$= 60 + 270$$
$$= \$330$$
$$\text{Profit}(N) = 20 \times \frac{26}{3} + 30 \times \frac{10}{3}$$
$$= \frac{520}{3} + \frac{300}{3}$$
$$= \frac{820}{3} = \$273\frac{1}{3}$$

As per the graph, from point M, increasing x will decrease y and increasing y decreases x. This clearly specifies, the point M is the maximum profit point.

Hence maximum profit that can be achieved is \$330 when 3 chairs and 9 tables are manufactured with all the three machines.