Chapter 2a

Number System

Decimal – 0-9
Hexadecimal 0-9, A,B, C, D, E, F
Octal – 0-7
Binary - 0,1

Decimal	0	1	2	3	4	5	6	7	8	9					
Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E
Binary	0	1													

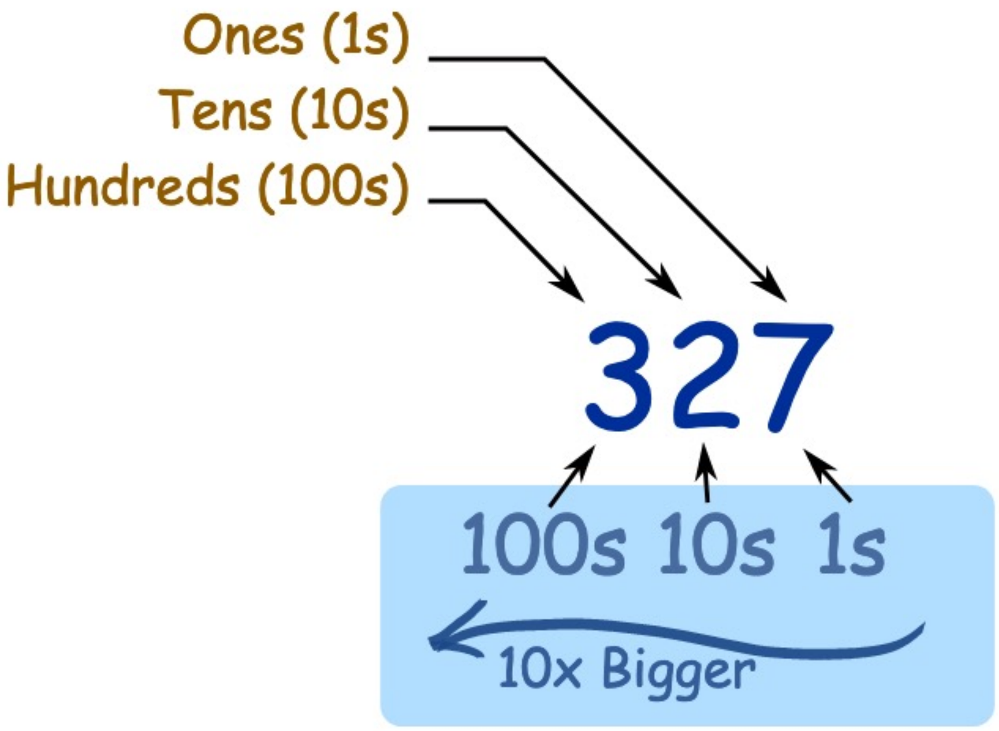
Definition of

Decimal Number System

more ...

The number system we use every day, based on 10 digits (0,1,2,3,4,5,6,7,8,9).

Position is important, with the first position being units, then next on the left being tens, then hundreds and so on.



Hexadecimal

16 Different Values

There are **16** Hexadecimal digits. They are the same as the decimal digits up to 9, but then there are the letters A, B, C, D, E and F in place of the decimal numbers 10 to 15:

Hexadecimal:	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

So a single Hexadecimal digit can show 16 different values instead of the normal 10.

Octal

An Octal Number uses only these 8 digits: 0, 1, 2, 3, 4, 5, 6 and 7

Examples:

- **7** in Octal equals 7 in the Decimal Number System
- **10** in Octal equals 8 in the Decimal Number System
- **11** in Octal equals 9 in the Decimal Number System

...

- **167** in Octal equals 119 in the Decimal Number System

Binary Number

A Binary Number is made up of only **0**s and **1**s.

110100

Example of a Binary Number

There is no 2, 3, 4, 5, 6, 7, 8 or 9 in Binary!

Base 10 (Denary)

0 1 2 3 4 5 6 7 8 9

digits

1000

Decimal System - Poistional Number System

10000	1000	100	10	1
0	0	5	3	7

(5 * 100) + (3 * 10) + (7 * 1)

Base 2 (Binary)

0 1

Electrial Signal – On & Off

bits

11111111

Binary System - Poistional Number System

128	64	32	16	8	4	2	1
0	0	1	1	0	1	0	1

(1 * 32) + (1 * 16) + (1 * 4) + (1 * 1) = 53

Largest Binary – 8 Bits

128	64	32	16	8	4	2	1
1	1	1	1	1	1	1	1

$(1 * 128) + (1 * 64) + (1 * 32) + (1 * 16) + (1 * 8) + (1 * 4) + (1 * 2) + (1 * 1) = 255$

Smallest Binary – 8 Bits

128	64	32	16	8	4	2	1
0	0	0	0	0	0	0	0

Convert the 8 bit binary number 00011010 into denary

128	64	32	16	8	4	2	1
0	0	0	1	1	0	1	0

$16 + 8 + 2 = 26$

$00011010_2 = 26_{10}$

Excerise - 1

00000101	01111111
01111001	10000000
01010100	10101010
01101101	11000011

Excerise - 2

17

44

132

101

Excerise - 3

What is the biggest denary number that you can represent in binary using 8 bits?

How many different denary numbers can you represent in binary using 8 bits?

The ASCII and Unicode Character Sets

List of characters a computer can understand

American Standard Code for Information Interchange (ASCII)

Number	Character		Number	Character	Number	Character	Number	Character
0	*NULL*	*NULL CHARACTER*	32	*SPACE*	64	@	96	`
1	*SOH*	*START OF HEADING*	33	!	65	A	97	a
2	*STX*	*START OF TEXT*	34	"	66	B	98	b
3	*ETX*	*END OF TEXT*	35	#	67	C	99	c
4	*EOT*	*END OF TRANSMISSION*	36	$	68	D	100	d
5	*ENQ*	*ENQUIRY*	37	%	69	E	101	e
6	*ACK*	*ACKNOWLEDGE*	38	&	70	F	102	f
7	*BEL*	*BELL*	39	'	71	G	103	g
8	*BS*	*BACKSPACE*	40	(	72	H	104	h
9	*HT*	*HORIZONTAL TAB*	41	)	73	I	105	i
10	*LF*	*LINE FEED*	42	*	74	J	106	j
11	*VT*	*VERTICAL TAB*	43	+	75	K	107	k
12	*FF*	*FORM FEED*	44	,	76	L	108	l
13	*CR*	*CARRIAGE RETURN*	45	-	77	M	109	m
14	*SO*	*SHIFT OUT*	46	.	78	N	110	n
15	*SI*	*SHIFT IN*	47	/	79	O	111	o
16	*DLE*	*DATALINK ESCAPE*	48	0	80	P	112	p
17	*DC1*	*DEVICE CONTROL 1*	49	1	81	Q	113	q
18	*DC2*	*DEVICE CONTROL 2*	50	2	82	R	114	r
19	*DC3*	*DEVICE CONTROL 3*	51	3	83	S	115	s
20	*DC4*	*DEVICE CONTROL 4*	52	4	84	T	116	t
21	*NAK*	*NEGATIVE ACKNOWLEDGE*	53	5	85	U	117	u
22	*SYN*	*SYNCHRONOUS IDLE*	54	6	86	V	118	v
23	*ETB*	*END OF TRANS, BLOCK*	55	7	87	W	119	w
24	*CAN*	*CANCEL*	56	8	88	X	120	x
25	*EM*	*END OF MEDIUM*	57	9	89	Y	121	y
26	*SUB*	*SUSTITUTE*	58	:	90	Z	122	z
27	*ESC*	*ESCAPE*	59	;	91	[	123	{
28	*FS*	*FILE SEPARATOR*	60	<	92	\	124	|
29	*GS*	*GROUP SEPARATOR*	61	=	93	]	125	}
30	*RS*	*RECORD SEPARATOR*	62	>	94	^	126	~
31	*US*	*UNIT SEPARATOR*	63	?	95	_	127	*DEL*

American Standard Code for Information Interchange (ASCII)

Decimal	Hex	Binary	Character		Decimal	Hex	Binary	Character	Decimal	Hex	Binary	Character	Decimal	Hex	Binary	Character
0	00	0000000	*NULL*	*NULL CHARACTER*	32	20	0100000	*SPACE*	64	40	1000000	@	96	60	1100000	`
1	01	0000001	*SOH*	*START OF HEADING*	33	21	0100001	!	65	41	1000001	A	97	61	1100001	a
2	02	0000010	*STX*	*START OF TEXT*	34	22	0100010	"	66	42	1000010	B	98	62	1100010	b
3	03	0000011	*ETX*	*END OF TEXT*	35	23	0100011	#	67	43	1000011	C	99	63	1100011	c
4	04	0000100	*EOT*	*END OF TRANSMISSION*	36	24	0100100	$	68	44	1000100	D	100	64	1100100	d
5	05	0000101	*ENQ*	*ENQUIRY*	37	25	0100101	%	69	45	1000101	E	101	65	1100101	e
6	06	0000110	*ACK*	*ACKNOWLEDGE*	38	26	0100110	&	70	46	1000110	F	102	66	1100110	f
7	07	0000111	*BEL*	*BELL*	39	27	0100111	'	71	47	1000111	G	103	67	1100111	g
8	08	0001000	*BS*	*BACKSPACE*	40	28	0101000	(	72	48	1001000	H	104	68	1101000	h
9	09	0001001	*HT*	*HORIZONTAL TAB*	41	29	0101001	)	73	49	1001001	I	105	69	1101001	i
10	0A	0001010	*LF*	*LINE FEED*	42	2A	0101010	*	74	4A	1001010	J	106	6A	1101010	j
11	0B	0001011	*VT*	*VERTICAL TAB*	43	2B	0101011	+	75	4B	1001011	K	107	6B	1101011	k
12	0C	0001100	*FF*	*FORM FEED*	44	2C	0101100	,	76	4C	1001100	L	108	6C	1101100	l
13	0D	0001101	*CR*	*CARRIAGE RETURN*	45	2D	0101101	-	77	4D	1001101	M	109	6D	1101101	m
14	0E	0001110	*SO*	*SHIFT OUT*	46	2E	0101110	.	78	4E	1001110	N	110	6E	1101110	n
15	0F	0001111	*SI*	*SHIFT IN*	47	2F	0101111	/	79	4F	1001111	O	111	6F	1101111	o
16	10	0010000	*DLE*	*DATALINK ESCAPE*	48	30	0110000	0	80	50	1010000	P	112	70	1110000	p
17	11	0010001	*DC1*	*DEVICE CONTROL 1*	49	31	0110001	1	81	51	1010001	Q	113	71	1110001	q
18	12	0010010	*DC2*	*DEVICE CONTROL 2*	50	32	0110010	2	82	52	1010010	R	114	72	1110010	r
19	13	0010011	*DC3*	*DEVICE CONTROL 3*	51	33	0110011	3	83	53	1010011	S	115	73	1110011	s
20	14	0010100	*DC4*	*DEVICE CONTROL 4*	52	34	0110100	4	84	54	1010100	T	116	74	1110100	t
21	15	0010101	*NAK*	*NEGATIVE ACKNOWLEDGE*	53	35	0110101	5	85	55	1010101	U	117	75	1110101	u
22	16	0010110	*SYN*	*SYNCHRONOUS IDLE*	54	36	0110110	6	86	56	1010110	V	118	76	1110110	v
23	17	0010111	*ETB*	*END OF TRANS. BLOCK*	55	37	0110111	7	87	57	1010111	W	119	77	1110111	w
24	18	0011000	*CAN*	*CANCEL*	56	38	0111000	8	88	58	1011000	X	120	78	1111000	x
25	19	0011001	*EM*	*END OF MEDIUM*	57	39	0111001	9	89	59	1011001	Y	121	79	1111001	y
26	1A	0011010	*SUB*	*SUSTITUTE*	58	3A	0111010	:	90	5A	1011010	Z	122	7A	1111010	z
27	1B	0011011	*ESC*	*ESCAPE*	59	3B	0111011	;	91	5B	1011011	[	123	7B	1111011	{
28	1C	0011100	*FS*	*FILE SEPARATOR*	60	3C	0111100	<	92	5C	1011100	\	124	7C	1111100	|
29	1D	0011101	*GS*	*GROUP SEPARATOR*	61	3D	0111101	=	93	5D	1011101	]	125	7D	1111101	}
30	1E	0011110	*RS*	*RECORD SEPARATOR*	62	3E	0111110	>	94	5E	1011110	^	126	7E	1111110	~
31	1F	0011111	*US*	*UNIT SEPARATOR*	63	3F	0111111	?	95	5F	1011111	_	127	7F	1111111	*DEL*

American Standard Code for Information Interchange (ASCII)

Decimal	Hex	Binary	Character		Decimal	Hex	Binary	Character	Decimal	Hex	Binary	Character	Decimal	Hex	Binary	Character
0	00	0000000	NULL	NULL CHARACTER	32	20	0100000	SPACE	64	40	1000000	@	96	60	1100000	`
1	01	0000001	SOH	START OF HEADING	33	21	0100001	!	65	41	1000001	A	97	61	1100001	a
2	02	0000010	STX	START OF TEXT	34	22	0100010	"	66	42	1000010	B	98	62	1100010	b
3	03	0000011	ETX	END OF TEXT	35	23	0100011	#	67	43	1000011	C	99	63	1100011	c
4	04	0000100	EOT	END OF TRANSMISSION	36	24	0100100	$	68	44	1000100	D	100	64	1100100	d
5	05	0000101	ENQ	ENQUIRY	37	25	0100101	%	69	45	1000101	E	101	65	1100101	e
6	06	0000110	ACK	ACKNOWLEDGE	38	26	0100110	&	70	46	1000110	F	102	66	1100110	f
7	07	0000111	BEL	BELL						47	1000111	G	103	67	1100111	g
8	08	0001000	BS	BACKSPACE						48	1001000	H	104	68	1101000	h
9	09	0001001	HT	HORIZONTAL TAB							1001001	I	105	69	1101001	i
10	0A	0001010	LF	LINE FEED							1001010	J	106	6A	1101010	j
11	0B	0001011	VT	VERTICAL TAB							1001011	K	107	6B	1101011	k
12	0C	0001100	FF	FORM FEED							1001100	L	108	6C	1101100	l
13	0D	0001101	CR	CARRIAGE RETURN							1001101	M	109	6D	1101101	m
14	0E	0001110	SO	SHIFT OUT							1001110	N	110	6E	1101110	n
15	0F	0001111	SI	SHIFT IN							1001111	O	111	6F	1101111	o
16	10	0010000	DLE	DATALINK ESCAPE						50	1010000	P	112	70	1110000	p
17	11	0010001	DC1	DEVICE CONTROL 1						51	1010001	Q	113	71	1110001	q
18	12	0010010	DC2	DEVICE CONTROL 2	50	32	0110010	2	82	52	1010010	R	114	72	1110010	r
19	13	0010011	DC3	DEVICE CONTROL 3	51	33	0110011	3	83	53	1010011	S	115	73	1110011	s
20	14	0010100	DC4	DEVICE CONTROL 4	52	34	0110100	4	84	54	1010100	T	116	74	1110100	t
21	15	0010101	NAK	NEGATIVE ACKNOWLEDGE	53	35	0110101	5	85	55	1010101	U	117	75	1110101	u
22	16	0010110	SYN	SYNCHRONOUS IDLE	54	36	0110110	6	86	56	1010110	V	118	76	1110110	v
23	17	0010111	ETB	END OF TRANS, BLOCK	55	37	0110111	7	87	57	1010111	W	119	77	1110111	w
24	18	0011000	CAN	CANCEL	56	38	0111000	8	88	58	1011000	X	120	78	1111000	x
25	19	0011001	EM	END OF MEDIUM	57	39	0111001	9	89	59	1011001	Y	121	79	1111001	y
26	1A	0011010	SUB	SUSTITUTE	58	3A	0111010	:	90	5A	1011010	Z	122	7A	1111010	z
27	1B	0011011	ESC	ESCAPE	59	3B	0111011	;	91	5B	1011011	[	123	7B	1111011	{
28	1C	0011100	FS	FILE SEPARATOR	60	3C	0111100	<	92	5C	1011100	\	124	7C	1111100	|
29	1D	0011101	GS	GROUP SEPARATOR	61	3D	0111101	=	93	5D	1011101	]	125	7D	1111101	}
30	1E	0011110	RS	RECORD SEPARATOR	62	3E	0111110	>	94	5E	1011110	^	126	7E	1111110	~
31	1F	0011111	US	UNIT SEPARATOR	63	3F	0111111	?	95	5F	1011111	_	127	7F	1111111	DEL

$$2^7 = 128$$

1 1 1 1 1 1 1

American Standard Code for Information Interchange (ASCII)

Decimal	Hex	Binary	Character		Decimal	Hex	Binary	Character	Decimal	Hex	Binary	Character	Decimal	Hex	Binary	Character
0	00	0000000	NULL	NULL CHARACTER	32	20	0100000	SPACE	64	40	1000000	@	96	60	1100000	`
1	01	0000001	SOH	START OF HEADING	33	21	0100001	!	65	41	1000001	A	97	61	1100001	a
2	02	0000010	STX	START OF TEXT	34	22	0100010	"	66	42	1000010	B	98	62	1100010	b
3	03	0000011	ETX	END OF TEXT	35	23	0100011	#	67	43	1000011	C	99	63	1100011	c
4	04	0000100	EOT	END OF TRANSMISSION	36	24	0100100	$	68	44	1000100	D	100	64	1100100	d
5	05	0000101	ENQ	ENQUIRY	37	25	0100101	%	69	45	1000101	E	101	65	1100101	e
6	06	0000110	ACK	ACKNOWLEDGE	38	26	0100110	&	70	46	1000110	F	102	66	1100110	f
7	07	0000111	BEL	BELL	39	27	0100111	'	71	47	1000111	G	103	67	1100111	g
8	08	0001000	BS	BACKSPACE	40	28	0101000	(	72	48	1001000	H	104	68	1101000	h
9	09	0001001	HT	HORIZONTAL TAB	41	29	0101001	)	73	49	1001001	I	105	69	1101001	i
10	0A	0001010	LF	LINE FEED	42	2A	0101010	*	74	4A	1001010	J	106	6A	1101010	j
11	0B	0001011	VT	VERTICAL TAB	43	2B	0101011	+	75	4B	1001011	K	107	6B	1101011	k
12	0C	0001100	FF	FORM FEED	44	2C	0101100	,	76	4C	1001100	L	108	6C	1101100	l
13	0D	0001101	CR	CARRIAGE RETURN	45	2D	0101101	-	77	4D	1001101	M	109	6D	1101101	m
14	0E	0001110	SO	SHIFT OUT	46	2E	0101110	.	78	4E	1001110	N	110	6E	1101110	n
15	0F	0001111	SI	SHIFT IN	47	2F	0101111	/	79	4F	1001111	O	111	6F	1101111	o
16	10	0010000	DLE	DATALINK ESCAPE	48	30	0110000	0	80	50	1010000	P	112	70	1110000	p
17	11	0010001	DC1	DEVICE CONTROL 1	49	31	0110001	1	81	51	1010001	Q	113	71	1110001	q
18	12	0010010	DC2	DEVICE CONTROL 2	50	32	0110010	2	82	52	1010010	R	114	72	1110010	r
19	13	0010011	DC3	DEVICE CONTROL 3	51	33	0110011	3	83	53	1010011	S	115	73	1110011	s
20	14	0010100	DC4	DEVICE CONTROL 4	52	34	0110100	4	84	54	1010100	T	116	74	1110100	t
21	15	0010101	NAK	NEGATIVE ACKNOWLEDGE	53	35	0110101	5	85	55	1010101	U	117	75	1110101	u
22	16	0010110	SYN	SYNCHRONOUS IDLE	54	36	0110110	6	86	56	1010110	V	118	76	1110110	v
23	17	0010111	ETB	END OF TRANS, BLOCK	55	37	0110111	7	87	57	1010111	W	119	77	1110111	w
24	18	0011000	CAN	CANCEL	56	38	0111000	8	88	58	1011000	X	120	78	1111000	x
25	19	0011001	EM	END OF MEDIUM	57	39	0111001	9	89	59	1011001	Y	121	79	1111001	y
26	1A	0011010	SUB	SUSTITUTE	58	3A	0111010	:	90	5A	1011010	Z	122	7A	1111010	z
27	1B	0011011	ESC	ESCAPE	59	3B	0111011	;	91	5B	1011011	[	123	7B	1111011	{
28	1C	0011100	FS	FILE SEPARATOR	60	3C	0111100	<	92	5C	1011100	\	124	7C	1111100	|
29	1D	0011101	GS	GROUP SEPARATOR	61	3D	0111101	=	93	5D	1011101	]	125	7D	1111101	}
30	1E	0011110	RS	RECORD SEPARATOR	62	3E	0111110	>	94	5E	1011110	^	126	7E	1111110	~
31	1F	0011111	US	UNIT SEPARATOR	63	3F	0111111	?	95	5F	1011111	_	127	7F	1111111	DEL

American Standard Code for Information Interchange (ASCII)

Decimal	Hex	Binary	Character		Decimal	Hex	Binary	Character	Decimal	Hex	Binary	Character	Decimal	Hex	Binary	Character
0	00	0000000	*NULL*	*NULL CHARACTER*	32	20	0100000	*SPACE*	64	40	1000000	@	96	60	1100000	`
1	01	0000001	*SOH*	*START OF HEADING*	33	21	0100001	!	65	41	1000001	A	97	61	1100001	a
2	02	0000010	*STX*	*START OF TEXT*	34	22	0100010	"	66	42	1000010	B	98	62	1100010	b
3	03	0000011	*ETX*	*END OF TEXT*	35	23	0100011	#	67	43	1000011	C	99	63	1100011	c
4	04	0000100	*EOT*	*END OF TRANSMISSION*	36	24	0100100	$	68	44	1000100	D	100	64	1100100	d
5	05	0000101	*ENQ*	*ENQUIRY*	37	25	0100101	%	69	45	1000101	E	101	65	1100101	e
6	06	0000110	*ACK*	*ACKNOWLEDGE*	38	26	0100110	&	70	46	1000110	F	102	66	1100110	f
7	07	0000111	*BEL*	*BELL*	39	27	0100111	'	71	47	1000111	G	103	67	1100111	g
8	08	0001000	*BS*	*BACKSPACE*						48	1001000	H	104	68	1101000	h
9	09	0001001	*HT*	*HORIZONTAL TAB*						49	1001001	I	105	69	1101001	i
10	0A	0001010	*LF*	*LINE FEED*						4A	1001010	J	106	6A	1101010	j
11	0B	0001011	*VT*	*VERTICAL TAB*						4B	1001011	K	107	6B	1101011	k
12	0C	0001100	*FF*	*FORM FEED*						4C	1001100	L	108	6C	1101100	l
13	0D	0001101	*CR*	*CARRIAGE RETURN*						4D	1001101	M	109	6D	1101101	m
14	0E	0001110	*SO*	*SHIFT OUT*						4E	1001110	N	110	6E	1101110	n
15	0F	0001111	*SI*	*SHIFT IN*						4F	1001111	O	111	6F	1101111	o
16	10	0010000	*DLE*	*DATALINK ESCAPE*						50	1010000	P	112	70	1110000	p
17	11	0010001	*DC1*	*DEVICE CONTROL 1*						51	1010001	Q	113	71	1110001	q
18	12	0010010	*DC2*	*DEVICE CONTROL 2*	50	32	0110010	2	82	52	1010010	R	114	72	1110010	r
19	13	0010011	*DC3*	*DEVICE CONTROL 3*	51	33	0110011	3	83	53	1010011	S	115	73	1110011	s
20	14	0010100	*DC4*	*DEVICE CONTROL 4*	52	34	0110100	4	84	54	1010100	T	116	74	1110100	t
21	15	0010101	*NAK*	*NEGATIVE ACKNOWLEDGE*	53	35	0110101	5	85	55	1010101	U	117	75	1110101	u
22	16	0010110	*SYN*	*SYNCHRONOUS IDLE*	54	36	0110110	6	86	56	1010110	V	118	76	1110110	v
23	17	0010111	*ETB*	*END OF TRANS, BLOCK*	55	37	0110111	7	87	57	1010111	W	119	77	1110111	w
24	18	0011000	*CAN*	*CANCEL*	56	38	0111000	8	88	58	1011000	X	120	78	1111000	x
25	19	0011001	*EM*	*END OF MEDIUM*	57	39	0111001	9	89	59	1011001	Y	121	79	1111001	y
26	1A	0011010	*SUB*	*SUSTITUTE*	58	3A	0111010	:	90	5A	1011010	Z	122	7A	1111010	z
27	1B	0011011	*ESC*	*ESCAPE*	59	3B	0111011	;	91	5B	1011011	[	123	7B	1111011	{
28	1C	0011100	*FS*	*FILE SEPARATOR*	60	3C	0111100	<	92	5C	1011100	\	124	7C	1111100	|
29	1D	0011101	*GS*	*GROUP SEPARATOR*	61	3D	0111101	=	93	5D	1011101	]	125	7D	1111101	}
30	1E	0011110	*RS*	*RECORD SEPARATOR*	62	3E	0111110	>	94	5E	1011110	^	126	7E	1111110	~
31	1F	0011111	*US*	*UNIT SEPARATOR*	63	3F	0111111	?	95	5F	1011111	_	127	7F	1111111	*DEL*

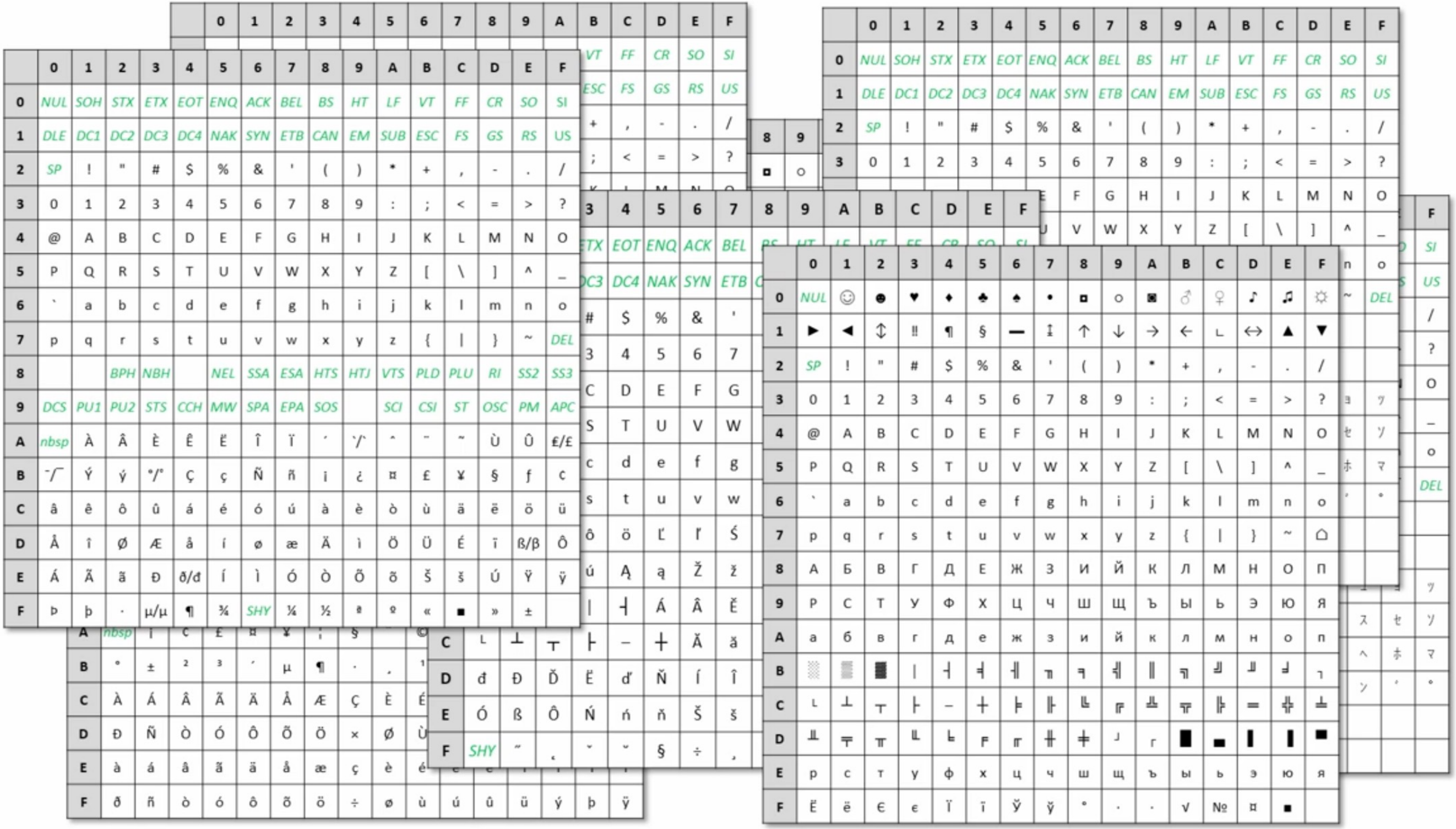
8	4	2	1
1	0	0	1

American Standard Code for Information Interchange (ASCII)

Decimal	Hex	Binary	Character		Decimal	Hex	Binary	Character	Decimal	Hex	Binary	Character	Decimal	Hex	Binary	Character
0	00	0000000	NULL	NULL CHARACTER	32	20	0100000	SPACE	64	40	1000000	@	96	60	1100000	`
1	01	0000001	SOH	START OF HEADING	33	21	0100001	!	65	41	1000001	A	97	61	1100001	a
2	02	0000010	STX	START OF TEXT	34	22	0100010	"	66	42	1000010	B	98	62	1100010	b
3	03	0000011	ETX	END OF TEXT	35	23	0100011	#	67	43	1000011	C	99	63	1100011	c
4	04	0000100	EOT	END OF TRANSMISSION	36	24	0100100	$	68	44	1000100	D	100	64	1100100	d
5	05	0000101	ENQ	ENQUIRY	37	25	0100101	%	69	45	1000101	E	101	65	1100101	e
6	06	0000110	ACK	ACKNOWLEDGE	38	26	0100110	&	70	46	1000110	F	102	66	1100110	f
7	07	0000111	BEL	BELL	39	27	0100111	'	71	47	1000111	G	103	67	1100111	g
8	08	0001000	BS	BACKSPACE	40	28	0101000	(	72	48	1001000	H	104	68	1101000	h
9	09	0001001	HT	HORIZONTAL TAB	41	29	0101001	)	73	49	1001001	I	105	69	1101001	i
10	0A	0001010	LF	LINE FEED	42	2A	0101010	*	74	4A	1001010	J	106	6A	1101010	j
11	0B	0001011	VT	VERTICAL TAB	43	2B	0101011	+	75	4B	1001011	K	107	6B	1101011	k
12	0C	0001100	FF	FORM FEED	44	2C	0101100	,	76	4C	1001100	L	108	6C	1101100	l
13	0D	0001101	CR	CARRIAGE RETURN	45	2D	0101101	-	77	4D	1001101	M	109	6D	1101101	m
14	0E	0001110	SO	SHIFT OUT	46	2E	0101110	.	78	4E	1001110	N	110	6E	1101110	n
15	0F	0001111	SI	SHIFT IN	47	2F	0101111	/	79	4F	1001111	O	111	6F	1101111	o
16	10	0010000	DLE	DATALINK ESCAPE	48	30	0110000	0	80	50	1010000	P	112	70	1110000	p
17	11	0010001	DC1	DEVICE CONTROL 1	49	31	0110001	1	81	51	1010001	Q	113	71	1110001	q
18	12	0010010	DC2	DEVICE CONTROL 2	50	32	0110010	2	82	52	1010010	R	114	72	1110010	r
19	13	0010011	DC3	DEVICE CONTROL 3	51	33	0110011	3	83	53	1010011	S	115	73	1110011	s
20	14	0010100	DC4	DEVICE CONTROL 4	52	34	0110100	4	84	54	1010100	T	116	74	1110100	t
21	15	0010101	NAK	NEGATIVE ACKNOWLEDGE	53	35	0110101	5	85	55	1010101	U	117	75	1110101	u
22	16	0010110	SYN	SYNCHRONOUS IDLE	54	36	0110110	6	86	56	1010110	V	118	76	1110110	v
23	17	0010111	ETB	END OF TRANS, BLOCK	55	37	0110111	7	87	57	1010111	W	119	77	1110111	w
24	18	0011000	CAN	CANCEL	56	38	0111000	8	88	58	1011000	X	120	78	1111000	x
25	19	0011001	EM	END OF MEDIUM	57	39	0111001	9	89	59	1011001	Y	121	79	1111001	y
26	1A	0011010	SUB	SUSTITUTE	58	3A	0111010	:	90	5A	1011010	Z	122	7A	1111010	z
27	1B	0011011	ESC	ESCAPE	59	3B	0111011	;	91	5B	1011011	[	123	7B	1111011	{
28	1C	0011100	FS	FILE SEPARATOR	60	3C	0111100	<	92	5C	1011100	\	124	7C	1111100	|
29	1D	0011101	GS	GROUP SEPARATOR	61	3D	0111101	=	93	5D	1011101	]	125	7D	1111101	}
30	1E	0011110	RS	RECORD SEPARATOR	62	3E	0111110	>	94	5E	1011110	^	126	7E	1111110	~
31	1F	0011111	US	UNIT SEPARATOR	63	3F	0111111	?	95	5F	1011111	_	127	7F	1111111	DEL

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	*NUL*	*SOH*	*STX*	*ETX*	*EOT*	*ENQ*	*ACK*	*BEL*	*BS*	*TAB*	*LF*	*VT*	*FF*	*CR*	*SO*	*SI*
1	*DLE*	*DC1*	*DC2*	*DC3*	*DC4*	*NAK*	*SYN*	*ETB*	*CAN*	*EM*	*SUB*	*ESC*	*FS*	*GS*	*RS*	*US*
2		!	"	#	$	%	&	'	(	)	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	'	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	*DEL*

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	*NUL*	☺	☻	♥	♦	♣	♠	•	◘	○	◙	♂	♀	♪	♫	☼
1	►	◄	↕	‼	¶	§	▬	↨	↑	↓	→	←	∟	↔	▲	▼
2	*SP*	!	"	#	$	%	&	'	(	)	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	⌂
8	Ç	ü	é	â	ä	à	å	ç	ê	ë	è	ï	î	ì	Ä	Å
9	É	æ	Æ	ô	ö	ò	û	ù	ÿ	Ö	Ü	¢	£	¥	₧	ƒ
A	á	í	ó	ú	ñ	Ñ	ª	º	¿	⌐	¬	½	¼	¡	«	»
B	░	▒	▓	│	┤	╡	╢	╖	╕	╣	║	╗	╝	╜	╛	┐
C	└	┴	┬	├	─	┼	╞	╟	╚	╔	╩	╦	╠	═	╬	╧
D	╨	╤	╥	╙	╘	╒	╓	╫	╪	┘	┌	█	▄	▌	▐	▀
E	α	ß	Γ	π	Σ	σ	µ	τ	Φ	Θ	Ω	δ	∞	φ	ε	∩
F	≡	±	≥	≤	⌠	⌡	÷	≈	°	∙	·	√	^{n}	^{2}	■	*nbsp*



After www – unicode standard was published in 1991.

The design goals of Unicode

- A unique code point for every possible character

$2^{16} = 65536$

The design goals of Unicode

- A unique code point for every possible character
- Backwards compatibility with ASCII
- Space efficient

00000000 00000000 00000000 01000001

The design goals of Unicode

- A unique code point for every possible character
- Backwards compatibility with ASCII
- Space efficient
- Allow for efficient data transmission

00000000 00000000 00000000 01000001

Unicode Transformation Format (UTF-8)

A	0	1	0	0	0	0	0	1	U+0041

Control Bit

				1024	512	256	128	64			32	16	8	4	2	1	
Ω	1	1	0	0	1	1	1	0	1	0	1	0	1	0	0	1	U+03A9

Unicode Transformation Format (UTF-8)

A	0	1	0	0	0	0	0	1	U+0041

				1024	512	256	128	64			32	16	8	4	2	1	
Ω	1	1	0	0	1	1	1	0	1	0	1	0	1	0	0	1	U+03A9

					32768	16384	8192	4096			2048	1024	512	256	128	64			32	16	8	4	2	1	
♫	1	1	1	0	0	0	1	0	1	0	0	1	1	0	0	1	1	0	1	0	1	0	1	1	U+266B

For Chinese, Koren + other symbols

Byte 1	Byte 2	Byte 3	Byte 4	Bits available
0XXXXXXX				7
110XXXXX	10XXXXXX			11
1110XXXX	10XXXXXX	10XXXXXX		16
11110XXX	10XXXXXX	10XXXXXX	10XXXXXX	21

Emojis

A	0	1	0	0	0	0	0	1	U+0041

A	1	1	1	0	0	0	0	0	1	0	0	0	[illegible]	0	0	0	0	1	0	0	0	0	0	1	U+0041

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
U+1F30x	🌀	🌁	🌂	🌃	🌄	🌅	🌆	🌇	🌈	🌉	🌊	🌋	🌌	🌍	🌎	🌏
U+1F31x	🌐	🌑	🌒	🌓	🌔	🌕	🌖	🌗	🌘	🌙	🌚	🌛	🌜	🌝	🌞	🌟
U+1F32x	🌠	🌡			🌤	🌥	🌦	🌧	🌨	🌩	🌪	🌫	🌬	🌭	🌮	🌯
U+1F33x	🌰	🌱	🌲	🌳	🌴	🌵	🌶	🌷	🌸	🌹	🌺	🌻	🌼	🌽	🌾	🌿
U+1F34x	🍀	🍁	🍂	🍃	🍄	🍅	🍆	🍇	🍈	🍉	🍊	🍋	🍌	🍍	🍎	🍏
U+1F35x	🍐	🍑	🍒	🍓	🍔	🍕	🍖	🍗	🍘	🍙	🍚	🍛	🍜	🍝	🍞	🍟
U+1F36x	🍠	🍡	🍢	🍣	🍤	🍥	🍦	🍧	🍨	🍩	🍪	🍫	🍬	🍭	🍮	🍯
U+1F37x	🍰	🍱	🍲	🍳	🍴	🍵	🍶	🍷	🍸	🍹	🍺	🍻	🍼	🍽	🍾	🍿
U+1F38x	🎀	🎁	🎂	🎃	🎄	🎅	🎆	🎇	🎈	🎉	🎊	🎋	🎌	🎍	🎎	🎏
U+1F39x	🎐	🎑	🎒	🎓			🎖	🎗		🎙	🎚	🎛			🎞	🎟
U+1F3Ax	🎠	🎡	🎢	🎣	🎤	🎥	🎦	🎧	🎨	🎩	🎪	🎫	🎬	🎭	🎮	🎯
U+1F3Bx	🎰	🎱	🎲	🎳	🎴	🎵	🎶	🎷	🎸	🎹	🎺	🎻	🎼	🎽	🎾	🎿
U+1F3Cx	🏀	🏁	🏂	🏃	🏄	🏅	🏆	🏇	🏈	🏉	🏊	🏋	🏌	🏍	🏎	🏏

Byte 1	Byte 2	Byte 3	Byte 4	Bits available
0XXXXXXX				7
110XXXXX	10XXXXXX			11
1110XXXX	10XXXXXX	10XXXXXX		16
11110XXX	10XXXXXX	10XXXXXX	10XXXXXX	21

More than 1 million possible characters

Summary

- ASCII is a 7 bit encoding system for a limited number of characters
- Extended ASCII resulted in lots of incompatible code pages
- Unicode allows every character in every written language to be encoded
- Unicode is backwards compatible with ASCII
- Unicode is space efficient
- Unicode Transformation Format (UTF-8) uses 1, 2, 3 or 4 bytes
- Unicode is universally supported

Representing Negative Integers in Binary

Two’s Complement

Convert 01011010 into denary

128	64	32	16	8	4	2	1
0	1	0	1	1	0	1	0

$64 + 16 + 8 + 2 = 90$

$01011010_2 = 90_{10}$

Convert 11011010 into denary

-128	64	32	16	8	4	2	1
1	1	0	1	1	0	1	0

$$-128 + 64 + 16 + 8 + 2 = -38$$

$$11011010_2 = -38_{10}$$

$$\begin{array}{rcc} & 9 & 0^{1} \\ - & 3^{1} & 8 \\ \hline & 5 & 2 \\ \hline \end{array}$$

$$\begin{array}{rcc} & 9 & 0 \\ + & -3 & 8 \\ \hline & 5 & 2 \\ \hline \end{array}$$

	0	1	0	1	1	0	1	0	90
+	1	1	0	1	1	0	1	0	-38
1	1		1	1		1			
	0	0	1	1	0	1	0	0	

Convert 00110100 into denary

-128	64	32	16	8	4	2	1
0	0	1	1	0	1	0	0

$32 + 16 + 4 = 52$

$00110100_2 = 52_{10}$

Saima Sattar MS CS 2019: Ok sir

Convert -6 into 8 bit binary

-128	64	32	16	8	4	2	1
1	1	1	1	1	0	1	0

$-128 + 64 + 32 + 16 + 8 + 2 = -6$

$-6_{10} = 11111010_2$

Convert -49 into 8 bit binary

-128	64	32	16	8	4	2	1
1	1	0	0	1	1	1	1

$-128 + 64 + 8 + 4 + 2 + 1 = -49$

$-49_{10} = 11001111_2$

-128	64	32	16	8	4	2	1
1	1	1	1	1	1	1	1

$$-128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 = -1$$

$$11111111_2 = -1_{10}$$

-128	64	32	16	8	4	2	1
0	1	1	1	1	1	1	1

$$64 + 32 + 16 + 8 + 4 + 2 + 1 = 127$$

$$01111111_2 = 127_{10}$$

-128	64	32	16	8	4	2	1
1	0	0	0	0	0	0	0

-128

$10000000_2 = -128_{10}$

0	00000000	32	00100000	64	01000000	96	01100000	128	10000000	160	10100000	192	11000000	224	11100000
1	00000001	33	00100001	65	01000001	97	01100001	129	10000001	161	10100001	193	11000001	225	11100001
2	00000010	34	00100010	66	01000010	98	01100010	130	10000010	162	10100010	194	11000010	226	11100010
3	00000011	35	00100011	67	01000011	99	01100011	131	10000011	163	10100011	195	11000011	227	11100011
4	00000100	36	00100100	68	01000100	100	01100100	132	10000100	164	10100100	196	11000100	228	11100100
5	00000101	37	00100101	69	01000101	101	01100101	133	10000101	165	10100101	197	11000101	229	11100101
6	00000110	38	00100110	70	01000110	102	01100110	134	10000110	166	10100110	198	11000110	230	11100110
7	00000111	39	00100111	71	01000111	103	01100111	135	10000111	167	10100111	199	11000111	231	11100111
8	00001000	40	00101000	72	01001000	104	01101000	136	10001000	168	10101000	200	11001000	232	11101000
9	00001001	41	00101001	73	01001001	105	01101001	137	10001001	169	10101001	201	11001001	233	11101001
10	00001010	42	00101010	74	01001010	106	01101010	138	10001010	170	10101010	202	11001010	234	11101010
11	00001011	43	00101011	75	01001011	107	01101011	139	10001011	171	10101011	203	11001011	235	11101011
12	00001100	44	00101100	76	01001100	108	01101100	140	10001100	172	10101100	204	11001100	236	11101100
13	00001101	45	00101101	77	01001101	109	01101101	141	10001101	173	10101101	205	11001101	237	11101101
14	00001110	46	00101110	78	01001110	110	01101110	142	10001110	174	10101110	206	11001110	238	11101110
15	00001111	47	00101111	79	01001111	111	01101111	143	10001111	175	10101111	207	11001111	239	11101111
16	00010000	48	00110000	80	01010000	112	01110000	144	10010000	176	10110000	208	11010000	240	11110000
17	00010001	49	00110001	81	01010001	113	01110001	145	10010001	177	10110001	209	11010001	241	11110001
18	00010010	50	00110010	82	01010010	114	01110010	146	10010010	178	10110010	210	11010010	242	11110010
19	00010011	51	00110011	83	01010011	115	01110011	147	10010011	179	10110011	211	11010011	243	11110011
20	00010100	52	00110100	84	01010100	116	01110100	148	10010100	180	10110100	212	11010100	244	11110100
21	00010101	53	00110101	85	01010101	117	01110101	149	10010101	181	10110101	213	11010101	245	11110101
22	00010110	54	00110110	86	01010110	118	01110110	150	10010110	182	10110110	214	11010110	246	11110110
23	00010111	55	00110111	87	01010111	119	01110111	151	10010111	183	10110111	215	11010111	247	11110111
24	00011000	56	00111000	88	01011000	120	01111000	152	10011000	184	10111000	216	11011000	248	11111000
25	00011001	57	00111001	89	01011001	121	01111001	153	10011001	185	10111001	217	11011001	249	11111001
26	00011010	58	00111010	90	01011010	122	01111010	154	10011010	186	10111010	218	11011010	250	11111010
27	00011011	59	00111011	91	01011011	123	01111011	155	10011011	187	10111011	219	11011011	251	11111011
28	00011100	60	00111100	92	01011100	124	01111100	156	10011100	188	10111100	220	11011100	252	11111100
29	00011101	61	00111101	93	01011101	125	01111101	157	10011101	189	10111101	221	11011101	253	11111101
30	00011110	62	00111110	94	01011110	126	01111110	158	10011110	190	10111110	222	11011110	254	11111110
31	00011111	63	00111111	95	01011111	127	01111111	159	10011111	191	10111111	223	11011111	255	11111111

0	00000000	32	00100000	64	01000000	96	01100000	-128	10000000	-96	10100000	-64	11000000	-32	11100000
1	00000001	33	00100001	65	01000001	97	01100001	-127	10000001	-95	10100001	-63	11000001	-31	11100001
2	00000010	34	00100010	66	01000010	98	01100010	-126	10000010	-94	10100010	-62	11000010	-30	11100010
3	00000011	35	00100011	67	01000011	99	01100011	-125	10000011	-93	10100011	-61	11000011	-29	11100011
4	00000100	36	00100100	68	01000100	100	01100100	-124	10000100	-92	10100100	-60	11000100	-28	11100100
5	00000101	37	00100101	69	01000101	101	01100101	-123	10000101	-91	10100101	-59	11000101	-27	11100101
6	00000110	38	00100110	70	01000110	102	01100110	-122	10000110	-90	10100110	-58	11000110	-26	11100110
7	00000111	39	00100111	71	01000111	103	01100111	-121	10000111	-89	10100111	-57	11000111	-25	11100111
8	00001000	40	00101000	72	01001000	104	01101000	-120	10001000	-88	10101000	-56	11001000	-24	11101000
9	00001001	41	00101001	73	01001001	105	01101001	-119	10001001	-87	10101001	-55	11001001	-23	11101001
10	00001010	42	00101010	74	01001010	106	01101010	-118	10001010	-86	10101010	-54	11001010	-22	11101010
11	00001011	43	00101011	75	01001011	107	01101011	-117	10001011	-85	10101011	-53	11001011	-21	11101011
12	00001100	44	00101100	76	01001100	108	01101100	-116	10001100	-84	10101100	-52	11001100	-20	11101100
13	00001101	45	00101101	77	01001101	109	01101101	-115	10001101	-83	10101101	-51	11001101	-19	11101101
14	00001110	46	00101110	78	01001110	110	01101110	-114	10001110	-82	10101110	-50	11001110	-18	11101110
15	00001111	47	00101111	79	01001111	111	01101111	-113	10001111	-81	10101111	-49	11001111	-17	11101111
16	00010000	48	00110000	80	01010000	112	01110000	-112	10010000	-80	10110000	-48	11010000	-16	11110000
17	00010001	49	00110001	81	01010001	113	01110001	-111	10010001	-79	10110001	-47	11010001	-15	11110001
18	00010010	50	00110010	82	01010010	114	01110010	-110	10010010	-78	10110010	-46	11010010	-14	11110010
19	00010011	51	00110011	83	01010011	115	01110011	-109	10010011	-77	10110011	-45	11010011	-13	11110011
20	00010100	52	00110100	84	01010100	116	01110100	-108	10010100	-76	10110100	-44	11010100	-12	11110100
21	00010101	53	00110101	85	01010101	117	01110101	-107	10010101	-75	10110101	-43	11010101	-11	11110101
22	00010110	54	00110110	86	01010110	118	01110110	-106	10010110	-74	10110110	-42	11010110	-10	11110110
23	00010111	55	00110111	87	01010111	119	01110111	-105	10010111	-73	10110111	-41	11010111	-9	11110111
24	00011000	56	00111000	88	01011000	120	01111000	-104	10011000	-72	10111000	-40	11011000	-8	11111000
25	00011001	57	00111001	89	01011001	121	01111001	-103	10011001	-71	10111001	-39	11011001	-7	11111001
26	00011010	58	00111010	90	01011010	122	01111010	-102	10011010	-70	10111010	-38	11011010	-6	11111010
27	00011011	59	00111011	91	01011011	123	01111011	-101	10011011	-69	10111011	-37	11011011	-5	11111011
28	00011100	60	00111100	92	01011100	124	01111100	-100	10011100	-68	10111100	-36	11011100	-4	11111100
29	00011101	61	00111101	93	01011101	125	01111101	-99	10011101	-67	10111101	-35	11011101	-3	11111101
30	00011110	62	00111110	94	01011110	126	01111110	-98	10011110	-66	10111110	-34	11011110	-2	11111110
31	00011111	63	00111111	95	01011111	127	01111111	-97	10011111	-65	10111111	-33	11011111	-1	11111111

Excersise -4

10000101	127
11111001	-56
01010110	-86
11101101	-61

Two’s Complement

Convert -6 into 8 bit binary

128	64	32	16	8	4	2	1
0	0	0	0	0	1	1	0
1	1	1	1	1	0	0	1

One’s Compliment

Convert -6 into 8 bit binary

Add 1 to One's Compliment to get its Two's Compliment

1	1	1	1	1	0	0	1
0	0	0	0	0	0	0 $_1$	1
1	1	1	1	1	0	1	0

Convert -6 into 8 bit binary

-128	64	32	16	8	4	2	1
1	1	1	1	1	0	1	0

$-128 + 64 + 32 + 16 + 8 + 2 = -6$

$11111010_2 = -6_{10}$

Convert -109 into 8 bit binary

128	64	32	16	8	4	2	1
0	1	1	0	1	1	0	1

Convert -109 into 8 bit binary

128	64	32	16	8	4	2	1
1	0	0	1	0	0	1	0

Convert -109 into 8 bit binary

1	0	0	1	0	0	1	0
0	0	0	0	0	0	0	1
1	0	0	1	0	0	1	1

Convert -109 into 8 bit binary

1	0	0	1	0	0	1	1

Convert -77 into 8 bit binary

128	64	32	16	8	4	2	1
0	1	0	0	1	1	0	1

Convert -77 into 8 bit binary

128	64	32	16	8	4	2	1
1	0	1	1	0	0	1	0

Convert -77 into 8 bit binary

1	0	1	1	0	0	1	0
0	0	0	0	0	0	0	1
1	0	1	1	0	0	1	1

Convert -77 into 8 bit binary

1	0	1	1	0	0	1	1

Excersise 5: Convert them using Two's Compliment.

-17

-44

-122

Convert -44 into 8 bit binary

128	64	32	16	8	4	2	1
0	0	1	0	1	1	0	0
1	1	0	1	0	1	0	0

$-44_{10} = 11010100_{2}$

Summary

- Computers use two's complement to represent negative numbers in binary
- Half of the available combinations of bits are used to represent negative numbers
- Three methods to convert negative denary numbers into binary

Decimal to Binary

Conversion steps:

1. Divide the number by 2.
2. Get the integer quotient for the next iteration.
3. Get the remainder for the binary digit.
4. Repeat the steps until the quotient is equal to 0.

Example #1

Convert 13_{10} to binary:

Division by 2	Quotient	Remainder	Bit #
13/2	6	1	0
6/2	3	0	1
3/2	1	1	2
1/2	0	1	3

So $13_{10} = 1101_2$

Binary to Decimal

For binary number with n digits:

$$d_{n-1} \ldots d_3\, d_2\, d_1\, d_0$$

The decimal number is equal to the sum of binary digits (d_n) times their power of 2 (2^n):

$$\text{decimal} = d_0 \times 2^0 + d_1 \times 2^1 + d_2 \times 2^2 + \ldots$$

Example

Find the decimal value of 111001_2:

binary number:	1	1	1	0	0	1
power of 2:	2^5	2^4	2^3	2^2	2^1	2^0

$$111001_2 = 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 57_{10}$$

Decimal to Hexadecmial

Conversion steps:

1. Divide the number by 16.
2. Get the integer quotient for the next iteration.
3. Get the remainder for the hex digit.
4. Repeat the steps until the quotient is equal to 0.

Example #1

Convert 7562_{10} to hex:

Division by 16	Quotient (integer)	Remainder (decimal)	Remainder (hex)	Digit #
7562/16	472	10	A	0
472/16	29	8	8	1
29/16	1	13	D	2
1/16	0	1	1	3

So $7562_{10} = 1D8A_{16}$

Representing Real Numbers in Binary

Fixed Point Binary Fractions

Decimal Point in Denary
Binary Point in Binary

-128	64	32	16	8	4	2	1	1/2	1/4	1/8	1/16
0	1	0	0	1	0	0	0	1	1	0	0

$$64 + 8 + 1/2 + 1/4 = 72\ 3/4$$

$$010010001100_2 = 72.75_{10}$$

-128	64	32	16	8	4	2	1	1/2	1/4	1/8	1/16
0	1	1	0	1	0	1	1	1	0	1	0

-128	64	32	16	8	4	2	1	0.5	0.25	0.125	0.0625
0	1	1	0	1	0	1	1	1	0	1	0

-128	64	32	16	8	4	2	1	0.5	0.25	0.125	0.0625
0	1	1	0	1	0	1	1	1	0	1	0

$$64 + 32 + 8 + 2 + 1 + 0.5 + 0.125 = 107.625$$

$$011010111010_2 = 107.625_{10}$$

-128	64	32	16	8	4	2	1	0.5	0.25	0.125	0.0625
1	0	1	0	0	1	1	0	1	1	0	0

$$-128 + 32 + 4 + 2 + 0.5 + 0.25 = -89.25$$

$$101001101100_2 = -89.25_{10}$$

Excersise 6

The following binary numbers are stored using two's complement in a 12 bit register with 4 bits after the binary point. Convert them into decimal fractions.

011111111111

111111111111

000001110010

Excersise 7

Using two's complement, convert the following denary numbers into fixed point binary to be stored in a 12 bit register with 4 bits after the binary point.

27.5

-55.75

-1.75

Given a 4 bit register, with 1 bit before and 3 bits after the binary point, using two's complement, calculate:

- The largest positive number that can be represented

-1	0.5	0.25	0.125
0	1	1	1

0.5 + 0.25 + 0.125 = 0.875

- The smallest positive number that can be represented (not including 0)

-1	0.5	0.25	0.125
0	0	0	1

0.125

- The smallest magnitude negative number that can be represented (closest to 0)

-1	0.5	0.25	0.125
1	1	1	1

-1 + 0.5 + 0.25 + 0.125 = -0.125

- The largest magnitude negative number that can be represented

-1	0.5	0.25	0.125
1	0	0	0

-1

Summary

- Fixed point binary is used in Digital Signal Processing
- Simpler and therefore cheaper processor hardware
- Greatly simplified arithmetic means much faster processing
- Trade off between range and precision
- Some numbers can never be represented accurately

Floating Point Binary

To represent very large values
To represent very small values
To represent values with great accuracy

Standard Scientific Notation

2.99×10^{8}	m/s	Speed of light
6.02×10^{23}	/mol	Avogadro's number
1.60×10^{-19}	C	Charge of electron
4.35×10^{17}	s	Age of the universe

mantissa exponent

$$6.02 \times 10^{23}$$

602000000000000000000000.

Governs the precisiion

Governs the range

$$6.022140857 \times 10^{23}$$

602214085700000000000000.

$$1.60 \times 10^{-19}$$

$$0.000000000000000000160$$

mantissa										exponent					
0	1	1	0	1	0	0	0	0	0	0	0	0	0	1	1

mantissa										exponent					
0	1	1	0	1	0	0	0	0	0	0	0	0	0	1	1

mantissa										exponent					
0	1	1	0	1	0	0	0	0	0	0	0	0	0	1	1

mantissa										exponent					
0	1	1	0	1	0	0	0	0	0	0	0	0	0	1	1

Floating-Point Binary

IEEE Short Real: 32 bits	1 bit for the sign, 8 bits for the exponent, and 23 bits for the mantissa. Also called *single precision*.
IEEE Long Real: 64 bits	1 bit for the sign, 11 bits for the exponent, and 52 bits for the mantissa. Also called *double precision*.

Both formats use essentially the same method for storing floating-point binary numbers, so we will use the Short Real as an example in this tutorial. The bits in an IEEE Short Real are arranged as follows, with the most significant bit (MSB) on the left:

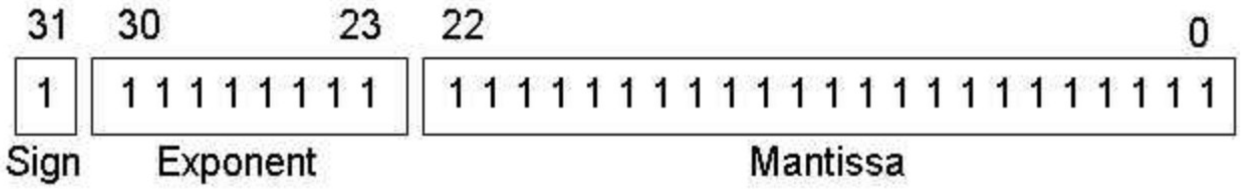
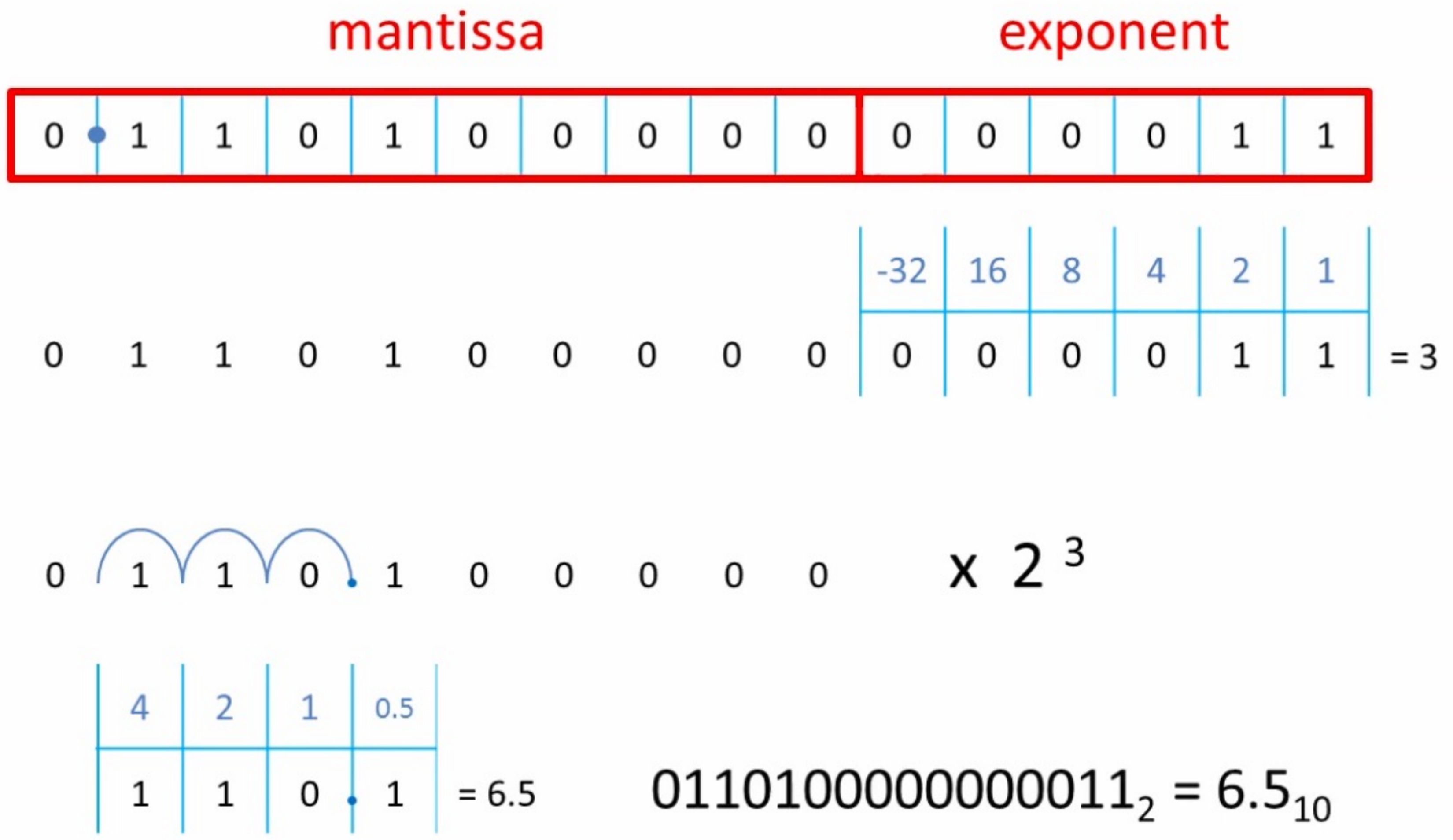


Fig. 1

http://cstl-csm.semo.edu/xzhang/Class%20Folder/CS280/Workbook_HTML/FLOATING_tut.htm

Floating-Point Binary



$0110100000000011_2 = 6.5_{10}$

Floating-Point Binary

mantissa										exponent					
0	1	0	1	0	1	0	0	0	0	0	0	0	0	1	0

0 1 0 1 0 1 0 0 0 0

-32	16	8	4	2	1
0	0	0	0	1	0

= 2

0 1 0 . 1 0 1 0 0 0 0 $\times\ 2^{2}$

2	1	0.5	0.25	0.125
1	0 .	1	0	1

= 2.625

$0101010000000010_2 = 2.625_{10}$

Floating-Point Binary

mantissa										exponent					
0	1	0	0	0	0	0	0	0	0	1	1	1	1	1	0

0 1 0 0 0 0 0 0 0 0

-32	16	8	4	2	1	
1	1	1	1	1	0	= -2

0 . 0 0 1 0 0 0 0 0 0 0 0 $\times\ 2^{-2}$

1	0.5	0.25	0.125	
0 .	0	0	1	= 0.125

$0100000000111110_2 = 0.125_{10}$

Exercise 8

Convert the following floating point binary numbers into denary. Assume 10 bits for the mantissa and 6 bits for the exponent, both in two's complement

0110110000000100

0101000000111111

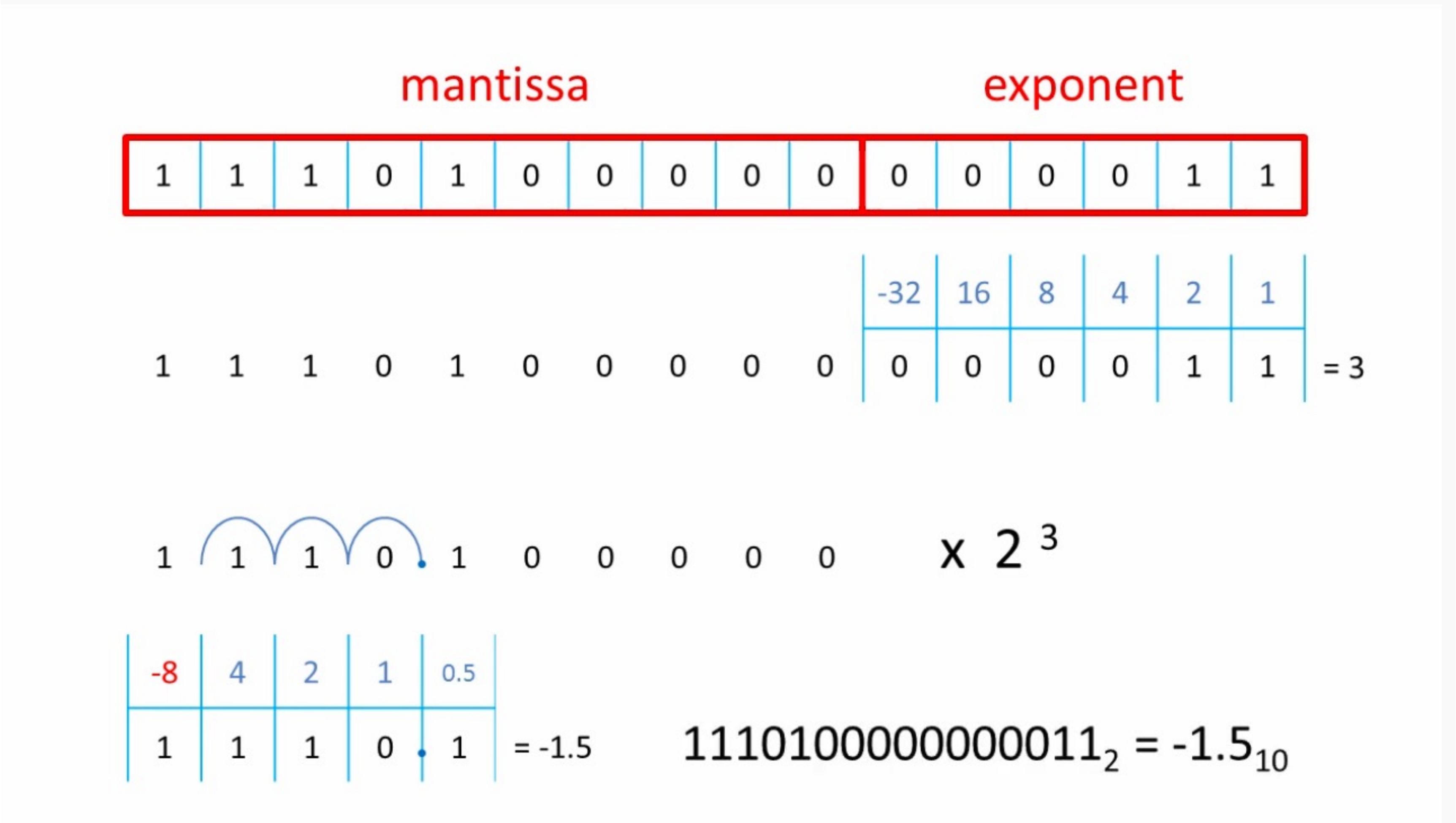
Exercise 9

Convert the following floating point binary numbers into denary. Assume 4 bits for the mantissa and 4 bits for the exponent, both in two's complement

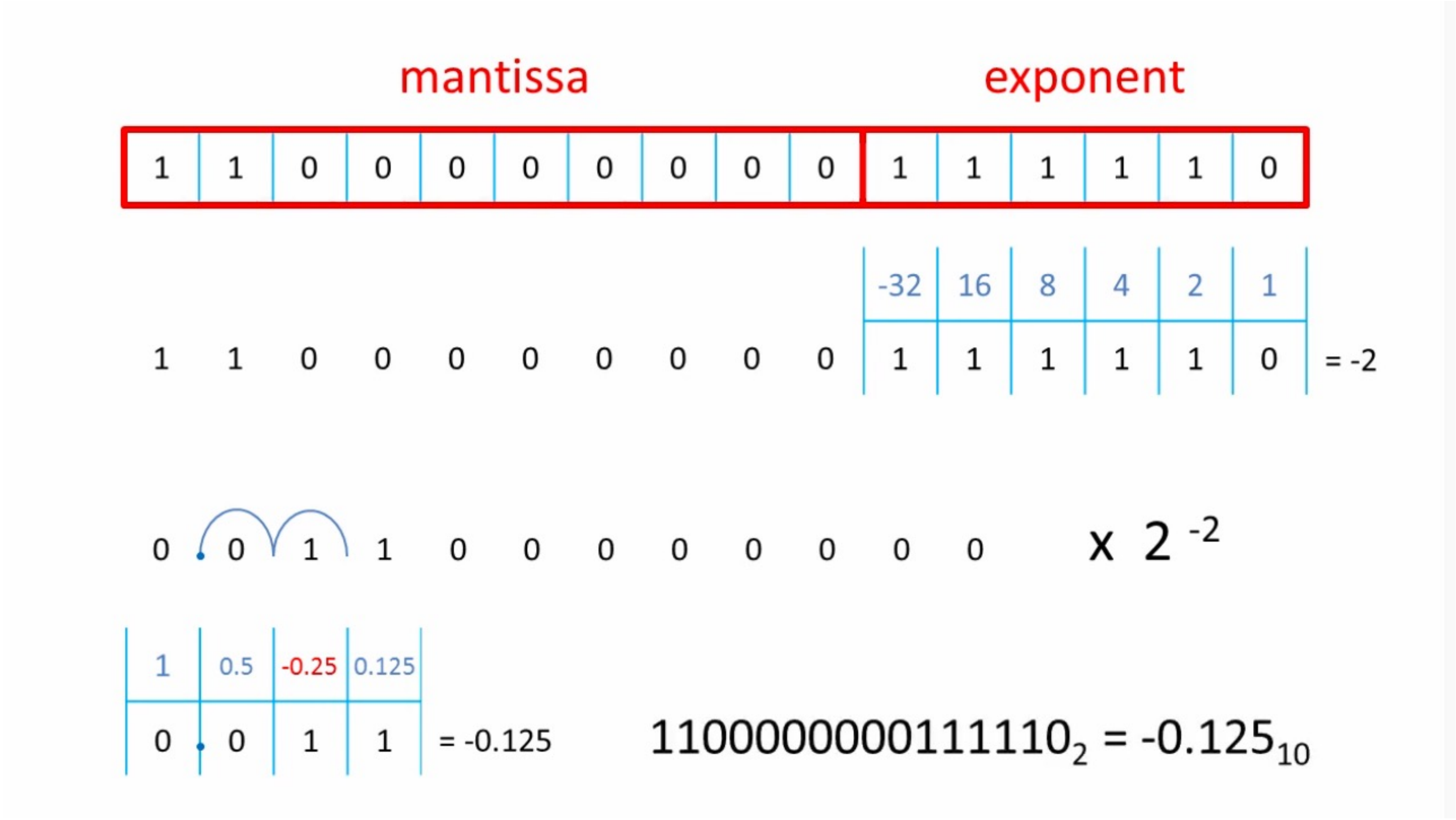
01110011

01111110

Floating-Point Binary



Floating-Point Binary



Exercise 10

Convert the following floating point binary numbers into denary. Assume 10 bits for the mantissa and 6 bits for the exponent, both in two's complement

1101000000 111111

1001101000000110

Floating Point Binary

Range versus Precision

$$0.125_{10} = 0.001_2$$

1	0.5	0.25	0.125	0.0625	0.03125	0.015625	0.0078125	0.00390625
0	0	0	1					

Precision relates to numbers of bits used to represent a binary.
These two values are exactly the same.
With the precision of 4 bits you can represent accuratley.

$$0.1_{10} = 0.0001100110011001100110011..._2$$

1	0.5	0.25	0.125	0.0625	0.03125	0.015625	0.0078125	0.00390625	
0	0	0	0	1	1	0	0	1	. . .

1/10 in Decimal cannot be accuratley represented in Binary as
it is recurring sequance of bits.
Cannot be respenested accurately in binary with any number of bits.

$$0.1_{10} = 0.00011_2$$

1	0.5	0.25	0.125	0.0625	0.03125	0.015625	0.0078125	0.00390625
0	0	0	0	1	1	0	0	

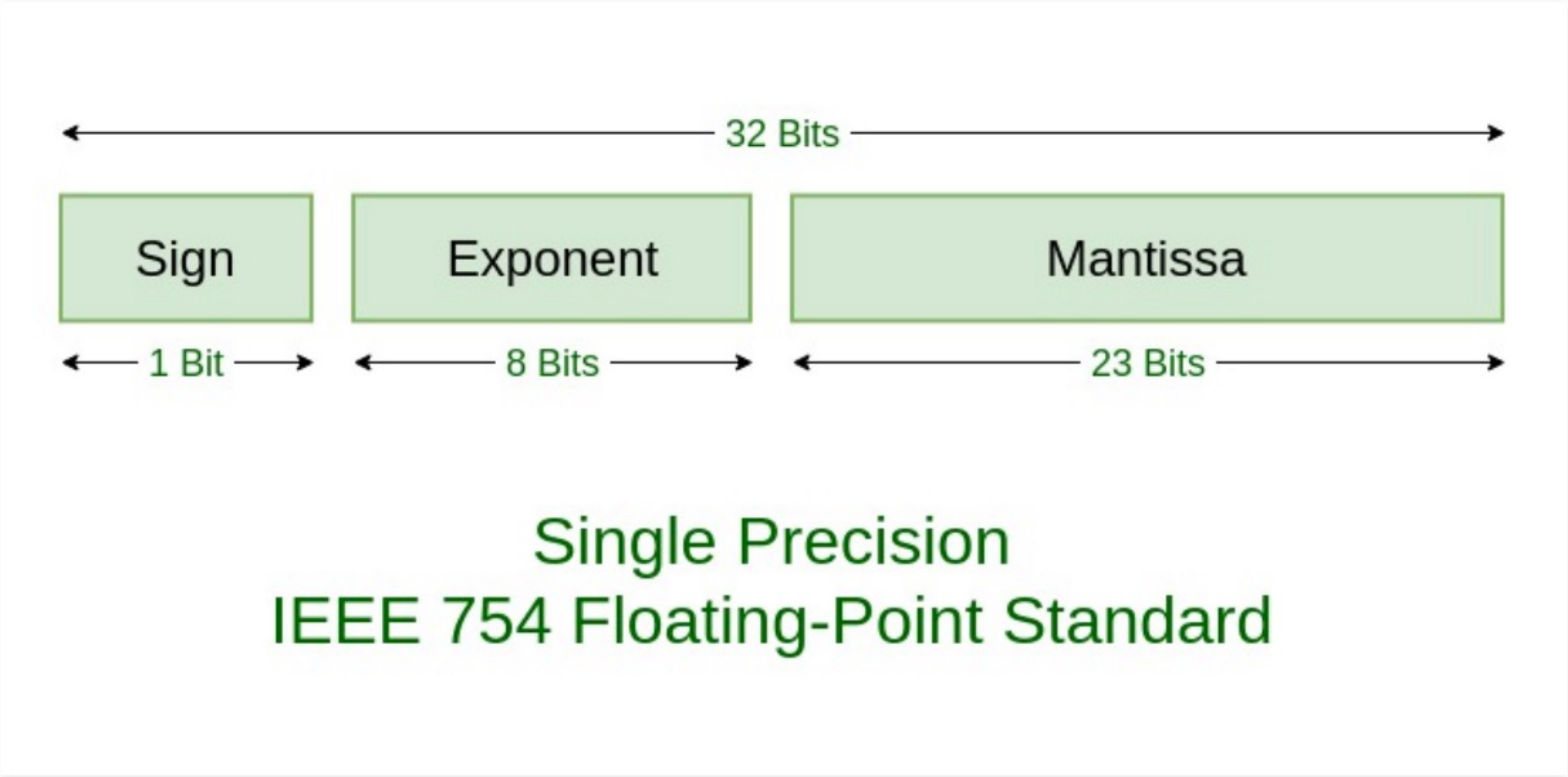
$$0.00011_2 = 0.0625 + 0.03125 = 0.09375_{10}$$

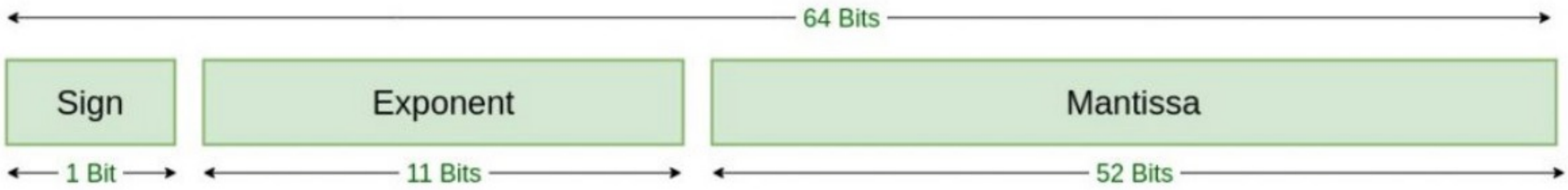
Only an approximation

Precission and Accuracy are used interchangeably.
Low precision means less accuracy. However, this not always true.
Forexample- 0.5 in Decimal any easily be represented accurately with 2 bits.

In floting point binary precision is governed by the no of bits allocated to The mantissa.

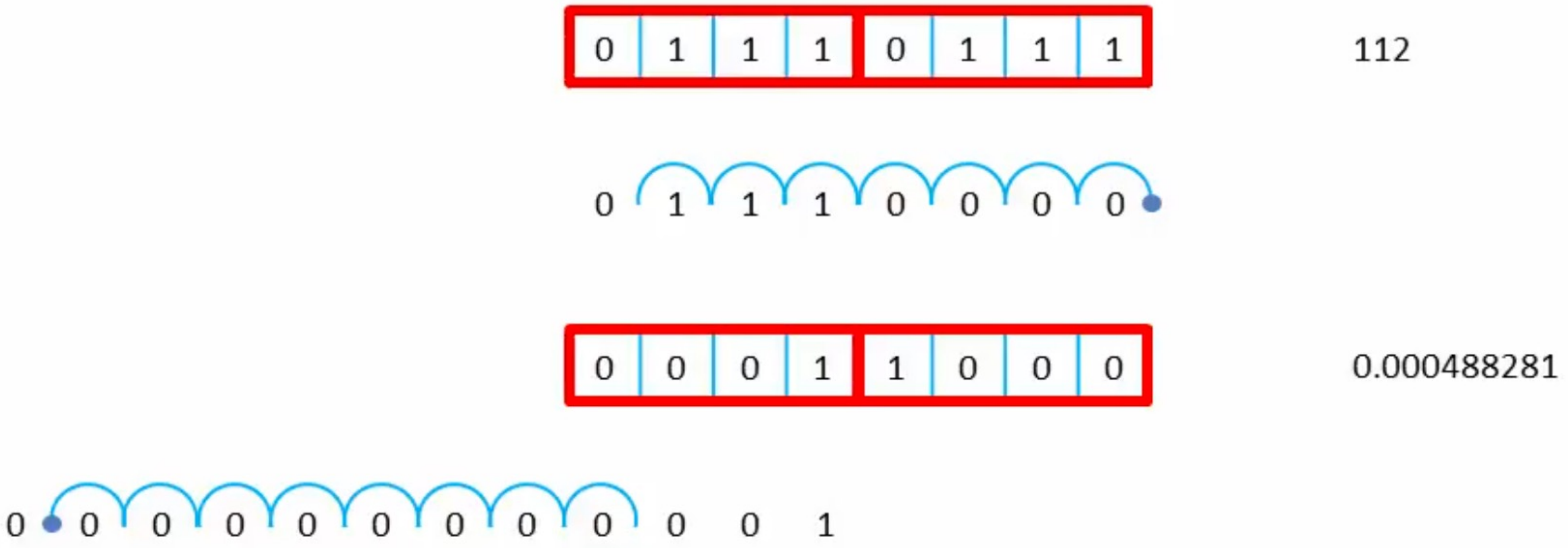
IEEE 754 numbers are divided into two based on the above three components: single precision and double precision.





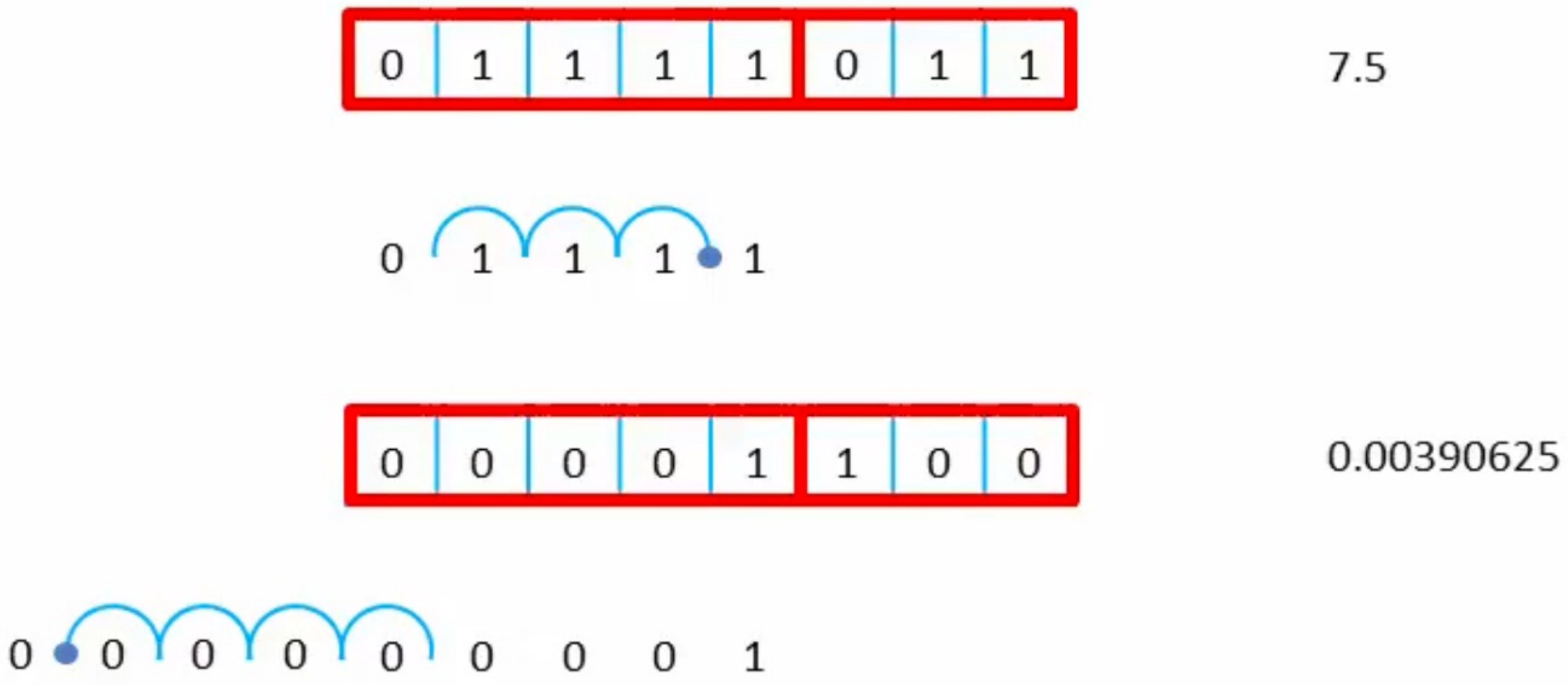
Double Precision
IEEE 754 Floating-Point Standard

Largest and Smallest Value Possible



Can we represent accuratley 7.5 in the register?

Largest and Smallest Value Possible



Summary

- For a given sized register, the number of values that can be represented is limited
- Greater precision comes at the expense of range
- Greater range comes at the expense of precision
- Accuracy often depends on precision, but not always
- There will always be values that can’t be represented accurately in binary
- Programmers should understand how floating point binary works

```csharp
using System;

namespace FloatingNumber
{
    class Program
    {
        static void Main(string[] args)
        {
            float a = 12.345F;
            float b = 12;
            float c = a - b;
            Console.WriteLine(c);
        }
    }
}
```

```csharp
using System;

namespace FloatingNumber
{
    class Program
    {
        static void Main(string[] args)
        {
            decimal a = 12.345M;
            decimal b = 12;
            decimal c = a - b;
            Console.WriteLine(c);

        }
    }
}
```

Floating Point Binary

Convert from Denary to Normalised Binary

$0\ 1\ 1\ 1\ 0\ 0\ 1\ 1$

$= 0\,.\,1\ 1\ 1\quad 0\ 0\ 1\ 1$

$= 0\,.\,1\ 1\ 1 \quad x\ 2^{3}$

$= 0\quad 1\ 1\ 1\,.$

$= 0\ 1\ 1\ 1\,.\,0$

$= 4 + 2 + 1$

$= 7$

$0\ 1\ 1\ 1\ 0\ 0\ 1\ 1_2 = 7_{10}$

$0\ 1\ 1\ 1\ 1\ 1\ 1\ 0$

$= 0\,.\,1\ 1\ 1\quad 1\ 1\ 1\ 0$

$= 0\,.\,1\ 1\ 1 \quad x\ 2^{-2}$

$= .\,0\ 0\quad 1\ 1\ 1$

$= 0\,.\,0\ 0\ 1\ 1\ 1$

$= 0.125 + 0.0625 + 0.03125$

$= 0.21875$

$0\ 1\ 1\ 1\ 1\ 1\ 1\ 0_2 = 0.21875_{10}$

Convert positive denary numbers into floating point binary

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value 7 into floating point binary

8	4	2	1	0.5	
0	1	1	1	. 0	= 7.0

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value 7 into floating point binary

0 1 1 1 . 0 = 7.0

0 . 1 1 1

0 . 1 1 1 x 2^3

0 . 1 1 1

8	4	2	1
0	0	1	1

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value 7 into floating point binary

0 1 1 1 . 0 = 7.0

0 . 1 1 1

0 . 1 1 1 x 2^3

0 . 1 1 1 0 0 1 1

$7_{10} = 01110011_2$

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value 1.5 into floating point binary

2	1	0.5	
0	1 .	1	= 1.5

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value 1.5 into floating point binary

0 1 . 1 = 1.5

0 . 1 1 0

0 . 1 1 0 $\times 2^1$

0 . 1 1 0

8	4	2	1
0	0	0	1

With 4 bits for the mantissa and 4 bits for the exponent, both in two’s complement, convert the value 1.5 into floating point binary

0 1 . 1 = 1.5

0 . 1 1 0

0 . 1 1 0 $\times 2^1$

0 . 1 1 0 0 0 0 1

$$1.5_{10} = 01100001_2$$

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value 0.25 into floating point binary

1	0.5	0.25	
0 .	0	1	= 0.25

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value 0.25 into floating point binary

0 . 0 1 = 0.25

0 0 . 1

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value 0.25 into floating point binary

0 . 0 1 = 0.25

0 . 1

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value 0.25 into floating point binary

0 . 0 1 = 0.25

0 . 1 0 0

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value 0.25 into floating point binary

0 . 0 1 = 0.25

0 . 1 0 0

0 . 1 0 0 x 2^{-1}

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value 0.25 into floating point binary

0 . 0 1 = 0.25

0 . 1 0 0

0 . 1 0 0 $\times 2^{-1}$

0 . 1 0 0

- 8	4	2	1
1	1	1	1

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value 0.25 into floating point binary

0 . 0 1 = 0.25

0 . 1 0 0

0 . 1 0 0 $\times 2^{-1}$

0 . 1 0 0 1 1 1 1

$$0.25_{10} = 01001111_2$$

Exercise 11

Convert the following denary numbers into floating point binary. Assume 4 bits for the mantissa and 4 bits for the exponent, both in two's complement

2.5 | 1.75 | 0.375

Exercise 12

Convert the following denary numbers into floating point binary. Assume 10 bits for the mantissa and 6 bits for the exponent, both in two's complement

19.25

0.0625

Convert negative denary numbers into floating point binary

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value -6 into floating point binary

-8	4	2	1	0.5	
1	0	1	0 .	0	= -6.0

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value -6 into floating point binary

1 0 1 0 . 0 = -6.0

1 . 0 1 0

1 . 0 1 0 $\times 2^3$

1 . 0 1 0   0 0 1 1

$-6_{10} = 10100011_2$

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value -1.5 into floating point binary

-2	1	0.5	
1	0 .	1	= -1.5

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value -1.5 into floating point binary

-2	1	0.5	
1	0 .	1	= -1.5

-4	2	1	0.5	
1	1	0 .	1	= -1.5

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value -1.5 into floating point binary

-2	1	0.5	
1	0 .	1	= -1.5

-8	4	2	1	0.5	
1	1	1	0 .	1	= -1.5

With 4 bits for the mantissa and 4 bits for the exponent, both in two's complement, convert the value -1.5 into floating point binary

$1 \quad 0 . 1 \quad = -1.5$

$1 . 0 \quad 1 \quad 0$

$1 . 0 \quad 1 \quad 0 \qquad \times 2^1$

$1 . 0 \quad 1 \quad 0 \qquad 0 \quad 0 \quad 0 \quad 1$

$$-1.5_{10} = 10100001_2$$

Exercise 13

Convert the following denary numbers into floating point binary. Assume 4 bits for the mantissa and 4 bits for the exponent, both in two's complement

-4 -0.25 -1.75

Exercise 14

Convert the following denary numbers into floating point binary. Assume 10 bits for the mantissa and 6 bits for the exponent, both in two's complement

-12.75

-0.125

Normalisation

4 bit Mantissa + 4 bit Exponent

0 1 0 0 1 1 1 1	0 0 0 1 0 0 0 1	0 0 1 0 0 0 0 0
= 0 . 1 0 0 1 1 1 1	= 0 . 0 0 1 0 0 0 1	= 0 . 0 1 0 0 0 0 0
= 0 . 1 0 0 $\times 2^{-1}$	= 0 . 0 0 1 $\times 2^{1}$	= 0 . 0 1 0 $\times 2^{0}$
= .0 1 0 0	= 0 0.0 1	= 0 . 0 1 0
= 0 . 0 1 0 0	= 0 0 . 0 1	= 0 . 0 1 0
= 0.25	= 0.25	= 0.25

Speed of Light in Vacuum (m/s)

2.99×10^8

299000000

29.9×10^7

$299. \times 10^6$

29.9×10^7

2.99×10^8

0.29×10^9

0.02×10^{10}

Normalized form – best balance between precision and range

$299. \times 10^6$

29.9×10^7

2.99×10^8

0.29×10^9

0.02×10^{10}

0 1 0 0 1 1 1 1	0 0 0 1 0 0 0 1	0 0 1 0 0 0 0 0
= 0 . 1 0 0 1 1 1 1	= 0 . 0 0 1 0 0 0 1	= 0 . 0 1 0 0 0 0 0
= 0 . 1 0 0 $\times 2^{-1}$	= 0 . 0 0 1 $\times 2^{1}$	= 0 . 0 1 0 $\times 2^{0}$
= .0 1 0 0	= 0 0.0 1	= 0 . 0 1 0
= 0 . 0 1 0 0	= 0 0 . 0 1	= 0 . 0 1 0
= 0.25	= 0.25	= 0.25

Normalized form – sign bit 0 is followed immediately by 1

Objectives of normalisation

- Maximise precision
- Unambiguous representation
- Simplify arithmetic

- For POSITIVE numbers, the normalised form starts with a 0 followed immediately by a 1
- For NEGATIVE numbers, the normalised form starts with a 1 followed immediately by a 0

The floating point binary number 00010001 is stored using 4 bits for the mantissa and 4 bits for the exponent, both in two's complement. Normalise it.

$$0\,0\,0\,1\,0\,0\,0\,1$$

$$= 0\,.\,0\,0\,1 \quad 0\,0\,0\,1$$

$$= 0\,.\,0\,0\,1 \quad \times 2^{1}$$

$$= 0 \quad 0\,0.1$$

$$= 0\,.\,1\,0\,0$$

$$= 0\,.\,1\,0\,0 \quad \times 2^{1 - 2 = -1}$$

$$= 0\,1\,0\,0\,1\,1\,1\,1$$

The floating point binary number 00111111 is stored using 4 bits for the mantissa and 4 bits for the exponent, both in two's complement. Normalise it.

$$0\,0\,1\,1\,1\,1\,1\,1$$

$$= 0\,.\,0\,1\,1 \quad 1\,1\,1\,1$$

$$= 0\,.\,0\,1\,1 \quad \times 2^{-1}$$

$$= 0 \quad 0.1\,1$$

$$= 0\,.\,1\,1\,0$$

$$= 0\,.\,1\,1\,0 \quad \times 2^{-1\,-\,1\,=\,-2}$$

$$= 0\,1\,1\,0\,1\,1\,1\,0$$

The floating point binary number 11000001 is stored using 4 bits for the mantissa and 4 bits for the exponent, both in two's complement. Normalise it.

$$11000001$$

$$= 1\,.\,1\,0\,0 \quad 0\,0\,0\,1$$

$$= 1\,.\,1\,0\,0 \quad \times 2^{1}$$

$$= 1 \quad 1.0\,0$$

$$= 1\,.\,0\,0\,0$$

$$= 1\,.\,0\,0\,0 \quad \times 2^{1-1=0}$$

$$= 1\,0\,0\,0\,0\,0\,0\,0$$

The floating point binary number 11101011 is stored using 5 bits for the mantissa and 3 bits for the exponent, both in two's complement. Normalise it.

$$11101011$$

$$= 1.1101 \quad 011$$

$$= 1.1101 \times 2^{3}$$

$$= 1 \quad 11.0 \quad 1$$

$$= 1.0100$$

$$= 1.0100 \times 2^{3-2=1}$$

$$= 10100001$$

Exercise 15

The following floating point binary numbers are stored using 5 bits for the mantissa and 3 bits for the exponent, Normalise them.

11011001

00011111

Exercise 16

The following floating point binary numbers are stored using 10 bits for the mantissa and 6 bits for the exponent, Normalise them.

0000000110000111

1111101000001011

Floating Point Binary Addition

$$5.2 \times 10^3 \quad + \quad 2.34 \times 10^3 \quad = \quad 7.54 \times 10^3$$

$$\begin{array}{r} 5.2 \\ +\ 2.34 \\ \hline 7.54 \\ \hline \end{array} \times 10^3$$

$$5.2 \times 10^4 \quad + \quad 2.34 \times 10^3 \quad = \quad 5.434 \times 10^4$$

$$0.234 \times 10^4$$

$$\begin{array}{r} 5.2 \\ +\ 0.234 \\ \hline 5.434 \\ \hline \end{array} \times 10^4$$

$$8.2 \times 10^4 \quad + \quad 123.25 \times 10^3 \quad = \quad 2.0525 \times 10^5$$

$$12.325 \times 10^{4}$$

$$\begin{array}{r} 8.2 \\ +\ 12.325 \\ \hline 20.525 \\ \hline \end{array} \times 10^4$$

$$2.0525 \times 10^{5}$$

Floating point binary addition

- Make sure both numbers are normalised
- Make exponents the same
- Add mantissas together
- Normalise result if necessary

Show how you would add 0100000011 to 0100100010. Both numbers are in floating point binary format using 6 bits for the mantissa and 4 bits for the exponent, both in two's complement. Show your result in the same format.

010000 0011	+	010010 0010	
0.10000×2^3	+	0.10010×2^2	
0.10000×2^3	+	$0.0.10010 \times 2^3$	Make exponents the same
0.10000×2^3	+	0.01001×2^3	

$$
\begin{array}{r}
0.10000 \\
+\ 0.01001 \\
\hline
0.11001 \\
\hline
\end{array}
$$

Add mantissas together

0.11001×2^3

011001 0011 — Result already normalised

Double check result

$0.10000 \times 2^3 = 100.0 = 4$

$0.10010 \times 2^2 = 10.01 = 2.25$

$4 + 2.25 = 6.25$

$6.25 = 110.01$

$110.01 = 0.11001 \times 2^3 = 011001\ \ 0011$

Show how you would add 010000001111 to 010101000100. Both numbers are in floating point binary format using 8 bits for the mantissa and 4 bits for the exponent, both in two's complement. Show your result in the same format.

01000000 1111 + 01010100 0100

0.1000000×2^{-1} + 0.1010100×2^{4}

0 0 0 0 0 0.1000000×2^{4} + 0.1010100×2^{4} Make exponents the same

0.0000010×2^{4} + 0.1010100×2^{4}

Add mantissas together

$$
\begin{array}{r}
0.0000010 \\
+\ 0.1010100 \\
\hline
0.1010110 \\
\hline
\end{array}
$$

0.1010110×2^{4}

01010110 0100 Result already normalised

Double check result

$0.1000000 \times 2^{-1} = 0.01 = 0.25$

$0.1010100 \times 2^{4} = 1010.1 = 10.5$

$0.25 + 10.5 = 10.75$

$10.75 = 1010.11$

$1010.11 = 0.101011 \times 2^{4} = 01010110\ \ 0100$

Show how you would add 0100100100 to 0100100010. Both numbers are in floating point binary format using 6 bits for the mantissa and 4 bits for the exponent, both in two's complement. Show your result in the same format.

010010 0100	+	010010 0010	
0.10010×2^4	+	0.10010×2^2	
0.10010×2^4	+	0.0 0.10010×2^4	Make exponents the same
0.10010×2^4	+	0.0010010×2^4	TRUNCATION ERROR

$$
\begin{array}{r}
0.10010 \\
+\ 0.00100 \\
\hline
0.10110 \\
\hline
\end{array}
$$

Add mantissas together

0.10110×2^4

010110 0100 — Normalise result

Double check result

$0.10010 \times 2^4 = 1001.0 = 9$

$0.10010 \times 2^2 = 10.010 = 2.25$

$9 + 2.25 = 11.25$

$11.25 = 1011.01$

$1011.01 = 0.101101 \times 2^4 = 0101101\ 0100$

Convert result to denary

$010110\ 0100 = 0.10110 \times 2^4 = 1011.0 = 11$

Exercise 17

Show how you would add 0101000110 to 0010100101. Both numbers are in floating point binary format using 6 bits for the mantissa and 4 bits for the exponent, both in two's complement. Show your result in the same format.

Show how you would add 0100100011 to 1001000010. Both numbers are in floating point binary format using 6 bits for the mantissa and 4 bits for the exponent, both in two's complement. Show your result in the same format.

Floating point binary addition

- Make sure both numbers are normalised
- Make smaller exponent match the larger exponent
- Add mantissas together
- Normalise result if necessary

Floating point binary subtraction

- Make sure both numbers are normalised
- Make exponents the same
- If performing subtraction, negate the number to subtract
- Add mantissas together
- Normalise result if necessary

Show how you would subtract 0110000010 from 0111000011. Both numbers are in floating point binary format using 6 bits for the mantissa and 4 bits for the exponent, both in two's complement. Show your result in the same format.

011100 0011	-	011000 0010	
0.11100×2^3	-	0.11000×2^2	
0.11100×2^3	-	$0\,0.11000 \times 2^3$	Make exponents the same
0.11100×2^3	-	0.01100×2^3	
	invert bits	1.10011	Negate the number to subtract
	add 1	+ 1	
		1.10100	
0.11100×2^3	+	1.10100×2^3	
		0.11100	Add mantissas together
	carry 1 overflow	+ 1.10100	
		0.10000	
		0.10000×2^3	
		010000 0011	Result already normalised

Double check result

$0.11100 \times 2^3 = 111.0 = 7$

$0.11000 \times 2^2 = 11.0 = 3$

$7 - 3 = 4$

$4 = 100.0$

$100.0 = 0.10000 \times 2^3 = 0.10000\ \ 0011$

Show how you would subtract 0100100010 from 0100100100. Both numbers are in floating point binary format using 6 bits for the mantissa and 4 bits for the exponent, both in two's complement. Show your result in the same format.

010010 0100 - 010010 0010

0.10010×2^4 - 0.10010×2^2

0.10010×2^4 - 0.0 0.10010×2^4 Make exponents the same

0.10010×2^4 - 0.00100×2^4 TRUNCATION ERROR

invert bits 1.11011 Negate the number to subtract

add 1 + 1

1.11100

0.10010×2^4 + 1.11100×2^4

0.10010 Add mantissas together

carry 1 overflow + 1.11100

0.01110

0.01110×2^4

0.01110×2^3 Normalise result

0.11100×2^3

011100 0011

Double check result

$0.10010 \times 2^4 = 1001.0 = 9$

$0.10010 \times 2^2 = 10.010 = 2.25$

$9 - 2.25 = 6.75$

$6.75 = 0110.11$

$0110.11 = 0.11011 \times 2^3 = 011011\ \ 0011$

Show how you would subtract 0100100010 from 0100100100. Both numbers are in floating point binary format using 6 bits for the mantissa and 4 bits for the exponent, both in two's complement. Show your result in the same format.

010010 0100 - 010010 0010

0.10010×2^4 - 0.10010×2^2

0.10010×2^4 - 0.0 0.10010×2^4 Make exponents the same

0.10010×2^4 - 0.00100×2^4 TRUNCATION ERROR

invert bits 1.11011 Negate the number to subtract

add 1 + 1

1.11100

0.10010×2^4 + 1.11100×2^4

0.10010 Add mantissas together

carry 1 overflow + 1.11100

0.01110

0.01110×2^4

0.01110×2^3 Normalise result

0.11100×2^3

011100 0011

Double check result

$0.10010 \times 2^4 = 1001.0 = 9$

$0.10010 \times 2^2 = 10.010 = 2.25$

$9 - 2.25 = 6.75$

$6.75 = 0110.11$

$0110.11 = 0.11011 \times 2^3 =$ 011011 0011

Convert result to denary

$0.11100 \times 2^3 = 111.0 = 7$

Exercise 18

Show how you would subtract 010000100101 from 010001000110. Both numbers are in floating point binary format using 8 bits for the mantissa and 4 bits for the exponent, both in two's complement. Show your result in the same format.

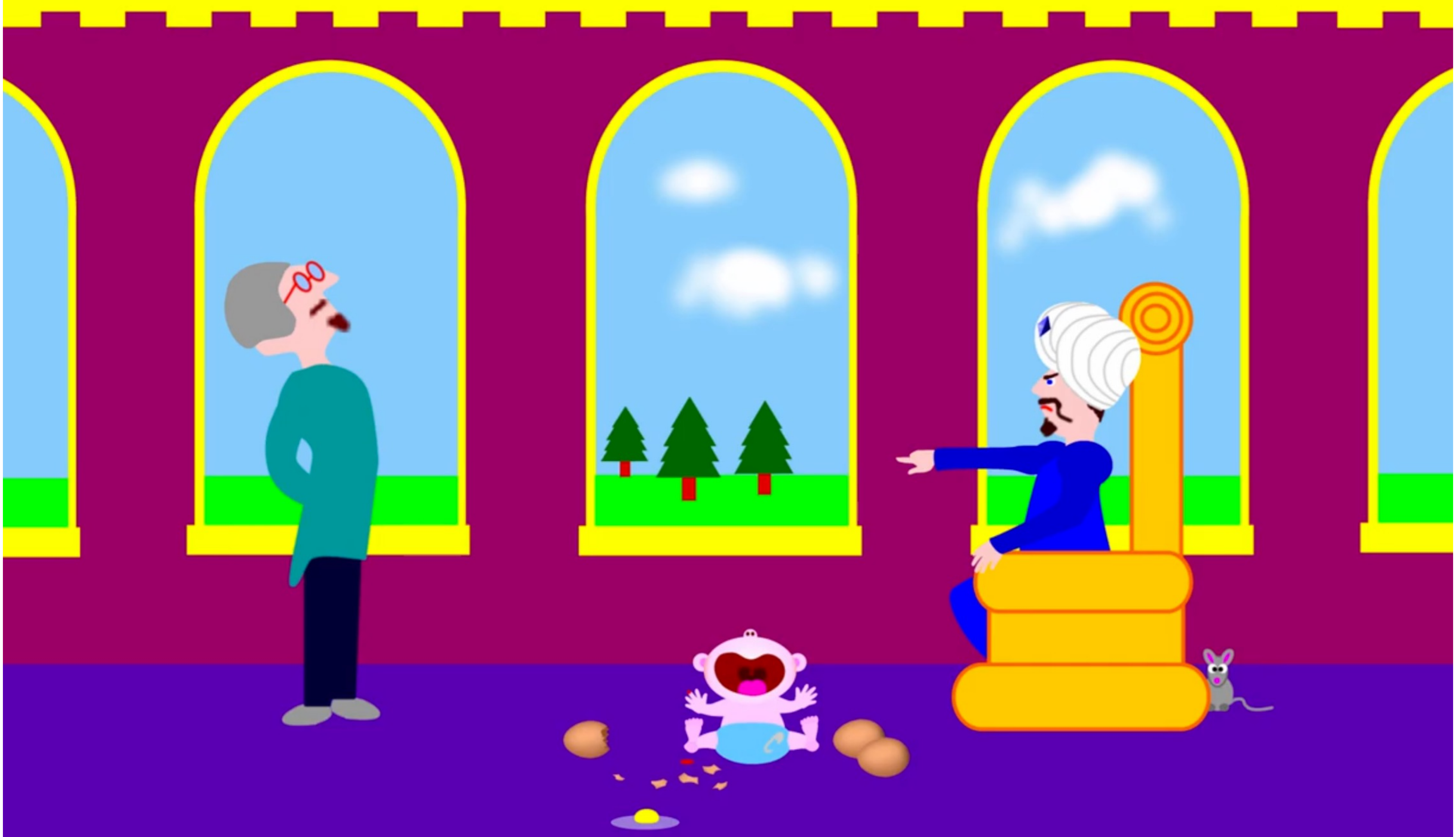
Show how you would subtract 010010000011 from 110100001111. Both numbers are in floating point binary format using 8 bits for the mantissa and 4 bits for the exponent, both in two's complement. Show your result in the same format.

Floating point binary addition

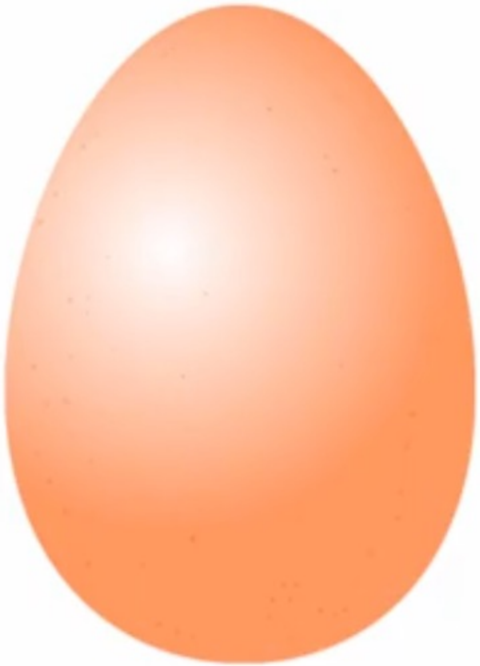
- Make sure both numbers are normalised
- Make smaller exponent match the larger exponent
- Add mantissas together
- Normalise result if necessary

Byte Ordering

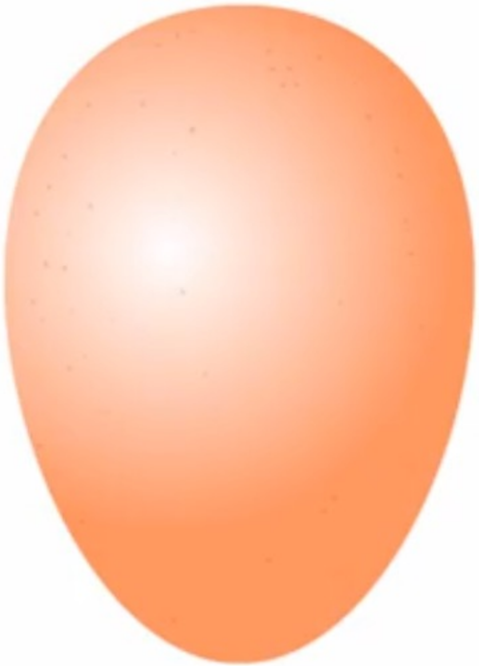
Endianness



ON HOLY WARS AND A PLEA FOR PEACE



Little Endian



Big Endian

1000	100	10	1
1	2	3	4

1000	100	10	1
1	2	3	4

1	10	100	1000
4	3	2	1

Decimal system, also called **Hindu-Arabic number system** or **Arabic number system**, in mathematics, positional numeral system employing 10 as the base and requiring 10 different numerals, the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. It also requires a dot (decimal point) to represent decimal fractions. In this scheme, the numerals used in denoting a number take different place values depending upon position. In a base-10 system the number 543.21 represents the sum $(5 \times 10^2) + (4 \times 10^1) + (3 \times 10^0) + (2 \times 10^{-1}) + (1 \times 10^{-2})$. *See* numerals and numeral systems.

1000	100	10	1
1	2	3	4

128	64	32	16	8	4	2	1
0	1	0	1	1	1	0	1

$64 + 16 + 8 + 4 + 1 = 93_{10}$

...	...	...	...	...	...	...	...
0	1	0	1	1	0	1	0

...	...	...	1048576	524288	262144	131072	65536
0	1	1	0	1	0	1	1

32768	16384	8192	4096	2048	1024	512	256
0	1	1	1	1	1	0	0

128	64	32	16	8	4	2	1
1	0	0	0	1	1	0	1

$= 1516993677_{10}$

$= 5A6B7C8D_{16}$

Hexadecimal and Binary

8	4	2	1	8	4	2	1
0	1	0	1	1	0	1	0
5				A			

8	4	2	1	8	4	2	1
0	1	1	0	1	0	1	1
6				B			

8	4	2	1	8	4	2	1
0	1	1	1	1	1	0	0
7				C			

8	4	2	1	8	4	2	1
1	0	0	0	1	1	0	1
8				D			

8	4	2	1	8	4	2	1
0	1	0	1	1	0	1	0

5 A

8	4	2	1	8	4	2	1
0	1	1	0	1	0	1	1

6 B

8	4	2	1	8	4	2	1
0	1	1	1	1	1	0	0

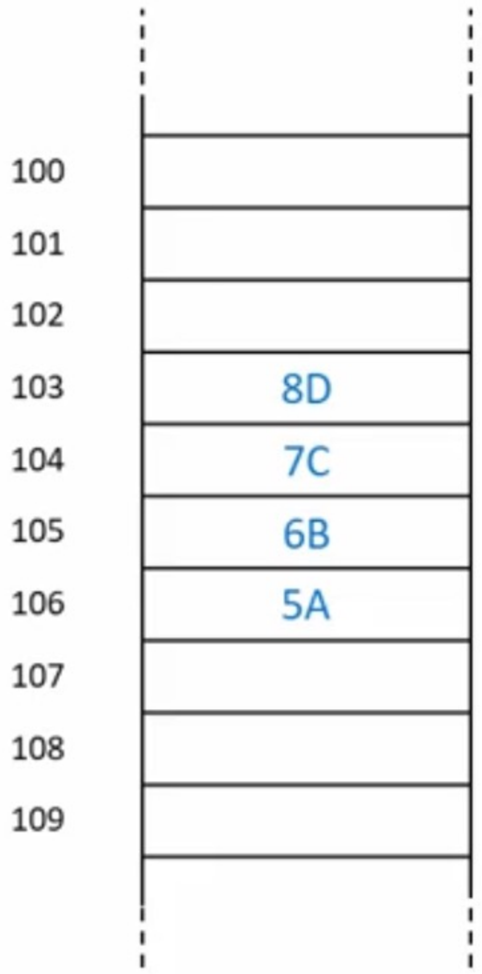
7 C

8	4	2	1	8	4	2	1
1	0	0	0	1	1	0	1

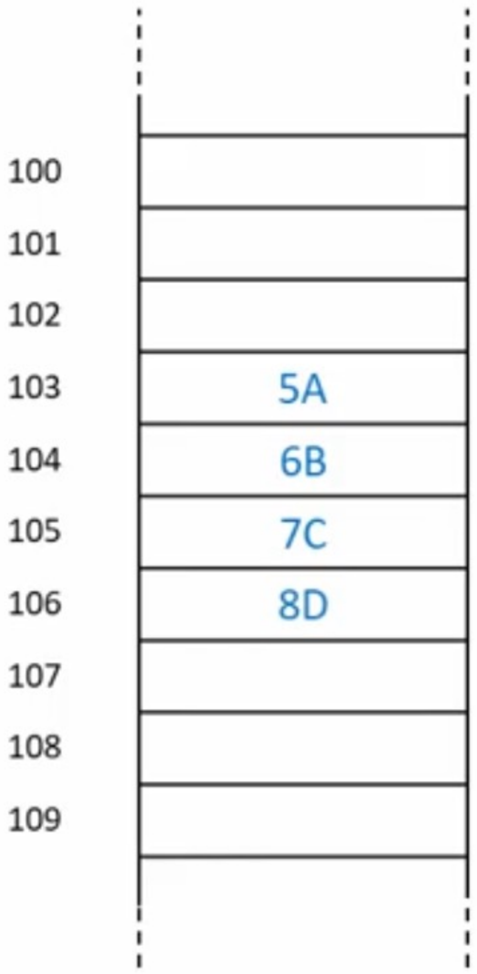
8 D

5A 6B 7C 8D

Least significant byte at lower address



Least significant byte at higher address



5A 6B 7C 8D

Little endian

Least significant byte at lower address

100	
101	
102	
103	8D
104	7C
105	6B
106	5A
107	
108	
109	

Big endian

Least significant byte at higher address

100	
101	
102	
103	5A
104	6B
105	7C
106	8D
107	
108	
109	

5A 6B 7C 8D

Little endian

Least significant byte at lower address

Address	Content
100	
101	
102	
103	1 0 0 0 1 1 0 1
104	0 1 1 1 1 1 0 0
105	0 1 1 0 1 0 1 1
106	0 1 0 1 1 0 1 0
107	
108	
109	

Big endian

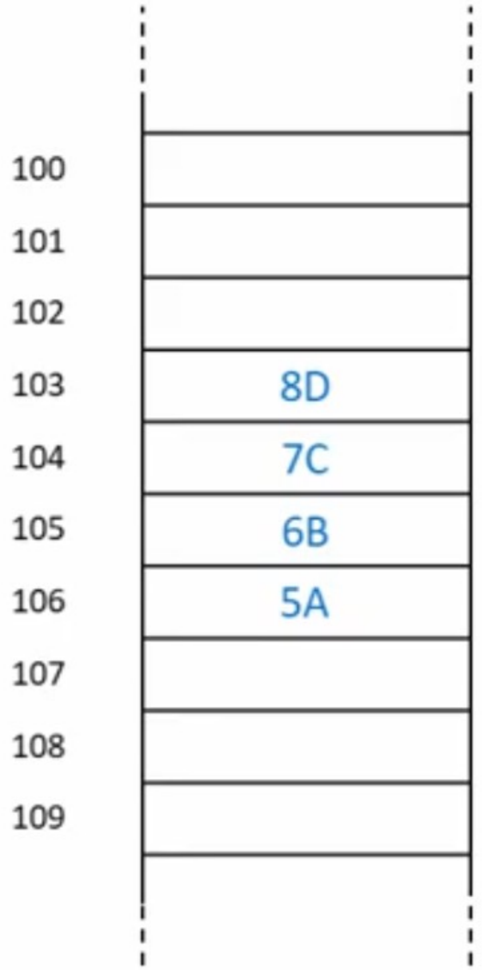
Least significant byte at higher address

Address	Content
100	
101	
102	
103	0 1 0 1 1 0 1 0
104	0 1 1 0 1 0 1 1
105	0 1 1 1 1 1 0 0
106	1 0 0 0 1 1 0 1
107	
108	
109	

5A 6B 7C 8D

Little endian

Least significant byte at lower address



Erlier Main Frame + IBM + Apple are Big Endian
Intel processor are Little Endian due to efficiency
Of some processes

106	105	104	103
5A	6B	7C	8D
0 1 0 1 1 0 1 0	0 1 1 0 1 0 1 1	0 1 1 1 1 1 0 0	1 0 0 0 1 1 0 1

206	205	204	203
B2	C2	D2	E2
1 0 1 1 0 0 1 0	1 1 0 0 0 0 1 0	1 1 0 1 0 0 1 0	1 1 1 0 0 0 1 0

	0	1	0	1	1	0	1	0
	1	0	1	1	0	0	1	0
1	1	1	1			1		
	0	0	0	0	1	1	0	1

	0	1	1	0	1	0	1	1
	1	1	0	0	0	0	1	0
1	1					1	1	
	0	0	1	0	1	1	1	0

	0	1	1	1	1	1	0	0
	1	1	0	1	0	0	1	0
1	1	1	1					
	0	1	0	0	1	1	1	1

	1	0	0	0	1	1	0	1
	1	1	1	0	0	0	1	0
1								
	0	1	1	0	1	1	1	1

```
Dim ShoeSize As Integer

ShoeSize = 5

Dim NewShoeSize as Short

NewShoeSize = CShort(ShoeSize)
```

Little endian

Least significant byte at lower address

100	
101	
102	05
103	00
104	00
105	00
106	
107	
108	05
109	00

Big endian

Least significant byte at higher address

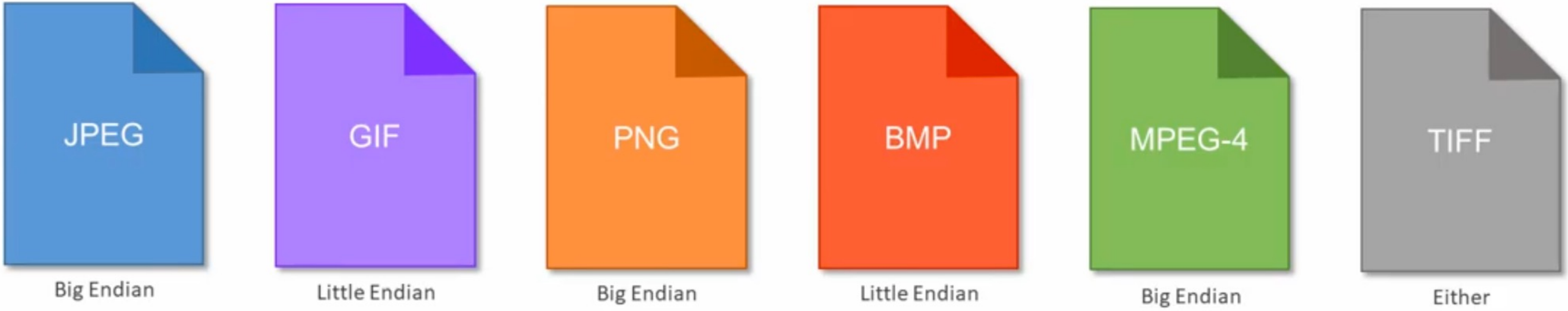
100	
101	
102	00
103	00
104	00
105	05
106	
107	
108	00
109	05

Network Byte Order (Big Endian)

Network Byte Order (Big Endian)

"We agree that the difference between sending eggs with the little- or the big-end first is trivial, but we insist that everyone must do it in the same way, to avoid anarchy. Since the difference is trivial we may choose either way, but a decision must be made." Danny Cohen 1 April 1980


JPEG
Big Endian
GIF
Little Endian
PNG
Big Endian
BMP
Little Endian
MPEG-4
Big Endian


JPEG
Big Endian
GIF
Little Endian
PNG
Big Endian
BMP
Little Endian
MPEG-4
Big Endian
TIFF
Either

Unicode Transformation Formats and Byte Order

Summary

- Normalisation maximises precision
- Normalisation allows for an unambiguous representation
- For positive numbers, the normalised form starts with 01
- For negative numbers, the normalised form starts with 10

IEEE 754 Standard for Floating-Point Arithmetic

Representing 32 bit Single Precision Numbers

Representing Real Numbers

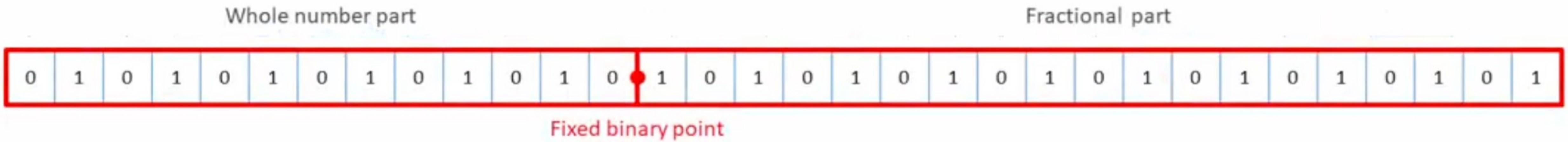
Fixed point binary

...	...	...	...	...	262144	131072	65536	32768	16384	8192	4096	2048	1024	512	256	128	64	32	16	8	4	2	1	0.5	0.25	0.125	0.0625	0.03125	0.015625	...	...
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Fixed binary point

Representing Real Numbers

Fixed point binary



Representing Real Numbers

Fixed point binary

Whole number part | Fractional part

0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Fixed binary point

Representing Real Numbers

Fixed point binary

Whole number part | Fractional part

0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Fixed binary point

Floating point binary

Mantissa | Exponent

0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1

Representing Real Numbers

Fixed point binary

Whole number part																			Fractional part												
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Fixed binary point

Floating point binary

Mantissa																								Exponent							
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1

Floating binary point

Representing Real Numbers

Fixed point binary

Whole number part | Fractional part

0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Fixed binary point

Floating point binary

Mantissa | Exponent = 15

0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1

Floating binary point

21845.375_{10}

Precision is govern by No of Bits available Mantisa

Range is govern by No of Bits available Exponent

Negative values are represented by the Tow's Compliment

IEEE 754 Standard for Single Precision 32 bit floating point binary

Convert the denary value 19.59375 into IEEE 754 standard 32 bit floating point binary

Sign bit	Exponent								Mantissa																						
0																															

Step 1: Determine the sign bit (0 if positive, 1 if negative)

Sign bit = 0

Step 2: Convert to pure binary

	16	8	4	2	1	0.5	0.25	0.125	0.0625	0.03125
19.59375_{10} =	1	0	0	1	1 •	1	0	0	1	1

19 ÷ 2 = 9	remainder 1	0.59375 x 2 = 1.1875	1
9 ÷ 2 = 4	remainder 1	0.1875 x 2 = 0.375	0
4 ÷ 2 = 2	remainder 0	0.375 x 2 = 0.75	0
2 ÷ 2 = 1	remainder 0	0.75 x 2 = 1.5	1
	remainder 1	0.5 x 2 = 1.0	1

Convert the denary value 19.25 into IEEE 754 standard 32 bit floating point binary

Sign bit	Exponent	Mantissa
0	1 0 0 0 0 0 1 1	0 0 1 1 1 0 0 1 1

Step 1: Determine the sign bit (0 if positive, 1 if negative)

Sign bit = 0

Step 2: Convert to pure binary

		16	8	4	2	1	0.5	0.25	0.125	0.0625	0.03125
19.59375_{10}	=	1	0	0	1	1 •	1	0	0	1	1

Step 3: Normalise to determine the mantissa and the unbiased exponent ***(place the binary point after leftmost 1)***

1 0 0 1 1 • 1 0 0 1 1 = 1.001110011×2^4

Convert the denary value 19.25 into IEEE 754 standard 32 bit floating point binary

Sign bit	Exponent								Mantissa																						
0	1	0	0	0	0	0	1	1	0	0	1	1	1	0	0	1	1														

Step 1: Determine the sign bit (0 if positive, 1 if negative)

Sign bit = 0

Step 2: Convert to pure binary

	16	8	4	2	1	0.5	0.25	0.125	0.0625	0.03125
19.59375_{10} =	1	0	0	1	1 •	1	0	0	1	1

Step 3: Normalise to determine the mantissa and the unbiased exponent ***(place the binary point after leftmost 1)***

1 0 0 1 1 • 1 0 0 1 1 = 1.001110011×2^4

Step 4: Determine the biased exponent ***(add 127 then convert to an unsigned binary integer)***

$4 + 127 = 131_{10} = 10000011_2$

Convert the denary value 19.25 into IEEE 754 standard 32 bit floating point binary

Sign bit | Exponent | Mantissa

0	1	0	0	0	0	0	1	1	0	0	1	1	1	0	0	1	1														

Step 1: Determine the sign bit (0 if positive, 1 if negative)

Sign bit = 0

Step 2: Convert to pure binary

	16	8	4	2	1	0.5	0.25	0.125	0.0625	0.03125
19.59375_{10} =	1	0	0	1	1 .	1	0	0	1	1

Step 3: Normalise to determine the mantissa and the unbiased exponent *(place the binary point after leftmost 1)*

1 0 0 1 1 . 1 0 0 1 1 = 1.001110011×2^4

Step 4: Determine the biased exponent *(add 127 then convert to an unsigned binary integer)*

$4 + 127 = 131_{10} = 10000011_2$

Left most bit of the mantisa will always be 1 – therefore no need to store it. Just put back when doing any calculations.

Step 5: Remove the leading 1 from the mantissa *(remove the leftmost 1)*

1.001110011 = 001110011

Convert the denary value 19.25 into IEEE 754 standard 32 bit floating point binary

Sign bit	Exponent							Mantissa																							
0	1	0	0	0	0	0	1	1	0	0	1	1	1	0	0	1	1														

Step 1: Determine the sign bit (0 if positive, 1 if negative)

Sign bit = 0

Step 2: Convert to pure binary

	16	8	4	2	1	0.5	0.25	0.125	0.0625	0.03125
19.59375_{10} =	1	0	0	1	1 •	1	0	0	1	1

Step 3: Normalise to determine the mantissa and the unbiased exponent ***(place the binary point after leftmost 1)***

1 0 0 1 1 • 1 0 0 1 1 = 1.001110011×2^4

Step 4: Determine the biased exponent ***(add 127 then convert to an unsigned binary integer)***

$4 + 127 = 131_{10} = 10000011_2$

Step 5: Remove the leading 1 from the mantissa ***(remove the leftmost 1)***

1.001110011 = 001110011

Convert the denary value 19.25 into IEEE 754 standard 32 bit floating point binary

Sign bit	Exponent								Mantissa																						
0	1	0	0	0	0	0	1	1	0	0	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Step 1: Determine the sign bit (0 if positive, 1 if negative)

Sign bit = 0

Step 2: Convert to pure binary

	16	8	4	2	1	0.5	0.25	0.125	0.0625	0.03125
19.59375_{10} =	1	0	0	1	1 •	1	0	0	1	1

Step 3: Normalise to determine the mantissa and the unbiased exponent *(place the binary point after leftmost 1)*

1 0 0 1 1 • 1 0 0 1 1 = 1.001110011×2^4

Step 4: Determine the biased exponent *(add 127 then convert to an unsigned binary integer)*

$4 + 127 = 131_{10} = 10000011_2$

Step 5: Remove the leading 1 from the mantissa *(remove the leftmost 1)*

1.001110011 = 001110011

Baised Exponent – 4 Bit Exponent

0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	0	8
1	0	0	1	9
1	0	1	0	10
1	0	1	1	11
1	1	0	0	12
1	1	0	1	13
1	1	1	0	14
1	1	1	1	15

0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	0	8
1	0	0	1	9
1	0	1	0	10
1	0	1	1	11
1	1	0	0	12
1	1	0	1	13
1	1	1	0	14
1	1	1	1	15

8	4	2	1	
0	0	1	1	$= 3_{10}$

0	0	0	0	0	0
0	0	0	1	1	1
0	0	1	0	2	2
0	0	1	1	3	3
0	1	0	0	4	4
0	1	0	1	5	5
0	1	1	0	6	6
0	1	1	1	7	7
1	0	0	0	8	-8
1	0	0	1	9	-7
1	0	1	0	10	-6
1	0	1	1	11	-5
1	1	0	0	12	-4
1	1	0	1	13	-3
1	1	1	0	14	-2
1	1	1	1	15	-1

-8	4	2	1	
1	0	1	1	$= -5_{10}$

0	0	0	0	0	0
0	0	0	1	1	1
0	0	1	0	2	2
0	0	1	1	3	3
0	1	0	0	4	4
0	1	0	1	5	5
0	1	1	0	6	6
0	1	1	1	7	7
1	0	0	0	8	-8
1	0	0	1	9	-7
1	0	1	0	10	-6
1	0	1	1	11	-5
1	1	0	0	12	-4
1	1	0	1	13	-3
1	1	1	0	14	-2
1	1	1	1	15	-1

-8	4	2	1	
1	0	1	1	$= -5_{10}$

0	0	0	0	0	0	-5	
0	0	0	1	1	1	-4	
0	0	1	0	2	2	-3	
0	0	1	1	3	3	-2	
0	1	0	0	4	4	-1	
0	1	0	1	5	5	0	
0	1	1	0	6	6	1	
0	1	1	1	7	7	2	
1	0	0	0	8	-8	3	
1	0	0	1	9	-7	4	
1	0	1	0	10	-6	5	
1	0	1	1	11	-5	6	
1	1	0	0	12	-4	7	
1	1	0	1	13	-3	8	
1	1	1	0	14	-2	9	
1	1	1	1	15	-1	10	

When we need more +ve values and few –ve values

$\boxed{0\ 0\ 0\ 0}$	0	0	$\boxed{-5}$
0 0 0 1	1	1	-4
0 0 1 0	2	2	-3
0 0 1 1	3	3	-2
0 1 0 0	4	4	-1
0 1 0 1	5	5	0
0 1 1 0	6	6	1
0 1 1 1	7	7	2
1 0 0 0	8	-8	3
1 0 0 1	9	-7	4
1 0 1 0	10	-6	5
1 0 1 1	11	-5	6
1 1 0 0	12	-4	7
1 1 0 1	13	-3	8
1 1 1 0	14	-2	9
1 1 1 1	15	-1	10

0	0	0	0	0	0	-5
0	0	0	1	1	1	-4
0	0	1	0	2	2	-3
0	0	1	1	3	3	-2
0	1	0	0	4	4	-1
0	1	0	1	5	5	0
0	1	1	0	6	6	1
0	1	1	1	7	7	2
1	0	0	0	8	-8	3
1	0	0	1	9	-7	4
1	0	1	0	10	-6	5
1	0	1	1	11	-5	6
1	1	0	0	12	-4	7
1	1	0	1	13	-3	8
1	1	1	0	14	-2	9
1	1	1	1	15	-1	10

0 0 0 0	0	0	-5
0 0 0 1	1	1	-4
0 0 1 0	2	2	-3
0 0 1 1	3	3	-2
0 1 0 0	4	4	-1
0 1 0 1	5	5	0
0 1 1 0	6	6	1
0 1 1 1	7	7	2
$\boxed{1\ 0\ 0\ 0}$	$\boxed{8}$	-8	$\boxed{3}$
1 0 0 1	9	-7	4
1 0 1 0	10	-6	5
1 0 1 1	11	-5	6
1 1 0 0	12	-4	7
1 1 0 1	13	-3	8
1 1 1 0	14	-2	9
1 1 1 1	15	-1	10

Need to add 5 to convert 3 into unsigned binary integer

0	0	0	0	0	0	-5
0	0	0	1	1	1	-4
0	0	1	0	$\boxed{2}$	2	$\boxed{-3}$
0	0	1	1	3	3	-2
0	1	0	0	4	4	-1
0	1	0	1	5	5	0
0	1	1	0	6	6	1
0	1	1	1	7	7	2
1	0	0	0	8	-8	3
1	0	0	1	9	-7	4
1	0	1	0	10	-6	5
1	0	1	1	11	-5	6
1	1	0	0	12	-4	7
1	1	0	1	13	-3	8
1	1	1	0	14	-2	9
1	1	1	1	15	-1	10

0	0	0	0	0	0	-5	-9
0	0	0	1	1	1	-4	-8
0	0	1	0	2	2	-3	-7
0	0	1	1	3	3	-2	-6
0	1	0	0	4	4	-1	-5
0	1	0	1	5	5	0	-4
0	1	1	0	6	6	1	-3
0	1	1	1	7	7	2	-2
1	0	0	0	8	-8	3	-1
1	0	0	1	9	-7	4	0
1	0	1	0	10	-6	5	1
1	0	1	1	11	-5	6	2
1	1	0	0	12	-4	7	3
$\boxed{1}$	$\boxed{1}$	$\boxed{0}$	$\boxed{1}$	$\boxed{13}$	-3	8	$\boxed{4}$
1	1	1	0	14	-2	9	5
1	1	1	1	15	-1	10	6

0	0	0	0	0	0	-5	-9	-8	-7
0	0	0	1	1	1	-4	-8	-7	-6
0	0	1	0	2	2	-3	-7	-6	-5
0	0	1	1	3	3	-2	-6	-5	-4
0	1	0	0	4	4	-1	-5	-4	-3
0	1	0	1	5	5	0	-4	-3	-2
0	1	1	0	6	6	1	-3	-2	-1
0	1	1	1	7	7	2	-2	-1	0
1	0	0	0	8	-8	3	-1	0	1
1	0	0	1	9	-7	4	0	1	2
1	0	1	0	10	-6	5	1	2	3
1	0	1	1	11	-5	6	2	3	4
1	1	0	0	12	-4	7	3	4	5
1	1	0	1	13	-3	8	4	5	6
1	1	1	0	14	-2	9	5	6	7
1	1	1	1	15	-1	10	6	7	8

$2^{(4-1)} - 1 = 7$ negative values

Offset of bais

IEEE 754 Format	Sign	Exponent	Mantissa	Exponent Bias
32 bit single precision	1 bit	8 bits	23 bits (+ 1 not stored)	$2^{(8-1)} - 1 = 127$
64 bit double precision	1 bit	11 bits	52 bits (+ 1 not stored)	$2^{(11-1)} - 1 = 1023$
128 bit quadruple precision	1 bit	15 bits	112 bits (+ 1 not stored)	$2^{(15-1)} - 1 = 16383$

m
b

From Text

From Other Sources

Existing Connections

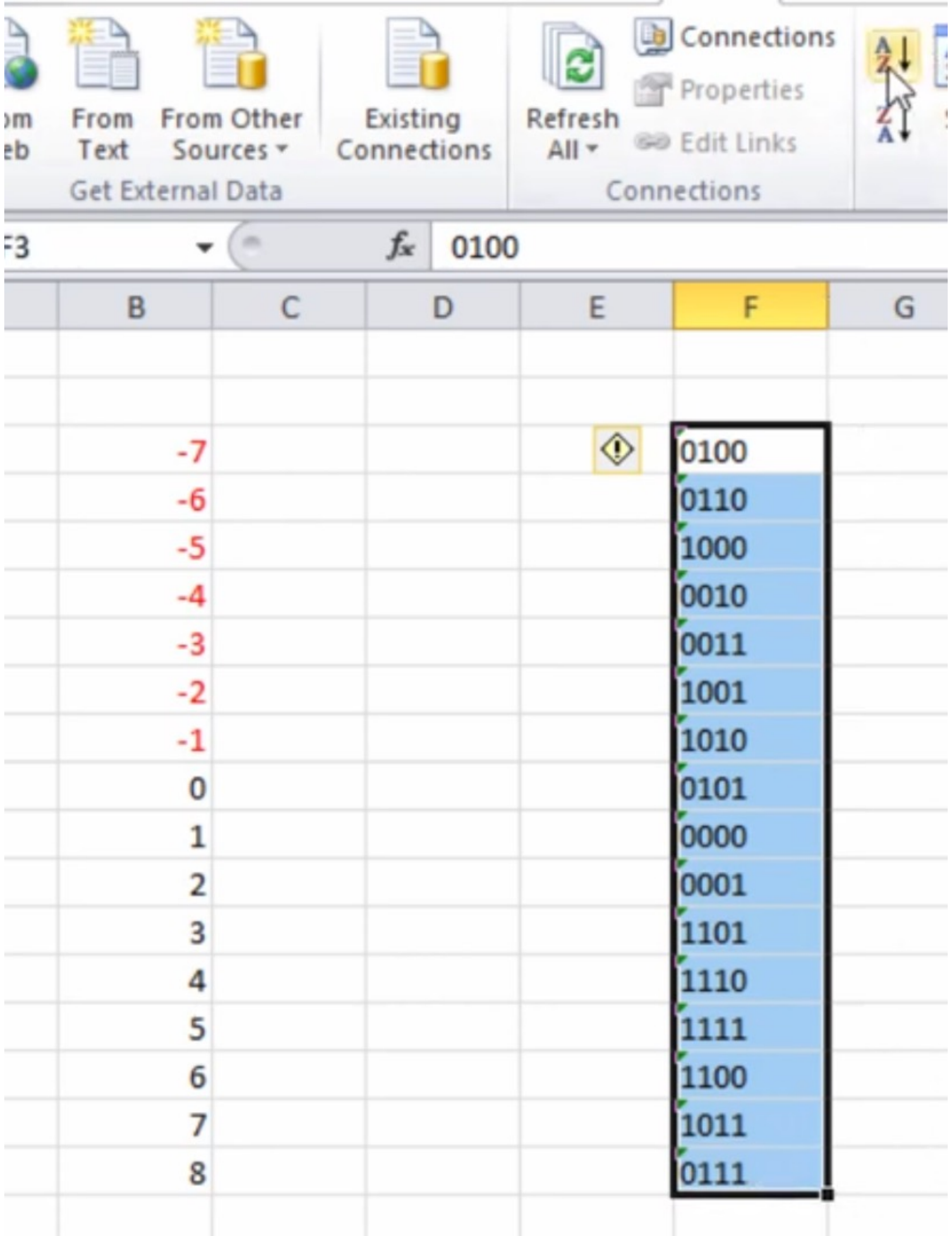
Get External Data

3

fx

B	C	D
-7		0000
-6		0001
-5		0010
-4		0011
-3		0100
-2		0101
-1		0110
0		0111
1		1000
2		1001
3		1010
4		1011
5		1100
6		1101
7		1110
8		1111

-7				0100
-6				0110
-5				1000
-4				0010
-3				0011
-2				1001
-1				1010
0				0101
1				0000
2				0001
3				1101
4				1110
5				1111
6				1100
7				1011
8				0111


From Text
From Other Sources
Existing Connections
Get External Data
Refresh All
Connections
Properties
Edit Links
Connections
F3
0100
B
C
D
E
F
G
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
0100
0110
1000
0010
0011
1001
1010
0101
0000
0001
1101
1110
1111
1100
1011
0111

-7	0000
-6	0001
-5	0010
-4	0011
-3	0100
-2	0101
-1	0110
0	0111
1	1000
2	1001
3	1010
4	1011
5	1100
6	1101
7	1110
8	1111

Exercise 19

Convert 0.09375 into IEEE 754 single precision floating point binary

Convert -123.3 into IEEE 754 single precision floating point binary

Exercise 20

Convert 01000010011010100000000000000000 into denary

Step 1: Determine the sign in denary

Convert 10000100100011001000000000000000 into denary