EE338: FILTER DESIGN ASSIGNMENT

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Filter Number : 36

1 - BandPass-Butterworth Filter

1.1 Un-normalized Discrete Time Filter Specifications

Filter Number = 36
Since filter number is < 36 , m = 68 and passband will be monotonic
q(m) = 3
r(m) = m - 10*q(m) = 36 - 10*3 = 6
BL(m) = 5 + 1.4 q(m) + 4 r(m) = 5 + 1.4*3 + 4*6 = 33.2
BH(m) = BL(m) + 10 = 43.2
The first filter is given to be a **Band-Pass fi**lter with passband from BL(m) kHz to BH(m) kHz.
Therefore the specifications are :-

Passband : 33.2 kHz to 43.2 kHz
Transition Band : 2 kHz on either side of passband
Stopband : 0-31.2 kHz and 45.2-130 kHz (* Sampling rate is 300 kHz)
Tolerance : 0.15 in magnitude for both Passband and Stopband
Passband Nature : Monotonic
Stopband Nature : Monotonic

1.2 Normalized Digital Filter Specifications

Sampling Rate = 300 kHz
In the normalized frequency axis, sampling rate corresponds to 2*pi
Thus, any frequency(Ω) upto 150 kHz(Sampling Rate/2) can be represented on the normalized axis(w) as :-
w = Ω * 2pi/(Ωs(Sampling Rate))

Therefore the corresponding normalized discrete filter specifications are :-

Passband : 0.2213*pi to 0.2880*pi
Transition Band : 0.0133*pi on either side of passband
Stopband : 0-0.2080*pi and 0.3013*pi-1pi

Tolerance : 0.15 in magnitude for both Passband and Stopband
Passband Nature : Monotonic
Stopband Nature : Monotonic

1.3 Discrete filter specifications to Analog filter specs using Bilinear Transformation

The bilinear transformation is given as :- $s = \frac{1-z^{-1}}{1+z^{-1}}$

$\Omega = \tan(\omega/2)$ where Ω = *analog frequency variable* , ω = *discrete frequency variable*

Applying the Bilinear transform to the frequencies at the band-edges, we get :-

ω	Ω
0.2080π	0.3389
0.2213π	0.3624
0.2880π	0.4860
0.3013π	0.5122
0	0
π	∞

Therefore the corresponding analog filter specifications for the same type of analog filter using the bilinear transformation are :-

Passband : 0.3624(ω_{P1}) to 0.4860(ω_{P2})
Transition Band : 0.3389 to 0.3624 & 0.4860 to 0.5122
Stopband : 0 to 0.3389(ω_{S1}) and 0.5122(ω_{S2}) to ∞
Tolerance : 0.15 in magnitude for both Passband and Stopband
Passband Nature : Monotonic
Stopband Nature : Monotonic

1.4 Frequency Transformation & Relevant Parameters

We need to transform a Bandpass analog filter to a Low pass analog filter. We require two parameters in such a case. We can make use of the Bandpass transformation which is given as

$$\Omega_L = \frac{\Omega^2 - \Omega_0{}^2}{B\Omega}$$

The two parameters of interest in the above equation are B and Ω_0. They can be determined using some constraints and specifications of the bandpass analog filter and the parameters come out to be as follows -

$\Omega_0 = \sqrt{\Omega_{p1}\Omega_{p2}}$ = sqrt(0.3624 * 0.4860) = 0.4197
$B = \Omega_{P2} - \Omega_{P1}$ = 0.4860 - 0.3624 = 0.1236

Ω	Ω_L
0^+	$-\infty$
0.3389(Ω_{s1})	-1.4631(Ω_{Ls1})
0.3624(Ω_{p1})	-1.000(Ω_{Lp1})
0.4197(Ω_0)	0
0.4860(Ω_{p2})	1.000(Ω_{Lp2})
0.5122(Ω_{s2})	1.3614(Ω_{Ls2})
∞	∞

1.5 Frequency Transformed low pass Analog Filter specifications

Passband Edge : 1 (Ω_{LP})
Stopband Edge : min($-\Omega_{LS1}$,Ω_{LS2}) = 1.3614
Tolerance : 0.15 in magnitude for both Passband and Stopband
Passband Nature : Monotonic
Stopband Nature : Monotonic

1.6 Analog low pass Transfer Function

We need an Analog Filter which has monotonic passband and a monotonic stopband. Therefore we need to design using the Butterworth approximation. Since the tolerance(δ) in both passband and stopband is 0.15, we define two new quantities in the following way :-

$$D1 = 1/(1-\delta^2) - 1 = 1/0.85^2 - 1 = 0.3841$$
$$D2 = 1/\delta^2 - 1 = 1/0.15^2 - 1 = 43.44$$

Now using the inequality on the order N of the filter for the Butterworth Approximation we get :-

$$N_{min} = \lceil \frac{\log \sqrt{\frac{D_2}{D_1}}}{\log \frac{\Omega_S}{\Omega_P}} \rceil$$

Nmin = 8

The cutoff frequency(c) of the Analog LPF should satisfy the following constraint :-

$$\frac{\Omega_P}{D_1^{\frac{1}{2N}}} \leq \Omega_c \leq \frac{\Omega_S}{D_2^{\frac{1}{2N}}}$$

1.0616 <= Ω_c <= 1.0755

We can free to choose Omega_c to be any value in the range [1.0616, 1.0755]

Let's choose $\Omega_c = 1.069$

Now putting the denominator equal to zero we get the poles of the butterworth filter,

$$1 + \left(\frac{s}{j\omega_c}\right)^{2N} = 1 + \left(\frac{s}{j1.069}\right)^{16} = 0$$

The poles (left half plane) obtained by solving the above equation are as follows:

P1 = -0.5939 + 0.8888i

P2 = -0.8888 + 0.5939i

P3 = -1.0485 + 0.2086i

P4 = -1.0485 - 0.2086i

P5 = -0.8888 - 0.5939i

P6 = -0.5939 - 0.8888i

P7 = -0.2086 - 1.0485i

P8 = -0.2086 + 1.0485i

Using the above poles which are in the left half plane we can write the Analog Low Pass Transfer Function as :-

$$H_{analog,\,LPF}(s_L) = \frac{\Omega_c{}^N}{(S_L - p_1)(S_L - p_2)(S_L - p_3)(S_L - p_4)(S_L - p_5)(S_L - p_6)(S_L - p_7)(S_L - p_8)}$$

$H_{analog,LPF}(s_L)$ =

```
                                        1.705
   ------------------------------------------------------------------------------------
   s^8 + 5.48 s^7 + 15.01 s^6 + 26.69 s^5 + 33.55 s^4 + 30.5 s^3 + 19.6 s^2 + 8.177 s + 1.705
```

1.7 Analog Bandpass Transfer Function

The transformation from *bandpass to lowpass* is given by :

$$s_L = (s^2 + \Omega_0^2)/Bs$$

Substituting the values of the parameters B(0.1236) and Ω_0(0.4197), we get :-

$$s_L = (s^2 + 0.1761)/ 0.1236*s$$

Substituting the value of s_L into $H_{analog,LPF}$ (s_L) we get $H_{analog,BPF}$ (s) as :-

$H_{analog,BPF}(s)$ =

Denominator coefficients of $H_{analog,BPF}(s)$

degree	s^16	s^15	s^14	S^13	S^12	s^11	s^10
coefficient	1	0.6774	1.638	0.8855	1.119	0.4865	0.4183

degree	s^9	s^8	s^7	S^6	S^5	s^4	s^3
coefficient	0.1456	0.0939	0.02565	0.0129	0.00265	0.00107	0.000150

degree	s^2	s^1	s^0
coefficient	4.89e-05	3.561e-06	9.258e-07

Denominator

Numerator coefficients of $H_{analog ,BPF}(s)$

degree	s^8
coefficient	9.298e-08

Numerator

1.8 Discrete Time Filter Transfer Function

To transform the analog domain transfer function into the discrete domain, we need to make use of the bilinear transformation which is of the form :

$$s = \frac{az + b}{cz + d}$$

One such transformation can be given by using below transformation where
a = 1 , b = -1 , c = 1 , d = 1

$$s = \frac{1-z^{-1}}{1+z^{-1}}$$

Using above equation we get $H_{discrete;BPF}(z)$ from $H_{analog;BPF}(s)$ as :-

$H_{discrete,BPF} =$

```
1.429e-08 s^16 - 1.143e-07 s^14 + 4.001e-07 s^12 - 8.002e-07 s^10 + 1e-06 s^8 - 8.002e-07 s^6 + 4.001e-07 s^4 - 1.143e-07 s^2
                                                                                                                  + 1.429e-08
------------------------------------------------------------------------------------------------------------------------------
  s^16 - 10.4 s^15 + 54.25 s^14 - 186 s^13 + 466.2 s^12 - 902.7 s^11 + 1393 s^10 - 1745 s^9 + 1791 s^8 - 1511 s^7
                                     + 1045 s^6 - 586.4 s^5 + 262.3 s^4 - 90.61 s^3 + 22.89 s^2 - 3.803 s + 0.3166
```

[Replace ‘s’ by ‘z’ to make it z-transform]**

1.9 Realization using Direct Form II

Direct Form II is obtained by treating the transfer function H(z) = N(z)/D(z) as a cascade of 1/D(z) followed by N(z). The intermediate signal formed is the one whose samples get stored in the buffer. Thus it is advantageous in comparison to Direct Form I since it saves memory space.

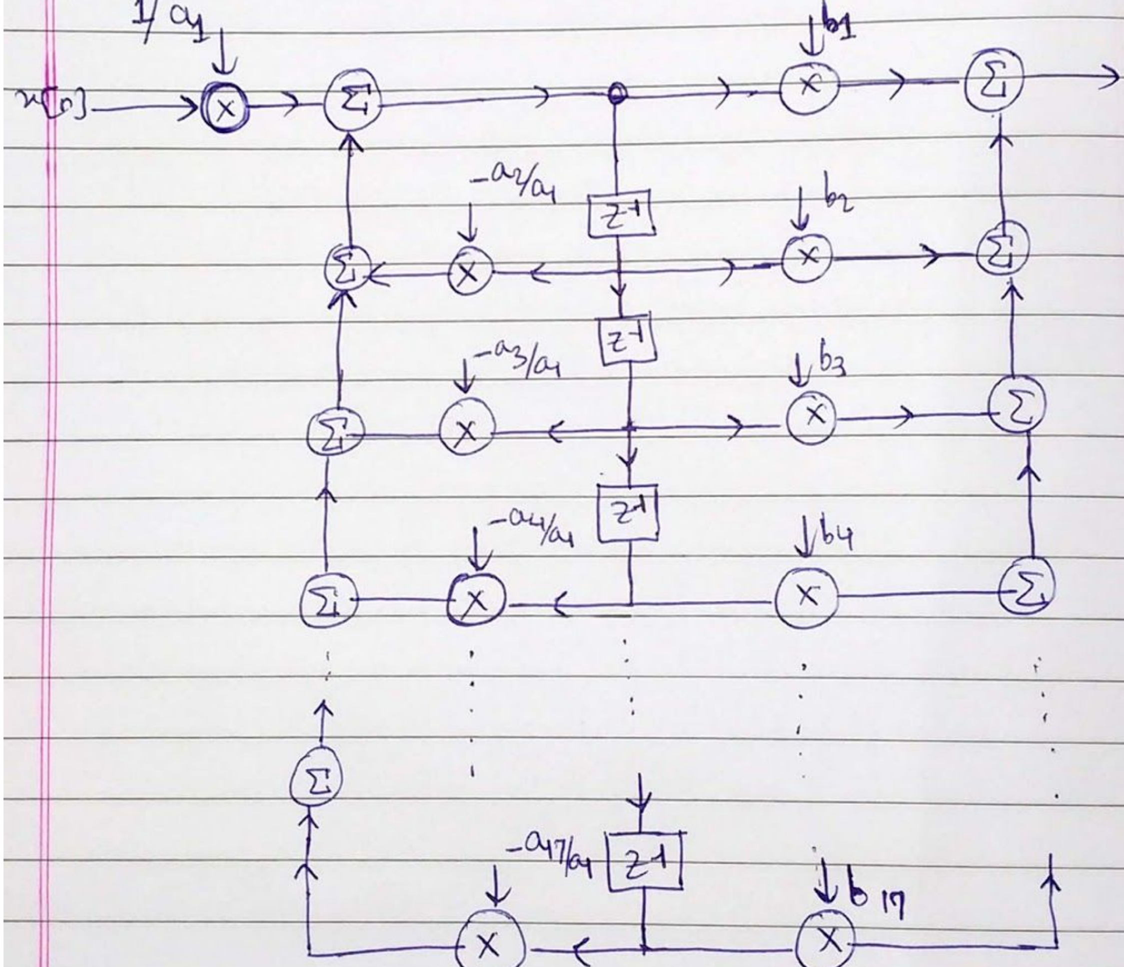
Realization using Direct Form II of Discrete Band Pass filter

$$H_{discrete,BPF}(z) = \frac{b_1 z^{16} + b_2 z^{15} + b_3 z^{14} + \dots + b_{16} z + b_{17}}{a_1 z^{16} + a_2 z^{15} + a_3 z^{14} + \dots + a_{16} z + a_{17}}$$

$$\frac{Y(z)}{X(z)} = \frac{b_1 + b_2 z^{-1} + b_3 z^{-2} + \dots + b_{16} z^{-15} + b_{17} z^{-16}}{a_1 + a_2 z^{-1} + a_3 z^{-2} + \dots + a_{16} z^{-15} + a_{17} z^{-16}}$$

by simplification and taking inverse z-transform.

$$a_1 y[n] + a_2 y[n-1] + a_3 y[n-2] + \dots + a_{16} y[n-15] + a_{17} y[n-16]$$
$$= b_1 x[n] + b_2 x[n-1] + b_3 x[n-2] + \dots + b_{16} x[n-15] + b_{17} x[n-16]$$



2 - BandPass FIR Filter using Kaiser Window

The tolerance in both the stopband and passband is given to be 0.15.
Therefore δ = 0.15 and we get the minimum stopband attenuation to be :-

A = -20 log(0.15) = 16.4782 dB

Since A < 21, we get β to be 0 where β is the shape parameter of Kaiser window.
Now to estimate the window length required, we use the empirical formula for the lower bound on the window length.

$$N \;>=\; \frac{0.5*(A-8)}{2.285*\Delta\omega_T}$$

Here $\Delta\omega_T$ is the minimum transition width. In our case, the transition width is the same on either side of the passband.

$$\Delta\omega_T \;=\; \frac{2\,kHz*2\pi}{samp\,freq} \;=\; 0.0419$$

$$N \;>=\; 45$$

The above equation gives a loose bound on the window length when the tolerance is not very stringent . On successive trials in MATLAB,it was found that a window length of **45** is required to satisfy the required constraints. Also, since is 0, the window is actually a rectangular window. The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. For generating the ideal impulse response a separate function was made to generate the impulse response of Low-Pass filter. It took the Cutoff value and the number of samples as input argument. The bandpass impulse response samples were generated as the difference between two low-pass filters as done in class.

2.1 Time domain sequence of $h_{FIR}[n]$

Columns 1 through 10

0.0018 0.0115 0.0143 0.0083 -0.0026 -0.0115 -0.0129 -0.0066 0.0028 0.0090

Columns 11 through 20

0.0088 0.0036 -0.0018 -0.0037 -0.0020 0.0000 -0.0008 -0.0043 -0.0067 -0.0037

Columns 21 through 30

0.0052 0.0146 0.0167 0.0068 -0.0113 -0.0264 -0.0268 -0.0088 0.0187 0.0385

Columns 31 through 40

0.0358 0.0092 -0.0268 -0.0497 -0.0429 -0.0081 0.0348 0.0587 0.0472 0.0055

Columns 41 through 50

-0.0416 -0.0646 -0.0484 -0.0019 0.0464 0 0.0464 -0.0019 -0.0484 -0.0646

Columns 51 through 60

-0.0416 0.0055 0.0472 0.0587 0.0348 -0.0081 -0.0429 -0.0497 -0.0268 0.0092

Columns 61 through 70

0.0358 0.0385 0.0187 -0.0088 -0.0268 -0.0264 -0.0113 0.0068 0.0167 0.0146

Columns 71 through 80

0.0052 -0.0037 -0.0067 -0.0043 -0.0008 0.0000 -0.0020 -0.0037 -0.0018 0.0036

Columns 81 through 90

0.0088 0.0090 0.0028 -0.0066 -0.0129 -0.0115 -0.0026 0.0083 0.0143 0.0115

Column 91
0.0018

3 - BandStop-Chebyshev Filter

3.1 Un-normalized Discrete Time Filter specifications

Filter Number = 36
Since filter number is < 36 , m = 36 and passband will be equiripple and stopband will be monotonic
q(m) = 3
r(m) = m - 10*q(m) = 36 - 10*3 = 6
BL(m) = 5 + 1.2 q(m) + 2.5 r(m) = 5+1.2*3+2.5*6 = 23.6
BH(m) = BL(m) + 6 = 29.6

The second filter is given to be a Band-Stop filter with stopband from BL(m) kHz to BH(m) kHz. Therefore the specifications are :-

Stopband : 23.6 kHz to 29.6 kHz
Transition Band : 2 kHz on either side of stopband
Passband : 0-21.6 kHz and 31.6 - 90 kHz (* Sampling rate is 200 kHz)
Tolerance : 0.15 in magnitude for both Passband and Stopband
Passband Nature : Equiripple
Stopband Nature : Monotonic

3.2 Normalized Digital Filter specifications

Sampling Rate = 200 kHz
In the normalized frequency axis, sampling rate corresponds to 2*pi
Thus, any frequency(Ω) upto 100 kHz(Sampling Rate/2) can be represented on the normalized axis(ω) as :-

ω = Ω * 2pi/(Ωs(Sampling Rate))

Therefore the corresponding normalized discrete filter specifications are :-
Stopband : 0.7414 to 0.9299
Transition Band : 0.02pi on either side of passband
Passband : 0 - 0.6786 and 0.9927 - pi
Tolerance : 0.15 in magnitude for both Passband and Stopband
Passband Nature : Equiripple
Stopband Nature : Monotonic

3.3 Analog Filter specifications for Band-stop analog filter using Bilinear Transformation

The bilinear transformation is given as :- $s = \frac{1-z^{-1}}{1+z^{-1}}$ => z = $e^{j\omega}$ => Ω = tan(ω/2)
Applying the Bilinear transform to the frequencies at the band-edges, we get :-

ω	Ω
0.6786	0.3529
0.7414	0.3887
0.9299	0.5016
0.9927	0.5416

0	0
π	∞

Therefore the corresponding analog filter specifications for the same type of analog lter using the bilinear transformation are :-

Stopband : 0.3887(ω_{s1}) to 0.5016(ω_{s2})
Passband : 0 to 0.3529(ω_{p1}) and 0.5416(ω_{p2}) to ∞
Tolerance : 0.15 in magnitude for both Passband and Stopband
Passband Nature : Equiripple
Stopband Nature : Monotonic

3.4 Frequency Transformation & Relevant Parameters

We need to transform a Band-Stop analog lter to a Low Pass analog Filter. We require two parameters in such a case. We can make use of the Bandstop transformation which is given as

$$\omega_L = B\Omega/(\Omega_0^2 - \Omega^2)$$

The two parameters in the above equation are B and Ω_0. They can be determined using the specifications of the bandpass analog filter using the following relations :-

$\Omega_0 = \sqrt{\Omega_{p1}\Omega_{p2}}$ = sqrt(0.3529*0.5416) = 0.4372
$B = \Omega_{P2} - \Omega_{P1}$ = 0.5416 - 0.3529 = 0.1887

Ω	Ω_L
0^+	0^+
0.3529(Ω_{P1})	1.00(Ω_{LP1})
0.3887(Ω_{S1})	1.8294(Ω_{LS1})
0.4372(Ω_0^-)	∞
0.4372(Ω_0^-)	-∞
0.5016(Ω_{S2})	-1.5646(Ω_{LS2})
0.5416(Ω_{P2})	-1.0(Ω_{LP2})
∞	0^-

3.5 Frequency Transformed Low pass Analog Filter specifications

Passband Edge : 1 (Ω_{LP})
Stopband Edge : min(Ω_{LS1} ,-Ω_{LS2}) = 1.5646 (Ω_{LS})
Tolerance : 0.15 in magnitude for both Passband and Stopband
Passband Nature : Equiripple
Stopband Nature : Monotonic

3.6 Analog Low pass Transfer Function

We need an Analog Filter which has an equiripple passband and a monotonic stopband. Therefore we need to design using the Chebyshev approximation. Since the tolerance(δ) in both passband and stopband is 0.15, we define two new quantities in the following way :-

D1 = $1/(1-\delta^2) - 1$ = $1/0.85^2 - 1 = 0.3841$
D2 = $1/\delta^2 - 1 = 1/0.15^2 - 1 = 43.44$

Now choosing the parameter ∈ of the Chebyshev FIlter to be $\sqrt{D1}$, we get the minimum value of N as :-

$$N_{min} = \lceil \frac{cosh^{-1}(\sqrt{\frac{D_2}{D_1}})}{cosh^{-1}(\frac{\Omega_{L_S}}{\Omega_{L_P}})} \rceil$$

Nmin = 4

Now, the poles of the transfer function can be obtained by solving the equation :-

$$1 + D_1 cosh^2(N_{min} cosh^{-1}(\frac{s}{j})) = 1 + 0.3841 cosh^2(4 cosh^{-1}(\frac{s}{j})) = 0$$

In order to get a stable Analog LPF, we must include the poles lying in the Left Half Plane in the Transfer Function (The poles are symmetric about origin and we can pick one from each pair to be a part of our Transfer Function).

P1 = -0.2949 + 0.4017i
P2 =   -0.2949 - 0.4017i
P3 = -0.1222 - 0.9698i
P4 = -0.1222 + 0.9698i

Using the above poles which are in the left half plane and the fact that N is even we can write the Analog Low Pass Transfer Function as :-

$$H_{analog,LPF}(s_L) = \frac{(-1)^4 p_1 p_2 p_3 p_4}{\sqrt{(1+D_1)}(s_L - p_1)(s_L - p_2)(s_L - p_3)(s_L - p_4)}$$

$H_{analog,LPF}(s_L)$ =

```
              (0.2017-4.129e-17i)
  -----------------------------------------------
  s^4 + 0.8342 s^3 + 1.348 s^2 + 0.6243 s + 0.2373
```

The transformation equation from analog LPF to analog bandstop filter is given by :-

$$s_L = \frac{Bs}{\Omega_0^2 + s^2}$$

Putting the value of S_L in $H_{analog,LPF}(s_L)$, we get

$H_{analog,BPF}(s)$ =

```
(0.85-1.74e-16i) s^8 + (0.6499-1.33e-16i) s^6 + (0.1864-3.815e-17i) s^4 + (0.02375-4.861e-18i) s^2 + (0.001135-2.323e-19i)
---------------------------------------------------------------------------------------------------------------------------
      s^8 + 0.4963 s^7 + 0.9668 s^6 + 0.3082 s^5 + 0.3019 s^4 + 0.05892 s^3 + 0.03533 s^2 + 0.003467 s + 0.001335
```

3.7 Discrete Time Filter Transfer Function

To transform the analog domain transfer function into the discrete domain, we need to make use of the Bilinear Transformation which is given as :-

$$s = (1 - z^{-1})/(1 + z^{-1})$$

Using above equation we get $H_{discrete;BPF}$ (z) from $H_{analog;BPF}$ (s) as :-

$H_{discrete,BSF}(s)$ =

```
(0.5394-1.104e-16i) s^8 - (2.93-5.998e-16i) s^7 + (8.127-1.664e-15i) s^6 - (14.2-2.906e-15i) s^5 + (17.01-3.482e
                  -15i) s^4 - (14.2-2.906e-15i) s^3 + (8.127-1.664e-15i) s^2 - (2.93-5.998e-16i) s + (0.5394-1.104e-16i)
------------------------------------------------------------------------------------------------------------------------------
            s^8 - 4.782 s^7 + 11.7 s^6 - 18.16 s^5 + 19.51 s^4 - 14.75 s^3 + 7.747 s^2 - 2.604 s + 0.4534
```

3.8 Realization using Direct Form II

The realization of Chebyshev bandstop filter will be similar to the one done above for bandpass butterworth filter.

4 - BandStop FIR Filter using Kaiser Window

The tolerance in both the stopband and passband is given to be 0.15.
Therefore δ = 0.15 and we get the minimum stopband attenuation to be :-

A = -20 log(0.15) = 16.4782 dB

Since A < 21, we get β to be 0 where β is the shape parameter of Kaiser window.
Now to estimate the window length required, we use the empirical formula for the lower bound on the window length.

$$N >= \frac{0.5*(A-8)}{2.285*\Delta\omega_T}$$

Here $\Delta\omega_T$ is the minimum transition width. In our case, the transition width is the same on either side of the passband.

$$\Delta\omega_T = \frac{2\ kHz * 2\pi}{samp\ freq} = 0.0628$$

$$N >= 30$$

Time domain sequence of $h_{FIR}[n]$

Columns 1 through 10

-0.0065	-0.0054	0.0018	0.0111	0.0151	0.0082	-0.0073	-0.0213	-0.0227	-0.0074

Columns 11 through 20

0.0162	0.0322	0.0276	0.0026	-0.0275	-0.0419	-0.0285	0.0061	0.0395	0.0485

Columns 21 through 30

0.0248	-0.0173	-0.0501	-0.0506	-0.0169	0.0294	0.0574	0.0477	0.0060	-0.0402

Columns 31 through 40

1.0000	-0.0402	0.0060	0.0477	0.0574	0.0294	-0.0169	-0.0506	-0.0501	-0.0173

Columns 41 through 50

0.0248   0.0485   0.0395   0.0061   -0.0285   -0.0419   -0.0275   0.0026   0.0276   0.0322

Columns 51 through 60

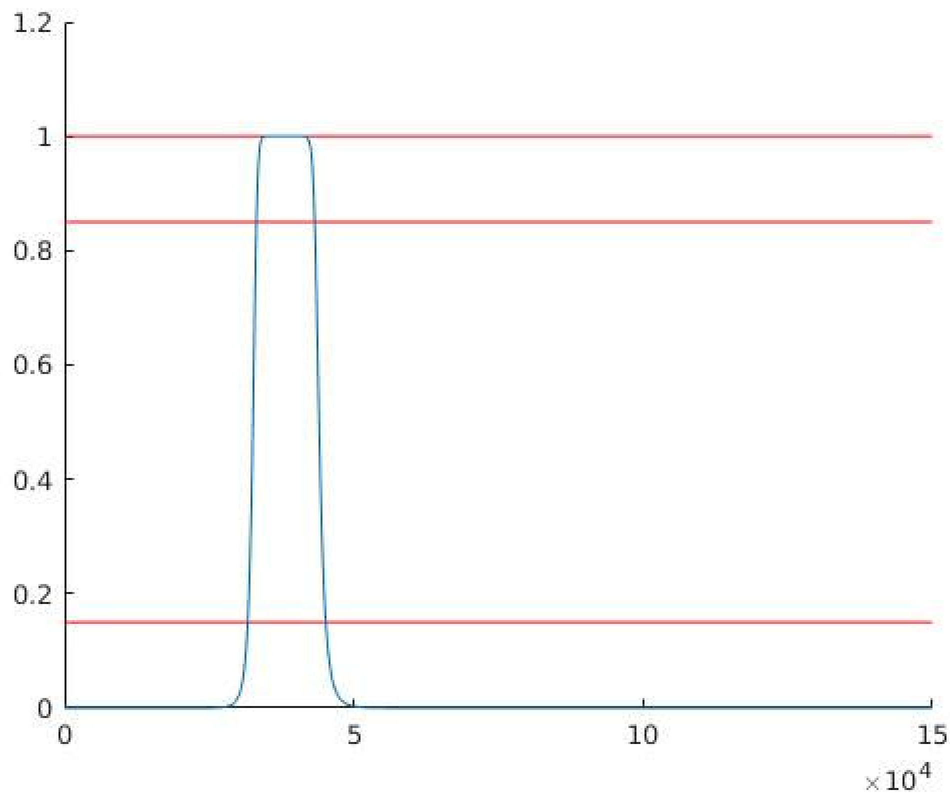
0.0162   -0.0074   -0.0227   -0.0213   -0.0073   0.0082   0.0151   0.0111   0.0018   -0.0054

Column 61

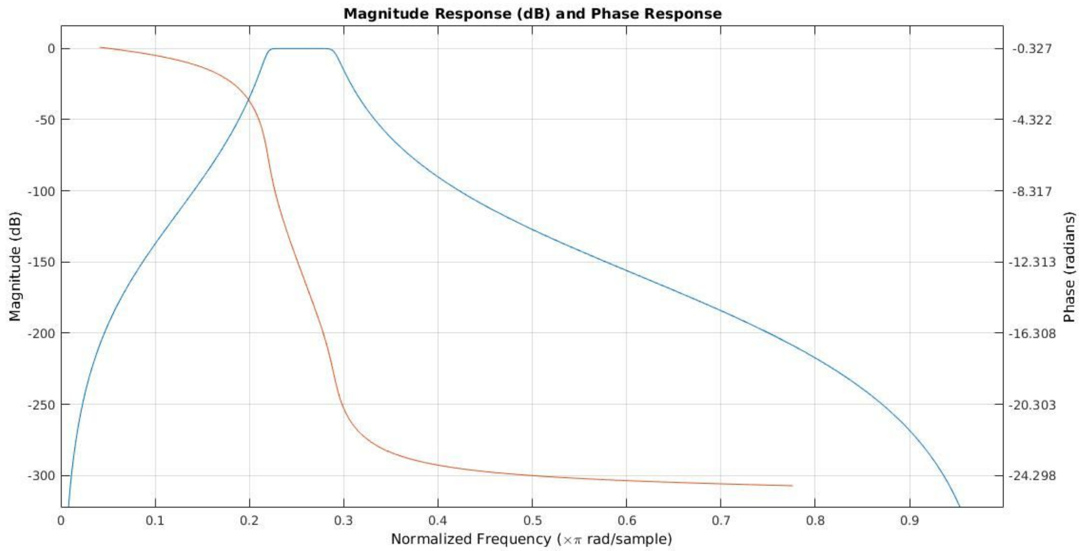
-0.0065

Matlab Plots

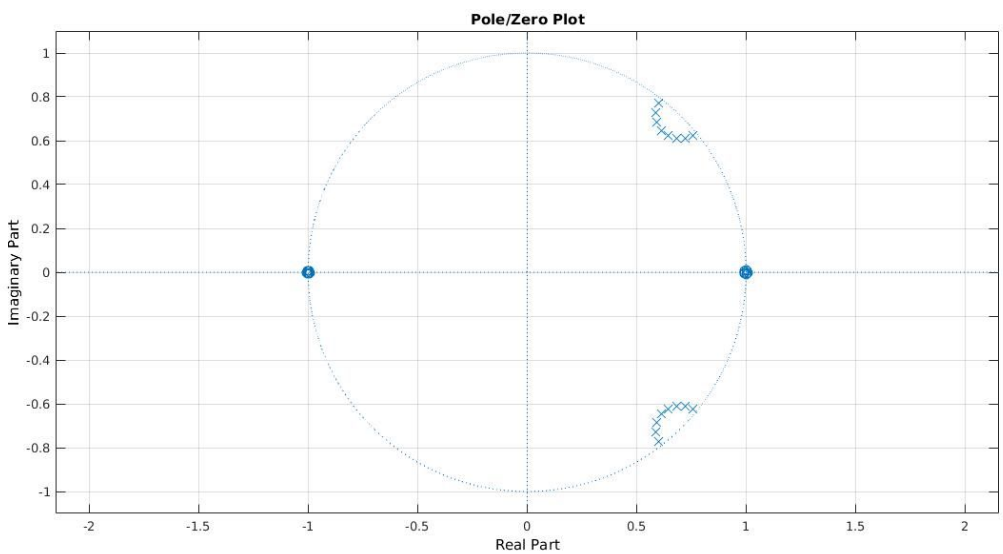
1. Butterworth Bandpass Filter



Magnitude Response

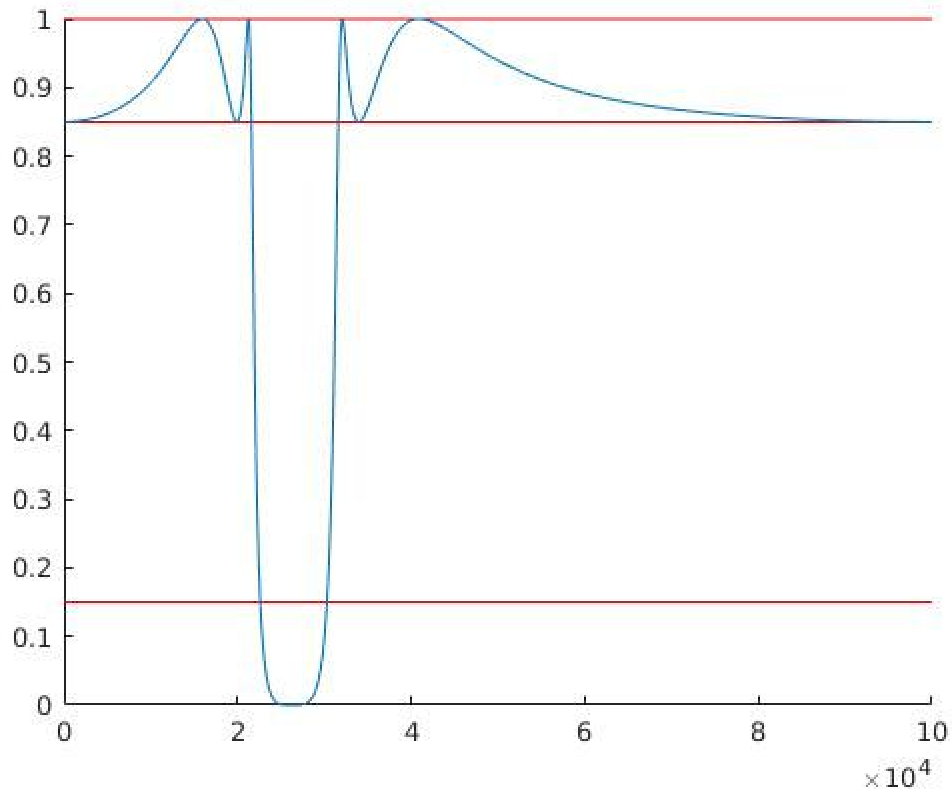


Magnitude(in dB) and Phase Response

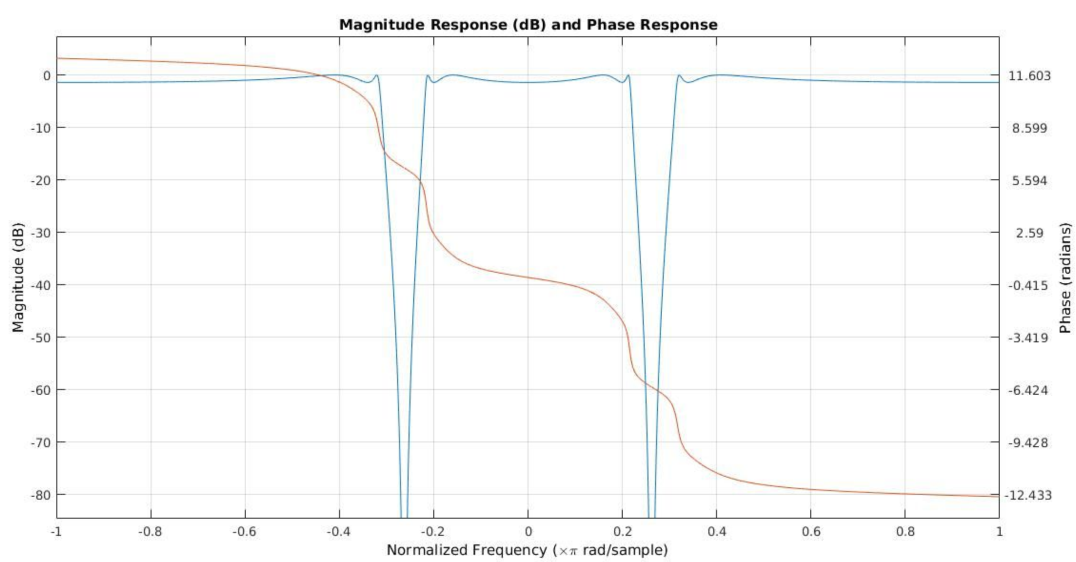


Pole-Zero Plot

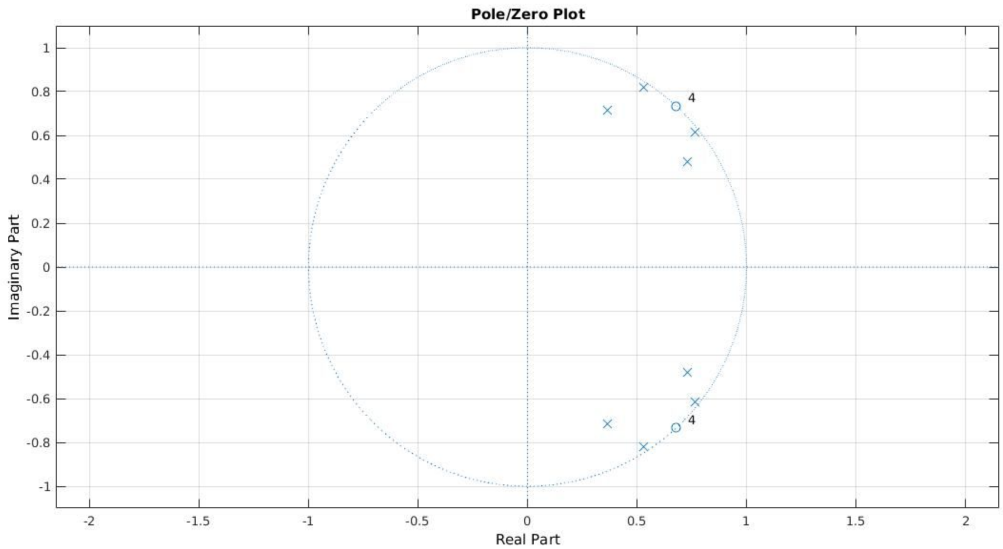
2. Chebyshev Bandstop Filter



Magnitude Response

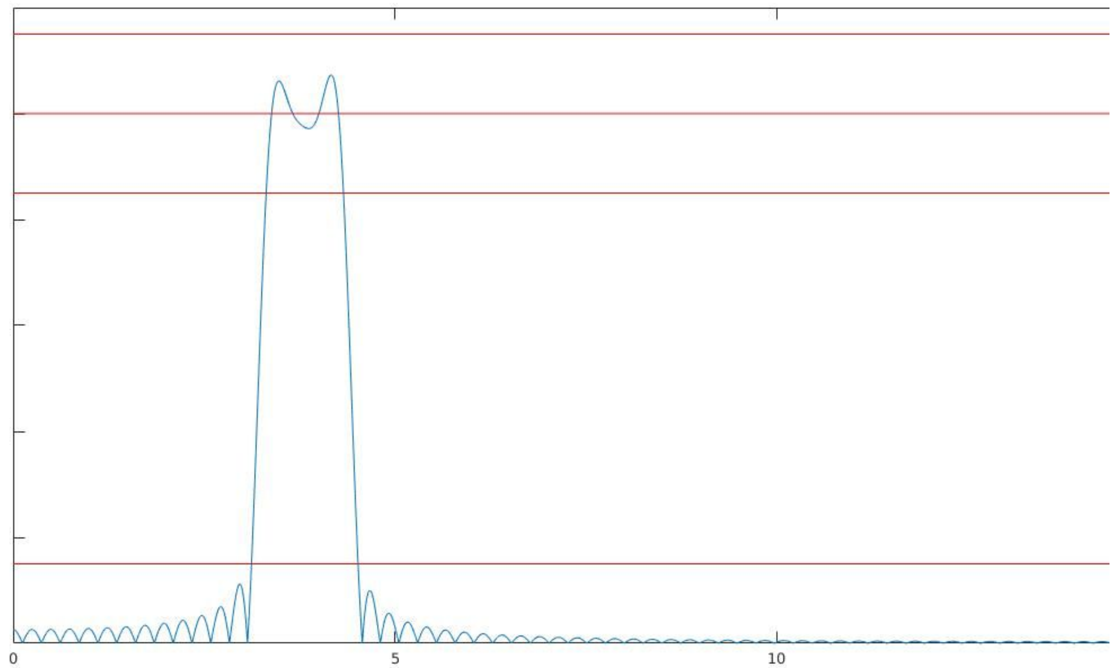


Magnitude(dB) and phase plot

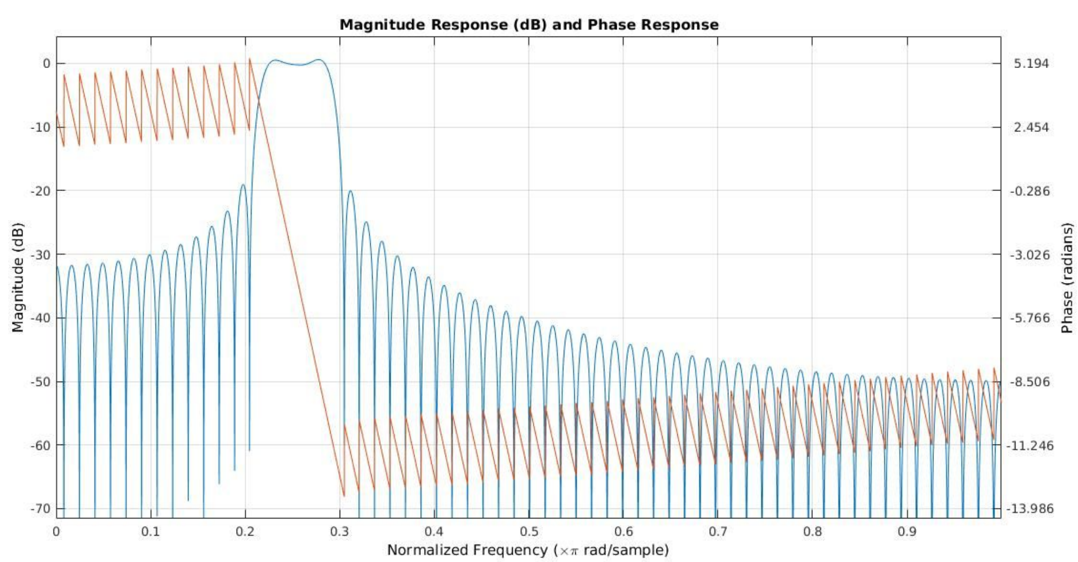


Pole-Zero Plot

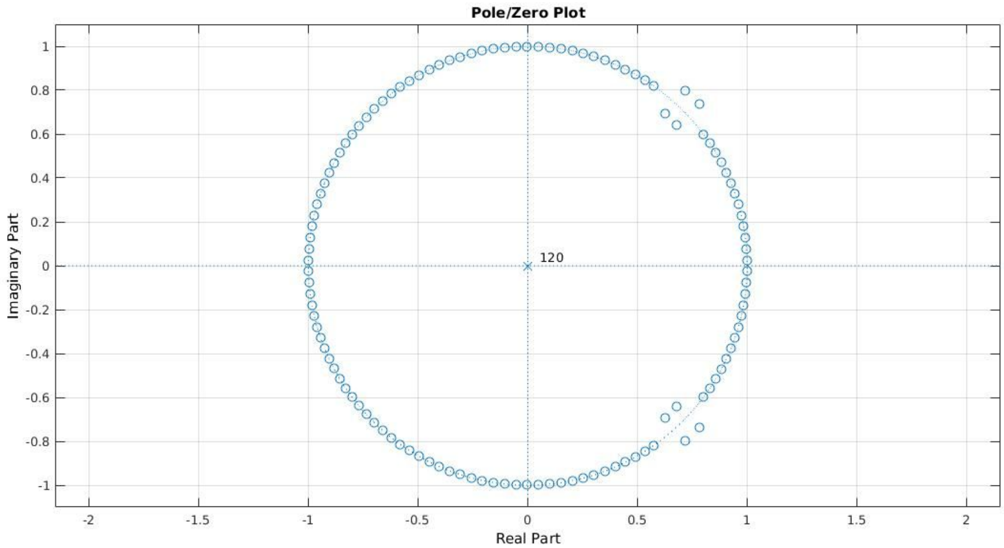
3. FIR - Bandpass Filter



Magnitude Response (with N = 121)

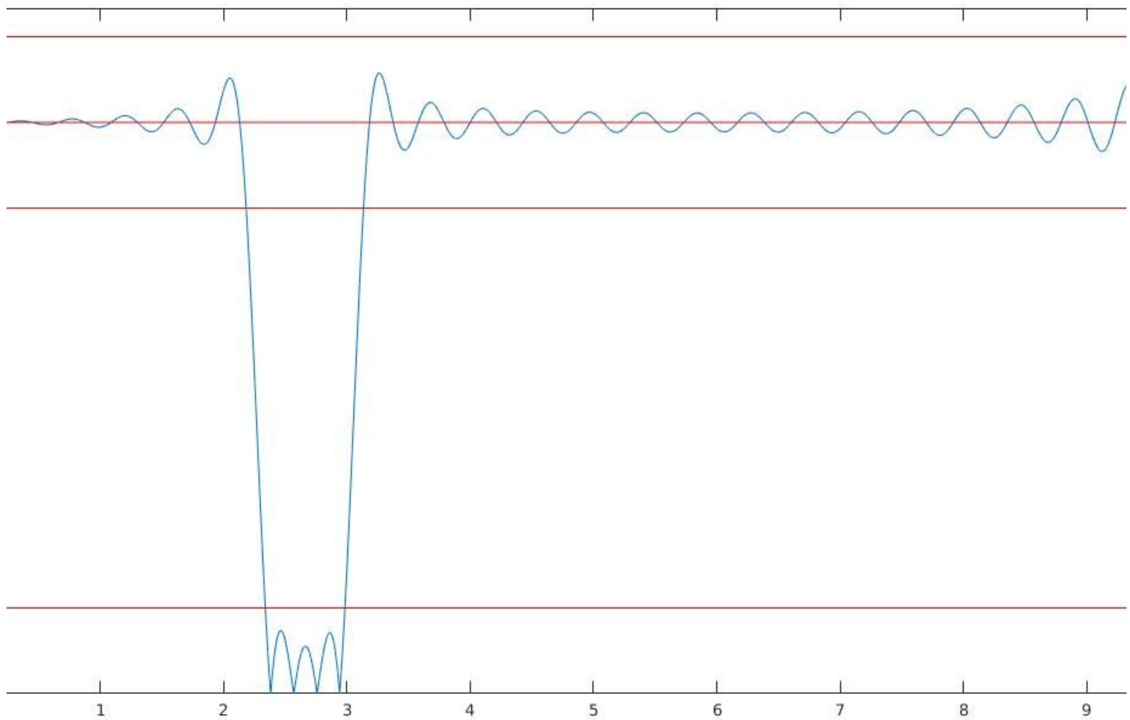


Magnitude (in dB) and Phase Response

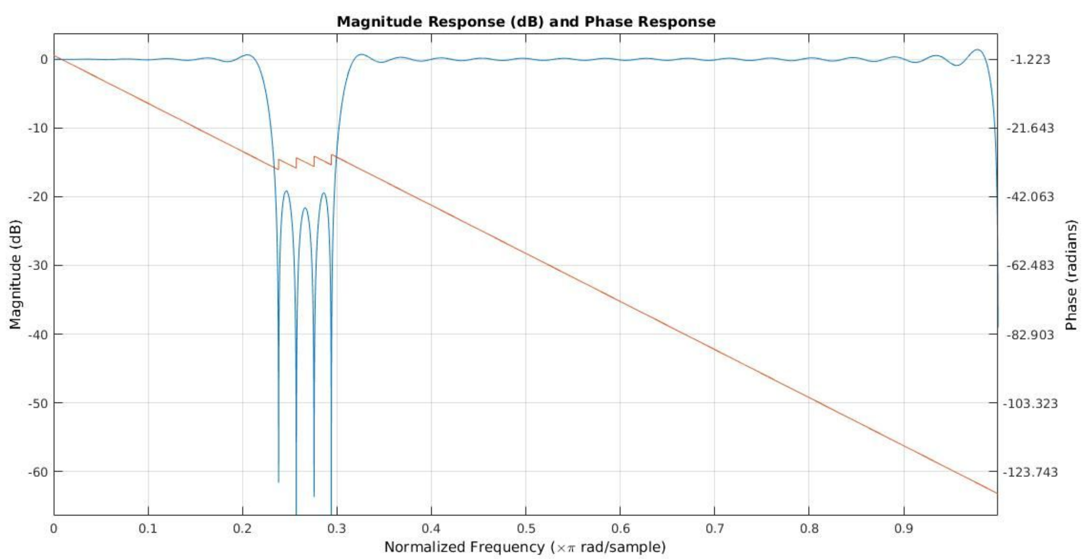


Pole - Zero Plot

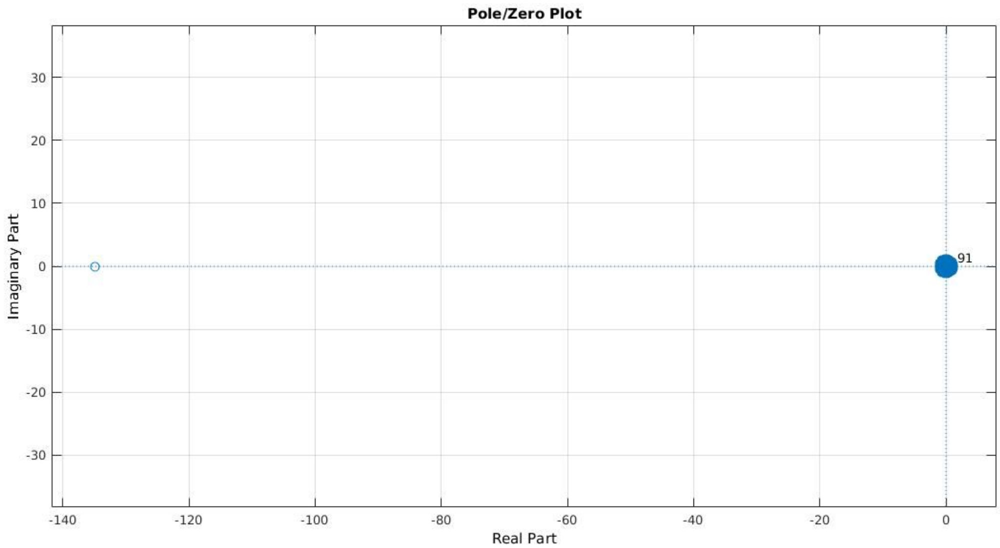
4. BandStop FIR Filter



Magnitude Response (with N = 92)



Magnitude (in dB) and phase plot



Pole Zero Plot