

Rising Intangible Capital and the Disappearance of Public Firms[†]

Sara Casella[‡] Hanbaek Lee[§] Sergio Villalvazo[¶]

University of Pennsylvania University of Tokyo Federal Reserve Board

September 22, 2022

[\(click here for the latest version\)](#)

Abstract

Since 1996, the number of listed firms in the U.S. has decreased by around 50%. Using U.S. Compustat and earnings surprises from I/B/E/S data, we document that the financial reports of listed firms required by the U.S. Securities and Exchange Commission's (SEC) regulation have become significantly less transparent over the same period. To theoretically and quantitatively analyze these secular trends, we develop a heterogeneous-firm equilibrium model where the endogenous choice to go public or private and the distribution of the firm-level allocations are characterized by closed-form solutions. In the model, each listed firm's publicly disclosed intangible is diffused to other firms' productivity as an externality. In the estimated model, the increased intangible share has substantially decreased the average transparency of the financial disclosure and the number of listed firms. This leads to significant losses in welfare and productivity due to reduced technology diffusion. Finally, we characterize a policy maker's dilemma between maximizing welfare and productivity. According to the estimation, the recent stricter SEC disclosure requirement has significantly mitigated the welfare cost induced by rising intangibles at the cost of productivity loss.

Keywords: Intangible capital, Corporate disclosures, Technology diffusion.

JEL codes: D24, G24, G38.

[†] We are grateful for Harold Cole, Jesús Fernández-Villaverde, Angus Foulis, Luca Guerrieri, Dirk Krueger, Ryo Jinnai, Skander Van den Heuvel, and participants of the seminar at the Federal Reserve Board, Lisbon Macro Workshop, and Hitotsubashi University for their constructive comments and discussions. The views expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. All remaining errors are our own.

[‡]Email: scas@sas.upenn.edu

[§]Email: hanbaeklee1@gmail.com

[¶]Email: sergio.villalvazo-martin@frb.gov

1 Introduction

Since 1996 the number of listed firms in the U.S. has decreased by around 50%. Over the same period, public firms' performance has become increasingly difficult to forecast and the financial reports of listed firms have become more opaque. What are the driving forces for these changes? What are their macroeconomic consequences? This paper answers these questions through the lens of a heterogeneous-firms general equilibrium model where a rich set of equilibrium allocations are characterized by a closed form solution.

We focus on one key feature that distinguishes private and public firms: the regulation on public disclosure. The U.S. Securities and Exchange Commission (SEC, hereafter) requires listed firms to publicly reveal their annual and quarterly financial information and disclose material events such as transactions involving shareholders and insiders, in order to protect investors and facilitate a fair capital market. However, disclosure may also reveal crucial information to competitors, which is especially detrimental to highly innovative firms.

By estimating our model on U.S. firm data from 1992 to 2016, we show that the critical factor that drives the disappearance of the public firms is the increased importance of intangible capital in production. As firms start adopting more intangible capital, which is subject to imitation risk to a greater extent, they have a stronger incentive to conceal information, leading to an increased tendency to remain privately held. This technological change also leads to less transparent financial reports for firms that stay public, as observed in the data. The estimated model also predicts that access to funds by private investors has become easier, which contributed to the reduction of public firms. We show that the disappearance of public firms and overall greater opacity in financial markets substantially reduce productivity and technological diffusion across firms.

The baseline version of our model envisions a key trade-off between mandated transparency and market output and productivity. We capture the government's regulation on information revelation by including a mandated minimum level of disclosure for listed firms in the model. Using the closed-form characterization of the equilibrium allocations, we are able to analyze globally how information regulation affects welfare, productivity, and output. In the baseline model, stricter regulation increases the welfare of risk-averse investors through more transparent information in the financial reports. Stricter regulation can however crowd out voluntary disclosure and even backfire through the extensive-margin channel, as more firms tend to stay in the private equity market, which is more opaque¹. This reduces

¹This is one of the core issue SEC is concerned about. For example, in a February 2017 speech, SEC Commissioner Kara Stein posed a question regarding additional disclosures and regulation around private market investment: "We also need to understand why more companies are staying private for longer periods of time. Should we apply enhanced disclosure laws to these private companies? Or perhaps they require a unique set of rules." See "The Markets in 2017: What's at Stake?" Commissioner Kara M. Stein, SEC

the technology diffusion across firms, which is followed by lower productivity and output.² We characterize this dilemma for the policy maker between welfare and productivity. The tradeoffs make it hard for the policymaker to consider a dramatic shift in policy in a certain direction. Our estimation captures that disclosure policy has become stricter in the recent period. We show that even small magnitudes in policy shift have a meaningful effect on the transparency of the financial reports, as much as a 9.4% increase in the transparency measure.

We start by collecting and analyzing macro and micro-level empirical evidence, on which our structural framework is based. On the macroeconomic side, we revisit some well-known facts in the literature; the level of intangible capital has risen, and the number of public firms has decreased almost by 50% in the U.S. since 1996. Then, using U.S. Compustat data and data on earnings surprises from I/B/E/S, we construct a transparency measure of the firm-level financial report. Our measure shows that the average transparency has significantly declined over the same period of the two aforementioned trends.

To further investigate the relationship between transparency and intangible capital, we run a panel regression of the transparency on the intangible capital with firm-level controls and fixed effects. We find that the transparency (opacity) of the financial report is significantly negatively (positively) correlated with the firm level of intangible capital. This result shows that a firm with a high reliance on intangible capital has a tendency to report information less transparently. This cross-sectional fact gives an important bridge to link the two macro facts: rising intangible capital and declining transparency in the financial report.

In order to analyze these empirical patterns and their impact on the macroeconomic allocations in a unified framework, we introduce a general equilibrium model of heterogeneous firms where financing decisions are endogenous. In the model, ex-ante homogeneous firms choose whether to go public or private, the level of intangible capital stock, and the transparency of their intangible capital. The intangible capital is subject to diffusion to other firms as an externality in the form of productivity gain. If a firm goes private, transparency is at the minimum, and there is no technology diffusion to the other firms. Due to the externality, a firm with a greater intangible capital stock has less incentive to choose high transparency. However, a risk-averse representative household values transparency, giving better funding opportunities for firms with greater transparency. This generates a clear trade-off in choosing low transparency: it reduces intangible leakage, but it worsens the funding opportunities. In equilibrium, the distribution of transparency and the portion

website, <https://www.sec.gov/news/speech/stein-secspeaks-whats-at-stake.html>

²We characterize the space of parameters for which this is true. In some parametric regions, this dilemma may not show up; all of the welfare, productivity, and output can improve by the change in the information regulation, as the divine coincidence in the literature on monetary policy.

of private firms are determined at the price and externality level where all firms become indifferent.

A policy maker sets a mandated minimum transparency level, and all the listed firms need to sustain a greater transparency level than the mandated level. Therefore, a higher mandated transparency level decreases the incentive to go public, leading to a greater portion of private firms in the equilibrium. However, a stricter policy would lower uncertainty for investors, achieving greater welfare. Therefore, the policy maker also faces a sharp trade-off.

To evaluate the consequences of the information disclosure policy, we provide three criteria: output, productivity, and investors' welfare³. In the estimated model, a regulation policy can achieve only either higher output and productivity or higher welfare, which shows the policy maker's dilemma. From the perspective of the protection of investors, we find the recent regulation has substantially improved welfare. However, we also document that it has led to a substantial loss in productivity in the production sector.

One of the advantages of our model is that these decisions have a closed-form solution, which allows us to characterize the model and optimal policy globally and cleanly. The model resembles to the one in ?, as it characterizes the general equilibrium distribution of endogenous objects in closed form. In their model, the wage distribution is endogenously determined as the model captures the endogenous wage postings from the firm side. Similarly, in our model, a risk-averse representative household with CARA utility endogenously chooses the amount of funding for each transparency level.

Contribution and literature Our paper delivers two main contributions to the literature. First, we provide a theoretical and quantitative model framework that analyzes the effect of rising intangible capital on the firm-level financing decision⁴. Using the estimated model, we show that the rising intangible has been the key driver of the disappearing public firms. Also, the qualitative aspect of our model is worth highlighting as it allows closed-form characterization of rich equilibrium allocations, including the distribution of public and private firms. This tractability promotes the transparent illustration of endogenous mechanisms in our model. Also, it enables a fast and accurate quantitative analysis.⁵

Second, we bring a novel policy angle, information regulation, to the table and analyze its macroeconomic trade-off. From the tractable general equilibrium model, we show that

³The mission of SEC is "The mission of the SEC is to protect investors, maintain fair, orderly, and efficient markets, and facilitate capital formation." See "Our Goals", SEC website, <https://www.sec.gov/our-goals>.

⁴? hinted at the possibility of the role of intangible capital in the observed declining trends of listed firms. However, the structural analysis of the channel has been missing in the literature.

⁵The portion of public firms are often substantially smaller than the private firms in many countries. Then, a computation error of 0.1% in the portion of public firms is a substantially large error. Therefore, a highly-computational model is easily subject to a high approximation error in capturing the portion of large firms.

in a reasonable range of parameters, a policymaker faces a dilemma between welfare and productivity. We believe the closed-form characterization of our model would serve as a useful tool for future research on the information regulation policy.

Two strands of the literature are closely related to this paper. The first is the literature that studies the rising importance of intangible capital. It was only around a decade ago that intangible capital was first recognized as an important macroeconomic factor that affects economic growth and the business cycle. For example, ? and ? highlight the importance of intangible capital as a key input factor for production and show how mismeasurement of intangible capital may mislead the neoclassical model predictions in terms of economic growth. Relatedly, ? and ? modeled plant-level intangible capital as an important input for production. Mainly, their intangible capital refers to organizational capital that is partly firm-specific and partly embodied in key labor inputs.

We contribute to this literature by analyzing a novel macroeconomic implication of the rising share of intangible capital. The intangible capital has become an important source of competitiveness, leaving firms to put a great effort into R&D or developing a productive corporate culture. However, the intangible capital has a strong spillover effect, which can benefit competitors besides the owner firm. Therefore, the rising importance of intangible capital has naturally increased a firm's incentive to stay opaque in its financial disclosure. Using our model, we theoretically and quantitatively analyze how this change affects the macroeconomy in terms of welfare and productivity.

The second literature is about the disappearance of the listed firms. Different explanations have been put forward to shed light on this issue. For example, ? point to the increase in mergers and acquisitions among U.S. firms; ? conjecture that as markets have become more globally integrated, the net benefits of going public in the U.S. versus in other markets have decreased; ? argue that the deregulation of securities laws (NSMIA 1996) improved the private equity market, which reduced the incentives for firms to go public.

In this paper, we propose a complementary explanation. We argue that the rise of intangible capital, especially the components of intangible capital that could benefit competitors besides the owner firm, has increased the cost of disclosing information and made staying private more attractive. The estimated model also predicts that access to funds by venture capital firms, private equity funds, and other private investors has become easier.

2 Empirical analysis

2.1 Data and measurement

Our main source of information on the number of U.S. publicly traded firms and their characteristics is Compustat. Despite the consensus on the rising importance of intangible capital in the modern economy, there is still no consensus on the measurement of intangible capital due to its complex nature. Among the key papers measuring intangible capital, ? showed that around 30% of intangible capital in the U.S. is organizational capital, the single largest category of intangible capital. These are consistent results with the notion that intangible capital is hard to trade as it is less valued outside of the plant it belongs to and thus hard to be collateralized (?). Recent papers in the literature applied the perpetual inventory method to U.S. Compustat data to capitalize the intangible-related expenditures (??). Those expenditures include the R&D cost and a portion of SG&A expenditure. ? measures the intangible capital stock with intangible capital price considered using the equity price and the exit price. We also followed this approach to compute the intangible capital stock.

Our baseline measure of internally generated intangible capital is the sum of two components: (i) estimated knowledge capital, calculated using research and development expenditure; and (ii) estimated organizational capital, calculated using selling, general, and administrative expenses. The measure is constructed using the perpetual inventory method, which aggregates net investment flows over the life of the firm.⁶

$$\begin{aligned} K_{i,t}^G &= (1 - \delta_G) K_{i,t-1}^G + \gamma_G R\&D_{it} \\ K_{i,t}^O &= (1 - \delta_O) K_{i,t-1}^O + \gamma_O SG\&A_{it} \end{aligned}$$

where $R\&D$ is research and development expenditure, $SG\&A$ is selling, general, and administrative expenses, both deflated by the price of intellectual property products; and the depreciation rates δ_G and δ_O are taken from ?.

Then, we calculate the acquired amount of intangibles from the changes in the book values of intangibles after the amortization in the balance sheet data. By combining the internally generated intangibles and acquired intangibles, we obtain the firm-level intangible capital.

In order to get measures of firms' transparency, we leverage information on earnings surprises. Following ?, earnings surprises are defined as the difference between a firm's earnings announcement and the earnings forecasts made by analysts for that firm, normalized by the price of a share. Data on analysts' forecasts come from the Institutional Brokers'

⁶see for example Cockburn and Griliches (1988), Eisfeldt and Papanikolaou (2013, 2014), Ewens, Peters, Wang (2020), Hall, Mairesse, and Mohnen (2010), Hulten and Hao (2008)

Estimate System (I/B/E/S). The dataset collects quarterly estimates made by stock analysts on the future earnings for publicly traded companies.

We use two different proxies for firm transparency. The first is the inverse of the absolute value of earnings surprises from the consensus analyst forecast, i.e. the median forecast among all the analysts that make a forecast in the last 30 calendar days before the earning announcement. Let $e_{t,k}$ be the earnings per share announced in quarter t for company k and $\hat{e}_{t,k}$ be the corresponding consensus analyst forecast. Indicate by $P_{t,k}$ the price of the shares of company k in quarter. Earnings surprise from the consensus is given by:

$$\widehat{ES}_{t,k} := \frac{e_{t,k} - \hat{e}_{t,k}}{P_{t,k}}$$

Assuming that more transparent firms have on average lower earning surprise, our first proxy is then:

$$t_{k,t}^1 := \frac{1}{|\widehat{ES}_{t,k}|}$$

Our second measure takes advantage of the fact that I/B/E/S collects multiple estimates made by different analysts for each firm, up to 35 and on average 3 in our dataset. Let $e_{t,k,j}$ be the earning forecast made by analyst j , then earning surprise for each firm-analyst is:

$$ES_{t,k,j} := \frac{e_{t,k} - e_{t,k,j}}{P_{t,k}}$$

Under the assumption that more transparent firms have lower disagreement among analysts, our second transparency proxy is:

$$t_{k,t}^2 := \frac{1}{\text{var}(ES_{t,k,j})}$$

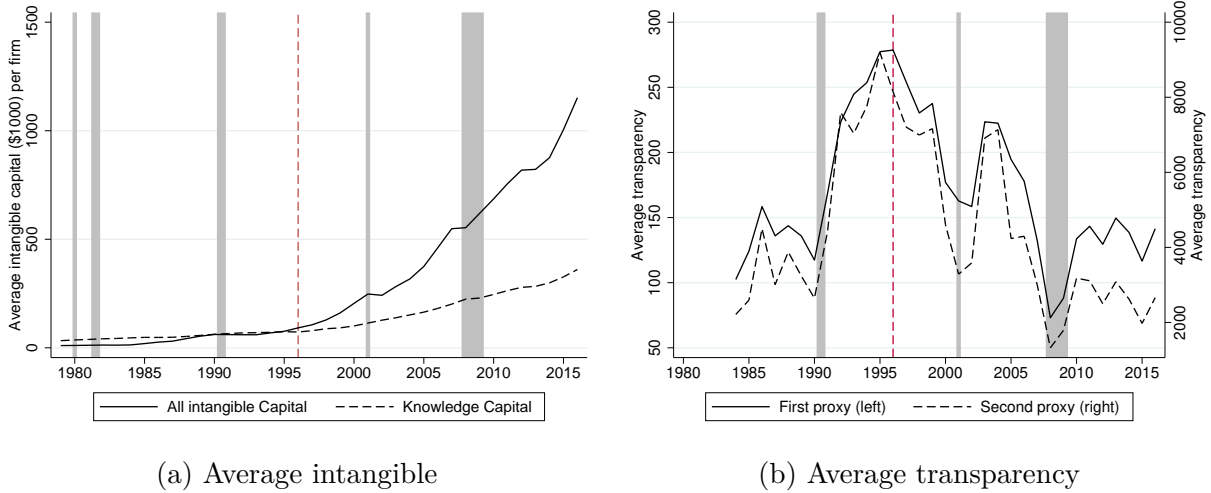
2.2 Trends in intangible capital and transparency

Figure (??) shows the change in the aggregate variables among publicly listed firms from 1980 through 2016. As seen in panel (a), the number of listed firms started to decrease from 1996, which the dotted vertical line indicates. Before 1996, there was no such trend of decreasing listed firms. One of the explanations for the decrease of listed firms points to the higher number of mergers and acquisitions observed in the U.S. (?). Panel (b) shows the number of M&A, which has been increasing over the whole sample period. We focus instead on an alternative and complementary explanation: Panel (c) shows the average intangible capital stock has increased sharply from the middle of the 1990s.

At the same time, the average transparency level of corporate's quarterly report has



Figure 1: Number of Listed Firms



(a) Average intangible

(b) Average transparency

Figure 2: Timeseries plot of aggregate corporate variables

decreased from the middle of 1990, as can be seen in panel (d). Recessions and especially the Great Recession represent a big shocks to earnings surprises. In order to take that into account, we also measure average transparency by excluding recession periods as measured by the NBER, and we still find that average transparency has been declining.

2.3 Cross-sectional analysis

In this section we describe cross-sectional evidence that links high reliance on intangible capital with the value of earning surprises. We run the following regression on our baseline sample, which includes all firms in Compustat from 1990 to 2016 for which information on

earnings forecasts by at least one analysts is available:

$$y_{f,t} = \theta_t + FEs + \beta \times \text{Intangible capital over total assets}_{f,t} + \gamma \times X_{f,t} + \varepsilon_{f,t}$$

where $y_{f,t}$ is either the absolute value of earning surprises from the consensus, or the variance of earning surprises when more than one analyst forecast is present. θ_t are year fixed effects and FEs include, depending on the specification, industry or firm fixed effects. $X_{f,t}$ represents firm controls.⁷ The firm-level controls include book-to-market ratio, return on asset, leverage (total debt over total asset), tangible capital, sales, age (from the IPO year), and the number of analysts. Intangible and tangible capital are normalized by the total asset. Since information on firms' characteristics is only available at the fiscal year level, we average all observations of earnings surprises for a given firm in a given fiscal year.

Table ?? reports the results. The regressions show that intangible capital and transparency are inversely related, i.e., firms that have a higher share of intangible capital compared to their size are more difficult to forecast. Specifically, an increase in one standard deviation in intangible capital over assets increases the absolute value of earnings surprises from consensus by 0.135 standard deviations, and the variance of analysts forecast by 0.075 standard deviations, and the effect resists the inclusion of firm fixed effects.

⁷Firm-level controls and regression specifications are based on ? and ?.

	ABSOLUTE VALUE OF EARNING SURPRISES		VARIANCE OF ANALYSTS FORECASTS	
	(1)	(2)	(3)	(4)
Intangible capital	0.135***	0.088***	0.075***	0.035***
Book to Market Ratio	0.172***	0.175***	0.082***	0.098***
Return on assets	-0.156***	-0.098***	-0.118***	-0.037***
Leverage	-0.087***	-0.137***	-0.026***	-0.064***
Tangible capital	0.000	0.018	-0.004	-0.001
Sales	-0.026***	-0.003	-0.001	-0.000
Age	-0.084***	-0.550	-0.044***	1.871*
Number of analysts	-0.037***	-0.009**	0.000	0.004
Year FE	✓	✓	✓	✓
Industry FE	✓		✓	
Firm FE		✓		✓
Adj. R^2	0.111	0.375	0.037	0.339
Observations	60015	58551	37633	35942

Table 1: The Relationship Between Intangible Capital and Earnings Forecasts

This table reports the estimates of the coefficients from the following regression using our baseline sample, which includes all firms in Compustat from 1990 to 2016 for which information on earnings forecasts by at least one analysts is available:

$$y_{f,t} = \theta_t + FE_s + \beta \times \text{Intangible capital over total assets}_{f,t} + \gamma \times X_{f,t} + \varepsilon_{f,t}$$

where $y_{f,t}$ is either the absolute value of earning surprises from the consensus, or the variance of earning surprises when more than one analyst forecast is present. θ_t are year fixed effects and FEs include, depending on the specification, industry or firm fixed effects. $X_{f,t}$ represents firm controls.

All the variables are standardized to have a standard deviation of one. ***, **, * indicate statistical significance at the 1%, 5% and 10%, respectively

3 Baseline model

In this section, we introduce a heterogeneous-firm general equilibrium model, where the equilibrium allocations are characterized in the closed form. Using this model, we will qualitatively and quantitatively analyze the cause and consequences of the empirical patterns we have shown in the previous section.

We consider a stand-in household and a continuum of measure one of firms that are ex-ante homogeneous. The model is static.⁸ A representative household decides its asset portfolio and consumes the payouts from the portfolio. An entrepreneur decides in which market the firm operates between the public and private equity markets. If a firm is listed, the entrepreneur chooses the disclosure level of the firm's intangible capital to the public, which we define as transparency. If a firm is private, the entrepreneur does not disclose any intangible to the public.

3.1 Household

A stand-in household decides the asset portfolio and consumes the portfolio return. The household is given with the wealth level $a > 0$. The household is risk-averse, and the utility takes the following constant absolute risk aversion form (CARA):

$$u(C) = -e^{-\Lambda C}$$

where $\Lambda > 0$ is the absolute risk aversion parameter.

In the listed market, the household forms a belief on the return $\tilde{r}(q)$ based on a balance sheet information of a listed firm with a transparency level q . The belief on the return is assumed as follows:

$$\begin{aligned} \tilde{r}(q) &\sim N(\bar{r}(q), (\bar{q} + q)^{-\chi}) \\ \text{s.t. } \bar{r} &= \frac{\pi(q)}{P(q)} \end{aligned}$$

where $q \in [0, 1 - \bar{q}]$ is a transparency of the balance sheet information⁹; \bar{q} is the mandated transparency required by the policy maker; $\pi(q)$ is the profit of the firm with transparency q ; $P(q)$ is the price of the firm with transparency q . Due to opacity in the information, a

⁸The model is intended to capture an equilibrium that is formed over long years. Therefore, the dynamic aspect is ignored. Also, the static setup gives a great degree of tractability in the model, as will be described in the equilibrium analysis.

⁹The range of transparency is assumed at our convenience. However, the qualitative and quantitative results of this paper are unaffected from this normalization assumption.

household believes probabilistic return, where the variance decreases in transparency q . χ is a structural parameter that governs the variance of the listed firms' return. We interpret this parameter as the illegibility of the financial information. As the illegibility of financial information increases, the variance of return increases.¹⁰

In the private equity market, the household forms the following belief on the non-listed firms:

$$\begin{aligned}\tilde{r}^N &\sim N(\bar{r}^N, \frac{1}{\xi}) \\ \text{s.t. } \bar{r}^N &= \frac{\pi^N}{P^N}\end{aligned}$$

where ξ is a structural parameter that governs the variance of the non-listed firms' return. π^N and P^N are the profit and price of a non-listed firms. As non-listed firms do not disclose any information publicly, the household do not distinguish a non-listed from another.

3.2 Technology

Ex-ante homogeneous firms produce output using two inputs: tangible (k_T) and intangible capital (k_I). Tangible's share in the production is α , and the intangible's share is θ , where $\alpha + \theta < 1$. In this economy, there are two types of production technologies. One is listed firms' production technology and the other is non-listed firms' production technology.

3.2.1 Production function of listed firms

While tangible capital is a standard input in production, intangible capital is different. The intangible is shared with other firms without additional cost if it is disclosed. Due to this externality, a firm in our model is reluctant to reveal much of their own intangible capital because they can free ride on the readily available shared knowledge already in the market. However, the trade-off is that if they conceal their intangible capital, investors discount their value due to the opaque information about the firm. Therefore, if a firm decides to be listed, the firm should decide how much intangible capital to reveal (q) to the public. Also, there is a required level of information disclosure (\bar{q}) imposed by the policy maker for the listed firms. The unit of q is assumed to be the same as the intangible capital stock.

The production function of a listed firm is as follows:

$$zk_T^\alpha (k_I(1 - \bar{q} - q))^\theta (\Phi^{ex})^\gamma$$

¹⁰This is because $q + \bar{q} \leq 1$.

where z is the aggregate TFP; \bar{q} is the exogenous minimum level of intangible disclosure imposed by the policy maker; q is the disclosed amount of intangible (the transparency level); Φ^{ex} is the shared intangible capital from all other firms. r is the capital rental rate, and p is the R&D cost per unit of intangible capital. γ is the scale parameter for the externality. If γ is beyond a certain threshold, the equilibrium does not exist due to a divergent externality effect. In particular, only when $\alpha + \theta + \gamma \leq 1$, the equilibrium exists, which we will formally discuss after we define the equilibrium.

Importantly, we assume the revealed portion of intangible capital is deducted from the private intangible stock. This is an assumption to avoid double counting. The revealed intangible is absorbed into the public knowledge stock. Then, by having the revealed portion deducted from the private stock, a firm benefits from the revealed intangibles only from the added component to the public stock. Also, this assumption gives a symmetry of benefits of the shared knowledge between the revealing firms and the free-riding firms: both parties get the same benefit from the shared knowledge. If this symmetry is not guaranteed, the information cannot be said as fully shared, which conceptually confuses what it means to share information as the intensive margin of shared information needs to be specified further. We simplify the model by assuming pure symmetry to avoid such conceptual hurdles.

Ex-post profit $\pi(q)$ is obtained after taking out the operational costs $rk_T + pk_I$ from the revenue:

$$\pi(q) := \max_{k_T, k_I} z k_T^\alpha (k_I (1 - \bar{q} - q))^\theta (\Phi^{ex})^\gamma - rk_T - pk_I$$

We assume a firm i 's disclosed intangible q_i is perfectly substitutable by the other disclosed intangible. Therefore, the shared intangible are aggregated in the following way:¹¹

$$\Phi^{ex} = \int_0^1 k_{I,i} \left(\underbrace{\bar{q}}_{\text{Disclosure mandated by the policy maker}} + \underbrace{q_i}_{\text{Voluntary disclosure}} \right) di$$

3.2.2 Production function of non-listed (private) firms

If a firm is private, they do not need to publicly disclose their intangible capital. The production function of a listed firm is as follows:

$$z k_T^\alpha (k_I)^\theta (\Phi^{ex})^\gamma$$

¹¹We also consider a finite elasticity of substitution, $\lambda < \infty$, across the disclosed information of each firm as an extension of the baseline model. However, the main results of this paper stay unaffected over different choices of λ .

Except for the disclosure of the intangible capital, the production function is assumed to take the same form and the same parameters. The profit is also similarly defined as listed firms:

$$\pi^N := \max_{k_T, k_I} z k_T^\alpha (k_I)^\theta (\Phi^{ex})^\gamma - r k_T - p k_I$$

3.3 Financial markets

In this section, we characterize the financial market in the model. The funding supply is driven by the representative household's portfolio choice problem. The funding demand is determined by each firm's value maximization problem.

3.3.1 Funding supply: The household's mean-variance portfolio

A household solves the following maximization problem:

$$\begin{aligned} \max_{x(q), x^N} \quad & \mathbb{E}(-e^{-\Lambda C}) \\ \text{s.t.} \quad & C = \int x(\tilde{q}) \tilde{r}(\tilde{q}) d\tilde{q} + x^N \tilde{r}^N, \quad \int x(\tilde{q}) d\tilde{q} + x^N = a \end{aligned}$$

where $x(q)$ is the funding supply for firms with transparency level q and x^N is the funding supply for non-listed firms. As the model does not include the inter-temporal decision of the household, all the payoffs from the equity investment are consumed.

Then, from the *i.i.d* assumption of the stock return uncertainty, the consumption (income) satisfies

$$C \sim N\left(\int x(\tilde{q}) \tilde{r}(\tilde{q}) d\tilde{q} + x^D \tilde{r}^D, \int x(\tilde{q})^2 (\bar{q} + q)^{-\chi} d\tilde{q} + (x^D)^2 \frac{1}{\xi}\right)$$

Then, the investors' expected utility maximization problem is translated into the following form:¹²

$$\max_{\int x(\tilde{q}) d\tilde{q} + x^N = a} -e^{-\Lambda \left(\int x(\tilde{q}) \frac{\pi(\tilde{q})}{P(\tilde{q})} d\tilde{q} + x^N \frac{\pi^N}{P^N} - \frac{\Lambda}{2} \int x(\tilde{q})^2 (\bar{q} + q)^{-\chi} d\tilde{q} - \frac{\Lambda}{2} (x^N)^2 \frac{1}{\xi} \right)}$$

¹²The derivation of the mean-variance portfolio objective function is as follows: consider a random variable, $y \sim N(\mu_y, \sigma_y^2)$. Then,

$$\mathbb{E}(-e^{-\Lambda y}) = -\mathbb{E}(e^{-\Lambda y}) = -e^{-\Lambda(\mu_y - \frac{\Lambda}{2} \sigma_y^2)}.$$

The last equation is derived from the moment generating function of the normal distribution.

This is equivalent to

$$\max_{\int x(\tilde{q})d\tilde{q}+x^N=a} \int x(\tilde{q}) \frac{\pi(\tilde{q})}{P(\tilde{q})} d\tilde{q} + x^N \frac{\pi^N}{P^N} - \frac{\Lambda}{2} \int x(\tilde{q})^2 (\bar{q} + q)^{-\chi} d\tilde{q} - \frac{\Lambda}{2} (x^N)^2 \frac{1}{\xi}$$

The first-order condition with respect to $x(q)$ yields

$$\frac{\pi(q)}{P(q)} - \Lambda x^*(q) (\bar{q} + q)^{-\chi} - \mu = 0$$

where μ is the Lagrange multiplier of the wealth constraint. From this equation, we can derive the following supply curve of funding for the listed market:

$$x^*(q) = \frac{\pi(q)/P(q) - \mu}{\Lambda(\bar{q} + q)^{-\chi}}$$

where $x^*(q)$ is the funding supply in a dollar amount for firms with the transparency level q . So, the household is willing to invest $\frac{\pi(q)/P(q) - \mu}{\Lambda(\bar{q} + q)^{-\chi}}$ in the firms with transparency level q .

Similarly, from the first-order condition with respect to x^N , the funding supply curve for non-listed firms is characterized as follows:

$$x^{D*} = \frac{\pi^N/P^N - \mu}{\Lambda/\xi}$$

From this point on, we assume the representative household has a large enough wealth a , as our interest is not on the household's constrained optimization. Thus, $\mu = 0$.

3.3.2 Funding demand: Listed firms' value maximization

The price of a firm, $P(q)$, is determined at the level where funding supply in the number of firms, $\frac{x^*(q)}{P(q)}$ meets the funding demand in the number of firms $\mathcal{M}(q)$. Thus, the market-clearing condition is as follows:

$$\frac{x^*(q)}{P(q)} = \mathcal{M}(q)$$

In the model, a firm's value is identical to the price of the firm. Therefore, a manager of a firm chooses the transparency level to maximize the price of the firm:

$$\max_{q \geq 0} P(q)$$

Given the funding demand and the market-clearing condition, this problem is equivalent to the following form:

$$\max_{q \geq 0} \sqrt{\frac{\pi(q)}{\Lambda(\bar{q} + q)^{-\chi} \mathcal{M}(q)}}$$

which is equivalent to

$$\max_{q \geq 0} \frac{\pi(q)}{(\bar{q} + q)^{-\chi} \mathcal{M}(q)}$$

Now, we define a net funding intensity $\phi^L(q)$ as follows:

$$\phi^L(q) := \frac{(\bar{q} + q)^\chi}{\mathcal{M}(q)}$$

Therefore, a listed firm's problem can be summarized as the following form:

$$\begin{aligned} J^L(\mathcal{M}) &= \max_{q, k_T, k_I} \left(z k_T^\alpha (k_I (1 - \bar{q} - q))^\theta (\Phi^{ex})^\gamma - r k_T - p k_I \right) \phi^L(q) \\ \text{s.t. } \phi^L(q) &= (\bar{q} + q)^\chi / \mathcal{M}(q) \end{aligned}$$

where J^L is the value of a listed firm given the distribution of listed firm \mathcal{M} . In equilibrium, the value is equal to the price of each of the listed firms: $J^L(\mathcal{M}) = P(q(\mathcal{M}))$ for $\forall q \geq 0$. The solution to this problem characterizes the funding demand of each firm.

3.3.3 Financial market for non-listed firms

The price of a non-listed firm, P^N , is determined at the level where funding supply in the number of firms, $\frac{x^{D*}}{P^N}$ is matched with the demand in a frictional private equity market. Especially, we assume the congestion among non-listed firms generates the attrition in the funding opportunity in the following way:

$$\frac{x^{D*}}{P^N} = M_N^{\nu_N}, \quad \nu_N > 1$$

where, M_N is the total number of non-listed firms. $\nu_N > 1$ is a structural parameter that captures the congestion effect in the private equity market.

Then, we define a net funding intensity $\phi^N(q)$ as follows:

$$\phi^N := \xi / M_N^{\nu_N}.$$

A non-listed firms' problem can be written down as follows, simliar to the listed firms' problem:

$$J^N(\mathcal{M}_N) = \max_{k_T, k_I} \left(zk_T^\alpha(k_I)^\theta (\Phi^{ex})^\gamma - rk_T - pk_I \right) \phi^N$$

$$\text{s.t. } \phi^N = \zeta / M_N^{\nu_N}$$

3.4 A firm's problem

A firms' manager should decide whether to go listed or non-listed before the operation. If a firm becomes non-listed, the manager does not have to worry about the leakage of their intangible through the disclosure. However, investors penalize the opacity of the non-listed firms by allowing only a low funding intensity.

If a firm becomes public, the manager should decide the level of transparency $q \geq 0$. If there are too many firms to choose the same transparency level, it will decrease the value of the firm in the listed market.

A firm's problem could be summarized as follows:

$$\begin{aligned} \text{[Entry decision]} \quad & V(\mathcal{M}, M_N) = \max\{J^L(\mathcal{M}), J^N(M_N)\} \\ \text{[Listed firm's problem]} \quad & J^L(\mathcal{M}) = \max_{q, k_T, k_I} \left(zk_T^\alpha(k_I(1 - \bar{q} - q))^\theta (\Phi^{ex})^\gamma - rk_T - pk_I \right) \phi^L(q) \\ & \text{s.t. } \phi^L(q) = (\bar{q} + q)^\chi / \mathcal{M}(q) \\ \text{[Non-listed firm's problem]} \quad & J^N(M_N) = \max_{k_T, k_I} \left(zk_T^\alpha(k_I)^\theta (\Phi^{ex})^\gamma - rk_T - pk_I \right) \phi^N \\ & \text{s.t. } \phi^N = \zeta / M_N^{\nu_N} \end{aligned}$$

4 Equilibrium

Here we define an equilibrium where the economy is given total intangible capital reserve K^I (fixed aggregate intangible supply). This endogenously determines the R&D cost of intangible capital p . Here, the R&D cost is not a price for a trade. Instead, it is a cost that increases if all the other firms invest in their own R&D. This is because developing new knowledge is harder if more firms are seeking new knowledge. The rental rate for the tangible capital r is exogenously given.

Definition 1. A collection of functions $(k_T, k_I, q, \mathcal{M}, M_N, p, \Phi^{ex})$ is an equilibrium if

1. $(k_T(q, \mathcal{M}), k_I(q, \mathcal{M}), q(\mathcal{M}))$ solves the listed firm's problem.

2. The measure of listed firms choosing a transparency level q is consistent with $M(q)$ for all $q \in [0, 1 - \bar{q}]$.
3. The measure of non-listed firms is M_N and satisfies

$$\int_0^{1-\bar{q}} \mathcal{M}(q) dq + M_N = 1$$

4. R&D cost of intangible capital p is determined by the following equation:

$$K^I = \int_0^1 k_{I,i} di$$

5. Aggregate shared knowledge satisfies

$$\Phi^{ex} = \int_0^1 k_{I,i}(\bar{q} + q_i) di$$

6. All the firm prices are identical:

$$P(q) = P^N, \text{ for } \forall q \in [0, 1 - \bar{q}]$$

With the endogenously determined distribution \mathcal{M} of firms for each q , we can re-write the market-clearing condition for intangible capital and the externality condition using \mathcal{M} . In the definition, each firm is aggregated over the index $i \in [0, 1]$. Instead, we aggregate firms over the distribution of firms at each q . This is doable since \mathcal{M} is endogenously obtained, and k_I is also a function of q and \mathcal{M} . Therefore, we re-write those two conditions in the following way.

$$I = \int_0^{1-\bar{q}} k_I(q, M) M(q) dq$$

$$\Phi^{ex} = \int_0^{1-\bar{q}} k_I(q, M) (\bar{q} + q) M(q) dq :$$

Among all possible equilibria, we are interested in the non-degenerate equilibrium where all the homogeneous firms use mixed strategies over the transparency level q . The mixed strategy leads to the distribution of firms at each level of q . And in the equilibrium, this distribution needs to be consistent with the distribution that a firm takes as a given state variable.

In the following section, we analytically characterize the equilibrium allocations in this economy.

4.1 A listed firm's decision

First, we solve a listed firm's problem backward from the decision on the transparency level and the other allocations. Then, we solve the firm's decision on which financial market to go between the public and private market.

Given a net funding intensity function, ϕ^L and the externality, Φ^{ex} , a listed firms' firm's problem is characterized as follows:

$$\max_{q, k_T, k_I} \left(zk_T^\alpha (k_I(1 - \bar{q} - q))^\theta (\Phi^{ex})^\gamma - rk_T - pk_I \right) \phi^L(q)$$

Then, this problem can be written as in the following form:

$$\begin{aligned} \max_q \quad & \overbrace{\left[\max_{k_T, k_I} \left(zk_T^\alpha (k_I(1 - \bar{q} - q))^\theta (\Phi^{ex})^\gamma - rk_T - pk_I \right) \phi^L(q) \right]}^{\text{Interim problem}} \\ \text{s.t.} \quad & \phi^L(q) = (\bar{q} + q)^\chi / \mathcal{M}(q) \end{aligned}$$

This tranformation is feasible because the original problem is unconstrained optimization with continuous objective function (given q) in the convex domain.¹³ Then, k_I is indexed by q at the interim problem specified above.

From the optimality conditions of the interim problem, we can derive the relationship among the transparency q , the regulation parameter \bar{q} , and the intangible capital k_I . The relationship is formally stated in the following proposition:

Proposition 1. (Intangibles and the transparency)

Given $\alpha + \theta < 1$, $k^I(q, \bar{q})$ decreases in both q and \bar{q} .

Proof.

See Appendix ??.

■

If a firm adopts a large amount of intangible capital, it has an incentive not to choose a high level of transparency, as the combination of high transparency and large intangible capital will lead to excessively large benefit to competitors through the externality. This result is consistent with the empirical observation we documented in Section ??.

Then, from the optimality condition with respect to the transparency, q , we can characterize an ordinary differential equation where the function of interest is the net funding intensity function $\phi(q)$. The ODE is specified in Appendix ??. By solving the ODE, we

¹³This is the reason why we can solve the model in two steps: 1) ex-post profit maximization and then 2) transparency and public/private choice.

characterize the closed form of the transparency distribution \mathcal{M} . We formally state the closed form of \mathcal{M} in the following proposition:

Proposition 2. (Transparency distribution)

The unnormalized probability density function \mathcal{M} of transparency q has the following closed form:

$$\mathcal{M}(q) = (\bar{q} + q)^\chi (1 - \bar{q} - q)^{\frac{\theta}{1-\alpha-\theta}} \frac{1}{\phi^N}.$$

Proof.

See Appendix ??.

■

In the multiplicative form of the closed-form endogenous distribution in Proposition ??, each component is directly interpretable.

$$\mathcal{M}(q) = \underbrace{(\bar{q} + q)^\chi}_{\text{funding supply}} \underbrace{(1 - \bar{q} - q)^{\frac{\theta}{1-\alpha-\theta}}}_{\text{funding demand}} \underbrace{\frac{1}{\phi^N}}_{\text{eq. normalizer}}.$$

The first component $(\bar{q} + q)^\chi$ comes from the household's preference for transparent firms. For a higher q , the household is willing to provide greater funding to the firm. This generates an incentive for a firm to choose high q . In contrast, the second term $(1 - \bar{q} - q)^{\frac{\theta}{1-\alpha-\theta}}$ captures firms' incentive to reveal less information. This is because greater revelation only benefits competitors at the firm's own cost. The third term $\frac{1}{\phi^N}$ is the equilibrium object that balances the measure of listed and non-listed firms.

The following corollary establishes that the equilibrium distribution is unique for the given support of the transparency, $[0, 1 - \bar{q}]$.

Corollary 1. (Uniqueness of the transparency distribution)

Given the support $[0, 1 - \bar{q}]$, the equilibrium unnormalized probability density function \mathcal{M} is unique.

Proof. The result is immediate from the uniqueness of the ODE solution that satisfies the boundary condition.

■

The probability density function $\mathcal{M}(q)$ belongs to a variant of a well-known class of density function: Beta distribution. In the following corollary, we prove that $\mathcal{M}(q)$ follows a shifted truncated beta distribution and provide the closed-form characterization of the net funding intensity of the private firms, ϕ^N . For the brevity of notation, I define $B := \frac{\theta}{1-\alpha-\theta} + 1$.

Corollary 2. (Truncated normalized Beta distribution)

The gross transparency, $y := q + \bar{q}$, follows a truncated normalized Beta distribution where the shape parameters are $\chi + 1$ and $B + 1$, and the support is $[\bar{q}, 1]$.

$$q + \bar{q} \sim \frac{\mathbb{I}\{q \in [0, 1 - \bar{q}]\}}{1 - M_N} \times \text{Beta}(\chi + 1, B + 1),$$

where $B = \frac{\theta}{1 - \alpha - \theta}$.

Proof.

See Appendix ??.

■

Even if we obtain the the closed form of the transparency distribution \mathcal{M} , the distribution is not readily matched with the data counterpart, as the firm-level transparency distribution is not observed. Instead, the residualized variance of the stock return $(\bar{q} + q)^{-\chi}$ can be measured from the variance of earnings' surprise. To discipline the parameters based on the proper observable counterparts in the data, we analyze the distribution of stock return variance \hat{M} instead of \mathcal{M} . The closed form of $\hat{M}(\hat{q})$, where $\hat{q} = (\bar{q} + q)^{-\chi}$ is stated in the following corollary. As the return variance decreases in the transparency, we name \hat{q} the opacity of the firms' information.

Corollary 3. (Opacity distribution)

The unnormalized probability density of return variances, $\hat{M}(\hat{q})$ has the following closed form:

$$\hat{M}(\hat{q}) = \frac{1}{\chi \phi^N} \hat{q}^{-\frac{1}{\chi} - 2} \left(1 - \hat{q}^{-\frac{1}{\chi}}\right)^B \text{ for } \forall \hat{q} \in [1, \bar{q}^{-\chi}]$$

where $B = \frac{\theta}{1 - \alpha - \theta}$.

Proof.

See Appendix ??.

■

In the proof, we use a useful property that \hat{q} is a monotone transformation of q . This relationship also helps characterize the corresponding boundaries of the support. Specifically, the probability density $\hat{M}(\hat{q})$ takes a bounded support, $[1, \bar{q}^{-\chi}]$.

It is worth noting that the probability density of q or \hat{q} depends on the net funding intensity of non-listed firms, ϕ^N . This net funding intensity is determined at the following identity that requires total measure of firms is unity:

$$\frac{1}{\phi^N} \int_0^{1 - \bar{q}} (\bar{q} + q)^\chi (1 - \bar{q} - q)^B dq = 1 - \left(\frac{\phi^N}{\xi}\right)^{-\frac{1}{\nu_N}}. \quad (1)$$

Equivalently, we can write down the identity in terms of the mass of non-listed firms as follows:

$$\frac{1}{\xi} M_N^{\nu_N} \int_0^{1-\bar{q}} (\bar{q} + q)^\chi (1 - \bar{q} - q)^B dq = 1 - M_N. \quad (2)$$

The equation (??) is the fundamental component of the model, which captures how the total measure of non-listed firms, M_N , behaves when the policy parameter \bar{q} changes. Using Corollary ??, we can integrate out the $M(q)$ in the left-hand side of the equation in the following steps, using $y = q + \bar{q}$:

$$\frac{1}{\xi} M_N^{\nu_N} \int_{\bar{q}}^1 (y)^\chi (1 - y)^B dy = 1 - M_N.$$

Then, we divide the both sides by a beta function, $\mathcal{B}(\chi + 1, B + 1)$.¹⁴

$$\frac{1}{\xi} M_N^{\nu_N} \frac{1}{\mathcal{B}(\chi + 1, B + 1)} \int_{\bar{q}}^1 (y)^\chi (1 - y)^B dy = \frac{1}{\mathcal{B}(\chi + 1, B + 1)} (1 - M_N).$$

We integrate the left-hand side using the cumulative distribution function of beta distribution, F :

$$\frac{1}{\xi} M_N^{\nu_N} (1 - F(\bar{q}; \chi + 1, B + 1)) = \frac{1}{\mathcal{B}(\chi + 1, B + 1)} (1 - M_N).$$

By rearranging the terms, we obtain the following equation:

$$\xi M_N^{\nu_N} (1 - M_N) = \mathcal{B}(\chi + 1, B + 1) (1 - F(\bar{q}; \chi + 1, B + 1)) \quad (3)$$

Equation (??) characterizes the measure of private firms in the closed-form. Importantly, the equation does not include either the price of intangible or the externality. That is, the measure of private firms are independently determined from the general equilibrium effects and externality. The intuition behind this result is that both the productivity shift through the externality and the general equilibrium effect uniformly affect the operating profit of each firm, so they do not affect the decision of how to finance their operating activities. Due to this separation, a measure of private firm M_N , is determined directly by Equation (??). M_N determines the funding intensity of private firm ϕ^N . Then, from Proposition ??, the

¹⁴The beta function is defined as follows:

$$\mathcal{B}(a, b) := \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \frac{(a-1)!(b-1)!}{(a+b-1)!} = \int_0^1 x^{a-1}(1-x)^{b-1} dx$$

distribution of firms over transparency is also independently determined from the general equilibrium effect and the externality. Therefore, the mandated transparency \bar{q} affects the firm-level distribution directly through Equation (??) without any feedback effects in the general equilibrium.

Also, it is worth noting that the right-hand side of Equation (??) strictly decreases in the mandated transparency \bar{q} . This characterization theoretically predicts that M_N decreases in \bar{q} .

Proposition 3. (The relationship between disclosure regulation and the measure of listed firms)

M_N strictly increases in $\bar{q} \in [0, 1]$.

Proof.

See Appendix ?? ■

Therefore, as the policy maker requires a stricter disclosure regulation on the financial information, the measure of non-listed firms increases. This is because a firm does not internalize the productivity gain from the shared information. As can be observed from the equation (??), the measure of non-listed firms is independent from the externality effect, Φ^{ex} .

However, the total measure of listed or non-listed firms cannot solely serve as a objective of the information regulation. The objective is stated clearly in the following mission of SEC in the U.S.: “the mission of the SEC is to protect investors, maintain fair, orderly, and efficient markets, and facilitate capital formation.”¹⁵ Therefore, the effect of regulation on the investors’ welfare and the productivity needs to be investigated.

4.2 The scoreboards: Welfare, productivity, and output

In this section, we define the three objectives of the information disclosure policy: welfare, productivity and output. First, we define the welfare measure. Besides the performance of the firms, the investor values the transparency of the disclosed information, as it is helpful for its portfolio. The representative investors utility can be monotonely transformed into the following mean-variance form:

$$\begin{aligned} Objective_{welfare} &= \int x(\tilde{q}) \frac{\pi(\tilde{q})}{p(\tilde{q})} d\tilde{q} + x^N \frac{\pi^N}{P^N} - \frac{\Lambda}{2} \int x(\tilde{q})^2 (\bar{q} + \tilde{q})^{-\chi} d\tilde{q} - \frac{\Lambda}{2} (x^N)^2 \frac{1}{\xi} \\ &= \int M(\tilde{q}) \pi(\tilde{q}) d\tilde{q} + M^N \pi^N - \frac{1}{2} \int \frac{\pi(\tilde{q})}{M(\tilde{q})} d\tilde{q} - \frac{1}{2} \frac{\pi^N}{M_N}. \end{aligned} \quad (4)$$

¹⁵The mission is from <https://www.sec.gov/our-goals>.

The welfare measure increases in the expected profits and decreases in the ex-ante variance of the profits.

The second measure is the productivity in the production sector. The productivity measure is defined as follows:

$$\begin{aligned} Objective_{productivity} &= (\Phi^{ex})^\gamma \\ &= \left(\int_0^{1-\bar{q}} (\tilde{A}(\bar{q} + q)(1 - \bar{q} - q)^{\frac{\theta}{1-\alpha-\theta}} \mathcal{M}(q))^{\frac{\lambda-1}{\lambda}} dq \right)^{\gamma \times \frac{\lambda}{\lambda-1} \times \frac{1-\alpha-\theta}{1-\alpha-\theta-\gamma}}, \end{aligned}$$

where $\tilde{A} := \left(\left(\frac{\alpha z}{r} \right)^{\frac{1}{1-\alpha-\theta}} \left(\frac{r\theta}{p\alpha} \right)^{\frac{1-\alpha}{1-\alpha-\theta}} \right)$. The productivity is identical to the externality effect, which is the aggregated shared knowledge in the economy. λ is the elasticity of substitution across the knowledge shared from each firm. In the baseline quantitative analysis, we assume $\lambda = \infty$ so the knowledge is perfectly substitutable across the firms. As a robustness check, we investigate the macroeconomic implications under $\lambda < \infty$. In the regulator's perspective, there is a trade-off in the productivity measure for increasing the strictness of the disclosure requirement. For higher \bar{q} , the amount of shared information is greater, while the pool of listed firms to share the information shrink due to the firm-level extensive-margin responses.

The third measure is the aggregate output in the economy. The output measure is defined in the following form:

$$Objective_{output} = \int_0^{1-\bar{q}} k_T(q)^\alpha (k_I(q)(1 - \bar{q} - q))^\theta (\Phi^{ex})^\gamma M(q) + k_{DT}^\alpha k_{DI}^\theta (\Phi^{ex})^\gamma M_N$$

As the regulation parameter \bar{q} increases, productivity varies, and total measure of listed firms changes. Therefore, the aggregate output can potentially feature a nonlinear curve over the variation in \bar{q} . In the quantitative analysis, we will quantitatively analyze the variation in these three measures.

5 Quantitative analysis

Based on the model we developed in the theory section, we quantitatively analyze the macroeconomic impact of rising importance of intangibles and the role of the information regulation policy. First, we estimate our model based on two different periods. One is from 1992 to 1996, which is the baseline period, and the other is the post-change period, from 2012 to 2016. As our model is static, we cannot analyze the dynamic response that might have happened right after a change in structural parameters. Therefore, we analyze a period just before the year of dramatic change in the trend of number of listed firms as a baseline and

compare this with a period several years after the change to assume that it has reached a stationary level.

5.1 Estimation

In this section, we elaborate on how we fit the firm-level data into the model. The core parameters to be estimated are as follows:

$$\{\bar{q}, \theta, \chi, \xi, \nu_N\}$$

where \bar{q} is the mandated transparency of disclosure; θ is the intangible share; χ is the residualized return variance parameter of listed firms; ξ is the residualized return variance parameter of non-listed firms, which we interpret as illegibility of the financial information; ν_N is the private equity market efficiency parameter.

The baseline estimates are from matching the average target moments between 1992 and 1996. The estimates of the post-change periods are from matching the average target moments between 2012 and 2016. The target moments and the simulated moments are reported in Table ???. We first normalize the friction parameter of the private equity market ν_N at unity in the baseline estimation. The post-change level is estimated.

The identification strategy of the parameter \bar{q} comes from the fraction of listed firms out of total firms.¹⁶ As studied in the model section, the higher required transparency increases incentive to stay in the private equity market.

We target the growth rate of the private equity funding relative to the GDP growth to identify ν^N in the post-change period. Thus, the post-change period includes an additional target moment as follows:¹⁷

$$\frac{x^{post}/GDP^{post}}{x^{pre}/GDP^{pre}} = \frac{29.18/7275.80}{143.9/17213.47} = 2.08,$$

where the model-side private equity funding supply x and GDP is defined as follows:

$$x = P^N M_N^{\nu_N} = \sqrt{\frac{\pi^N M_N^{\nu_N}}{\xi}}$$

$$GDP = \int z k_T^\alpha (k_I(1 - \bar{q} - q))^\theta (\Phi^{ex})^\gamma dM(q) + z k_T^\alpha k_I^\theta (\Phi^{ex})^\gamma M^N.$$

¹⁶The private equity market congestion parameter ν_N also significantly affects the fraction of listed firms (?). As we normalize $\nu_N = 1$ in the pre-change period, the fraction of listed firms solely identifies the policy parameter \bar{q} .

¹⁷The data is from private equity funding data from Thompson Reuters.

Lastly, θ is from the intangible to tangible ratio. χ is identified from the funding intensity variation by matching the ex-ante intangible-to-profit ratio.

In the model, the households form a belief on a stock return that follows a normal distribution:

$$\tilde{r}(q) \sim N(\bar{r}(q), (\bar{q} + q)^{-\chi})$$

Analysts' forecast dispersion is a natural data counterpart to the dispersion in the ex-ante stock return, $q^{-\chi}$. Specifically, earnings surprise is defined as:

$$ES(q) := \bar{r}(q) - \tilde{r}(q) \sim N(0, (\bar{q} + q)^{-\chi}).$$

Once χ is identified, a firm i 's transparency level can be recovered as:

$$\hat{q}_i = \text{var}(ES_i)^{-\frac{1}{\chi}}.$$

We internally calibrate ξ using the top 1% opaque firms' average forecast variance in the equilibrium outcome. This is from an additional assumption that non-listed firms' opacity is at a similar level with the most opaque firms among the listed.

We use the method of simulated moments to estimate the parameters. The weight matrix is chosen to be identity matrix. However, the choice of the weight matrix is not an issue in our estimation, as the parameters are exactly identified at the level where the level of moments are exactly matched.

Moments	Data	Model	Reference
Baseline (1992 ~ 1996)			
Fraction of listed firms with 100+ emp. (%)	8.02	8.02	Compustat & Census BDS
Intangible/Tangible (%)	27.31	27.31	Compustat
Intangible/Profit (%)	36.20	36.20	Compustat
Post-change periods (2012 ~ 2016)			
Fraction of listed firms with 100+ emp. (%)	4.39	4.39	Compustat & Census BDS
Intangible/Tangible (%)	36.34	36.34	Compustat
Intangible/Profit (%)	44.37	44.37	Compustat
Δ private equity funding (%)	2.08	2.08	Thomson Reuters PE data

Table 2: Fitted Moments

Table ?? reports the estimated parameters. The estimated mandated transparency parameter, \bar{q} , slightly increased in the post-change period, which implies the information regu-

lation has become stricter, which is consistent with the policy maker's intended direction of the reform. The share of intangible, θ , is around 50% greater in the post-change period. This is consistent with the empirical fact that the importance of intangible input has risen significantly. The the stock return uncertainty parameter of listed firms, χ has increased, and the stock return uncertainty parameter of non-listed firms, ξ has decreased over the same period, which implies the variance return variance on both listed and non-listed markets increased. And the friction parameter ν_N has increased, which implies the private equity market has improved, reflecting the impact of NSMIA 1996 (?).

Parameters	Description	Value
Baseline (1992 ~ 1996)		
\bar{q}	Mandated transparency	0.792
θ	Intangible share	0.137
χ	Dispersion of stock returns	8.560
ν_N	PE market efficiency	1.000
$1/\xi$	PE return uncertainty*	1/0.365
Post-change periods (2012 ~ 2016)		
\bar{q}	Mandated transparency	0.802
θ	Intangible share	0.182
χ	Dispersion of stock returns	12.231
ν_N	PE market efficiency	2.350
$1/\xi$	PE return uncertainty*	1/0.267

Notes: The parameter with *, ξ is internally calibrated using the top 1% opaque firms' average forecast variance in the equilibrium outcome, assuming that non-listed firms' opacity is at a similar level with the most opaque firms among the listed.

Table 3: Estimated parameters

Besides the estimated parameters, we fix the following parameters before the estimation.

$$\{\alpha, \gamma, K^I\}$$

Capital share, α , is set to be 0.50. Because our model is abstract from a labor input, the capital share in the model needs to be interpreted as an after-labor-adjustment capital share, as in the following formulation:

$$\begin{aligned}
Ak^\alpha &= \max_L \tilde{A} k^{\tilde{\alpha}} L^\epsilon - wL \\
&= (1 - \epsilon) \tilde{A}^{\frac{1}{1-\epsilon}} \left(\frac{\epsilon}{w} \right)^{\frac{\epsilon}{1-\epsilon}} k^{\frac{\tilde{\alpha}}{1-\epsilon}} = Ak^{\frac{\tilde{\alpha}}{1-\epsilon}}
\end{aligned}$$

where $A = (1 - \epsilon) \tilde{A}^{\frac{1}{1-\epsilon}} \left(\frac{\epsilon}{w}\right)^{\frac{\epsilon}{1-\epsilon}}$. Therefore, our model's α is equivalent to a standard model's $\frac{\tilde{\alpha}}{1-\epsilon}$. We assume $\tilde{\alpha} = 0.2$, and $\epsilon = 0.6$, leading to $\alpha = 0.50$. Public intangible share, γ , is assumed to be equal as the private intangible share, θ . The total intangible capital stock, K^I , is normalized to 1.

Parameters	Description	Value
α	Capital share	0.50
γ	Public intangible share	$= \theta$
r	Rental rate tangible capital	0.02
K^I	Total intangible supply	1
z	TFP level	1

Table 4: Fixed parameters

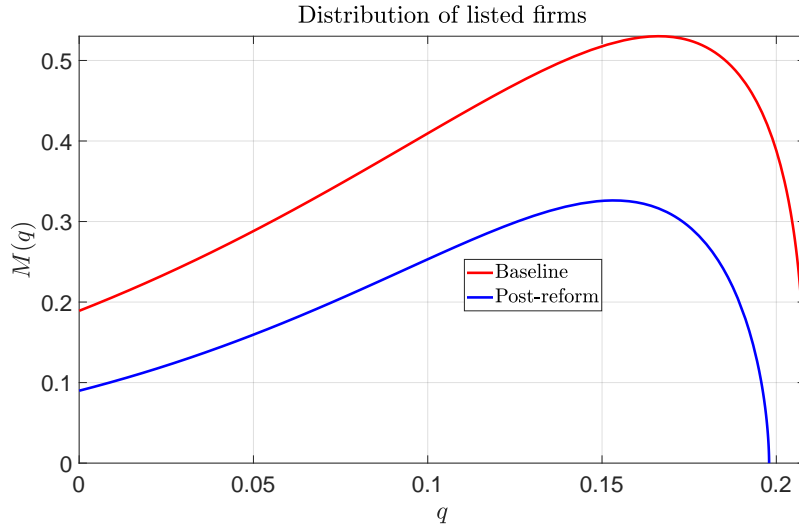


Figure 3: Distribution of listed firms over transparency

Figure ?? shows the unnormalized distribution of listed firms over the transparency level for the baseline and the post-change periods. The entire distribution shrinks in the post-change period, reflecting the reduced number of listed firms. Also, the distribution shifts towards the left, leading to reduced average transparency in the post-change period.

5.2 Decomposition analysis

In this section, we calculated the average contributions of each parameter to the decrease in the measure of listed firms and the decrease in the average transparency. The contributions are obtained in the following way. First, we keep the estimated parameters at their baseline

values and change only one of the parameters to their post-change value. In this way we obtain the counterfactual measure of listed firms and average transparency if only that specific parameter changed. Second, we keep the estimated parameters at their post-change values and change only one of the parameters to their baseline value. Similarly, we obtain the counterfactual measure of listed firm and average transparency if only that specific parameter remained at the baseline value. We do this for all 5 estimated parameters. Then, we average both numbers from each parameter to obtain the average contributions to the decrease in the measure of listed firms and the decrease in the average transparency. In Table ??, we report the result of the decomposition analysis in the annualized percentage.¹⁸

Param.	Channel	Contribution to the change in (%):			
		#listed	transparency	productivity	welfare
	Total change (p.a.)	-2.3	-0.9	-1.5	-2.7
θ	Rising intangible share (p.a.)	-1.9	-0.4	-1.3	-1.7
χ	Stock return dispersion (p.a.)	-1.2	-0.6	-0.5	-1.9
ν_N	PE market improvement (p.a.)	-0.3	0	-0.1	-0.1
\bar{q}	SEC regulation (p.a.)	-0.1	0.1	-0.1	0.4
ζ	Opacity of private firms (p.a.)	1.2	0	0.4	0.7

Table 5: Decomposition of the channels in the macroeconomic changes

As shown in Table ??, the measure of listed firms went from 8.02% in the baseline period to 4.39% in the post-change period, which is around a 45% drop over 20 years. In the annual term, this is a 2.3% drop on average. Likewise, transparency has dropped annually by 0.9% on average; productivity has declined annually by 1.5% on average, and welfare has decreased annually by 2.7% on average.

For the total change in the number of listed firms, the rising intangible share, θ , contributed to -1.9; the stock return dispersion, χ , to -1.2; the PE market improvement, ν_N , to -0.3; the SEC regulation, \bar{q} , to -0.1; and the opacity of private firms, ζ , to 1.2 percentage.

These contributions suggest that a larger share of intangible capital, together with a higher stock return dispersion, accounts for most of the observed decrease in the number of listed firms. Also, it is worth noting that the improvement in private market funding has significantly contributed to the trend of the declining number of listed firms, consistent with ?. For other macroeconomic allocations, a similar pattern is observed: the dominant drivers of lowered transparency, productivity, and welfare are rising intangible share and the

¹⁸The two periods of comparison are 20 years apart from each other. So, we annualize the total change by a division of 20.

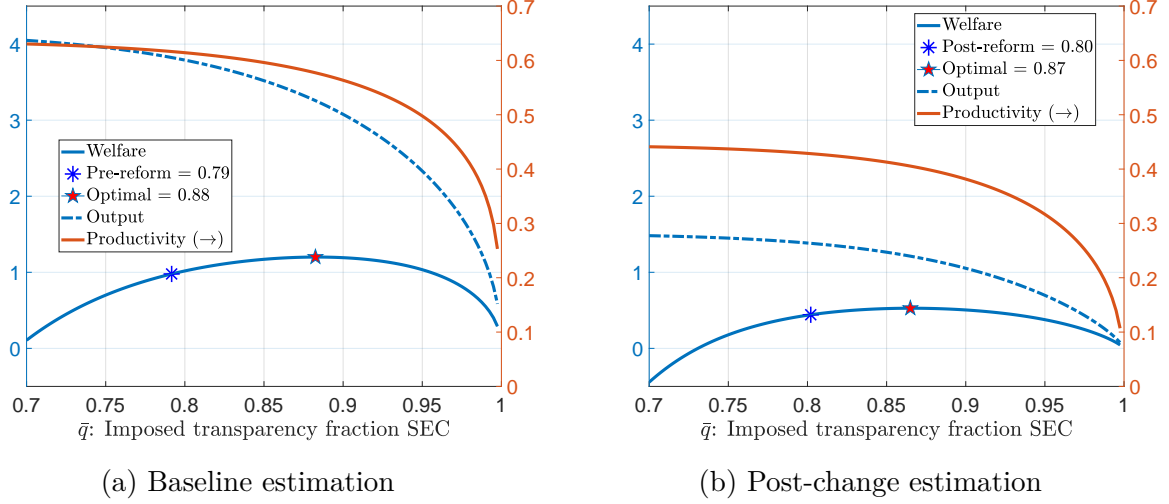


Figure 4: Optimal level of mandated transparency

increased stock return dispersion.

5.3 Optimal Policies

In this section, we use the proposed model to analyze the welfare optimizing level of imposed transparency. As shown in the previous section, the policy maker can choose the imposed transparency level \bar{q} . However, since welfare is obtained from the utility maximization problem of the household, \bar{q} will have two effects on welfare. On the one hand, lower imposed transparency increases the measure of listed firms which will have more access to finance relative to the private firms, increasing output and consumption. However, on the other hand, lower imposed transparency also increases the output's variance, lowering the welfare of the risk-averse household. Hence there is a trade-off between the level of consumption and its volatility. In Figure ?? we show the Laffer-type curve for the transparency policy for both periods.

The estimated level of transparency in the pre-change period is 0.75 (Table ??) and the optimal level is 0.89. Suggesting the mandated transparency was below the optimal level in the pre-change period. In the post-change period estimation, the results suggest that while the optimal level decreased slightly to 0.87, the estimated level increased to 0.87 (Table ??), shortening the gap between the optimal and estimated values. Moreover, although welfare at the optimal points decreased in the post-change period, we can see that welfare at the estimated values slightly increased. Also, it is worth mentioning that output and productivity also show an U-inverse shape.¹⁹ This property of the model suggests that

¹⁹Figure ?? only shows the region where output and productivity decrease monotonically with respect to

depending on the value of the estimated parameters, moving \bar{q} towards the welfare-optimal point could increase both output and productivity as well, achieving a divine coincidence. With the current estimated parameters, such divine coincidence happens when \bar{q} is above the welfare optimal point: decreasing \bar{q} towards the optimal would increase welfare, output and productivity.

6 Concluding remarks

This paper analyzes how the rising intangible has affected the secular trend of disappearing public firms and declining transparency of the financial report and its macroeconomic impact. From the empirical analysis, we show that the number of listed firms and the average transparency (the inverse of the earnings forecast dispersion) of listed firms' financial reports have substantially decreased in recent years. Then, from the cross-sectional regression of the transparency (opacity) on the firm-level intangible stocks, we document a negative correlation between the transparency and the intangible stock of a firm. To theoretically and quantitatively analyze the observed patterns in the data, we develop a heterogeneous-firm equilibrium model where the firm-level decision of going public or private and the distribution of rich equilibrium allocations are characterized in a closed form. Using the model, we theoretically show that the number of listed firms decreases in the strictness of the SEC's requirement on the listed firms' financial disclosure. From the estimated model, we quantitatively show that the rising importance of intangible capital is the crucial driving factor of the recent changes. Furthermore, the change has led to significant losses in productivity and welfare.

Then, we analyze the optimal regulation policy and on which side the current policy parameter is located with respect to the optimal level in terms of output, productivity, and welfare. Each of the macroeconomic allocations displays the inverted-U-shaped changes over the shifts in the regulation parameter. According to the estimated model, the SEC regulation parameter has been at a level lower than the optimal level in terms of welfare, while it is at a greater level than the optimal levels for productivity and output. The recent changes in the disclosure regulation, including Sarbanes-Oxley Act, slightly contributed to the trend of disappearing listed firms and a drop in productivity. However, it has significantly contributed to mitigating the welfare loss dominantly driven by the rising intangibles.

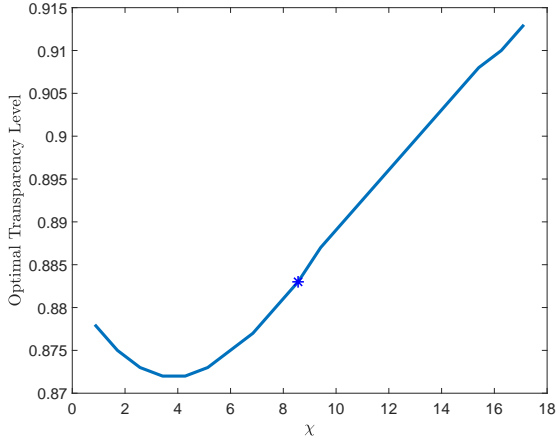
Our approach broadens the scope of structural policy analysis to the regulation on information disclosure. In the future research of this area, our analytical framework will serve as a useful tool to analyze the impact of information regulation change on the productivity

\bar{q} .

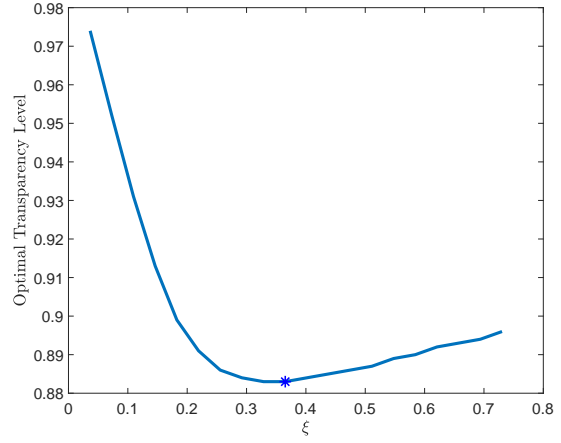
and the welfare.

Appendices

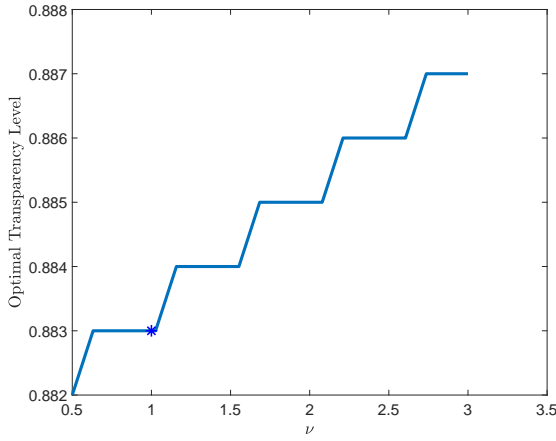
A Comparative Statics



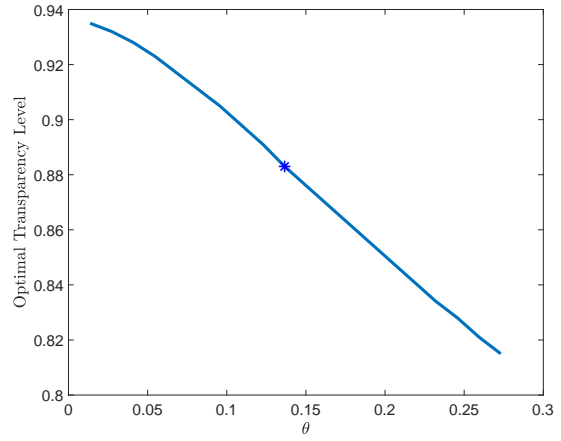
(a) Dispersion of stock returns χ



(b) Uncertainty private returns ξ

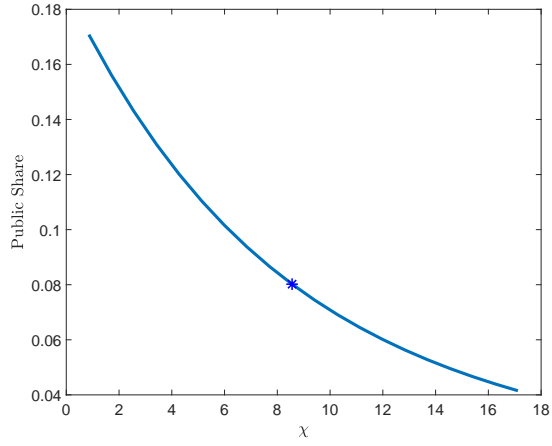


(c) Congestion private funds ν_N

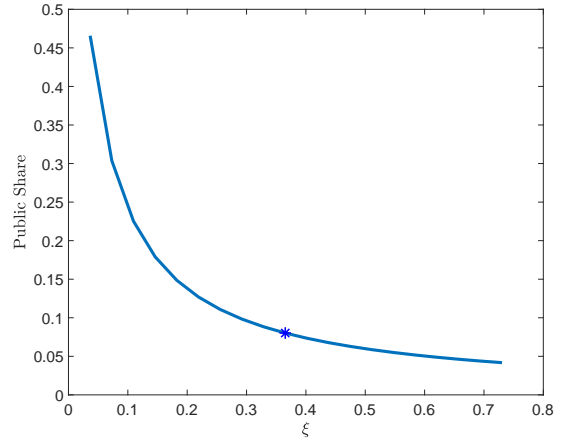


(d) Intangible share θ

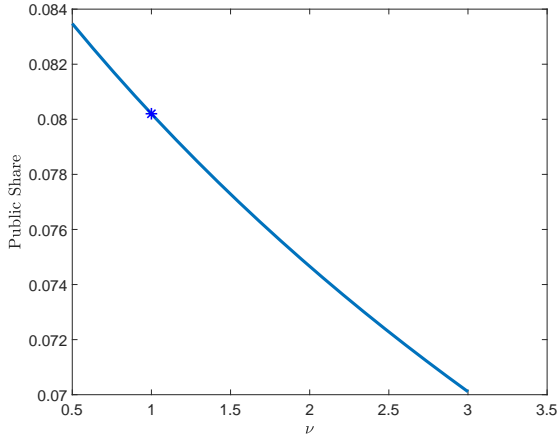
Figure A.1: Comparative statics on optimal transparency level with respect to each parameter. We change each single parameter, keeping the others constant at their baseline value, and calculate the resulting optimal transparency level.



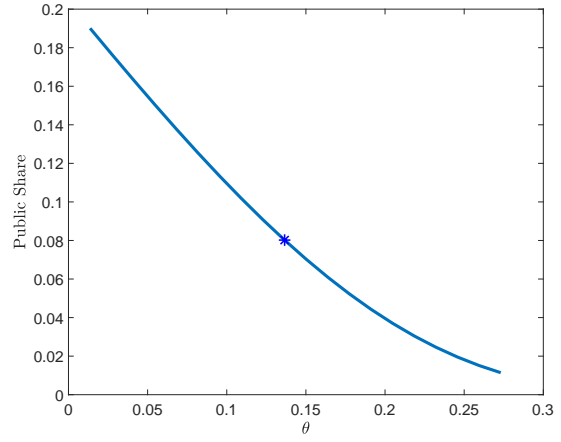
(a) Dispersion of stock returns χ



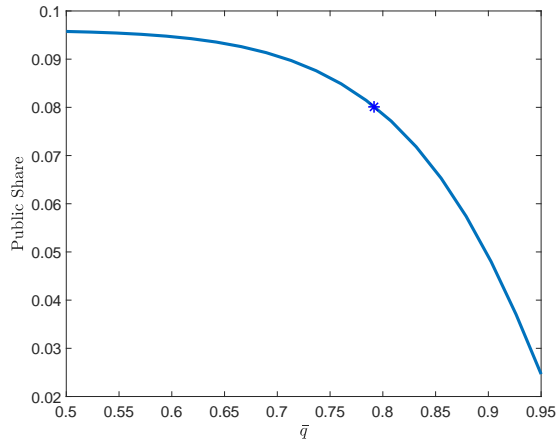
(b) Uncertainty private returns ξ



(c) Congestion private funds ν_N



(d) Intangible share θ



(e) Mandated minimum transparency \bar{q}

Figure A.2: Comparative statics on fraction of listed firms with respect to each parameter. We change each single parameter, keeping the others constant at their baseline value, and calculate the resulting number of listed firms.

B Proofs

B.1 Proof for Proposition ??

Proposition 1. (Intangibles and the transparency)

Given $\alpha + \theta < 1$, $k^I(q, \bar{q})$ decreases both in q and in \bar{q} .

Proof.

From FOC

$$\begin{aligned} [k_T] : \quad & z\alpha k_T^{\alpha-1} (k_I(1 - \bar{q} - q))^{\theta} (\Phi^{ex})^{\gamma} = r \\ [k_I] : \quad & z\theta k_T^{\alpha} (k_I(1 - \bar{q} - q))^{\theta-1} (\Phi^{ex})^{\gamma} (1 - \bar{q} - q) = p \\ [q] : \quad & -z\theta k_T^{\alpha} (k_I(1 - \bar{q} - q))^{\theta-1} (\Phi^{ex})^{\gamma} \phi^L(q) k_I \\ & + \left(z k_T^{\alpha} (k_I(1 - \bar{q} - q))^{\theta} (\Phi^{ex})^{\gamma} - r k_T - p k_I \right) \phi'^L(q) = 0 \end{aligned}$$

From the first-order conditions with respect to k_T and k_I , we obtain

$$\frac{r}{p} = \left(\frac{\alpha}{\theta} \right) \frac{k_I}{k_T}.$$

Substituting this relation into the first-order condition with respect to k_T , we get

$$r = \alpha z \left(\frac{\alpha p}{\theta r} \right)^{\alpha-1} (k_I)^{\alpha+\theta-1} (1 - \bar{q} - q)^{\theta} (\Phi^{ex})^{\gamma}.$$

Thus,

$$k_I = \left(\left(\frac{\alpha z (\Phi^{ex})^{\gamma}}{r} \right)^{\frac{1}{1-\alpha-\theta}} \left(\frac{r\theta}{p\alpha} \right)^{\frac{1-\alpha}{1-\alpha-\theta}} \right) (1 - \bar{q} - q)^{\frac{\theta}{1-\alpha-\theta}} = A (1 - \bar{q} - q)^{\frac{\theta}{1-\alpha-\theta}},$$

where $A := \left(\left(\frac{\alpha z (\Phi^{ex})^{\gamma}}{r} \right)^{\frac{1}{1-\alpha-\theta}} \left(\frac{r\theta}{p\alpha} \right)^{\frac{1-\alpha}{1-\alpha-\theta}} \right)$. As $\alpha + \theta < 1$, the proposition is immediate from the last equation. ■

B.2 Proof for Proposition ??

Proposition 2. (Transparency distribution)

The probability density function \mathcal{M} of transparency q has the following closed form:

$$\mathcal{M}(q) = (\bar{q} + q)^{\chi} (1 - \bar{q} - q)^{\frac{\theta}{1-\alpha-\theta}} \frac{1}{\phi^N}.$$

Proof.

We derive the following equations using the first-order condition with respect to q :

$$\begin{aligned}\frac{\phi'^L(q)}{\phi^L(q)} &= \frac{z\theta k_T^\alpha (k_I(1 - \bar{q} - q))^{\theta-1} (\Phi^{ex})^\gamma k_I}{zk_T^\alpha (k_I(1 - \bar{q} - q))^\theta (\Phi^{ex})^\gamma - rk_T - pk_I} \\ &= \frac{z\theta k_T^\alpha (k_I(1 - \bar{q} - q))^{\theta-1} (\Phi^{ex})^\gamma k_I}{(1 - \alpha - \theta)zk_T^\alpha (k_I(1 - \bar{q} - q))^\theta (\Phi^{ex})^\gamma} \\ &= \frac{\theta}{1 - \alpha - \theta} \left(\frac{1}{1 - \bar{q} - q} \right)\end{aligned}$$

From $\frac{\partial}{\partial q} \log(\phi^L(q)) = \frac{\phi'^L(q)}{\phi^L(q)}$, the solution of the first-order differential equation is as follows:

$$\phi^L(q) = (1 - \bar{q} - q)^n \tilde{C},$$

for some $n \in \mathbb{R}$ and some $\tilde{C} \in \mathbb{R}$. From the indifference condition in the equilibrium, $\pi^L \phi^L(q)$ does not depend on q .

$$\pi^L \phi^L(q) = \left(z(1 - \alpha - \theta) \left(\frac{\alpha p}{\theta r} \right)^\alpha A^{\alpha+\theta} (1 - \bar{q} - q)^{\frac{\theta}{1-\alpha-\theta}} \Phi^\gamma \right) (1 - \bar{q} - q)^n \tilde{C}$$

Therefore,

$$n = -\frac{\theta}{1 - \alpha - \theta}$$

This leads to $\phi^L(q) = (1 - \bar{q} - q)^{-\frac{\theta}{1-\alpha-\theta}} \tilde{C}$.

Then, the distribution of listed firms is as follows:

$$\begin{aligned}\mathcal{M}(q) &= (\bar{q} + q)^\chi / \phi^L(q) \\ &= (\bar{q} + q)^\chi (1 - \bar{q} - q)^{\frac{\theta}{1-\alpha-\theta}} \frac{1}{\tilde{C}}.\end{aligned}$$

From the indifference condition between listed and non-listed,

$$\begin{aligned}\phi^N &= \frac{\phi^L(q)}{\phi^N} \\ &= \frac{\left(z(1 - \alpha - \theta) \left(\frac{\alpha p}{\theta r} \right)^\alpha A^{\alpha+\theta} (1 - \bar{q} - q)^{\frac{\theta}{1-\alpha-\theta}} \Phi^\gamma \right) (1 - \bar{q} - q)^{-\frac{\theta}{1-\alpha-\theta}} \tilde{C}}{\left(z(1 - \alpha - \theta) \left(\frac{\alpha p}{\theta r} \right)^\alpha A^{\alpha+\theta} \Phi^\gamma \right)} \\ &= \tilde{C}\end{aligned}$$

Therefore, $\mathcal{M}(q) = (\bar{q} + q)^\chi (1 - \bar{q} - q)^{\frac{\theta}{1-\alpha-\theta}} \frac{1}{\phi^N}$

In the equilibrium, $\phi^N (= \tilde{C})$ is determined at the level where the following equation holds:

$$\int_0^{1-\bar{q}} \mathcal{M}(q) dq = 1 - M_N.$$

■

B.3 Proof for Corollary ??

Corollary 1. (Truncated Beta distribution)

The gross transparency, $y := q + \bar{q}$, follows a truncated Beta distribution where the shape parameters are $\chi + 1$ and $B + 1$, and the support is $[\bar{q}, 1]$.

$$q + \bar{q} \sim \frac{\mathbb{I}\{q \in [0, 1 - \bar{q}]\}}{1 - M_N} \times \text{Beta}(\chi + 1, B + 1),$$

where $B = \frac{\theta}{1-\alpha-\theta}$.

Proof.

We define $M_Y(y)$ as the probability density function of the random variable $y = q + \bar{q}$.

$$M_Y(y) \propto y^\chi (1 - y)^B \quad \text{and} \quad y \in [\bar{q}, 1].$$

Also, $\int_{\bar{q}}^1 M_Y(y) dy = 1 - M_N$. Therefore, $y \sim \frac{\mathbb{I}\{y \in [\bar{q}, 1]\}}{1 - M_N} \times \text{Beta}(\chi + 1, B + 1)$.

■

B.4 Proof for Corollary ??

Corollary 2. (Opacity distribution)

The probability density of return variances, $\hat{M}(\hat{q})$ has the following closed form:

$$\hat{M}(\hat{q}) = \frac{1}{\chi \phi^N} \hat{q}^{-\frac{1}{\chi}-2} \left(1 - \hat{q}^{-\frac{1}{\chi}}\right)^B \quad \text{for } \forall \hat{q} \in [1, \bar{q}^{-\chi}]$$

where $B = \frac{\theta}{1-\alpha-\theta}$.

Proof.

From $\hat{q} = (q + \bar{q})^{-\chi}$,

$$\frac{d\hat{q}}{dq} = -\chi(q + \bar{q})^{-\chi-1}.$$

As \widehat{q} is the strictly monotone trasformation of q , the following equation holds:

$$\begin{aligned}
\widehat{M}(\widehat{q}) &= \mathcal{M}(q) \left| \frac{dq}{d\widehat{q}} \right| \\
&= (\bar{q} + q)^\chi (1 - \bar{q} - q)^{\frac{\theta}{1-\alpha-\theta}} \frac{1}{\phi^N} \left(\frac{1}{\chi} \right) (q + \bar{q})^{\chi+1} \\
&= \widehat{q}^{-1} \left(1 - \widehat{q}^{-\frac{1}{\chi}} \right)^B \frac{1}{\phi^N} \left(\frac{1}{\chi} \right) \widehat{q}^{-\frac{1}{\chi}-1} \\
&= \frac{1}{\chi \phi^N} \widehat{q}^{-\frac{1}{\chi}-2} \left(1 - \widehat{q}^{-\frac{1}{\chi}} \right)^B \text{ for } \forall \widehat{q} \in [1, \bar{q}^{-\chi}]
\end{aligned}$$

■

B.5 Proof for Proposition ??

Proposition 3. (The relationship between disclosure regulation and the measure of listed firms)

M_N strictly increases in $\bar{q} \in (0, 1)$.

Proof.

The following equation holds for all $\bar{q} \in (0, 1)$:

$$\xi M_N^{-\nu_N} (1 - M_N) = \mathcal{B}(\chi + 1, B + 1) (1 - F(\bar{q}; \chi + 1, B + 1)).$$

We take the derivative with respect to \bar{q} on both sides:

$$\xi \left(-\nu_N M_N^{-(\nu_N+1)} - (1 - \nu_N) M_N^{-\nu_N} \right) \frac{dM_N}{d\bar{q}} = -\mathcal{B}(\chi + 1, B + 1) f(\bar{q}; \chi + 1, B + 1)$$

where f is the probability density function corresponding to the cumulative distribution function, F . Then, by rearranging the terms in the left-hand side of the equation,

$$\underbrace{-\xi M_N^{-\nu_N}}_{<0} \underbrace{\left(\nu_N (M_N^{-1} - 1) + 1 \right)}_{>0 \text{ (} : 0 < M_N < 1 \text{)}} \frac{dM_N}{d\bar{q}} = \underbrace{-\mathcal{B}(\chi + 1, B + 1) f(\bar{q}; \chi + 1, B + 1)}_{<0}.$$

Therefore, we conclude

$$\frac{dM_N}{d\bar{q}} = \frac{\overbrace{-\mathcal{B}(\chi+1, B+1)f(\bar{q}; \chi+1, B+1)}^{<0}}{\underbrace{-\xi M_N^{-\nu_N} \left(\nu_N (M_N^{-1} - 1) + 1 \right)}_{<0}} > 0.$$

■