Assignment - 1

Ques 4)

$\phi \rightarrow m \times n$ matrix $\rightarrow$ all rows normalized

$\psi \rightarrow$ orthonormal matrix

$$\mu(\phi, \psi) = \sqrt{n} \max_{i,j \in \{0,1,\dots n-1\}} |\phi^{i^T} \psi_j|$$

Consider a unit vector g. As ψ is an orthonormal basis, g can be written as $g = \sum_{k=1}^{n} \alpha_k \psi_k$

$g \rightarrow$ unit vector $\Rightarrow \|g\| = 1$

$$\Rightarrow \sum_{k=1}^{n} \alpha_k^2 \|\psi_k\|^2 = 1 \quad \left(\because \psi_i^T \cdot \psi_j = 0 \text{ for } i \neq j \atop = 1 \text{ for } i = j \right)$$

$$\Rightarrow \sum_{k=1}^{n} \alpha_k^2 = 1$$

$$\mu(g, \psi) = \sqrt{n} \max_{i \in \{0,1,\dots n-1\}} |g, \psi_i| = \sqrt{n} \max_{i \in \{0,1,\dots n-1\}} |\alpha_i \psi_i, \psi_i|$$

$$(\because |\psi_j, \psi_i| = 0 \text{ for } i \neq j)$$

$$= \sqrt{n} \max_{i \in \{0,1,\dots n-1\}} |\alpha_i| \quad (\because |\psi_i, \psi_i| = \|\psi_i\|^2 = 1)$$

Now, as g was a unit vector. We can have every row of ϕ as g.

$\therefore$ Lower value of $\mu(\phi, \psi) = \sqrt{n} \max_{i \in \{0,1,\dots n-1\}} |\alpha_i|$ subject to constraints $\sum_{k=1}^{n} \alpha_k^2 = 1$

$= \sqrt{n} \max_{i \in \{0,1,\dots n-1\}} B_i$ with $\sum_{k=1}^{n} B_k^2 = 1$ & $B_i > 0 \ \forall i$.

is achieved when all B_i are equal, i.e., $n B_i^2 = 1 \Rightarrow B_i = \frac{1}{\sqrt{n}}$

because if max is $< \frac{1}{\sqrt{n}}$, then $\sum B_i^2 < 1$ & hence not possible.

$\therefore$ lowest value of $\mu = \sqrt{n} \times \frac{1}{\sqrt{n}} = \underline{\underline{1}}$

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Highest value $= \sqrt{n} \max\limits_{i \in \{0,1,\dots n-1\}} |\alpha_i|$ when $\sum\limits_{k=1}^{n} \alpha_k^2 = 1$

is achieved when one of $\alpha_i = 1$ & other $= 0$

$= \sqrt{n} \times 1 = \sqrt{n}$

$\therefore$ μ lies between $\underline{\underline{1 \text{ \& } \sqrt{n}}}$.