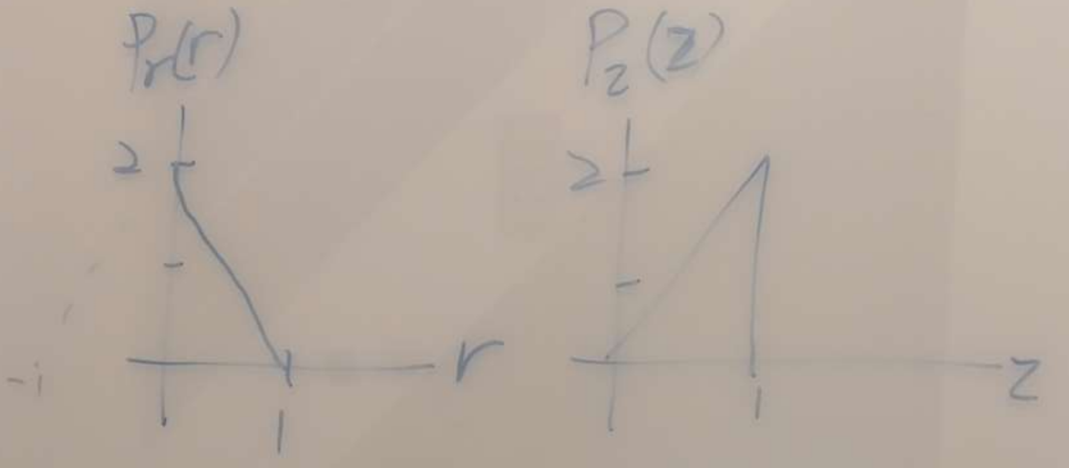
3.12



$P_r(r) = -2r + r$

$P_z(z) = 2z$

$$s = T(r) = \int_0^r P_r(w)\cdot dw = \int_0^r (-2w+2)\cdot dw$$

$$= -r^2 + 2r$$

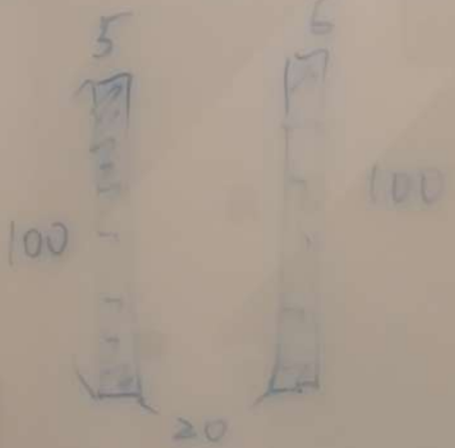
$$v = G(z) = \int_0^z P_z(w)\, dw = \int_0^z 2w \cdot dw$$

$$= z^2$$

令 $s = v$ 得

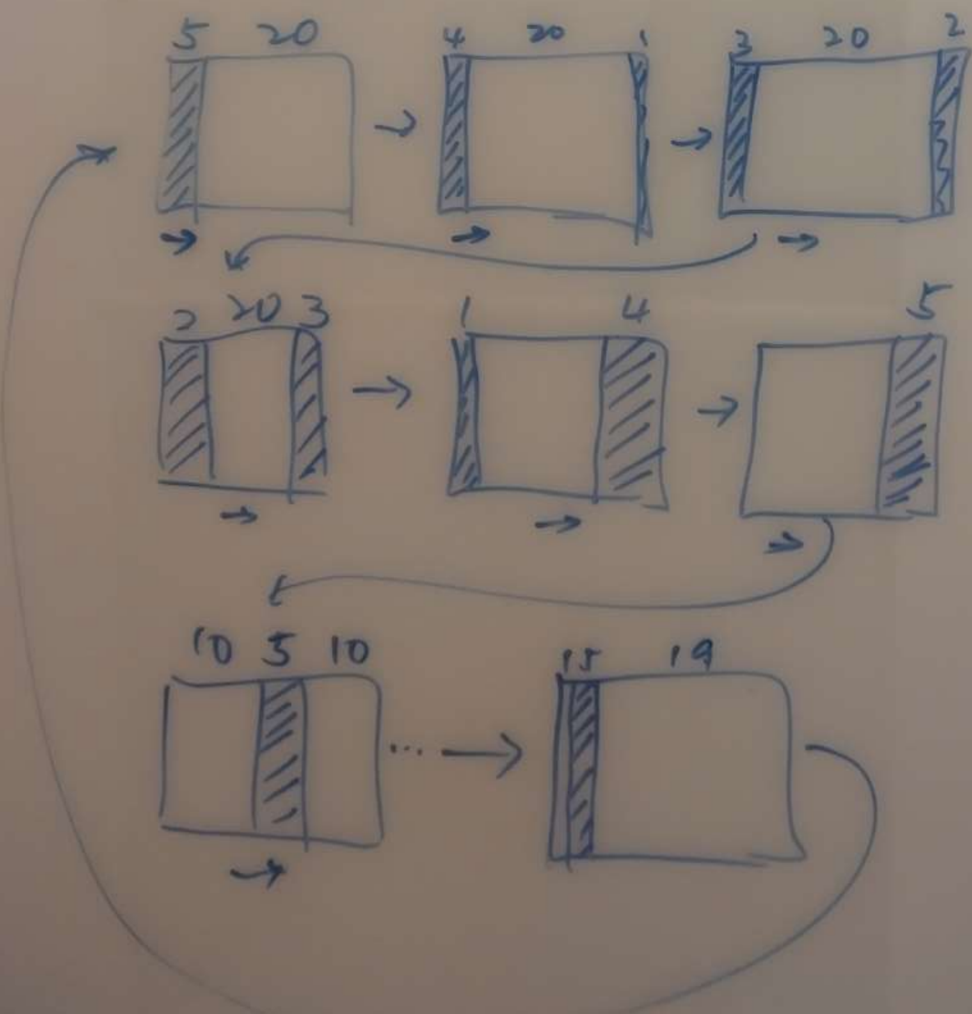
$z = G^{-1}(v) = \pm\sqrt{v} = \pm\sqrt{-r^2+2r}$, $r = [0, 1]$

3.3 (4)



照下圖所示
Square Box kernel 25
移動过程永遠是
Bar 5. Blank 20 在做平均
所以最終灰度都一樣

Square Box Kernel
25



3.42

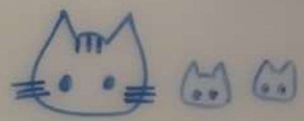
(3-55) Unsharp Masking

$$g_{mask} = f(x.y) - \bar{f}(x.y)$$

$$g(x.y) = f(x.y) + k\,g_{mask}(x.y)$$

(3-53)

$$\nabla f^2(x.y) = f(x+1,y) + f(x-1,y) + f(x.y+1) + f(x.y-1) - 4f(x.y)$$



② $$g(x.y) = f(x.y) + c[\nabla^2 f(x.y)]$$

$$\bar{f}(x.y) = \frac{1}{5}\left[f(x,y) + f(x+1.y) + f(x-1,y) + f(x.y+1) + f(x.y-1)\right]$$

$$g_{mask} = f(x.y) - \bar{f}(x.y)$$

$$= \frac{1}{5}\left[\cancel{f(x.y)} + f(x.y) + f(x.y) + f(x.y) + f(x.y)\right]$$

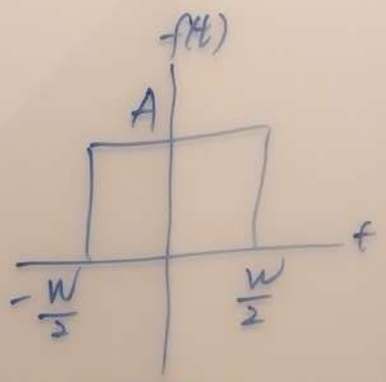
$$- \frac{1}{5}\left[\cancel{f(x.y)} + f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)\right]$$

$$= \frac{-1}{5}\left[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x.y-1) - 4f(x.y)\right]$$

$$= \frac{-1}{5}\left[\nabla f^2(x.y)\right]$$

4.2

Example 4.1



$$F(\mu)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j2\pi\mu t}\,dt$$

$$=\int_{-\frac{W}{2}}^{\frac{W}{2}} A\cdot e^{-j2\pi\mu t}\,dt$$

$$=\frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]\Big|_{-W/2}^{W/2}$$

$$=\frac{-A}{j2\pi\mu}\left[e^{-j\pi\mu W}-e^{j\pi\mu W}\right]$$

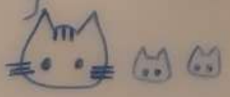
$$=\frac{A}{j2\pi\mu}\left[e^{j\pi\mu W}-e^{-j\pi\mu W}\right]$$

$$=AW\frac{\sin(\pi\mu W)}{\pi\mu W}=AW\,\mathrm{sinc}(\mu W)$$

$$\int_0^T A\cdot e^{-j2\pi\mu t}\,dt$$

$$=\frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]_0^T$$

$$=\frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu T}-e^{-j2\pi\mu 0}\right]$$



令 $\left.\begin{aligned}\theta+\phi&=2\pi\mu T\\ \theta-\phi&=2\pi\mu 0=0\end{aligned}\right\}\begin{aligned}\theta&=\pi\mu T\\ \phi&=\pi\mu T\end{aligned}$

$$[\cos(\theta+\phi)-j\sin(\theta+\phi)]$$

$$-[\cos(\theta-\phi)-j\sin(\theta-\phi)]$$

$$=[(\cancel{\cos\theta\cos\phi}-\sin\theta\sin\phi)-j(\cancel{\sin\theta\cos\phi}+\cos\theta\sin\phi)]$$

$$-[(\cancel{\cos\theta\cos\phi}+\sin\theta\sin\phi)-j(\cancel{\sin\theta\cos\phi}-\cos\theta\sin\phi)]$$

$$=-2\sin\theta\sin\phi-2j\cos\theta\sin\phi$$

$$=-2j\sin\phi[\cos\theta-j\sin\theta]$$

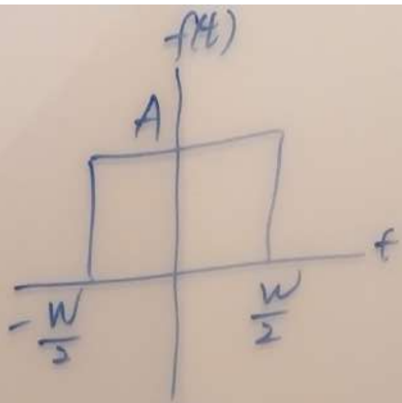
$$\frac{-A}{j2\pi\mu}\cdot\left[-2j\sin(\pi\mu T)\cdot e^{-j\pi\mu T}\right]$$

$$=\frac{AT}{\pi\mu T}\cdot\sin(\pi\mu T)\cdot e^{-j\pi\mu T}=AT\,\mathrm{sinc}(\mu T)\boxed{e^{-j\pi\mu T}}$$

$$e^{-j2\pi\mu\frac{T}{2}} = e^{-j\pi\mu T}$$

4.2

Example 4.1



$$F(\mu) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j2\pi\mu t}\cdot dt$$

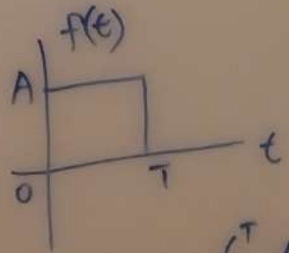
$$= \int_{-\frac{W}{2}}^{\frac{W}{2}} A\cdot e^{-j2\pi\mu t}\cdot dt$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]\Bigg|_{-W/2}^{W/2}$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j\pi\mu W} - e^{j\pi\mu W}\right]$$

$$= \frac{A}{j2\pi\mu}\left[e^{j\pi\mu W} - e^{-j\pi\mu W}\right]$$

$$= AW\frac{\sin(\pi\mu W)}{\pi\mu W} = AW \operatorname{sinc}(\mu W)$$



$$F(\mu) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j2\pi\mu t}\cdot dt$$

$$\Rightarrow F(\mu) = \int_{0}^{T} A\cdot e^{-j2\pi\mu t}\, dt = \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]\Bigg|_{0}^{T}$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu T} - e^{-j2\pi\mu 0}\right]$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu\frac{T}{2}} - e^{-j2\pi\mu\frac{-T}{2}}\right]\cdot e^{-j2\pi\mu\frac{T}{2}}$$

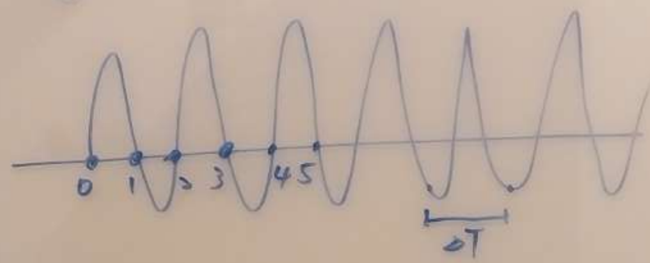
$$= \frac{+A}{j2\pi\mu}\left[e^{-j\pi\mu T} - e^{-j\pi\mu T}\right]\cdot e^{-j\pi\mu T}$$

$$= \frac{AT}{\pi\mu T}\left[\frac{e^{-j\pi\mu T} - e^{-j\pi\mu T}}{2j}\right] e^{-j\pi\mu T}$$

$$= AT\cdot \operatorname{sinc}(\mu T)\cdot e^{-j\pi\mu T}$$

4.6

Fig. 4-11

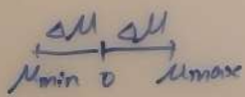


(b) $\Delta T \fallingdotseq 2.25$

$\therefore$ sampling rate $= \frac{1}{\Delta T} = \frac{1}{2.25} = \frac{4}{9}$

(a) Nyquist rate $= 2\mu_{max}$

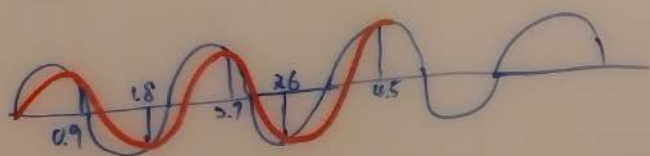
$\Delta\mu = \frac{1}{T} = \frac{1}{2}$



$\therefore$ Nyquist rate $= 2 \cdot \frac{1}{2} = 1$

Let sampling rate slight exceeds

$\frac{1}{\Delta T} > 1 \Rightarrow \Delta T = 0.9$, $\frac{1}{\Delta T} \fallingdotseq 1.1$



(c) $\because$ Nyquist rate $= 1$, $\Rightarrow \frac{1}{\Delta T} > 1$, $\Delta T < 1$

sampling rate $= \frac{1}{\Delta T}$

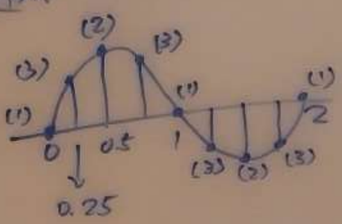
Fig 4.11 是週期2秒的 sin波

若要重建則

(1) 要有过零点

(2) 要有轉折点

(3) 要有中間点



$\Delta T \fallingdotseq 0.25$

sampling rate $\frac{1}{\Delta T} = \frac{1}{0.25} = 4/s$

4.9

令 $z = \cos\theta + i\sin\theta$

$$\frac{dz}{d\theta} = -\sin\theta + i\cos\theta$$
$$= i(\cos\theta + i\sin\theta)$$
$$= i\cdot z$$

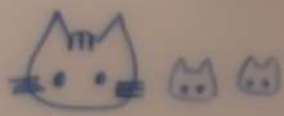
$\therefore$ $\frac{1}{z}\cdot dz = i\cdot d\theta \Rightarrow \int \frac{1}{z}\cdot dz = \int i\cdot d\theta$

$\Rightarrow \ln z = i\theta \Rightarrow e^{\ln z} = e^{i\theta}$

$\Rightarrow z = e^{i\theta}$

$\therefore$ $e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$



$$\cos\theta = \frac{1}{2}\left[e^{i\theta} + e^{-i\theta}\right]$$
$$\sin\theta = \frac{1}{2i}\left[e^{i\theta} - e^{-i\theta}\right]$$

令 $\theta = 2\pi\mu_0 t)$

$$\cos(2\pi\mu_0 t) = \frac{1}{2}\left[e^{i2\pi\mu_0 t} + e^{-i2\pi\mu_0 t}\right]$$

$$\mathcal{F}(\cos(2\pi\mu_0 t)) = \frac{1}{2}\int_{-\infty}^{\infty}\left[e^{i2\pi\mu_0 t} + e^{-i2\pi\mu_0 t}\right]e^{-i2\pi\mu t}\cdot dt$$
$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{-i2\pi(\mu-\mu_0)t}\cdot dt + \int_{-\infty}^{\infty} e^{-i2\pi(\mu+\mu_0)t}\cdot dt\right]$$
$$= \frac{1}{2}\left[\delta(\mu-\mu_0) + \delta(\mu+\mu_0)\right]$$

$$\mathcal{F}(\sin(2\pi\mu_0 t)) = \frac{1}{2}\left[e^{i2\pi\mu_0 t} - e^{-i2\pi\mu_0 t}\right]$$
$$= \frac{1}{2}\left[\delta(\mu-\mu_0) - \delta(\mu+\mu_0)\right]$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$\cos\theta = \frac{1}{2}\left[e^{i\theta} + e^{-i\theta}\right]$$

$$\sin\theta = \frac{1}{2i}\left[e^{i\theta} - e^{-i\theta}\right]$$

☆

$$e^{j2\pi t_0 t} \xrightarrow{F} \delta(u - t_0)$$

$$\theta = 2\pi t_0 t$$

$$\cos\theta \xrightarrow{F} \frac{1}{2}\left[F(e^{i\theta}) + F(e^{-i\theta})\right]$$

$$\frac{1}{2}\left[F(e^{j2\pi t_0 t}) + F(e^{j2\pi - t_0 t})\right]$$

$$\delta(u - t_0) + \delta(u + t_0)$$

$$\sin\theta \longrightarrow \frac{1}{2}\left[\delta(u - t_0) - \delta(u + t_0)\right]$$

$$☆\ e^{j2\pi t_0 t} \xrightarrow{F} \delta(u - t_0)$$

$$F(e^{j2\pi t_0 t}) = \int_{-\infty}^{\infty} e^{j2\pi t_0 t} \cdot e^{-j2\pi \mu t} \cdot dt$$

$$= \int_{-\infty}^{\infty} e^{j2\pi(t_0 - \mu)t} \cdot dt$$

$$= \delta(t_0 - \mu) = \delta(\mu - t_0)$$