Advertising on the Web

Mining of Massive Datasets
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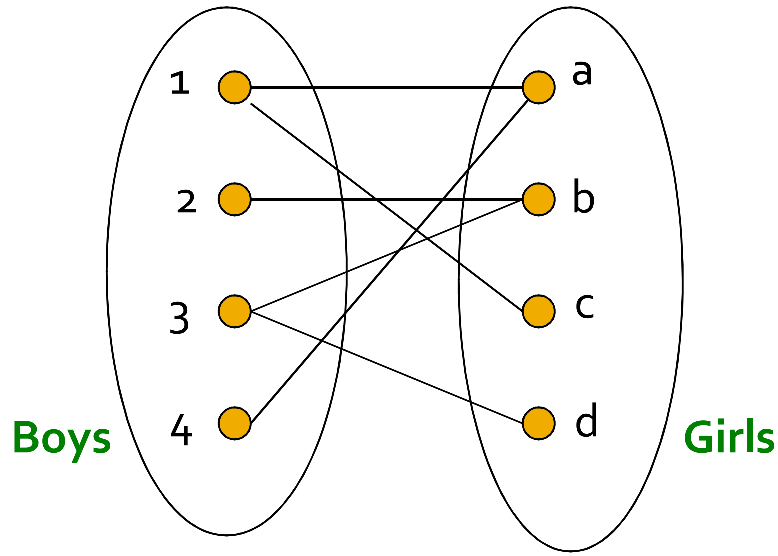
Online Algorithms

- **Classic model of algorithms**
  - You get to see the entire input, then compute some function of it
  - In this context, "offline algorithm"
- **Online Algorithms**
  - You get to see the input one piece at a time, and need to make irrevocable decisions along the way
  - **Similar to the data stream model**

Online Bipartite Matching

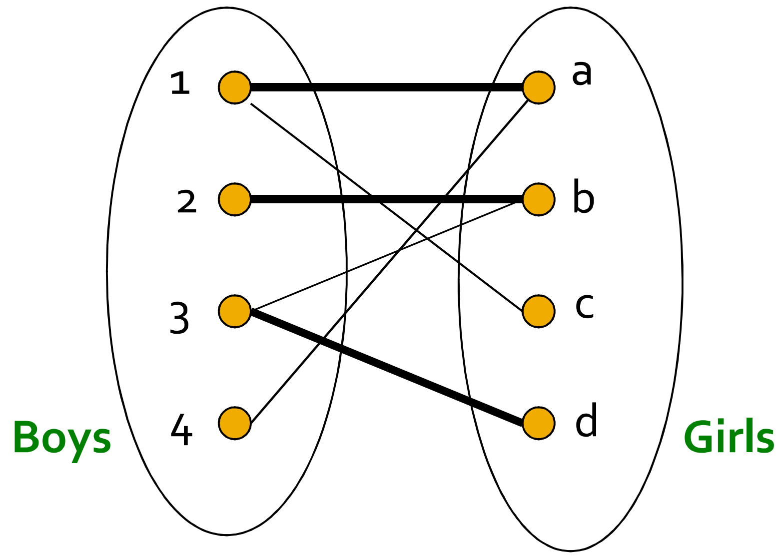
Example: Bipartite Matching



Nodes: Boys and Girls; Edges: Preferences

Goal: Match boys to girls so that maximum number of preferences is satisfied

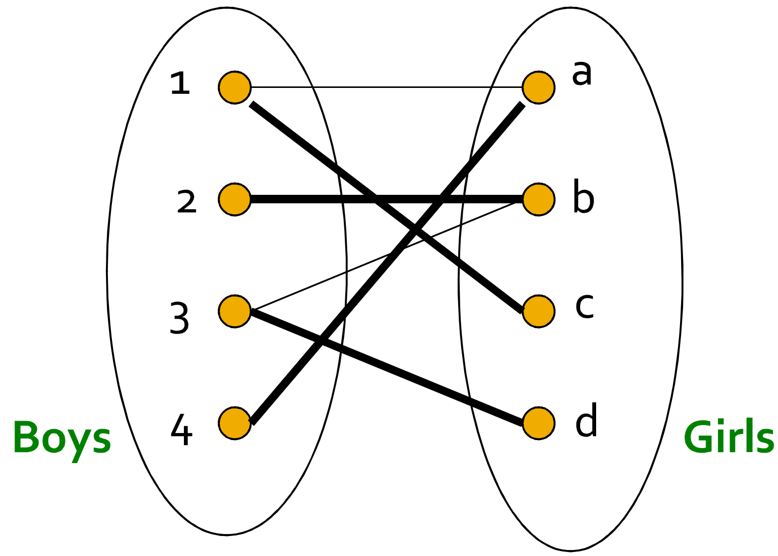
Example: Bipartite Matching



M = {(1,a),(2,b),(3,d)} is a **matching**

Cardinality of matching = |M| = 3

Example: Bipartite Matching



M = {(1,c),(2,b),(3,d),(4,a)} is a
perfect matching

Perfect matching … all vertices of the graph are matched
Maximum matching … a matching that contains the largest possible number of matches

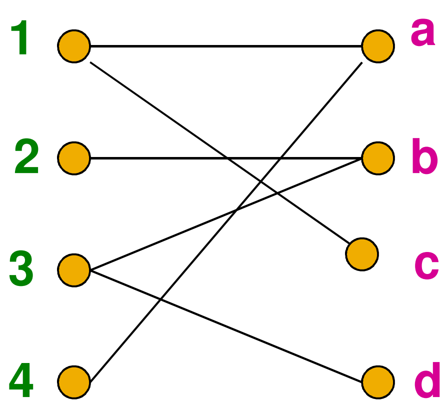
Matching Algorithm

- **Problem: Find a maximum matching for a given bipartite graph**
  - A perfect one if it exists
- There is a polynomial-time offline algorithm based on augmenting paths (Hopcroft & Karp 1973, see http://en.wikipedia.org/wiki/Hopcroft-Karp_algorithm)
- **But what if we do not know the entire graph upfront?**

Online Graph Matching Problem

- Initially, we are given the set **boys**
- In each **round**, **one girl's choices are revealed**
  - That is, girl's **edges** are revealed
- **At that time, we have to decide to either:**
  - Pair the **girl** with a **boy**
  - Do not pair the **girl** with any **boy**
- **Example of application:**
  Assigning tasks to servers

Online Graph Matching: Example



(1,a)
(2,b)
(3,d)

Greedy Algorithm

- **Greedy algorithm for the online graph matching problem:**
  - Pair the new girl with **any** eligible boy
    - If there is none, do not pair girl
- **How good is the algorithm?**

Competitive Ratio

- For input I, suppose greedy produces matching M_{greedy} while an optimal matching is M_{opt}

Competitive ratio =

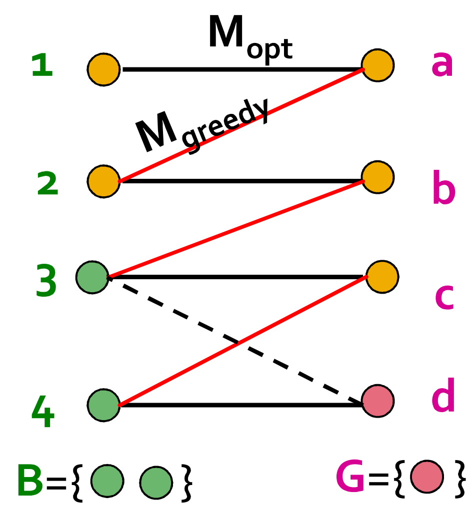
$$\min_{\text{all possible inputs } I} (|M_{greedy}| / |M_{opt}|)$$

(what is greedy's <u>worst</u> performance <u>over all possible</u> inputs *I*)

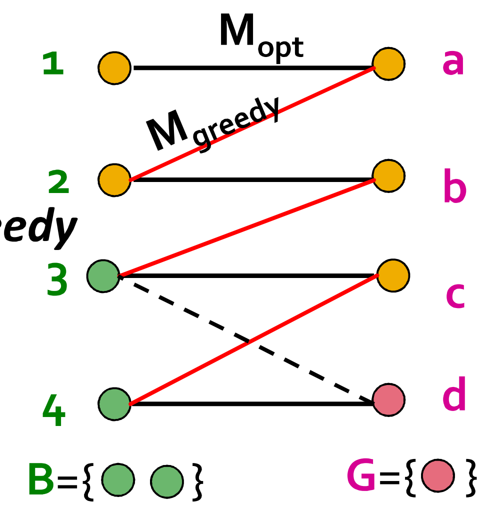
Analyzing the Greedy Algorithm

- Consider a case: $M_{greedy} \neq M_{opt}$
- Consider the set G of girls matched in M_{opt} but not in M_{greedy}
- Then every boy B adjacent to girls in G is already matched in M_{greedy}:
  - If there would exist such non-matched (by M_{greedy}) boy adjacent to a non-matched girl then greedy would have matched them
- Since boys B are already matched in M_{greedy} then

(1) $|M_{greedy}| \geq |B|$

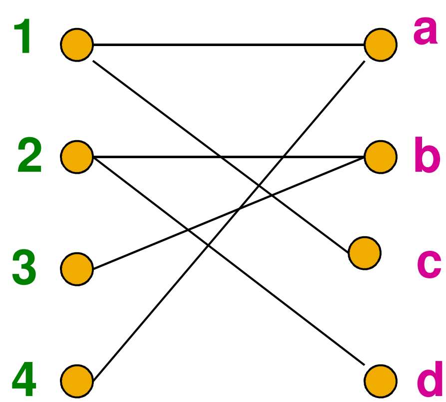


Analyzing the Greedy Algorithm



- **Summary so far:**
  - Girls G matched in M_{opt} but not in M_{greedy}
  - (1) $|M_{greedy}| \geq |B|$
- There are at least $|G|$ such boys ($|G| \leq |B|$) otherwise the optimal algorithm couldn't have matched all girls in G
  - **So:** $|G| \leq |B| \leq |M_{greedy}|$
- By definition of G also: $|M_{opt}| \leq |M_{greedy}| + |G|$
  - Worst case is when $|G| = |B| = |M_{greedy}|$
- $|M_{opt}| \leq 2|M_{greedy}|$ **then** $|M_{greedy}|/|M_{opt}| \geq 1/2$

Worst-case Scenario



(1,a)
(2,b)

Web Advertising

History of Web Advertising

- **Banner ads (1995-2001)**
  - Initial form of web advertising
  - Popular websites charged *X*$ for every 1,000 "impressions" of the ad
    - Called "**CPM**" rate (Cost per thousand impressions)
    - Modeled similar to TV, magazine ads
  - From **untargeted** to **demographically targeted**
  - **Low click-through rates**
    - Low ROI for advertisers

CPM…cost per *mille*
Mille…thousand in Latin

Performance-based Advertising

- **Introduced by Overture around 2000**
  - Advertisers **bid** on **search keywords**
  - When someone searches for that keyword, the **highest bidder's ad is shown**
  - Advertiser is charged only if the ad is clicked on
- Similar model adopted by Google with some changes around 2002
  - Called **Adwords**

Ads vs. Search Results

Web Results **1** - **10** of about **2,230,000** for **geico**. (**0.04** secc

GEICO Car Insurance. Get an auto insurance quote and save today ...
GEICO auto insurance, online car insurance quote, motorcycle insurance quote, online insurance sales and service from a leading insurance company.
www.**geico**.com/ - 21k - Sep 22, 2005 - Cached - Similar pages
Auto Insurance - Buy Auto Insurance
Contact Us - Make a Payment
More results from www.geico.com »

Geico, Google Settle Trademark Dispute
The case was resolved out of court, so advertisers are still left without legal guidance on use of trademarks within ads or as keywords.
www.clickz.com/news/article.php/3547356 - 44k - Cached - Similar pages

Google and **GEICO** settle AdWords dispute | The Register
Google and car insurance firm **GEICO** have settled a trade mark dispute over ... Car insurance firm **GEICO** sued both Google and Yahoo! subsidiary Overture in ...
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... involving a lawsuit filed by Government Employees Insurance Company (**GEICO**). **GEICO** has filed suit against two major Internet search engine operators, ...
www.consumeraffairs.com/news04/**geico**_google.html - 19k - Cached - Similar pages

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Great Car Insurance Rates
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See Your Rate with an Instant Quote
www.Safeco.com

Free Insurance Quotes
Fill out one simple form to get
multiple quotes from local agents.
www.HometownQuotes.com

5 Free Quotes. 1 Form.
Get 5 Free Quotes In Minutes!
You Have Nothing To Lose. It's Free
sayyessoftware.com/Insurance
Missouri

Web 2.0

- **Performance-based advertising works!**
  - Multi-billion-dollar industry

- **Interesting problem:**
  What ads to show for a given query?
  - (Today's lecture)

- **If I am an advertiser, which search terms should I bid on and how much should I bid?**
  - (Not focus of today's lecture)

Adwords Problem

- **Given:**
  - **1.** A set of bids by advertisers for search queries
  - **2.** A click-through rate for each advertiser-query pair
  - **3.** A budget for each advertiser (say for 1 month)
  - **4.** A limit on the number of ads to be displayed with each search query
- **Respond to each search query with a set of advertisers such that:**
  - **1.** The size of the set is no larger than the limit on the number of ads per query
  - **2.** Each advertiser has bid on the search query
  - **3.** Each advertiser has enough budget left to pay for the ad if it is clicked upon

Adwords Problem

- A stream of queries arrives at the search engine: $q_1, q_2, \ldots$
- Several advertisers bid on each query
- When query q_i arrives, search engine must pick a subset of advertisers whose ads are shown

- **Goal: Maximize search engine's revenues**
  - **Simple solution:** Instead of raw bids, use the "**expected revenue per click**" (i.e., **Bid*CTR**)
- **Clearly we need an online algorithm!**

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
A	$1.00	1%	1 cent
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents

Click through rate

Expected revenue

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents
A	$1.00	1%	1 cent

Complications: Budget

- **Two complications:**
  - **Budget**
  - **CTR of an ad is unknown**

- **Each advertiser has a limited budget**
  - **Search engine guarantees that the advertiser will not be charged more than their daily budget**

Complications: CTR

- **CTR: Each ad has a different likelihood of being clicked**
  - **Advertiser 1** bids \$2, click probability = 0.1
  - **Advertiser 2** bids \$1, click probability = 0.5
  - **Clickthrough rate (CTR)** is measured **historically**
    - **Very hard problem: Exploration vs. exploitation**
      Exploit: Should we keep showing an ad for which we have good estimates of click-through rate
      or
      Explore: Shall we show a brand new ad to get a better sense of its click-through rate

Greedy Algorithm

- **Our setting: Simplified environment**
  - There is **1** ad shown for each query
  - All advertisers have the same budget ***B***
  - All ads are equally likely to be clicked
  - Value of each ad is the same (=**1**)
- **Simplest algorithm is greedy:**
  - For a query pick any advertiser who has bid **1** for that query
  - **Competitive ratio of greedy is 1/2**

Bad Scenario for Greedy

- **Two advertisers A and B**
  - ***A*** bids on query ***x***, ***B*** bids on ***x*** and ***y***
  - Both have budgets of **$4**
- **Query stream:** ***x x x x y y y y***
  - Worst case greedy choice: ***B B B B*** **_ _ _ _**
  - Optimal: **A A A A B B B B**
  - **Competitive ratio = ½**
- **This is the worst case!**
  - **Note:** Greedy algorithm is deterministic – it always resolves draws in the same way

BALANCE Algorithm [MSVV]

- **BALANCE** Algorithm by Mehta, Saberi, Vazirani, and Vazirani
  - **For each query, pick the advertiser with the largest unspent budget**
    - Break ties arbitrarily (**but in a deterministic way**)

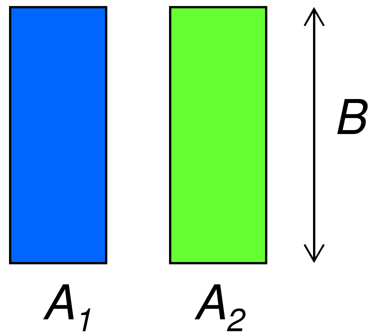
Example: BALANCE

- **Two advertisers A and B**
  - **A** bids on query ***x***, **B** bids on ***x*** and ***y***
  - Both have budgets of **$4**
- **Query stream:** ***x x x x y y y y***
- **BALANCE choice:** **A B A B B B _ _**
  - Optimal: **A A A A B B B B**
- **In general:** For **BALANCE** on **2** advertisers
  Competitive ratio = ¾

Analyzing BALANCE

- **Consider simple case (w.l.o.g.):**
  - **2** advertisers, $\mathbf{A_1}$ and $\mathbf{A_2}$, each with budget **B** (≥1)
  - Optimal solution exhausts both advertisers' budgets
- **BALANCE must exhaust at least one advertiser's budget:**
  - **If not, we can allocate more queries**
    - Whenever BALANCE makes a mistake (both advertisers bid on the query), advertiser's unspent budget only decreases
    - Since optimal exhausts both budgets, one will for sure get exhausted
  - Assume BALANCE exhausts A_2's budget, but allocates x queries fewer than the optimal
  - **Revenue:** $BAL = 2B - x$

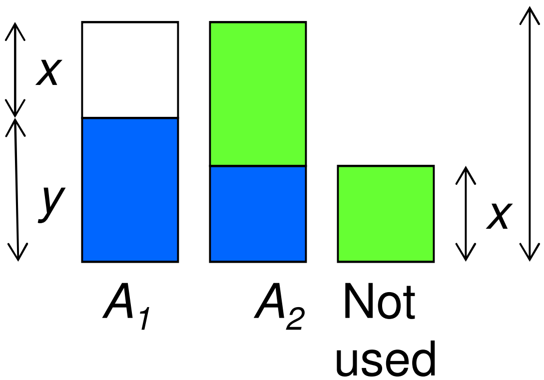
Analyzing Balance



Queries allocated to A_1 in the optimal solution

Queries allocated to A_2 in the optimal solution

Optimal revenue = **2B**
Assume Balance gives revenue **= 2B-x = B+y**



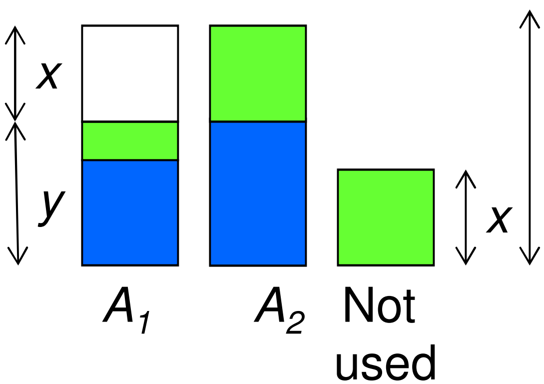
Unassigned queries should be assigned to A_2
(if we could assign to A_1 we would since we still have the budget)

Goal: Show we have **y ≥ x**

Case 1) ≤ ½ of A_1's queries got assigned to A_2
then $y \geq B/2$

Case 2) > ½ of A_1's queries got assigned to A_2
then $x \leq B/2$ and $x + y = B$

Balance revenue is minimum for $x = y = B/2$
Minimum Balance revenue = $3B/2$
Competitive Ratio = 3/4



BALANCE exhausts A_2's budget

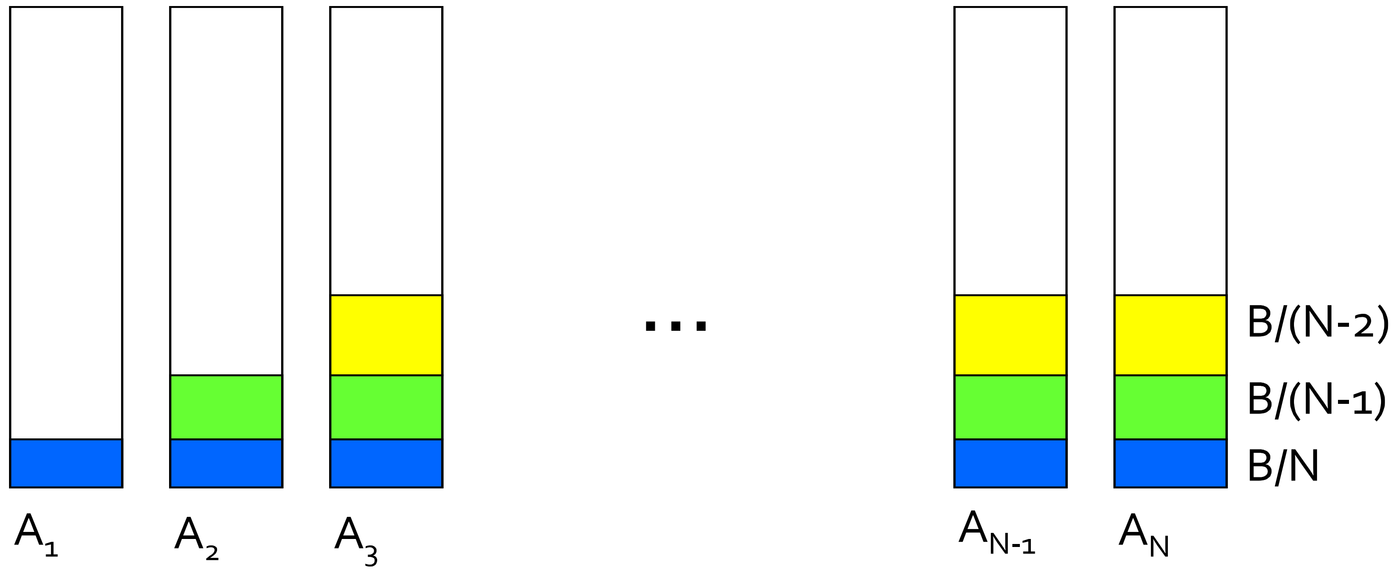
BALANCE: General Result

- **In the general case, worst competitive ratio of BALANCE is 1–1/e = approx. 0.63**
  - Interestingly, no online algorithm has a better competitive ratio!
- **Let's see the worst case example that gives this ratio**

Worst case for BALANCE

- ***N* advertisers:** $A_1, A_2, \ldots A_N$
  - Each with budget $B > N$
- **Queries:**
  - $N \cdot B$ queries appear in ***N*** rounds of ***B*** queries each
- **Bidding:**
  - Round **1** queries: bidders $A_1, A_2, \ldots, A_N$
  - Round **2** queries: bidders $A_2, A_3, \ldots, A_N$
  - Round ***i*** queries: bidders $A_i, \ldots, A_N$
- **Optimum allocation:**
  Allocate round ***i*** queries to A_i
  - Optimum revenue $N \cdot B$

BALANCE Allocation



BALANCE assigns each of the queries in round 1 to **N** advertisers. After ***k*** rounds, sum of allocations to each of advertisers $\boldsymbol{A_k,\dots,A_N}$ is

$$\boldsymbol{S_k = S_{k+1} = \cdots = S_N = \sum_{i=1}^{k-1} \frac{B}{N-(i-1)}}$$

If we find the smallest *k* such that $S_k \geq B$, then after *k* rounds we cannot allocate any queries to any advertiser

BALANCE: Analysis

B/1 B/2 B/3 … B/(N-(k-1)) … B/(N-1) B/N

S_1

S_2

S_k **= B**

1/1 1/2 1/3 … 1/(N-(k-1)) … 1/(N-1) 1/N

S_1

S_2

S_k **= 1**

BALANCE: Analysis

- **Fact:** $H_n = \sum_{i=1}^{n} 1/i \approx \ln(n)$ for large n
  - Result due to Euler

1/1   1/2   1/3 … 1/(N-(k-1)) … 1/(N-1)   1/N

ln(N)

ln(N)-1 $S_k = 1$

- $S_k = 1$ implies: $H_{N-k} = ln(N) - 1 = ln(\frac{N}{e})$
- We also know: $H_{N-k} = ln(N-k)$
- **So:** $N - k = \frac{N}{e}$
- **Then:** $k = N(1 - \frac{1}{e})$

N terms sum to ln(*N*).
Last *k* terms sum to 1.
First *N*-*k* terms sum
to ln(*N*-*k*) but also to ln(*N*)-1

BALANCE: Analysis

- So after the first **$k=N(1-1/e)$** rounds, we cannot allocate a query to any advertiser
- **Revenue = $B \cdot N\ (1-1/e)$**
- **Competitive ratio = $1-1/e$**

General Version of the Problem

- **Arbitrary bids and arbitrary budgets!**
- Consider we have 1 query $\boldsymbol{q}$, advertiser $\boldsymbol{i}$
  - Bid = $\boldsymbol{x_i}$
  - Budget = $\boldsymbol{b_i}$
- **In a general setting BALANCE can be terrible**
  - Consider two advertisers $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$
  - $\boldsymbol{A_1}$: $\boldsymbol{x_1}$ = **1**, $\boldsymbol{b_1}$ = **110**
  - $\boldsymbol{A_2}$: $\boldsymbol{x_2}$ = **10**, $\boldsymbol{b_2}$ = **100**
  - Consider we see **10** instances of **q**
  - BALANCE always selects $\boldsymbol{A_1}$ and earns **10**
  - Optimal earns **100**

Generalized BALANCE

- **Arbitrary bids:** consider query q, bidder i
  - Bid = x_i
  - Budget = b_i
  - Amount spent so far = m_i
  - Fraction of budget left over $f_i = 1-m_i/b_i$
  - Define $\psi_i(q) = x_i(1-e^{-f_i})$
- Allocate query q to bidder i with largest value of $\psi_i(q)$
- **Same competitive ratio (1-1/e)**