24.11 TOTAL ENERGY DENSITY IN ELECTROMAGNETIC WAVES

Since an electromagnetic wave consists of both components, the electric field vector $\vec{E}$ and magnetic field vector $\vec{B}$ (or $\vec{H}$), the total energy density is the sum of electrostatic and magnetic energy density.

For a plane polarized electromagnetic wave travelling along X- direction, the two components E_z and H_y are related by the equation

$$\frac{Ez}{Hy} = \sqrt{\frac{\mu}{\epsilon}} \qquad \text{... } (i)$$

where μ is the magnetic permeability and ϵ, the electric pemittivity of the dielectric medium. Squaring Eq. (*i*), we get

$$\epsilon E_z^2 = \mu H_y^2$$

Now the quantity $\in E_z^2$ has the dimension of $\frac{\text{Farads}}{\text{metre}} \times \frac{\text{Volts}^2}{\text{metre}^2} = \frac{\text{Joule}}{\text{metre}^3}$ *i.e.* energy per unit volume.

Therefore, the quantity $\frac{1}{2} \in E_z^2$ is the electrostatic energy per unit volume for a dielectric and $\frac{1}{2}\mu H_y^2$ is the magnetic energy per unit volume. We know that energy stored per unit volume is called the energy density (U).

Hence,

$$\text{Electrostatic energy density, } U_e = \frac{1}{2} \in E_z^2$$

$$\text{Magnetic energy density, } U_m = \frac{1}{2}\mu H_y^2$$

$\therefore$ Total energy density of a plane-polarized E.M wave will be

$$U_p = \frac{1}{2} \in E_z^2 + \frac{1}{2}\mu H_y^2 \qquad \text{... (ii)}$$

In general, for an electromagnetic wave

$$U = \frac{1}{2} \in E^2 + \frac{1}{2}\mu H^2 \qquad \text{... (iii)}$$

Moreover, it is observed that electrostatic energy density U_e is equal to the magnetic energy density U_m.

$$\therefore \quad \frac{U_e}{U_m} = \frac{\frac{1}{2} \in E^2}{\frac{1}{2}\mu H^2} = \frac{\in}{\mu}\frac{E^2}{H^2}$$

$$\text{but} \quad \frac{E}{H} = \sqrt{\frac{\mu}{\in}} \quad \text{or} \quad \frac{E^2}{H^2} = \frac{\mu}{\in}$$

$$\therefore \quad \frac{U_e}{U_m} = \frac{\in}{\mu} \cdot \frac{\mu}{\in} = 1$$

$$\text{or} \quad U_e = U_m \qquad \text{... (iv)}$$

From Eq (*iii*), we have

$$U = U_e + U_m = \frac{1}{2} \in E^2 + \frac{1}{2}\mu H^2$$

$$= \frac{1}{2} \in E^2 + \frac{1}{2} \in E^2 = \in E^2$$

$$= (\in_r)(\in_0 E^2) \qquad [\because \in = \in_0 . \in_r]$$

$$= \in_r \times [\text{energy density in free space}] \qquad \text{... (v)}$$

Thus, the energy density in a dielectric is $\in_r$ times the energy denisty of the same wave in vacuum (*i.e.* free space).

24.12 REFLECTION AND TRANSMISSION OF ELECTROMAGNETIC WAVES

Consider an electromagnetic wave incident normally on an infinite plane boundary separating two media of impedances Z_1 and Z_2. These impedances will be real for dielectric media and complex quantities for conducting media. The electric field vectors for the incident, reflected and transmitted waves are indicated as E_i, E_r and E_t and the corresponding values for the magnetic field vectors, H_i, H_r and H_t respectively. The vector direction $\vec{E}_r \times \vec{H}_r$ must be opposite to that of $\vec{E}_i \times \vec{H}_i$ to satisfy the energy flow condition of poynting vector. Therefore, if the incident wave is travelling along the positive direction of X-axis, the reflected wave will travel in the negative direction of *X*-axis *i.e.*, the incident wave will be reflected normally backward. If Z_2 is less than Z_1 it is the electric vector which is reversed and if Z_2 is greater than Z_1 the magnetic vector will be reversed. We are considering a case $Z_2 < Z_1$.

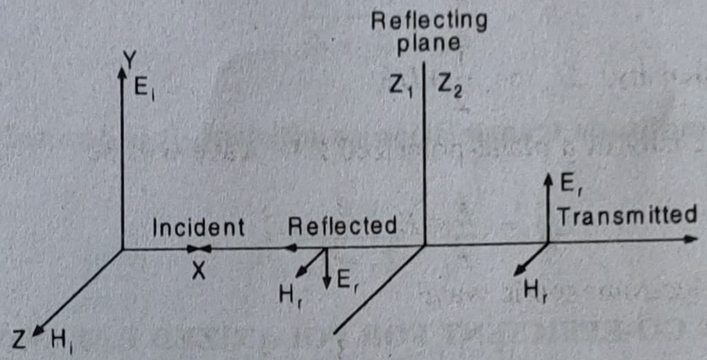

Fig. 24.4

According to boundary conditions from electromagnetic theory, the components of field vectors $\vec{E}$ and $\vec{H}$ tangential or parallel to the boundary are continuous across the boundary.

$$\therefore \qquad E_i + E_r = E_t \qquad \text{... (i)}$$

and

$$H_i + H_r = H_t \qquad \text{... (ii)}$$

Again

$$Z_1 = \frac{E_i}{H_i}, -Z_1 = \frac{E_r}{H_r} \text{ and } Z_2 = \frac{E_t}{H_t}$$

or

$$H_i = \frac{E_i}{Z_1}, H_r = -\frac{E_r}{Z_1} \text{ and } H_t = \frac{E_t}{Z_2}$$

Substituting in (*ii*) we have

$$\frac{E_i}{Z_1} - \frac{E_r}{Z_1} = \frac{E_t}{Z_2}$$

or

$$E_i - E_r = E_t \frac{Z_1}{Z_2} \qquad \text{... (iii)}$$

Dividing (*i*) by (*iii*), we have

$$\frac{E_i + E_r}{E_i - E_r} = \frac{Z_2}{Z_1} \qquad \text{... (iv)}$$

By componendo and dividendo

$$E_r^2 = E_i^2 - \frac{\eta_1}{\eta_2} \cdot \frac{E_t^2 \cos\theta_2}{\cos\theta_1}$$

Dividing through out by E_i^2, we get

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\eta_1 E_t^2 \cos\theta_2}{\eta_2 E_i^2 \cos\theta_1}$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\epsilon_2}\, E_t^2 \cos\theta_2}{\sqrt{\epsilon_1}\, E_i^2 \cos\theta_1} \qquad \text{... } (ii)$$

Horizontal Polarization

In this case, the electric vector $\vec{E}$ is perpendicular to the plane of incidence and parallel to the reflecting surface. Let the electric field strength E_i of the incident wave be in the + ve X-direction, then applying boundary condition that the tangential component of $\vec{E}$ is continuous across the boundary,

$$E_i + E_r = E_t$$

Dividing by E_i, we get

$$\frac{E_t}{E_i} = 1 + \frac{E_r}{E_i} \qquad \text{... } (iii)$$

Substituting in equation (ii), we have

$$\frac{E_r^2}{E_i^2} = 1 - \sqrt{\frac{\epsilon_2}{\epsilon_1}} \left(1 + \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_2}{\cos\theta_2}$$

$$1 - \frac{E_r^2}{E_i^2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \left(1 + \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_2}{\cos\theta_1}$$

$$\left(1 - \frac{E_r}{E_i}\right)\left(1 + \frac{E_r}{E_i}\right) = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \cdot \left(1 + \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_2}{\cos\theta_1}$$

$$1 - \frac{E_r}{E_i} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \cdot \left(1 + \frac{E_r}{E_i}\right) \frac{\cos\theta_2}{\cos\theta_1}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\epsilon_1}\cos\theta_1 - \sqrt{\epsilon_2}\cos\theta_2}{\sqrt{\epsilon_1}\cos\theta_1 + \sqrt{\epsilon_2}\cos\theta_2} \qquad \text{... } (iv)$$

From equation (i),

$$\sqrt{\epsilon_2}\cos\theta_2 = \sqrt{\epsilon_2 (1 - \sin^2\theta_2)} = \sqrt{\epsilon_2 - \epsilon_2 \sin^2\theta_2}$$

$$\sqrt{\epsilon_2}\cos\theta_2 = \sqrt{\epsilon_2 - \epsilon_1 \sin^2\theta_1} \qquad (\because \epsilon_2 \sin^2\theta_1 = \epsilon_1 \sin^2\theta_1)$$

$$\therefore \quad \frac{E_r}{E_i} = \frac{\sqrt{\epsilon_1}\cos\theta_1 - \sqrt{\epsilon_2 - \epsilon_1\sin^2\theta_1}}{\sqrt{\epsilon_1}\cos\theta_1 + \sqrt{\epsilon_2 - \epsilon_1\sin^2\theta_1}} \qquad \text{... (v)}$$

Thus, reflection co-efficient is given by

$$R = \frac{E_r}{E_i} = \frac{\cos\theta_1 - \sqrt{\left(\frac{\epsilon_2}{\epsilon_1}\right) - \sin^2\theta_1}}{\cos\theta_1 + \sqrt{\left(\frac{\epsilon_2}{\epsilon_1}\right) - \sin^2\theta_1}} \qquad \text{... (vi)}$$