Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676 --

   The work begin after 10 min, so the average time increase from 45min to 55min.

   for normal distribution :-

   z = (X-mean)/std

   = (60-55)/8

   = 0.625

   import scipy.stats as st

`1-st.norm.cdf(0.625)= 0.2659855`

   C. 0.5
   D. 0.6987

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   Ans) False – with mean = 38 and std dev= 6
   P(X<44) = st.norm.cdf(44,38,6) = 84% which implies that there is only 16% probability that the might be 44+ age

   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.
   Ans) True, P(X>30) = 1- st.norm.cdf(30,38,6) = 91% which implies that there is 9% probability that employees might be of age less than 30 years.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans) According to the **Central Limit Theorem**, any **large sum** of **independent, identically distributed(iid)** random variables is approximately **Normal**.

The **Normal distribution** is defined by two parameters, the **mean μ**, and the **variance σ^2**, and written as .**$X \sim N(\mu, \sigma^2)$**

Given **$X1 \sim N(\mu, \sigma^2)$ and $X2 \sim N(\mu, \sigma^2)$** are two independent identically distributed random variables.

From the properties of **normal random variables**,

if **$X \sim N(\mu 1, \sigma^2 1$ and $Y \sim N(\mu 2, \sigma^2 2)$**are two independent identically distributed random variables then

- the **sum** of normal random variables is given by

  ,**$X+Y \sim N(\mu 1+ \mu 2, \sigma^2 1, \sigma^2 2)$**

- and the **difference** of normal random variables is given by

  , **$X-Y \sim N(\mu 1- \mu 2, \sigma^2 1+\sigma^2 2)$**

- When ,$Z=aX$ the **product** of X is given by

  $Z \sim N(a\mu 1,a^2 \sigma^2 1)$

- When , **$Z=aX+bY$** the **linear combination** of X and Y is given by

  $Z \sim N(a\mu 1+b \mu 2, a^2 \sigma^2 1+ b^2\sigma^2 2)$

Given to find, **2X1**

Thus, following the property of multiplication, we get

$2X1 \sim N(2\mu,2^2 \sigma^2) \Rightarrow 2X1 \sim N(2 \mu,4 \sigma^2)$

and following the property of addition,

$X1+X2 \sim N(\mu+ \mu, \sigma^2+ \sigma^2) \sim N(2 \mu ,2 \sigma^2)$

And the difference between the two is given by

$2X1-(X1+X2) \sim N(2 \mu -2 \mu,2 \sigma^2 1+4 \sigma^2 2) \sim N(0,6 \sigma^2)$

The mean of **2X1and X1+X2** is same but the **var(σ^2)** of **2X1** is **2 times** more than the variance of **X1+X2**

The difference between the two says that the two given variables

are **identically** and **independently** distributed.

4. Let X ~ N(100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5

   Ans) p(a<x<b) = 0.99 , mean =100, std= 20

   we have to remove area of .005 in each of the left and right tails of Normal Distribution giving as 0.5 and 0.995 percentage Z scores

z score at 99.5 given as => st.norm.ppf(0.995)= 2.5758

z core at 0.005 fiven as =>st.norm.ppf(0.005)= -2.5758

Z = (x - 100)/20 = > x = 20z+100

a = -(20*2.5758) + 100= 48.484

b = (20*2.5758)+100= 151.516 hence two values a,b are 48.5, 151.5

   E. 90.1, 109.9

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1$ ~ N(5, 3^2) and $Profit_2$ ~ N(7, 4^2) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

   Ans) Total mean profit of the company from 2 divisions 5+7=12 as N(mean,variance)

   converting it in rupees million 12*45rs=540 million rupees

   Total Variance is (9+16)=25*45rs= 1125 million rs

   Total Standard Deviation is sqrt(25)*45rs =225 million rs

95% prob Rupee Range on annual profit of the company
st.norm.interval(0.95,mean=540,std=225)
(99.008, 980.991) million rupees

   B. Specify the 5th percentile of profit (in Rupees) for the company

Ans)For 5th percentile z= st.norm.ppf(0.05)=-1.6448

X=mean + Zstd = 540+((-1.6448)*(225))
X=169.92

C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans)**Probability of Division 1 making the loss is P(X<0)**

st.norm.cdf(0,5,3)= 0.0477

Probability of Division 2 making loss is P(X<0)
st.norm.cdf(0,7,4)=0.0400