

Individual-level analysis: mixed effects models

Part of the final project for AQMSS II

Polikanov Stepan and Okisheva Vera

Mixed effects models

Nested Logit

```
source(here::here("utilities", "check_packages.R"))
source(here::here("utilities", "functions.R"))

conflicts_prefer(dplyr::filter)
```

```
ep_raw_dep <- read_rds(here("data", "data_built", "ep_raw_dep.rds"))
data_country <- read_rds(here("data", "data_built", "data_country.rds"))
```

```
load(here("scripts", "models", "feme_bobyqa.RData"))
load(here("scripts", "models", "nlme_bobyqa.RData"))
load(here("scripts", "models", "me_allfit.RData"))
load(here("scripts", "models", "nl_fe.RData"))
```

```
# Recode no data back to native NA
model_data <- ep_raw_dep |>
  mutate(across(c(sex, age_bin, time_to_vs.less_than_hour,
                  time_to_vs.less_than_hour, out_of_Russia_time,
                  result_trust_bin),
            ~ if_else(. %in% c("No Data", "Declined to answer"), NA, .)),
         vote = relevel(as.factor(vote), ref = "Putin"),
         sex = relevel(as.factor(sex), ref = "Male"),
         age_bin = relevel(as.factor(age_bin), ref = "25-44"),
         out_of_Russia_time = relevel(as.factor(out_of_Russia_time), ref = "Before annexation"),
         result_trust_bin = relevel(as.factor(result_trust_bin), ref = "Yes")) |>
  filter(!countryname_en %in% c("New Zealand", "Australia"))
```

```

# Drop Australia and New Zealand for this because they have very skewed and low
# n observations. Might also help with convergence
nested_me_data <- model_data |>
  filter(vote != "Tore up/took", !countryname_en %in% c("Australia",
                                                         "New Zealand")) |>
  mutate(non_answer = if_else(vote == "Declined to answer", 1, 0),
         putin_else = case_when(vote == "Putin" ~ 1,
                                vote %in% c("Davankov", "Spoiled ballot",
                                              "Slutsky", "Haritonov") ~ 0,
                                .default = NA),
         nonsys_sys = case_when(vote %in% c("Davankov", "Spoiled ballot") ~ 1,
                                vote %in% c("Slutsky", "Haritonov") ~ 0,
                                .default = NA),
         davankov_spoiled = case_when(vote == "Davankov" ~ 1,
                                       vote == "Spoiled ballot" ~ 0,
                                       .default = NA),
         slutsky_haritonov = case_when(vote == "Slutsky" ~ 1,
                                       vote == "Haritonov" ~ 0,
                                       .default = NA)) |>
  left_join(select(data_country,
                  -countryname_en),
            by = c("countrycode_n", "countrycode_c"))

```

We fit the model consecutively for each dichotomy this time separating data by hand in the same way as the `nestedLogit` package does.

For some models we get non-convergence. We diagnose which optimizers work best in those case and update the model to achieve convergence.

```

# Answer (1) vs non-answer (0)
m4a.nested <- glmer(non_answer ~ sex + age_bin + time_to-vs.less_than_hour
                  + out_of_Russia_time + result_trust_bin
                  + (1 | countryname_en),
                  data = nested_me_data, family = binomial,
                  control = glmerControl(optimizer = "bobyqa"))

```

Not answer v answer

```

# Use bobyqa
m5a.nested <- glmer(non_answer ~ sex + age_bin + time_to-vs.less_than_hour
                  + out_of_Russia_time + result_trust_bin
                  + orthodox_share + vdem_polyarchy_2022
                  + log(mad_gdppc_2018) + obl_type + export_share
                  + import_share + friendly_status + help + military_dummy
                  + log(dist) + (1 | countryname_en),
                  data = nested_me_data, family = binomial,
                  control = glmerControl(optimizer = "bobyqa",
                                         optCtrl = list(maxfun = 1e9)))

# Doesn't converge

```

```
# Fit with all optimizers
m5a.allfit <- allFit(m5a.nested, maxfun = 1e9)
```

```
bobyqa : [OK]
Nelder_Mead : [OK]
nlminbwrap : [OK]
optimx.L-BFGS-B : [OK]
nloptwrap.NLOPT_LN_NELDERMEAD : [OK]
nloptwrap.NLOPT_LN_BOBYQA : [OK]
```

```
# Convergence results

## Export models
m5a.allfit_OK <- m5a.allfit[sapply(m5a.allfit, is, "merMod")]

## Generate warnings encountered
data.frame(lapply(m5a.allfit_OK, function(x) x@optinfo$conv$lme4$messages)) |>
  pivot_longer(everything(), names_to = "Method",
               values_to = "Estimation result") |>
  arrange(`Method`, `Estimation result`) |>
  distinct() |>
  mutate(`Method` = if_else(lag(`Method`) != `Method` | is.na(lag(`Method`)),
                           `Method`, ""),
         `Estimation result` = gsub("\\|", " ", `Estimation result`)) |>
  kable(booktabs = T,
        caption = paste("Convergence results for Answer/non-Answer dichotomy,",
                        "ME with level 2 variables and all optimizers")) |>
  column_spec(1, width = "8cm") |>
  column_spec(2, width = "8cm")
```

Table 1: Convergence results for Answer/non-Answer dichotomy, ME with level 2 variables and all optimizers

Method	Estimation result
Nelder_Mead	Model failed to converge with max grad = 0.0288314 (tol = 0.002, component 1)
bobyqa	Model failed to converge with max grad = 0.00240407 (tol = 0.002, component 1)
nlminbwrap	Model failed to converge with max grad = 0.071349 (tol = 0.002, component 1)
nloptwrap.NLOPT_LN_BOBYQA	Model failed to converge with max grad = 0.0450179 (tol = 0.002, component 1)
nloptwrap.NLOPT_LN_NELDERMEAD	Model failed to converge: degenerate Hessian with 1 negative eigenvalues unable to evaluate scaled gradient

optimx.L-BFGS-B

Model failed to converge: degenerate Hessian
with 1 negative eigenvalues
unable to evaluate scaled gradient

```
# Log-Likelihoods
(llikes <- sort(sapply(m5a.allfit_OK, logLik))) |>
  kable(col.names = "Log-likelihood", booktabs = T, digits = 3)
```

	Log-likelihood
optimx.L-BFGS-B	-8472.303
Nelder_Mead	-8472.302
nloptwrap.NLOPT_LN_BOBYQA	-8472.302
nlminbwrap	-8472.302
bobyqa	-8472.302
nloptwrap.NLOPT_LN_NELDERMEAD	-8472.302

```
# Coefficients from different optimizers

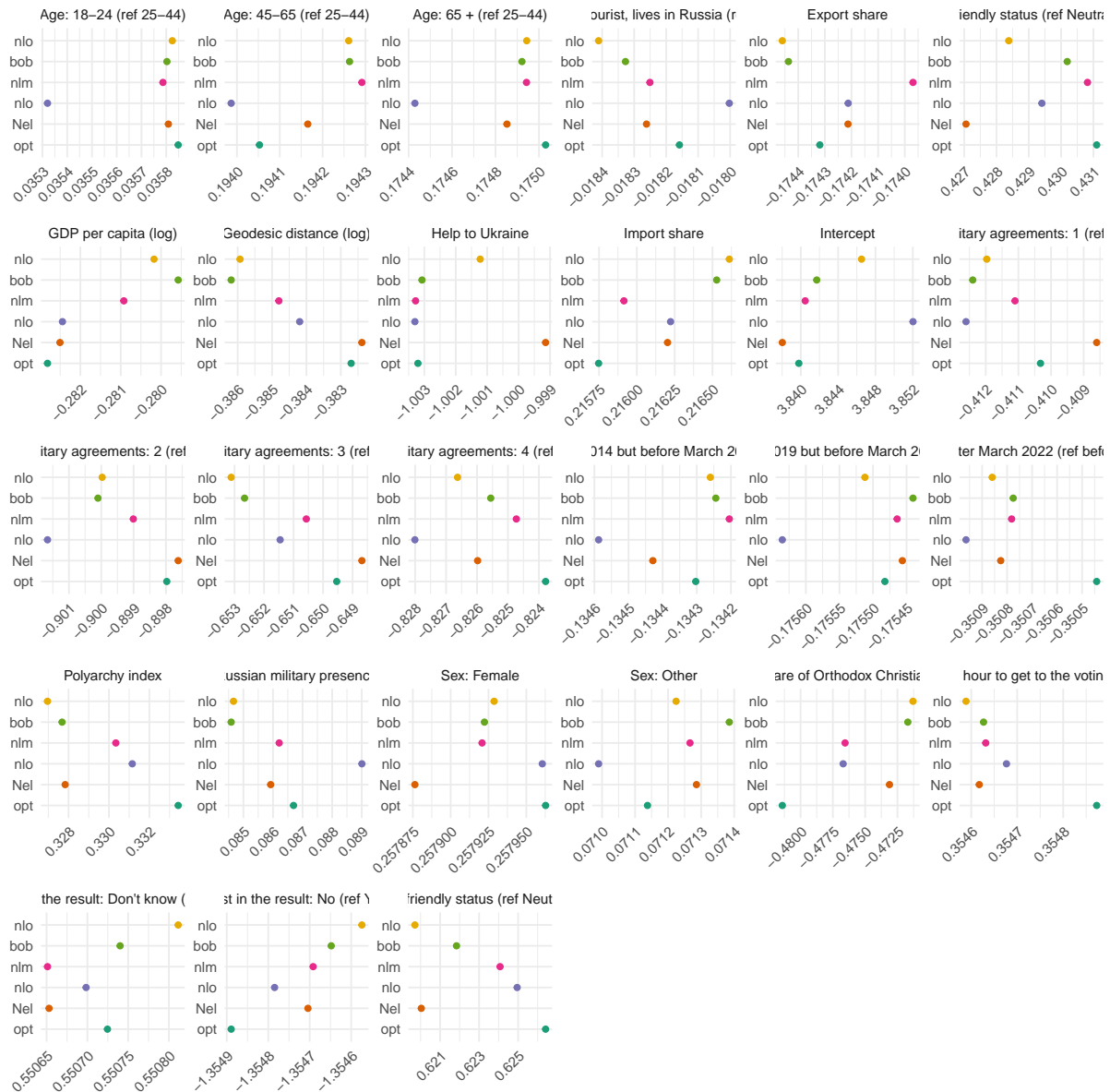
## Export fixef and melt into single dataframe
models <- levels(melt(t(sapply(m5a.allfit_OK, fixef))))$Var1
m5a.allfit.fixef.m <- transform(melt(t(sapply(m5a.allfit_OK, fixef))),
                               Var1 = factor(Var1, levels = names(lliks))) |>
  transmute(`Method` = Var1,
            `Coefficient` = value,
            `Variable` = case_when(
              Var2 == "(Intercept)" ~ "Intercept",
              Var2 == "sexFemale" ~ "Sex: Female",
              Var2 == "sexOther" ~ "Sex: Other",
              Var2 == "age_bin18-24" ~ "Age: 18-24 (ref 25-44)",
              Var2 == "age_bin45-64" ~ "Age: 45-65 (ref 25-44)",
              Var2 == "age_bin65+" ~ "Age: 65 + (ref 25-44)",
              Var2 == "time_to_vs.less_than_hourYes" ~
                "Took < 1 hour to get to the voting station",
              Var2 == "out_of_Russia_timeAfter invasion" ~
                "Moved after March 2022 (ref before 2014)",
              Var2 == "out_of_Russia_time2 - 5 years" ~
                paste("Moved after March 2019 but before",
                    "March 2022 (ref before 2014)"),
              Var2 == "out_of_Russia_timeAfter annexation" ~
                paste("Moved after March 2014 but before",
                    "March 2019 (ref before 2014)"),
              Var2 == "out_of_Russia_timeTourist (lives in Russia)" ~
                paste("Didn't move - tourist, lives",
                    "in Russia (ref before 2014)"),
              Var2 == "result_trust_binDon't know" ~
                "Trust in the result: Don't know (ref Yes)",
              Var2 == "result_trust_binNo" ~
                "Trust in the result: No (ref Yes)",
              Var2 == "orthodox_share" ~ "Share of Orthodox Christians",
              Var2 == "vdem_polyarchy_2022" ~ "Polyarchy index",
              Var2 == "log(mad_gdppc_2018)" ~ "GDP per capita (log)",
```

```

Var2 == "obl_type1" ~ "Military agreements: 1 (ref 0)",
Var2 == "obl_type2" ~ "Military agreements: 2 (ref 0)",
Var2 == "obl_type3" ~ "Military agreements: 3 (ref 0)",
Var2 == "obl_type4" ~ "Military agreements: 4 (ref 0)",
Var2 == "export_share" ~ "Export share",
Var2 == "import_share" ~ "Import share",
Var2 == "friendly_statusUnfriendly" ~
  "Unfriendly status (ref Neutral)",
Var2 == "friendly_statusFriendly" ~
  "Friendly status (ref Neutral)",
Var2 == "help" ~ "Help to Ukraine",
Var2 == "military_dummy" ~ "Russian military presence",
Var2 == "log(dist)" ~ "Geodesic distance (log)",
)

## Plot
ggplot(m5a.allfit.fixef.m,
  aes(x = `Coefficient`, y = `Method`, colour = `Method`)) +
  geom_point() +
  facet_wrap(~ `Variable`, scale = "free") +
  scale_colour_brewer(palette = "Dark2") +
  scale_y_discrete(breaks = models,
    labels = substr(models, 1, 3)) +
  labs(x = "", y = "") +
  theme_minimal() +
  theme(legend.position = "none",
    axis.text.x = element_text(angle = 45, hjust = 1))

```



```
# So judging by this I leave the model alone - the coefficients seem to be okay
# between different optimizers and since bobyqa comes close to the tolerance
# threshold, I will use it.

# Check m5a-specific diagnostics
# Gradient 1
derivs1 <- m5a.nested@optinfo$derivs
sc_grad1 <- with(derivs1, solve(Hessian, gradient))
gradres1 <- max(pmin(abs(sc_grad1), abs(derivs1$gradient)))

# Gradient 2
dd <- update(m5a.nested, devFunOnly = TRUE)
pars <- unlist(getME(m5a.nested, c("theta", "fixef")))
```

```

grad2 <- grad(dd, pars)
hess2 <- hessian(dd, pars)
sc_grad2 <- solve(hess2, grad2)
gradres2 <- max(pmin(abs(sc_grad2), abs(grad2)))

## The gradient cutoffs for Nelder Mead (nlopt), bobyqa and nlminwrap seem to
## be close to what I am getting.

# Check for singular fit
singres <- if_else(isSingular(m5a.nested), "Yes", "No")

# Display diagnostics
kable(cbind(gradres1, gradres2, singres),
      booktabs = T, digits = 3,
      col.names = c("Gradient, method I", "Gradient, method II",
                    "Singular fit?"),
      caption = paste("Diagnostics for `Answer vs No answer to poll`,
                    "model (with preferred optimizer)"))

```

Table 3: Diagnostics for Answer vs No answer to poll model (with preferred optimizer)

Gradient, method I	Gradient, method II	Singular fit?
0.00240406734519638	0.00243011504717848	No

The model doesn't converge.

Putin v everyone else

```

# Putin (1) vs everyone else (0), declined to answer NA
m4p.nested <- glmer(putin_else ~ sex + age_bin + time_to_vs.less_than_hour
  + out_of_Russia_time + result_trust_bin
  + (1 | countryname_en),
  data = nested_me_data, family = binomial,
  control = glmerControl(optimizer = "bobyqa"))

# Converges!

```

```

m5p.nested <- glmer(putin_else ~ sex + age_bin + time_to_vs.less_than_hour
  + out_of_Russia_time + result_trust_bin
  + orthodox_share + vdem_polyarchy_2022
  + log(mad_gdppc_2018) + obl_type + export_share
  + import_share + friendly_status + help + military_dummy
  + log(dist) + (1 | countryname_en),
  data = nested_me_data, family = binomial,
  control = glmerControl(optimizer = "bobyqa",
    optCtrl = list(maxfun = 1e9)))

```

```
# Fit with all optimizers (loaded with other models)
m5p.allfit <- allFit(m5p.nested, maxfun = 1e9)
```

```
bobyqa : [OK]
Nelder_Mead : [OK]
nlminbwrap : [OK]
optimx.L-BFGS-B : [OK]
nloptwrap.NLOPT_LN_NELDERMEAD : [OK]
nloptwrap.NLOPT_LN_BOBYQA : [OK]
```

```
# Convergence results

## Export models
m5p.allfit_OK <- m5p.allfit[sapply(m5p.allfit, is, "merMod")]

## Generate warnings encountered
data.frame(lapply(m5p.allfit_OK, function(x) x@optinfo$conv$lme4$messages)) |>
  pivot_longer(everything(), names_to = "Method",
               values_to = "Estimation result") |>
  arrange(`Method`, `Estimation result`) |>
  distinct() |>
  mutate(`Method` = if_else(lag(`Method`) != `Method` | is.na(lag(`Method`)),
                           `Method`, ""),
         `Estimation result` = gsub("\\|", " ", `Estimation result`)) |>
  kable(booktabs = T,
        caption = paste("Convergence results for Answer/non-Answer dichotomy,",
                        "ME with level 2 variables and all optimizers")) |>
  column_spec(1, width = "8cm") |>
  column_spec(2, width = "8cm")
```

Table 4: Convergence results for Answer/non-Answer dichotomy, ME with level 2 variables and all optimizers

Method	Estimation result
Nelder_Mead	Model failed to converge with max grad = 0.0439094 (tol = 0.002, component 1)
bobyqa	Model failed to converge with max grad = 0.00486188 (tol = 0.002, component 1)
nlminbwrap	Model failed to converge with max grad = 0.0198981 (tol = 0.002, component 1)
nloptwrap.NLOPT_LN_BOBYQA	Model failed to converge with max grad = 0.0277748 (tol = 0.002, component 1)
nloptwrap.NLOPT_LN_NELDERMEAD	Model failed to converge with max grad = 0.0240685 (tol = 0.002, component 1)

optimx.L.BFGS.B

Model failed to converge with max grad =
0.0818325 (tol = 0.002, component 1)

```
# Log-Likelihoods
(llikes <- sort(sapply(m5p.allfit_OK, logLik))) |>
  kable(col.names = "Log-likelihood", booktabs = T, digits = 3)
```

	Log-likelihood
Nelder_Mead	-3178.423
nloptwrap.NLOPT_LN_BOBYQA	-3178.423
optimx.L-BFGS-B	-3178.423
nlminbwrap	-3178.423
nloptwrap.NLOPT_LN_NELDERMEAD	-3178.423
bobyqa	-3178.423

```
# Coefficients from different optimizers

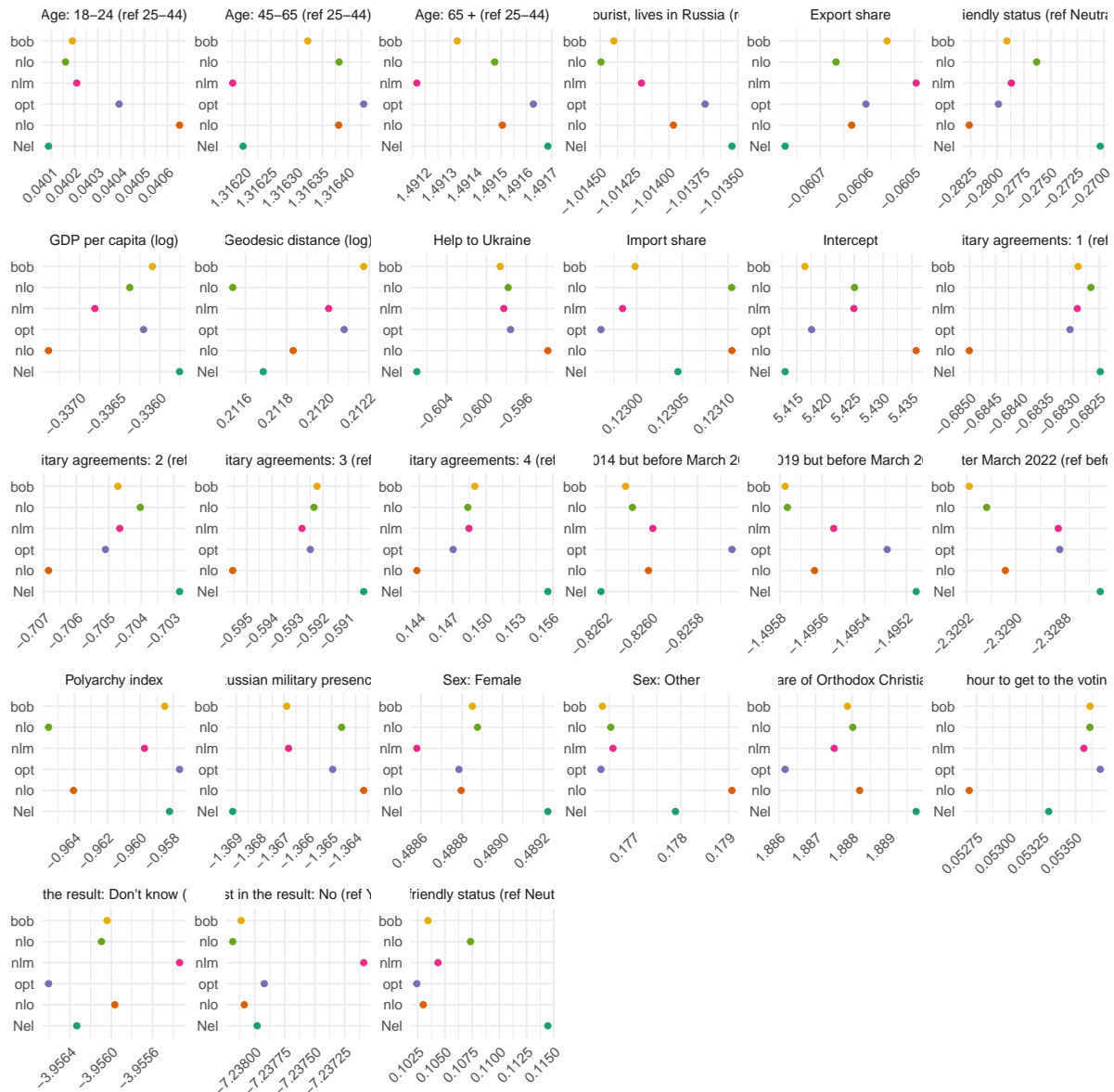
## Export fixef and melt into single dataframe
models <- levels(melt(t(sapply(m5p.allfit_OK, fixef)))$Var1)
m5p.allfit.fixef.m <- transform(melt(t(sapply(m5p.allfit_OK, fixef))),
  Var1 = factor(Var1, levels = names(llikes))) |>
  transmute(`Method` = Var1,
    `Coefficient` = value,
    `Variable` = case_when(
      Var2 == "(Intercept)" ~ "Intercept",
      Var2 == "sexFemale" ~ "Sex: Female",
      Var2 == "sexOther" ~ "Sex: Other",
      Var2 == "age_bin18-24" ~ "Age: 18-24 (ref 25-44)",
      Var2 == "age_bin45-64" ~ "Age: 45-65 (ref 25-44)",
      Var2 == "age_bin65+" ~ "Age: 65 + (ref 25-44)",
      Var2 == "time_to_vs.less_than_hourYes" ~
        "Took < 1 hour to get to the voting station",
      Var2 == "out_of_Russia_timeAfter invasion" ~
        "Moved after March 2022 (ref before 2014)",
      Var2 == "out_of_Russia_time2 - 5 years" ~
        paste("Moved after March 2019 but before",
          "March 2022 (ref before 2014)"),
      Var2 == "out_of_Russia_timeAfter annexation" ~
        paste("Moved after March 2014 but before",
          "March 2019 (ref before 2014)"),
      Var2 == "out_of_Russia_timeTourist (lives in Russia)" ~
        paste("Didn't move - tourist, lives",
          "in Russia (ref before 2014)"),
      Var2 == "result_trust_binDon't know" ~
        "Trust in the result: Don't know (ref Yes)",
      Var2 == "result_trust_binNo" ~
        "Trust in the result: No (ref Yes)",
      Var2 == "orthodox_share" ~ "Share of Orthodox Christians",
      Var2 == "vdem_polyarchy_2022" ~ "Polyarchy index",
      Var2 == "log(mad_gdppc_2018)" ~ "GDP per capita (log)",
      Var2 == "obl_type1" ~ "Military agreements: 1 (ref 0)",
      Var2 == "obl_type2" ~ "Military agreements: 2 (ref 0)",
```

```

Var2 == "obl_type3" ~ "Military agreements: 3 (ref 0)",
Var2 == "obl_type4" ~ "Military agreements: 4 (ref 0)",
Var2 == "export_share" ~ "Export share",
Var2 == "import_share" ~ "Import share",
Var2 == "friendly_statusUnfriendly" ~
  "Unfriendly status (ref Neutral)",
Var2 == "friendly_statusFriendly" ~
  "Friendly status (ref Neutral)",
Var2 == "help" ~ "Help to Ukraine",
Var2 == "military_dummy" ~ "Russian military presence",
Var2 == "log(dist)" ~ "Geodesic distance (log)",
)

## Plot
ggplot(m5p.allfit.fixef.m,
  aes(x = `Coefficient`, y = `Method`, colour = `Method`)) +
  geom_point() +
  facet_wrap(~ `Variable`, scale = "free") +
  scale_colour_brewer(palette = "Dark2") +
  scale_y_discrete(breaks = models,
    labels = substr(models, 1, 3)) +
  labs(x = "", y = "") +
  theme_minimal() +
  theme(legend.position = "none",
    axis.text.x = element_text(angle = 45, hjust = 1))

```



```
# Check m5p-specific diagnostics

# Gradient 1
derivs1 <- m5p.nested@optinfo$derivs
sc_grad1 <- with(derivs1, solve(Hessian, gradient))
gradres1 <- max(pmin(abs(sc_grad1), abs(derivs1$gradient)))

# Gradient 2
dd <- update(m5p.nested, devFunOnly = TRUE)
pars <- unlist(getME(m5p.nested, c("theta", "fixef")))
grad2 <- grad(dd, pars)
hess2 <- hessian(dd, pars)
sc_grad2 <- solve(hess2, grad2)
```

```

gradres2 <- max(pmin(abs(sc_grad2), abs(grad2)))

## The gradient cutoffs for Nelder Mead (nlopt), bobyqa and nlminwrap seem to
## be close to what I am getting.

# Check for singular fit
singres <- if_else(isSingular(m5p.nested), "Yes", "No")

# Display diagnostics
kable(cbind(gradres1, gradres2, singres),
      booktabs = T, digits = 3,
      col.names = c("Gradient, method I", "Gradient, method II",
                    "Singular fit?"),
      caption = paste("Diagnostics for `Putin vs Everyone",
                      "else` model (with preferred optimizer)"))

```

Table 6: Diagnostics for Putin vs Everyone else model (with preferred optimizer)

Gradient, method I	Gradient, method II	Singular fit?
0.0132050081447233	0.0131996664023155	No

Non-systemic v systemic opposition

```

# Non-systemic - Davankov or Spoiled (1) vs systemic - Haritonov, Slutsky (0)
# opposition, declined to answer and Putin are NA
m4s.nested <- glmer(nonsys_sys ~ sex + age_bin + time_to_vs.less_than_hour
                  + out_of_Russia_time + result_trust_bin
                  + (1 | countryname_en),
                  data = nested_me_data, family = binomial,
                  control = glmerControl(optimizer = "bobyqa"))

# Converges

```

```

m5s.nested <- glmer(nonsys_sys ~ sex + age_bin + time_to_vs.less_than_hour
                  + out_of_Russia_time + result_trust_bin
                  + orthodox_share + vdem_polyarchy_2022
                  + log(mad_gdppc_2018) + obl_type + export_share
                  + import_share + friendly_status + help + military_dummy
                  + log(dist) + (1 | countryname_en),
                  data = nested_me_data, family = binomial,
                  control = glmerControl(optimizer = "bobyqa",
                                          optCtrl = list(maxfun = 1e9)))

# Singular fit!

# The predictors most likely to cause issues are obl_type and friendly_status
# as they are broadly (and correlated between themselves and other variables)
# defined dichotomous predictors. Remove them from the model and try again

```

```
m5s.red <- update(m5s.nested, ~ . - obl_type - friendly_status)

# Works great and no issues with convergence either
```

Davankov v spoiled

```
# Davankov (1) vs Spoiled (0) declined to answer, Haritonov, Slutsky and Putin
# are NA
m4d.nested <- glmer(davankov_spoiled ~ sex + age_bin + time_to_vs.less_than_hour
  + out_of_Russia_time + result_trust_bin
  + (1 | countryname_en),
  data = nested_me_data, family = binomial,
  control = glmerControl(optimizer = "bobyqa"))
```

```
m5d.nested <- glmer(davankov_spoiled ~ sex + age_bin + time_to_vs.less_than_hour
  + out_of_Russia_time + result_trust_bin
  + orthodox_share + vdem_polyarchy_2022
  + log(mad_gdppc_2018) + obl_type + export_share
  + import_share + friendly_status + help + military_dummy
  + log(dist) + (1 | countryname_en),
  data = nested_me_data, family = binomial,
  control = glmerControl(optimizer = "bobyqa",
    optCtrl = list(maxfun = 1e9)))
```

```
m5d.allfit <- allFit(m5d.nested, maxfun = 1e9)
```

```
bobyqa : [OK]
Nelder_Mead : [OK]
nlminbwrap : [OK]
optimx.L-BFGS-B : [OK]
nloptwrap.NLOPT_LN_NELDERMEAD : [OK]
nloptwrap.NLOPT_LN_BOBYQA : [OK]
```

```
# Convergence results

## Export models
m5d.allfit_OK <- m5d.allfit[sapply(m5d.allfit, is, "merMod")]

## Generate warnings encountered
data.frame(lapply(m5d.allfit_OK, function(x) x@optinfo$conv$lme4$messages)) |>
  pivot_longer(everything(), names_to = "Method",
    values_to = "Estimation result") |>
  arrange(`Method`, `Estimation result`) |>
  distinct() |>
  mutate(`Method` = if_else(lag(`Method`) != `Method` | is.na(lag(`Method`)),
    `Method`, ""),
    `Estimation result` = gsub("\\\\|", " ", `Estimation result`)) |>
  kable(booktabs = T,
    caption = paste("Convergence results for Answer/non-Answer dichotomy,"))
```

```

"ME with level 2 variables and all optimizers")) |>
column_spec(1, width = "8cm") |>
column_spec(2, width = "8cm")

```

Table 7: Convergence results for Answer/non-Answer dichotomy, ME with level 2 variables and all optimizers

Method	Estimation result
Nelder_Mead	Model failed to converge with max grad = 0.425244 (tol = 0.002, component 1)
bobyqa	Model failed to converge with max grad = 0.0157886 (tol = 0.002, component 1)
nlminbwrap	Model failed to converge with max grad = 1.66793 (tol = 0.002, component 1)
nloptwrap.NLOPT_LN_BOBYQA	Model failed to converge with max grad = 0.0678025 (tol = 0.002, component 1)
nloptwrap.NLOPT_LN_NELDERMEAD	Model failed to converge with max grad = 0.081166 (tol = 0.002, component 1)
optimx.L.BFGS.B	Model failed to converge with max grad = 0.0682182 (tol = 0.002, component 1)

```

# Log-Likelihoods
(llikes <- sort(sapply(m5d.allfit_OK, logLik))) |>
kable(col.names = "Log-likelihood", booktabs = T, digits = 3)

```

	Log-likelihood
Nelder_Mead	-20537.76
nlminbwrap	-20537.73
optimx.L-BFGS-B	-20537.73
nloptwrap.NLOPT_LN_BOBYQA	-20537.73
nloptwrap.NLOPT_LN_NELDERMEAD	-20537.73
bobyqa	-20537.73

```

# Coefficients from different optimizers

## Export fixef and melt into single dataframe
models <- levels(melt(t(sapply(m5d.allfit_OK, fixef)))$Var1)
m5d.allfit.fixef.m <- transform(melt(t(sapply(m5d.allfit_OK, fixef))),
                               Var1 = factor(Var1, levels = names(llikes))) |>
  transmute(`Method` = Var1,
            `Coefficient` = value,
            `Variable` = case_when(
              Var2 == "(Intercept)" ~ "Intercept",

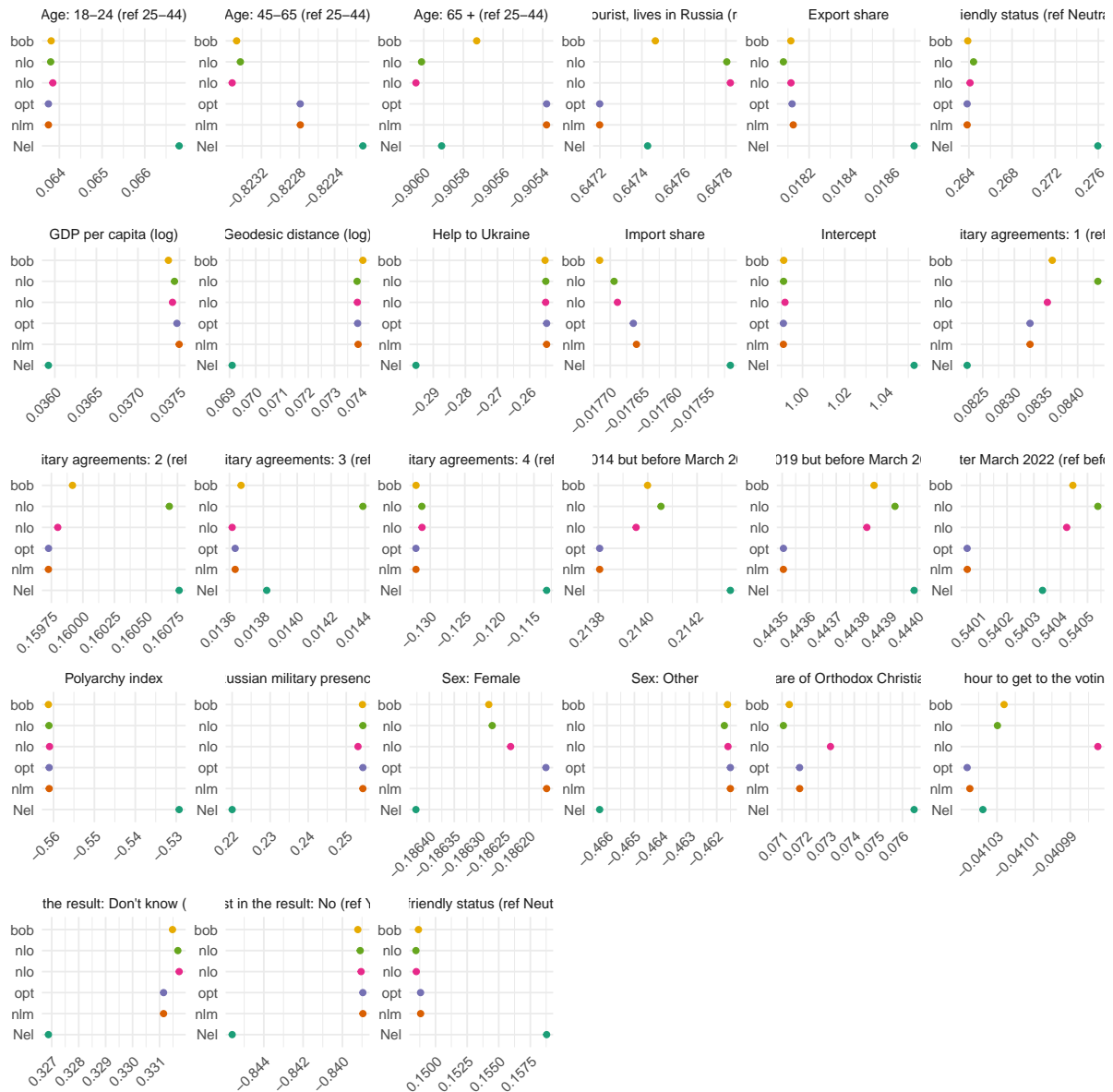
```

```

Var2 == "sexFemale" ~ "Sex: Female",
Var2 == "sexOther" ~ "Sex: Other",
Var2 == "age_bin18-24" ~ "Age: 18-24 (ref 25-44)",
Var2 == "age_bin45-64" ~ "Age: 45-65 (ref 25-44)",
Var2 == "age_bin65+" ~ "Age: 65 + (ref 25-44)",
Var2 == "time_to_vs.less_than_hourYes" ~
  "Took < 1 hour to get to the voting station",
Var2 == "out_of_Russia_timeAfter invasion" ~
  "Moved after March 2022 (ref before 2014)",
Var2 == "out_of_Russia_time2 - 5 years" ~
  paste("Moved after March 2019 but before",
    "March 2022 (ref before 2014)"),
Var2 == "out_of_Russia_timeAfter annexation" ~
  paste("Moved after March 2014 but before",
    "March 2019 (ref before 2014)"),
Var2 == "out_of_Russia_timeTourist (lives in Russia)" ~
  paste("Didn't move - tourist, lives",
    "in Russia (ref before 2014)"),
Var2 == "result_trust_binDon't know" ~
  "Trust in the result: Don't know (ref Yes)",
Var2 == "result_trust_binNo" ~
  "Trust in the result: No (ref Yes)",
Var2 == "orthodox_share" ~ "Share of Orthodox Christians",
Var2 == "vdem_polyarchy_2022" ~ "Polyarchy index",
Var2 == "log(mad_gdppc_2018)" ~ "GDP per capita (log)",
Var2 == "obl_type1" ~ "Military agreements: 1 (ref 0)",
Var2 == "obl_type2" ~ "Military agreements: 2 (ref 0)",
Var2 == "obl_type3" ~ "Military agreements: 3 (ref 0)",
Var2 == "obl_type4" ~ "Military agreements: 4 (ref 0)",
Var2 == "export_share" ~ "Export share",
Var2 == "import_share" ~ "Import share",
Var2 == "friendly_statusUnfriendly" ~
  "Unfriendly status (ref Neutral)",
Var2 == "friendly_statusFriendly" ~
  "Friendly status (ref Neutral)",
Var2 == "help" ~ "Help to Ukraine",
Var2 == "military_dummy" ~ "Russian military presence",
Var2 == "log(dist)" ~ "Geodesic distance (log)",
)

## Plot
ggplot(m5d.allfit.fixef.m,
  aes(x = `Coefficient`, y = `Method`, colour = `Method`)) +
  geom_point() +
  facet_wrap(~ `Variable`, scale = "free") +
  scale_colour_brewer(palette = "Dark2") +
  scale_y_discrete(breaks = models,
    labels = substr(models, 1, 3)) +
  labs(x = "", y = "") +
  theme_minimal() +
  theme(legend.position = "none",
    axis.text.x = element_text(angle = 45, hjust = 1))

```



```
# Check m5d-specific diagnostics

# Gradient 1
derivs1 <- m5d.nested@optinfo$derivs
sc_grad1 <- with(derivs1, solve(Hessian, gradient))
gradres1 <- max(pmin(abs(sc_grad1), abs(derivs1$gradient)))

# Gradient 2
dd <- update(m5d.nested, devFunOnly = TRUE)
pars <- unlist(getME(m5d.nested, c("theta", "fixef")))
grad2 <- grad(dd, pars)
hess2 <- hessian(dd, pars)
sc_grad2 <- solve(hess2, grad2)
```



```

gradres2 <- max(pmin(abs(sc_grad2), abs(grad2)))

## The gradient cutoffs for Nelder Mead (nlopt), bobyqa and nlminwrap seem to
## be close to what I am getting.

# Check for singular fit
singres <- if_else(isSingular(m5d.nested), "Yes", "No")

# Display diagnostics
kable(cbind(gradres1, gradres2, singres),
      booktabs = T, digits = 3,
      col.names = c("Gradient, method I", "Gradient, method II",
                    "Singular fit?"),
      caption = paste("Diagnostics for `Davankov vs Spoil the ballot`,
                      "model (with preferred optimizer)"))

```

Table 9: Diagnostics for Davankov vs Spoil the ballot model (with preferred optimizer)

Gradient, method I	Gradient, method II	Singular fit?
0.0040686829458804	0.00394873324194497	No

```

save(list = c("m5a.nested", "m5p.nested", "m5s.nested", "m5s.red", "m5d.nested"),
     file = "nlme_bobyqa.RData")

save(list = c("m4a.nested", "m4p.nested", "m4s.nested", "m4d.nested"),
     file = "feme_bobyqa.RData")

save(list = c("m5a.allfit", "m5p.allfit", "m5d.allfit"),
     file = "me_allfit.RData")

```

```

coef_map_me <- c("(Intercept)", "sexFemale" = "Sex: Female",
                "sexOther" = "Sex: Other",
                "age_bin18-24" = "Age: 18-24 (ref 25-44)",
                "age_bin45-64" = "Age: 45-65 (ref 25-44)",
                "age_bin65+" = "Age: 65 + (ref 25-44)",
                "time_to_vs.less_than_hourYes" =
                  "Took < 1 hour to get to the voting station",
                "out_of_Russia_timeAfter invasion" =
                  "Moved after March 2022 (ref before 2014)",
                "out_of_Russia_time2 - 5 years" =
                  paste("Moved after March 2019 but before",
                        "March 2022 (ref before 2014)"),
                "out_of_Russia_timeAfter annexation" =
                  paste("Moved after March 2014 but before",
                        "March 2019 (ref before 2014)"),
                "out_of_Russia_timeTourist (lives in Russia)" =
                  paste("Didn't move - tourist, lives",
                        "in Russia (ref before 2014)"),
                "result_trust_binDon't know" =
                  "Trust in the result: Don't know (ref Yes)",
                "result_trust_binNo" =
                  "Trust in the result: No (ref Yes)",
                "orthodox_share" = "Share of Orthodox Christians",
                "vdem_polyarchy_2022" = "Polyarchy index",

```

Table 10: Mixed effects results, no level 2 variables

	Declined to answer vs answer	Putin vs everyone else	Non-systemic vs systemic opposition	Davankov vs Spoiled
Sex: Female	0.275*** (0.044)	0.482*** (0.071)	0.022 (0.056)	-0.199*** (0.024)
Sex: Other	0.064 (0.237)	0.171 (0.368)	-0.375 (0.296)	-0.470** (0.144)
Age: 18-24 (ref 25-44)	-0.003 (0.081)	0.026 (0.124)	-0.095 (0.086)	0.062+ (0.037)
Age: 45-65 (ref 25-44)	0.180** (0.059)	1.322*** (0.086)	-0.302*** (0.087)	-0.826*** (0.039)
Age: 65 + (ref 25-44)	0.170* (0.077)	1.507*** (0.131)	-0.957*** (0.155)	-0.931*** (0.106)
Took < 1 hour to get to the voting station	0.376*** (0.052)	0.094 (0.081)	-0.099 (0.061)	-0.034 (0.026)
Moved after March 2022 (ref before 2014)	-0.359*** (0.069)	-2.330*** (0.101)	0.766*** (0.084)	0.565*** (0.040)
Moved after March 2019 but before March 2022 (ref before 2014)	-0.207** (0.075)	-1.480*** (0.109)	0.409*** (0.091)	0.454*** (0.042)
Moved after March 2014 but before March 2019 (ref before 2014)	-0.159* (0.080)	-0.806*** (0.120)	0.296** (0.102)	0.219*** (0.045)
Didn't move - tourist, lives in Russia (ref before 2014)	-0.039 (0.096)	-0.975*** (0.143)	0.041 (0.144)	0.652*** (0.087)
Trust in the result: Don't know (ref Yes)	0.499*** (0.082)	-3.960*** (0.109)	0.718*** (0.127)	0.387*** (0.139)
Trust in the result: No (ref Yes)	-1.328*** (0.060)	-7.218*** (0.101)	1.903*** (0.094)	-0.806*** (0.105)
Num.Obs.	53 824	51 202	42 363	40 946
R2 Marg.	0.139	0.781	0.070	0.054
R2 Cond.	0.227	0.813	0.075	0.070
AIC	18 314.4	6766.2	11 756.3	45 344.3
BIC	18 438.9	6890.0	11 877.5	45 465.0
ICC	0.1	0.1	0.0	0.0
RMSE	0.21	0.13	0.18	0.43

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

```

"log(mad_gdppc_2018)" = "GDP per capita (log)",
"obl_type1" = "Military agreements: 1 (ref 0)",
"obl_type2" = "Military agreements: 2 (ref 0)",
"obl_type3" = "Military agreements: 3 (ref 0)",
"obl_type4" = "Military agreements: 4 (ref 0)",
"export_share" = "Export share",
"import_share" = "Import share",
"friendly_statusUnfriendly" =
  "Unfriendly status (ref Neutral)",
"friendly_statusFriendly" =
  "Friendly status (ref Neutral)",
"help" = "Help to Ukraine",
"military_dummy" = "Russian military presence",
"log(dist)" = "Geodesic distance (log)"

```

```

modelsummary(list("Declined to answer vs answer" = m4a.nested,
                  "Putin vs everyone else" = m4p.nested,
                  "Non-systemic vs systemic opposition" = m4s.nested,
                  "Davankov vs Spoiled" = m4d.nested),
  output = "kableExtra", stars = T,
  coef_map = coef_map_me) |>
  kable_styling(latex_options = "scale_down")

```

```

m3.answer <- models(m3.nested.fe, 1)
m3.putin <- models(m3.nested.fe, 2)
m3.nonsys <- models(m3.nested.fe, 3)
m3.davankov <- models(m3.nested.fe, 4)

```

Table 11: Comparison: logistic vs mixed effects models (no lvl 2 variables)

	Answer vs No answer		Putin vs everyone else		Non-systemic vs systemic opposition		Davankov vs Spoiled	
	Logistic	Mixed effects	Logistic	Mixed effects	Logistic	Mixed effects	Logistic	Mixed effects
Sex: Female	0.275*** (0.044)	0.275*** (0.044)	0.481*** (0.072)	0.482*** (0.071)	0.019 (0.057)	0.022 (0.056)	-0.195*** (0.024)	-0.199*** (0.024)
Sex: Other	0.039 (0.240)	0.064 (0.237)	0.208 (0.369)	0.171 (0.368)	-0.371 (0.301)	-0.375 (0.296)	-0.455** (0.146)	-0.470** (0.144)
Age: 18-24 (ref 25-44)	-0.008 (0.082)	-0.003 (0.081)	0.019 (0.126)	0.026 (0.124)	-0.114 (0.088)	-0.095 (0.086)	0.072+ (0.037)	0.062+ (0.037)
Age: 45-65 (ref 25-44)	0.172** (0.059)	0.180** (0.059)	1.304*** (0.087)	1.322*** (0.086)	-0.294*** (0.088)	-0.302*** (0.087)	-0.827*** (0.039)	-0.826*** (0.039)
Age: 65 + (ref 25-44)	0.161* (0.078)	0.170* (0.077)	1.501*** (0.132)	1.507*** (0.131)	-0.994*** (0.159)	-0.957*** (0.155)	-0.952*** (0.107)	-0.931*** (0.106)
Took < 1 hour to get to the voting station	0.310*** (0.055)	0.376*** (0.052)	0.071 (0.085)	0.094 (0.081)	-0.086 (0.067)	-0.099 (0.061)	-0.028 (0.027)	-0.034 (0.026)
Moved after March 2022 (ref before 2014)	-0.363*** (0.069)	-0.359*** (0.069)	-2.357*** (0.103)	-2.330*** (0.101)	0.820*** (0.090)	0.766*** (0.084)	0.539*** (0.040)	0.565*** (0.040)
Moved after March 2019 but before March 2022 (ref before 2014)	-0.217** (0.075)	-0.207** (0.075)	-1.467*** (0.110)	-1.480*** (0.109)	0.419*** (0.093)	0.409*** (0.091)	0.438*** (0.042)	0.454*** (0.042)
Moved after March 2014 but before March 2019 (ref before 2014)	-0.179* (0.081)	-0.159* (0.080)	-0.798*** (0.121)	-0.806*** (0.120)	0.299** (0.103)	0.296** (0.102)	0.213*** (0.046)	0.219*** (0.045)
Didn't move - tourist, lives in Russia (ref before 2014)	-0.002 (0.098)	-0.039 (0.096)	-0.997*** (0.146)	-0.975*** (0.143)	0.134 (0.150)	0.041 (0.144)	0.633*** (0.088)	0.652*** (0.087)
Trust in the result: Don't know (ref Yes)	0.521*** (0.083)	0.499*** (0.082)	-3.955*** (0.110)	-3.960*** (0.109)	0.716*** (0.129)	0.718*** (0.127)	0.392*** (0.141)	0.387** (0.139)
Trust in the result: No (ref Yes)	-1.293*** (0.060)	-1.328*** (0.060)	-7.204*** (0.102)	-7.218*** (0.101)	1.891*** (0.095)	1.903*** (0.094)	-0.804*** (0.106)	-0.806*** (0.105)
Num.Obs.	53 824	53 824	51 202	51 202	42 363	42 363	40 946	40 946
R2 Marg.		0.139		0.781		0.070		0.054
R2 Cond.		0.227		0.813		0.075		0.070
AIC	18 166.4	18 314.4	6724.4	6766.2	11 769.4	11 756.3	45 257.7	45 344.3
BIC	18 833.4	18 438.9	7387.7	6890.0	12 418.5	11 877.5	45 904.2	45 465.0
ICC		0.1		0.1		0.0		0.0
RMSE	0.21	0.21	0.13	0.13	0.18	0.18	0.43	0.43

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

```

modelsummary(list("Logistic" = m3.answer,
                  "Mixed effects" = m4a.nested,
                  "Logistic" = m3.putin,
                  "Mixed effects" = m4p.nested,
                  "Logistic" = m3.nonsys,
                  "Mixed effects" = m4s.nested,
                  "Logistic" = m3.davankov,
                  "Mixed effects" = m4d.nested),
  output = "kableExtra", stars = T,
  coef_map = coef_map_me) |>
kable_styling(latex_options = "scale_down") |>
add_header_above(c(" " = 1, "Answer vs No answer" = 2,
                  "Putin vs everyone else" = 2,
                  "Non-systemic vs systemic opposition" = 2,
                  "Davankov vs Spoiled" = 2))

```

```

modelsummary(list("Declined to answer vs answer" = m5a.nested,
                  "Putin vs everyone else" = m5p.nested,
                  "Non-systemic vs systemic opposition" = m5s.red,
                  "Davankov vs Spoiled" = m5d.nested),
  output = "kableExtra", stars = T,
  coef_map = coef_map_me) |>
kable_styling(latex_options = "scale_down")

```

```

modelsummary(list("No lvl 2" = m4a.nested,
                  "With lvl 2" = m5a.nested,
                  "No lvl 2" = m4p.nested,

```

Table 12: Mixed effects results, with level 2 variables

	Declined to answer vs answer	Putin vs everyone else	Non-systemic vs systemic opposition	Davankov vs Spoiled
Sex: Female	0.258*** (0.045)	0.489*** (0.073)	0.015 (0.059)	-0.186*** (0.025)
Sex: Other	0.071 (0.240)	0.176 (0.376)	-0.232 (0.328)	-0.462** (0.148)
Age: 18-24 (ref 25-44)	0.036 (0.083)	0.040 (0.125)	-0.117 (0.088)	0.064+ (0.038)
Age: 45-65 (ref 25-44)	0.194** (0.060)	1.316*** (0.089)	-0.299*** (0.090)	-0.823*** (0.040)
Age: 65 + (ref 25-44)	0.175* (0.078)	1.491*** (0.134)	-0.930*** (0.159)	-0.906*** (0.108)
Took < 1 hour to get to the voting station	0.355*** (0.054)	0.054 (0.084)	-0.081 (0.065)	-0.041 (0.027)
Moved after March 2022 (ref before 2014)	-0.351*** (0.071)	-2.329*** (0.103)	0.817*** (0.090)	0.540*** (0.040)
Moved after March 2019 but before March 2022 (ref before 2014)	-0.175* (0.076)	-1.496*** (0.111)	0.402*** (0.093)	0.444*** (0.042)
Moved after March 2014 but before March 2019 (ref before 2014)	-0.134+ (0.081)	-0.826*** (0.121)	0.318** (0.103)	0.214*** (0.045)
Didn't move - tourist, lives in Russia (ref before 2014)	-0.018 (0.099)	-1.014*** (0.148)	0.148 (0.153)	0.647*** (0.090)
Trust in the result: Don't know (ref Yes)	0.551*** (0.084)	-3.956*** (0.111)	0.700*** (0.133)	0.331* (0.146)
Trust in the result: No (ref Yes)	-1.355*** (0.062)	-7.238*** (0.103)	1.910*** (0.098)	-0.839*** (0.111)
Share of Orthodox Christians	-0.472 (0.468)	1.888*** (0.412)	-0.360* (0.142)	0.071 (0.163)
Polyarchy index	0.328 (0.668)	-0.959 (0.674)	0.532* (0.250)	-0.561* (0.233)
GDP per capita (log)	-0.280 (0.174)	-0.336+ (0.176)	0.036 (0.088)	0.037 (0.072)
Military agreements: 1 (ref 0)	-0.412 (0.471)	-0.683 (0.454)		0.084 (0.162)
Military agreements: 2 (ref 0)	-0.900+ (0.507)	-0.705 (0.485)		0.160 (0.171)
Military agreements: 3 (ref 0)	-0.653 (0.463)	-0.592 (0.427)		0.014 (0.162)
Military agreements: 4 (ref 0)	-0.826 (0.814)	0.149 (0.739)		-0.132 (0.278)
Export share	-0.174** (0.062)	-0.061 (0.058)	0.030 (0.024)	0.018 (0.016)
Import share	0.217*** (0.063)	0.123* (0.056)	-0.020 (0.020)	-0.018 (0.018)
Unfriendly status (ref Neutral)	0.622 (0.819)	0.103 (0.794)		0.149 (0.284)
Friendly status (ref Neutral)	0.430 (0.480)	-0.279 (0.466)		0.264 (0.183)
Help to Ukraine	-1.003* (0.476)	-0.599 (0.532)	-0.124 (0.194)	-0.255 (0.196)
Russian military presence	0.085 (0.728)	-1.367* (0.678)	0.224 (0.195)	0.254 (0.264)
Geodesic distance (log)	-0.386* (0.151)	0.212 (0.144)	-0.150** (0.057)	0.074 (0.050)
Num.Obs.	48 964	46 494	37 827	36 523
R2 Marg.	0.199	0.817	0.076	0.075
R2 Cond.	0.244	0.823	0.076	0.079
AIC	17 000.6	6412.8	10 728.8	41 131.5
BIC	17 247.0	6657.8	10 916.7	41 369.6
ICC	0.1	0.0	0.0	0.0
RMSE	0.21	0.13	0.18	0.43

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

```

        "With lvl 2" = m5p.nested,
        "No lvl 2" = m4s.nested,
        "With lvl 2" = m5s.red,
        "No lvl 2" = m4d.nested,
        "With lvl 2" = m5d.nested),
    output = "kableExtra", stars = T,
    coef_map = coef_map_me) |>
kable_styling(latex_options = "scale_down") |>
add_header_above(c(" " = 1, "Answer vs No answer" = 2,
                    "Putin vs everyone else" = 2,
                    "Non-systemic vs systemic opposition" = 2,
                    "Davankov vs Spoiled" = 2))

```

Table 13: Comparison: mixed effects (no lvl 2) vs mixed effects models (with lvl 2)

	Answer vs No answer		Putin vs everyone else		Non-systemic vs systemic opposition		Davankov vs Spoiled	
	No lvl 2	With lvl 2	No lvl 2	With lvl 2	No lvl 2	With lvl 2	No lvl 2	With lvl 2
Sex: Female	0.275*** (0.044)	0.258*** (0.045)	0.482*** (0.071)	0.489*** (0.073)	0.022 (0.056)	0.015 (0.059)	-0.199*** (0.024)	-0.186*** (0.025)
Sex: Other	0.064 (0.237)	0.071 (0.240)	0.171 (0.368)	0.176 (0.376)	-0.375 (0.296)	-0.232 (0.328)	-0.470** (0.144)	-0.462** (0.148)
Age: 18-24 (ref 25-44)	-0.003 (0.081)	0.036 (0.083)	0.026 (0.124)	0.040 (0.125)	-0.095 (0.086)	-0.117 (0.088)	0.062+ (0.037)	0.064+ (0.038)
Age: 45-65 (ref 25-44)	0.180** (0.059)	0.194** (0.060)	1.322*** (0.086)	1.316*** (0.089)	-0.302*** (0.087)	-0.299*** (0.090)	-0.826*** (0.039)	-0.823*** (0.040)
Age: 65 + (ref 25-44)	0.170* (0.077)	0.175* (0.078)	1.507*** (0.131)	1.491*** (0.134)	-0.957*** (0.155)	-0.930*** (0.159)	-0.931*** (0.106)	-0.906*** (0.108)
Took < 1 hour to get to the voting station	0.376*** (0.052)	0.355*** (0.054)	0.094 (0.081)	0.054 (0.084)	-0.099 (0.061)	-0.081 (0.065)	-0.034 (0.026)	-0.041 (0.027)
Moved after March 2022 (ref before 2014)	-0.359*** (0.069)	-0.351*** (0.071)	-2.330*** (0.101)	-2.329*** (0.103)	0.766*** (0.084)	0.817*** (0.090)	0.565*** (0.040)	0.540*** (0.040)
Moved after March 2019 but before March 2022 (ref before 2014)	-0.207** (0.075)	-0.175* (0.076)	-1.480*** (0.109)	-1.496*** (0.111)	0.409*** (0.091)	0.402*** (0.093)	0.454*** (0.042)	0.444*** (0.042)
Moved after March 2014 but before March 2019 (ref before 2014)	-0.159* (0.080)	-0.134+ (0.081)	-0.806*** (0.120)	-0.826*** (0.121)	0.296** (0.102)	0.318** (0.103)	0.219*** (0.045)	0.214*** (0.045)
Didn't move - tourist, lives in Russia (ref before 2014)	-0.039 (0.096)	-0.018 (0.099)	-0.975*** (0.143)	-1.014*** (0.148)	0.041 (0.144)	0.148 (0.153)	0.652*** (0.087)	0.647*** (0.090)
Trust in the result: Don't know (ref Yes)	0.499*** (0.082)	0.551*** (0.084)	-3.960*** (0.109)	-3.956*** (0.111)	0.718*** (0.127)	0.700*** (0.133)	0.387** (0.139)	0.331* (0.146)
Trust in the result: No (ref Yes)	-1.328*** (0.060)	-1.355*** (0.062)	-7.218*** (0.101)	-7.238*** (0.103)	1.903*** (0.094)	1.910*** (0.098)	-0.806*** (0.105)	-0.839*** (0.111)
Share of Orthodox Christians		-0.472 (0.468)		1.888*** (0.412)		-0.360* (0.142)		0.071 (0.163)
Polyarchy index		0.328 (0.668)		-0.959 (0.674)		0.532* (0.250)		-0.561* (0.233)
GDP per capita (log)		-0.280 (0.174)		-0.336+ (0.176)		0.036 (0.088)		0.037 (0.072)
Military agreements: 1 (ref 0)		-0.412 (0.471)		-0.683 (0.454)				0.084 (0.162)
Military agreements: 2 (ref 0)		-0.900+ (0.507)		-0.705 (0.485)				0.160 (0.171)
Military agreements: 3 (ref 0)		-0.653 (0.463)		-0.592 (0.427)				0.014 (0.162)
Military agreements: 4 (ref 0)		-0.826 (0.814)		0.149 (0.739)				-0.132 (0.278)
Export share		-0.174** (0.062)		-0.061 (0.058)		0.030 (0.024)		0.018 (0.016)
Import share		0.217*** (0.063)		0.123* (0.056)		-0.020 (0.020)		-0.018 (0.018)
Unfriendly status (ref Neutral)		0.622 (0.819)		0.103 (0.794)				0.149 (0.284)
Friendly status (ref Neutral)		0.430 (0.480)		-0.279 (0.466)				0.264 (0.183)
Help to Ukraine		-1.003* (0.476)		-0.599 (0.532)		-0.124 (0.194)		-0.255 (0.196)
Russian military presence		0.085 (0.728)		-1.367* (0.678)		0.224 (0.195)		0.254 (0.264)
Geodesic distance (log)		-0.386* (0.151)		0.212 (0.144)		-0.150** (0.057)		0.074 (0.050)
Num.Obs.	53 824	48 964	51 202	46 494	42 363	37 827	40 946	36 523
R2 Marg.	0.139	0.199	0.781	0.817	0.070	0.076	0.054	0.075
R2 Cond.	0.227	0.244	0.813	0.823	0.075	0.076	0.070	0.079
AIC	18 314.4	17 000.6	6766.2	6412.8	11 756.3	10 728.8	45 344.3	41 131.5
BIC	18 438.9	17 247.0	6890.0	6657.8	11 877.5	10 916.7	45 465.0	41 369.6
ICC	0.1	0.1	0.1	0.0	0.0	0.0	0.0	0.0
RMSE	0.21	0.21	0.13	0.13	0.18	0.18	0.43	0.43

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001