Problem 3.12.
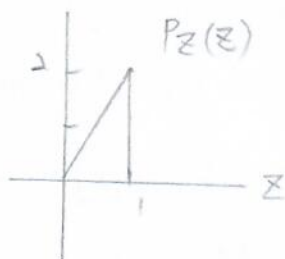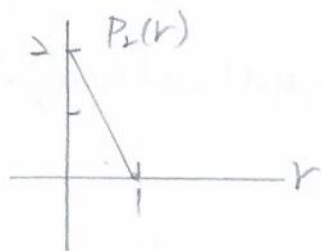

$$\begin{cases} P_r(r) = -2r + 2 \\ P_z(z) = 2z \end{cases} \Rightarrow P_r(r),\ P_z(z) \text{ 的線性方程}$$

$$S = T(r) = \int_0^r P_r(w)\cdot dw = \int_0^r (-2w+2)\cdot dw = -r^2 + 2r \ \Big\}\ \text{轉換成 CDF}$$

$$V = G(z) = \int_0^z P_z(w)\,dw = \int_0^z 2w\cdot dw = z^2$$
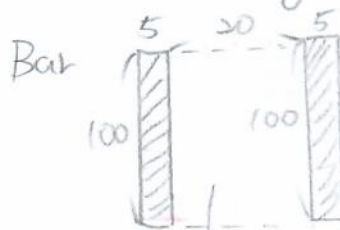
令 $S = V$ 得

$$Z = G^{-1}(V) = \pm\sqrt{V} = \pm\sqrt{-r^2+2r},\quad r\in[0,1]$$
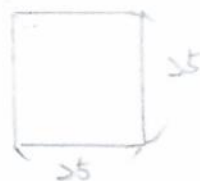
$$\because Z\in[0,1],\ \therefore\ Z = \sqrt{-r^2+2r},\ r\in[0,1],\ Z\in[0,1]$$

Problem 3.34.

已知 Bar 跟 Square Box Kernel with 25 pixel 如下

Bar   5   20   5   Square Box kernel   25
100   100   ↓空白間格.   25
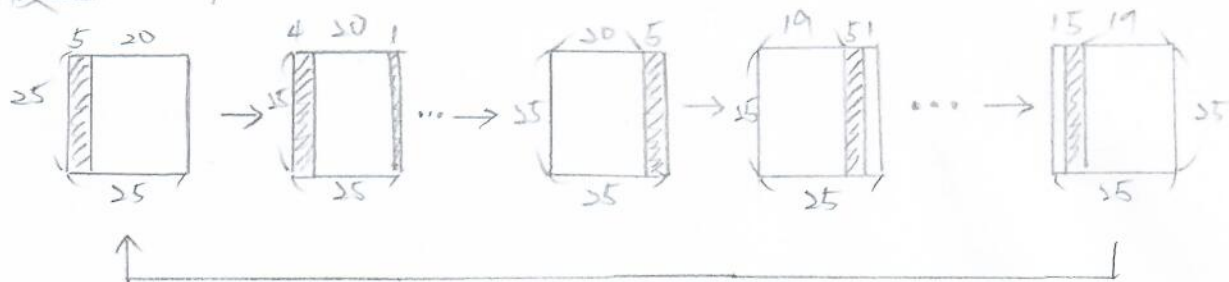
因此, Square Box kernel with 25 pixel 在移動過程 会一直
覆蓋到 5 pixel 的 Bar 與 20 pixel 的空白間格, 以致最終呈現一樣的灰色

Problem 3.42

已知 公式 (3-53) . 拉普拉斯

$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

$$g(x,y) = f(x,y) + c[\nabla^2 f(x,y)] \quad ---- \textcircled{1}$$

又知 公式 (3-55). Unsharp Masking

$$g_{mask} = f(x,y) - \overline{f}(x,y)$$

$$g(x,y) = f(x,y) + k g_{mask}(x,y) \quad ----- \textcircled{2}$$

其中 $g_{mask} = f(x,y) - \overline{f}(x,y)$

$$= \frac{1}{5}\left[f(x,y) + f(x,y) + f(x,y) + f(x,y) + f(x,y) \right]$$

$$- \frac{1}{5}\left[f(x,y) + f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) \right]$$

$$= \frac{1}{5}\left[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y) \right]$$

$$= \frac{1}{5}\left[\nabla f^2(x,y) \right]$$
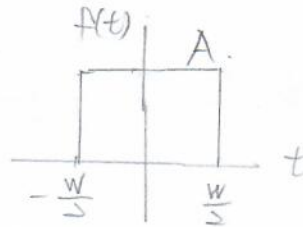
因此，減去 Laplacion 的結果 等比例於 套用 Unsharp Mask 的結果

Problem 4.2

已知 Example 4.1

$$F(\mu) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j2\pi\mu t} \cdot dt$$

$$= \int_{-\frac{W}{2}}^{\frac{W}{2}} A \cdot e^{-j2\pi\mu t} \cdot dt$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]\Big|_{-\frac{W}{2}}^{\frac{W}{2}} = \frac{-A}{j2\pi\mu}\left[e^{-j\pi\mu W} - e^{j\pi\mu W}\right] = \frac{A}{j2\pi\mu}\left[e^{j\pi\mu W} - e^{-j\pi\mu W}\right]$$

$$= AW \frac{\sin(\pi\mu W)}{\pi\mu W} = \underline{AW \, \text{sinc}(\mu W)}$$

如今 $f(t) = A$ for $0 \le t \le T$ and $f(t) = 0$ for all other values of t

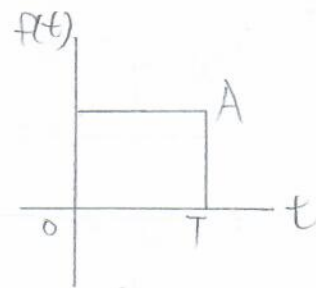$$F(\mu) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j2\pi\mu t} \cdot dt$$

$$= \int_{0}^{T} A \cdot e^{-j2\pi\mu t} \cdot dt$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]\Big|_{0}^{T} = \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu T} - e^{j2\pi\mu 0}\right]$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu\frac{T}{2}} - e^{-j2\pi\mu(\frac{T}{2})}\right] \cdot e^{-j2\pi\mu\frac{T}{2}}$$

$$= \frac{A}{j2\pi\mu}\left[e^{j\pi\mu T} - e^{-j\pi\mu T}\right] \cdot e^{-j\pi\mu T}$$

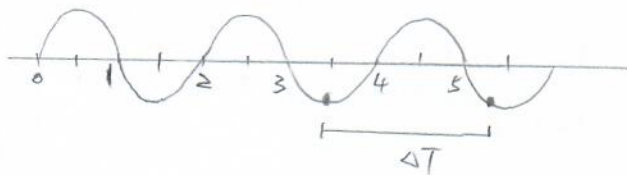$$= \underline{AT \, \text{sinc}(\mu T) \cdot e^{-j\pi\mu T}}$$

因此,差了一個 $e^{-j\pi\mu T}$. 其原因是、兩者雖然都是理想盒式濾波器.
但後者卻平移了一半週期、也就是 $\frac{T}{2}$.
所以,轉到頻域後,也就差了 $e^{-j2\pi\mu \cdot \frac{T}{2}}$ 的相位.

$$e^{-j2\pi\mu\frac{T}{2}} = e^{-j\pi\mu T}$$
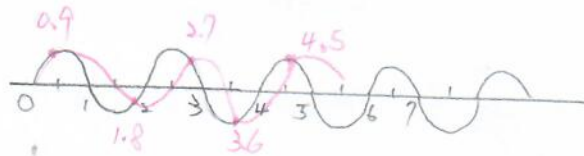
最終多了一個 $e^{-j\pi\mu T}$

Problem 46

Fig 4-11 如右

(a) Nyquist Rate $= 2\mu_{max}$

$$\Delta u = \frac{1}{T}, \quad T = 2 \quad \Rightarrow \quad \Delta u = \frac{1}{2}, \quad \text{而} \quad$$

$$\therefore \text{Nyquist Rate} = 2 \cdot \frac{1}{2} = 1$$

To let sampling rate slightly exceed

$$\frac{1}{\Delta T} > 1 \quad \Rightarrow \quad 取 \Delta T = 0.9, \quad \frac{1}{\Delta T} \cong 1.11 \quad \Rightarrow$$
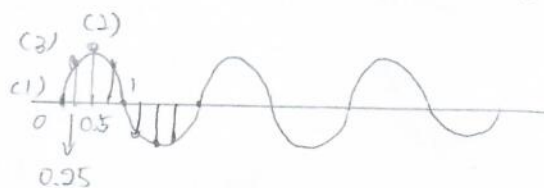

(b) $\Delta T \cong 0.25$

$$\therefore \text{sampling rate} = \frac{1}{\Delta T} = \frac{1}{0.25} = \frac{4}{9}$$

(c) 因為 Nyquist Rate $= 1$, Sampling Rate $= \frac{1}{\Delta T} \quad \Rightarrow \quad \frac{1}{\Delta T} > 1, \quad \Delta T < 1$

Fig 4.11 是週期 2 秒的 sin 波.

若要重建則需 $\begin{cases} 過零點 & ① \\ 轉折點 & ② \\ 中繼點 & ③ \end{cases}$

取 $\Delta T \cong 0.25$, sampling rate $= \frac{1}{\Delta T} = \frac{1}{0.25} = 4$ /每秒

Problem 4.9

尤拉公式 $\begin{cases} e^{i\theta} = \cos\theta + i\sin\theta \\ e^{-i\theta} = \cos\theta - i\sin\theta \end{cases} \Rightarrow \begin{cases} \cos\theta = \frac{1}{2}[e^{i\theta} + e^{-i\theta}] \\ \sin\theta = \frac{1}{2i}[e^{i\theta} - e^{-i\theta}] \end{cases}$

已知傅立葉轉換為線性, 且 $e^{j2\pi\mu_0 t} \xrightarrow{F} \delta(\mu - \mu_0)$
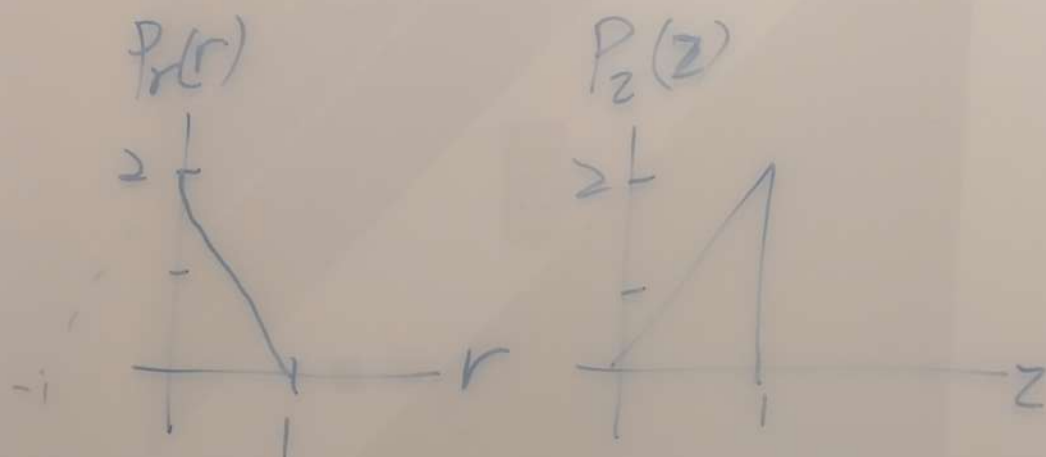
令 $\theta = 2\pi\mu_0 t$, 則

$$\cos(2\pi\mu_0 t) = \frac{1}{2}[e^{j2\pi\mu_0 t} + e^{-j2\pi\mu_0 t}]$$

$$F \downarrow \qquad\qquad\qquad F \downarrow$$

$$\underline{F[\cos(2\pi\mu_0 t)] = \frac{1}{2}[\delta(\mu - \mu_0) + \delta(\mu + \mu_0)]}$$

同理可證

$$\underline{F[\sin(2\pi\mu_0 t)] = \frac{1}{2j}[\delta(\mu - \mu_0) - \delta(\mu + \mu_0)]}$$

3.12

$P_r(r)$

$P_z(z)$

$P_r(r) = -2r + r$

$P_z(z) = 2z$

$S = T(r) = \int_0^r P_r(w) \cdot dw = \int_0^r (-2w+2) \cdot dw$

$= -r^2 + 2r$
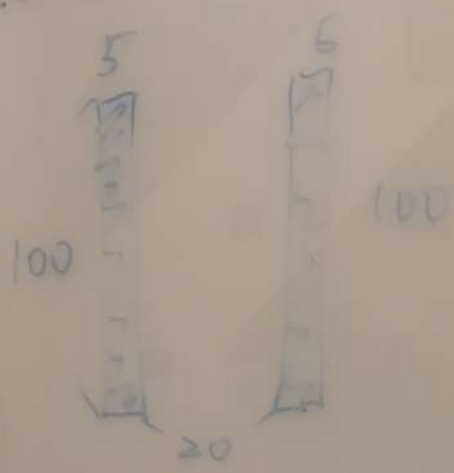
$V = G(z) = \int_0^z P_z(w) \, dw = \int_0^z 2w \cdot dw$

$= z^2$

令 $S = V$ 得

$z = G^{-1}(V) = \pm\sqrt{V} = \pm\sqrt{-r^2+2r}$, $r = [0.1]$
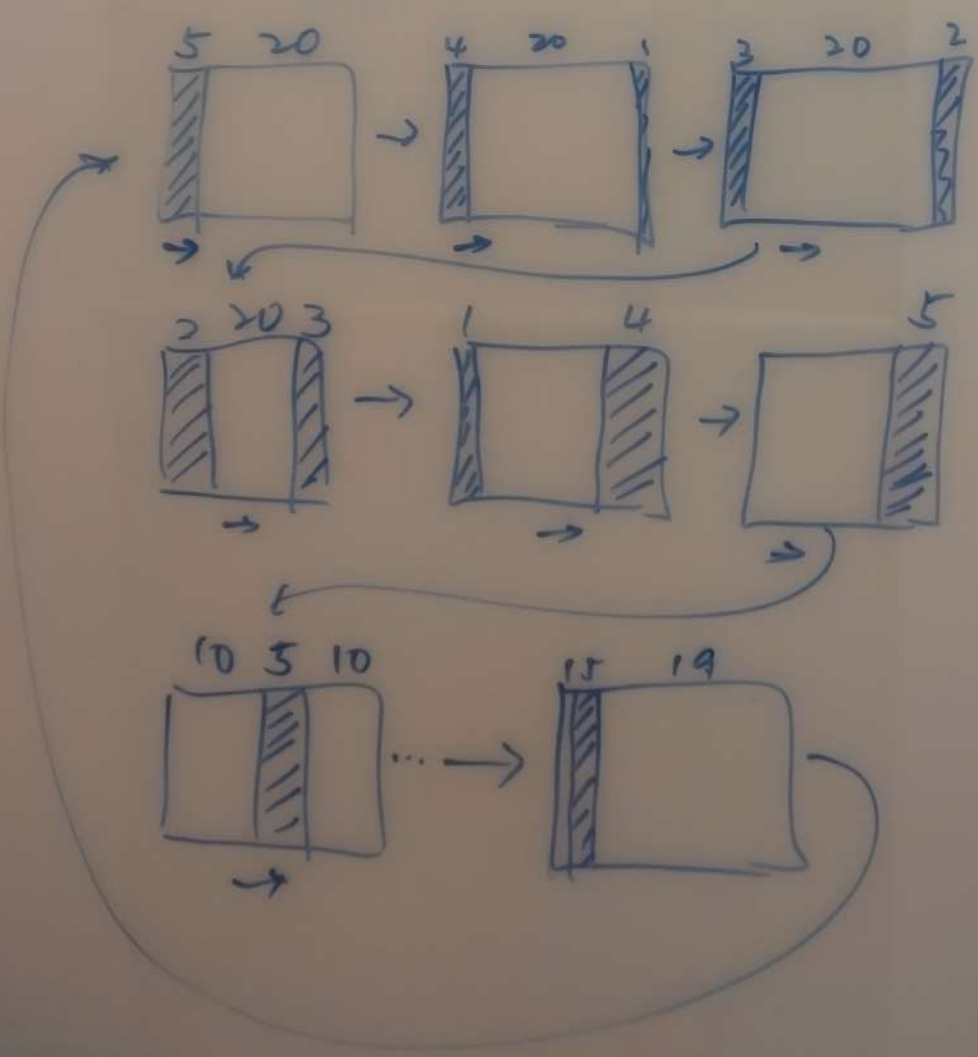
3.34

照下圖所示.
Square Box kernel 25
移動过程 永遠是
Bar 5. Blank 20 在做平均
所以最終夾度都一樣

Box Square Box Kernel

≥5

(3-55) Unsharp Masking

$$g_{mask} = f(x,y) - \bar{f}(x,y)$$

$$g(x,y) = f(x,y) + k\,g_{mask}(x,y)$$

(3-53)

$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y)$$
$$+ f(x,y+1) + f(x,y-1)$$
$$- 4f(x,y)$$

→ $$g(x,y) = f(x,y) + c[\nabla^2 f(x,y)]$$

$$\bar{f}(x,y) = \frac{1}{5}\left[f(x,y) + f(x+1,y) + f(x-1,y) \right.$$
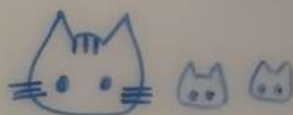$$\left. + f(x,y+1) + f(x,y-1) \right]$$

$$g_{mask} = f(x,y) - \bar{f}(x,y)$$
$$= \frac{1}{5}\left[f(x,y) + f(x,y) + f(x,y) + f(x,y) + f(x,y) \right]$$
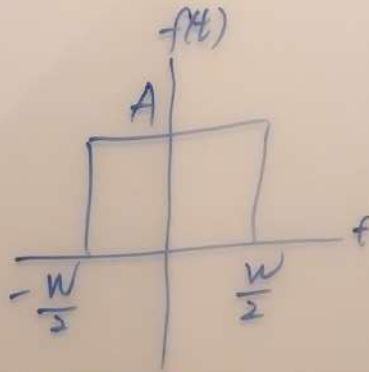$$- \frac{1}{5}\left[f(x,y) + f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) \right]$$
$$= \frac{-1}{5}\left[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) \right.$$
$$\left. - 4f(x,y) \right]$$
$$= \frac{-1}{5}\left[\nabla^2 f(x,y) \right]$$

4.2

Example 4.1

$$F(\mu) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j2\pi\mu t} \, dt$$

$$= \int_{-\frac{W}{2}}^{\frac{W}{2}} A \cdot e^{-j2\pi\mu t} \, dt$$

$$= \frac{-A}{j2\pi\mu} \left[e^{-j2\pi\mu t} \right]_{-\frac{W}{2}}^{\frac{W}{2}}$$

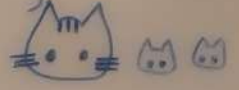$$= \frac{-A}{j2\pi\mu} \left[e^{-j\pi\mu W} - e^{j\pi\mu W} \right]$$

$$= \frac{A}{j2\pi\mu} \left[e^{j\pi\mu W} - e^{-j\pi\mu W} \right]$$

$$= AW \frac{\sin(\pi\mu W)}{\pi\mu W} = AW \, \text{sinc}(\mu W)$$

$$\int_{0}^{T} A \cdot e^{-j2\pi\mu t} \, dt$$

$$= \frac{-A}{j2\pi\mu} \left[e^{-j2\pi\mu t} \right]_{0}^{T}$$

$$= \frac{-A}{j2\pi\mu} \left[e^{-j2\pi\mu T} - e^{-j2\pi\mu 0} \right]$$

$$\xrightarrow{\tfrac{T}{2}} \quad \begin{aligned} \theta + \phi &= 2\pi\mu T \\ \theta - \phi &= 2\pi\mu v = 0 \end{aligned} \left\} \begin{aligned} \theta &= \pi\mu T \\ \phi &= \pi\mu T \end{aligned} \right.$$

$$\begin{aligned} & \left[\cos(\theta+\phi) - j\sin(\theta+\phi) \right] \\ - & \left[\cos(\theta-\phi) - j\sin(\theta-\phi) \right] \end{aligned}$$

$$= \left[(\cos\theta\cos\phi - \sin\theta\sin\phi) - j(\sin\theta\cos\phi + \cos\theta\sin\phi) \right]$$
$$- \left[(\cos\theta\cos\phi + \sin\theta\sin\phi) - j(\sin\theta\cos\phi - \cos\theta\sin\phi) \right]$$

$$= -2\sin\theta\sin\phi - 2j\cos\theta\sin\phi$$

$$= -2j\sin\phi \left[\cos\theta - j\sin\theta \right]$$

$$\frac{-A}{j2\pi\mu} \cdot \left[-2j\sin(\pi\mu T) \cdot e^{-j\pi\mu T} \right] \qquad \overset{e^{-j2\pi\mu\frac{T}{2}}}{\underset{\sqrt{j\pi\mu T}}{=}}$$

$$= \frac{AT}{\pi\mu T} \cdot \sin(\pi\mu T) \cdot e^{-j\pi\mu T} = AT \, \text{sinc}(\mu T) e^{-j\pi\mu T}$$

4.2

Example 4.1
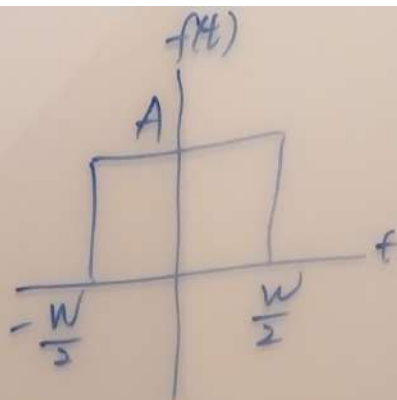
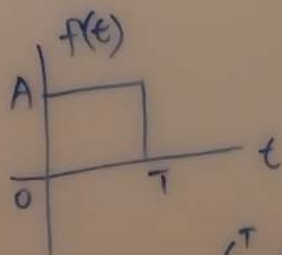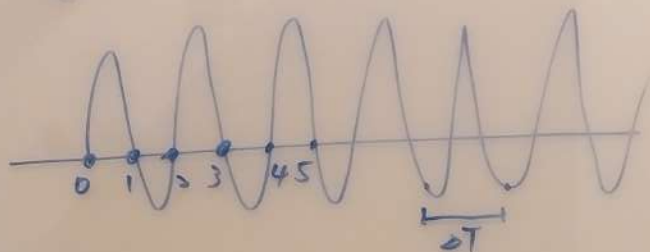$$F(\mu) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j2\pi\mu t} \, dt$$

$$= \int_{-\frac{W}{2}}^{\frac{W}{2}} A \cdot e^{-j2\pi\mu t} \, dt$$

$$= \frac{-A}{j2\pi\mu} \left[e^{-j2\pi\mu t} \right] \Big|_{-W/2}^{W/2}$$

$$= \frac{-A}{j2\pi\mu} \left[e^{-j\pi\mu W} - e^{j\pi\mu W} \right]$$

$$= \frac{A}{j2\pi\mu} \left[e^{j\pi\mu W} - e^{-j\pi\mu W} \right]$$

$$= AW \frac{\sin(\pi\mu W)}{\pi\mu W} = AW \, \text{sinc}(\mu W)$$

$$F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} \, dt$$

$$\Rightarrow F(\mu) = \int_{0}^{T} A \cdot e^{-j2\pi\mu t} \, dt = \frac{-A}{j\pi\mu} \left[e^{-j2\pi\mu t} \right] \Big|_{0}^{T}$$

$$= \frac{-A}{j\pi\mu} \left[e^{-j2\pi\mu T} - e^{-j2\pi\mu 0} \right]$$

$$= \frac{-A}{j\pi\mu} \left[e^{-j2\pi\mu \frac{T}{2}} - e^{-j2\pi\mu \frac{T}{5}} \right] \cdot e^{-j2\pi\mu \frac{T}{2}}$$

$$= \frac{+A}{j2\pi\mu} \left[e^{j\pi\mu T} - e^{-j\pi\mu T} \right] \cdot e^{-j\pi\mu T}$$

$$= \frac{AT}{\pi\mu T} \left[\frac{e^{-j\pi\mu T} - e^{-j\pi\mu T}}{2j} \right] \cdot e^{-j\pi\mu T}$$

$$= AT \cdot \text{sinc}(\mu T) \cdot e^{-j\pi\mu T}$$
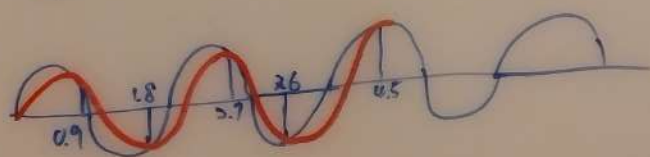
4.6

Fig. 4-11

(b) $\Delta T \cong 2.25$

$\therefore$ sampling rate $= \dfrac{1}{\Delta T} = \dfrac{1}{2.25} = \dfrac{4}{9}$

(a) Nyquist rate $= 2\mu_{max}$

$\circ \mu = \dfrac{1}{T} = \dfrac{1}{5}$ $\underset{\mu_{min} \quad 0 \quad \mu_{max}}{\underbrace{\Delta\mu \quad \Delta\mu}}$

$\therefore$ Nyquist rate $= 2 \cdot \dfrac{1}{5} = 1$
Let sampling rate slight exceeds

$\dfrac{1}{\Delta T} > 1 \Rightarrow \Delta T = 0.9, \dfrac{1}{\Delta T} \cong 1.1$
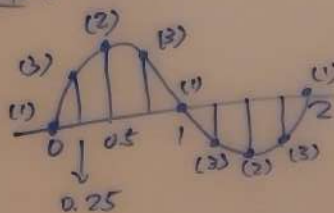

(c) $\because$ Nyquist rate $= 1$, $\Rightarrow \dfrac{1}{\Delta T} > 1$, $\Delta T < 1$
sampling rate $= \dfrac{1}{\Delta T}$

Fig. 4.11 是週期 2 秒的 sin 波.
若要重建則
(1) 要有过零点
(2) 要有轉折点
(3) 要有中間点

$\downarrow$
0.25

$\Delta T \cong 0.25$
sampling rate $\dfrac{1}{\Delta T} = \dfrac{1}{0.25} = 4/s$

4.9

$\frac{1}{2}$ $z = \cos\theta + i\sin\theta$

$\frac{dz}{d\theta} = -\sin\theta + i\cos\theta$

$= i(\cos\theta + i\sin\theta)$

$= i \cdot z$

$\therefore \quad \frac{1}{z}dz = i \cdot d\theta \Rightarrow \int \frac{1}{z}\cdot dz = \int i\cdot d\theta$

$\Rightarrow \ln z = i\theta \Rightarrow e^{\ln z} = e^{i\theta}$

$\Rightarrow \quad z = e^{i\theta}$

$\therefore \quad e^{i\theta} = \cos\theta + i\sin\theta$

$\quad e^{-i\theta} = \cos\theta - i\sin\theta$

$\cos\theta = \frac{1}{2}\left[e^{i\theta} + e^{-i\theta}\right]$

$\sin\theta = \frac{1}{2j}\left[e^{i\theta} - e^{-i\theta}\right]$

$\frac{1}{2} \theta = 2\pi\mu_0 t)$

$\cos(2\pi\mu_0 t) = \frac{1}{2}\left[e^{i2\pi\mu_0 t} + e^{-i2\pi\mu_0 t}\right]$

$\mathcal{J}(\cos 2\pi\mu_0 t) = \frac{1}{2}\int_{-\infty}^{\infty}\left[e^{i2\pi\mu_0 t} + e^{-i2\pi\mu_0 t}\right]e^{-i2\pi\mu t}\,dt$

$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{-i2\pi(\mu-\mu_0)t}\,dt + \int_{-\infty}^{\infty} e^{-i2\pi(\mu+\mu_0)t}\,dt\right]$

$= \frac{1}{2}\left[\delta(\mu-\mu_0) + \delta(\mu+\mu_0)\right]$

$\mathcal{J}(\sin(2\pi\mu_0 t)) = \frac{1}{2}\left[e^{i2\pi\mu_0 t} - e^{-i2\pi\mu_0 t}\right]$

$= \frac{1}{2j}\left[\delta(\mu-\mu_0) - \delta(\mu+\mu_0)\right]$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

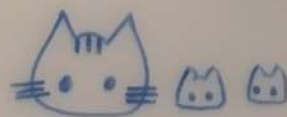$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$\cos\theta = \frac{1}{2}\left[e^{i\theta} + e^{-i\theta}\right]$$

$$\sin\theta = \frac{1}{2i}\left[e^{i\theta} - e^{-i\theta}\right]$$

⊛ $$e^{j2\pi t_0 t} \xrightarrow{F} \delta(u - t_0)$$

$$\theta = 2\pi t_0 t$$

$$\cos\theta \xrightarrow{F} \frac{1}{2}\left[F(e^{i\theta}) + F(e^{-i\theta})\right]$$

$$\frac{1}{2}\left[F(e^{j2\pi t_0 t}) + F(e^{j2\pi - t_0 t})\right]$$

$$\delta(u - t_0) + \delta(u + t_0)$$

$$\sin\theta \longrightarrow \frac{1}{2i}\left[\ \delta(u - t_0) - \delta(u + t_0)\right]$$

✿ $$e^{j2\pi t_0 t} \xrightarrow{F} \delta(u - t_0)$$

$$F(e^{j2\pi t_0 t}) = \int_{-\infty}^{\infty} e^{j2\pi t_0 t}\, e^{-j2\pi u t}\cdot dt$$

$$= \int_{-\infty}^{\infty} e^{j2\pi(t_0 - u)t}\cdot dt$$

$$= \delta(t_0 - u) = \delta(u - t_0)$$