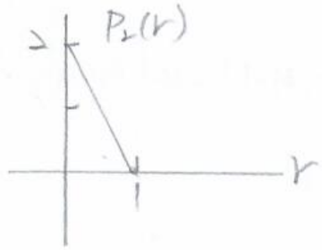
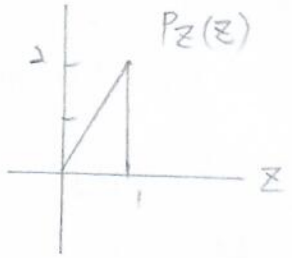
Problem 3.12.





$$\begin{cases} P_r(r) = -2r+2 \\ P_z(z) = 2z \end{cases} \Rightarrow P_r(r),\ P_z(z)\text{的線性方程}$$

$$\left.\begin{aligned} S = T(r) &= \int_0^r P_r(w)\cdot dw = \int_0^r (-2w+2)\cdot dw = -r^2+2r \\ V = G(z) &= \int_0^z P_z(w)\,dw = \int_0^z 2w\cdot dw = z^2 \end{aligned}\right\} \text{轉換成 CDF}$$

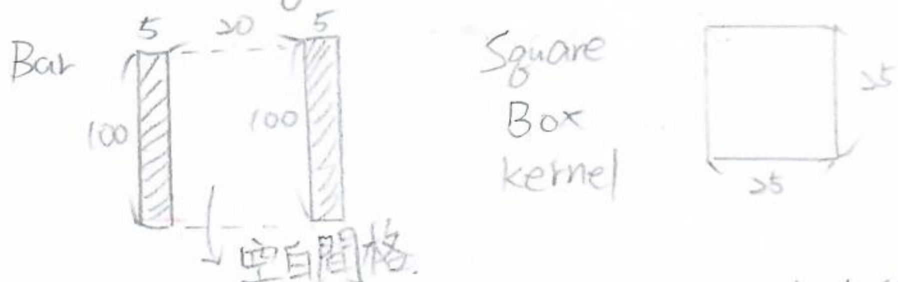
令 $S=V$，得

$$z = G^{-1}(V) = \pm\sqrt{V} = \pm\sqrt{-r^2+2r}, \quad r \in [0,1]$$

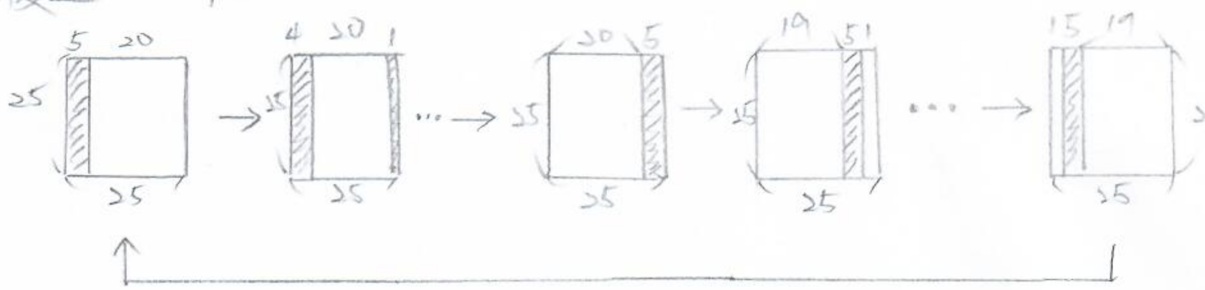
$$\because z \in [0,1],\ \therefore z = \sqrt{-r^2+2r},\ r \in [0,1],\ z \in [0,1]$$

Problem 3.34.

已知 Bar 跟 Square Box Kernel with 25 pixel 如下



因此，Square Box kernel with 25 pixel 在移動過程会一直覆蓋到 5 pixel 的 Bar 與 20 pixel 的空白間格，以致最終呈現一樣的灰色



Problem 3.42

已知 公式 (3-53). 拉普拉斯

$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

$$g(x,y) = f(x,y) + c[\nabla^2 f(x,y)] \quad ----- ①$$

又知 公式 (3-55). Unsharp Masking

$$g_{mask} = f(x,y) - \bar{f}(x,y)$$

$$g(x,y) = f(x,y) + k g_{mask}(x,y) \quad ----- ②$$

其中 $g_{mask} = f(x,y) - \bar{f}(x,y)$

$$= \frac{1}{5}[f(x,y) + f(x,y) + f(x,y) + f(x,y) + f(x,y)]$$

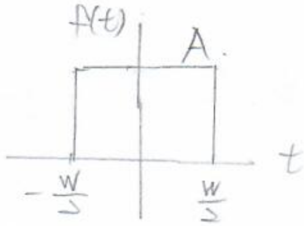
$$- \frac{1}{5}[f(x,y) + f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)]$$

$$= \frac{-1}{5}[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)]$$

$$= \frac{-1}{5}[\nabla f^2(x,y)]$$

因此, 減去 Laplacion 的結果 等比例於 套用 Unsharp Mask 的結果

Problem 4.2



已知 Example 4.1

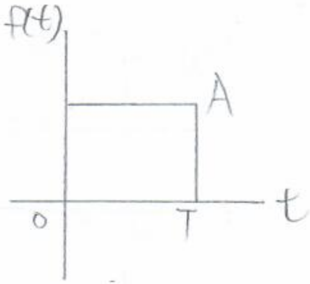
$$F(\mu) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j2\pi\mu t}\cdot dt$$

$$= \int_{-\frac{W}{2}}^{\frac{W}{2}} A\cdot e^{-j2\pi\mu t}\cdot dt$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]\Big|_{-\frac{W}{2}}^{\frac{W}{2}} = \frac{-A}{j2\pi\mu}\left[e^{-j\pi\mu W} - e^{j\pi\mu W}\right] = \frac{A}{j2\pi\mu}\left[e^{j\pi\mu W} - e^{-j\pi\mu W}\right]$$

$$= AW\,\frac{\sin(\pi\mu W)}{\pi\mu W} = \underline{\underline{AW\,\mathrm{sinc}(\mu W)}}$$

如今 $f(t)=A$ for $0\le t\le T$ and $f(t)=0$ for all other values of t



$$F(\mu) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j2\pi\mu t}\cdot dt$$

$$= \int_{0}^{T} A\cdot e^{-j2\pi\mu t}\cdot dt$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]\Big|_{0}^{T} = \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu T} - e^{j2\pi\mu 0}\right]$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu\frac{T}{2}} - e^{-j2\pi\mu(-\frac{T}{2})}\right]\cdot e^{-j2\pi\mu\frac{T}{2}}$$

$$= \frac{A}{j2\pi\mu}\left[e^{j\pi\mu T} - e^{-j\pi\mu T}\right]\cdot e^{-j\pi\mu T}$$

$$= \underline{\underline{AT\,\sin(\mu T)\cdot e^{-j\pi\mu T}}}$$

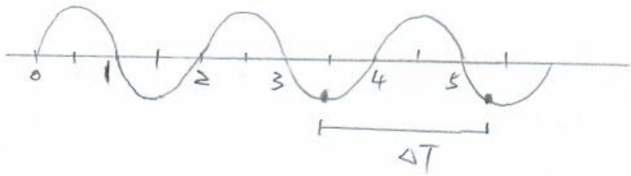
因此，差了一個 $e^{-j\pi\mu T}$，其原因是，兩者雖然都是理想盒式濾波器，
但後者卻平移了一半周期，也就是 $\frac{T}{2}$，
所以，轉到頻域後，也就差了 $e^{-j2\pi\mu\cdot\frac{T}{2}}$ 的相位。

$$e^{-j2\pi\mu\frac{T}{2}} = e^{-j\pi\mu T}$$

最終多了一個 $e^{-j\pi\mu T}$

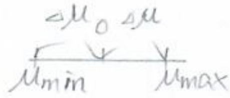
Problem 4.6

Fig 4-11 如右



(a) Nyquist Rate $= 2\mu_{max}$

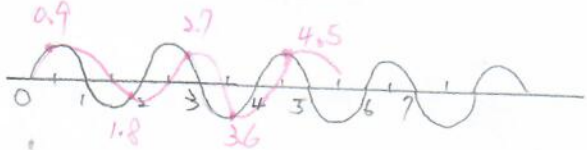
$\Delta\mu = \frac{1}{T}$, $T=2 \Rightarrow \Delta\mu = \frac{1}{2}$, 而



∴ Nyquist Rate $= 2 \cdot \frac{1}{2} = 1$

To let sampling rate slightly exceed

$\frac{1}{\Delta T} > 1 \Rightarrow$ 取 $\Delta T = 0.9$, $\frac{1}{\Delta T} \cong 1.11 \Rightarrow$



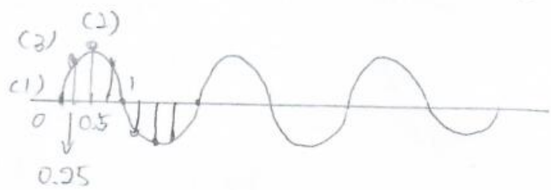
(b) $\Delta T \cong 2.25$

∴ sampling rate $= \frac{1}{\Delta T} = \frac{1}{2.25} = \frac{4}{9}$

(c) 因為 Nyquist Rate $= 1$, Sampling Rate $= \frac{1}{\Delta T} \Rightarrow \frac{1}{\Delta T} > 1$, $\Delta T < 1$

Fig 4.11 是週期2秒的 sin 波

若要重建,則需 $\left\{ \begin{array}{l} \text{過零點 ①} \\ \text{轉折點 ②} \\ \text{中繼點 ③} \end{array} \right.$



取 $\Delta T \cong 0.25$, sampling rate $= \frac{1}{\Delta T} = \frac{1}{0.25} = 4$ /每秒

Problem 4.9

尤拉公式 $\left\{ \begin{array}{l} e^{i\theta} = \cos\theta + i\sin\theta \\ e^{-i\theta} = \cos\theta - i\sin\theta \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \cos\theta = \frac{1}{2}[e^{i\theta} + e^{-i\theta}] \\ \sin\theta = \frac{1}{2i}[e^{i\theta} - e^{-i\theta}] \end{array} \right.$

已知傅立葉轉換為線性,且 $e^{j2\pi\mu_0 t} \xrightarrow[F]{} \delta(\mu - \mu_0)$

令 $\theta = 2\pi\mu_0 t$,則

$$\cos(2\pi\mu_0 t) = \frac{1}{2}\left[e^{j2\pi\mu_0 t} + e^{-j2\pi\mu_0 t}\right]$$

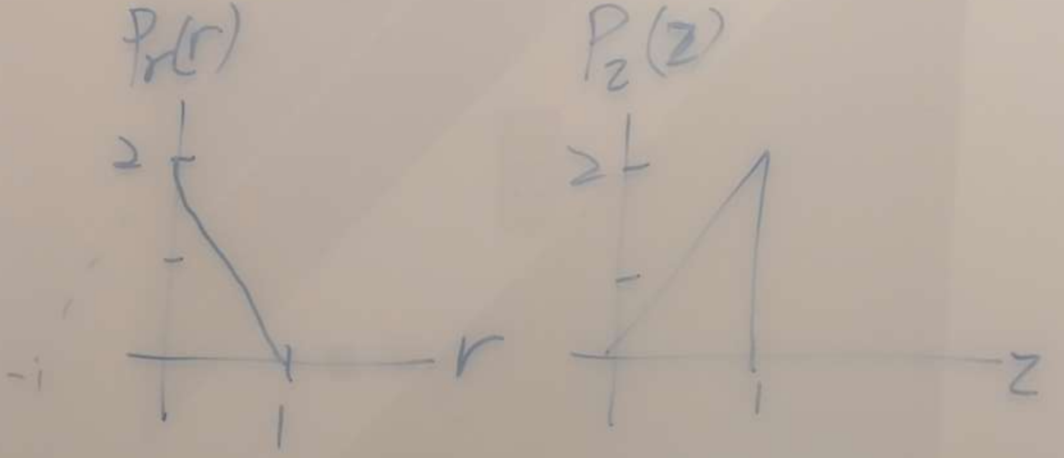
$$\downarrow F$$

$$F[\cos(2\pi\mu_0 t)] = \frac{1}{2}[\delta(\mu - \mu_0) + \delta(\mu + \mu_0)]$$

同理可證

$$F[\sin(2\pi\mu_0 t)] = \frac{1}{2}[\delta(\mu - \mu_0) - \delta(\mu + \mu_0)]$$

3.12



$p_r(r) = -2r + r$

$p_z(z) = 2z$

$$s = T(r) = \int_0^r p_r(w)\cdot dw = \int_0^r (-2w+2)\cdot dw$$

$$= -r^2 + 2r$$

$$v = G(z) = \int_0^z p_z(w)\, dw = \int_0^z 2w \cdot dw$$

$$= z^2$$

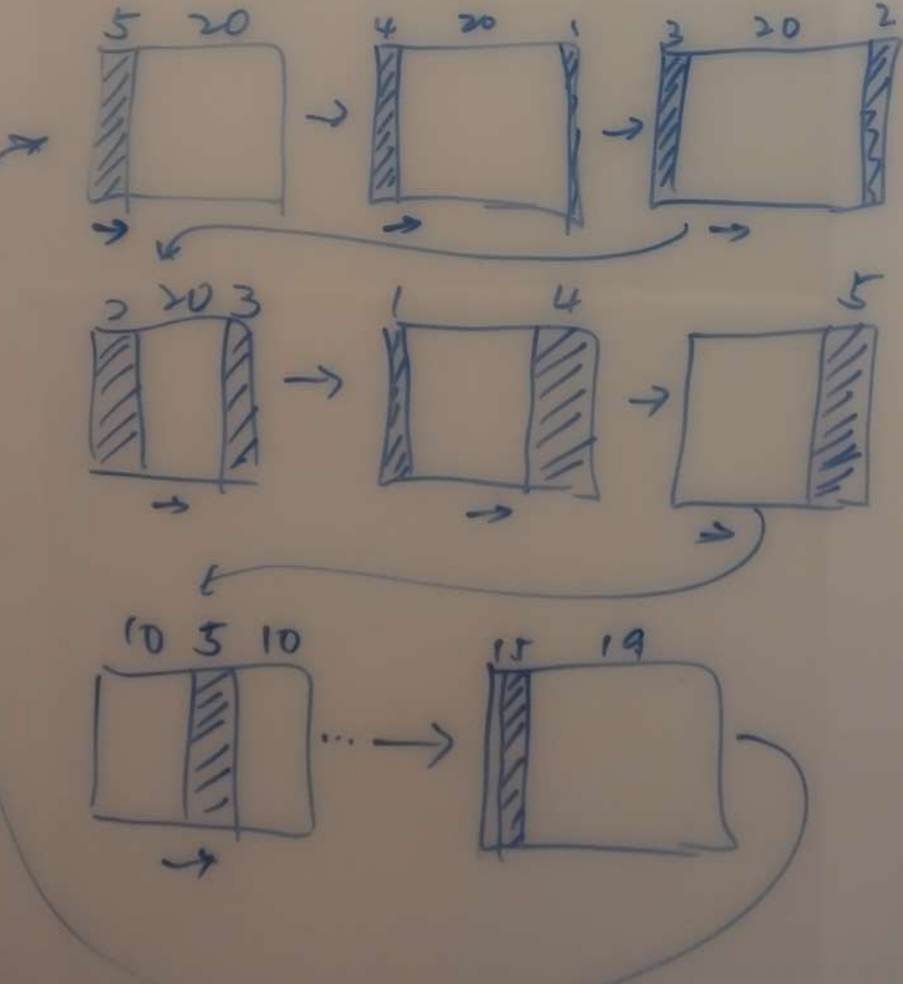
令 $s = v$ 得

$$z = G^{-1}(v) = \pm\sqrt{v} = \pm\sqrt{-r^2+2r}, \quad r = [0, 1]$$

3.3

照下圖所示.
Square Box kernel 25
移動过程永遠是
Bar 5. Blank 20 在做平均
所以最終灰度都一樣

Square Box Kernel
25



3.42

(3-55) Unsharp Masking

$$g_{mask} = f(x.y) - \bar{f}(x.y)$$

$$g(x.y) = f(x.y) + k\,g_{mask}(x.y)$$

(3-53)

$$\nabla f^2(x.y) = f(x+1,y) + f(x-1,y) + f(x.y+1) + f(x.y-1) - 4f(x.y)$$

2

$$g(x.y) = f(x.y) + c[\nabla^2 f(x.y)]$$

$$\bar{f}(x.y) = \frac{1}{5}\Big[f(x.y) + f(x+1.y) + f(x-1,y) + f(x.y+1) + f(x.y-1)\Big]$$

$$g_{mask} = f(x.y) - \bar{f}(x.y)$$

$$= \frac{1}{5}\big[\cancel{f(x.y)} + f(x.y) + f(x.y) + f(x.y) + f(x.y)\big]$$

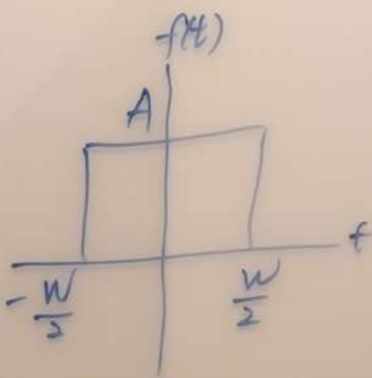
$$- \frac{1}{5}\big[\cancel{f(x.y)} + f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)\big]$$

$$= \frac{-1}{5}\big[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x.y-1) - 4f(x.y)\big]$$

$$= \frac{-1}{5}[\nabla f^2(x.y)]$$

4.2

Example 4.1



$$F(\mu)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j2\pi\mu t}\cdot dt$$

$$=\int_{-\frac{W}{2}}^{\frac{W}{2}} A\cdot e^{-j2\pi\mu t}\cdot dt$$

$$=\frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]\Big|_{-W/2}^{W/2}$$

$$=\frac{-A}{j2\pi\mu}\left[e^{-j\pi\mu W}-e^{j\pi\mu W}\right]$$

$$=\frac{A}{j2\pi\mu}\left[e^{j\pi\mu W}-e^{-j\pi\mu W}\right]$$

$$=AW\frac{\sin(\pi\mu W)}{\pi\mu W}=AW\,\mathrm{sinc}(\mu W)$$

$$\int_0^T A\cdot e^{-j2\pi\mu t}\,dt$$

$$=\frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]\Big|_0^T$$

$$=\frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu T}-e^{-j2\pi\mu 0}\right]$$

$$令\quad \left.\begin{aligned}\theta+\phi&=2\pi\mu T\\ \theta-\phi&=2\pi\mu 0=0\end{aligned}\right\}\quad \begin{aligned}\theta&=\pi\mu T\\ \phi&=\pi\mu T\end{aligned}$$

$$\left[\cos(\theta+\phi)-j\sin(\theta+\phi)\right]$$

$$-\left[\cos(\theta-\phi)-j\sin(\theta-\phi)\right]$$

$$=\left[(\cancel{\cos\theta\cos\phi}-\sin\theta\sin\phi)-j(\cancel{\sin\theta\cos\phi}+\cos\theta\sin\phi)\right]$$

$$-\left[(\cancel{\cos\theta\cos\phi}+\sin\theta\sin\phi)-j(\cancel{\sin\theta\cos\phi}-\cos\theta\sin\phi)\right]$$

$$=-2\sin\theta\sin\phi-2j\cos\theta\sin\phi$$

$$=-2j\sin\phi\left[\cos\theta-j\sin\theta\right]$$

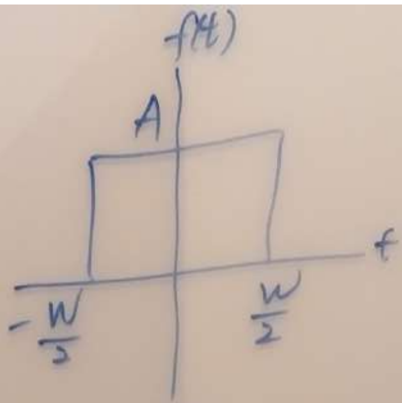
$$\frac{-A}{j2\pi\mu}\cdot\left[-2j\sin(\pi\mu T)\cdot e^{-j\pi\mu T}\right]$$

$$=\frac{AT}{\pi\mu T}\cdot\sin(\pi\mu T)\cdot e^{-j\pi\mu T}=AT\,\mathrm{sinc}(\mu T)\boxed{e^{-j\pi\mu T}}$$

$$e^{-j2\pi\mu\frac{T}{2}}\;"$$

4.2

Example 4.1



$$F(\mu) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j2\pi\mu t} \cdot dt$$

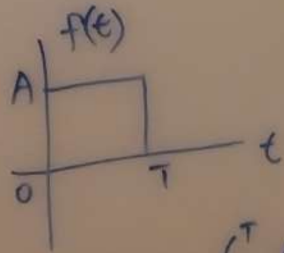
$$= \int_{-\frac{W}{2}}^{\frac{W}{2}} A \cdot e^{-j2\pi\mu t} \cdot dt$$

$$= \frac{-A}{j2\pi\mu} \left[e^{-j2\pi\mu t} \right] \Big|_{-W/2}^{W/2}$$

$$= \frac{-A}{j2\pi\mu} \left[e^{-j\pi\mu W} - e^{j\pi\mu W} \right]$$

$$= \frac{A}{j2\pi\mu} \left[e^{j\pi\mu W} - e^{-j\pi\mu W} \right]$$

$$= AW \frac{\sin(\pi\mu W)}{\pi\mu W} = AW \operatorname{sinc}(\mu W)$$



$$F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} \cdot dt$$

$$\Rightarrow F(\mu) = \int_0^T A \cdot e^{-j2\pi\mu t} dt = \frac{-A}{j2\pi\mu} \left[e^{-j2\pi\mu t} \right] \Big|_0^T$$

$$= \frac{-A}{j2\pi\mu} \left[e^{-j2\pi\mu T} - e^{-j2\pi\mu 0} \right]$$

$$= \frac{-A}{j2\pi\mu} \left[e^{-j2\pi\mu \frac{T}{2}} - e^{-j2\pi\mu \frac{T}{2}} \right] \cdot e^{-j2\pi\mu \frac{T}{2}}$$

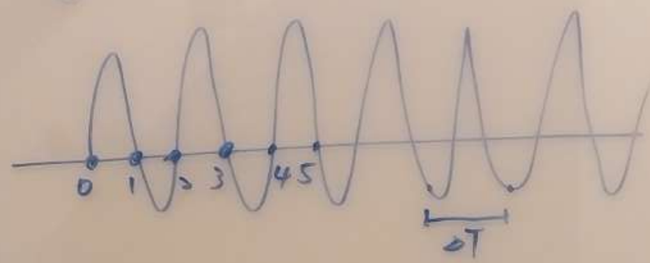
$$= \frac{+A}{j2\pi\mu} \left[e^{-j\pi\mu T} - e^{-j\pi\mu T} \right] \cdot e^{-j\pi\mu T}$$

$$= \frac{AT}{\pi\mu T} \left[\frac{e^{-j\pi\mu T} - e^{-j\pi\mu T}}{2j} \right] e^{-j\pi\mu T}$$

$$= AT \cdot \operatorname{sinc}(\mu T) \cdot e^{-j\pi\mu T}$$

4.6

Fig. 4-11



(b) $\Delta T \fallingdotseq 2.25$

$\therefore$ sampling rate $= \frac{1}{\Delta T} = \frac{1}{2.25} = \frac{4}{9}$

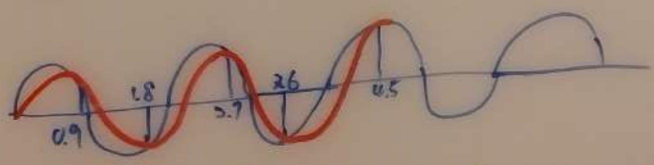
(a) Nxquist rate $= 2\mu_{max}$

$\Delta\mu = \frac{1}{T} = \frac{1}{2}$ $\quad \mu_{min} \xleftrightarrow{\Delta\mu} 0 \xleftrightarrow{\Delta\mu} \mu_{max}$

$\therefore$ Nyquist rate $= 2 \cdot \frac{1}{2} = 1$

Let sampling rate slight exceeds

$\frac{1}{\Delta T} > 1 \Rightarrow \Delta T = 0.9$, $\frac{1}{\Delta T} \fallingdotseq 1.1$



(c) $\because$ Nyquist rate $= 1$, $\Rightarrow \frac{1}{\Delta T} > 1$, $\Delta T < 1$

sampling rate $= \frac{1}{\Delta T}$

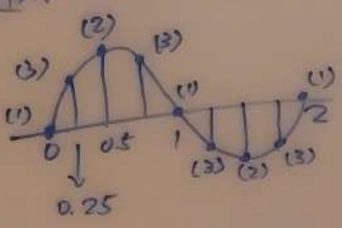
Fig 4.11 是週期2秒的sin波

若要重建則

(1)要有过零点

(2)要有轉折点

(3)要有中間点



$\Delta T \fallingdotseq 0.25$

sampling rate $\frac{1}{\Delta T} = \frac{1}{0.25} = 4/s$

4.9

令 $z = \cos\theta + i\sin\theta$

$$\frac{dz}{d\theta} = -\sin\theta + i\cos\theta$$
$$= i(\cos\theta + i\sin\theta)$$
$$= i \cdot z$$

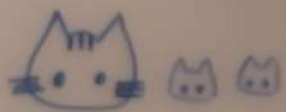
∴ $\frac{1}{z} \cdot dz = i \cdot d\theta \Rightarrow \int \frac{1}{z} \cdot dz = \int i \cdot d\theta$

$\Rightarrow \ln z = i\theta \Rightarrow e^{\ln z} = e^{i\theta}$

$\Rightarrow z = e^{i\theta}$

∴ $e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$



$$\cos\theta = \frac{1}{2}\left[e^{i\theta} + e^{-i\theta}\right]$$
$$\sin\theta = \frac{1}{2i}\left[e^{i\theta} - e^{-i\theta}\right]$$

令 $\theta = 2\pi\mu_0 t)$

$$\cos(2\pi\mu_0 t) = \frac{1}{2}\left[e^{i2\pi\mu_0 t} + e^{-i2\pi\mu_0 t}\right]$$

$$\mathcal{F}(\cos(2\pi\mu_0 t)) = \frac{1}{2}\int_{-\infty}^{\infty}\left[e^{i2\pi\mu_0 t} + e^{-i2\pi\mu_0 t}\right]e^{-i2\pi\mu t} \cdot dt$$
$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{-i2\pi(\mu-\mu_0)t} \cdot dt + \int_{-\infty}^{\infty} e^{-i2\pi(\mu+\mu_0)t} \cdot dt\right]$$
$$= \frac{1}{2}\left[\delta(\mu-\mu_0) + \delta(\mu+\mu_0)\right]$$

$$\mathcal{F}(\sin(2\pi\mu_0 t)) = \frac{1}{2}\left[e^{i2\pi\mu_0 t} - e^{-i2\pi\mu_0 t}\right]$$
$$= \frac{1}{2}\left[\delta(\mu-\mu_0) - \delta(\mu+\mu_0)\right]$$

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

$\cos\theta = \frac{1}{2}\left[e^{i\theta} + e^{-i\theta}\right]$

$\sin\theta = \frac{1}{2i}\left[e^{i\theta} - e^{-i\theta}\right]$

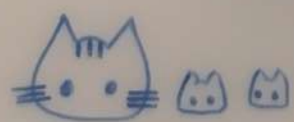
☆ $e^{j2\pi t_0 t} \xrightarrow{F} \delta(u - t_0)$

$\theta = 2\pi t_0 t$

$\cos\theta \xrightarrow{F} \frac{1}{2}\left[F(e^{i\theta}) + F(e^{-i\theta})\right]$

$\frac{1}{2}\left[F(e^{j2\pi t_0 t}) + F(e^{j2\pi - t_0 t})\right]$

$\delta(u - t_0) + \delta(u + t_0)$



$\sin\theta \longrightarrow \frac{1}{2}\left[\delta(u - t_0) - \delta(u + t_0)\right]$

☆ $e^{j2\pi t_0 t} \xrightarrow{F} \delta(u - t_0)$

$$F(e^{j2\pi t_0 t}) = \int_{-\infty}^{\infty} e^{j2\pi t_0 t} \cdot e^{-j2\pi \mu t} \cdot dt$$

$$= \int_{-\infty}^{\infty} e^{j2\pi(t_0 - \mu)t} \cdot dt$$

$$= \delta(t_0 - \mu) = \delta(\mu - t_0)$$