Ans. Fourier's theorem : According to this theorem "Any periodic function can be expressed as a sum of a number of sine and cosine functions having frequencies which are multiples of that of the given function."

Fourier's theorem cab be mathematically expressed as—

$$y = A_0 + A_1 \cos \omega t + A_2 \cos 2\omega t + \ldots\ldots$$
$$+ A_n \cos n\omega t + B_1 \sin \omega t + B_2 \sin 2\omega t$$
$$+ \ldots\ldots + B_n \sin n\omega t \qquad \ldots(1)$$

where y is the displacement of a complex periodic motion of frequency $\omega / 2\pi$ and A_0 is a constant and is a measure of this displacement of the vibration curve from the axis of co-ordinates.

In order to use this theorem for the analysis of a complex wave, the three constants A_0, A_n and B_n have to be evaluated first.

Evaluation of the constant A_n : For the evaluation of A_0 we integrate the equation (1) with respect to t, for a complex vibration (of period $T = 2\pi/\omega$ and we get,

$$\int_0^T y\,dt = A_0\int_0^T dt + An\int_0^T \cos n\omega t\,dt + B_n\int_0^T \sin n\omega t\,dt.$$

But for a complete cycle the values of the two integrals, i,e.,of

$\int_0^T \cos n\omega t\,dt$ and $\int_0^T \sin n\omega t\,dt$. are each equal to zero.

$$\therefore \quad \int_0^T y\,dt = A_0\int_0^T dt = A_0 T$$

$$\text{or,} \quad A_0 = \frac{1}{T}\int_0^T dt \qquad \ldots(2)$$

Evaluation of constant A_n : For the evaluation of A_n we multiply both the sides of equation (1) by $\cos n\omega t\,dt$ and then integrate both the sides with respect to t for a complete period of vibration T.

$$\therefore \quad \int_0^T y\cos n\omega t\,dt = A_0\int_0^T \cos n\omega t\,dt + \ldots..$$

$$+A_n\int_0^T \cos^2 n\omega t\,dt + \ldots.$$

$$+B_n\int_0^T \sin n\omega t.\cos n\omega t.dt.$$

But for a complete cycle

$$\int_0^T \sin n\omega t.\cos n\omega t\,dt = 0 \text{ and } \int_0^T \cos n\omega t\,dt = 0 \text{ etc.}$$

$$\therefore \quad \int_0^T y\cos n\omega t\,dt = A_n\int_0^T \cos^2 n\omega t\,dt$$

$$= A_n\int_0^T \frac{1+\cos 2n\omega t}{2}dt = \frac{A_n}{2}\int_0^T (1+\cos 2n\omega t)\,dt$$