Women's Labor Force Participation and the Business Cycle[*]

Sara Casella[†]

University of Pennsylvania

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Abstract

This paper studies the macroeconomic implications of the rise in participation and attachment to the labor force of women and secondary earners. I develop a business cycle model of couples that features labor market frictions, endogenous labor supply, and human capital accumulation. Households face unemployment risk over the business cycle, and secondary earners adjust their labor supply to respond to this risk, so that they are more likely to participate when primary earners face a high risk of job loss. I validate the model using novel empirical evidence documenting that women with more previous labor market experience and higher income are more likely to respond. A large mass of marginal secondary earners will dampen fluctuations in aggregate employment if the income effect induced by unemployment is greater than the substitution effect due to lower wages. The magnitude of the counter-cyclical effect is proportional to the distance from the participation frontier of secondary earners, which in turn depends on the gap in net wages between partners. For values of the wage gap smaller than 20%, women's aggregate labor supply elasticity converges to men's, and the dampening effect wanes.

1   Introduction

Over the course of the twentieth century, women have been catching up dramatically in terms of pay and labor force participation in the U.S., and today they contribute to a much larger share of total employment than in the past. As a consequence, the U.S. has gone from having a majority of households with one breadwinner to a structure in which the most common type of household is a dual-earner household.

In this paper, I show that the extent to which secondary earners participate in the labor markets significantly impacts how households' labor supply and aggregate employment respond to the business cycle. If unemployment risk in downturns is high enough relative to movements in the wage rate, a large mass of flexible secondary earners available to enter the labor market will dampen fluctuations in aggregate employment. However, the dampening effect wanes when women and secondary earners have little room to increase their labor supply.

The dampening effect of secondary earners relies on the fact that they may respond to their partners becoming unemployed by entering the labor force and finding a job. This mechanism, sometimes called the "added worker effect" (Lundberg 1985), is a way by which non-working partners can provide insurance to the family against income shocks, and it induces a negative correlation between spouses' employment statuses. Importantly, I offer new empirical evidence showing that this effect is stronger when the secondary earner's previous employment period is more recent and her income is high. This means that not all secondary earners respond in the same way to partners' job loss: Individuals closer to participation and with higher potential earnings are more effective added workers. Understanding the evolution of women's attachment to the labor force is, therefore, a crucial factor for the business cycle.

Motivated by the micro-evidence at the individual level, I develop a business cycle model with singles and couples and fluctuations in labor market flows. The model features imperfectly uninsurable uncertainty, safe assets accumulation, and

job-finding and losing frictions, as in Krusell et al. (2011), Krusell et al. (2017), McKay and Reis (2021), Mankart and Oikonomou (2017). Households jointly decide on consumption, labor supply, and savings. The labor market is subject to search frictions: workers may lose jobs, and unemployed workers must wait for job offers. Not all individuals are always part of the labor force because searching and working are costly in terms of utility. The human capital of employed workers increases over time due to returns to experience, while the human capital of workers who are out of employment depreciates, either because they are suffering an unemployment spell or are out of the labor force.

Within couples, none, one, or both individuals can choose to work. The model generates gender differences in participation and attachment through various channels. First, women face an exogenous wage gap that manifests as lower efficiency units of labor. The wage gap in the model is chosen to match the unconditional average hourly wage of employed women relative to employed men. It could reflect differences in education, occupation, misallocation, discrimination, or a combination of different factors. Second, couples are subject to joint progressive taxation, which has been shown to significantly holds back female labor supply by subjecting secondary earners to high marginal tax rates[1]. Because of this, women participate less (and less often) and hence accumulate less human capital than men.

The economy is subject to shocks to aggregate total factor productivity and shocks to frictions in the labor market, which raise unemployment risk in downturns. In the absence of time-varying unemployment risk, marginal secondary earners and workers would quit their job or stop searching during recessions when wages are low, and fewer workers would desire to enter the labor markets. This effect is present in many standard models of labor supply. However, if unemployment risk in downturns is high enough relative to movements in the wage rate, secondary earners who are just below the participation frontier will enter the labor market. The theory highlights the importance of cyclical idiosyncratic risk: how single and

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[1]See, for example, (Borella, De Nardi, and Yang, 2022), among others.

married men and women respond differently to it heavily influences the properties of aggregate labor supply.

The baseline model is calibrated on data between 1975 and 1985 and can match labor force participation rates and unemployment rates by different groups and the level and cyclicality of job finding and separation rates by men and women. To quantitatively account for the changing importance of married women's participation from the 1970s to today, I conduct a comparative statics exercise using the calibrated model and compare the business cycle in different economies that differ only by the extent of the gap in wages between partners.

The model predicts that (i) the income effect coming from the family insurance motive is larger than the substitution effect from lower wages in downturns, and (ii) hours and consumption volatility respond non-monotonically to changes in the wage gap. As the wage gap grows smaller, women have a higher earnings capacity, both because they face higher wages and because they accumulate more work experience before quitting the labor force. Consequently, they find it easier to enter during recessions, acting as a dampening force on the severity of recessions. I show quantitatively that a reduction in the wage gap from 50% to 30% has translated into a 6% decrease volatility of hours per capita and a 10% decrease in consumption volatility. For levels of the wage gap lower than 20%, women's participation and labor supply elasticity look more similar to men's. Most women are left with little room to increase their labor supply, and the dampening effect on employment disappears. Most households rely on self-insuring through assets.

Related Literature   This paper relates to several strands of literature. First, the literature that studies the determinants of aggregate labor supply in the presence of an extensive margin. Keane (2011) and Keane and Rogerson (2012) advocate the importance of studying how labor supply responses interact with some of the features studied in this paper (e.g., incomplete markets, human capital, and the extensive margin). When the extensive margin is operative, the slope of the aggregate

labor supply schedule is determined by the distribution of reservation wages (see Chang and Kim (2006)). Individuals closer to the reservation wage will be more sensitive to temporary wage changes, especially at monthly or quarterly frequencies (Erosa, Fuster, and Kambourov (2016)).

A related literature that explores business cycles implications has therefore identified the importance of marginal workers for aggregate employment fluctuations: A large mass of marginal individuals typically implies large fluctuations in employment and hours (Hansen (1985), Rogerson (1988) , Hagedorn and Manovskii (2008), Jaimovich and Siu (2009), Ljungqvist and Sargent (2017)). However, this paper shows that, contrary to what would emerge in a model without couples or without pro-cyclical unemployment risk, a larger proportion of marginal secondary workers has the opposite effect of stabilizing fluctuations. The model highlights that this is especially true if there are many marginal workers just *below* the participation frontier.

This paper also belongs to a literature that incorporates families into macroeconomics, showing that accounting for the family leads to new answers to classic macroeconomic questions (see Doepke and Tertilt (2016) for a review). In particular, I focus on how couples provide each other with insurance from labor market risk and how the large changes in the nature of the family affect the evolution of aggregate labor supply over the business cycle.

This paper is most related and builds on recent works that explore how spousal insurance can affect the aggregate economy. Mankart and Oikonomou (2017) pioneered the introduction of couples in models of labor market flows and participation, which Krusell, Mukoyama, Rogerson, and Sahin developed in the context of "bachelor" households (Krusell et al. (2011), Krusell et al. (2017)). They analyze the cyclicality of the labor force in U.S. data for different groups and show that spousal insurance contributes to its average acyclicality. Bardoczy (2022) shows that spouses can provide significant active and passive consumption insurance in a similar framework, where recessions are caused by movements in the discount factor and are

accompanied by higher rates of job loss. In particular, he makes the important point that even if secondary earners earn only a smaller fraction of household income, it is those who gain the most from doing so that select into "adding a worker". Since in his framework recessions do not come with movements in the marginal product of labor, the trade-off with the substitution effect caused by lower wages is not present.

Contributing to this body of work, this paper models secular trends in labor force participation and attachment. The resulting insight is that movements in spousal labor supply have a larger impact on aggregates when there are more marginal secondary earners; i.e., the strength of the counter-cyclical effect is proportional to the distance from the participation frontier of secondary earners. Quantitatively, getting a realistic mass of marginal women (and men) is, therefore, especially important. Relatively to previous papers, two novel features are important in this sense: progressive taxation and human capital accumulation and depreciation. Borella, De Nardi, and Yang (2022), among others, show that progressive taxation significantly holds back female labor supply in dual-earner couples. Moreover, without human capital depreciation – for example, if productivity follows a mean-reverting process – many non-participating women with high productivity would be ready to enter at any point, even if the wage gap is relatively high.

At the heart of the micro-foundation of the model lies the fact that partners jointly decide their labor supply and are subject to involuntary labor market separations. The so called added worker effect, i.e., a worker joining the labor force in response to their spouse's job loss, has been extensively studied in the empirical labor literature. Lundberg (1985), Stephens (2002), Parker and Skoufias (2004), and Juhn and Potter (2007) are examples. Bacher, Grubener, and Nord (2022) show that counter-cyclical entry is much more pronounced for young than for old households. Guner, Kulikova, and Valladares-Esteban (2020) use a decomposition method based on joint transition probabilities to show that the share of households in which both members are non-employed would be around 16% higher in the absence of the added worker effect. I confirm the existence of the added worker effect in PSID data and offer new empirical

evidence on the importance of the previous experience of married women.

There is also evidence that the added worker effect has strengthened over time among single earner couples. Mankart and Oikonomou (2016) and Guner, Kulikova, and Valladares-Esteban (2020) find so in CPS data. Mankart, Oikonomou, and Pascucci (2021) argue that this is because structural changes – such as declining gender wage gap – increased the insurance value of the added worker effect and not because of declining search frictions. A similar effect is present in my model. Finally, recent work finds that women respond not only to the realization of their husband displacement but also to a higher risk of displacement. For example, Park and Shin (2020) show that wives increase their labor supply significantly in response to increases in the variance of husbands' permanent wage shocks, reducing the welfare cost of increased earnings risk since the early 1970s. Ellieroth (2022) argues that women respond to increases in unemployment risk by quitting less.

The last strand of literature that this paper contributes to is the literature that considers how changes in demographics, and especially in women's participation, change the business cycle. Jaimovich and Siu (2009) perform a cross-country analysis and document that a larger mass of young workers contributes to amplifying fluctuations. Fukui, Nakamura, and Steinsson (Forthcoming), and Olsson (2020) show that recoveries for men have always been slow, and only recoveries for women have slowed since the early 1990s as a result of a slower or zero increase in the labor force participation of women. Finally, (Albanesi, 2019) provides evidence in an estimated DSGE model that women's employment plays a crucial role in phenomena such as jobless recoveries, the productivity slowdown, and the great moderation. Compared to these papers, I focus explicitly on the interplay between insurance motives and labor supply. In my model, the strength of this motive is the main driver of fluctuations in hours.

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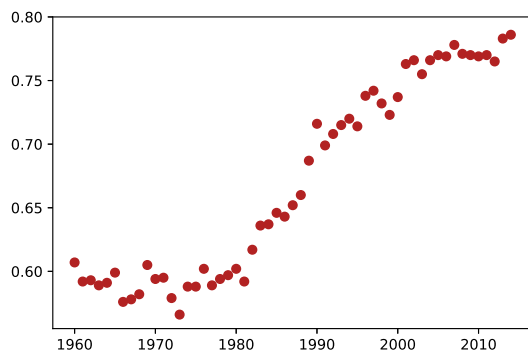
(a) Female to male earnings ratio, workers working year-round, full-time.
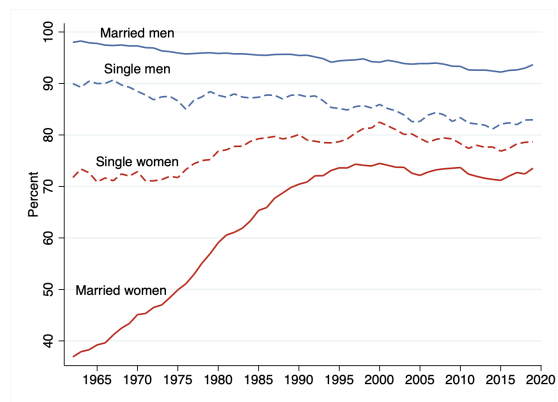
(b) Labor Force Participation by different groups

Figure 1: Source: U.S. Census Bureau Current Population Report, Income and Poverty in the United States: 2020, Table A-7. and Current Population Survey.

2 Empirical Evidence

I first discuss some long-run trends in labor supply by men and women, and by single and married individuals. Then I bring some new evidence of the existence and extent of the so called added worker effect using micro data.

2.1 Trends in Labor Supply

Data on trends in employment and income come from Current Population Survey March Annual Social and Economic Supplement (ASEC), ranging from 1962 to 2019. I focus on prime-age workers and exclude respondents younger than 25 or older than 55. The choice of schooling and retirement is therefore mostly not considered. All individuals are weighted by the individual weights provided by the survey.

Figure (1b) shows the trends in labor force participation by gender and marital status. The most striking feature is that participation among married women rose from around 40% in the sixties to around 70% in recent years. At the same time, participation among single women slightly increased, while that of married men decreased from 98% to around 93%.

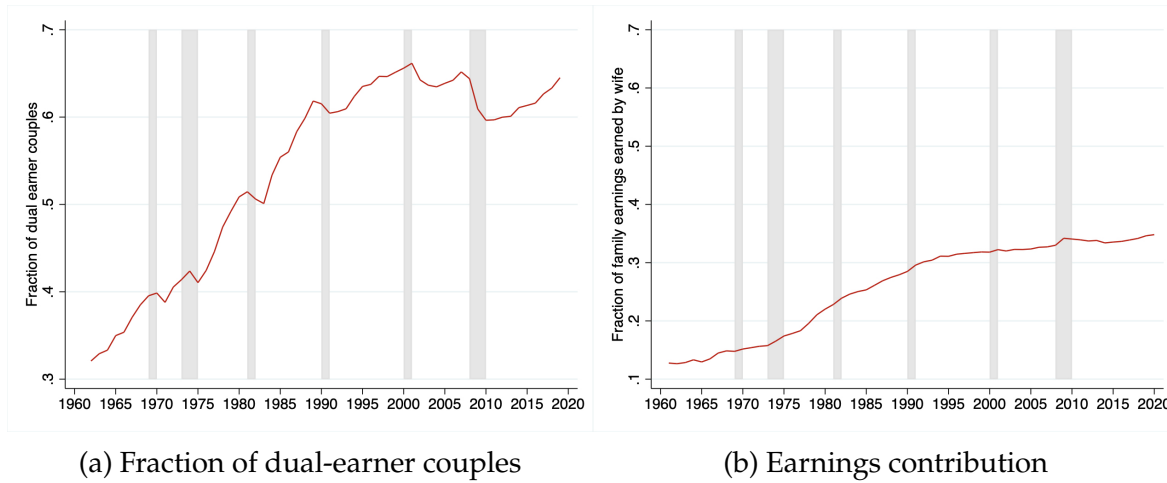(a) Fraction of dual-earner couples   (b) Earnings contribution

Figure 2: The number of dual-earner couples over all couples increased to 65%, and married women now contribute to around 35% of couple's earnings. Source: CPS ASEC.

As a consequence, the most common type of household in the U.S. today is a dual-earner household. Specifically, the number of dual-earner couples increased to 70% of all couples, and married women now contribute to around 35% of couple's earnings (see Figures (2a) and (2b)).

The increase in participation among married women has coincided with a strong decrease of the wage gap. Figure (1a) plots the ratio of median earnings of women working full-time, year-round and of median earnings of women working full-time, year-round. While until around 1980 women only earned 60% of men working comparable hours, they earned around 78% in the 2010s. It is important to notice that this is a measure of the *unconditional* wage gap, which reflects differences in education, job experience, occupation, child penalties, on top of possible overt and covert discrimination. That is, this is the difference in the wage that a woman and a man will face on average at different points in time.

2.2   Micro evidence: Active Spousal Insurance

The study uses data from the Panel Study of Income Dynamics (PSID), covering years from 1968-1997. These are the years for which PSID has annual coverage of

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households[2]. The main advantage of the PSID compared to other publicly available datasets is that it follows families over time. Therefore, it allows me to (i) document whether spouses who enter the labor force in respond to their partner becoming unemployed do eventually find a job, and (ii) explore heterogeneity with respect to previous experience. On the other hand, one of the main disadvantages is the relatively low sample size. For example, the Current Population Survey (CPS), which has been recently used to study the added worker effect for example by Mankart and Oikonomou (2016, 2017) and Guner, Kulikova, and Valladares-Esteban (2020) has a larger sample size, but only follows people for four consecutive months.

For this analysis, the sample is restricted to cohabiting families in which both the husband and the wife are between the ages of 25 and 60. The resulting sample contains 10,454 couples. I create a variable that indicates whether the husband becomes unemployed using the Employment Status variable. Specifically, the variable is equal to one if the husband goes from "Working Now" in the previous year to "Unemployed, looking for work" in the current year. I exclude individuals who go from employment to out of the labor force (for example, retirement), or who go from employment to "Only temporarily laid off or sick leave". I end up with 1,871 such events. PSID also asks unemployed people the reason why they have lost their previous job. Of those 1,871 workers who became unemployed, 80% were laid-off or their employer went out of business, 15% quit, 5% are unemployed because their seasonal work ended.

I first document the impact of unemployment on husband's earnings in recessions and expansions for the years studied in my sample using a dynamic diff-in-diff centered around husband's unemployment spell as in Jacobson, LaLonde, and Sullivan (1993), Stevens (1997), David and Von Wachter (2011), and most recently Birinci

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[2]Including the years 1999-2019 presents some challenges because households are only interviewed every two years. Since most of the adjustment for spouses happens the year that the husband becomes unemployed, such a lower frequency is likely to miss it. Nonetheless, I conduct a robustness analysis on those years in Appendix A.
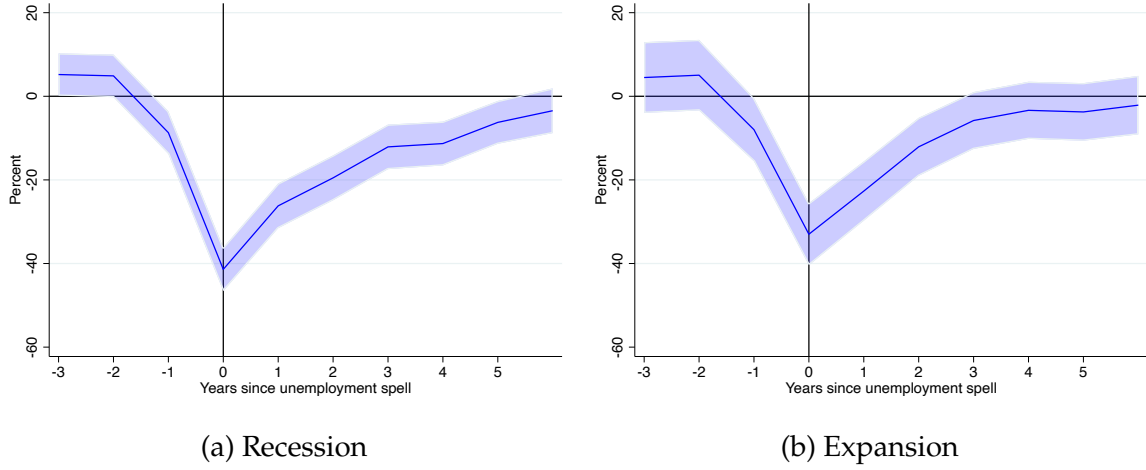
| (a) Recession | (b) Expansion |

Figure 3: Average difference in labor earnings between married men who experience an unemployment spell and married men who do not. The blue line reports point estimates with 95% confidence interval bands around. Source: PSID.

(2020):

$$Earnings_{it} = \alpha_i + \gamma_t + \beta X_{it} + \sum_{k \geq -3}^{5} \psi_R^k D_{it}^{R,k} + \sum_{k \geq -3}^{5} \psi_E^k D_{it}^{E,k} + \epsilon_{it} \qquad (1)$$

where $Earnings_{it}$ are husband's annual labor earnings, and is D_{kit} an indicator variable that equals one when husband in family i is k years from becoming unemployed either in a recession year (R) or expansion year (E), where recession years are defined by the NBER.

The main variable of interest is then the effect on spousal labor supply. To investigate the reaction of spouses, I first divide them into two groups: Those who were not working in the previous year and those who were already working in the previous year, as defined by self-reported employment status and hours[3].

I then report the impact of the husband becoming unemployed on three variables. The first is related to the extensive margin and is whether the spouse goes from being out of the labor force to employed (only for the group of women who were not previously working). The second is spousal hours, and the third is spousal income.

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[3]Where employment status is not available, as is the case before 1978, I classify individuals as not employed in the previous year if they reported working less than 400 hours.
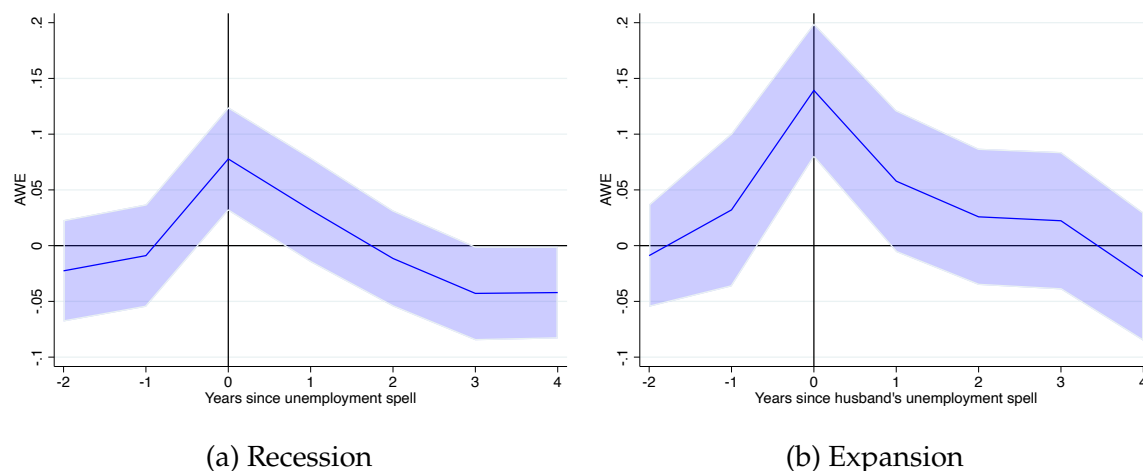
| (a) Recession | (b) Expansion |

Figure 4: Average difference in the probability of entering the labor market and finding a job between women whose husband experiences unemployment and women who do not. The blue line reports point estimates with 95% confidence interval bands around. Source: PSID.

Long-term Effects of Unemployment on Husband's Income Coefficients are reported as a percent of average earnings of employed workers. Married men who become unemployed lose about 40% of labor earnings on impact compared to similar individuals who are not unemployed, and it takes around 3 to 5 years to recover completely. Becoming unemployed in a recession has more severe consequences than in an expansion. This is in line with results in Davis and Von Wachter (2011) and Birinci (2020)[4].

Effect of Unemployment on Spouses In Figure 4 I report the difference in the probability of entering the labor market and finding a job between women whose husband experiences unemployment and women who do not. If the wife of a man who becomes unemployed was not working, she is 8 to 14 percentage points more likely to get a job. Most of the action happens on impact, which indicates that women either enter immediately or do not enter, and there is no response delayed by more than a year.

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[4]Since this dataset does not include the years after 1997 and the Great Recession, the differences between recessions and expansions are less pronounced than in those two studies.

These numbers are mostly in line with previous results on the added worker effect, although with a difference in specifications. Previous work on CPS data follows people for four months, so most of the action is from out of the labor force to *searching for a job*, but it is hard to see how many spouses actually get a job. PSID follows people for longer, so I focus on the transition to employment and confirm that it is significantly larger than 0. In Appendix A I report the differential impact on hours and income.

I then turn to analyze the heterogeneity in this effect by experience and income using a triple interaction specification in which the dummies D in equation 1 are interacted with a variable that encodes either how often the spouse has worked in the past or her average income when working in the past.[5]

The first measure is a proxy for distance from participation. If a spouse has never worked in five years, I interpret this as her being quite far from the participation frontier, and hence unlikely to enter even if the face. The second measure is a proxy for potential earnings. As can be seen in Figures 5 and 6, the probability of finding a job is much larger for women who have worked more often in the past and earned higher income when working. If a spouse has never worked in the five years preceding the husband's unemployment spell, the effect is not statistically significant from zero at the 95% confidence level. On the other hand, the response keeps growing with previous experience. The effect is similar for income. I interpret these findings as evidence that individuals closer to participation and with higher potential earnings are more effective added workers.

Finally, Figure 7 shows that the spouse of a man who becomes unemployed was already working, her hours do not significantly change compared to other women. This points to the fact that it is harder to increase hours if you are already working.

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[5]Both specifications only include women who are observed for at least six years prior to the unemployment spell. The specification with income in the past is restricted to having observed at least one reported income, which excludes women who are never observed working. I only report the coefficients at time 0.
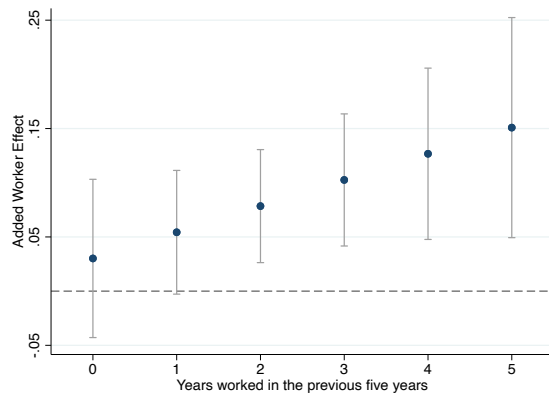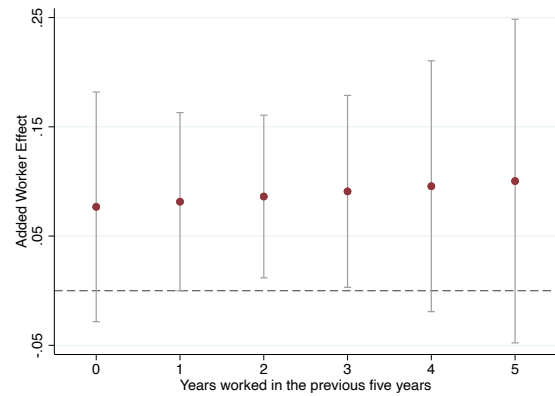
(a) Recession   (b) Expansion

Figure 5: The difference in the probability of finding a job becomes larger if the wife has worked more often in the past. The dots report point estimates at time 0 with 95% confidence interval bands around. Source: PSID.
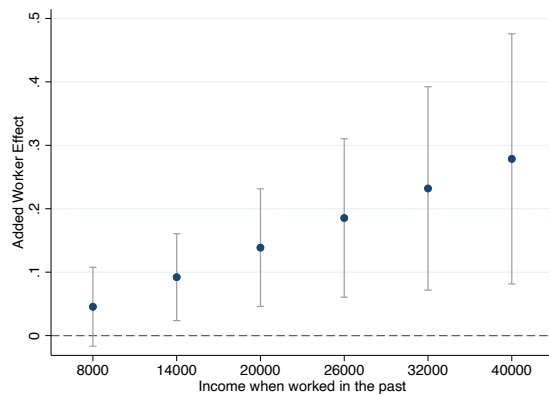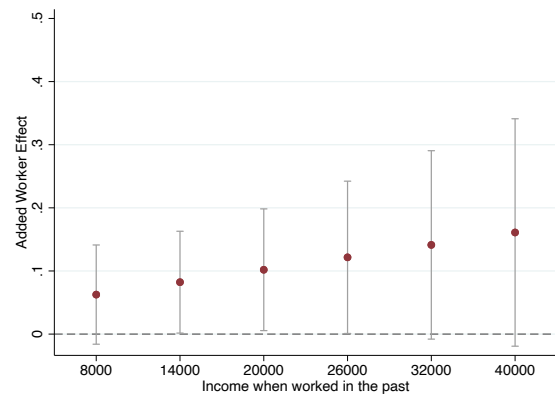

(a) Recession   (b) Expansion

Figure 6: The difference in the probability of finding a job becomes larger if the wife had higher income when working in the past. The dots report point estimates at time 0 with 95% confidence interval bands around. Source: PSID.

(a) Recession   (b) Expansion

Figure 7: No difference in hours for women already working. The blue line reports point estimates with 95% confidence interval bands around. Source: PSID.

3 The Model

3.1 Overview

The core model is one of singles and couples' consumption and labor supply in the presence of imperfectly insurable uncertainty and job-finding and losing frictions.
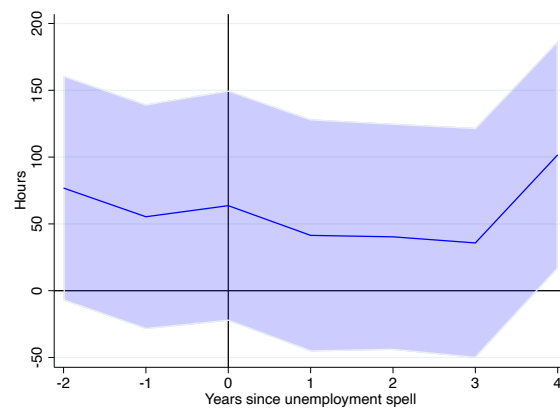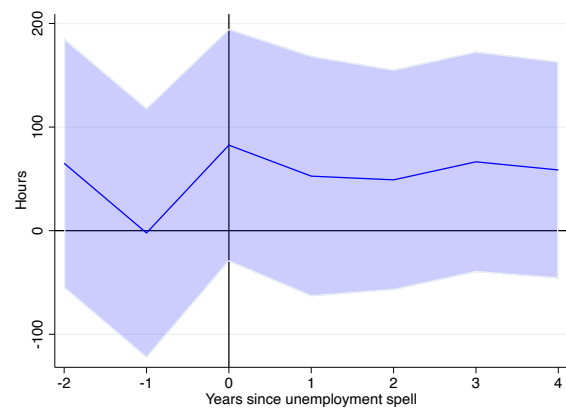
3.2 Environment

Agents and Preferences   The economy is populated by a measure one of households. They differ in terms of their marital status (married and single of two genders), labor market status and human capital. Marital status and gender are permanent types, and they determine the stochastic processes for labor market shocks and human capital accumulation.

Households derive utility from consumption and derive disutility from working for pay (n) and from searching for work (q). Labor supply and search decisions are at the extensive margin only. The period utility function for a single of gender g is given by:

$$u_{single,g}(c,n,q) = \frac{c^{1-\sigma}}{1-\sigma} - \phi_g \, n \, - \kappa_g \, q \tag{2}$$

and the period utility function for couples is:

$$u_{couple}(c,n_f,n_m,q_f,q_m) = \frac{c^{1-\sigma}}{1-\sigma} - \eta_f \, n_f - \eta_m n_m - \kappa_f \, q_f - \kappa_m \, q_m \tag{3}$$

The two members of each couple are assumed to make joint decisions on consumption and labor supply. Discussion on unitary model and perfect commitment HERE. Finally, each household discounts the future at factor β.

Wages   The effective wage of an individual depends on the aggregate wage in the economy w and the number of efficiency units the individual is endowed with. The latter depends accumulated human capital, and on the individual's gender. Labor

income y for employed individuals i, j of gender $g \in \{m, f\}$ working n hours is:

$$y_{i,m} = h_i w(n_i), \qquad y_{j,f} = (1 - \delta) h_j w(n_j) \tag{4}$$

Two things are worth noting: first, δ acts as a wedge between women's underlying productivity and their market productivity. Therefore, it is the main driver of the wage gap in the model. Second, individuals with a higher human capital have a higher effective wage.

Labor market frictions   One of the main sources of risk in the model is unemployment. Every period a fraction s of employed households loses employment and must search to regain employment. Following Krusell, Mukoyama, Rogerson, and Sahin (2017) I abstract from vacancy creation and instead specify a process in which workers find jobs with an exogenous probability f.

At the beginning of the period, individuals are characterized by their work status and by whether they receive unemployment insurance or not. Individuals can be employed (e), out of the labor force (o), or unemployed. Unemployed individuals either receive unemployment benefits (u_b) or are unemployed without benefits (u_{nb}). Couples are composed of two individuals and are therefore characterized by the individual state of each partner, for a total of sixteen possible combinations.

The transitions across states depend endogenously on individuals' labor supply and search effort decisions and exogenously on labor market finding and separation shocks. Specifically, the employment state at the beginning of the period determines the probability that the worker receives a job offer, and that she will be eligible to receive unemployment benefits.

Agents have to receive an offer to be able to work. If a job offer is received, the worker can decide whether to accept the offer and, if so, whether to work full-time or part-time. If she accepts the offer, she will become employed (e). If she decides to turn down the offer, she becomes non-employed. Non-employed individuals can

	Offer	No offer, non UI eligible	No offer, UI eligible
Employed	$(1-s)$	0	s
Out of labor force	f	$(1-f)$	0
Unemployed, no UI	f^*	$(1-f^*)$	0
Unemployed, UI	f^*	$\lambda(1-f^*)$	$(1-\lambda)(1-f^*)$

(a) Given their current employment and eligibility status, individuals exogenously receive a new offer and/or lose their unemployment benefits. Employed individuals lose their jobs with probability s. Out of labor force and unemployed individuals find a job with probabilities f and f^* respectively, with $f^* > f$. UI benefits expire at rate λ. All finding and separation rates are gender and marital status specific, and depend on the aggregate Total Factor Productivity Z.

	Employed'	Out of labor force'	Unemployed, no UI'	Unemployed, UI'
Offer	$\mathbb{1}(n=1)$	$\mathbb{1}(n=0,q=0)$	$\mathbb{1}(n=0,q=1)$	0
No offer, non UI eligible	0	$\mathbb{1}(q=0)$	$\mathbb{1}(q=1)$	0
No offer, UI eligible	0	$\mathbb{1}(q=0)$	0	$\mathbb{1}(q=1)$

(b) Given their offer status, individuals make labor supply decisions (n) and search effort decisions (q), which endogenously determine next period's employment status.

Table 1: Transition from current employment status to future employment status

decide whether to engage in costly search activity to increase their future job-finding probability. Non-employed individuals who do not exert search effort are considered out of the labor force (o); if they exert search effort they are considered unemployed (u)[6]. If a job offer is not received, individuals are automatically non-employed.

Employed agents experience involuntary separation with probability s. Only agents who become unemployed after the arrival of a separation shock can claim benefits. If they turn down an offer or voluntarily quit a job, they do not have access to unemployment benefits. For as long as they receive benefits, they must exert search effort. Notice that all unemployed agents exert the same search effort, whether they receive benefits or not. Finally, unemployment benefits expire at a constant rate in every period. A complete description of the transition probabilities can be found in Table 1.

The exogenous finding rates, separation rates, and unemployment benefit duration all depend on the aggregate state of the economy (Z). Specifically, during a recession, finding rates go down and separation rates go up, leading to higher involuntary unemployment. The duration of unemployment benefits is also longer.

Cyclical idiosyncratic risk is at the heart of the theory: how single and married men and women respond differently to it heavily influences the cyclical properties of labor supply. Cyclical idiosyncratic risk is also important because it leads business cycles to have substantial welfare costs.

Human capital accumulation process   An individual's human capital h is discrete and defined on a grid H. The human capital accumulation and decumulation process is stochastic and follows a first-order Markov chain that depends on hours worked n. If an individual works positive hours, human capital is more likely to go up, especially if she works full-time. If an individual works zero hours, either because she is out of the labor force or unemployed, human capital is more likely to go down

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[6]This classification closely follows the criterion of the CPS whereby individuals are considered unemployed if they are actively searching for a job.

(depreciate). This process captures both random shocks to productivity and returns to experience.

Summary: Timeline of the model   Figure 8 summarizes the timeline of the model.

0	1	2
Beginning of period: **Assets** a, **Employments status** $e_f, e_m \in \{E, U_b, U_{nb}, N\}$, **Human capital** h_f, h_m **Aggregate state** $\mathcal{S}$	Aggregate shocks to TFP and to labor market frictions probabilitites are realized $\mathcal{S} \to \mathcal{S}'$	Individual labor market finding and separation shocks are realized $e_f, e_m \to s_f, s_m \in \{O, NO_b, NO_{nb}\}$

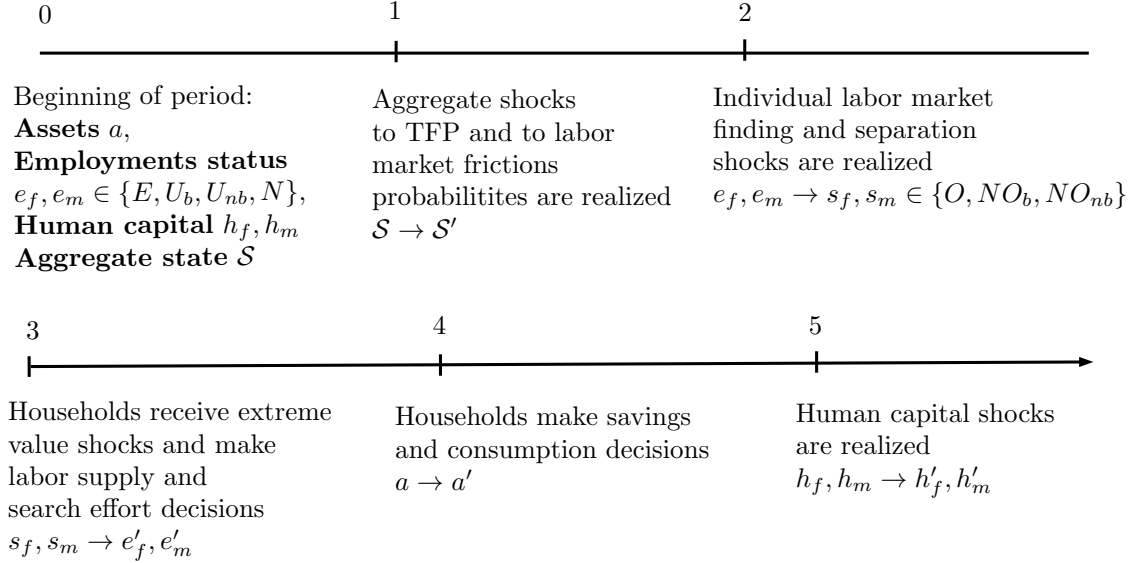| 3	4	5
Households receive extreme value shocks and make labor supply and search effort decisions $s_f, s_m \to e_f', e_m'$	Households make savings and consumption decisions $a \to a'$	Human capital shocks are realized $h_f, h_m \to h_f', h_m'$

Figure 8: Timeline of the model

3.2.1   The decision problem for singles

Values for individuals can be measured at different points in time: for example, at the beginning of the period, at step 0 before they receive offers to work, or in the middle of the period at step 2, after they have received the offers, or at step 4 when they take labor supply, consumption, and search effort decisions depending on their offer status (see timeline in figure 8). In what follows, it is convenient to define a *middle of the period* state variable $s \in \{$Offer, No Offer UI eligible, No Offer non-UI eligible$\}$, which determines the individual's offer status.

At any given time, a single individual is also characterized by their gender g, human capital h, assets a. After observing the aggregate state of the economy and her offer, she receives a specific taste for each different possible alternative in the set of possible alternatives $e \in \Gamma(s)$. For example, an individual who has an offer can

choose from $s \in$ {Employed, Unemployed without benefits, Out of the labor force}. The full set of alternatives is described in Table (1b). Finally, she chooses how much to consume and how much to save in a one-period bond.

For given pricing functions and aggregate law of motion $\Phi' = H(Z, \Phi)$, the recursive problem of a single of gender g at step 3 is:

$$V_g^3(a, h, s, \varepsilon; \mathcal{S}) = \max_{e \in \Gamma(s)} \left\{ V_g^4(a, h, e; \mathcal{S}) + \sigma_\varepsilon \varepsilon(e) \right\} \tag{5}$$

The value function at step 4 inside the max operator is:

$$V_g^4(a, h, e; \mathcal{S}) = \max_{c > 0, a' \geq 0} \left\{ u_g(c, n, q) + \tag{6} \right.$$

$$\left. \beta \sum_{h', s'} \int_{Z'} \pi(h' \mid h, e) \, \pi(s' \mid s, g, e, Z') \, \pi(Z'|Z) \left[EV_g^2(a', h', s'; \mathcal{S}') \right] \right\}$$

subject to

$$c + a' = \mathcal{T}\left(y(g, h, e; \mathcal{S}); \mathcal{S}\right) + (1 + r(\mathcal{S}))a + T$$

where I denote by $\mathcal{S}$ the collection of all aggregate variables. Earnings are subject to progressive taxation according to a tax schedule $\mathcal{T}(y, \mathcal{S})$. Earnings $y(g, h, s, e; \mathcal{S})$ can take two forms: labor earnings or unemployment insurance.

Specifically, for given w, and r the constraint for employed singles is as follows:

$$c + a' = \tau_0 \left(h \, w_g\right)^{(1-\tau_1)} + (1 + r)a + T \tag{7}$$

Where w_g is the wage paid to either men (w) or women ($(1 - \delta)w$). The constraint for singles who are unemployed and eligible to receive benefits is:

$$c + a' = \tau_0 \left(bh \, w_g\right)^{(1-\tau_1)} + (1 + r)a + T \tag{8}$$

Here b denotes the unemployment benefit replacement rate relative to potential income $(1 - \delta^g) hw$. Finally, the constraint for non-employed singles who are not

eligible to receive benefits is simply:

$$c + a' = (1 + r)a + T \tag{9}$$

When the individual is out of the labor force or unemployed without benefits, labor earnings are zero and the only two sources of income are asset income and government transfers T. Notice that an individual who decides not to work faces the same constraint as an individual who does not currently have an offer, and hence is forced not to work.

Because the taste shocks are independent extreme value distributed random variables, the expected value function $EV_g^2(a', h', s'; \mathcal{S}')$ is given by the well known log-sum formula in McFadden (1973).

Finally, this definition allows us to define the *beginning of the period* at step 0 value function as simply the expectation over the possible values in the middle of the period, where probabilities to receive an offer or not are defined in Table 1a.

3.2.2 The decision problem for couples

For given pricing functions and aggregate law of motion $\Phi' = H(Z, \Phi)$, the value function in the middle of the period for two spouses is given by:

$$
W^4\left(a, h^f, h^m, e_f, e_m; \mathcal{S}\right) = \max_{c>0, a' \geq 0} \left\{ u\left(c, n_f, n_m, q_f, q_m\right) + \right. \tag{10}
$$

$$
+ \beta \sum_{h^{f\prime}, h^{m\prime}, s^{f\prime}, s^{m\prime}} \int_{Z'} \pi\left(h^{f\prime} \mid h_f, e_f\right) \cdot \pi\left(h^{m\prime} \mid h_m, e_m\right) \cdot \pi\left(Z'|Z\right) \cdot
$$

$$
\left. \pi\left(s^{f\prime}, s^{m\prime} \mid s_f, s_m, e_f, e_m, Z'\right) \cdot \left[EW^2\left(a', (h^f)', (h^m)', (s_f)', (s_m)'; \mathcal{S}'\right)\right] \right\}
$$

with budget constraint

$$
c + a' = \mathcal{T}\left(y(h_f, h_m, e_f, e_m; \mathcal{S}); \mathcal{S}\right) + (1 + r(\mathcal{S}))a + T
$$

When both spouses are employed, for given τ_0 and w, the couple's labor income

net of taxes is given by:

$$\mathcal{T}(y) = \tau_0 \left((1-\delta)\, h^f\, w + h^m\, w \right)^{1-\tau_1} \tag{11}$$

Two things are worth noting: discussion of productivity wedge $(1-\delta)$ here. Discussion of joint taxation.

If the wife is out of the labor force, then the only source of labor income for the couple comes from the husband's salary:

$$\mathcal{T}(y) = \tau_0 \left(h^m\, w \right)^{1-\tau_1} \tag{12}$$

If only the wife is employed, while the husband is unemployed and eligible for unemployment insurance, net labor income is given by:

$$\mathcal{T}(y) = \tau_0 \left((1-\delta)\, h^f\, w + b\, h^m\, w \right)^{1-\tau_1} \tag{13}$$

and so on.

3.2.3 Firms

There is a representative firm hiring labor and producing output according to the following constant returns to scale production technology:

$$Y = Z\, K^\alpha L^{1-\alpha}$$

For the firm, hours worked by men and women are perfect substitutes. Therefore, in every period the aggregate wage per efficiency unit of labor is $w = Z(1-\alpha)\left(\frac{K}{L}\right)^\alpha$. Differences in the effective wage of an individual are driven by the number of efficiency units they are endowed with.

3.2.4 Business cycle fluctuations

The main driver of business cycle fluctuations is total factor productivity Z. Z is assumed to follow an AR(1) process in logs:

$$\log(Z_t) = \rho_z \log(Z_{t-1}) + \sigma_z \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0,1) \tag{14}$$

The exogenous finding rates and separation rates also depend on the aggregate state of the economy Z, and are therefore stochastic. Specifically, during a recession, finding rates go down and separation rates go up, leading to higher involuntary unemployment, as will be described in section 4.3.

3.2.5 The Government

The government provides two social insurance programs. The first is a progressive labor income tax. The tax function $\mathcal{T}(y)$ depends on two parameters that separately govern scale and progressivity, such that if y is pre-tax income, $\tau_0 y^{1-\tau_1}$ is after-tax income, as in Benabou (2002), Heathcote, Storesletten, and Violante (2017) and Holter, Krueger, Stepanchuk (2019). Couples' income is taxed jointly, which has been shown to be important for the decision to participate of secondary earners (References).

The second program is unemployment insurance. An individual classifies for unemployment benefits when she is unemployed and eligible for the program (u_b) and receives benefits proportionally to what her earnings would be if she was employed full-time. If y are potential earnings of an unemployed worker, then after-tax unemployment benefit is $b\tau_0 y^{1-\tau_1}$. The parameter b controls the generosity of unemployment insurance. Finally, unemployment benefits expire stochastically at rate $\lambda(Z)$.

The income obtained from tax revenues is used to pay for exogenous government expenditure G, which does not to provide utility to individuals and does not vary over the business cycle. Here discussion on don't want to focus on effects of time-

varying govt expenditure.

In every period, the government adjusts the level of public expenditure G in order to keep a balanced budget:

$$
\tau_0 w^{(1-\tau_1)} \underbrace{\int_{E_i} \left(\sum_g \left((1-\delta_g) h_{i,g,s} n_{i,g} \right)^{(1-\tau_1)} + \left((1-\delta) h_{i,f} n_{i,f} + h_{i,m} n_{i,m} \right)^{(1-\tau_1)} \right)}_{\text{Total taxes paid by employed workers}} + \quad (15)
$$

$$
-(1-\tau_0)(bw)^{(1-\tau_1)} \underbrace{\int_{U_i} \left(\sum_g \left((1-\delta_g) h_{i,g,s} \right)^{(1-\tau_1)} + \left((1-\delta) h_{i,f} + h_{i,m} \right)^{(1-\tau_1)} \right)}_{\text{Total UI claims net of taxes}} +
$$

$$
+ \quad \underbrace{T}_{\text{Direct transfers}} \quad - \quad \underbrace{G}_{\text{Government expenditure}} \quad = 0
$$

where wages, the measure of employed and unemployed workers, and hours worked all depend on the aggregate state of the economy $\mathcal{S}$.

3.3 Competitive Equilibrium

A recursive competitive equilibrium consists of K, L, w, r , τ_0, $\pi_c^{g,single}$, π_c^{couple}, $\pi_n^{g,single}$, $\pi_{n_f}^{couple}$, $\pi_{n_m}^{couple}$, $\pi_a^{g,single}$, π_a^{couple}, V_g, W, Φ, H, satisfying:

1. Given pricing functions $r(Z, \Phi)$, $w(Z, \Phi,)$, tax policy, and aggregate law of motion $H(Z, \Phi)$, $V^g(a, h, s; Z, \Phi)$ solves the optimization problem of the single household (6), and $\pi_c^{g,single}$, $\pi_n^{g,single}$, $\pi_q^{g,single}$, $\pi_a^{g,single}$ are associated decision rules.

2. Given pricing functions $r(Z, \Phi)$, $w(Z, \Phi)$, tax policy, and aggregate law of motion $H(Z, \Phi)$, $W(a, h^f, h^m, s_f, s_m; Z, \Phi)$ solves the optimization problem of

24

the married household (10), and π_c^{couple}, $\pi_{n_f}^{couple}$, $\pi_{n_m}^{couple}$, $\pi_{q_f}^{couple}$, $\pi_{q_m}^{couple}$, π_a^{couple} are associated decision rules.

3. Labor market clears:

$$L = \sum_g \int (1 - \delta_g) h \pi_n^{g,single}(a, h, s; \Phi, Z) \, d\Phi + \tag{16}$$

$$\int (1 - \delta) h_f \pi_{n_f}^{couple}(a, h^f, h^m, s_f, s_m; Z, \Phi) \, d\Phi +$$

$$\int h_m \pi_{n_m}^{couple}(a, h^f, h^m, s_f, s_m; Z, \Phi) \, d\Phi$$

4. Asset market clears:

$$K = \sum_g \int \pi_a^{g,single}(a, h, s; \Phi, Z) \, d\Phi + \tag{17}$$

$$\int \pi_a^{couple}(a, h_f, h_m, s_f, s_m; Z, \Phi) \, d\Phi$$

5. Wage and interest rate are consistent with optimal decision of firms:

$$w = Z(1 - \alpha) \left(\frac{K}{L} \right)^{\alpha} \tag{18}$$

$$r = Z\alpha \left(\frac{L}{K} \right)^{(1-\alpha)} \tag{19}$$

6. Balanced budget: for given level of transfers T, tax rates τ_0 and τ_1 and UI replacement rate b, government expenditure G adjusts to keep a balanced budget.

7. The aggregate law of motion H is induced by the optimal decision rules $\pi_a^{g,single}$ and $\pi_a^{couple} \in A$, transition probabilities of individual productivity $h \in H$, transition probability of labor market status $s \in \{$ Offer, No Offer UI Eligible, No Offer non UI eligible $\}$ and the distribution of gender, singles and couples.
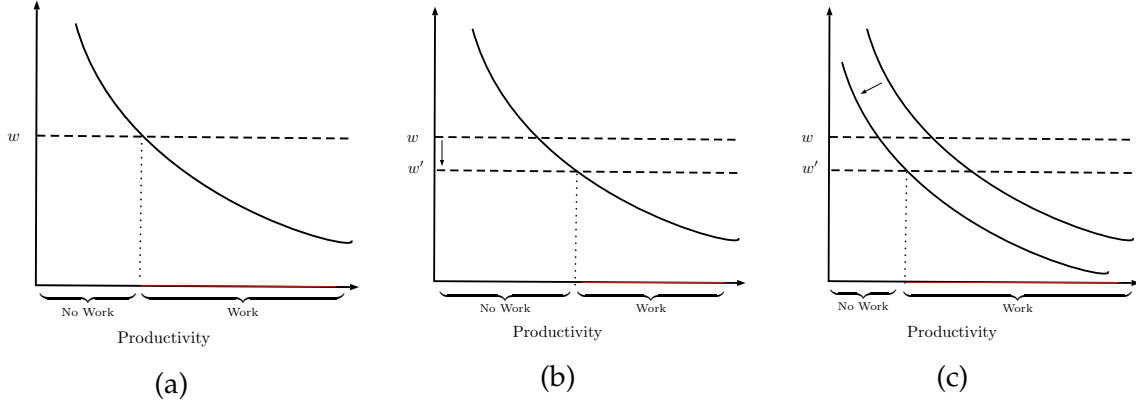
3.4 Discussion

Unpacking the model

Figure 9: Intuition

4 Calibration

The baseline model is calibrated on data between 1975 and 1985. The set of parameters is divided into two groups. The first one is calibrated following direct empirical observations or existing literature. The second group of parameters is estimated inside the model by the method of simulated moments. The estimation aims at matching (i) labor force participation rates and unemployment rates by different groups (ii) earning losses upon displacement and (iii) the level and cyclicality of job finding and separation rates by men and women.

4.1 Data

Data for the calibration comes from Current Population Survey (CPS) and Panel Study of Income Dynamics (PSID). The unit of time is assumed to be a month, in line with the CPS data on worker flows.

4.2 Preferences and technology

The period utility function for couples is:

$$u_{couple}(c, n_f, n_m, q_f, q_m) = \frac{c^{1-\sigma}}{1-\sigma} - \eta_f \, n_f - \eta_m n_m - \kappa_f \, q_f - \kappa_m \, q_m \qquad (20)$$

I set σ to be 1 (log utility). The disutility of working for married women and men, η_f and η_m, directly influences labor force participation, and is calibrated to match the labor force participation ratios of the respective groups (55% and 97%). The disutility of search κ_f and κ_m are calibrated to match unemployment rates (4.5% and 6%). I pick the discount factor β to be 0.97.

The capital share is $\alpha = 0.3$, and the depreciation rate is chosen to be $\delta_k = 0.0083$, which corresponds to 2.5% quarterly. I set the persistence of aggregate TFP shocks to be 0.7 and the variance of innovations to be 0.004.

4.3 Search technology

Search frictions and separations I calibrate search frictions and separation shocks separately for men and women in the model to match average gross flows between employment states, similarly to Krusell, Mukoyama, Rogerson, Sahin (2017), Albanesi and Sahin (2017), Mankart and Oikonomou (2017). E-U rate and U-E rate close between model and data. E-N rate given by extreme value shocks. N-E rate is the hardest moment to match, as discussed in Krusell, Mukoyama, Rogerson, Sahin (2017).

Changes in frictions over the cycle Functional form: $s(Z) = s^* \cdot Z^{\epsilon_s}$ (constant elasticity ϵ_s). This can equivalently be written in percentage deviations from steady states: $\log(s(Z)/s^*) = \epsilon_s \cdot \log(Z/Z_{ss})$, with $Z_{ss} = 1$. The level s^* is chosen to match average E-U rates for men and women separately. The elasticity is chosen to match the standard deviation of E-U rates over the business cycle. The same procedure is used for finding rates out of unemployment, which are chosen to match U-E

rates. Finally, finding rates out of the labor force could also potentially vary over the business cycle. As Mankart and Oikonomou (2017), I assume that they vary at the same rate as finding rates out of unemployment. This is because the model must be free to get movements in and out of the labor force endogenously.

4.4 Human capital accumulation process

Human capital is stochastic, but its underlying process depends on whether the individual is working or not, making it therefore partly endogenous. The process for human capital accumulation and depreciation is described in Table 2. Human capital is defined on a grid, and in a given month individuals can stay at their current human capital level, or move one point to the left or to the right. If an individual works positive hours, human capital is more likely to go up ($p_{up}^{w} > p_{up}^{nw}$). If an individual works zero hours, either because she is out of the labor force or unemployed, human capital is more likely to go down ($p_{down}^{w} < p_{down}^{nw}$). This process captures both random shocks to productivity and returns to experience.

I choose five points for human capital grid for each individual, so households have a total of twenty-five possible combinations. Points are spaced so that the difference between each point is a constant 1.4 log points. The probabilities are partly endogenous and partly exogenously chosen to match (i) expected earning losses during unemployment, (ii) expected wage growth when employed, and (iii) the labor earnings distribution across employed individuals. I set p_{up}^{w} to match a 1.1 percent average quarterly return to labor market experience (ADOT 2020) and p_{down}^{nw} to match wage losses upon unemployment (around 2.5% quarterly).

	h_1	h_2	h_3	h_4	h_5
h_1	$(1 - p^w_{up})$	p^w_{up}	0	0	0
h_2	p^w_{down}	p^w_{stay}	p^w_{up}	0	0
h_3	0	p^w_{down}	p^w_{stay}	p^w_{up}	0
h_4	0	0	p^w_{down}	p^w_{stay}	p^w_{up}
h_5	0	0	0	p^w_{down}	$(1 - p^w_{down})$

(a) Transition probabilities when $n = 1$, with $p^w_{stay} = (1 - p^w_{down} - p^w_{up})$.

	h_1	h_2	h_3	h_4	h_5
h_1	$(1 - p^{nw}_{up})$	p^{nw}_{up}	0	0	0
h_2	p^{nw}_{down}	p^{nw}_{stay}	p^{nw}_{up}	0	0
h_3	0	p^{nw}_{down}	p^{nw}_{stay}	p^{nw}_{up}	0
h_4	0	0	p^{nw}_{down}	p^{nw}_{stay}	p^{nw}_{up}
h_5	0	0	0	p^{nw}_{down}	$(1 - p^{nw}_{down})$

(b) Transition probabilities when $n = 0$, with $p^{nw}_{stay} = (1 - p^{nw}_{down} - p^{nw}_{up})$.

Table 2: Human capital accumulation and depreciation process. If an individual works positive hours, human capital is more likely to go up ($p^w_{up} > p^{nw}_{up}$). If an individual works zero hours, either because she is out of the labor force or unemployed, human capital is more likely to go down ($p^w_{down} < p^{nw}_{down}$). This process captures both random shocks to productivity and returns to experience.

4.5 Taxes and social insurance programs

As discussed in the model section, the tax function depends on two parameters that separately govern scale and progressivity: If y is pre-tax income, $\tau_0 y^{1-\tau_1}$ is after-tax income. The government also gives a lump sum transfer to all citizens. Daruich and Fernandez (2020) and Boar and Midrigan (2020) also allow for progressive taxation and a lump-sum transfer to better match the extent of redistribution in the United States. Daruich and Fernandez estimate the level of lump-sum transfer that is consistent with the difference between after-tax-and-transfers annual income and pre-tax annual income for low-income households to be around \$1600 per year in 2000 dollars. I follow them and set $\tau_0 = 0.8$ and $\tau_1 = 0.18$ as in Heathcote, Storesletten, Violante (2017), and $T = 0.02$. Finally, I set unemployment benefits replacement rate $b = 0.5$ (see, e.g. Krueger, Mitman, Perri (2016)) and their expiration rate $\lambda = 1/6$ per month so that the average duration of benefits is equal to six months.

Parameter	Description	Value	Source/Target
Preferences and Demographics			
β	Discount factor	0.97	Standard
σ	EIS	1	
μ_f, μ_m	Proportion single women and men	15%, 15%	U.S. data
Human capital			
Δh	Distance between grid points	1.4 log points	
p_{up}^w	Prob. h goes up when working	0.02	Wage growth
p_{down}^{nw}	Prob. h goes down when not working	0.06	Wage losses
Productivity wedge			
δ	Wedge women's productivity	0.4	U.S. data
Technology			
α	Share of capital	0.3	U.S. data
δ	Depreciation rate	0.0083	
ρ_z	TFP autocorrelation	0.7	
σ_z	TFP shock variance	0.004	
Government Policy			
τ_0	Level income tax	0.8	HSV (2019)
τ_1	Progressivity of income tax	0.18	HSV (2019)
b	UI replacement rate	0.5	KMP (2016)
T	Unconditional transfers	0.02	DF (2022)
λ	Benefits expiration rate	1/6	U.S. Data

Table 3: Externally calibrated parameters

Parameter	Description	Target
Preferences		
$\eta_{f,c}$	Disutility of working for married women	
$\eta_{f,s}$	Disutility of working for single women	
$\eta_{m,c}$	Disutility of working for married men	Participation rates
$\eta_{m,s}$	Disutility of working for single men	
$\kappa_{f,c}$	Disutility of search for married women	
$\kappa_{f,s}$	Disutility of search for single women	
$\kappa_{m,c}$	Disutility of search for married men	Unemployment rates
$\kappa_{m,s}$	Disutility of search for single men	
σ_ϵ	Variance of EV shocks	N-E rate
Human capital		
p^w_{down}	Prob. h goes down when working	
p^{nw}_{up}	Prob. h goes up when not working	
Search frictions		
s_m	Separation rate women	E-U rates
s_f	Separation rate men	
f_f	Finding rate unemployed women	U-E rates
f_m	Finding rate unemployed men	
f^*_f	Finding rate out of labor force women	N-E rates
f^*_m	Finding rate out of labor force men	
$\epsilon^m(s)$	Elasticity separation to TFP men	s.d.(E-U rates)
$\epsilon^f(s)$	Elasticity separation to TFP women	
$\epsilon^m(f)$	Elasticity finding rate unemployed men	s.d.(U-E rates)
$\epsilon^f(f)$	Elasticity finding rate unemployed women	
$\epsilon^m(f^*)$	Elasticity finding rate olf women	See text
$\epsilon^f(f^*)$	Elasticity finding rate olf men	

Table 4: Internally calibrated parameters

	Data	Model
Employment rates		
Married Women	0.51	0.51
Married Men	0.9	0.9
Single Women	0.7	0.7
Single Men	0.78	0.78
Fraction of Couples		
Both members working	0.51	0.0527
Only female working	0.04	0.037
Only male working	0.41	0.4
Neither working	0.04	0.036

Table 5: Labor Market Steady State. Data comes from CPS, averages 1970-1985.

5   Quantitative Results

5.1   Steady-state in the Baseline Economy

In this section I characterize the steady-state of the baseline economy. Table 5 reports statistics on the labor supply by different groups in the data and in the model. The data come from CPS and are averages of the period 1970 to 1985. Notice that the disutility of leisure and search are chosen to match labor force participation rates and unemployment rates, and hence employment rates as a residual. On the other hand, the division of labor for couples is not explicitly targeted. In the data, most of the couples are either dual earner couples or couples in which only the husband is working. There are 4% of all couples in which only the wife is working, and in 4% of all couples neither partner is working. The model presents similar patterns.

Figure 10 compares married men's earnings losses upon job displacement in recessions and in expansions between the data and the model. Since the empirical analysis is conducted at an annual frequency, I aggregate the model simulated data at a yearly period, and run the same regression in section 2.2 in PSID data and in the model simulated data. The model is able to match the large loss in labor earnings,
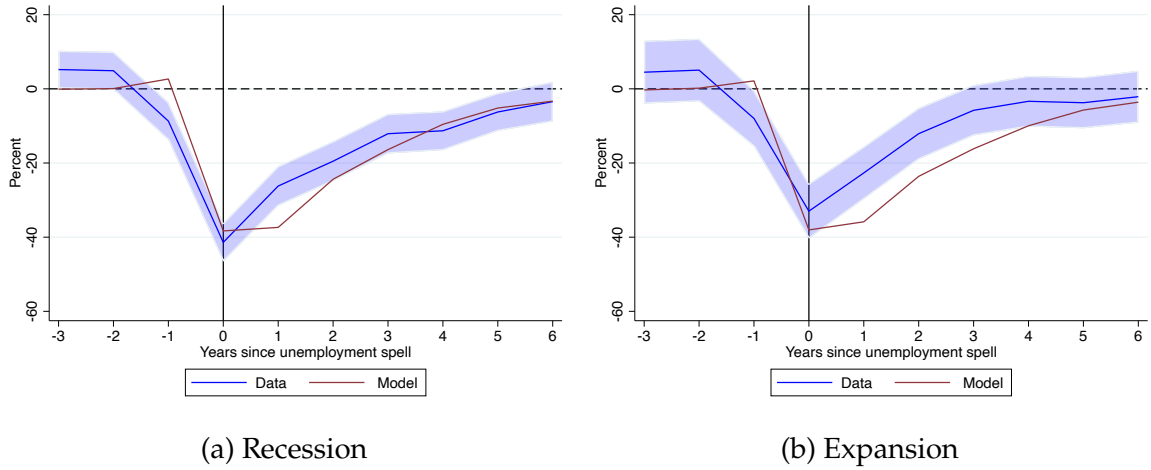
(a) Recession   (b) Expansion

Figure 10: This figure plots the fall in labor earnings after unemployment spell for married men in PSID and in model simulated data. I estimate the fall in earnings using a dynamic diff-in-diff on PSID data and on model simulated data. The blue line shows the point estimates on impact in PSID, and the shaded area is the 95% confidence interval. The red lines correspond to the model generated estimates.

although it overestimates persistence in both recessions and expansions. This is likely because the returns to experience are taken from the data on all workers rather than on recently unemployed workers, who might climb the job ladder faster. Notice that the only difference between recessions and expansions are changes in job market frictions. Expansions therefore have a lower scarring effect because unemployed workers can find a job faster.

5.2 Labor Supply Elasticities

The reservation wage broadly refers to the wage that makes an individual indifferent between working and not working. Since the model is dynamic, there is a meaningful distinction between a temporary and permanent change in wages. Similarly to Preston and Mui (2021), I define the reservation wage as the one-period wage that would make an individual indifferent between working and not working. For any individual at a given point in time, I calculate the change in the current wage required to make them indifferent and assume that the wage returns to its steady-state level

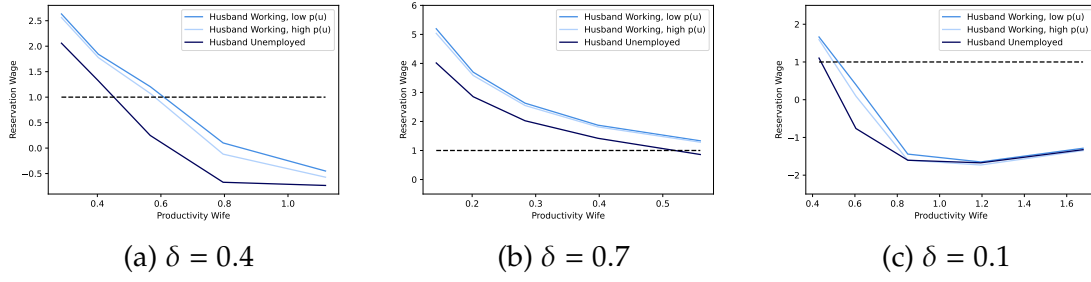| (a) $\delta = 0.4$ | (b) $\delta = 0.7$ | (c) $\delta = 0.1$ |

Figure 11: Reservation wages in different economies for a mean level of assets. The dark blue and purple lines correspond to the baseline economy, when the husband is employed and when the husband is unemployed respectively. The light blue line is obtained by shocking the same baseline economy with a higher and persistent probability that married men separate from their jobs.

forever after one period.

Assuming that ties are broken in favor of working, for each individual the extensive-margin labor supply can be formulated as a standard reservation wage rule:

$$n = \mathbb{1}\left(w \geq \overline{w}\right)$$

In Figure 11 I am plotting the reservation wages in different economies for a mean level of assets as a function of human capital. The dark blue and purple lines correspond to the baseline economy, when the husband is employed and when the husband is unemployed respectively. The light blue line is obtained by shocking the same baseline economy with a higher and persistent probability that married men separate from their jobs.

The reservation wage curve shifts to the left both if the husband is unemployed, or if the husband is currently employed but faces a higher risk of separation. During recessions, more husbands will become unemployed and all husbands will face a higher risk of unemployment. If movements in the wage are smaller than movements in these curves, secondary earners will act as a dampening mechanism. The strength of the mechanism depends on the strength of precautionary motives and on the distribution of human capital.
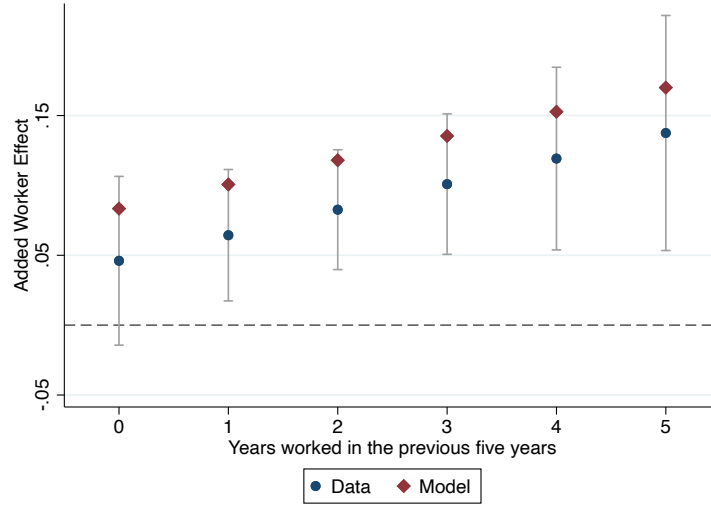
35

Figure 12: This figure plots the change in the probability of going from out of the labor force to employed when husband becomes unemployed depending on previous labor market experience, both in the model and in the data. I estimate the probability using a dynamic diff-in-diff on PSID data and on model simulated data. The unemployment dummy is interacted with a variable that encodes how often the wife worked in the previous five years. The blue circles show the point estimates on impact, and the whiskers are the 95% confidence interval. The red diamonds correspond to the model generated estimates.

They key intuition for understanding the role of the wage gap as a function of productivity wedge δ is that movements in δ shift the entire curves up and down. As a reminder, women's wages are given by $w_f = (1 - \delta)w$. For very high value of δ, since such a large cut of the wage is taken away, the overall economy wage would have to be very high for women to participate, regardless of unemployment risk. For low values of δ, most women are now deep inside the work region and are not affected by movements in men's unemployment either.

5.3 Model Validation

The added worker effect is not targeted by the calibration, which only uses information on average aggregate individual gross flows. In this section, I test the model's ability to replicate the probability that a married woman becomes employed upon

her husband becoming unemployed. Since the empirical analysis is conducted at an annual frequency, I aggregate the model simulated data at a yearly period and run the same regression in section 2.2 in PSID data and in the model simulated data.

In Figure 12, I plot the coefficients generated from the PSID sample (blue circles) and those obtained using model-generated data (red diamonds). The model is able to match the empirical evidence on the level and gradient of the added worker effect by previous experience quite well. Although it somewhat overestimates the magnitude compared to the point estimates, especially for low experience levels, it always falls within the 95% confidence bands.

Two channels allow the model to match the positive gradient with respect to experience found in the data. The first is human capital accumulation: Women who have worked very little in their past mostly sit at the bottom of the human capital grid. Lower human capital and hence effective wage means their incentive to act as an active secondary earner is dampened. The second is that asset-rich households tend at the same time to have low wife's participation and to rely more on their assets to insure against unemployment risk.

It is important to notice that in the model differences in previous participation act exclusively through incentives and not through frictions. In the model, once a woman has decided to join the labor force, she has the same probability of finding a job as anyone else searching. In practice, women who have been out of the labor force for a longer time might face higher frictions, for example, due to lower connections or knowledge of the labor market. This might lower the probability of finding a job on impact.
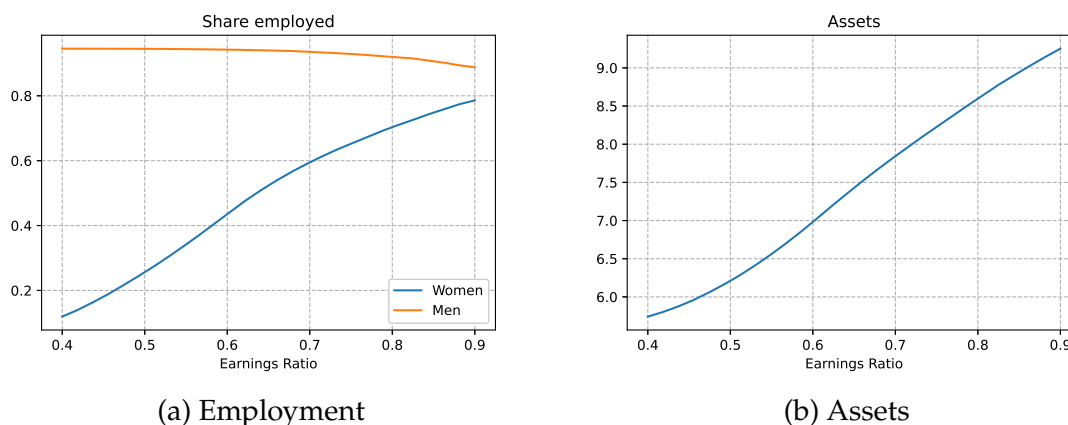
(a) Employment   (b) Assets

Figure 13: Share employed married men, employed married women, and assets as the earnings ratio goes up. Each point refers to a steady state in which the earning ratio stays at that specific level.

6 Changes in Women's Employment and Business Cycles

How do changes in the level of female employment affect the business cycle? In order to answer this question, I conduct a comparative statics exercise using the calibrated model. Specifically, I will compare the business cycle (stochastic steady state[7]) in different economies, where all parameters remain the same but women's productivity wedge δ is lowered, thereby exogenously raising female productivity. As a results, the ratio of the effective wage that women face and the effective wage for men (w_f/w_m) raises over time. This exercise exclusively teases out the effect of the wage gap as a driver of female employment.

Fukui, Nakamura, Steinsson (2021), and Olsson (2020) share the focus on women's productivity as a structural driver of higher female employment. The increase in female productivity can have different interpretations: For example, an increase in women's educational attainment, an increase in female-biased technical change such as the rise of the service sector (see Ngai and Petrongolo (2017)), or a decrease in discrimination and misallocation of talent (Hsie Klenow).

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[7]For details on how the stochastic stead state is computed, see Appendix D.

First, I will evaluate whether a change in women's productivity can generate an increase in female employment. As shown in Figure 12, a higher earning ratio leads to a large increase in female employment and a slight decrease in male employment. These results are similar to Olsson (2020), who shows that decreasing the wage gap leads to an increase in married women's participation, a slight decrease in married men's participation, and only minor changes in the labor supply of singles. This is because, with balanced growth type preferences, income and substitution effects roughly cancel for single women, and the labor supply is approximately the same. In my model, the presence of human capital accumulation makes the wage gap a more significant driver of participation than it would be otherwise: Small differences in the wage gap are amplified by the presence of learning on the job.

This exercise is equivalent to asking: what would the business cycle look like in a world where the only difference is that women are more productive? Naturally, one drawback of this approach is that it does not allow me to study what the business cycle would look like *along* a transition in the wage gap. This has been the focus of other mentioned recent papers that find that a faster increase in women's labor force participation results in shorter recoveries from recessions. I abstract from this point which has been extensively studied to focus on the relationship between the mass of marginal workers and the amplification of business cycle shocks.

Figures 13, 14 and 15 show the comparative statics with respect to the earnings ratio (w_f/w_m). Women's participation is counter-cyclical (panel a), and the largest movements in participation occur when the earnings ratio is between 0.6 and 0.7 (panel b). As a consequence, the volatility of total effective hours per capita decreases as more women start participating, until it arrives to it minimum at around 0.68 (panel c). For reference, this is the value that the gender gap reached in the 1990s. After that, the effect on the volatility of hours per capita keeps waning. On the other hand, the volatility of consumption does not go back to the previous levels (panel d). This indicates that the economy is now relying on other forms of insurance: wealth accumulation, and income pooling. Figure 15 shows an important distinction: the
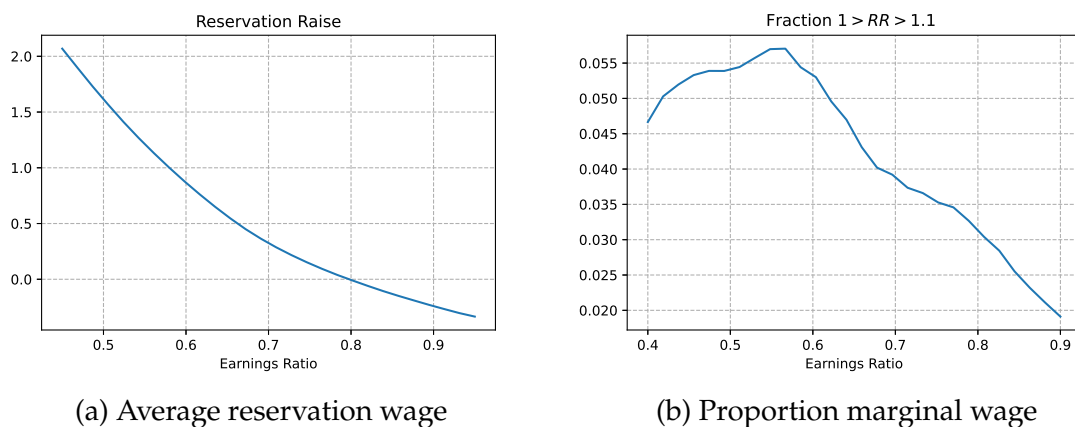
(a) Average reservation wage   (b) Proportion marginal wage

Figure 14: Reservation wage in the economy as the earnings ratio (w_f/w_m) grows. The current wage in the economy is always normalized to 1.

U-shaped effect exists when looking at the standard deviation of total hours per capita. However, as women become a larger part of the labor force, a mechanic effect is also at play. Since women's hours are less volatile (partly by construction, because the probability of getting fired in downturns is lower, and partly endogenously), the standard deviation of *percentage deviations* from the steady state keeps decreasing.
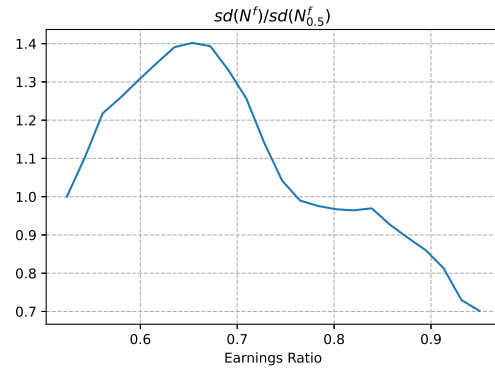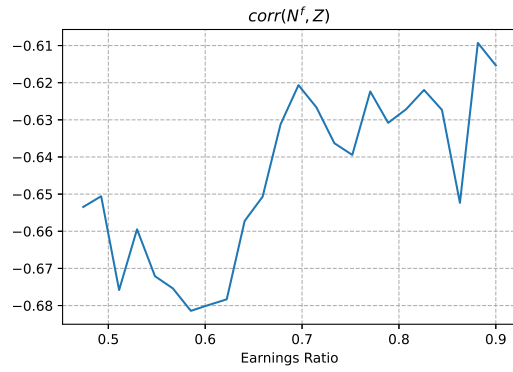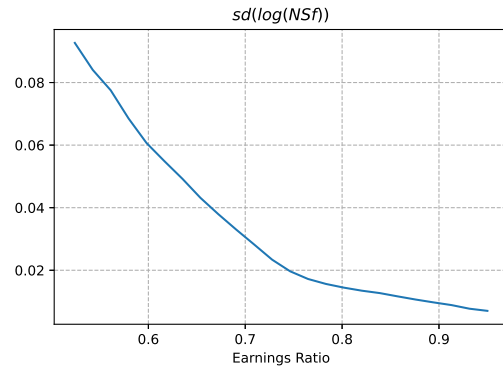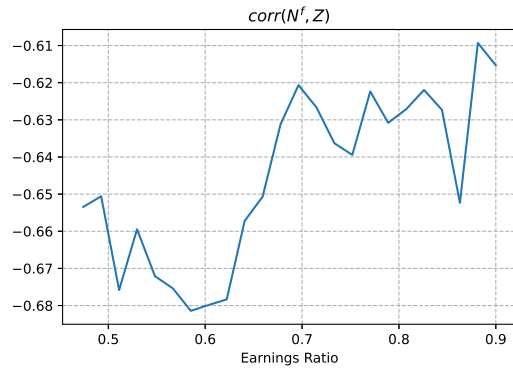
(a) Correlation Participation Women and TFP (b) Standard Deviation Participation Women
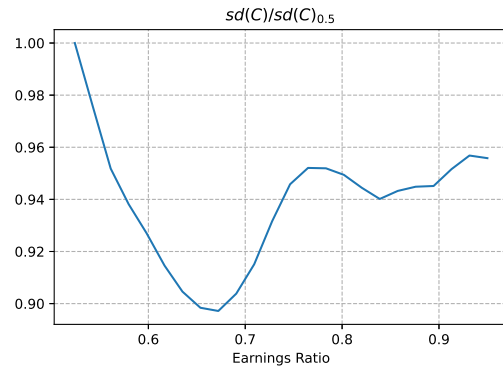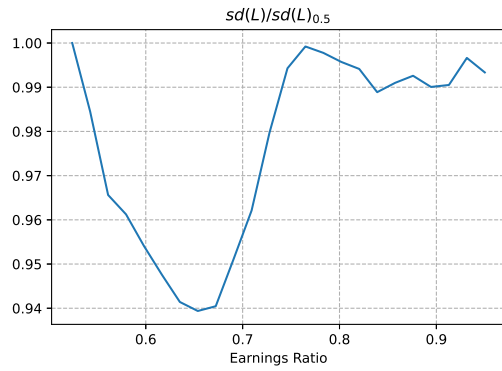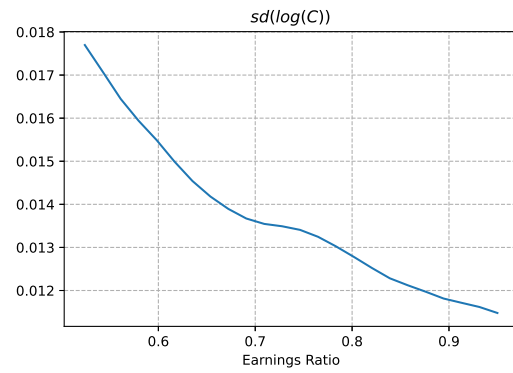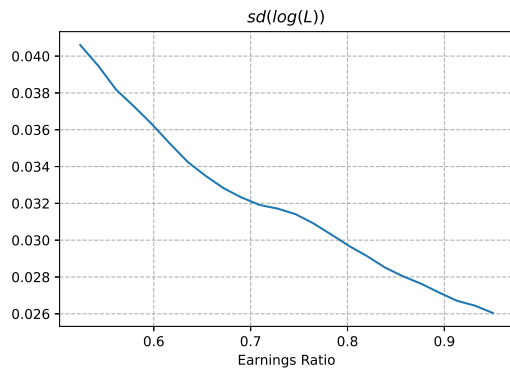

(c) Standard Deviation Total Hours   (d) Standard Deviation Consumption

Figure 15: Business cycle volatility in different steady states

(a) Correlation Participation Women and TFP (b) SD Log Deviations Participation Women

(c) SD Log Deviations Total Hours (d) SD Log Deviations Consumption

Figure 16: Business cycle volatility in different steady states, compositional effect

7 Conclusions

The degree to which women participate in the labor markets has increased substantially in the United States. Consequently, now there are many more couples in which both partners routinely work. In this paper, I show that the extent to which women participate in the labor markets significantly impacts how households' labor supply and aggregate employment respond to the business cycle. I build a model in which households decide their work arrangements, and there are fluctuations in productivity and search frictions. The model replicates key joint movements in employment of partners.

The quantitative model shows that (i) the family insurance motive is an important dampening mechanism of downturns, and (ii) hours and consumption volatility respond non-monotonically to changes in the wage gap. As the wage gap grows smaller, women have a higher earnings capacity, both because they face higher wages and because they accumulate more work experience before quitting the labor force. Consequently, they find it easier to enter during recessions, acting as a dampening force on the severity of recessions. I show quantitatively that a reduction in the wage gap from 50% to 30% has translated into a 6% decrease volatility of hours per capita and a 10% decrease in consumption volatility. For levels of the wage gap lower than 20%, women's participation and labor supply elasticity look more similar to men's. Most women are left with little room to increase their labor supply, and the dampening effect on employment disappears.

This paper leaves some important questions open for future research. First, this paper shows that the importance of the so called active spousal insurance – that is, the counter-cyclical entry of secondary earners – tends to vanish in advanced economies in which both partners in a couple are strongly attached to the labor force. However, this does not mean that partners do not provide insurance to each other anymore. If there is substantial income pooling of couples, and if partners incomes are sufficiently uncorrelated, then couples could still enjoy more insurance

against income shocks than singles. My model is not built to understand how couples pool their income and decide how much each partner consumes, because it takes a unitary approach to the couple. Further improving our understanding of the degree to which one partner's income loss translates into their own consumption could provide insight into the strength of this mechanism.

Second, the level to which secondary earners participate in the labor market could have important consequences for the design optimal stabilizers over the business cycle. Taking into account the existence of secondary earners can have consequences for the optimal level and cyclicality of automatic stabilizers such as unemployment insurance and progressive taxation. Interestingly, through the lens of the paper, the strength of these social insurance programs – and especially progressive taxation – affects how much secondary earners participate in normal times, how much human capital they accumulate, and therefore how good of an added worker they can be if needed.

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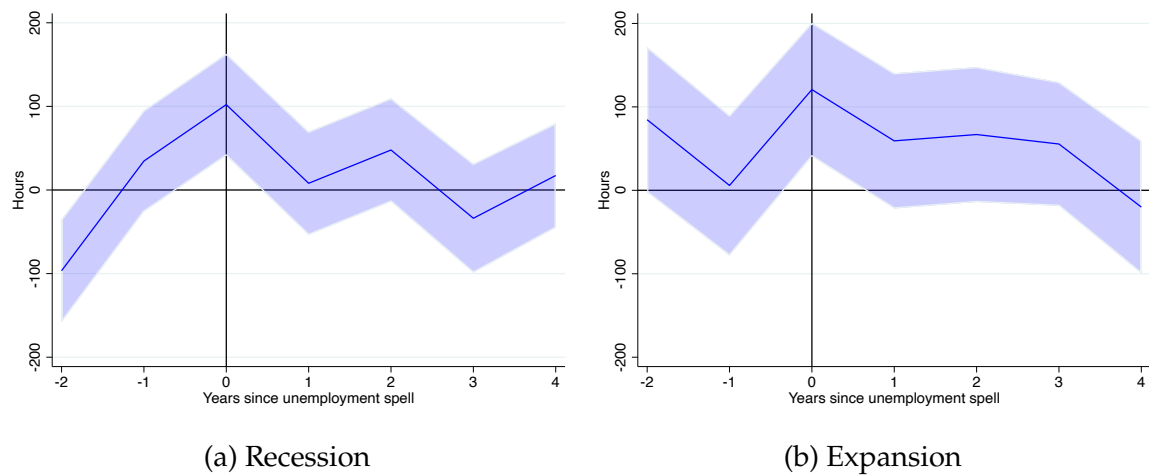
Appendices

A   Additional Empirical Results

(a) Recession

(b) Expansion

Figure 17: Added worker effect: Hours. Source: PSID.

(a) Recession

(b) Expansion
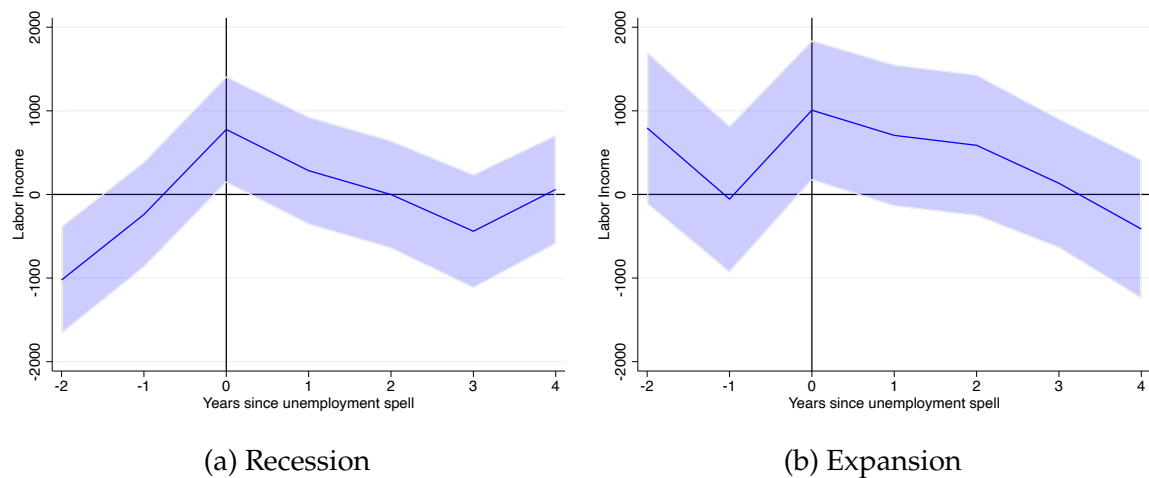
Figure 18: Added worker effect: Income. Source: PSID.
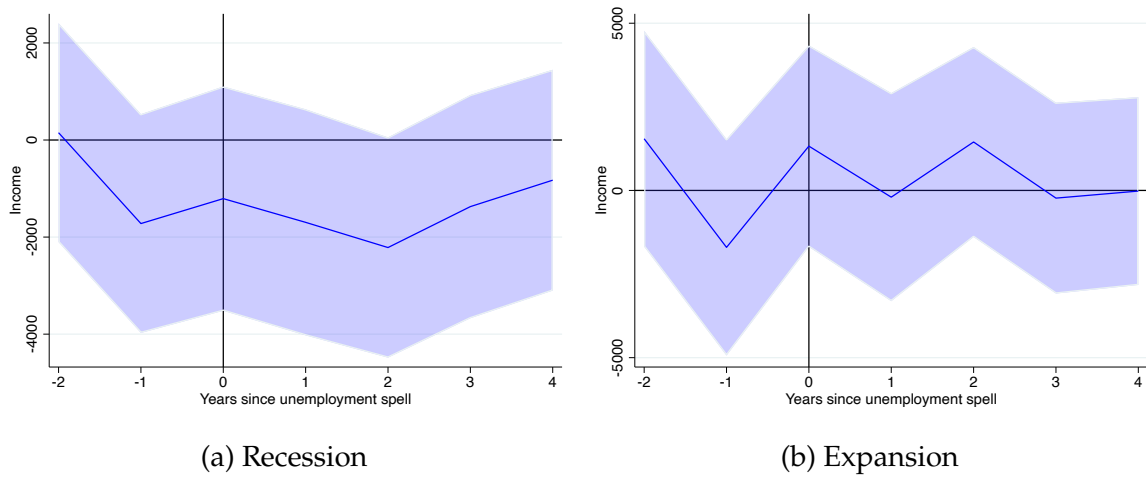
(a) Recession　　　　　　　　　　　(b) Expansion

Figure 19: No changes in income for women already working. Source: PSID.
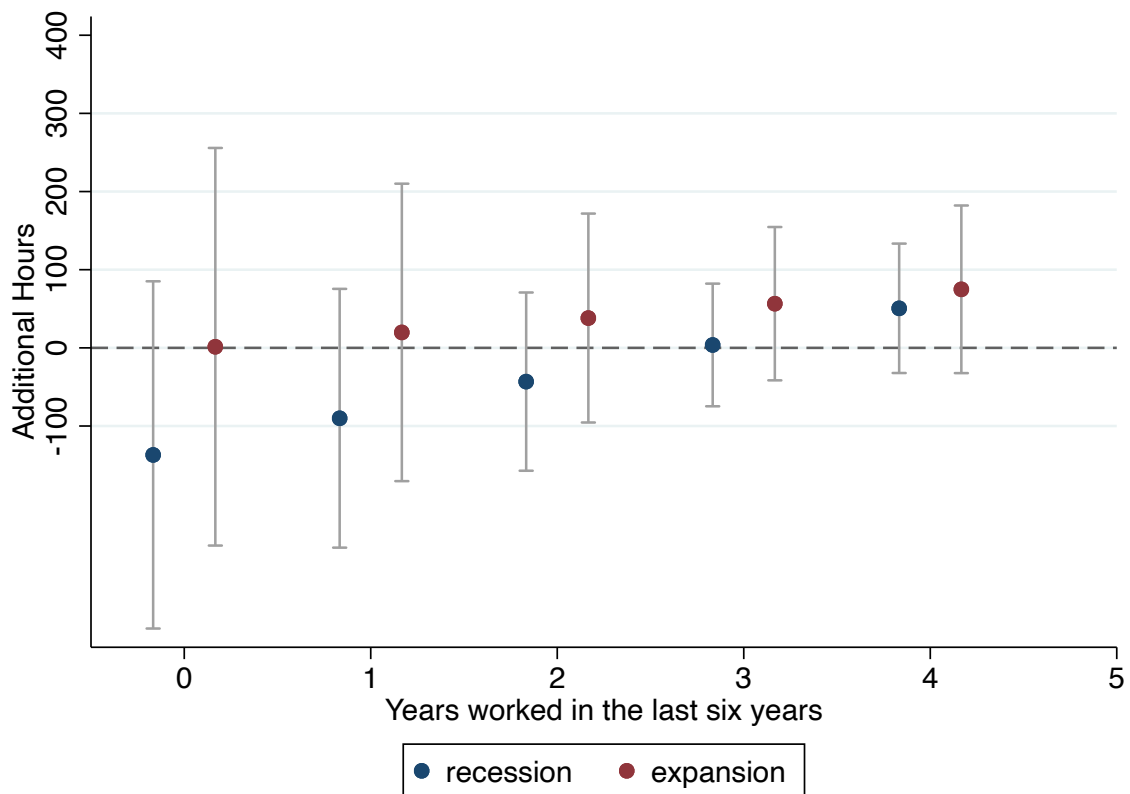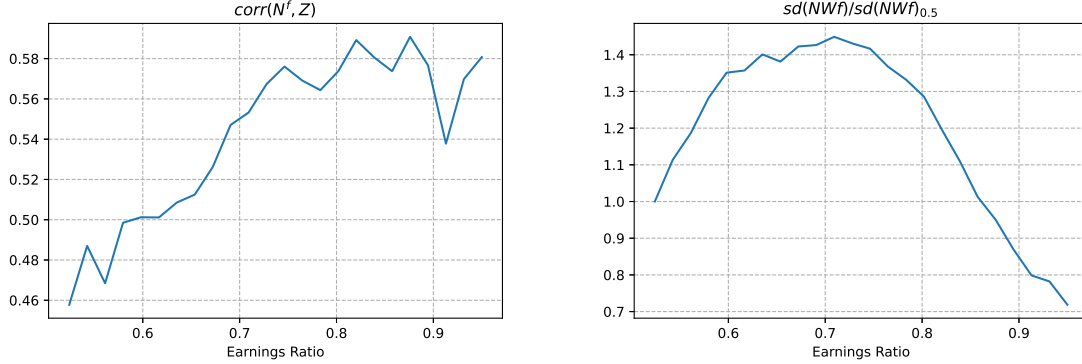

Figure 20: No changes in hours for women already working, conditioning on work experience.

B Business Cycle, No Movement in Unemployment Risk

(a) Correlation Participation Women and TFP (b) Standard Deviation Participation Women
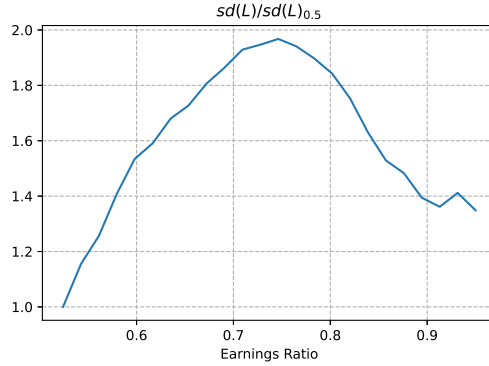
(c) Standard Deviation Total Hours

Figure 21: Business cycle volatility in different steady states, NO movements in unemployment risk, only wage

C A Simple Static Model

Environment   The economy is populated with a measure one of married/cohabiting couples and a measure one of singles. The utility function for a single of gender g is given by:

$$U_g(c, l) = \log(c) - \eta_g l$$

where c is consumption, $l \in \{0,1\}$ is labor supply, and $\eta_g \geq 0$ is the relative weight of leisure in utility. The utility function for a couple is:

$$U\left(c, l_m, l_f\right) = \log\left(c\right) - \eta_m l_m - \eta_f l_f$$

where c is consumption[8], $l_m, l_f \in \{0,1\}$ is labor supply of husband and wife respectively, and η_m, $\eta_f \geq 0$ are the relative weights of leisure in utility.

Assume that with probability u_m, a given men in the economy suffers an unemployment shock and is unable to work ($l_m = 0$), and with probability u_f women suffer an unemployment shock ($l_f = 0$). The realization of the shock is independent across spouses.

Single individuals solve the problem:

$$\max \sum_s \pi(s) \left[\log\left(c(s)\right) - \eta_g l(s)\right]$$

subject to the budget constraint:

$$c(s) + \psi l(s) = w_g l(s) + y$$

where $\psi > 0$ represents the fixed cost of running a household when working and y is unearned income. The assumption is that a person who does not work can replace the cost ψ through costless home production.

For a married couple, individuals jointly solve the problem:

$$\max \sum_s \pi(s) \left[\log\left(c(s)\right) - \eta_f l_f(s) - \eta_m l_m(s)\right]$$

subject to the budget constraint:

$$c(s) + \psi \min\left(l_f(s), l_m(s)\right) = w_f l_f(s) + w_m l_m(s) + y$$

---

[8]Here I assume that all consumption in the family is a public good. In reality there are some private and some public elements in household consumption; see Salcedo, Schoellmann, and Tertilt (2012) for a detailed analysis.

here $\psi > 0$ represents the fixed cost of running a household when both partners are working and y is unearned income. Male and female wages, denoted by w_m and w_f, as well as unearned income can potentially vary in the population, reflecting potential difference in human capital, wealth and transfers.

Reservation wages and aggregate hours   Assume for simplicity that parameters are such that single and married men are always working conditional on being employable. Let's define the *reservation wage* $\overline{w}$ as the wage that makes a women indifferent between working and not working.

Assuming that ties are broken in favor of working, for each woman who is not unemployed the extensive-margin labor supply can be formulated as a standard reservation wage rule:

$$n_f = \mathbb{1}\left(w_f \geq \overline{w}\right)$$

For single women, $\overline{w}^S$ is simply given by:

$$\log\left(\overline{w}^S + y - \psi\right) - \eta_f = \log\left(y\right)$$

Simplifying:

$$\overline{w}^S = \widetilde{\eta}_f y + \psi$$

where I defined $\widetilde{\eta}_f \equiv \exp(\eta_f) - 1$. Notice that if $\eta_f = 0$, then the reservation wage $\overline{w}^S$ is simply equal to the participation cost ψ. When $\eta_f > 0$, it increases linearly with unearned income y.

Married women have two different reservation wages: one when the husband is employed, and one when he is unemployed. When the husband is employed, $\overline{w}^{M,E}$ is implicitly given by:

$$\log\left(\overline{w}^{M,E} + w_m + y - \psi\right) - \eta_f = \log\left(w_m + y\right)$$

Simplifying:

$$\overline{w}^{M,E} = \widetilde{\eta}_f \left(w_m + y \right) + \psi$$

Again, notice that if $\eta_f = 0$, then the reservation wage $\overline{w}_e$ is simply equal to the participation cost ψ. When $\eta_f > 0$, it increases linearly with total disposable income, given by the sum of male wages w_m and unearned income y.

When the husband is unemployed, $\overline{w}^{M,U}$ is given by:

$$\log \left(\overline{w}^{M,U} + y \right) - \eta_f = \log \left(y \right)$$

or

$$\overline{w}^{M,U} = \widetilde{\eta}_f y$$

It is easy to see that the reservation wage when the husband is employed is always higher than the reservation wage when the husband is unemployed. That is because overall disposable income is lower and the cost ψ does not have to be paid. Intuitively, there is a fraction of women who do not work if their husband is working, but choose to enter the labor force if the husband is unemployed.

Women are therefore more likely to work when female wage w_f is high, and when the utility cost η_f, the monetary cost ψ, the male wage w_m, or unearned income y are low, and when husband's unemployment risk is high. Aggregate hours worked by single women equals the fraction of single women who are not unemployed and have a higher wage than the reservation wage:

$$N_f^S = \int_i n_{f,i}^S \, di = (1 - u_f) \int_i \mathbb{1} \left(w_{f,i} \geq \overline{w}_i^S \right) di$$

Aggregate hours worked by married women equals the fraction of women who are not unemployed and have a higher wage than the reservation wage, considering the two husband's employment cases:

$$N_f = \int_i n_{f,i} \, di = (1 - u_f) \left((1 - u_m) \int_i \mathbb{1} \left(w_{f,i} \geq \overline{w}_i^{M,E} \right) di + u_m \int_i \mathbb{1} \left(w_{f,i} \geq \overline{w}_i^{M,U} \right) di \right)$$

Elasticities Consider one possible case where the only dimension of heterogeneity in the population is unearned income y, the distribution of which is described by a distribution function $F(y)$ with continuous marginal density $f(y) = F'(y)$. I assume that the density satisfies the assumptions $F(0) = 0$, $F'(y) > 0$, $\lim_{y \to 0} f(y) = 0$, and $\lim_{y \to \infty} f(y) = 0$. That is, all individuals have at least some positive unearned income and the distribution thins out at each tail (one example is a log-normal distribution for y). This corresponds to the case where wages are homogenous for everyone and there is wealth inequality.

Aggregate hours worked by single women are then given by:

$$N_f^S = (1 - u_f)F\left(\frac{w_f - \psi}{\widetilde{\eta}_f}\right)$$

while aggregate hours worked by married women are given by:

$$N_f^M = (1 - u_f)\left((1 - u_m)F\left(\frac{w_f - \widetilde{\eta}_f w_m - \psi}{\widetilde{\eta}_f}\right) + u_m F\left(\frac{w_f}{\widetilde{\eta}_f}\right)\right)$$

Let's compare elasticities with respect to own wage, cross wage and unemployment.

The elasticity of labor supply with respect to own wage for single women is:

$$\frac{\partial N_f^S}{\partial w_f}\frac{w_f}{N_f^S} = \frac{w_f}{\widetilde{\eta}_f}\frac{F'\left(\frac{w_f - \psi}{\widetilde{\eta}_f}\right)}{F\left(\frac{w_f - \psi}{\widetilde{\eta}_f}\right)}$$

which is unambiguously positive. The elasticity of single women labor supply with respect to male wages and unemployment is simply zero.

The elasticity of married women labor supply with respect to own wage is:

$$\frac{\partial N_f}{\partial w_f}\frac{w_f}{N_f} = \frac{w_f}{\widetilde{\eta}_f}\frac{(1 - u_m)F'\left(\frac{w_f - \widetilde{\eta}_f w_m - \psi}{\widetilde{\eta}_f}\right) + u_m F'\left(\frac{w_f}{\widetilde{\eta}_f}\right)}{(1 - u_m)F\left(\frac{w_f - \widetilde{\eta}_f w_m - \psi}{\widetilde{\eta}_f}\right) + u_m F\left(\frac{w_f}{\widetilde{\eta}_f}\right)}$$

which is also unambiguously positive.

Proposition: for low enough levels of male unemployment, provided that at least some married women are working, married women's own wage elasticity is always higher than single women's own wage elasticity.

This results may seem to suggest that a higher proportion of married households should make aggregate labor supply more variable. However, let's consider the possibility of insurance within the family, and hence the elasticities with respect to husband's wage and unemployment risk.

The elasticity of labor supply with respect to husband's wage is:

$$\frac{\partial N_f}{\partial w_m} \frac{w_m}{N_f} = -w_m \frac{(1-u_m)F'\left(\frac{w_f - \widetilde{\eta}_f w_m - \psi}{\widetilde{\eta}_f}\right) + u_m F'\left(\frac{w_f}{\widetilde{\eta}_f}\right)}{(1-u_m)F\left(\frac{w_f - \widetilde{\eta}_f w_m - \psi}{\widetilde{\eta}_f}\right) + u_m F\left(\frac{w_f}{\widetilde{\eta}_f}\right)}$$

which is unambiguously negative. Finally, the elasticity of labor supply with respect to husband's unemployment is:

$$\frac{\partial N_f}{\partial u_m} \frac{u_m}{N_f} = u_m \frac{F\left(\frac{w_f}{\widetilde{\eta}_f}\right) - F\left(\frac{w_f - \widetilde{\eta}_f w_m - \psi}{\widetilde{\eta}_f}\right)}{(1-u_m)F\left(\frac{w_f - \widetilde{\eta}_f w_m - \psi}{\widetilde{\eta}_f}\right) + u_m F\left(\frac{w_f}{\widetilde{\eta}_f}\right)}$$

which is positive since $F\left(\frac{w_f}{\widetilde{\eta}_f}\right) > F\left(\frac{w_f - \widetilde{\eta}_f w_m - \psi}{\widetilde{\eta}_f}\right)$.

Effects of a recession, depending on average distance from the participation frontier

Recessions Define a recession as:

- a decrease in w_f

- a decrease in w_m

- an increase in unemployment probability u

55

For single women, the effect on hours worked is unambigously negative. For married women, it depends on the relative strength of the three channels.

Note that since this is a static model, $\overline{w}$ today depends only on the realization of unemployment and not on increase in uncertainty tomorrow.

Effects of distance from the participation frontier   First, let's define a notion of *distance from the participation frontier x* as the increase of reduction in the wage needed to make a woman indifferent between participating or not:

$$(1 + x)w_f = \overline{w}_f$$

What happens to aggregate labor supply in a recession as a function of x?

Main point: if married women act as a dampening force, that is more true if there are more marginal women.

D   Hours Volatility and Responses to Aggregate Shocks

Doepke and Tertilt (2016), Olsson (2020), among others, document that women's hours are less volatile than men's at a business cycle frequency. I provide a summary here.

Sector accounting exercise: How much of this lower volatility can be explained by the fact that women work disproportionately in less volatile occupations and sectors, such as the service sector? Answer: about two thirds explained, one third left unexplained.

Extend the analysis in Ellieroth (2019) to include all years in the CPS. Since there are big changes in trends, especially for E-N flows, I detrend the data using Hamilton filter. The main findings from Ellieroth are confirmed. Description. Table.

E Computational Details

The individual problem of the household is solved through the endogenous grid method. Since the model features discrete choices, the value function may not be globally concave, and the first order conditions are necessary but not sufficient. The consumption choices implied by the endogenous grid method must therefore be adjusted to get rid of suboptimal points. For a review of endogenous grid method with continuous and discrete choices, see Iskhakov, Jorgensen, Rust and Schjerning (2017) and Druedahl (2020). I follow the upper envelope algorithm proposed by Druedahl (2020) to solve efficiently for the policy functions.

For calculating stochastic steady states, I use the method proposed by Boppart Krusell Mitman (2017), modified as proposed by Reiter (2018). With this method, individual agents' decisions are preserved as a fully non-linear function of their own individual states. Then, a impulse response function in response to a small unforeseen (MIT) shock is calculated. The model solution X_t and its stochastic properties are then calculated as a linear superposition of the impulse response functions:

$$X_t = \sum_{k=0}^{K} \widehat{x}_k z_{t-k}$$

where $\widehat{x}_k$ are the elements of the impulse response function appropriately scaled and z_t is a long sequence of shocks. If the dependency on the vector of aggregate states is linear, under certainty equivalence, the model solution is exact. Otherwise, it is only an approximation.

Reiter (2018) proposes a generalization of the method that takes advantage of the fact that impulse response captures some of the non-linearities of the model. Specifically, instead of calculating just one impulse response function to a small shock, we can calculate many impulse response functions for shocks of different sizes and magnitudes. The model solution X_t can then be written as a moving

average of past shocks:

$$X_t = \sum_{k=0}^{K} \widehat{x}_k^{\mathscr{I}(z_{t-k})} z_{t-k}$$

where $\mathscr{I}(z)$ is defined as the indicator of the interval into which the shock z falls. The method allows for improvements in accuracy provided the model is not strongly non-linear.