

# Signal 158

$$x(t) = \begin{cases} 1, & \text{ak } t \in [-2; -1) \\ t, & \text{ak } t \in [-1; 0) \\ t, & \text{ak } t \in [0, 1) \\ 1, & \text{ak } t \in [1, 2] \end{cases}$$

1)  $x(t) = x(t + T_0) = x(t + 4) \Rightarrow x(t) = \sum_{n=-\infty}^{\infty} C_n e^{-jn\omega_0 t}$

folie  $\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$ ;  $C_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) e^{-jn\omega_0 t} dt =$   
 $= \frac{1}{4} \int_{-2}^2 x(t) e^{-jn\omega_0 t} dt = \frac{1}{4} \left[ \int_{-2}^{-\frac{1}{2}} e^{-jn\omega_0 t} dt + I_1 + I_2 + I_3 + I_4 \right] = (x)$

$$I_1 = \frac{j}{n\omega_0} e^{-jn\omega_0 t} \Big|_{-2}^{-1} = \frac{j}{n\omega_0} (e^{jn\omega_0} - e^{-j2n\omega_0}) =$$

$$= \frac{je^{jn\omega_0} (1 - e^{-jn\omega_0})}{n\omega_0}; I_2 = \int_{-1}^0 t e^{-jn\omega_0 t} dt = \begin{cases} u = t & dv = e^{-jn\omega_0 t} dt \\ du = dt & v = \frac{j}{n\omega_0} e^{-jn\omega_0 t} \end{cases} =$$

$$= \frac{jt \cdot e^{-jn\omega_0 t}}{n\omega_0} \Big|_{-1}^0 - \frac{j}{n\omega_0} \int_{-1}^0 e^{-jn\omega_0 t} dt = \frac{j \cdot e^{jn\omega_0}}{n\omega_0} +$$

$$+ \frac{1 - e^{-jn\omega_0}}{(n\omega_0)^2} = \frac{e^{jn\omega_0} (jn\omega_0 - 1) + 1}{(n\omega_0)^2};$$

$$I_3 = \frac{je^{jn\omega_0 t}}{n\omega_0} \Big|_0^1 + \frac{e^{-jn\omega_0 t}}{(n\omega_0)^2} \Big|_0^1 = \frac{j \cdot e^{-jn\omega_0}}{n\omega_0} +$$

$$+ \frac{e^{-jn\omega_0} - 1}{(n\omega_0)^2} = \frac{e^{-jn\omega_0} (jn\omega_0 + 1) - 1}{(n\omega_0)^2};$$

$$I_4 = \frac{j}{n\omega_0} \cdot e^{-jn\omega_0 t} \Big|_0^2 = \frac{j}{n\omega_0} (e^{-j2n\omega_0} - e^{-jn\omega_0}) =$$

$$= \frac{je^{-jn\omega_0} (e^{-jn\omega_0} - 1)}{n\omega_0}; \quad (*) = \frac{1}{4} \sum_{i=1}^4 I_i = \frac{1}{4} \left[ \frac{je^{jn\omega_0} (1 - e^{jn\omega_0})}{n\omega_0} + \right.$$

$$+ \frac{e^{jn\omega_0} (jn\omega_0 - 1) + 1}{(n\omega_0)^2} + \frac{e^{-jn\omega_0} (jn\omega_0 + 1) - 1}{(n\omega_0)^2} + \frac{je^{-jn\omega_0} (e^{-jn\omega_0} - 1)}{n\omega_0}$$

$$= \frac{e^{-j2n\omega_0} (-jn\omega_0 \cdot e^{jn\omega_0} + jn\omega_0 + e^{jn\omega_0} + e^{3jn\omega_0} (2jn\omega_0 - 1))}{4n^2 \cdot \omega_0^2};$$

• Po dosadzeniu  $\omega_0 = \frac{\omega}{2}$  otrzymujemy:

$$c_n = \frac{(-1)^n (j^n + (-j)^n (jn\omega - 1))}{n^2 \omega^2};$$

$$c_1 = \frac{-(j - j(j\omega - 1))}{\omega^2} = -\frac{1}{\omega} - j \cdot \frac{2}{\omega^2} \approx -0,318 - j 0,202$$

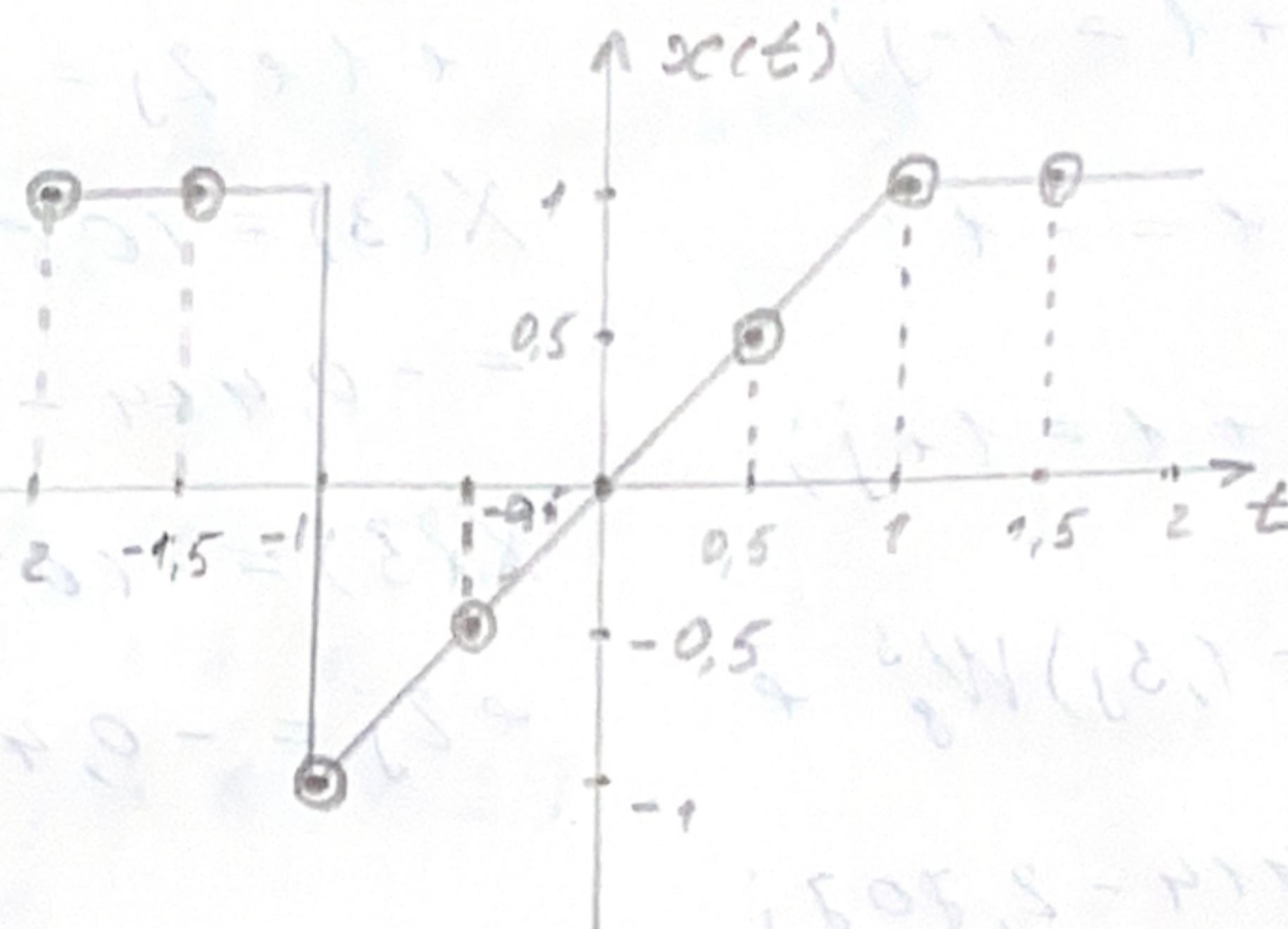
$$3) P_s = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} |x(t)|^2 dt = \frac{1}{4} \int_{-2}^2 |x(t)|^2 dt = \frac{1}{4} \left[ \int_{-1}^1 1 dt + \right.$$

$$\left. + \int_{-2}^{-1} t^2 dt + \int_{-1}^1 1 dt \right] = \frac{1}{4} \left[ 1 + \frac{2}{3} + 1 \right] = \frac{2}{3} \approx 0,666 W;$$

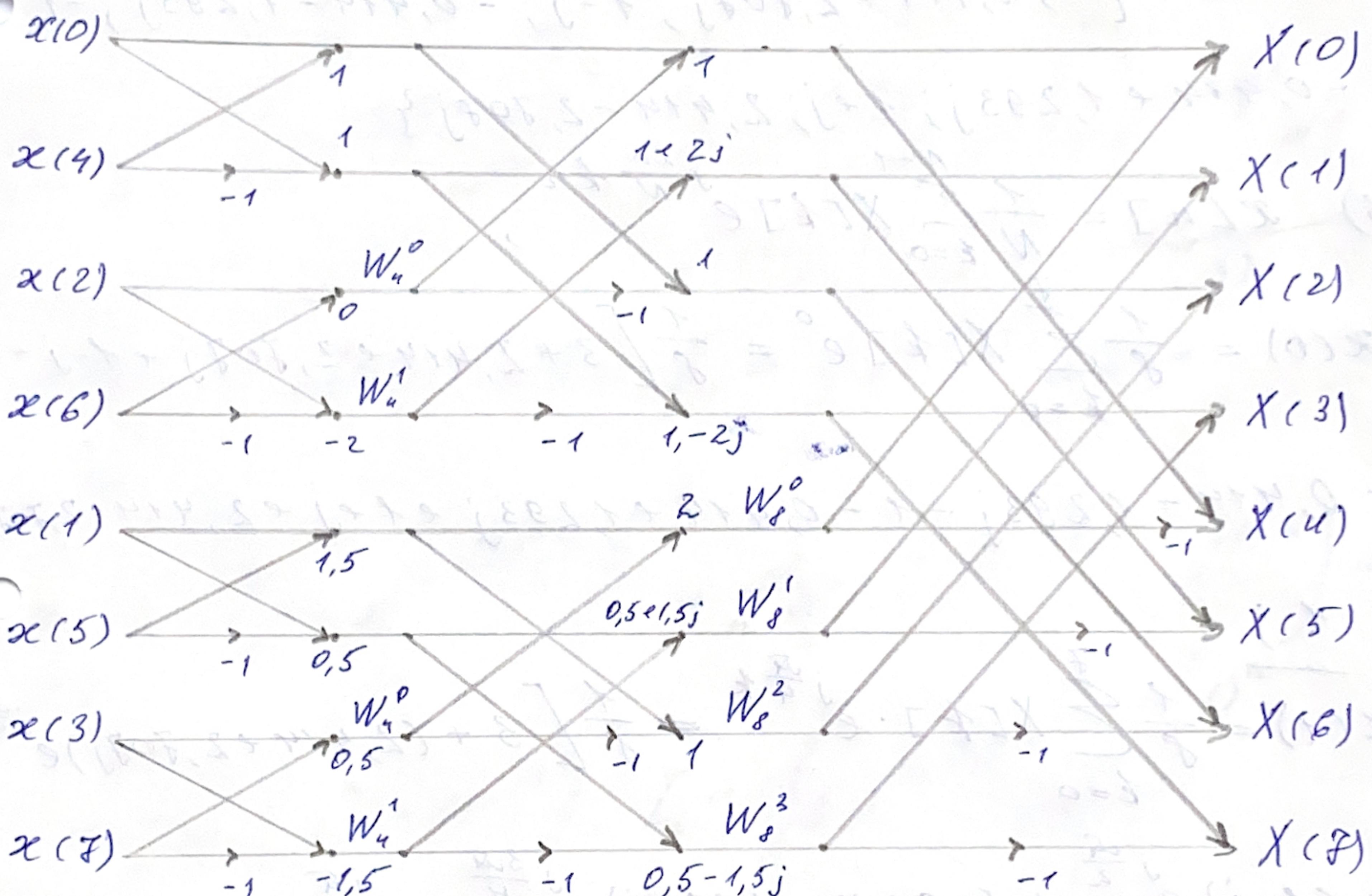
$$P_c = 2|c_1|^2 = 2 \left| -\frac{1}{\omega} - j \cdot \frac{2}{\omega^2} \right|^2 = 2 \left( \frac{4}{\omega^4} + \frac{1}{\omega^2} \right) \approx 0,285 W;$$

$$\frac{P_c}{P_s} \cdot 100\% = \frac{0,285}{0,666} \cdot 100\% \approx 42,792\%$$

$$1) P_0 = 4; N = 8; F_{VZ} = \frac{N}{P_0} = 2 \Rightarrow P_{VZ} = 0,5$$



$$2) x[n] = \{1; 1; -1; -0,5; 0; 0,5; 1; 1\}$$



$$W_4^0 = 1; \quad W_8^1 = 0,807 - 0,807j;$$

$$W_4^1 = -j; \quad W_8^2 = -j;$$

$$W_8^0 = 1; \quad W_8^3 = -0,807 - 0,807j$$

$$X(0) = 2 \cdot W_8^0 + 1 = 3 ;$$

$$X(1) = (0,5 + 1,5j) W_8^1 +$$

$$X(2) = W_8^2 + 1 = 1-j ;$$

$$+ 1 + 2j = 2,414 + 2,707j$$

$$X(4) = -2 + 1 = -1 ;$$

$$X(3) = (0,5 - 1,5j) W_8^3 + 1 - 2j =$$

$$= -0,414 - 1,293j$$

$$X(6) = -W_8^2 + 1 = 1+j ;$$

$$X(5) = -(0,5 - 1,5j) W_8^4 + 1 +$$

$$+ 2j = -0,414 + 1,293j$$

$$e^{-1-2j} = 2,414 - 2,707j$$

$$X[k] = \{3; 2,414 + 2,707j; 1-j; -0,414 - 1,293j; -1;$$

$$-0,414 + 1,293j; 1+j; 2,414 - 2,707j\}$$

$$3) x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j \frac{2\pi}{N} kn} ;$$

$$x(0) = \frac{1}{8} \sum_{k=0}^7 X[k] e^0 = \frac{1}{8} [3 + 2,414 + 2,707j + 1-j -$$

$$-0,414 - 1,293j - 1 - 0,414 + 1,293j + 1+j + 2,414 - 2,707j]$$

$$= 1 ;$$

$$x(1) = \frac{1}{8} \sum_{k=0}^7 X[k] \cdot e^{j \frac{\pi}{4} k} = \frac{1}{8} [3 + (2,414 + 2,707j) e^{j \frac{\pi}{4}} +$$

$$+ (1-j) e^{j \frac{\pi}{2}} + (-0,414 - 1,293j) e^{j \frac{3\pi}{4}} - e^{j\pi} + (-0,414 +$$

$$+ 1,293j) e^{j \frac{5\pi}{4}} + ((1+j) e^{j \frac{\pi}{2}} + (2,414 - 2,707j) e^{j \frac{7\pi}{4}}] =$$

$$= 1$$

$$x(2) = \frac{1}{8} \sum_{k=0}^7 X[k] e^{j \frac{\omega}{2} k} = \frac{1}{8} \left[ 3 + (2,414 + 2,707j) e^{j \frac{\omega}{2}} + \right.$$

$$+ (1-j) e^{j \frac{3\omega}{2}} + (-0,414 - 1,293j) e^{j \frac{5\omega}{2}} - e^{j \frac{7\omega}{2}} + (0,414 +$$

$$+ 1,293j) e^{j \frac{9\omega}{2}} + ((-1)e^{j \frac{3\omega}{2}} + (2,414 - 2,707j) e^{j \frac{11\omega}{2}} \right] =$$

$$= \underline{-1}$$

$$x(3) = \frac{1}{8} \sum_{k=0}^7 X[k] e^{j \frac{3\omega}{4} k} = \frac{1}{8} \left[ 3 + (2,414 + 2,707j) e^{j \frac{3\omega}{4}} + \right.$$

$$+ (1-j) e^{j \frac{5\omega}{4}} + (-0,414 - 1,293j) e^{j \frac{9\omega}{4}} + ((-1)e^{j \frac{3\omega}{4}} +$$

$$+ (-1)e^{j \frac{15\omega}{4}} + (-0,414 + 1,293j) e^{j \frac{15\omega}{4}} + (2,414 - 2,707j) \cdot$$

$$\cdot e^{j \frac{21\omega}{4}} \right] = \underline{-0,5}$$

$$x(4) = \frac{1}{8} \sum_{k=0}^7 X[k] \cdot e^{j \omega k} = \frac{1}{8} \left[ 3 + (2,414 + 2,707j) e^{j \omega} + \right.$$

$$+ (1-j) e^{j 2\omega} + (-0,414 - 1,293j) e^{j 3\omega} - e^{j 4\omega} + (0,414 +$$

$$+ 1,293j) e^{j 5\omega} + ((-1)e^{j 3\omega} + (2,414 - 2,707j) e^{j 7\omega}) = \underline{0}$$

$$x(5) = \frac{1}{8} \sum_{k=0}^7 X[k] e^{j \frac{5\omega}{4} k} = \frac{1}{8} \left[ 3 + (2,414 + 2,707j) e^{j \frac{5\omega}{4}} + \right.$$

$$+ (1-j) e^{j \frac{13\omega}{8}} + (-0,414 - 1,293j) e^{j \frac{15\omega}{8}} + ((-1)e^{j 5\omega} + (0,414 +$$

$$+ 1,293j) e^{j \frac{25\omega}{8}} + ((-1)e^{j \frac{15\omega}{8}} + (2,414 - 2,707j) e^{j \frac{35\omega}{8}} \right] =$$

$$= \underline{0,5};$$

$$x(6) = \frac{1}{8} \sum_{k=0}^7 X[k] \cdot e^{j \frac{3\omega}{2} k} = \frac{1}{8} \left[ 3 + (2,414 - 2,707j) \cdot e^{j \frac{3\omega}{2}} + (1-j) e^{j 3\omega} + (-0,414 - 1,293j) e^{j \frac{9\omega}{2}} - e^{j 6\omega} + \right. \\ \left. e^{j \frac{21\omega}{2}} \right] = 1$$

$$x(7) = \frac{1}{8} \sum_{k=0}^7 X[k] e^{j \frac{7\omega}{4} k} = \frac{1}{8} \left[ 3 + (2,414 - 2,707j) \cdot e^{j \frac{7\omega}{4}} + (1-j) e^{j \frac{7\omega}{2}} + (-0,414 - 1,293j) e^{j \frac{21\omega}{4}} - e^{j 7\omega} + \right. \\ \left. + (-0,414 - 1,293j) e^{j \frac{35\omega}{4}} + (1-ej) e^{j \frac{21\omega}{2}} + (2,414 - \right. \\ \left. - 2,707j) e^{j \frac{49\omega}{4}} \right] = 1$$

$$x[n] = \{ 1; 1; -1; -0,5; 0; 0,5; 1; 1 \}$$

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