

Individual-level analysis: mixed effects models

Part of the final project for AQMSS II

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Mixed effects models

Nested Logit

```
source(here::here("utilities", "check_packages.R"))
source(here::here("utilities", "functions.R"))

ep_raw_dep <- read_rds(here("data", "data_built", "ep_raw_dep.rds"))
data_country <- read_rds(here("data", "data_built", "data_country.rds"))

load(here("scripts", "models", "feme_bobyqa.RData"))
load(here("scripts", "models", "nlme_bobyqa.RData"))
load(here("scripts", "models", "me_allfit.RData"))
load(here("scripts", "models", "nl_fe.RData"))

# Recode no data back to native NA
model_data <- ep_raw_dep |>
  mutate(across(c(sex, age_bin, time_to_vs.less_than_hour,
                  time_to_vs.less_than_hour, out_of_Russia_time,
                  result_trust_bin),
    ~ if_else(. %in% c("No Data", "Declined to answer"), NA, .)),
  vote = relevel(as.factor(vote), ref = "Putin"),
  sex = relevel(as.factor(sex), ref = "Male"),
  age_bin = relevel(as.factor(age_bin), ref = "25-44"),
  out_of_Russia_time = relevel(as.factor(out_of_Russia_time), ref = "Before annexation"),
  result_trust_bin = relevel(as.factor(result_trust_bin), ref = "Yes")) |>
  filter(!countryname_en %in% c("New Zealand", "Australia"))

# Drop Australia and New Zealand for this because they have very skewed and low
# n observations. Might also help with convergence
nested_me_data <- model_data |>
  filter(vote != "Tore up/took", !countryname_en %in% c("Australia",
                                                       "New Zealand")) |>
```

```

mutate(non_answer = if_else(vote == "Declined to answer", 1, 0),
       putin_else = case_when(vote == "Putin" ~ 1,
                              vote %in% c("Davankov", "Spoiled ballot",
                                             "Slutsky", "Haritonov") ~ 0,
                              .default = NA),
       nonsys_sys = case_when(vote %in% c("Davankov", "Spoiled ballot") ~ 1,
                              vote %in% c("Slutsky", "Haritonov") ~ 0,
                              .default = NA),
       davankov_spoiled = case_when(vote == "Davankov" ~ 1,
                                     vote == "Spoiled ballot" ~ 0,
                                     .default = NA),
       slutsky_haritonov = case_when(vote == "Slutsky" ~ 1,
                                     vote == "Haritonov" ~ 0,
                                     .default = NA)) |>

left_join(select(data_country,
                 -countryname_en),
          by = c("countrycode_n", "countrycode_c"))

```

We fit the model consecutively for each dichotomy this time separating data by hand in the same way as the `nestedLogit` package does.

For some models we get non-convergence. We diagnose which optimizers work best in those case and update the model to achieve convergence.

```

# Answer (1) vs non-answer (0)
m4a.nested <- glmer(non_answer ~ sex + age_bin + time_to_vs.less_than_hour
                  + out_of_Russia_time + result_trust_bin
                  + (1 | countryname_en),
                  data = nested_me_data, family = binomial,
                  control = glmerControl(optimizer = "bobyqa"))

```

Not answer v answer

```

# Use bobyqa
m5a.nested <- glmer(non_answer ~ sex + age_bin + time_to_vs.less_than_hour
                  + out_of_Russia_time + result_trust_bin
                  + orthodox_share + vdem_polyarchy_2022
                  + log(mad_gdppc_2018) + obl_type + export_share
                  + import_share + friendly_status + help + military_dummy
                  + log(dist) + (1 | countryname_en),
                  data = nested_me_data, family = binomial,
                  control = glmerControl(optimizer = "bobyqa",
                                          optCtrl = list(maxfun = 1e9)))

# Doesn't converge

# Fit with all optimizers
m5a.allfit <- allFit(m5a.nested, maxfun = 1e9)

```

bobyqa : [OK]

```

Nelder_Mead : [OK]
nlminbwrap : [OK]
optimx.L-BFGS-B : [OK]
nloptwrap.NLOPT_LN_NELDERMEAD : [OK]
nloptwrap.NLOPT_LN_BOBYQA : [OK]

# Convergence results

## Export models
m5a.allfit_OK <- m5a.allfit[sapply(m5a.allfit, is, "merMod")]

## Generate warnings encountered
data.frame(lapply(m5a.allfit_OK, function(x) x@optinfo$conv$lme4$messages)) |>
  pivot_longer(everything(), names_to = "Method",
               values_to = "Estimation result") |>
  arrange(`Method`, `Estimation result`) |>
  distinct() |>
  mutate(`Method` = if_else(lag(`Method`) != `Method` | is.na(lag(`Method`)),
                           `Method`, ""),
         `Estimation result` = gsub("\\|", " ", `Estimation result`)) |>
  kable(booktabs = T,
        caption = paste("Convergence results for Answer/non-Answer dichotomy,",
                        "ME with level 2 variables and all optimizers")) |>
  column_spec(1, width = "8cm") |>
  column_spec(2, width = "8cm")

```

Table 1: Convergence results for Answer/non-Answer dichotomy, ME with level 2 variables and all optimizers

Method	Estimation result
Nelder_Mead	Model failed to converge with max grad = 0.0288314 (tol = 0.002, component 1)
bobyqa	Model failed to converge with max grad = 0.00240407 (tol = 0.002, component 1)
nlminbwrap	Model failed to converge with max grad = 0.071349 (tol = 0.002, component 1)
nloptwrap.NLOPT_LN_BOBYQA	Model failed to converge with max grad = 0.0450179 (tol = 0.002, component 1)
nloptwrap.NLOPT_LN_NELDERMEAD	Model failed to converge: degenerate Hessian with 1 negative eigenvalues
optimx.L-BFGS.B	unable to evaluate scaled gradient Model failed to converge: degenerate Hessian with 1 negative eigenvalues unable to evaluate scaled gradient

```
# Log-Likelihoods
(llikes <- sort(sapply(m5a.allfit_OK, logLik))) |>
  kable(col.names = "Log-likelihood", booktabs = T, digits = 3)
```

	Log-likelihood
optimx.L-BFGS-B	-8472.303
Nelder_Mead	-8472.302
nloptwrap.NLOPT_LN_BOBYQA	-8472.302
nlminbwrap	-8472.302
bobyqa	-8472.302
nloptwrap.NLOPT_LN_NELDERMEAD	-8472.302

```
# Coefficients from different optimizers
```

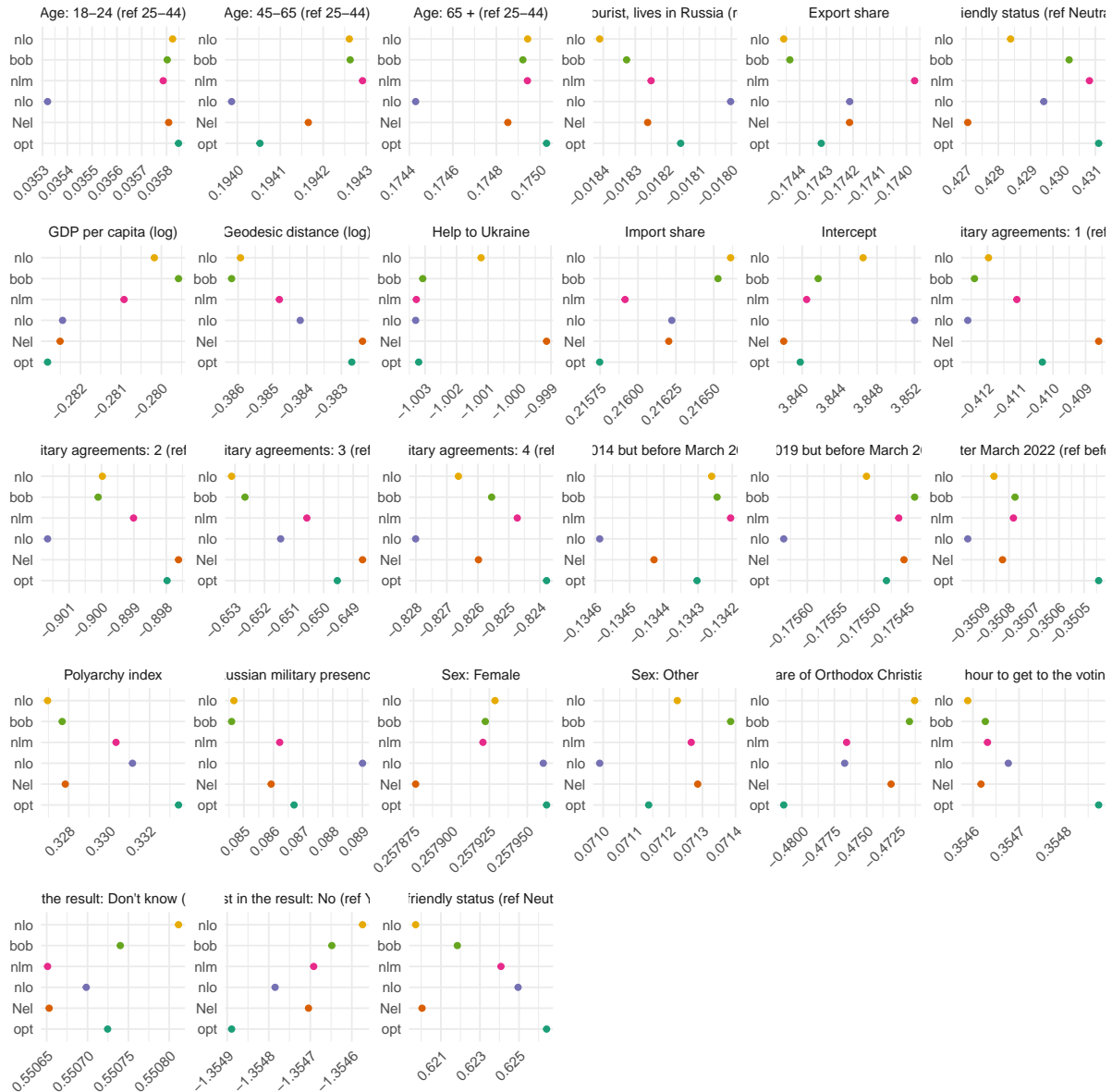
```
## Export fixef and melt into single dataframe
models <- levels(melt(t(sapply(m5a.allfit_OK, fixef)))$Var1)
m5a.allfit.fixef.m <- transform(melt(t(sapply(m5a.allfit_OK, fixef))),
  Var1 = factor(Var1, levels = names(llikes))) |>
  transmute(`Method` = Var1,
    `Coefficient` = value,
    `Variable` = case_when(
      Var2 == "(Intercept)" ~ "Intercept",
      Var2 == "sexFemale" ~ "Sex: Female",
      Var2 == "sexOther" ~ "Sex: Other",
      Var2 == "age_bin18-24" ~ "Age: 18-24 (ref 25-44)",
      Var2 == "age_bin45-64" ~ "Age: 45-65 (ref 25-44)",
      Var2 == "age_bin65+" ~ "Age: 65 + (ref 25-44)",
      Var2 == "time_to_vs.less_than_hourYes" ~
        "Took < 1 hour to get to the voting station",
      Var2 == "out_of_Russia_timeAfter invasion" ~
        "Moved after March 2022 (ref before 2014)",
      Var2 == "out_of_Russia_time2 - 5 years" ~
        paste("Moved after March 2019 but before",
          "March 2022 (ref before 2014)"),
      Var2 == "out_of_Russia_timeAfter annexation" ~
        paste("Moved after March 2014 but before",
          "March 2019 (ref before 2014)"),
      Var2 == "out_of_Russia_timeTourist (lives in Russia)" ~
        paste("Didn't move - tourist, lives",
          "in Russia (ref before 2014)"),
      Var2 == "result_trust_binDon't know" ~
        "Trust in the result: Don't know (ref Yes)",
      Var2 == "result_trust_binNo" ~
        "Trust in the result: No (ref Yes)",
      Var2 == "orthodox_share" ~ "Share of Orthodox Christians",
      Var2 == "vdem_polyarchy_2022" ~ "Polyarchy index",
      Var2 == "log(mad_gdppc_2018)" ~ "GDP per capita (log)",
      Var2 == "obl_type1" ~ "Military agreements: 1 (ref 0)",
      Var2 == "obl_type2" ~ "Military agreements: 2 (ref 0)",
      Var2 == "obl_type3" ~ "Military agreements: 3 (ref 0)",
      Var2 == "obl_type4" ~ "Military agreements: 4 (ref 0)",
      Var2 == "export_share" ~ "Export share",
      Var2 == "import_share" ~ "Import share",
      Var2 == "friendly_statusUnfriendly" ~
```

```

        "Unfriendly status (ref Neutral)",
Var2 == "friendly_statusFriendly" ~
        "Friendly status (ref Neutral)",
Var2 == "help" ~ "Help to Ukraine",
Var2 == "military_dummy" ~ "Russian military presence",
Var2 == "log(dist)" ~ "Geodesic distance (log)",
    )

## Plot
ggplot(m5a.allfit.fixef.m,
       aes(x = `Coefficient`, y = `Method`, colour = `Method`)) +
  geom_point() +
  facet_wrap(~ `Variable`, scale = "free") +
  scale_colour_brewer(palette = "Dark2") +
  scale_y_discrete(breaks = models,
                  labels = substr(models, 1, 3)) +
  labs(x = "", y = "") +
  theme_minimal() +
  theme(legend.position = "none",
        axis.text.x = element_text(angle = 45, hjust = 1))

```



```
# So judging by this I leave the model alone - the coefficients seem to be okay
# between different optimizers and since bobyqa comes close to the tolerance
# threshold, I will use it.
```

```
# Check m5a-specific diagnostics
# Gradient 1
derivs1 <- m5a.nested@optiminfo$derivs
sc_grad1 <- with(derivs1, solve(Hessian, gradient))
gradres1 <- max(pmin(abs(sc_grad1), abs(derivs1$gradient)))

# Gradient 2
dd <- update(m5a.nested, devFunOnly = TRUE)
pars <- unlist(getME(m5a.nested, c("theta", "fixef")))
```

```

grad2 <- grad(dd, pars)
hess2 <- hessian(dd, pars)
sc_grad2 <- solve(hess2, grad2)
gradres2 <- max(pmin(abs(sc_grad2), abs(grad2)))

## The gradient cutoffs for Nelder Mead (nlopt), bobyqa and nlminwrap seem to
## be close to what I am getting.

# Check for singular fit
singres <- if_else(isSingular(m5a.nested), "Yes", "No")

# Display diagnostics
kable(cbind(gradres1, gradres2, singres),
      booktabs = T, digits = 3,
      col.names = c("Gradient, method I", "Gradient, method II",
                    "Singular fit?"),
      caption = paste("Diagnostics for `Answer vs No answer to poll`",
                      "model (with preferred optimizer)"))

```

Table 3: Diagnostics for Answer vs No answer to poll model (with preferred optimizer)

Gradient, method I	Gradient, method II	Singular fit?
0.00240406734519638	0.00243011504717848	No

The model doesn't converge.

Putin v everyone else

```

# Putin (1) vs everyone else (0), declined to answer NA
m4p.nested <- glmer(putin_else ~ sex + age_bin + time_to_vs.less_than_hour
  + out_of_Russia_time + result_trust_bin
  + (1 | countryname_en),
  data = nested_me_data, family = binomial,
  control = glmerControl(optimizer = "bobyqa"))

# Converges!

m5p.nested <- glmer(putin_else ~ sex + age_bin + time_to_vs.less_than_hour
  + out_of_Russia_time + result_trust_bin
  + orthodox_share + vdem_polyarchy_2022
  + log(mad_gdppc_2018) + obl_type + export_share
  + import_share + friendly_status + help + military_dummy
  + log(dist) + (1 | countryname_en),
  data = nested_me_data, family = binomial,
  control = glmerControl(optimizer = "bobyqa",
    optCtrl = list(maxfun = 1e9)))

```

```

# Fit with all optimizers (loaded with other models)
m5p.allfit <- allFit(m5p.nested, maxfun = 1e9)

bobyqa : [OK]
Nelder_Mead : [OK]
nlminbwrap : [OK]
optimx.L-BFGS-B : [OK]
nloptwrap.NLOPT_LN_NELDERMEAD : [OK]
nloptwrap.NLOPT_LN_BOBYQA : [OK]

# Convergence results

## Export models
m5p.allfit_OK <- m5p.allfit[sapply(m5p.allfit, is, "merMod")]

## Generate warnings encountered
data.frame(lapply(m5p.allfit_OK, function(x) x@optinfo$conv$lme4$messages)) |>
  pivot_longer(everything(), names_to = "Method",
               values_to = "Estimation result") |>
  arrange(`Method`, `Estimation result`) |>
  distinct() |>
  mutate(`Method` = if_else(lag(`Method`) != `Method` | is.na(lag(`Method`)),
                           `Method`, ""),
         `Estimation result` = gsub("\\|", " ", `Estimation result`)) |>
  kable(booktabs = T,
        caption = paste("Convergence results for Answer/non-Answer dichotomy,",
                        "ME with level 2 variables and all optimizers")) |>
  column_spec(1, width = "8cm") |>
  column_spec(2, width = "8cm")

```

Table 4: Convergence results for Answer/non-Answer dichotomy, ME with level 2 variables and all optimizers

Method	Estimation result
Nelder_Mead	Model failed to converge with max grad = 0.0439094 (tol = 0.002, component 1)
bobyqa	Model failed to converge with max grad = 0.00486188 (tol = 0.002, component 1)
nlminbwrap	Model failed to converge with max grad = 0.0198981 (tol = 0.002, component 1)
nloptwrap.NLOPT_LN_BOBYQA	Model failed to converge with max grad = 0.0277748 (tol = 0.002, component 1)
nloptwrap.NLOPT_LN_NELDERMEAD	Model failed to converge with max grad = 0.0240685 (tol = 0.002, component 1)

optimx.L.BFGS.B

Model failed to converge with max grad =
0.0818325 (tol = 0.002, component 1)

```
# Log-Likelihoods
(llikes <- sort(sapply(m5p.allfit_OK, logLik))) |>
  kable(col.names = "Log-likelihood", booktabs = T, digits = 3)
```

	Log-likelihood
Nelder_Mead	-3178.423
nloptwrap.NLOPT_LN_BOBYQA	-3178.423
optimx.L-BFGS-B	-3178.423
nlminbwrap	-3178.423
nloptwrap.NLOPT_LN_NELDERMEAD	-3178.423
bobyqa	-3178.423

```
# Coefficients from different optimizers
```

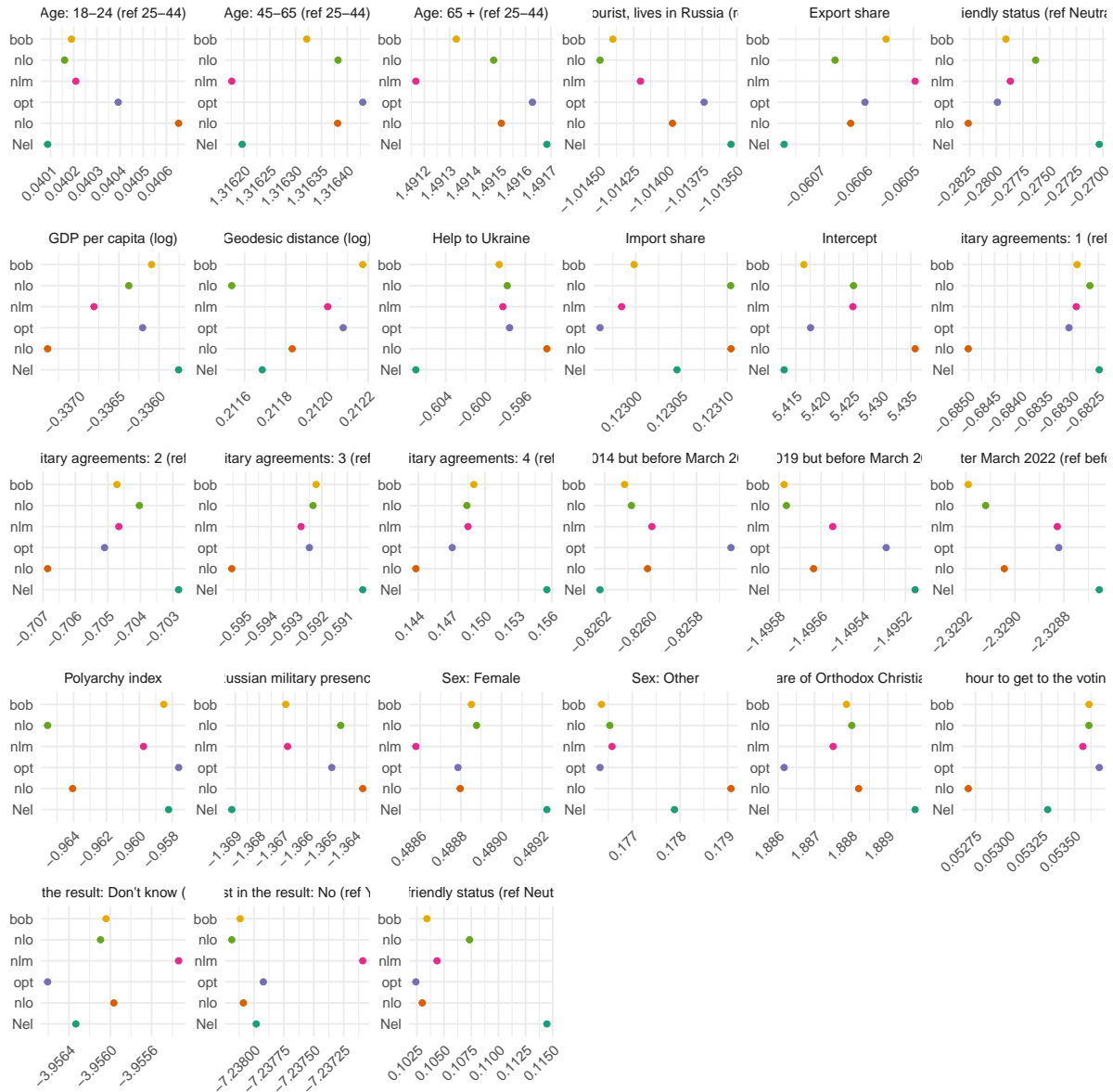
```
## Export fixef and melt into single dataframe
models <- levels(melt(t(sapply(m5p.allfit_OK, fixef)))$Var1)
m5p.allfit.fixef.m <- transform(melt(t(sapply(m5p.allfit_OK, fixef))),
  Var1 = factor(Var1, levels = names(lliks))) |>
  transmute(`Method` = Var1,
    `Coefficient` = value,
    `Variable` = case_when(
      Var2 == "(Intercept)" ~ "Intercept",
      Var2 == "sexFemale" ~ "Sex: Female",
      Var2 == "sexOther" ~ "Sex: Other",
      Var2 == "age_bin18-24" ~ "Age: 18-24 (ref 25-44)",
      Var2 == "age_bin45-64" ~ "Age: 45-65 (ref 25-44)",
      Var2 == "age_bin65+" ~ "Age: 65 + (ref 25-44)",
      Var2 == "time_to_vs.less_than_hourYes" ~
        "Took < 1 hour to get to the voting station",
      Var2 == "out_of_Russia_timeAfter invasion" ~
        "Moved after March 2022 (ref before 2014)",
      Var2 == "out_of_Russia_time2 - 5 years" ~
        paste("Moved after March 2019 but before",
          "March 2022 (ref before 2014)"),
      Var2 == "out_of_Russia_timeAfter annexation" ~
        paste("Moved after March 2014 but before",
          "March 2019 (ref before 2014)"),
      Var2 == "out_of_Russia_timeTourist (lives in Russia)" ~
        paste("Didn't move - tourist, lives",
          "in Russia (ref before 2014)"),
      Var2 == "result_trust_binDon't know" ~
        "Trust in the result: Don't know (ref Yes)",
      Var2 == "result_trust_binNo" ~
        "Trust in the result: No (ref Yes)",
      Var2 == "orthodox_share" ~ "Share of Orthodox Christians",
      Var2 == "vdem_polyarchy_2022" ~ "Polyarchy index",
      Var2 == "log(mad_gdppc_2018)" ~ "GDP per capita (log)",
      Var2 == "obl_type1" ~ "Military agreements: 1 (ref 0)",
      Var2 == "obl_type2" ~ "Military agreements: 2 (ref 0)",
```

```

Var2 == "obl_type3" ~ "Military agreements: 3 (ref 0)",
Var2 == "obl_type4" ~ "Military agreements: 4 (ref 0)",
Var2 == "export_share" ~ "Export share",
Var2 == "import_share" ~ "Import share",
Var2 == "friendly_statusUnfriendly" ~
  "Unfriendly status (ref Neutral)",
Var2 == "friendly_statusFriendly" ~
  "Friendly status (ref Neutral)",
Var2 == "help" ~ "Help to Ukraine",
Var2 == "military_dummy" ~ "Russian military presence",
Var2 == "log(dist)" ~ "Geodesic distance (log)",
)

## Plot
ggplot(m5p.allfit.fixef.m,
  aes(x = `Coefficient`, y = `Method`, colour = `Method`)) +
  geom_point() +
  facet_wrap(~ `Variable`, scale = "free") +
  scale_colour_brewer(palette = "Dark2") +
  scale_y_discrete(breaks = models,
    labels = substr(models, 1, 3)) +
  labs(x = "", y = "") +
  theme_minimal() +
  theme(legend.position = "none",
    axis.text.x = element_text(angle = 45, hjust = 1))

```



```
# Check m5p-specific diagnostics
```

```
# Gradient 1
derivs1 <- m5p.nested@optinfo$derivs
sc_grad1 <- with(derivs1, solve(Hessian, gradient))
gradres1 <- max(pmin(abs(sc_grad1), abs(derivs1$gradient)))
```

```
# Gradient 2
dd <- update(m5p.nested, devFunOnly = TRUE)
pars <- unlist(getME(m5p.nested, c("theta", "fixef")))
grad2 <- grad(dd, pars)
hess2 <- hessian(dd, pars)
sc_grad2 <- solve(hess2, grad2)
```

```

gradres2 <- max(pmin(abs(sc_grad2), abs(grad2)))

## The gradient cutoffs for Nelder Mead (nlopt), bobyqa and nlminwrap seem to
## be close to what I am getting.

# Check for singular fit
singres <- if_else(isSingular(m5p.nested), "Yes", "No")

# Display diagnostics
kable(cbind(gradres1, gradres2, singres),
      booktabs = T, digits = 3,
      col.names = c("Gradient, method I", "Gradient, method II",
                    "Singular fit?"),
      caption = paste("Diagnostics for `Putin vs Everyone",
                      "else` model (with preferred optimizer)"))

```

Table 6: Diagnostics for Putin vs Everyone else model (with preferred optimizer)

Gradient, method I	Gradient, method II	Singular fit?
0.0132050081447233	0.0131996664023155	No

Non-systemic v systemic opposition

```

# Non-systemic - Davankov or Spoiled (1) vs systemic - Haritonov, Slutsky (0)
# opposition, declined to answer and Putin are NA
m4s.nested <- glmer(nonsys_sys ~ sex + age_bin + time_to_vs.less_than_hour
                  + out_of_Russia_time + result_trust_bin
                  + (1 | countryname_en),
                  data = nested_me_data, family = binomial,
                  control = glmerControl(optimizer = "bobyqa"))

# Converges

m5s.nested <- glmer(nonsys_sys ~ sex + age_bin + time_to_vs.less_than_hour
                  + out_of_Russia_time + result_trust_bin
                  + orthodox_share + vdem_polyarchy_2022
                  + log(mad_gdppc_2018) + obl_type + export_share
                  + import_share + friendly_status + help + military_dummy
                  + log(dist) + (1 | countryname_en),
                  data = nested_me_data, family = binomial,
                  control = glmerControl(optimizer = "bobyqa",
                                          optCtrl = list(maxfun = 1e9)))

# Singular fit!

# The predictors most likely to cause issues are obl_type and friendly_status
# as they are broadly (and correlated between themselves and other variables)
# defined dichotomous predictors. Remove them from the model and try again

```

```
m5s.red <- update(m5s.nested, ~ . - obl_type - friendly_status)

# Works great and no issues with convergence either
```

Davankov v spoiled

```
# Davankov (1) vs Spoiled (0) declined to answer, Haritonov, Slutsky and Putin
# are NA
m4d.nested <- glmer(davankov_spoiled ~ sex + age_bin + time_to_vs.less_than_hour
  + out_of_Russia_time + result_trust_bin
  + (1 | countryname_en),
  data = nested_me_data, family = binomial,
  control = glmerControl(optimizer = "bobyqa"))

m5d.nested <- glmer(davankov_spoiled ~ sex + age_bin + time_to_vs.less_than_hour
  + out_of_Russia_time + result_trust_bin
  + orthodox_share + vdem_polyarchy_2022
  + log(mad_gdppc_2018) + obl_type + export_share
  + import_share + friendly_status + help + military_dummy
  + log(dist) + (1 | countryname_en),
  data = nested_me_data, family = binomial,
  control = glmerControl(optimizer = "bobyqa",
    optCtrl = list(maxfun = 1e9)))

m5d.allfit <- allFit(m5d.nested, maxfun = 1e9)

bobyqa : [OK]
Nelder_Mead : [OK]
nlminbwrap : [OK]
optimx.L-BFGS-B : [OK]
nloptwrap.NLOPT_LN_NELDERMEAD : [OK]
nloptwrap.NLOPT_LN_BOBYQA : [OK]
```

```
# Convergence results
```

```
## Export models
m5d.allfit_OK <- m5d.allfit[sapply(m5d.allfit, is, "merMod")]

## Generate warnings encountered
data.frame(lapply(m5d.allfit_OK, function(x) x@optinfo$conv$lme4$messages)) |>
  pivot_longer(everything(), names_to = "Method",
    values_to = "Estimation result") |>
  arrange(`Method`, `Estimation result`) |>
  distinct() |>
  mutate(`Method` = if_else(lag(`Method`) != `Method` | is.na(lag(`Method`)),
    `Method`, ""),
    `Estimation result` = gsub("\\|", " ", `Estimation result`)) |>
  kable(booktabs = T,
    caption = paste("Convergence results for Answer/non-Answer dichotomy,"))
```

```

"ME with level 2 variables and all optimizers")) |>
column_spec(1, width = "8cm") |>
column_spec(2, width = "8cm")

```

Table 7: Convergence results for Answer/non-Answer dichotomy, ME with level 2 variables and all optimizers

Method	Estimation result
Nelder_Mead	Model failed to converge with max grad = 0.425244 (tol = 0.002, component 1)
bobyqa	Model failed to converge with max grad = 0.0157886 (tol = 0.002, component 1)
nlminbwrap	Model failed to converge with max grad = 1.66793 (tol = 0.002, component 1)
nloptwrap.NLOPT_LN_BOBYQA	Model failed to converge with max grad = 0.0678025 (tol = 0.002, component 1)
nloptwrap.NLOPT_LN_NELDERMEAD	Model failed to converge with max grad = 0.081166 (tol = 0.002, component 1)
optimx.L.BFGS.B	Model failed to converge with max grad = 0.0682182 (tol = 0.002, component 1)

```

# Log-Likelihoods
(llikes <- sort(sapply(m5d.allfit_OK, logLik))) |>
  kable(col.names = "Log-likelihood", booktabs = T, digits = 3)

```

	Log-likelihood
Nelder_Mead	-20537.76
nlminbwrap	-20537.73
optimx.L-BFGS-B	-20537.73
nloptwrap.NLOPT_LN_BOBYQA	-20537.73
nloptwrap.NLOPT_LN_NELDERMEAD	-20537.73
bobyqa	-20537.73

```

# Coefficients from different optimizers

## Export fixef and melt into single dataframe
models <- levels(melt(t(sapply(m5d.allfit_OK, fixef)))$Var1)
m5d.allfit.fixef.m <- transform(melt(t(sapply(m5d.allfit_OK, fixef))),
                                Var1 = factor(Var1, levels = names(llikes))) |>
  transmute(`Method` = Var1,
            `Coefficient` = value,
            `Variable` = case_when(
              Var2 == "(Intercept)" ~ "Intercept",

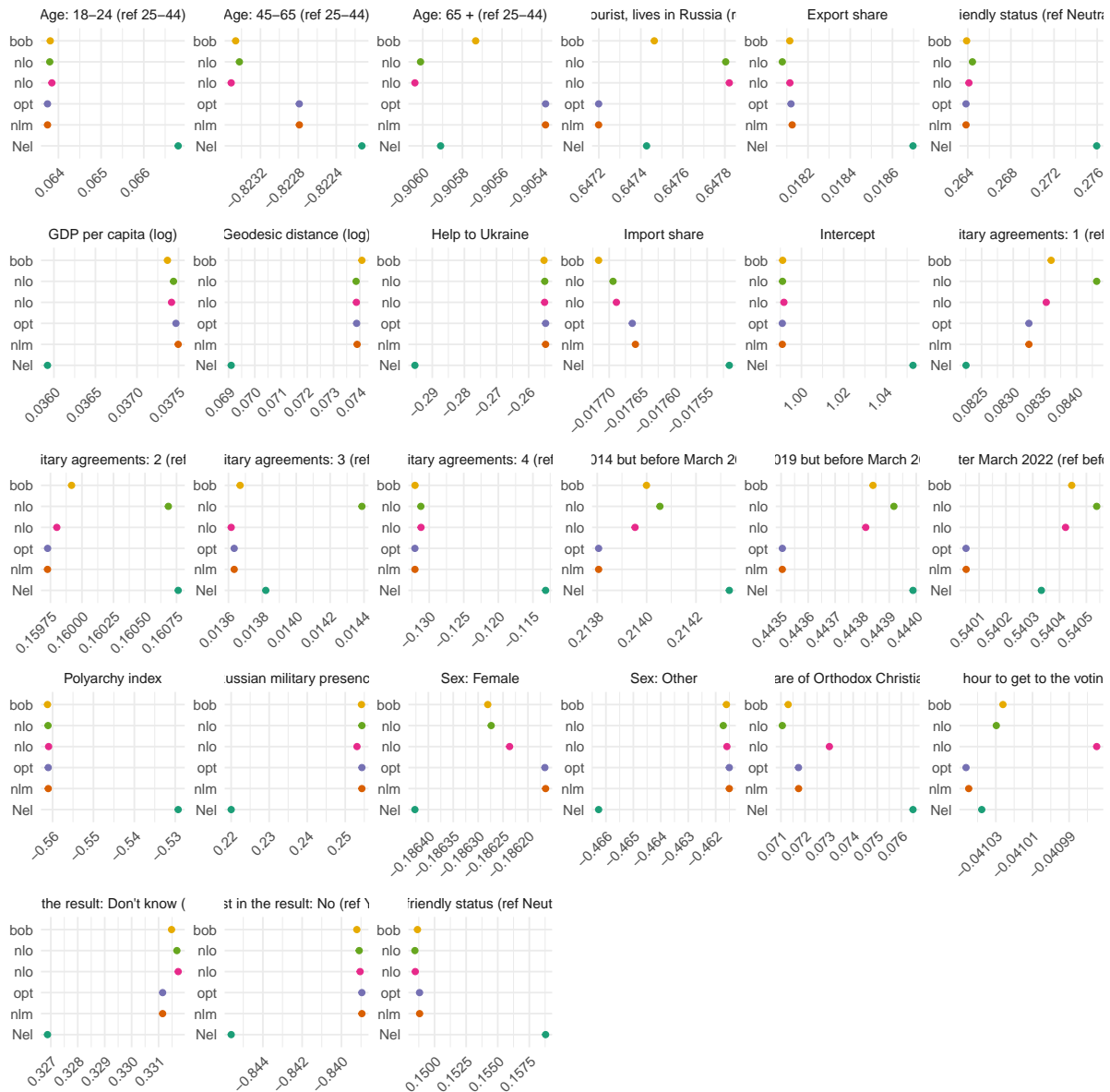
```

```

Var2 == "sexFemale" ~ "Sex: Female",
Var2 == "sexOther" ~ "Sex: Other",
Var2 == "age_bin18-24" ~ "Age: 18-24 (ref 25-44)",
Var2 == "age_bin45-64" ~ "Age: 45-65 (ref 25-44)",
Var2 == "age_bin65+" ~ "Age: 65 + (ref 25-44)",
Var2 == "time_to_vs.less_than_hourYes" ~
  "Took < 1 hour to get to the voting station",
Var2 == "out_of_Russia_timeAfter invasion" ~
  "Moved after March 2022 (ref before 2014)",
Var2 == "out_of_Russia_time2 - 5 years" ~
  paste("Moved after March 2019 but before",
    "March 2022 (ref before 2014)"),
Var2 == "out_of_Russia_timeAfter annexation" ~
  paste("Moved after March 2014 but before",
    "March 2019 (ref before 2014)"),
Var2 == "out_of_Russia_timeTourist (lives in Russia)" ~
  paste("Didn't move - tourist, lives",
    "in Russia (ref before 2014)"),
Var2 == "result_trust_binDon't know" ~
  "Trust in the result: Don't know (ref Yes)",
Var2 == "result_trust_binNo" ~
  "Trust in the result: No (ref Yes)",
Var2 == "orthodox_share" ~ "Share of Orthodox Christians",
Var2 == "vdem_polyarchy_2022" ~ "Polyarchy index",
Var2 == "log(mad_gdppc_2018)" ~ "GDP per capita (log)",
Var2 == "obl_type1" ~ "Military agreements: 1 (ref 0)",
Var2 == "obl_type2" ~ "Military agreements: 2 (ref 0)",
Var2 == "obl_type3" ~ "Military agreements: 3 (ref 0)",
Var2 == "obl_type4" ~ "Military agreements: 4 (ref 0)",
Var2 == "export_share" ~ "Export share",
Var2 == "import_share" ~ "Import share",
Var2 == "friendly_statusUnfriendly" ~
  "Unfriendly status (ref Neutral)",
Var2 == "friendly_statusFriendly" ~
  "Friendly status (ref Neutral)",
Var2 == "help" ~ "Help to Ukraine",
Var2 == "military_dummy" ~ "Russian military presence",
Var2 == "log(dist)" ~ "Geodesic distance (log)",
)

## Plot
ggplot(m5d.allfit.fixef.m,
  aes(x = `Coefficient`, y = `Method`, colour = `Method`)) +
  geom_point() +
  facet_wrap(~ `Variable`, scale = "free") +
  scale_colour_brewer(palette = "Dark2") +
  scale_y_discrete(breaks = models,
    labels = substr(models, 1, 3)) +
  labs(x = "", y = "") +
  theme_minimal() +
  theme(legend.position = "none",
    axis.text.x = element_text(angle = 45, hjust = 1))

```



```
# Check m5d-specific diagnostics
```

```
# Gradient 1
derivs1 <- m5d.nested@optinfo$derivs
sc_grad1 <- with(derivs1, solve(Hessian, gradient))
gradres1 <- max(pmin(abs(sc_grad1), abs(derivs1$gradient)))
```

```
# Gradient 2
dd <- update(m5d.nested, devFunOnly = TRUE)
pars <- unlist(getME(m5d.nested, c("theta", "fixef")))
grad2 <- grad(dd, pars)
hess2 <- hessian(dd, pars)
sc_grad2 <- solve(hess2, grad2)
```



```

gradres2 <- max(pmin(abs(sc_grad2), abs(grad2)))

## The gradient cutoffs for Nelder Mead (nlopt), bobyqa and nlminwrap seem to
## be close to what I am getting.

# Check for singular fit
singres <- if_else(isSingular(m5d.nested), "Yes", "No")

# Display diagnostics
kable(cbind(gradres1, gradres2, singres),
      booktabs = T, digits = 3,
      col.names = c("Gradient, method I", "Gradient, method II",
                    "Singular fit?"),
      caption = paste("Diagnostics for `Davankov vs Spoil the ballot`,
                    "model (with preferred optimizer)"))

```

Table 9: Diagnostics for Davankov vs Spoil the ballot model (with preferred optimizer)

Gradient, method I	Gradient, method II	Singular fit?
0.0040686829458804	0.00394873324194497	No

```

save(list = c("m5a.nested", "m5p.nested", "m5s.nested", "m5s.red", "m5d.nested"),
     file = "nlme_bobyqa.RData")

save(list = c("m4a.nested", "m4p.nested", "m4s.nested", "m4d.nested"),
     file = "feme_bobyqa.RData")

save(list = c("m5a.allfit", "m5p.allfit", "m5d.allfit"),
     file = "me_allfit.RData")

resizebox.stargazer(m4a.nested, m4p.nested, m4s.nested, m4d.nested,
  title = "Mixed effects models, level 1", header = F,
  dep.var.labels = c("Declined to answer vs answer",
                    "Putin vs everyone else",
                    "Non-systemic vs systemic opposition",
                    "Davankov vs Spoiled"),
  tab.height = "\\textheight", tab.width= "\\textwidth")

m3.answer <- models(m3.nested.fe, 1)
m3.putin <- models(m3.nested.fe, 2)
m3.nonsys <- models(m3.nested.fe, 3)
m3.davankov <- models(m3.nested.fe, 4)

resizebox.stargazer(m4a.nested, m3.answer, m4p.nested, m3.putin,
  title = "Mixed effects models, comparison I", header = F,
  omit = "as.factor",
  dep.var.labels = c("Declined to answer vs answer",
                    "Declined to answer vs answer",
                    "Putin vs everyone else",
                    "Putin vs everyone else"
                    ),
  tab.height = "\\textheight", tab.width= "\\textwidth")

```

Table 10: Mixed effects models, level 1

	<i>Dependent variable:</i>			
	Declined to answer vs answer	Putin vs everyone else	Non-systemic vs systemic opposition	Davankov vs Spoiled
	(1)	(2)	(3)	(4)
sexFemale	0.275*** (0.044)	0.482*** (0.071)	0.022 (0.056)	−0.199*** (0.024)
sexOther	0.064 (0.237)	0.171 (0.368)	−0.375 (0.296)	−0.470*** (0.144)
age_bin18-24	−0.003 (0.081)	0.026 (0.124)	−0.095 (0.086)	0.062* (0.037)
age_bin45-64	0.180*** (0.059)	1.322*** (0.086)	−0.302*** (0.087)	−0.826*** (0.039)
age_bin65+	0.170** (0.077)	1.507*** (0.131)	−0.957*** (0.155)	−0.931*** (0.106)
time_to_vs.less_than_hourYes	0.376*** (0.052)	0.094 (0.081)	−0.099 (0.061)	−0.034 (0.026)
out_of_Russia_time2 - 5 years	−0.207*** (0.075)	−1.480*** (0.109)	0.409*** (0.091)	0.454*** (0.042)
out_of_Russia_timeAfter annexation	−0.159** (0.080)	−0.806*** (0.120)	0.296*** (0.102)	0.219*** (0.045)
out_of_Russia_timeAfter invasion	−0.359*** (0.069)	−2.330*** (0.101)	0.766*** (0.084)	0.565*** (0.040)
out_of_Russia_timeTourist (lives in Russia)	−0.039 (0.096)	−0.975*** (0.143)	0.041 (0.144)	0.652*** (0.087)
result_trust_binDon't know	0.499*** (0.082)	−3.960*** (0.109)	0.718*** (0.127)	0.387*** (0.139)
result_trust_binNo	−1.328*** (0.060)	−7.218*** (0.101)	1.903*** (0.094)	−0.806*** (0.105)
Constant	−2.658*** (0.123)	1.992*** (0.158)	1.267*** (0.123)	1.558*** (0.117)
Observations	53,824	51,202	42,363	40,946
Log Likelihood	−9,143.189	−3,369.086	−5,864.162	−22,658.160
Akaike Inf. Crit.	18,314.380	6,766.172	11,756.320	45,344.320
Bayesian Inf. Crit.	18,438.890	6,889.982	11,877.480	45,465.000

Note:

*p<0.1; **p<0.05; ***p<0.01

Table 11: Mixed effects models, comparison I

	<i>Dependent variable:</i>			
	Declined to answer vs answer	Declined to answer vs answer	Putin vs everyone else	Putin vs everyone else
	<i>generalized linear mixed-effects</i>	<i>logistic</i>	<i>generalized linear mixed-effects</i>	<i>logistic</i>
	(1)	(2)	(3)	(4)
sexFemale	0.275*** (0.044)	0.275*** (0.044)	0.482*** (0.071)	0.481*** (0.072)
sexOther	0.064 (0.237)	0.039 (0.240)	0.171 (0.368)	0.208 (0.369)
age_bin18-24	-0.003 (0.081)	-0.008 (0.082)	0.026 (0.124)	0.019 (0.126)
age_bin45-64	0.180*** (0.059)	0.172*** (0.059)	1.322*** (0.086)	1.304*** (0.087)
age_bin65+	0.170** (0.077)	0.161** (0.078)	1.507*** (0.131)	1.501*** (0.132)
time_to_vs.less_than_hourYes	0.376*** (0.052)	0.310*** (0.055)	0.094 (0.081)	0.071 (0.085)
out_of_Russia_time2 - 5 years	-0.207*** (0.075)	-0.217*** (0.075)	-1.480*** (0.109)	-1.467*** (0.110)
out_of_Russia_timeAfter annexation	-0.159** (0.080)	-0.179** (0.081)	-0.806*** (0.120)	-0.798*** (0.121)
out_of_Russia_timeAfter invasion	-0.359*** (0.069)	-0.363*** (0.069)	-2.330*** (0.101)	-2.357*** (0.103)
out_of_Russia_timeTourist (lives in Russia)	-0.039 (0.096)	-0.002 (0.098)	-0.975*** (0.143)	-0.997*** (0.146)
result_trust_binDon't know	0.499*** (0.082)	0.521*** (0.083)	-3.960*** (0.109)	-3.955*** (0.110)
result_trust_binNo	-1.328*** (0.060)	-1.293*** (0.060)	-7.218*** (0.101)	-7.204*** (0.102)
Constant	-2.658*** (0.123)	-2.146*** (0.144)	1.992*** (0.158)	1.379*** (0.247)
Observations	53,824	53,824	51,202	51,202
Log Likelihood	-9,143.189	-9,008.215	-3,369.086	-3,287.206
Akaike Inf. Crit.	18,314.380	18,166.430	6,766.172	6,724.412
Bayesian Inf. Crit.	18,438.890		6,889.982	

Note:

*p<0.1; **p<0.05; ***p<0.01

```

resizebox.stargazer(m4s.nested, m3.nonsys, m4d.nested, m3.davankov,
  title = "Mixed effects models, comparison II", header = F,
  omit = "as.factor",
  dep.var.labels = c("Non-systemic vs systemic opposition",
    "Non-systemic vs systemic opposition",
    "Davankov vs Spoiled",
    "Davankov vs Spoiled"
  ),
  tab.height = "\\textheight", tab.width= "\\textwidth")

```

Table 12: Mixed effects models, comparison II

	<i>Dependent variable:</i>			
	Non-systemic vs systemic opposition	Non-systemic vs systemic opposition	Davankov vs Spoiled	Davankov vs Spoiled
	<i>generalized linear mixed-effects</i>	<i>logistic</i>	<i>generalized linear mixed-effects</i>	<i>logistic</i>
	(1)	(2)	(3)	(4)
sexFemale	0.022 (0.056)	0.019 (0.057)	-0.199*** (0.024)	-0.195*** (0.024)
sexOther	-0.375 (0.296)	-0.371 (0.301)	-0.470*** (0.144)	-0.455*** (0.146)
age_bin18-24	-0.095 (0.086)	-0.114 (0.088)	0.062* (0.037)	0.072* (0.037)
age_bin45-64	-0.302*** (0.087)	-0.294*** (0.088)	-0.826*** (0.039)	-0.827*** (0.039)
age_bin65+	-0.957*** (0.155)	-0.994*** (0.159)	-0.931*** (0.106)	-0.952*** (0.107)
time_to_vs.less_than_hourYes	-0.099 (0.061)	-0.086 (0.067)	-0.034 (0.026)	-0.028 (0.027)
out_of_Russia_time2 - 5 years	0.409*** (0.091)	0.419*** (0.093)	0.454*** (0.042)	0.438*** (0.042)
out_of_Russia_timeAfter annexation	0.296*** (0.102)	0.299*** (0.103)	0.219*** (0.045)	0.213*** (0.046)
out_of_Russia_timeAfter invasion	0.766*** (0.084)	0.820*** (0.090)	0.565*** (0.040)	0.539*** (0.040)
out_of_Russia_timeTourist (lives in Russia)	0.041 (0.144)	0.134 (0.150)	0.652*** (0.087)	0.633*** (0.088)
result_trust_binDon't know	0.718*** (0.127)	0.716*** (0.129)	0.387*** (0.139)	0.392*** (0.141)
result_trust_binNo	1.903*** (0.094)	1.891*** (0.095)	-0.806*** (0.105)	-0.804*** (0.106)
Constant	1.267*** (0.123)	1.223*** (0.198)	1.558*** (0.117)	1.401*** (0.130)
Observations	42,363	42,363	40,946	40,946
Log Likelihood	-5,864.162	-5,809.709	-22,658.160	-22,553.850
Akaike Inf. Crit.	11,756.320	11,769.420	45,344.320	45,257.690
Bayesian Inf. Crit.	11,877.480		45,465.000	

Note:

*p<0.1; **p<0.05; ***p<0.01

```

resizebox.stargazer(m5a.nested, m5p.nested, m5s.red, m5d.nested,
  title = "Mixed effects models with level 2", header = F,
  omit = "as.factor",
  dep.var.labels = c("Declined to answer vs answer",
    "Putin vs everyone else",

```

```

        "Non-systemic vs systemic opposition",
        "Davankov vs Spoiled"
    ),
    tab.height = "\\textheight", tab.width= "\\textwidth")

resizebox.stargazer(m4a.nested, m5a.nested, m4p.nested, m5p.nested,
    title = "Mixed effects models with level 2, comparison I", header = F,
    omit = "as.factor",
    dep.var.labels = c("Declined to answer vs answer",
        "Declined to answer vs answer",
        "Putin vs everyone else",
        "Putin vs everyone else"
    ),
    tab.height = "\\textheight", tab.width= "\\textwidth")

resizebox.stargazer(m4s.nested, m5s.red, m4d.nested, m5d.nested,
    title = "Mixed effects models with level 2, comparison II", header = F,
    omit = "as.factor",
    dep.var.labels = c("Non-systemic vs systemic opposition",
        "Non-systemic vs systemic opposition",
        "Davankov vs Spoiled",
        "Davankov vs Spoiled"
    ),
    tab.height = "\\textheight", tab.width = "\\textwidth")

```

Table 13: Mixed effects models with level 2

	<i>Dependent variable:</i>			
	Declined to answer vs answer	Putin vs everyone else	Non-systemic vs systemic opposition	Davankov vs Spoiled
	(1)	(2)	(3)	(4)
sexFemale	0.258*** (0.045)	0.489*** (0.073)	0.015 (0.059)	-0.186*** (0.025)
sexOther	0.071 (0.240)	0.176 (0.376)	-0.232 (0.328)	-0.462*** (0.148)
age_bin18-24	0.036 (0.083)	0.040 (0.125)	-0.117 (0.088)	0.064* (0.038)
age_bin45-64	0.194*** (0.060)	1.316*** (0.089)	-0.299*** (0.090)	-0.823*** (0.040)
age_bin65+	0.175** (0.078)	1.491*** (0.134)	-0.930*** (0.159)	-0.906*** (0.108)
time_to_vs.less_than_hourYes	0.355*** (0.054)	0.054 (0.084)	-0.081 (0.065)	-0.041 (0.027)
out_of_Russia_time2 - 5 years	-0.175** (0.076)	-1.496*** (0.111)	0.402*** (0.093)	0.444*** (0.042)
out_of_Russia_timeAfter annexation	-0.134* (0.081)	-0.826*** (0.121)	0.318*** (0.103)	0.214*** (0.045)
out_of_Russia_timeAfter invasion	-0.351*** (0.071)	-2.329*** (0.103)	0.817*** (0.090)	0.540*** (0.040)
out_of_Russia_timeTourist (lives in Russia)	-0.018 (0.099)	-1.014*** (0.148)	0.148 (0.153)	0.647*** (0.090)
result_trust_binDon't know	0.551*** (0.084)	-3.956*** (0.111)	0.700*** (0.133)	0.331** (0.146)
result_trust_binNo	-1.355*** (0.062)	-7.238*** (0.103)	1.910*** (0.098)	-0.839*** (0.111)
orthodox_share	-0.472 (0.468)	1.888*** (0.412)	-0.360** (0.142)	0.071 (0.163)
vdem_polyarchy_2022	0.328 (0.668)	-0.959 (0.674)	0.532** (0.250)	-0.561** (0.233)
log(mad_gdppc_2018)	-0.280 (0.174)	-0.336* (0.176)	0.036 (0.088)	0.037 (0.072)
obl_type1	-0.412 (0.471)	-0.683 (0.454)		0.084 (0.162)
obl_type2	-0.900* (0.507)	-0.705 (0.485)		0.160 (0.171)
obl_type3	-0.653 (0.463)	-0.592 (0.427)		0.014 (0.162)
obl_type4	-0.826 (0.814)	0.149 (0.739)		-0.132 (0.278)
export_share	-0.174*** (0.062)	-0.061 (0.058)	0.030 (0.024)	0.018 (0.016)
import_share	0.217*** (0.063)	0.123** (0.056)	-0.020 (0.020)	-0.018 (0.018)
friendly_statusUnfriendly	0.622 (0.819)	0.103 (0.794)		0.149 (0.284)
friendly_statusFriendly	0.430 (0.480)	-0.279 (0.466)		0.264 (0.183)
help	-1.003** (0.476)	-0.599 (0.532)	-0.124 (0.194)	-0.255 (0.196)
military_dummy	0.085 (0.728)	-1.367** (0.678)	0.224 (0.195)	0.254 (0.264)
log(dist)	-0.386** (0.151)	0.212 (0.144)	-0.150*** (0.057)	0.074 (0.050)
Constant	3.842* (1.965)	5.416*** (2.091)	1.739* (1.051)	0.991 (0.907)
Observations	48,964	46,494	37,827	36,523
Log Likelihood	-8,472.302	-3,178.423	-5,342.410	-20,537.730
Akaike Inf. Crit.	17,000.600	6,412.846	10,728.820	41,131.460
Bayesian Inf. Crit.	17,246.970	6,657.764	10,916.720	41,369.610

Note:

*p<0.1; **p<0.05; ***p<0.01

Table 14: Mixed effects models with level 2, comparison I

	<i>Dependent variable:</i>			
	Declined to answer vs answer		Declined to answer vs answer	
	(1)	(2)	(3)	(4)
sexFemale	0.275*** (0.044)	0.258*** (0.045)	0.482*** (0.071)	0.489*** (0.073)
sexOther	0.064 (0.237)	0.071 (0.240)	0.171 (0.368)	0.176 (0.376)
age_bin18-24	-0.003 (0.081)	0.036 (0.083)	0.026 (0.124)	0.040 (0.125)
age_bin45-64	0.180*** (0.059)	0.194*** (0.060)	1.322*** (0.086)	1.316*** (0.089)
age_bin65+	0.170** (0.077)	0.175** (0.078)	1.507*** (0.131)	1.491*** (0.134)
time_to_vs.less_than_hourYes	0.376*** (0.052)	0.355*** (0.054)	0.094 (0.081)	0.054 (0.084)
out_of_Russia_time2 - 5 years	-0.207*** (0.075)	-0.175** (0.076)	-1.480*** (0.109)	-1.496*** (0.111)
out_of_Russia_timeAfter annexation	-0.159** (0.080)	-0.134* (0.081)	-0.806*** (0.120)	-0.826*** (0.121)
out_of_Russia_timeAfter invasion	-0.350*** (0.069)	-0.351*** (0.071)	-2.330*** (0.101)	-2.329*** (0.103)
out_of_Russia_timeTourist (lives in Russia)	-0.039 (0.096)	-0.018 (0.099)	-0.975*** (0.143)	-1.014*** (0.148)
result_trust_binDon't know	0.499*** (0.082)	0.551*** (0.084)	-3.960*** (0.109)	-3.956*** (0.111)
result_trust_binNo	-1.328*** (0.060)	-1.355*** (0.062)	-7.218*** (0.101)	-7.238*** (0.103)
orthodox_share		-0.472 (0.468)		1.888*** (0.412)
vdem_polyarchy_2022		0.328 (0.668)		-0.959 (0.674)
log(mad_gdppe_2018)		-0.280 (0.174)		-0.336* (0.176)
obl_type1		-0.412 (0.471)		-0.683 (0.454)
obl_type2		-0.900* (0.507)		-0.705 (0.485)
obl_type3		-0.653 (0.463)		-0.592 (0.427)
obl_type4		-0.826 (0.814)		0.149 (0.739)
export_share		-0.174*** (0.062)		-0.061 (0.058)
import_share		0.217*** (0.063)		0.123** (0.056)
friendly_statusUnfriendly		0.622 (0.819)		0.103 (0.794)
friendly_statusFriendly		0.430 (0.480)		-0.279 (0.466)
help		-1.003** (0.476)		-0.599 (0.532)
military_dummy		0.085 (0.728)		-1.367** (0.678)
log(dist)		-0.386** (0.151)		0.212 (0.144)
Constant	-2.658*** (0.123)	3.842* (1.965)	1.992*** (0.158)	5.416*** (2.091)
Observations	53,824	48,964	51,202	46,494
Log Likelihood	-9,143.189	-8,472.302	-3,369.086	-3,178.423
Akaike Inf. Crit.	18,314.380	17,000.600	6,766.172	6,412.846
Bayesian Inf. Crit.	18,438.890	17,246.970	6,889.982	6,657.764

Note:

*p<0.1; **p<0.05; ***p<0.01

Table 15: Mixed effects models with level 2, comparison II

	<i>Dependent variable:</i>			
	Non-systemic vs systemic opposition		Non-systemic vs systemic opposition	
	(1)	(2)	(3)	(4)
sexFemale	0.022 (0.056)	0.015 (0.059)	-0.199*** (0.024)	-0.186*** (0.025)
sexOther	-0.375 (0.296)	-0.232 (0.328)	-0.470*** (0.144)	-0.462*** (0.148)
age_bin18-24	-0.095 (0.086)	-0.117 (0.088)	0.062* (0.037)	0.064* (0.038)
age_bin45-64	-0.302*** (0.087)	-0.299*** (0.090)	-0.826*** (0.039)	-0.823*** (0.040)
age_bin65+	-0.957*** (0.155)	-0.930*** (0.159)	-0.931*** (0.106)	-0.906*** (0.108)
time_to_vs.less_than_hourYes	-0.099 (0.061)	-0.081 (0.065)	-0.034 (0.026)	-0.041 (0.027)
out_of_Russia_time2 - 5 years	0.409*** (0.091)	0.402*** (0.093)	0.454*** (0.042)	0.444*** (0.042)
out_of_Russia_timeAfter annexation	0.296*** (0.102)	0.318*** (0.103)	0.219*** (0.045)	0.214*** (0.045)
out_of_Russia_timeAfter invasion	0.766*** (0.084)	0.817*** (0.090)	0.565*** (0.040)	0.540*** (0.040)
out_of_Russia_timeTourist (lives in Russia)	0.041 (0.144)	0.148 (0.153)	0.652*** (0.087)	0.647*** (0.090)
result_trust_binDon't know	0.718*** (0.127)	0.700*** (0.133)	0.387*** (0.139)	0.331** (0.146)
result_trust_binNo	1.903*** (0.094)	1.910*** (0.098)	-0.806*** (0.105)	-0.839*** (0.111)
orthodox_share		-0.360** (0.142)		0.071 (0.163)
vdem_polyarchy_2022		0.532** (0.250)		-0.561** (0.233)
log(mad_gdppc_2018)		0.036 (0.088)		0.037 (0.072)
obl_type1				0.084 (0.162)
obl_type2				0.160 (0.171)
obl_type3				0.014 (0.162)
obl_type4				-0.132 (0.278)
export_share		0.030 (0.024)		0.018 (0.016)
import_share		-0.020 (0.020)		-0.018 (0.018)
friendly_statusUnfriendly				0.149 (0.284)
friendly_statusFriendly				0.264 (0.183)
help		-0.124 (0.194)		-0.255 (0.196)
military_dummy		0.224 (0.195)		0.254 (0.264)
log(dist)		-0.150*** (0.057)		0.074 (0.050)
Constant	1.267*** (0.123)	1.739* (1.051)	1.558*** (0.117)	0.991 (0.907)
Observations	42,363	37,827	40,946	36,523
Log Likelihood	-5,864.162	-5,342.410	-22,658.160	-20,537.730
Akaike Inf. Crit.	11,756.320	10,728.820	45,344.320	41,131.460
Bayesian Inf. Crit.	11,877.480	10,916.720	45,465.000	41,369.610

Note:

*p<0.1; **p<0.05; ***p<0.01