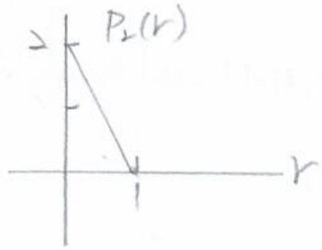
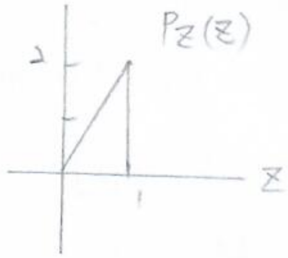
Problem 3.12





$$\begin{cases} p_r(r) = -2r+2 \\ p_z(z) = 2z \end{cases} \Rightarrow p_r(r),\ p_z(z) \text{的線性方程}$$

$$\left.\begin{aligned} s = T(r) &= \int_0^r p_r(w)\cdot dw = \int_0^r (-2w+2)\cdot dw = -r^2+2r \\ v = G(z) &= \int_0^z p_z(w)\,dw = \int_0^z 2w\cdot dw = z^2 \end{aligned}\right\} \text{轉換成 CDF}$$

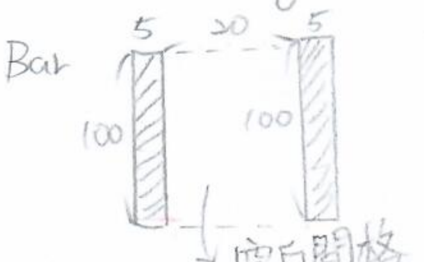
令 $s=v$，得

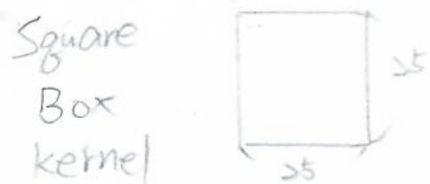
$$z = G^{-1}(v) = \pm\sqrt{v} = \pm\sqrt{-r^2+2r}, \quad r\in[0,1]$$

$$\because z\in[0,1], \ \therefore z = \sqrt{-r^2+2r}, \ r\in[0,1], \ z\in[0,1]$$

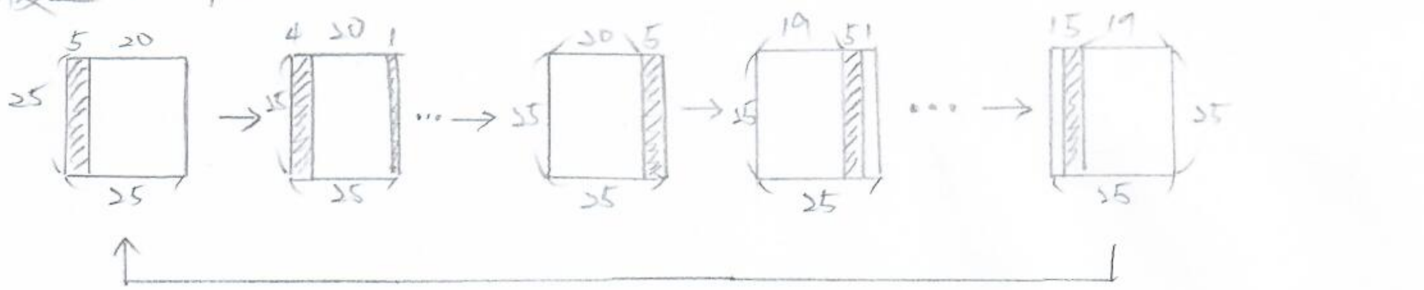
Problem 3.34

已知 Bar 跟 Square Box Kernel with 25 pixel 如下





因此，Square Box kernel with 25 pixel 在移動過程會一直覆蓋到 5 pixel 的 Bar 與 20 pixel 的空白間格，以致最終呈現一樣的灰色



Problem 3.42

已知 公式 (3-53). 拉普拉斯

$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

$$g(x,y) = f(x,y) + c[\nabla^2 f(x,y)] \quad \text{----- ①}$$

又知 公式 (3-55). Unsharp Masking

$$g_{mask} = f(x,y) - \bar{f}(x,y)$$

$$g(x,y) = f(x,y) + k g_{mask}(x,y) \quad \text{----- ②}$$

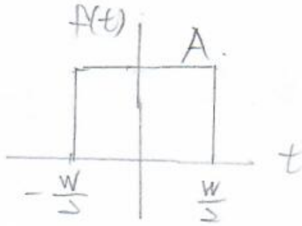
其中 $g_{mask} = f(x,y) - \bar{f}(x,y)$

$$= \frac{1}{5}[f(x,y) + f(x,y) + f(x,y) + f(x,y) + f(x,y)]$$
$$- \frac{1}{5}[f(x,y) + f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)]$$
$$= \frac{-1}{5}[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)]$$
$$= \frac{-1}{5}[\nabla f^2(x,y)]$$

因此，減去 Laplacion 的結果等比例於套用 Unsharp Mask 的結果

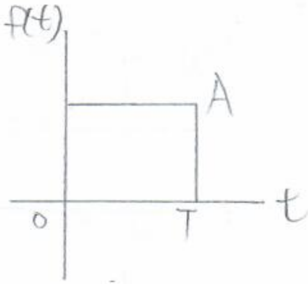
Problem 4.2

已知 Example 4.1



$$F(\mu)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j2\pi\mu t}\cdot dt$$
$$=\int_{-\frac{W}{2}}^{\frac{W}{2}} A\cdot e^{-j2\pi\mu t}\cdot dt$$
$$=\frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]\Big|_{-\frac{W}{2}}^{\frac{W}{2}}=\frac{-A}{j2\pi\mu}\left[e^{-j\pi\mu W}-e^{j\pi\mu W}\right]=\frac{A}{j2\pi\mu}\left[e^{j\pi\mu W}-e^{-j\pi\mu W}\right]$$
$$=AW\frac{\sin(\pi\mu W)}{\pi\mu W}=AW\,\mathrm{sinc}(\mu W)$$

如今 $f(t)=A$ for $0\le t\le T$ and $f(t)=0$ for all other values of t



$$F(\mu)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j2\pi\mu t}\cdot dt$$
$$=\int_{0}^{T} A\cdot e^{-j2\pi\mu t}\cdot dt$$
$$=\frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]\Big|_{0}^{T}=\frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu T}-e^{j2\pi\mu 0}\right]$$
$$=\frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu\frac{T}{2}}-e^{-j2\pi\mu(-\frac{T}{2})}\right]\cdot e^{-j2\pi\mu\frac{T}{2}}$$
$$=\frac{A}{j2\pi\mu}\left[e^{j\pi\mu T}-e^{-j\pi\mu T}\right]\cdot e^{-j\pi\mu T}$$
$$=AT\sin(\mu T)\cdot e^{-j\pi\mu T}$$

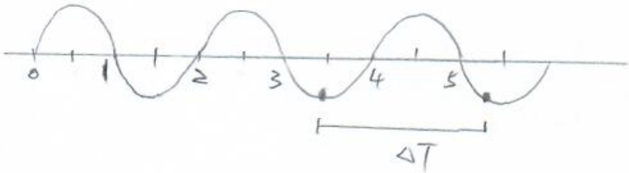
因此，差了一個 $e^{-j\pi\mu T}$，其原因是，兩者雖然都是理想盒式濾波器，
但後者卻平移了一半週期，也就是 $\frac{T}{2}$，
所以，轉到頻域後，也就差了 $e^{-j2\pi\mu\cdot\frac{T}{2}}$ 的相位。

$$e^{-j2\pi\mu\frac{T}{2}}=e^{-j\pi\mu T}$$

最終多了一個 $e^{-j\pi\mu T}$

Problem 4.6

Fig 4-11 如右



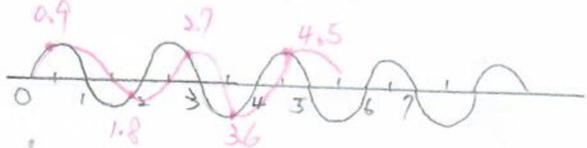
(a) Nyquist Rate $= 2\mu_{max}$

$\Delta u = \frac{1}{T}$, $T=2 \Rightarrow \Delta u = \frac{1}{2}$ ，而 $\underset{\mu_{min}}{\vdash} \overset{\Delta\mu \quad \mu_0 \quad \Delta\mu}{\longleftrightarrow} \underset{\mu_{max}}{\dashv}$

$\therefore$ Nyquist Rate $= 2 \cdot \frac{1}{2} = 1$

To let sampling rate slightly exceed

$\frac{1}{\Delta T} > 1 \Rightarrow$ 取 $\Delta T = 0.9$, $\frac{1}{\Delta T} \cong 1.11 \Rightarrow$



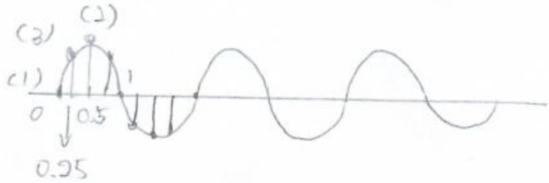
(b) $\Delta T \cong 0.25$

$\therefore$ sampling rate $= \frac{1}{\Delta T} = \frac{1}{0.25} = \frac{4}{9}$

(c) 因為 Nyquist Rate $= 1$, Sampling Rate $= \frac{1}{\Delta T} \Rightarrow \frac{1}{\Delta T} > 1$, $\Delta T < 1$

Fig 4.11 是週期 2 秒的 sin 波

若要重建,則需 $\begin{cases} \text{過零點 ①} \\ \text{轉折點 ②} \\ \text{中繼點 ③} \end{cases}$



取 $\Delta T \cong 0.25$, sampling rate $= \frac{1}{\Delta T} = \frac{1}{0.25} = 4$ /每秒

Problem 4.9

尤拉公式 $\begin{cases} e^{i\theta} = \cos\theta + i\sin\theta \\ e^{-i\theta} = \cos\theta - i\sin\theta \end{cases} \Rightarrow \begin{cases} \cos\theta = \frac{1}{2}[e^{i\theta} + e^{-i\theta}] \\ \sin\theta = \frac{1}{2i}[e^{i\theta} - e^{-i\theta}] \end{cases}$

已知傅立葉轉換為線性,且 $e^{j2\pi\mu_0 t} \xrightarrow[F]{} \delta(\mu - \mu_0)$

令 $\theta = 2\pi\mu_0 t$,則

$$\cos(2\pi\mu_0 t) = \frac{1}{2}[e^{j2\pi\mu_0 t} + e^{-j2\pi\mu_0 t}]$$

$$\downarrow F \qquad\qquad F \downarrow$$

$$\underline{F[\cos(2\pi\mu_0 t)] = \frac{1}{2}[\delta(\mu - \mu_0) + \delta(\mu + \mu_0)]}$$

同理可證

$$\underline{F[\sin(2\pi\mu_0 t)] = \frac{1}{2}[\delta(\mu - \mu_0) - \delta(\mu + \mu_0)]}$$