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Evaluating the importance of nodes in complex networks

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HIGHLIGHTS

- A Node Importance ranking method (DIL) is proposed based on local information.
- The importance of line is considered to evaluate the importance of node.
- DIL can well identify the importance of nodes especially the bridge nodes.
- DIL can be used in large-scale networks with lower computational complexity.

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ABSTRACT

Evaluating the importance of nodes for complex networks is of great significance to the research of survivability and robusticity of networks. This paper proposes an effective ranking method based on degree value and the importance of lines. It can well identify the importance of bridge nodes with lower computational complexity. Firstly, the properties of nodes that are connected to a line are used to compute the importance of the line. Then, the contribution of nodes to the importance of lines is calculated. Finally, degree of nodes and the contribution of nodes to the importance of lines are considered to rank the importance of nodes. Five real networks are used as test data. The experimental results show that our method can effectively evaluate the importance of nodes for complex networks.

1. Introduction

Complex network is an important research field of complexity science [1]. Network science has greatly developed in the past ten years, and is currently a leading scientific field in the description of complex networks [2]. Recently, with the construction of smart city, the application of complex network is more and more popular. Complex networks can be seen everywhere [3,4], such as traffic network [5–7], power grid [8,9], social network [10], etc. These networks provide great convenience for our life. On the other hand, it will lead to great damage of the whole networks in most cases due to a key node's failure. It is well known that many mechanisms such as spreading, cascading, and synchronizing are highly affected by a tiny fraction of key nodes [11–16]. In recent years, a series of cascading large blackouts have occurred in some countries, which caused huge social economy loss. In 2003, the August 14th blackout in the United States and Canada caused global attention. But identifying and protecting the key nodes of power grid could have prevented the blackout. Similarly, at the beginning of 2008, an infrequent and serious ice cover disaster in southern China led to large-scale blackout owing to great damage to the main transmission lines and the key towers. Therefore, identifying key nodes of networks is of great

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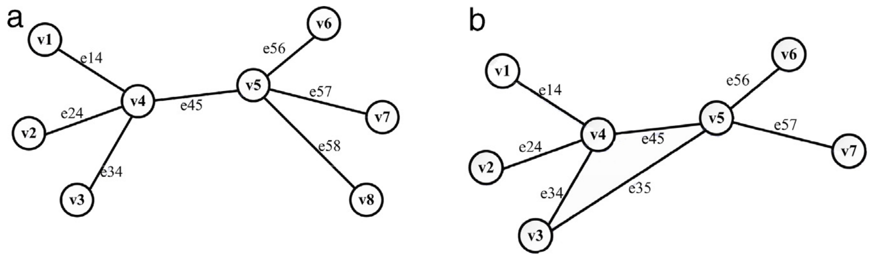


Fig. 1. The topologies of two simple networks.

significance. For example, in the spread of the virus network, it can help to find and control the key dissemination source so as to prevent further spread of the virus. In large-scale computer network, it is significant to backup servers and make redundant design according to the importance of servers, which can not only improve the robustness of computer network but also save the resources. In a word, identifying the key nodes can help to enhance the survivability and robusticity of complex networks by protecting key nodes of complex networks [17].

Identifying key nodes in complex networks has attracted increasing attention in recent years [18–25]. There are many methods to evaluate the importance of nodes [26]. Degree Centrality (DC), Betweenness Centrality (BC) and Closeness Centrality (CC) are the methods commonly used in complex networks [27]. DC emphasizes the number of lines linked to the node directly. It can explain the importance of nodes to some extent, but nodes owning the same degree may not play the same important role in a complex network. In addition, a bridge node connecting two important nodes is also very important though its degree is lower. BC describes the abilities of nodes or lines to control the information of networks. It requires that information should spread through the shortest way, but sometimes the information does not spread through the shortest way in most real networks. CC reflects distance between one node and the others, which cannot reflect the importance of nodes comprehensively. Moreover, BC and CC need the global information to calculate the shortest path of any pair of nodes. So the algorithm's time complexity is very high, and also the global information is not readily available in large-scale network. It is more significant to explore a method to calculate node's importance using local information. Many researchers put forward some new methods to improve accuracy of node importance ranking. Considering that the importance of nodes is related to the degrees of their neighborhood nodes in addition to the degree of the nodes themselves, Wang et al. [28] proposed a method based on degree and degree of their neighborhoods (DDN). But DDN cannot effectively identify the importance of the bridge node. Chen et al. [19] proposed a semi-local centrality measure (LC) and it ignores the importance of bridge nodes similarly. Ren et al. [29] present their research based on degree and clustering coefficient (DCC) to measure the node importance. DCC not only takes into account the neighbor size but also measures the closeness among the neighbors. However, it cannot effectively identify the importance of bridge node, either.

In view of the above, we would like to propose a new ranking method based on local information instead of global information. And it is expected to well identify the importance of bridge nodes. To evaluate the algorithmic performance, we use the decline rate of network efficiency to examine the importance of the nodes ranked by different centrality measures. The simulations on five real networks prove that our method can well identify the key nodes. Comparing with DC, DDN, DCC and LC, the current method can well identify the importance of bridge node. Meanwhile, comparing with BC and CC, it requires local information instead of global information. So it can be used in large-scale network without the global information. Furthermore, we research the relations between the current method and BC, CC and LC.

2. Measurement of node importance based on degree and the importance of lines (DIL)

We assume that a network $G = (V, E)$ is an undirected and unweighted network with $N = |V|$ nodes and $M = |E|$ edges. $V = \{v_1, v_2, \cdots, v_N\}, E = \{e_1, e_2, \ldots, e_M\}$.

The line between node v_m and node v_n is e_{mn}. Its importance is defined as

$$I_{emn} = \frac{U}{\lambda}, \tag{1}$$

where, $U = (k_m - p - 1) \cdot (k_n - p - 1)$ reflects the connectivity ability of line e_{mn}, k_m is the degree of node v_m, k_n is the degree of node v_n, p is the number of triangle, one edge of the triangle is e_{mn}. λ is alternative index of line e_{mn}, which is defined as $\lambda = \frac{p}{2} + 1$.

Fig. 1 shows two simple networks, and the method above is used to evaluate the importance of line e_{45}. In the network of Fig. 1(a):

$$p = 0, \qquad U = (4 - 1) \cdot (4 - 1) = 9, \qquad \lambda = 1, \qquad I_{e45} = \frac{U}{\lambda} = 9.$$

In the network of Fig. 1(b):

$$p = 1, \qquad U = (4 - 1 - 1) \cdot (4 - 1 - 1) = 4, \qquad \lambda = \frac{1}{2} + 1 = 1.5, \qquad I_{e45} = \frac{U}{\lambda} = \frac{8}{3} \approx 2.6667.$$

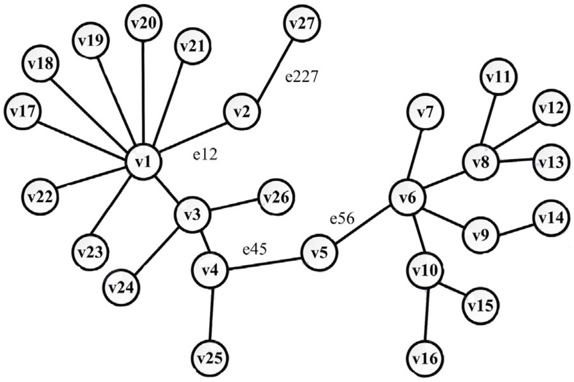


Fig. 2. The topology of a simple network.

One can see that the endpoints of e_{45} own the same degree in the two networks. However, calculating results show that line e_{45} in Fig. 1(a) is more important than line e_{45} in Fig. 1(b). In fact, there are nine combination ways between v_1 v_2 v_3 and v_6 v_7 v_8 that are connected by the line e_{45} of network (a), and the line e_{45} of network (a) is irreplaceable. But there are only four combination ways between v_1 v_2 and v_6 v_7 that are connected by the line e_{45} of network (b), and the line e_{45} of network (b) can be replaced by e_{34} e_{35}. So the calculation results are reasonable.

Generally, the more important a line is, the more important the node connected to the line is. But some measures must be taken if one wants to get more suitable result.

In Fig. 2, the degree of v_2 is equal to the degree of v_5, e_{12} and e_{227} are the lines which are connected to v_2 directly. According to Eq. (1)

$$I_{e12} = \frac{U}{\lambda} = 8, \qquad I_{e227} = \frac{U}{\lambda} = 0$$

I_{e12} stands for the importance value of e_{12}, I_{e227} stands for the importance value of e_{227}.

For the same reason, e_{45} and e_{56} are the lines which are connected to v_5 directly.

$$I_{e45} = \frac{U}{\lambda} = 2, \qquad I_{e56} = \frac{U}{\lambda} = 4.$$

According to the above, one can easily get the following result:

$$I_{e12} + I_{e227} = 8, \qquad I_{e45} + I_{e56} = 6.$$

It seems that v_2 is more important than v_5. But from Fig. 2 one can see that v_5 is more important than v_2. This is because v_5 is a bridge node, and v_5 is more closer to the network center than v_2. So this is not a reasonable result if we simply sum up the importance value of edges which are connected to the node directly.

In order to get a more reasonable result, we introduce $W_{v_i v_j}$.

$$W_{v_i v_j} = I_{eij} \cdot \frac{k_i - 1}{k_i + k_j - 2}, \tag{2}$$

where, $W_{v_i v_j}$ stands for the contribution that v_i makes to the importance of e_{ij}, k_i is the degree of v_i, k_j is the degree of v_j, v_j is the neighborhood node of v_i.

The importance of node v_i is defined thus:

$$L_{vi} = k_i + \sum_{v_j \in \Gamma_i} W_{v_i v_j}, \tag{3}$$

where Γ_i is the set of neighbors of v_i.

Here, we use the above method to evaluate the importance of node in Fig. 2 again.

$$L_{v2} = \frac{26}{9}, \qquad L_{v5} = \frac{52}{15}, \quad L_{v5} > L_{v2}.$$

From the calculation result, one can see that node v_5 is more important than node v_2, and that satisfies the fact. In order to highlight the effectiveness of this new method, we rank the importance of v_2 and v_5 using DC, DDN, DCC and LC. According to DC, v_2 and v_5 are equally important. However, according to DDN, DCC and LC, v_2 is more important than v_5. It is contrary to the fact.

3. Evaluation

There are two kinds of methods which are used to evaluate the accurate of ranking method [3]. One is based on transmission dynamics, and the other is based on the theory that the network damage caused by deleting node is equivalent to the importance of node. The latter is used in this paper to evaluate the ranking method.

Network efficiency [29] reflects the network connectivity. The better the network connectivity is, the better the network efficiency is. Network efficiency η is defined thus:

$$\eta = \frac{1}{N(N-1)} \sum_{v_i \neq v_j \in V} \eta_{ij}, \tag{4}$$

where η_{ij} is the efficiency between v_i and v_j, $\eta_{ij} = 1/d_{ij}$, d_{ij} is the shortest way between v_i and v_j, N is the number of network nodes.

μ is the decline rate of network efficiency, which is defined as

$$\mu = 1 - \frac{\eta}{\eta_0}, \tag{5}$$

where η is the efficiency of the network which is attacked by removing node. η_0 is the initial efficiency of the network.

The bigger the μ is, the worse the network connectivity destroyed by removing nodes is and the more important the node removed is.

4. Experiments and discussions

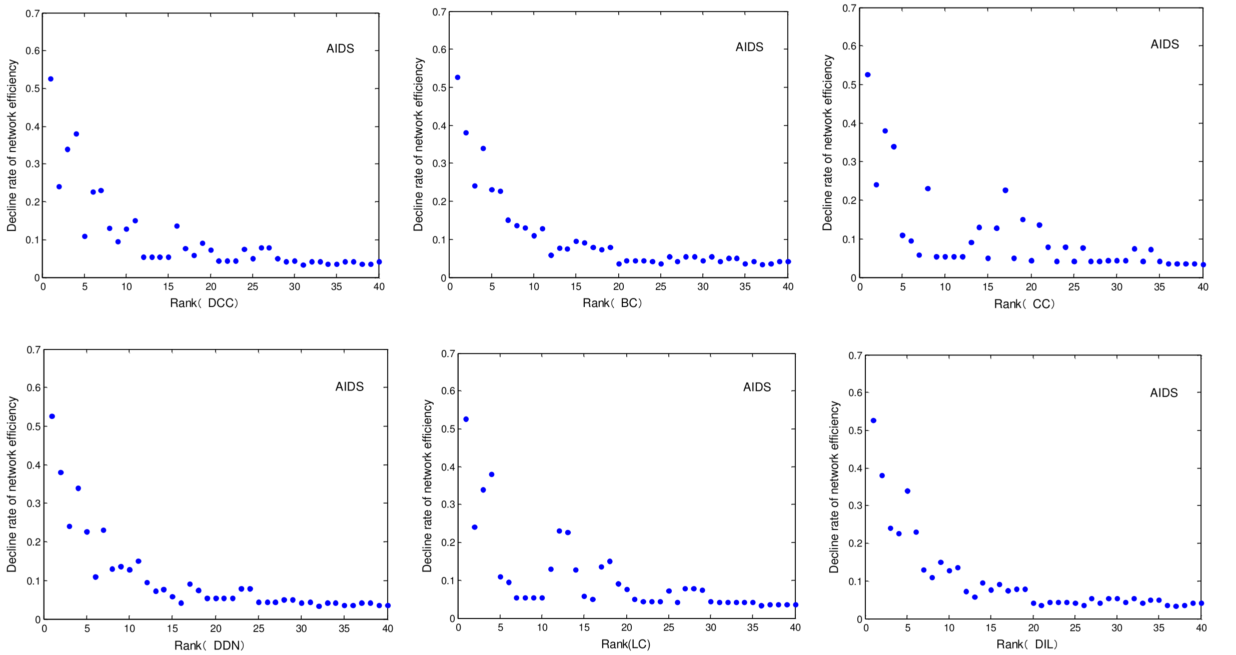
4.1. Data sources

Five networks are introduced to validate the proposed approach. The selected networks are AIDS patients' sexual relationship network [28], dolphins' social network [30], USA airport network [31], email network [19] and the ARPA (Advanced Research Project Agency) network [32]. There are 40 nodes in AIDS patients' sexual relationship network, the nodes represent the AIDS patients, and the lines between nodes represent the sexual relationship between the two AIDS patients. There are 62 nodes in dolphins' social network, the nodes represent dolphins and the lines between nodes represent associations between dolphin pairs occurring more often than expected by chance. There are 500 nodes in USA airport network, the nodes represent the airport, and the lines between nodes represent that there are flights between the two airports. There are 1133 nodes and 10 904 lines in email network.

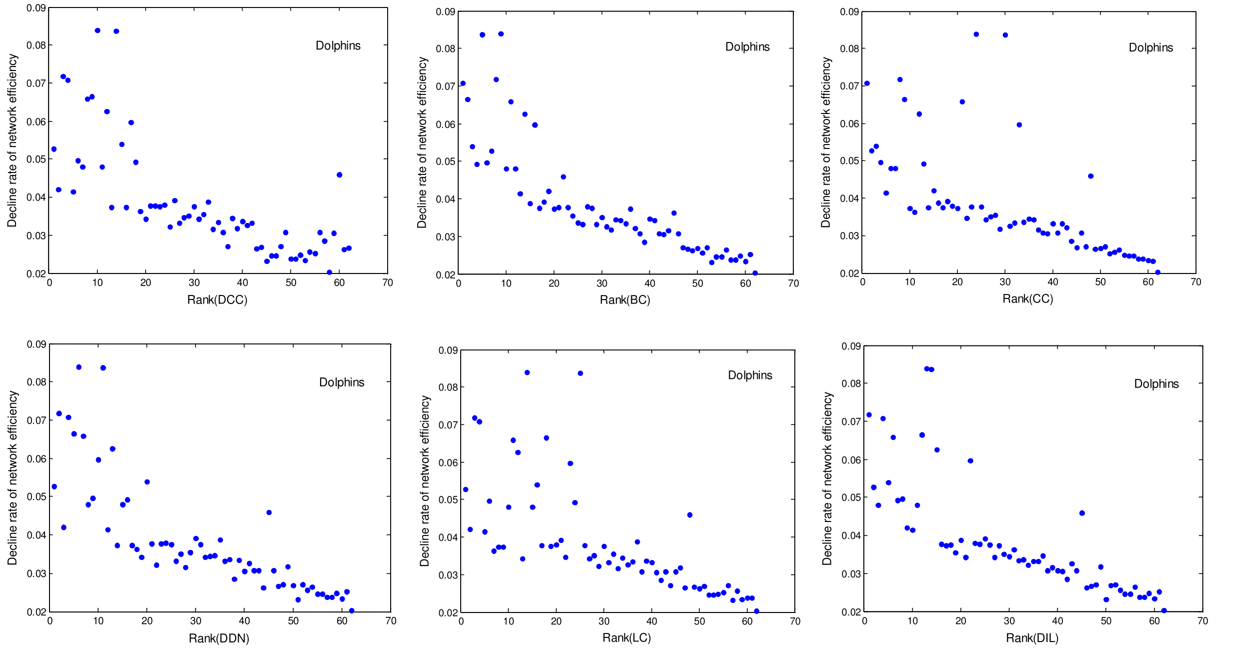
4.2. Experimental analysis

In our research, we compare the proposed DIL method with BC, CC, DDN, DCC and LC on some real networks, including AIDS patients' sexual relationship network, dolphins' social network, USA airport network and email network. In each implementation only one node is removed according to the importance ranking list. And then we calculate the decline rate of the network efficiency. After n implementations, we investigate the relation between the decline rate of the network efficiency and the importance of node measured by five ranking methods. In principle, the more important the node removed is, the bigger the decline rate of the network efficiency is. In Fig. 3, it shows the experimental results of six ranking methods on four networks. In AIDS, there is a clear correlation between the importance of node and the decline rate of the network efficiency for BC and DIL method, and CC performs worst. In Dolphins, in most cases, the decline rate of the network efficiency is descending with the decline of the importance of the node removed for DIL, BC and CC, and DIL performs better. In Airport, there is no clear correlation between the decline rate of the network efficiency and the importance of node. Similarly, there is no clear correlation between the decline rate of the network efficiency and the importance of node in email network. Comparatively, BC and DIL perform better. In a word, by testing the correlation between the importance of node and the decline rate of the network efficiency, it is true that DIL performs competitively well.

To evaluate the performance of the DIL method, the top ten percent of the nodes are removed one by one from the networks according to the importance ranking lists produced by DCC, BC, CC, DDN, LC and DIL, which can cause a decline in network efficiency. And we calculate the decline rate of network efficiency. In Fig. 4, it shows the relationship between the decline rate of network efficiency and the number of nodes removed from the networks. From Fig. 4, one can see that the decline rate of efficiency is rising with the increase of the number of nodes removed. In AIDS and dolphins networks, DIL performs the best among all six methods. The detail information is shown in Tables 1 and 2. In airport network, BC performs the best before the sixteenth node is removed, and then DIL performs the best with the increase of the number of nodes removed. This implies that DIL identifies the vast majority of important nodes. In email network, BC performs the best. However, based on local information, DIL can still give comparatively good performance with lower computational complexity, and is much better than DCC, CC, DDN and LC. From Fig. 4, one can also see that the four networks show different invulnerability with the top 10% important nodes deleted. The dolphins' social network performs the best, the



(a) AIDS patients' sexual relationship network.

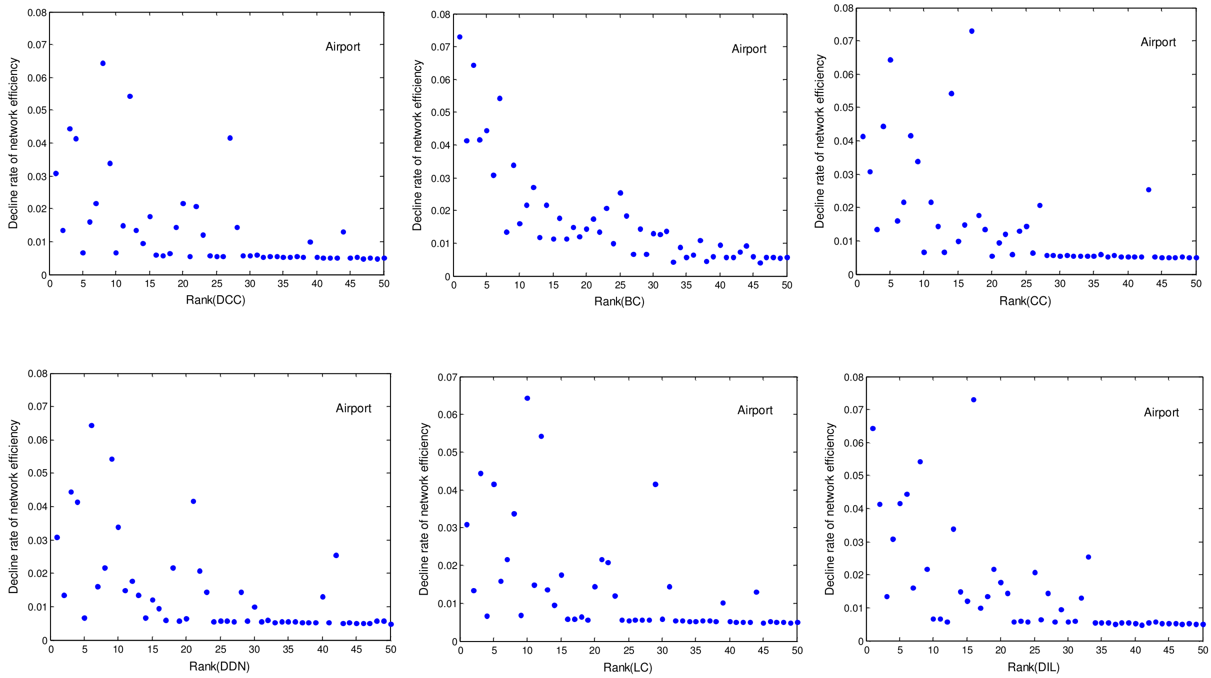


(b) Dolphins' social network.

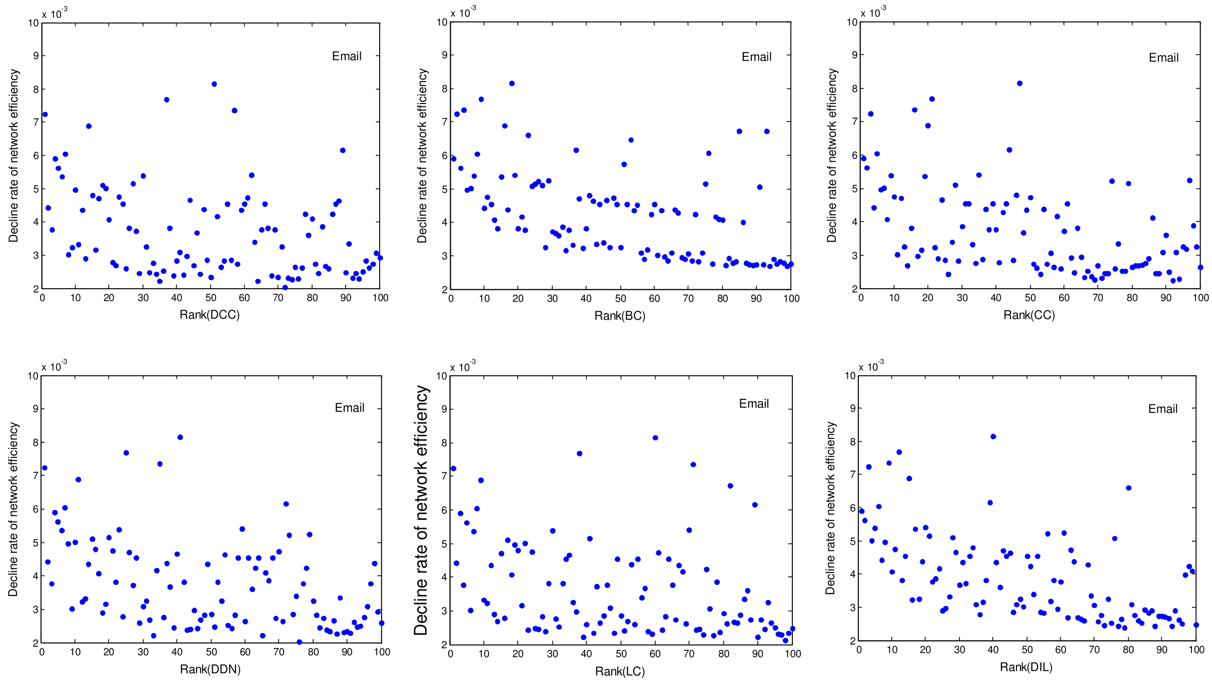
Fig. 3. The relation between decline rate of network efficiency and the importance ranking of nodes evaluated by DCC, BC, CC, DDN, LC and DIL methods on four networks.

decline rate of network efficiency is less than 37%, and the USA airport network performs the worst, the decline rate of network efficiency is higher than 90%. It is reasonable because networks of different topology structures have different invulnerability.

In addition, we investigate the relations between DIL and other methods. Fig. 5 shows the relation between these methods on four real networks. In Fig. 5, each point indicates a node in the network, and its color represents the decline rate of the network efficiency caused by removing the node. τ is used to measure the correlation between DIL and BC, CC and LC. τ is



(c) USA airport network.



(d) Email network.

Fig. 3. (*continued*)

defined as [19]:

$$\tau = \frac{N_c - N_d}{\frac{1}{2}N(N-1)},$$

where N_c and N_d are the number of concordant and discordant pairs, respectively. N is the number of network nodes. Let $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ be a set of joint observations from two random variables X and Y respectively. Any pair of observations (x_i, y_i) and (x_j, y_j) are said to be concordant if both $x_i > x_j$ and $y_i > y_j$ or if both $x_i < x_j$ and $y_i < y_j$. They are

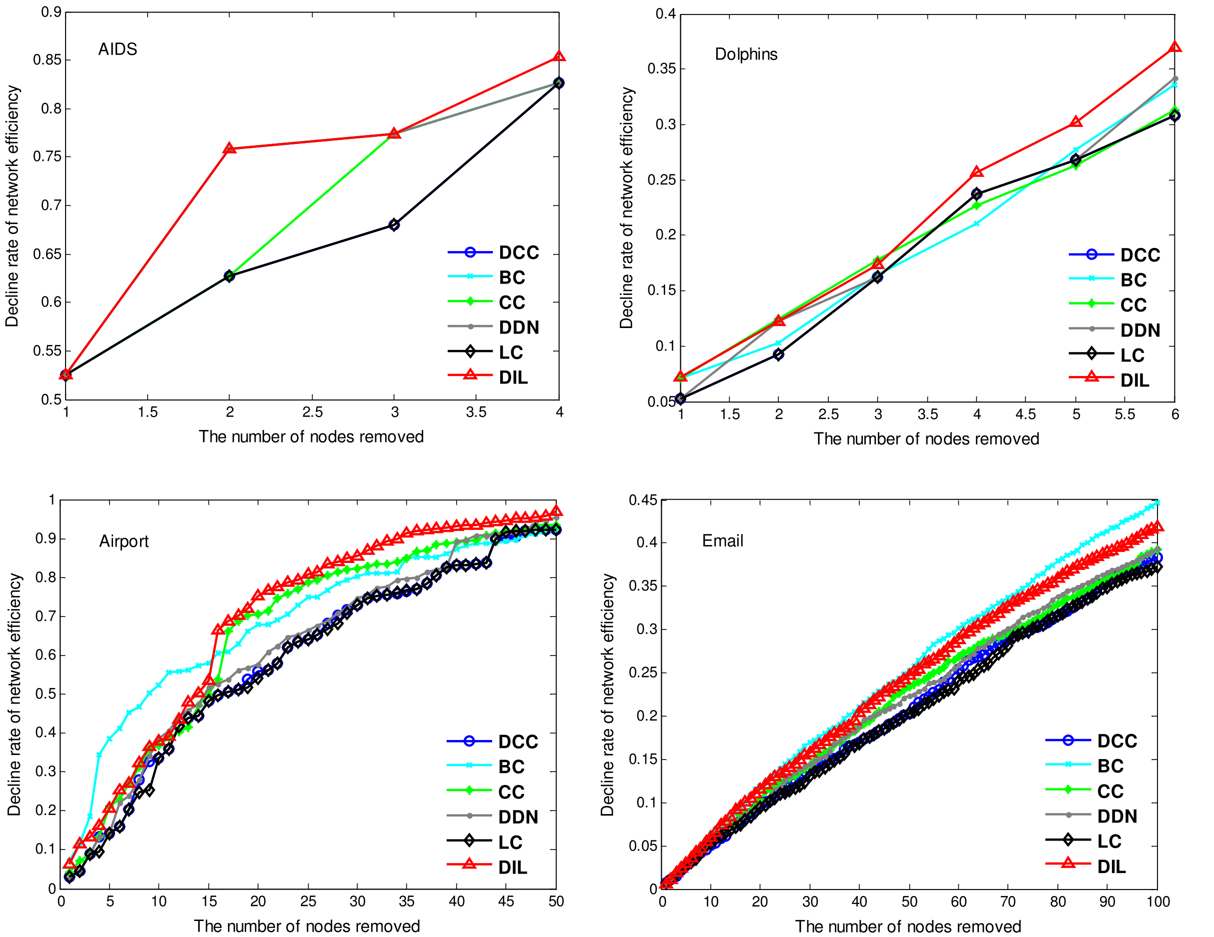


Fig. 4. The relation between decline rate of network efficiency and the number of nodes removed from the network. The top ten percent of the nodes are removed one by one from four real networks (AIDS, Dolphins, Airport and Email) according to the importance ranking lists produced by DCC, BC, CC, DDN, LC and DIL.

Table 1
Decline rate of efficiency for AIDS network with top 4 important nodes removed.

Method	1	2	3	4
DCC	0.5257	0.6270	0.6800	0.8267
BC	0.5257	0.7590	0.7737	0.8267
CC	0.5257	0.6270	0.7737	0.8267
DDN	0.5257	0.7590	0.7737	0.8267
LC	0.5257	0.6270	0.6800	0.8267
DIL	0.5257	0.7590	0.7737	0.8539

Table 2
Decline rate of efficiency for Dolphins network with top 6 important nodes removed.

Method	1	2	3	4	5	6
DCC	0.0526	0.0930	0.1627	0.2370	0.2682	0.3080
BC	0.0709	0.1031	0.1648	0.2102	0.2769	0.3362
CC	0.0709	0.1241	0.1778	0.2273	0.2629	0.3132
DDN	0.0526	0.1228	0.1627	0.2370	0.2676	0.3424
LC	0.0526	0.0930	0.1627	0.2370	0.2682	0.3080
DIL	0.0718	0.1228	0.1736	0.2572	0.3022	0.3698

said to be discordant, if $x_i > x_j$ and $y_i < y_j$ or if $x_i < x_j$ and $y_i > y_j$. If $x_i = x_j$ or $y_i = y_j$, the pair is neither concordant nor discordant.

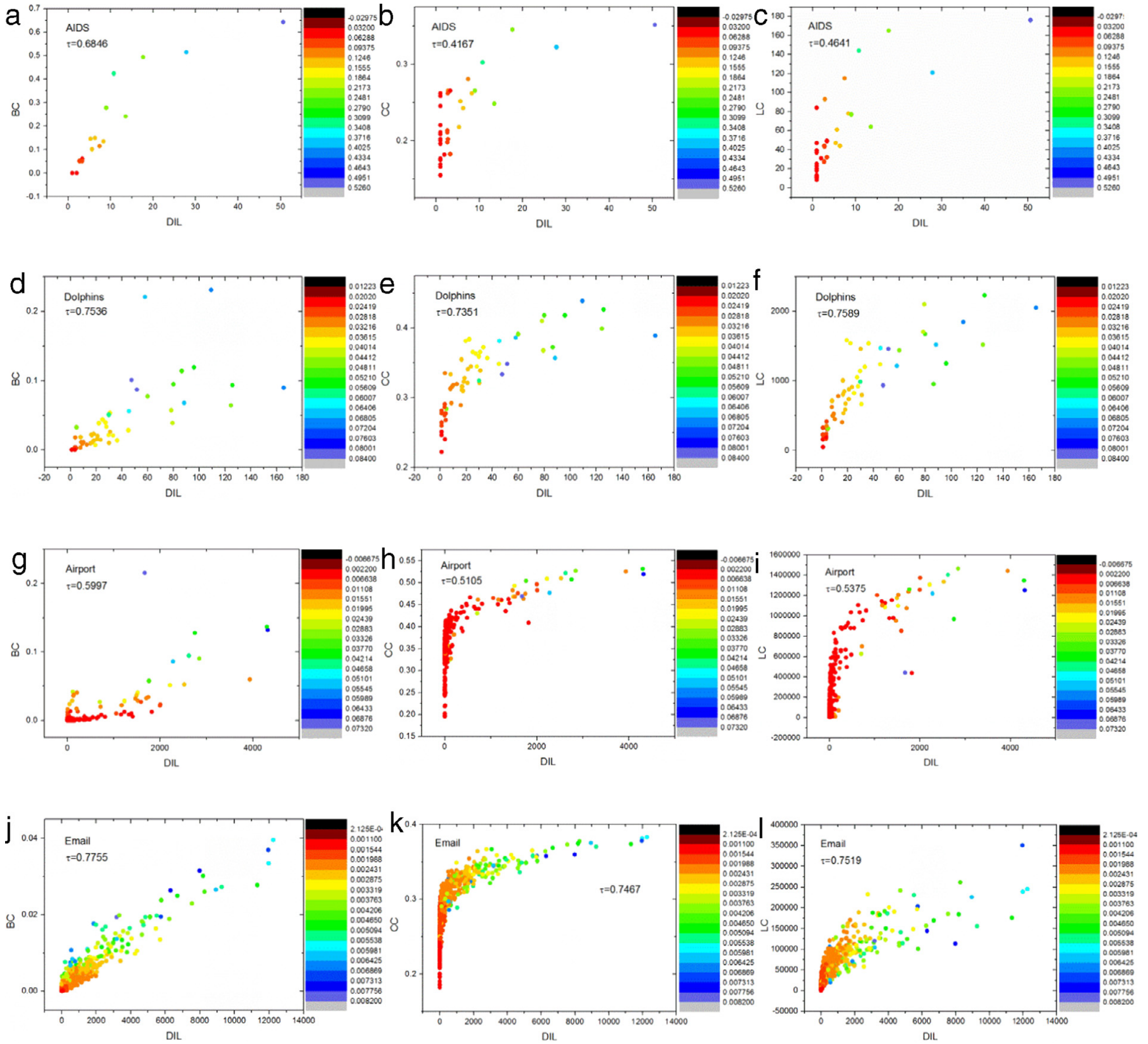


Fig. 5. The relations between DIL and BC, CC and LC on four real networks (AIDS, Dolphins, Airport, Email). Each data point denotes a node, and its color represents the decline rate of the network efficiency caused by removing the node. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

From Fig. 5(a)–(c), one can see that BC, CC and LC are all positively correlated with DIL in AIDS patients' sexual relationship network. Furthermore, BC is strongly positively correlated with DIL. And the node causing higher decline rate of network efficiency (as indicated by the color) tends to own larger value calculated by BC and DIL. There is the similar phenomenon in USA airport network and email network, which is shown in Fig. 5(g)–(i) and Fig. 5(j)–(l). In Dolphins' social network, BC, CC and LC are also positively correlated with DIL, and all these methods perform comparatively well. It indicates that it is easy to identify the key nodes in Dolphins' social network due to its special topology, and all these methods can work well on it. On the whole, the correlation between BC and DIL is stronger than other two cases (CC vs. DIL and LC vs. DIL). That is to say, the nodes with large betweenness centralities tend to have large values calculated based on DIL. However, different from BC, DIL is based on local information and owns lower computational complexity. To further illuminate the effectiveness of DIL, we apply it to analysis the ARPA (Advanced Research Project Agency) network, and compare the results with DC, BC and CC. In Fig. 6, it shows the topology graph of ARPA network. There are twenty-one nodes and twenty-three lines.

The ranking results of DIL, DC, BC and CC methods which evaluate the nodes importance of ARPA are listed in Table 3. From Table 3, one can see that four results are slightly different, mainly because the principle of each method assessing the nodes importance is different.

The ranking top six important nodes which are evaluated by the above four methods are removed from ARPA network. In Fig. 7, it shows the rest graph. Fig. 7(a) is the result produced by deleting the top six importance nodes which are evaluated

Table 3
The ranking results of DIL, DC, BC and CC methods on ARPA network.

DC		BC		CC		DIL	
ID	value	ID	value	ID	value	ID	value
2	4	3	0.3158	3	0.3571	3	15.25
3	4	12	0.2706	19	0.3448	14	14.35
14	4	19	0.2154	12	0.3390	2	11.55
6	3	6	0.2013	18	0.3333	12	7.6667
12	3	4	0.1794	4	0.3175	19	7.6667
15	3	14	0.1746	13	0.3175	6	7
19	3	13	0.1588	14	0.3175	15	5.9333
1	2	5	0.1566	17	0.3077	16	3.5
4	2	11	0.1557	2	0.2985	17	3.5
5	2	2	0.1491	20	0.2985	13	3.4167
7	2	18	0.1197	5	0.2941	18	3.4167
8	2	10	0.1075	6	0.2941	4	3.25
9	2	7	0.1066	11	0.2857	5	3.1667
10	2	20	0.0974	15	0.2778	7	3.1667
11	2	21	0.0719	16	0.2740	11	3.1667
13	2	9	0.0719	21	0.2703	20	3.1667
16	2	8	0.0689	1	0.2532	21	3.1667
17	2	17	0.0474	7	0.2532	8	3
18	2	15	0.0461	10	0.2532	9	3
20	2	16	0.0118	9	0.2326	10	3
21	2	1	0	8	0.2299	1	2

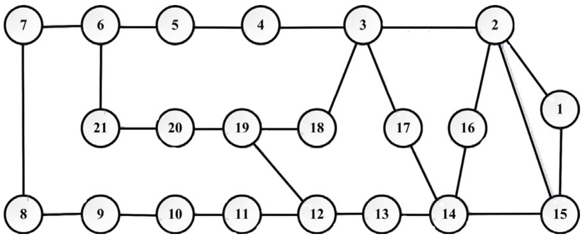


Fig. 6. The topology graph of ARPA network.

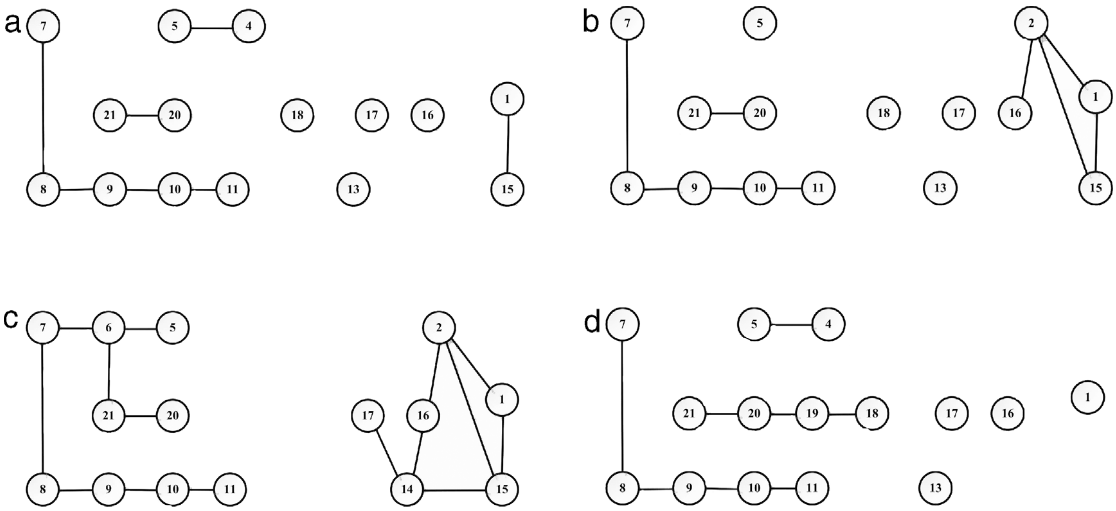


Fig. 7. ARPA network with the top six important nodes removed according to four ranking lists produced by four methods, including DIL, BC, CC and DC. (a) corresponds to DIL. (b) corresponds to BC. (c) corresponds to CC. (d) corresponds to DC.

by DIL. It can be seen that the graph is divided into eight parts after the top six importance nodes are removed. Fig. 7(b)–(d) are the graphs based on BC, CC and DC, respectively, which all do the same thing as Fig. 7(a). From Fig. 7, one can see that Fig. 7(b) and (d) are respectively divided into seven independent communities, and there are two independent communities owning more than three nodes. Fig. 7(c) is only divided into two independent parts. The contrasted results show that DIL is superior to other three methods in the nodes importance evaluation on ARPA network.

Table 4
The computational complexity of six methods.

Method	Information	Computational complexity
BC	Global information	$O(n^3)$
CC	Global information	$O(n^2)$
DCC	Local information	$O(m + n\langle k\rangle)$
DDN	Local information	$O(n\langle k\rangle^2)$
LC	Local information	$O(n\langle k\rangle^2)$
DIL	Local information	$O(n\langle k\rangle^2)$

4.3. Complexity analysis

In normal conditions, there are many nodes in real networks. So the node importance ranking methods should be high-efficiency in addition to reasonable. The computational complexity of the six methods is shown in Table 4, where n is the total number of nodes in network, m is the number of lines, $\langle k\rangle$ is the average degree of the network.

From Table 4, we can see that the computational complexity of DCC is $O(m + n\langle k\rangle)$, which is the lowest, but from the experimental results one can see that it performs the worst. The computational complexity of DIL is $O(n\langle k\rangle^2)$, which is equal to that of DDN and LC. And it is lower than that of BC and CC, which indicates that DIL has higher computation efficiency and can be used in large-scale networks. Moreover, the results of contrast experiments show that DIL can give a reasonable result.

5. Conclusions

Ranking the importance of nodes in complex networks is of theoretical and practical significance. In this paper, we proposed the DIL method to evaluate the importance of nodes in complex networks. The proposed method based on degree and the importance of lines only needs the local characteristics of nodes to evaluate the importance of the nodes. Comparing with the methods using global information, DIL is more suitable for large-scale networks. And from Table 4, one can see that Computational complexity of DIL is relatively low. Comparing with other local centralities, such as DC, DDN, DCC and LC, our method can well identify the importance of bridge nodes according to the calculation results on the network shown in Fig. 2. We evaluated the effectiveness of our method by comparing the decline rate of network efficiency caused by removing nodes. The higher the decline rate of network efficiency caused by removing the node is, the more important the node is. In Fig. 4, the experimental results on AIDS, Dolphins and Airport networks show that DIL method can evaluate the importance of nodes more reasonably than DCC, BC, CC, DDN and LC. In email network, BC performs the best, but DIL can still give comparatively good performance with lower computational complexity. And from Fig. 6, one can see that BC is strongly positively correlated with DIL. And they both perform better than CC and LC. Finally, the experimental result on ARPA network also illustrates the effectiveness of DIL. In a word, comparing global centralities, such as BC and CC, DIL can give comparatively better results using less information and lower computational complexity. And comparing with local centralities, such as DC, DDN, DCC and LC, our method can effectively identify the importance of bridge nodes. We believe that this paper may shed some light on this direction.

Acknowledgments

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