

Math 6643, Numerical Linear Algebra
HW 3, Due on Wednesday, Nov. 1

Notice: For the computational problems, you must design your own ways, such as using tables and plots etc, to present the results you obtain. And you must show the procedure you take, and explain the results you get, and draw conclusions if you have any. Computer code is not considered as an explanation. Please upload your code in T-square, and return your HW in class on or before the Due date.

- **Problem 1:** Use inverse power iteration to compute the eigenvector of matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

that corresponds to the eigenvalue close to $\mu = 1.268$. You can use calculators, but not computers to help you. Compute at least three iterations and comment on the accuracy of your results.

- **Problem 2:** Use power iteration to design an algorithm that computes the largest, in the absolute value sense, root of a polynomial $p(z) = z^n + a_1 z^{n-1} + \cdots + a_{n-1} z + a_n$. Justify your answer.
- **Problem 3:** Suppose $A \in \mathbb{R}^{m \times n}$ with $m > n$ and $y \in \mathbb{R}^m$. Construct an augmented matrix $\bar{A} = [A, y]$. Show that the condition number, measured in the induced matrix 2-norm, of \bar{A} is larger than that of A .
- **Problem 4:** Let A be a matrix in the following form

$$\begin{bmatrix} a_1 & b_2 & b_3 & \cdots & \cdots & b_n \\ c_2 & a_2 & 0 & \cdots & \cdots & 0 \\ c_3 & 0 & a_3 & \ddots & & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & & \ddots & a_{n-1} & 0 \\ c_n & 0 & \cdots & \cdots & 0 & a_n \end{bmatrix}.$$

Give an detailed algorithm using Givens rotation to compute the QR factorization of A . Write your own code to Implement your algorithm. Apply the code to matrices with randomly generated a_i , b_i and c_i values. Test your code with $n = 100, 200, 400, 800, \dots$. Comment on the computational complexity of your tests.

- **Problem 5:** Use the matrix A , with $\lambda = 0$ and $\lambda = 2$ respectively, from of Problem 5 in HW1 to finish the following work. You should run your code for matrix size $m = 256, 512, 1024, \dots$. You need comment on your results as well as computation cost.
 - (a) Write your code for the power iteration or the inverse iteration to identify the largest and smallest eigenvalues of A . How do you speed up the computation? What is the condition number of A ?

(b) Write your code for QR algorithm to compute the eigenvalues and eigenvectors.