ASSIGNMENT-1

Q. 2 Lower bound on the number of measurements for exact reconstruction of a signal in R^n that is S-sparse in some orthonormal basis ψ.

ⓐ Basis-Pursuit Algorithm

Number of measurements $m \geq C \log(n/\delta) \|\theta\|_0 \mu^2(\theta, \psi)$

Exact Reconstruction guaranteed with probability $(1-\delta)$

ⓑ A combinatorial algorithm (Problem P0)

$y = \phi\psi x = Ax$ Let x be a k-sparse signal.

Null space of matrix A $N(A) = \{x : Ax = 0\}$

In order to provide a unique solution, any two k-sparse vectors x and x' should not result in same measurement vector.

i.e, $A(x - x') \neq 0$ their difference should not lie in the null space of A.

Now,

Since the difference between two k-sparse vectors is at-most 2k-sparse, then a k-sparse vector x is uniquely reconstructed if null space of A contains no 2k sparse vectors which means that any 2K columns of A ~~are~~ linearly independent.

$$\text{spark}(A) > 2K$$

where $\text{spark}(A) \in [2, m+1]$

$$\Rightarrow m+1 > 2K \Rightarrow \boxed{m > 2K - 1}$$

Page No.

Date : / /

Maximum Bound allowed on the RIC of matrix $\phi\psi$

$$\boxed{\delta_{2S} < 0.41}$$