Functional Distribution, Land Ownership and Industrial Takeoff: The Role of Effective Demand*

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15th December 2007

Abstract

In this paper we analyze how the distribution of land property rights affects industrial takeoff and aggregate income through its impact on effective demand. We apply a modified version of the model provided in Murphy et al. (1989a) which allows us to analyze the role of land distribution when it is independent of the distribution of firm ownership. We extend the result of Murphy et al. (1989a) by showing that industrialization and income depend non-monotonically on the distribution of land and by demonstrating that this result is due to the way land distribution affects the distribution of profits among firms. Moreover, we show that there may be a tradeoff between industrialization and income, the latter being associated with a distribution of land which is more equal than that associated with maximum industrialization.

JEL classification: D33; O14; Q15
Keywords: Industrialization; Increasing Returns; Effective Demand; Functional Distribution; Land Ownership.

*We would really like to thank Andrea Battinelli, Leonardo Boncinelli, Samuel Bowels, Francesco Drago, Massimo De Vito, Davide Fiaschi, Mario Lavezzi, Gabriel Porcile, Fabio Petri, Martina Pignatti Morano and Neri Salvadori for their useful comments and suggestions. All mistakes remain ours.
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In this paper we analyze how the distribution of land property rights affects industrial takeoff and aggregate income through its impact on effective demand. In particular, we focus on the effects of the distribution of land on the domestic demand for manufactures, investigating how the former shapes the composition of the latter both in a direct way and through the distribution of profits among firms.

Our analysis extends that developed in Murphy et al. (1989a) where industrial takeoff is shown to depend on the composition of domestic demand for manufactures which, in turn, is shown to depend on the distribution of income. The basic assumption of the analysis in Murphy et al. (1989a) are that i) individuals have hierarchical preferences, ii) industrial production involves a fixed set up cost, and iii) a fraction of the labour force receives, besides wages, a given share of profits and rents. The main finding of Murphy et al. (1989a) is that both a too concentrated and a too diffused distribution of the land and firm shares can prevent industrialization and, hence, be detrimental to aggregate income. The central message of their analysis is that a sufficiently thick "middle class" is needed to obtain the buying power for domestic manufactures which is necessary to trigger industrialization.[1]

Assumption iii) may seem to be at odd with the historical evidence of countries in their early stage of industrialization. Indeed, economies undergoing their industrial takeoff are often characterized by a sharp functional division of income.[2] The typical case is that of a majority of workers earning just a subsistence wage, hereditary landowners earning rents and newly formed capitalists earning profits. However, assumption iii) is a reasonable enough approximation of reality if one is interested in studying the impact of income distribution on industrialization. On the contrary, if one wants to study – as in this case – the impact of the distribution of land property rights then assumption iii) is inappropriate. The reason is not its lack of realism but the fact that, since an owner of a given quota of rents also owns the *same* quota of profits, the distribution of profits is necessarily equal to the distribution of rents and, hence, the impact of land distribution *per se* cannot be investigated. Under iii), comparative statics in terms of land distribution only is impossible.

Therefore, we follow Murphy et al. (1989a) in assuming i) and ii) but instead of assuming iii) we give explicit role to land ownership and entrepreneurship and posit that some individuals in the population do not work but act as landowners or entrepreneurs. We assume that the number of workers and landowners is exogenously given – e.g. by birth right or government rule – while we leave entrepreneurship as a matter of choice. This alternative set of assumptions allows us to isolate the impact of land distribution and to investigate how it affects profits and their distribution among firms. The latter thing will turn out to have an important role in determining industrialization and aggregate income.

We provide two main results. First we prove that what Murphy et al. (1989a) have shown for the distribution of income in general – i.e. that industrialization and income depends non-monotonically on the distribution of land and firms shares – also holds when only the the distribution of land is considered, but it does for partly different reasons. As in Murphy

[1] See Murphy et al. (1989a, p.538).

[2] Examples of industrializing countries reported in Murphy et al. (1989a, p.539-40) also seem to support this view.

et al. (1989a), a too concentrated ownership of land determines a too varied demand for manufactures which fails to make mass production profitable. However, differently from Murphy et al. (1989a), we show that a too diffused distribution of land ownership may be detrimental not because it fails to provide people with the necessary buying power but because it creates a concentration of profits into the hand of few entrepreneurs which, in turn, produces a demand for manufactures which is very dispersed and hence not sufficient to trigger industrialization in many markets. In other terms, a too equal distribution of rents creates a too unequal distribution of income in favor of the owners of firms that, in turn, creates a negative situation which is similar to that of a too concentrated distribution of land. Our second result is the possibility of having a tradeoff between industrialization and income. We show that in many cases industrialization – either measured as the number of markets that adopt industrial production or as the number of people employed as industrial workers – does not coincide with the maximum aggregate income and that the latter requires a more intense exploitation of the economies of scale. This means that, in general, maximum income obtains for a distribution of land that is less concentrated than that associated with the maximum industrialization.

The paper is organized as follows. Next subsection reviews the related literature. Section II presents the basic model. Section III first provides results about existence and uniqueness of the equilibrium of the economy and then illustrates the impact of land distribution on industrialization and income. First, the case where industrialization does not take place is studied. Three different kinds can be distinguished: a) subsistence economies where only food is produced and consumed and there are only landowners and land workers, b) small economies where a manufacturing sector exists but the population is too small to make entrepreneurship and mass production profitable and c) traditional economies where wages are at subsistence level but there is a manufacturing sector producing only for landowners. The latter are taken as a negative benchmark and in the study of how the distribution of land property rights can trigger industrialization. Assume that wages are at the subsistence level allows to isolate the impact of land distribution and greatly simplifies the analysis while it does not imply a loss of generality. Section IV shows why and how a tradeoff between industrialization and aggregate income can arise. Section V contains a few comments about the scope of our results.

1 Related literature

The interest in the impact of land distribution on economic development has been recently revived by, among others, Deininger and Squire (1998) who produced a new data set and an important piece of empirical evidence. They found that initial inequality of land distribution has a significant negative effect on subsequent growth rates. Only two of the 15 developing countries with land Gini coefficient above 70 grew at more than 2.5% over the 1960-1992 period.[3] Deininger and Squire (1998) provide two possible explanations for this relationship. The first recognizes that, whenever there are imperfections in asset markets, people without the necessary collateral may be prevented from undertaking the efficient level of investment. The second looks at the interaction between land ownership distribution and the political system, showing that individual ownership of assets affects people's preferences as to political outcomes: under a democratic regime, inequality would be detrimental to growth because it induces preferences for higher taxation, therefore reducing incentives for investments.[4] Galor et al. (2004) analyse how the distribution of land property rights affects early growth via education. They argue that the more unequal is the distribution of land ownership the later educational reforms are introduced, with a strong negative impact on the accumulation of human capital.[5]

Murphy et al. (1989a), Baland and Ray (1991), Eswaran and Kotwal (1993) and Matsuyama (2002) are all attempts to investigate the link between inequality – in term of income or income-generating property – and industrialization taking into account the composition of demand.[6] The basic productive structure which is assumed in these contributions is that of the dual economy studied in Rosestein-Rodan (1943), Lewis (1954, 1967) and Fleming (1955) between the 1940s and 1960s. A part from Murphy et al. (1989a) these papers mainly focus on the persistent effects of productivity improvements.

The interest in land distributional issues indirectly steamed also from Lucas (1993). In the paper he reported that in the early 1960s South Korea and Philippine exhibited similar macroeconomic backgrounds under many respects having about the same GDP per capita, schooling levels, population and urbanization. Nevertheless, during the following twenty-five years the former experienced sustained growth – about 6% – being fully committed to the industrialization process, while the latter grew at a speed of about one third – less than 2% – remaining mainly an agricultural economy. Lucas classified the case of Korea as a sort of *productivity miracle* As Bénabou (1996) pointed out, on shifting the focus from standard macroeconomic magnitudes to the distribution of income and land ownership, one finds no such similarities. Indeed, the two countries were appreciably different with South Korea showing a much more equal distribution of both land property rights and income than Philippines. Remarkably, the ratio between income of the top 20% population and that of the bottom 20% – or even 40% – was nearly twice bigger in Philippines. The Gini coefficient for land ownership was 38.7 in Korea in 1961 and 53.4 in the Philippines in 1960.[7] These distributive differences contribute to explain the better economic performance

[3]See Deininger and Squire (1998, p.260). Greater details about the data set can be found in Deininger and Squire (1996).

[4]See Bénabou (1996) and Bollettini and Ottaviano (2005) for references to recent contributions on the general link between inequality and growth.

[5]Galor et al. (2004) provide empirical evidence from the US in the period 1880-1920.

[6]More recently, Zweimüller (2001) and Mani (2001) sought to consider explicitly the growth process by investigating how hierarchical demand influences technological progress.

[7]This latter difference is the effect of the land reform undertaken by the Government of South Korea in 1949 which took the name of Agricultural Land Reform Amendment Act (ALRAA). It consisted mostly of the redistribution of land previously owned by Japanese people. ALRAA reduced the number of tenants to

of Korea, particularly in the early years of industrialization. A more equal distribution of income and land ownership gave Korea a greater and more stable domestic demand for basic manufactures which made investments in mass production technologies more profitable.[8]

2 The Model

2.1 Commodities and Consumption Patterns

There is a single homogeneous and divisible agricultural good. For simplicity we label it *food* and use it as numeraire. Moreover, there is a continuum of manufactured goods represented by the open interval $[0, \infty) \in \Re$. Each good is denoted by its distance q from the origin. The consumption pattern – or tastes, if one prefers – is assumed to be the same for each individual. There is a subsistence level of food consumption $\bar{\omega}$. After that, any unit of income is spent to buy the manufactured goods following their order in the interval.

This assumption is intended to be a simple way of introducing a common ranking of necessities: people first need to buy what is necessary to survive, then basic manufactures and durables which allow better life standards and, only after that, they buy luxuries. For simplicity, we assume that only one unit is bought of any manufactured good. In other terms, any individual with income $\omega \geq \bar{\omega}$ uses her first $\bar{\omega}$ of income to purchase food needed to survive and $(\omega - \bar{\omega})$ to purchase the manufactured goods. Any individual with $\omega < \bar{\omega}$ starves.[9]

It is worth pointing out the intuitive consequences of our assumptions. First, individuals are almost identical in terms of their consumption decisions: they only differ in income. Thus, a landowner and her servants would consume the same if given the same income. Second, any increase of income results in an increase of consumption variety. In particular, richer people buy the same bundle of poorer people plus some other commodities.

nearly zero in a couple of years (Jeon and Kim, 2000).

[8] Chenery and Syrquin (1975), Chenery et al. (1986) provide further empirical evidence of the relevance of domestic demand for industrialization. Using a sample of rapidly growing economies they show that the expansion of domestic demand accounts for a large part of the increase of domestic income. For the biggest countries in their sample, domestic demand explains more than 70 % of the increase in domestic income, while in small countries (under 20 million people) the percentage falls to a minimum of 50 %. See also Murphy et al. (1989b, section II).

[9] As shown in Murphy et al. (1989a), this consumption behaviour can be rationalized by means of the following utility function which captures the idea of hierarchical preferences

$$U = \begin{cases} c & \text{if } c \leq \bar{\omega} \\ \bar{\omega} + e^{\int_0^1 (1-x(q))^{\frac{1}{q}} \mathrm{d}q + \int_0^\infty x(q)^{\frac{1}{q}} \mathrm{d}q} & \text{if } c > \bar{\omega} \end{cases}$$

where c is the consumption of food and $x(q)$ is equal to 1 if good q is consumed.

2.2 The Agricultural Sector

In order to produce food it is necessary to use land and labour. We abstract from land and assume it is always fully utilized in production. For the sake of simplicity, we also assume all workers have the same skills – i.e. labour is homogenous – and perfect competition in the output side – i.e. no profits are earned.

Technology and Incomes. Given the amount of land utilized, labour has decreasing marginal productivity. Total production is determined by the function $F(L_f)$ where L_f is the number of workers employed in agriculture. It is assumed that $F' > 0$, $F'' < 0$. Agricultural wage w_f is a function of agricultural employment with $w'_f(L_F) < 0$. This formalization is consistent with the case in which labour is paid its marginal product.

Since profits are nil, income generated in agriculture is exhausted by the wage of land workers and the rents of landowners. Denoting with R the total amount of rents earned, we have the account equation

$$R = F(L_f) - w_f L_f \tag{1}$$

Land Ownership. Differently from Murphy et al. (1989a), we assume property rights of the land stock to be equally distributed among M landowners. We also assume that the income of each landowner is equal to R/M and, hence, is negatively related to their number.[10] The idea is that, on average, the greater the number of landowners, the smaller is the area of land they possess and, therefore, the lower the rent they earn. Although a non-uniform distribution of land property rights is the norm, our simplification works well as long as the average concentration is the relevant feature. In this sense, M should be interpreted as a rough index of land property concentration. Finally, we abstract from the issue of productivity change due to variations in the distribution of land ownership, such as that described in Banerjee et al. (2002).[11]

2.3 The Manufacturing Sector

The continuum of goods defines a continuum of markets. The number of workers employed in the manufacturing sector as a whole is denoted by L_m while the ruling wage is w_m.

Technology and Markets. Each commodity q is produced with the same cost structure. Two technologies are available. The first, labelled *traditional technology* or TT, requires α units of labour in order to produce a unit of output. This represents the case in which

[10] Murphy et al. (1989a) do not consider the existence of landowners as individuals: in their model, agricultural production – like industrial production – is organized by firms which divide their profits among a certain number of shareholders.

[11] The qualitative results of our model can be obtained also by allowing for an increase in productivity due to the reduced size of land property. However, the analysis would become more complicated and would somehow obscure the mechanism we seek to highlight.

commodities are produced by artisans who, at the same time, both supervise production and work as wage-paid labourers. For this reason, the number of workers in TT markets also includes artisans. The second, labelled *industrial technology* or IT, requires k units of labour to start up plus β units of labour per unit of output produced, with $0 < \beta < \alpha$.

We assume $(k+1) > (\alpha - \beta)$ which means that the amount $(\alpha - \beta)$ of labour saved producing one unit of output using IT is less than the fixed amount k needed to introduce the IT plus the unit of labour provided by the artisan. Clearly, this is the only interesting case because if $(k+1) \leq (\alpha - \beta)$ IT never requires more units of labour with respect to TT and, hence, it is always preferred. Lastly, we denote by E the number of entrepreneurs.

Notice that TT shows constant returns to scale while IT shows increasing returns. The difference between these two technologies represents the economic advantage of industrialization.

Competition and Income. A group of competing artisans is assumed to operate in each market q of the economy. Given a wage w_m, any amount of commodities can be produced and sold at the unit price αw_m. No profits are earned by artisans. Besides, in each market q there exists one and only one artisan who knows IT. If she decides to be an entrepreneur she can become a monopolist by slightly undercutting the price αw_m. In this case nobody buys the good produced with TT and profits of market q are

$$\pi(q) = (p_q - \beta w_m)D_q - k w_m \tag{2}$$

where p_q is the price and D_q is the demand.

2.4 Population and Labour Market .

Agricultural employment determines the ruling wage w_f. We assume perfect mobility of labour among sectors and markets so that $w_f = w_m = w$.

The active population is denoted by L and each worker either supplies inelastically one unit of labour or becomes an entrepreneur. The total supply of labour is hence equal to $L - E$. Finally, the population is assumed to be fixed and equal to $N = L + M$ where $L = L_f + L_m + E$.

3 Industrialization, Income and Land Ownership Distribution

In this section we provide results about existence and uniqueness of the equilibrium of the economy. Then, we sketch the mechanism through which the distribution of land property rights affects the equilibrium level of income and industrialization.

Since we want the economy to actually produce commodities, we make the following assumptions. First, the ruling wage w is not less than the subsistence level $\bar{\omega}$. Second, the economy can sustain the whole population, i.e. $F(L) \geq \bar{\omega}N$. For the sake of realism, we further assume that rent of a single landowner, R/M, cannot be lower than the ruling wage, w. Notice that the same holds for profits as artisans knowing the IT decide to become entrepreneurs if and only if $\pi_q \geq w$.

Under such assumptions the following proposition holds.

PROPOSITION 1 *For any given combination of population size, N, number of landowners, M, agricultural technology, $F(\cdot)$, and manufacturing technology, $\tau \equiv (\alpha, \beta, k)$, there exists one and only one combination of the extent of industrialization, $Q^* \geq 0$, and agricultural employment, $L_f^* > 0$, such that IT is used in all markets in the interval $[0, Q^*]$, and all markets – food market and manufacture markets – clear.*

Basically, Proposition 1 says that for any combination of population, technology and land distribution there exists one and only one equilibrium combination of industrialization and income level. It must be noted that Q^* and L_f^* are sufficient to determine any other variable of the economy and hence fully characterize the equilibrium (the Appendix contains the proof of the proposition and further details on economic variables other than Q^* and L_f^*).

Before giving a detailed analysis of how the distribution of land ownership influences income and industrialization, we find it convenient to give a brief description of the economic mechanism which is responsible for such an influence. Consider an economy whose agricultural sector is already in equilibrium. We can do this without loss of generality because demand for food is exogenously given. Denote with Ω_m the total expenditure in manufactures and with ω the income of a generic individual. Since every consumer who has already bought $\bar{\omega}$ units of food spends her remaining income to get a unit of each manufacture in the specified order, the demand D_q faced by a generic market q is determined by the number of individuals who earn enough income to buy at least commodity q, namely the number of individuals who satisfy $(\omega - \bar{\omega})/\alpha w > q$.

Assume, for the sake of the argument, that workers are poor and consume only food, i.e. $w = \bar{\omega}$. Hence, the distribution of land property rights shapes the demand for manufactures by determining the income and the number of individuals who buy manufactures. If, for instance, there are only few rich landowners, then the extent of the manufacturing sector will be quite large and the demand faced by each market will be relatively small. If, on the contrary, landowners are many but each with a low income, then the extent of the manufacturing sector will be quite small and the demand faced by each of these markets will be relatively large. In addition, total expenditures on manufactures, Ω_m, decreases in the concentration of land property rights because a smaller number of landowners implies that a smaller fraction of rents is spent on manufactures.

We now turn our attention to industrialization. Obviously, a large Ω_m favors industrialization. However, since IT is introduced only if demand goes over a certain profitability threshold, a too concentrated ownership of land may prevent the takeoff even if Ω_m is large. Otherwise if land ownership is sufficiently distributed then the profitability threshold may be exceeded. In such a case, some artisans becomes entrepreneurs, earn positive profits and the market in which they operate industrialize. The new earnings obtained by entrepreneurs start a multiplicative process of demand for manufactures. New demand generates new profits and new profits generate new demand.[12] Such a feedback process can take place several times but in each round the amount of new profits diminishes because only a fraction of the new demand becomes new profits – the remaining part going to cover production costs. The process ends when new generated profits fail to industrialize new markets or to generate new demand for markets already industrialized.

3.1 Non-industrial Economies

In order for an artisan of market q to become an entrepreneur profits π_q must not be less than the ruling wage w. Since technology is the same in every market and $q' < q''$ implies $D_{q'} \geq D_{q''}$, a necessary and sufficient condition for no industrialization is $\pi_0 < w$. From equation (2) we get that $\pi_0 < w$ if and only if $D_0 < \rho$ where $\rho \equiv (k+1)/(\alpha - \beta)$. In equilibrium no market industrializes if and only if the demand faced by the first market is less than the value which covers start-up costs plus the opportunity cost faced by the artisan. Any consumer who earns more than $\bar{\omega}$ demands at least the 0-commodity which implies that, for $R/M > \bar{\omega}$, $D_0 \geq M$ and, for $R/M \geq w > \bar{\omega}$, $D_0 \geq M + L$. So, the industrialization may fail under a variety of combinations of $F(\cdot)$,L,M, and τ. We group these into three classes of interest.

Subsistence economy. In a subsistence economy only food is produced and consumed and there is no manufacturing sector. The ruling wage is $w = \bar{\omega}$ and L, F and M are such that $M + L = F(L)/\bar{\omega}$. Clearly, $L_f^* = L$ meaning that all the labour force of the economy is employed in agriculture. Hence, we get $L_m^* = 0$, $E = Q^* = 0$.

Given the level of agricultural productivity, the number of landowners with respect to population is too high to sustain industrialization. The excessive dispersion of land property rights makes the individual rent of landowners, R/M, as low as $\bar{\omega}$, fully offsetting the landowners' benefits due to a very low wage. As a consequence, no one demands manufactured goods and there is no manufacturing sector.

Traditional economy. In a traditional economy workers earn just what is needed to survive while few landowners are rich enough to demand manufactures. There exists a manufacturing sector but mass production is not profitable. Formally, $w = \bar{\omega}$ and L, $F(L_f)$ and M are such that $M + L < F(L)/\bar{\omega}$ and $M < \rho$. Clearly, $L_f^* < L$ and $R/M > \bar{\omega}$.[13] Landowners spend $(R/M - \bar{\omega})$ in manufactures consuming commodities in $[0, Q_R]$ where $Q_R = (R - \bar{\omega}M)/\alpha\bar{\omega}M$. Since each operating market faces a demand $D_q = M < \rho$, no market industrializes. Hence, the extent of the manufacturing sector coincides with the extent of landowners demand, i.e. $\bar{Q} = Q_R$ (see Figure 1).

[12]The precise outcome depends on how profits are distributed among entrepreneurs. This issue is investigated in detail in the following sections and in the Appendix.

[13]Since $L_f^* < L$, in equilibrium we have that $M < L + M - L_f^*$ if an only if $\bar{\omega} < (F(L_f^*) - L_f^*\bar{\omega})/M = R/M$

In this economy land concentration prevents industrialization because, although landowners are rich enough to demand manufactures, their number is not sufficient to make the introduction of IT profitable for entrepreneurs.

Small economy. In a small economy both workers and landowners are rich enough to demand manufactures but population is so small that IT is not profitable. Formally, $R/M \geq w > \bar{\omega}$ and $M + L < \rho$. As before, $L_f^* < L$. Moreover, there is an upper bound for w constituted by the level of wages which reduces the rent of each landowner to the level of wages, i.e. $\bar{\omega}L/(1 + L_f^*)$. For $w < \bar{\omega}L/(1 + L_f^*)$ both workers and landowners demand manufactures. Let $Q_L \equiv (w - \bar{\omega})/\alpha w$ be the extent of workers' demand and, again, Q_R be the extent of landowners demand. In a small economy markets in $[0, Q_L]$ face a demand equal to $D_q = (L + M) < \rho$ while markets in $(Q_L, Q_R]$ get a demand equal to $M < \rho$ (see Figure 2). Hence, no market industrializes and the extent of the manufacturing sector is $\bar{Q} = Q_R$.[14]

In this economy, industrialization is prevented by the small population size and not by distribution. Indeed, even if agricultural productivity is high enough to grant both workers and landowners a very high income, their small number makes mass production unprofitable. In this case the manufacturing sector may flourish producing manufactures of great quality and value but no artisan find it convenient to become an entrepreneur.

3.2 The Impact of Land Ownership Distribution

In order to isolate the impact of land ownership distribution we assume that $w = \bar{\omega}$. Under this assumption workers spend all their income on food and hence demand for manufactures can only come from landowners and entrepreneurs. Of course, we are aware that this is not necessarily the case in real world economies. However, this assumption allows us to better illustrate the impact of land ownership distribution and, more important, since wages are determined by labour productivity in the agricultural sector and demand for food is exogenously given by population, the results obtained for $w = \bar{\omega}$ can be easily extended to the case where $w > \bar{\omega}$ without any substantial change in their quality. Besides this, in most cases countries in their early stage of industrialization are in a situation where workers mostly spend their income on subsistence goods. So, although our assumption may not be satisfied in a strict sense, it may still be a good approximation.[15]

We investigate how land distribution affects the income level and the extent of industrialization of the economy by comparing the equilibrium values of aggregate income, industrial extent and industrial employment which are associated with different degrees of land concen-

[14]For the sake of completeness, note that by assuming $R/M \geq w$ we have ruled out the case of $\bar{\omega} < w = L\bar{\omega}/L_f^*$ where only workers demand manufactures. In such a case, $D_q = L$ for any q in $[0, Q_L]$ and an extent of the manufacturing sector would be equal to $\bar{Q} = Q_L$.

[15]An interesting issue is what happens to income and industrialization in case of an exogenous factor which rises the wage level – e.g. an institutional shock which modifies the norms of product sharing in favor of agricultural workers. We investigate such a case in great detail in a companion paper.

tration. Since calculations are not particularly enlightening and are rather long, we collect them in the Appendix while providing here only the main results and their interpretation

As we have pointed out for traditional economies, if $M < \rho$ no artisan introduces the IT. Hence, both industrial extent and employment are nil. In this case, the income of the economy is $Y^* = R^* + \bar{\omega}N - \bar{\omega}M$. Since N and R^* are constant, Y^* decreases in M meaning that a more equal distribution of land property rights reduces aggregate income. The reason is that there are more landowners and hence less people work. On the demand side, aggregate demand of manufactures decreases in M because the quota of rents spent on food increases.

For $M = \rho$ we have a sharp change. Assuming for simplicity that IT is introduced whenever it is not disadvantageous to do so, then industrial extent jumps to $Q^* = Q_R = \bar{Q}$ and no commodity is produced with the TT. Artisans operating in markets $[0, Q_R]$ who know the IT become entrepreneurs, although they still earn as much as a worker. Similarly, industrial employment is $L_{IT}^* = \beta M Q_R + k Q_R$ where the first term stands for workers in direct production and the second for those involved in start-up tasks. On the contrary aggregate income, which is still equal to $R^* + \bar{\omega}(N - M)$, is not greater than the income obtained for any $M < \rho$. This happens because increasing returns are exploited just enough to compensate start-up costs while the negative effect due to M is lager.

For $M > \rho$, we also have positive industrialization. In addition, some entrepreneurs are rich enough to spend part of their profits in manufactures. This generates new demand, starting the multiplicative mechanism described in the previous section. Different distributive scenarios may take place depending on M. This is illustrated by the following lemma which will turn out to be useful in the proof of Proposition 1.

LEMMA 1 *For any given combination of population size, N, agricultural technology, $F(\cdot)$, and manufacturing technology, $\tau = (\alpha, \beta, k)$, such that industrialization is feasible, i.e. $N - F^{-1}(\bar{\omega}N) > \rho$, there exists $\mu > 0$ such that*

- *i) $M = \mu$ implies that all entrepreneurs earn the same profits which are equal to landowners' rents,*
- *ii) $\rho < M < \mu$ implies that entrepreneurs earn different levels of profits but no one earns profits that are equal or greater than landowners' rent,*
- *iii) $\mu < M < N - F^{-1}(\bar{\omega}N)$ implies that entrepreneurs earn different levels of profits; moreover, those operating in $[0, Q_R]$ earn profits which are greater than landowners' rents while the remaining entrepreneurs may earn profits which are smaller than landowners' rents,*

In other terms if the economy can sustain the industrialization of some markets then there exists a level of M, denoted by μ, such that for $M = \mu$ all entrepreneurs and landowners are equally rich, for $M < \mu$ entrepreneurs make heterogeneous profits but all are poorer than landowners, and for $M > \mu$ entrepreneurs again make heterogeneous profits but those operating in $[0, Q_R]$ are richer than landowners while the remaining may well be poorer.

For $M \leq \mu$ the extent of industrialization Q^* decreases with the number of landowners M. This happens because landowners are the richest consumers and, hence, the extent of industrialization coincides with the extent of landowners' demand Q_R which decreases in M. Since no one demands commodities beyond Q_R, no good is produced with the TT and $Q^* = Q_R = \bar{Q}$. As regards industrial employment we have

$$L_{IT}^* = \frac{R^*}{\bar{\omega}} - (M + Q_R) \tag{3}$$

Notice that L_{IT}^* is equal to the number of people who are not employed as agricultural workers, $R^*/\bar{\omega} = (N - L_f^*)$, minus the sum of landowners and entrepreneurs, $(M + Q_R)$. Since Q_R diminishes in M at a decreasing rate, then, from equation (3), L_{IT}^* can increase, decrease, or first increase and then decrease. Similarly, aggregate income is

$$Y^* = R^* + \bar{\omega}N + \Pi^* - \bar{\omega}(M + Q_R) \tag{4}$$

and is not, in general, monotonic in M.[16] In the range under consideration it can either increase or first increase and then decrease. The term $\Pi^* - \bar{\omega}(M + Q_R)$, which determines the actual behaviour of Y^*, depends on two opposite effects. On the one hand, the concentration of landowners' demand in fewer markets allows better exploitation of increasing returns and hence increases profits. On the other, a higher quota of rents spent on subsistence reduces aggregate landowners' demand for manufactures and decreases the extent of industrialization possibly affecting profits negatively. The impact on Y^* of these two opposite effects is ambiguous.

For $M > \mu$ entrepreneurs in $[0, Q_R]$ demand commodities beyond Q_R. If their number is high enough – namely $Q_R \geq \rho$ – also some markets beyond Q_R industrialize. Moreover, if $Q_R > \rho$, also entrepreneurs of these markets demand manufactures which, in turn, increases the earnings of some entrepreneurs in $[0, Q_R]$. These entrepreneurs are the richest among all entrepreneurs since their products are demanded by all who buy manufactures. Their additional profits transform into demand for a new variety of commodities. Hence, if the number os these entrepreneurs is at least ρ, then also the artisans producing such new commodities adopt the IT and become entrepreneurs.

This chain of events – industrialization, new profits, new demand – may take place several times, each round corresponding to the industrialization of a new interval of markets. We refer to such intervals as *steps* and we denote with i^* their number in equilibrium. Notice that, i^* is a non-increasing step function of M. However, a greater i^* does not imply a greater Q^*. In particular, Q^* is determined by the income of the richest group of entrepreneurs among those of dimension not less than ρ and can be written as

$$Q^* = (M - \rho) \sum_{j=1}^{i^*} \left(\frac{\alpha - \beta}{\alpha} \right)^j + Q_R \left(\frac{\alpha - \beta}{\alpha} \right)^{i^*} \tag{5}$$

[16] The expression for Π^* is quite complicated and adds very little by itself. It can be found in the Appendix.

The first term accounts for the positive effect produced by the concentration of landowners' demand for basic manufactures which allows the richest entrepreneurs to make more profits and, hence, to extend their demand. The magnitude of this effect increases, *ceteris paribus*, with the number of steps as more steps means that more groups of entrepreneurs buy commodities produced with IT. The second term accounts for the negative effect produced by the reduction in the extent of landowners' demand which, *ceteris paribus*, reduces the number of industrialized markets and, hence, the demand faced by the richest group of entrepreneurs. The magnitude of this negative effect decreases with the number of steps since it is partially compensated by the industrialization of more markets which do not receive landowners' demand.[17] As a result, the extent of industrialization can both increase and/or decrease in M, possibly showing discontinuous variations when M reaches values which imply a decrease in i^*. Moreover, a part from such points of discontinuity, we have $\bar{Q} > Q^*$. Indeed, the few richest entrepreneurs demand commodities produced with the TT and a traditional production survives.[18]

In such a case, industrial employment is equal to

$$L^*_{IT} = \frac{R^*}{\bar{\omega}} - (M + Q^*) - L^*_{TT} \tag{6}$$

where L^*_{TT} is the number of workers producing with TT. For any feasible value of i^* and the associated range of M, L^*_{TT} can both increase or decrease in M. Thus, taking into account the behaviour of Q^*, then also L^*_{IT} can increase and/or decrease and possibly show discontinuities in coincidence with the reduction of i^*.

As for $M < \mu$, Y^* can either decrease or first increase and then decrease. The intuition is fundamentally the same given for that case, although here income is more likely to decrease in M. For $\rho \leq M < \mu$ landowners are the richest group in society and no one demands commodities produced with TT. As a consequence, all profits are transformed into extra demand for industrial goods except what is spent in subsistence. On the contrary, for $M > \mu$ some entrepreneurs are the richest group in society and demand commodities produced with TT . Hence, the fraction of profits which generates additional income is lower which suggests that Y^* is more likely to be decreasing in M.

4 Tradeoff between Income and Industrialization

So far, we have shown that aggregate income, industrial extent and industrial employment have a non-monotonic relationship with land concentration. The next step is to identify the values of M which gives the maximal level of these variables. Quite interestingly, it turns out that maxima are not achieved for the same distribution of land ownership. This suggests

[17] Of course, there exists a level of M such that $i^* = 0$. In such a case no market beyond Q_R industrializes and $Q^* = Q_R$ as for $M < \mu$; moreover, there are commodities produced with TT and the extent of the manufacturing sector $\bar{Q}$ is still equal to the extent of entrepreneurs' demand (all earn the same profits).

[18] In the discontinuity points where a change in I^* takes place, then the richest group of entrepreneurs has no less than ρ members such that $\bar{Q} = Q^*$ and production with TT disappears.

that there may be a trade off between income and industrialization during the early stages of industrialization.

PROPOSITION 2 *Let N be the population of the economy and τ and $F(\cdot)$ be, respectively, the manufacturing technology and the agricultural technology. Let also $M \in [\rho, N]$. Then, we have*

i) the maximum extent of industrialization, $\widehat{Q^*}$, is obtained for $M = \rho$,

ii) the maximum industrial employment, $\widehat{L}^*_{IT}$, is obtained for some M in $[\rho, \mu]$; more precisely, for $M = \rho$ if $\sqrt{R/\alpha\bar{\omega}} \leq \rho$, for $M = \sqrt{R/\alpha\bar{\omega}}$ if $\rho < \sqrt{R/\alpha\bar{\omega}} < \mu$ and for $M = \mu$ if $\sqrt{R/\alpha\bar{\omega}} \geq \mu$,

iii) the maximum aggregate income, $\widehat{Y}^*$, is obtained for $M = \sqrt{(k+1)R/\alpha\bar{\omega}}$ whenever $\mu > \sqrt{(k+1)R/\alpha\bar{\omega}}$; otherwise it is obtained for some $M \in [\mu, N]$

The proof of Proposition 2 is found in the appendix. In the following we try to give the intuition of such results.

Let us consider see why $\widehat{Q^*}$ is obtained for $M = \rho$. Notice that for $M = \rho$ all workers of the manufacturing sector produce with the IT and industrial employment is the minimum which allows industrialization of these markets. Notice also that the maximum number of people employable as industrial workers is $(N - F^{-1}(\bar{\omega}N) - M - Q^*)$. Therefore, for $M > \rho$ it is impossible to have a greater extent of industrialization because there are not enough workers available for operating IT in more markets. So, the maximum extent of industrialization is obtained for the distribution of land which produces a demand for manufactures just sufficient to industrialize markets in $[0, Q_R]$, making entrepreneurs earn as much as workers while landowners are the richest in society. Clearly the greater the start-up costs or the smaller the difference in marginal cost between TT e IT the greater M which produces the maximum industrial extent.

Maximum industrial employment $\widehat{L}^*_{IT}$ is obtained for some M in $[\rho, \mu]$. The exact value depends on τ and F. To see why $\widehat{L}^*_{IT}$ cannot happen for $M > \mu$ recall that L^*_{IT} always decreases in $(M + Q^*)$ and that for $M = \mu$ we have $L^*_{TT} = 0$. Hence, a necessary condition to have $\widehat{L}^*_{IT}$ with $M > \mu$ is that $M + Q^*(M) \leq \mu + Q^*(\mu)$ which implies $Q^*(M) < Q^*(\mu)$. So, the richest entrepreneurs must earn less than what they earn when $M = \mu$. This implies that the total demand faced by the industrial sector as a whole cannot be greater than in $M = \mu$ while employment in start-up tasks is certainly lower. Therefore, the value of M which maximizes industrial employment cannot be greater than μ. More precisely, it depends on the behaviour of $(M + Q_R)$ in the interval $[\rho, \mu]$. If, for instance, a greater number of landowners induces a shrinking of the range of industrialized markets that *never* frees enough labour force to compensate for the decrease in total labour force – due to a higher M – then $\widehat{L}^*_{IT}$ is obtained for $M = \rho$. If, instead, we are in the opposite case where the shrinking of the range of industrialized markets *always* frees enough labour force to compensate for the total labour force reduction, then $\widehat{L}^*_{IT}$ is obtained for $M = \mu$. In all other cases $\widehat{L}^*_{IT}$ is achieved for $M = \sqrt{R/\alpha\bar{\omega}}$. Hence, a part from the two extreme cases, a greater R^* implies a that $\widehat{L}^*_{IT}$ is obtained for a greater M. This is because a greater agricultural surplus extends the extent of the demand of manufactures and, hence, it creates the room for a better exploitation of increasing returns through the concentration of demand in fewer markets. A smaller α have a similar effect because, by decreasing the price of manufactures, it extends the interval of markets which face landowners' demand.

Finally, the value of M which gives $\widehat{Y}^*$ depends on both τ and F. Whatever the technology, however, $\widehat{Y}^*$ is achieved for an M which is greater than that associated with $\widehat{Q^*}$. In fact, for $M = \rho$ increasing returns are not exploited at all and income is even lower than in the traditional economy case. A more equal distribution of land increases Y^* because, by inducing a greater concentration of demand in basic manufactures, it allows a better exploitation of increasing returns. On the other hand, too wide a distribution of land property rights may be detrimental. The concentration of landowners' demand in few basic manufactures has the effect of concentrating most of the profits into the hands of few entrepreneurs. Since these are very rich with respect to the size of the industrial sector, they spend a substantial part of their earnings on manufactures produced with TT which is detrimental to income.

Recapitulating, maximum income Y^* may be obtained when land is concentrated and landowners are the richest in society, $M < \mu$, when land is more equally distributed and the richest group is constituted by some entrepreneurs, $M > \mu$, or when landowners and entrepreneurs earn exactly the same, $M = \mu$. For the case $M > \mu$ the exact value of M it quite complicated to obtain analytically and, hence, we compute it numerically. Nonetheless, from a comparative statics exercise we can understand a few things about how the level of M which gives $\widehat{Y}^*$ varies with the exogenous parameters of the model. A higher k increases the optimal M because the profitability threshold of IT increases and, hence, requires a greater demand concentration for optimality. A higher R^* has the same effect as it increases the relative advantage of concentrating demand in fewer manufactures. On the contrary, a higher α reduces the optimal M because it increases the relative price of manufactures, having the same effect on landowners' demand as a reduction in rents. A higher β may or may not have an effect but certainly increases μ because it reduces the profits earned for each unit sold and, hence, it increases the range of M for which landowners are the richest.[19]

This discussion highlights some interesting implications of Proposition 2 which we collect in the following

COROLLARY 1 *If $\sqrt{R/\alpha\bar{\omega}} > \mu$ then for $M \in [\rho, \sqrt{R/\alpha\bar{\omega}}]$ there is a tradeoff between Q^* and L^*_{IT}. Moreover, if $\sqrt{(k+1)R/\alpha\bar{\omega}} \le \mu$ then*

i) for $M \in [\rho, \sqrt{(k+1)R/\alpha\bar{\omega}}]$ there is tradeoff between Y^* and Q^*,

ii) for $M \in [\sqrt{R/\alpha\bar{\omega}}, \sqrt{(k+1)R/\alpha\bar{\omega}}]$ there is a tradeoff between Y^* and L^*_{IT}.

Finally, if $\sqrt{(k+1)R/\alpha\bar{\omega}} > \mu$ then

[19] See the Appendix for more details on this point.

i) either Y^* and L^*_{IT} jointly attain their maximum for $M = \mu$ or there exists $\epsilon > 0$ such that for $M \in [\sqrt{R/\alpha\bar{\omega}}, \mu + \epsilon]$ there is a tradeoff between Y^* and L^*_{IT},

ii) there exists $\epsilon' > 0$ such that for $M \in [\rho, \mu + \epsilon']$ there is a tradeoff between Y^* and Q^*.

The proof of Corollary 1 is straightforward from the proof of Proposition 2 (see the Appendix). In order to give the flavour of these findings we depict an example in Figure 3. It exemplifies the non-monotonic relation between land distribution and industrialization/income as well as the fact that maximal income, industrial extent and employment are not obtained for the same distribution of land property rights. Moreover, it illustrates that in the range $[\rho, \mu]$ there may be a tradeoff between industrialization and income. In this example until λ the tradeoff is between industrial extent on the one hand and industrial employment and income on the other.[20] For $M \in [\rho, \lambda]$, a more equal distribution of land concentrates landowners' demand in such a way that the number of new workers needed for direct production is greater than that previously needed for the start-up tasks of markets that no longer industrialize. Thus, income and industrial employment go the same way. By contrast, for $M \in [\lambda, \mu]$ a tradeoff exists between income and both industrial extent and employment. In this range, income increases despite the decrease in industrial employment because the total number of workers employed in direct production is still rising and industrial surplus grows. Industrial employment decreases because the reduction of workers hired for start-up tasks exceeds new hirings for direct production, so that better exploitation of increasing returns no longer coincides with a greater number of industrial workers.

For $M \in [\mu, \eta]$, land is distributed so widely that some entrepreneurs are richer than landowners.[21] A part from discontinuity points, there is again a tradeoff between industrial extent on the one hand and industrial employment and income on the other but with the opposite sign. Besides, industrialization of new markets intervals is induced, creating up to five different earning groups of entrepreneurs.

Finally, for M greater than η land ownership is so dispersed – and consequently landowners' demand so concentrated – that there are a few and very rich entrepreneurs. Their number is so small that they require many different manufactures but no single market receives enough demand to industrialize. In this range there is no longer a tradeoff between income, industrial employment and industrial extent as they all decrease in M until the latter reaches is upper bound $N - F^{-1}(\bar{\omega}N)$ (the vertical line of Figure 3).

5 Concluding Remarks

In this paper we have analyzed how the distribution of land property rights may affect income and industrialization through the demand side. To this aim we have developed a

[20]When $M = \lambda$, the industrial employment $\hat{L}^*_{IT}$ is maximized and lies inbetween the maximum level of income and the maximum level of industrial extent.

[21]When $M = \eta$, we have $Q_R = \rho$. A higher M would imply that the number of entrepreneurs which receives the demand of landowners would not be sufficient to industrialize markets beyond Q_R.

modified version of the model of Murphy et al. (1989a). The novel feature of our model is that property of land and property of firms are independent and given on a functional basis.

We found that the general results about the distribution of land property *and* firm property provided in Murphy et al. (1989a) can be particularized to the distribution of land property alone. More precisely, we found that the degree of land ownership concentration is in a non-monotonic relation with both aggregate income and industrialization. However, our finding is due to partly different reasons with respect to Murphy et al. (1989a). On the one side, as in Murphy et al. (1989a) a too concentrated land distribution may determine a demand for manufactures which is too dispersed among the different commodity markets and, hence, may fail to make mass production profitable. On the other, differently from Murphy et al. (1989a) we found that a too diffused land distribution may be detrimental not only because it may fail to generate the necessary aggregate demand for manufactures but also because it creates a concentration of profits into the hand of few entrepreneurs which, in turn, produces a demand for manufactures which is so dispersed that is not sufficient to trigger industrialization in many markets. In other words, a too equal distribution of rents creates a too unequal distribution of income in favor of the owners of firms that, in turn, induces a distribution of income which is detrimental for the *same* reason that makes detrimental a too concentrated distribution of land.

The other important finding is that, against conventional wisdom, there may be a tradeoff between industrialization and income. In fact, in many cases both industrial extent and industrial employment does not coincide with the maximum aggregate income, the latter requiring a more intense exploitation of the economies of scale. More precisely, in general maximum aggregate income is associated with a distribution of land which is more diffused than that associated with either maximum industrial extent or employment. In particular, such a tradeoff arises and is substantial when fixed start-up costs are high because this creates he possibility to obtain large benefits by concentrating demand for manufactures in few markets.

Finally, a few remarks on the nature of our results are necessary. In our analysis there is no dynamics and all findings rest on a comparative statics exercise. Therefore, this study does not offer any reliable prediction about the impact of *changes* in land ownership distribution. Thus, although we recognize that land redistribution is a major source of reduction in land concentration, it has not been an explicit issue here. Indeed, a policy of land redistribution triggers a number of economic and social mechanisms which are not captured by our model and clearly require a dynamic analysis. In this sense, the present study can provide only weak policy suggestions. Nevertheless, the comparative statics that we carried out tells us something important. If a country is on the threshold of industrial takeoff we expect that, *ceteris paribus*, countries with a very concentrated land ownership perform worse than countries with a mild concentration. Going back to the example about South Korea and the Philippines mentioned in the introduction, we understand that the more equal distribution of land in South Korea has helped its industrial takeoff by providing domestic demand for basic manufactures since the very beginning of its development. In other words, we expect those countries that have *successfully* carried out a land reform to be in a better position for industrial takeoff with respect to those which have not. Finally, since our analysis abstracts from the effects of industrialization in the long run, our findings must be intended as restricted only to countries in their early phase of industrialization.

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A Appendix

Since proofs are quite simple but rather long and tedious, they do not deserve much attention. For this reason we provide here only the most complicated steps and the basic intuition behind them. All missing details are either trivial or can be found in Bilancini and D'Alessandro (2005).

A.1 Proof of Proposition 1

The demand for food is given by $D_f = (L_f + L_m + E + M)\,\bar{\omega} = \bar{\omega}N$ while the supply of food is $S_f = F(L_f)$. For what concerns the manufacturing sector we have to take into account how prices influence both aggregate demand and supply. The price of commodities produced with TT is αw as a consequence of competition among artisans. The price of commodities produced with IT is set by entrepreneurs in order to maximize profits. Since consumers buy manufactured goods following a well specified order and at most one of each kind, in any market the elasticity of demand with respect to the price is 0.[22] Hence, entrepreneurs find it worth rising prices as high as possible. However, the level αw constitutes an upper boundary because, for any price greater than that, nobody would buy commodities from them. Therefore, the price of each manufacturing commodity is αw regardless of how many markets industrialize and which is the technology applied.

Besides, since poorer people simply consume a bundle of commodities which is a subset of richer ones, it cannot happen that for two markets q' and q'', such that $q' < q''$, we have $D_{q'} < D_{q''}$. Therefore, the demand faced by each manufacturing market is non-increasing in q. Moreover, entrepreneurs face the same cost structure, so in each sector they find convenient to start their business for the same level of D_q. The last two observations imply that there is a separating market Q^* such that IT is introduced in any $0 \leq q \leq Q^*$ while in the remaining markets production is carried out by means of TT. Thus the aggregate demand of the manufacturing sector as a whole is

$$D_m = \frac{1}{\alpha w}\left[(R - \bar{\omega}M) + (L_f + L_m)(w - \bar{\omega}) + \int_0^{Q^*} (\pi(q, \tau, w) - \bar{\omega})\mathrm{d}q\right] \tag{7}$$

and aggregate supply is

$$S_m = \int_0^{\bar{Q}} S_q \mathrm{d}q \tag{8}$$

where $\bar{Q}$ denotes the extent of the manufacturing sector and S_q the supply of the market q. Finally, the demand for labour is $D_l = L_f + L_m$ while the supply is $S_l = L - Q^*$ where the number of entrepreneurs is $E = Q^*$.

[22] Notice that as the manufacturing sector is a continuum of markets, the consumer income is always entirely spent.

In equilibrium it must simultaneously hold that $D_f = S_f$, $D_m = S_m$ and $D_l = S_l$. Since the economy can sustain the whole population $N = (L + M)$, from $D_f = S_f$ we get the equilibrium value of employment in agriculture $L_f^* = F^{-1}\big(\bar{\omega}N\big)$ which is fully determined as $F(L_f)$ is invertible with respect to L_f and the parameters N and $\bar{\omega}$ are given. In particular the equilibrium levels of wage, employment and output in the agricultural sector are determined, unique and independent of the equilibrium of the manufacturing sector since the aggregate demand of food is exogenously given and equal to $\bar{\omega}N$. From $D_l = S_l$ and L_f^* we get the equilibrium value $(L - L_f^*)$ of people with a job in the manufacturing sector (workers, artisans or entrepreneurs). From L_f^* we obtain $w(L_f^*)$; then, M, $F(\cdot)$ and τ determine the extent of the manufacturing sector $\bar{Q}$ which is also unique. We are left with only two unknowns, namely Q^* and L_m^*. Exploiting equilibrium conditions, equation (7) can be written as

$$\begin{aligned} D_m &= \frac{1}{\alpha w}\left[R^* + (L_f^* + L_m)w + \int_0^{Q^*} \pi(q,\tau,w)\mathrm{d}q - (L_f^* + L_m + Q^* + M)\bar{\omega}\right] = \\ &= \frac{1}{\alpha w}\left[F(L_f^*) + L_m w + \int_0^{Q^*} \pi(q,\tau,w)\mathrm{d}q - (L + M)\bar{\omega}\right] = \\ &= \frac{1}{\alpha w}\left[L_m w + \int_0^{Q^*} \pi(q,\tau,w)\mathrm{d}q\right] \end{aligned} \tag{9}$$

where R^* is the equilibrium level of aggregate rents. Now, from $D_m = S_m$, equation (2), $D_q = S_q$ for each $q \in [0, \bar{Q}]$, and equations (8) and (9) we obtain

$$\begin{aligned} L_m^* &= \alpha \int_0^{\bar{Q}} S_q \mathrm{d}q - \int_0^{Q^*} \big((\alpha - \beta)D_q - k\big)\mathrm{d}q \\ &= \alpha \int_{Q^*}^{\bar{Q}} D_q \mathrm{d}q + \beta \int_0^{Q^*} D_q \mathrm{d}q + kQ^* \end{aligned} \tag{10}$$

The first term of equation (10) represents the labour employed in markets using the TT while the sum of the second and third terms represents the labour employed in the industrialized markets.

Since any entrepreneur in q starts her business depending on the value of D_q, the extent of industrialization Q^* is univocally determined by the continuum of demands in $[0, \bar{Q}]$. For any given continuum of demands in $[0, \bar{Q}]$, equation (10) identifies a unique value of L_m^*. In the following we show that the continuum of demands in $[0, \bar{Q}]$ is univocally determined by N, M, $F(\cdot)$ and τ, hence completing the proof. However, since calculations are quite long and moreover contains derivations of other economic magnitudes of interest that are used in the text, we have organized the remaining part of the proof in paragraphs which follows the proof of Lemma 1.

Proof of Lemma 1. We first consider the case where entrepreneurs and landowners have the same income. For any $M > \rho$, the Q_R entrepreneurs who receive the demand of landowners will demand manufactures in the interval $[0, Q_1]$ where $Q_1 = (M - \rho)(\alpha - \beta)/\alpha$. Assuming that $Q_1 \leq Q_R$, entrepreneurs in $[0, Q_1]$ face additional demand and their profits will be equal to $[(M + Q_R)(\alpha - \beta) - k]\bar{\omega}$. Equalizing the latter expression to the income of each landowner, R/M, we obtain

$$[(M + Q_R)(\alpha - \beta) - k]\bar{\omega} = \frac{R}{M} \quad \Longleftrightarrow \quad \alpha(\alpha - \beta)M^2 - [\alpha(k+1) - \beta]M - \beta\frac{R}{\bar{\omega}} = 0 \quad (11)$$

The solutions to equation (11) are

$$\frac{\alpha(k+1) - \beta \pm \sqrt{(\alpha(k+1) - \beta)^2 + 4\ \alpha\beta(\alpha - \beta)\frac{R}{\bar{\omega}}}}{2(\alpha - \beta)\alpha} \quad (12)$$

Notice that $(\alpha(k+1) - \beta)$ is positive and greater than one by assumptions about technology, so we have one strictly positive and one strictly negative solution. We name μ the positive one. Substituting μ in the expression for Q_1 we also get that $Q_1 = Q_R$ which concludes the proof of part i). Part ii) and part iii) follows, respectively, by noticing that for $\rho < M < \mu$ rents are greater than for $M = \mu$ while profits must be strictly lower than for $M = \mu$, and that for $\mu < M < N - F^{-1}(\bar{\omega})$ rents are decreasing in M while the profits of those receiving landowners' demand – the entrepreneurs in in $[0, Q_R]$ – must be increasing since the number of landowner is increasing.

The case of $M \leq \mu$. Define Q_2 the extent of demand of the group of entrepreneurs in $[0, Q_1]$. Then, $M \leq \mu$ implies $Q_2 \leq Q_R$. Since entrepreneurs in $[0, Q_1]$ face the highest demand, and hence earn the highest profits, no entrepreneur demands manufactures beyond Q_R. Moreover, in equilibrium entrepreneurs in $[0, Q_1]$ receive the demand of every entrepreneur, including themselves, so their total demand is $(Q_R + M)$. Hence, entrepreneurs in $(Q_1, Q_2]$ earn additional profits and demand manufactures until $Q_3 = (M + Q_2)(\alpha - \beta)/\alpha$. By iterating this mechanism, we can calculate the equilibrium demand and profits of each market.

The generic Q_{2i} (even index) and Q_{2i+1} (odd index) can be written, respectively, as

$$\begin{aligned} Q_{2i} &= (M - \rho)\sum_{j=1}^{i}\left(\frac{\alpha - \beta}{\alpha}\right)^j + Q_R\left(\frac{\alpha - \beta}{\alpha}\right)^i \\ &= (M - \rho)\sum_{j=0}^{i}\left(\frac{\alpha - \beta}{\alpha}\right)^j + Q_R\left(\frac{\alpha - \beta}{\alpha}\right)^i - M^k \qquad (13) \\ Q_{2i+1} &= (M - \rho)\sum_{j=1}^{i+1}\left(\frac{\alpha - \beta}{\alpha}\right)^j = (M - \rho)\sum_{j=0}^{i+1}\left(\frac{\alpha - \beta}{\alpha}\right)^j - (M - \rho) \qquad (14) \end{aligned}$$

where $Q_0 \equiv Q_R$. As i goes to ∞ expressions (13) and (14) converge to the same value from above and below, respectively

$$
\begin{aligned}
\lim_{i\to\infty} Q_{2i} &= \lim_{i\to\infty} (M-\rho)\sum_{j=0}^{i}\left(\frac{\alpha-\beta}{\alpha}\right)^j + Q_R\left(\frac{\alpha-\beta}{\alpha}\right)^i - (M-\rho) = (M-\rho)\left(\frac{\alpha-\beta}{\beta}\right) \\
\lim_{i\to\infty} Q_{2i+1} &= \lim_{i\to\infty} (M-\rho)\sum_{j=0}^{i+1}\left(\frac{\alpha-\beta}{\alpha}\right)^j - (M-\rho) = (M-\rho)\left(\frac{\alpha-\beta}{\beta}\right)
\end{aligned}
$$

Therefore, for any value of $M \leq \mu$ the equilibrium demand of each market is uniquely determined and the calculation of the equilibrium profits of each entrepreneur is straightforward. In the case $M = \mu$ we get $Q_R = Q_2 i$ for every $i \geq 1$.

Figure 4 and Figure 5 give a graphical representation of industrialization for $M < \mu$ and for $M = \mu$ respectively.

Industrial Extent, Industrial Employment and Aggregate Income when $M \leq \mu$. Since no entrepreneur demands beyond market Q_R the extent of industrialization is $Q^* = Q_R$. Moreover, industrial employment is given by $L_{IT} = N - L_f - L_{TT} - E - M$. In an equilibrium with positive industrialization, since $L^*_{TT} = 0$ and $E = Q^* = Q_R$ we have $L^*_{IT} = N - L^*_f - Q_R - M$. Taking into account equation (1) and the fact that $F(L^*_f) = \bar{\omega}N$ we obtain $N - L^*_f = \frac{R}{\bar{\omega}}$. From these it is straightforward to get equation (3).

Finally, aggregate income is $Y = R + \Pi + w(L_f + L_m)$. Since $(L_f + L_m) = N - E - M$, in equilibrium we obtain equation (4). The expression for Π^* is derived by adding the profits of each entrepreneur, calculated on the basis of the demand she faces in equilibrium. In order to calculate aggregate demand, we first derive the length of the generic intervals

$$
\begin{aligned}
Q_{2i} - Q_{2i+2} &= (M-\rho)\left(\frac{\alpha-\beta}{\alpha}\right)^i + Q_R\frac{\beta}{\alpha}\left(\frac{\alpha-\beta}{\alpha}\right)^i && (15) \\
Q_{2i+1} - Q_{2i-1} &= (M-\rho)\left(\frac{\alpha-\beta}{\alpha}\right)^{i+1} && (16)
\end{aligned}
$$

where $Q_{-1} \equiv 0$. Multiplying the length of each interval of markets by the demand exceeding ρ that they face, we get the aggregate demand which generates profits exceeding subsistence, which we denote with D^π

$$
D^\pi(M \leq \mu) = \sum_{i=0}^{\infty}\left[(Q_{2i+1} - Q_{2i-1})((M-\rho) + Q_{2i}) + (Q_{2i} - Q_{2i+2})((M-\rho) + Q_{2i-1})\right] \quad (17)
$$

Solving the series we get

$$D^{\pi}(M \leq \mu) = (M-\rho)Q_R\left(\frac{\alpha(\alpha-\beta)}{\beta(2\alpha-\beta)} + \frac{\alpha^2}{\beta(2\alpha-\beta)}\right) = (M-\rho)Q_R\frac{\alpha}{\beta}$$

Since profits are equal to the units of manufactures demanded beyond those needed to introduce IT, i.e. $(M-\rho)Q_R$, times the profit earned for each unit sold, i.e. $(\alpha-\beta)\bar{\omega}$, times α/β which accounts for the multiplicative process we have described plus the subsistence of the entrepreneurs $\bar{\omega}Q_R$ we get

$$\Pi^* = (\alpha-\beta)\frac{\alpha}{\beta}(M-\rho)\,\bar{w}Q_R + \bar{w}Q_R$$

The case of $M > \mu$. For $M > \mu$ we have $Q_2 > Q_R$. As before, markets in $[0, Q_R]$ industrialize. In order to determine whether or not other markets introduce the IT, we must compare the demand they face with the threshold value ρ.

The sequence of Q_{2i+1} is formally as in 14 but stops beyond Q_R. The sequence of Q_{2i} is constant and equal to

$$Q_{2i} = Q_2 = (M-\rho)\left(\frac{\alpha-\beta}{\alpha}\right) + Q_R\left(\frac{\alpha-\beta}{\alpha}\right)$$

and it stops as soon as Q_{2i+1} stops. So far, each market in $[0, Q_R]$ faces the same demand $(Q_R + M)$ and each entrepreneur in $[0, Q_R]$ earns the same profits. In order to take into account the multiplicative process triggered by industrialization beyond Q_R, let us simplify notation and preserve the intuition about the sequence of Qs. Both Q_1 and Q_0 are set equal to Q_R, and Q_2 denotes the extent of demand of the richest entrepreneurs, no matter where the "Q_1" defined for the previous case falls. So, $(Q_R, Q_2]$ is the first interval to receive only entrepreneurs demand, which industrializes if and only if $Q_R > \rho$. If this happens, we call Q_3 the extent of demand of entrepreneurs in $(Q_R, Q_2]$. Similarly, Q_4 indicate the new extent of demand of entrepreneurs in $[0, Q_3]$, and so on. Therefore, the generic Q_{2i} and Q_{2i+1}

$$Q_{2i} = (M-\rho)\sum_{j=1}^{i}\left(\frac{\alpha-\beta}{\alpha}\right)^j + Q_R\left(\frac{\alpha-\beta}{\alpha}\right)^i \tag{18}$$

$$Q_{2i+1} = Q_R\left(\frac{\alpha-\beta}{\alpha}\right)^i - \rho\sum_{j=1}^{i}\left(\frac{\alpha-\beta}{\alpha}\right)^j \tag{19}$$

As previously mentioned, markets in $(Q_R, Q_2]$ industrialize if and only if $Q_R \geq \rho$. If the latter holds with strict inequality, then new demand is generated and entrepreneurs in $[0, Q_3]$ earn new profits and extend their demand of manufactures beyond Q_2 until Q_4. In this case markets in $(Q_2, Q_4]$ industrialize if and only if $Q_3 \geq \rho$ which holds if and only if $Q_R \geq \rho[1 + \alpha(\alpha-\beta)]$.

By iteration, we get that the number of steps – that is the number of new industrialized intervals of markets – is given by the minimum value of i, denoted by i^*, such that

$$Q_R < \rho \sum_{j=0}^{i} \left(\frac{\alpha}{\alpha - \beta} \right)^j \tag{20}$$

Since $\alpha/(\alpha - \beta)$ is greater than 1 there exists a finite value of i such that inequality (20) is satisfied. Moreover, i^* is a non-increasing step function afterwards.

Industrial Extent, Industrial Employment and Aggregate Income when $M > \mu$. Given i^*, the extent of industrialization is

$$Q^* = Q_{2i^*} = (M - \rho) \sum_{j=1}^{i^*} \left(\frac{\alpha - \beta}{\alpha} \right)^j + Q_R \left(\frac{\alpha - \beta}{\alpha} \right)^{i^*} \tag{21}$$

Similarly the extent of the manufacturing sector is

$$\bar{Q} = Q_{2(i^*+1)} = (M - \rho) \sum_{j=1}^{i^*+1} \left(\frac{\alpha - \beta}{\alpha} \right)^j + Q_R \left(\frac{\alpha - \beta}{\alpha} \right)^{i^*+1} \tag{22}$$

and the last group of markets which receive only the demand of the richest entrepreneurs employs the TT. However, if the last group of entrepreneurs who industrialize faces a demand equal to ρ, then the extent of the manufacturing sector coincides with the extent of industrialization and traditional production disappears.

The equilibrium level of L_{TT}^* is equal to the number of units produced with TT multiplied by the labour coefficient α, that is

$$\begin{aligned} L_{TT}^* &= \alpha(\bar{Q} - Q^*) Q_{2i^*+1} = \\ &= \left(\frac{\alpha - \beta}{\alpha} \right)^{i^*} [(M - \rho)(\alpha - \beta) - \beta Q_R] \left[Q_R \left(\frac{\alpha - \beta}{\alpha} \right)^{i^*} - \rho \sum_{j=1}^{i^*} \left(\frac{\alpha - \beta}{\alpha} \right)^j \right] \end{aligned} \tag{23}$$

where $(\bar{Q} - Q^*) Q_{2i^*+1}$ is the number of traditional manufactures demanded. By plugging equation (23) into (6) we get L_{IT}^*.

For any i^*, aggregate profits are obtained by summing the profits earned by each entrepreneur. If there are no steps then $D_0^\pi = Q_R(M + Q_R - \rho)$. With one step we have $D_1^\pi = D_0^\pi + (Q_2 - Q_R)[Q_R(1 + x) - \rho(1 + x)]$ where, $x = (\alpha - \beta)/\alpha$. By iteration, for the generic i^* we get

$$D_{i^*}^\pi = Q_R(M + Q_R - \rho) + (Q_2 - Q_R) \left(Q_R \sum_{j=0}^{2i^*-1} x^j - \rho \sum_{j=0}^{i^*-1} x^j \sum_{j=0}^{i^*} x^j \right) \tag{24}$$

Therefore, we have

$$\Pi^* = (\alpha - \beta)\left[Q_R(M + Q_R - \rho) + (Q_2 - Q_R)\left(Q_R \sum_{j=0}^{2i^*-1} x^j - \rho \sum_{j=0}^{i^*-1} x^j \sum_{j=0}^{i^*} x^j\right)\right]\bar{\omega} + \bar{\omega}Q^* \tag{25}$$

A.2 Proof of Proposition 2

As we have already pointed out in the paper, it is easily seen that the maximum extent of industrialization is obtained for $M = \rho$. In such a case we get

$$\widehat{Q}^* = \frac{R - \bar{\omega}\rho}{\alpha\bar{\omega}\rho} = \frac{1}{\alpha}\left(\frac{R}{\bar{\omega}}\frac{\alpha - \beta}{k+1} - 1\right) \tag{26}$$

Turning our attention to maximum income, we notice that we have two different functions for aggregate income depending on whether M is greater than μ or not. Taking into account equations (4) and (18), for $M < \mu$ we have

$$\begin{aligned} Y^* &= R + \bar{\omega}N + (\alpha - \beta)(M - \rho)\frac{R - \bar{\omega}M}{\beta M} - \bar{\omega}M \\ &= \frac{\alpha}{\beta}R + \bar{\omega}N - \frac{\alpha}{\beta}\bar{\omega}M - \frac{k+1}{\beta}\frac{R}{M} + \frac{k+1}{\beta}\bar{\omega} \end{aligned} \tag{27}$$

In order to maximize (27) with respect to M we calculate the first order condition.

$$\frac{\partial Y^*}{\partial M} = \frac{k+1}{\beta}\frac{R}{M^2} - \frac{\alpha}{\beta}\bar{\omega} = 0 \quad \Longleftrightarrow \quad M^2 = \frac{k+1}{\alpha}\frac{R}{\bar{\omega}} \tag{28}$$

which has the unique positive solution

$$M = \sqrt{\frac{k+1}{\alpha}\frac{R}{\bar{\omega}}} \tag{29}$$

If $\mu > \sqrt{(k+1)R/\alpha\bar{\omega}}$ then from equations (27) and (29) we get that maximum income $\widehat{Y}^*$ is equal to

$$\begin{aligned} \widehat{Y}^* &= \frac{\alpha}{\beta}R + \bar{\omega}N + \frac{k+1}{\beta}\bar{\omega} - 2\frac{\sqrt{\alpha\bar{\omega}(k+1)R}}{\beta} \\ &= \bar{\omega}N + \left(\sqrt{\frac{\alpha}{\beta}R} - \sqrt{\frac{k+1}{\beta}\bar{\omega}}\right)^2 \end{aligned} \tag{30}$$

If instead $\mu \leq \sqrt{(k+1)R/\alpha\bar{\omega}}$ then the maximum income is reached for some $M \geq \mu$. In this case, the value of M which maximizes aggregate income must be calculated numerically since the function changes depending on the number of steps.

As anticipated in the paper, maximum industrial employment $\widehat{L}^*_{IT}$ is obtained for M in $[\rho, \mu]$. *Ab absurdo* consider that there is a distribution of land ownership such that $\widehat{L}^*_{IT}$ is obtained for $M > \mu$. A necessary condition for M to maximize L^*_{IT} when $M > \mu$ is that $(M + Q^*)$ does not increase in M, since i) $L^*_{IT} = N - L^*_f - L^*_{TT} - (M + Q^*)$, ii) $L^*_{TT} = 0$ for $M \leq \mu$ and iii) $L^*_{TT} \geq 0$ for $M > \mu$. For $M = \mu$ industrial labour amounts to the units of labour necessary to produce the demanded units of industrial goods, hence

$$L^*_{IT}(\mu) = \beta Q^*_\mu (Q^*_\mu + \mu) + kQ^*_\mu \tag{31}$$

where $Q^*_\mu (Q^*_\mu + \mu)$ is aggregate demand of manufactures. When $M > \mu$ the richest group of entrepreneurs receives demand by $(M + Q^*_M)$ individuals, but other industrialized markets will receive less. However we overestimate the aggregate industrial demand and, as before, we write industrial labour as

$$L^*_{IT}(M) = \beta Q^*_M (Q^*_M + M) + kQ^*_M \tag{32}$$

We can say with certainty that (31) is greater then (32) since as M increases $(M + Q^*)$ does not increase and Q^* decreases. Therefore, $Q^*_\mu > Q^*_M$ for $M > \mu$ implying that it is impossible that $\widehat{L}^*_{IT}$ is obtained for $M > \mu$.

In the interval $[\rho, \mu]$ there is no market operating with the TT, therefore industrial employment is given by equation (3). In order to maximize (3) with respect to M we derive the first order condition

$$\frac{\partial L^*_{IT}}{\partial M} = 0 \Longleftrightarrow M = \sqrt{\frac{R}{\alpha\bar{\omega}}}$$

If $\sqrt{R/(\alpha\bar{\omega})} \leq \rho$, then maximal industrial employment is obtained for $M = \rho$ also maximizing the extent of industrialization. If $\sqrt{R/(\alpha\bar{\omega})} \geq \mu$, then $\widehat{L}^*_{IT}$ is obtained for $M = \mu$ which may also maximize aggregate income. Finally if $\rho < \sqrt{R/(\alpha\bar{\omega})} < \mu$ the maximum industrial employment is obtained for a level of M strictly lower than that which maximizes aggregate income, that is $\sqrt{(k+1)R/\alpha\bar{\omega}}$. In this last case

$$\widehat{L}^*_{IT} = \left(\sqrt{\frac{R}{\bar{\omega}}} - \sqrt{\frac{1}{\alpha}}\right)^2 \tag{33}$$

B List of Figures

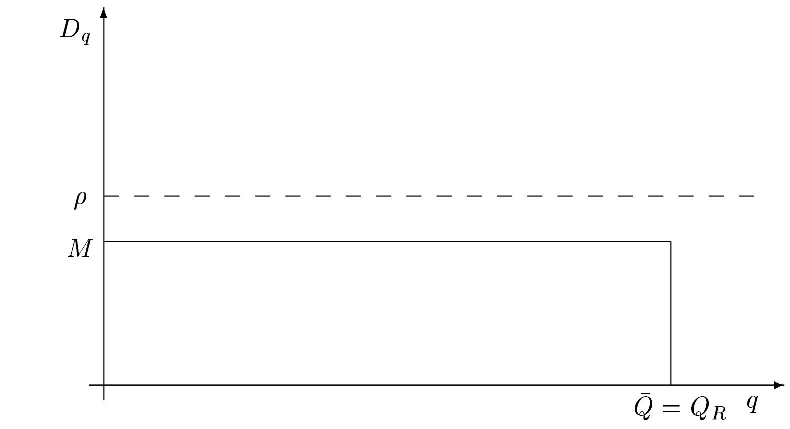

Figure 1. Traditional economy

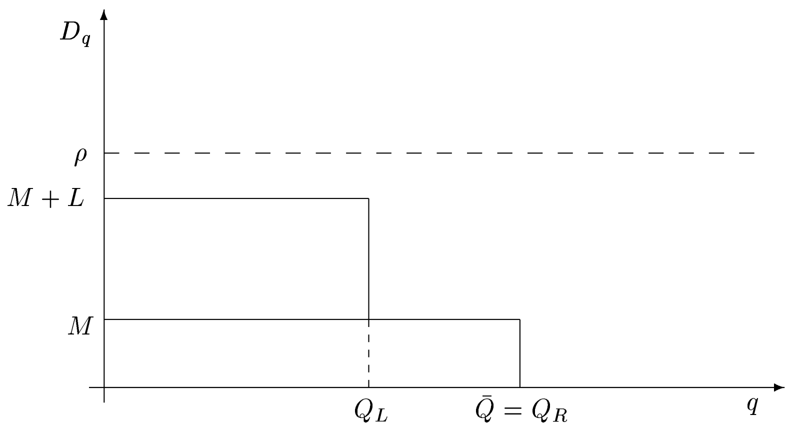

Figure 2. Small economy ($Q_R > Q_L$)

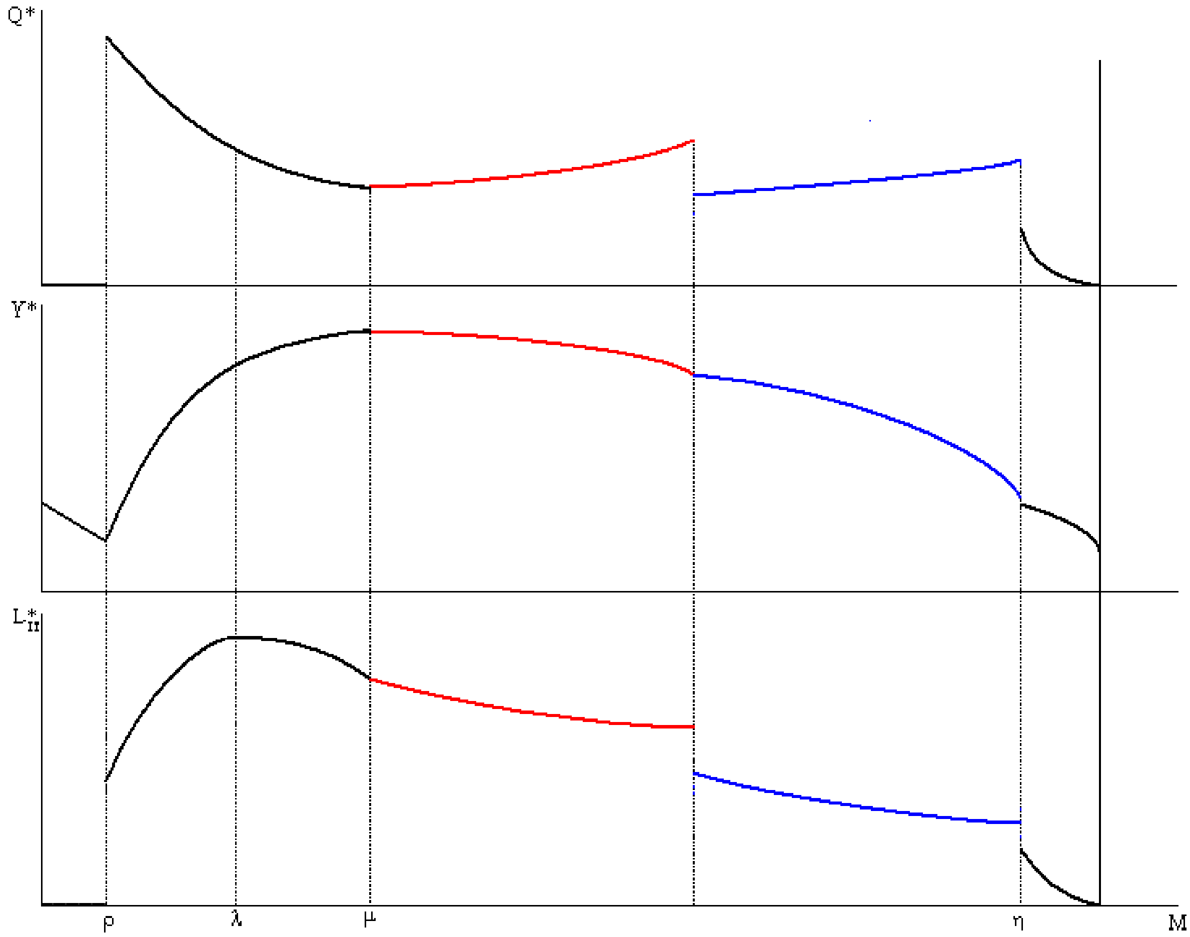

Figure 3. An example with a maximum i^* of 2.

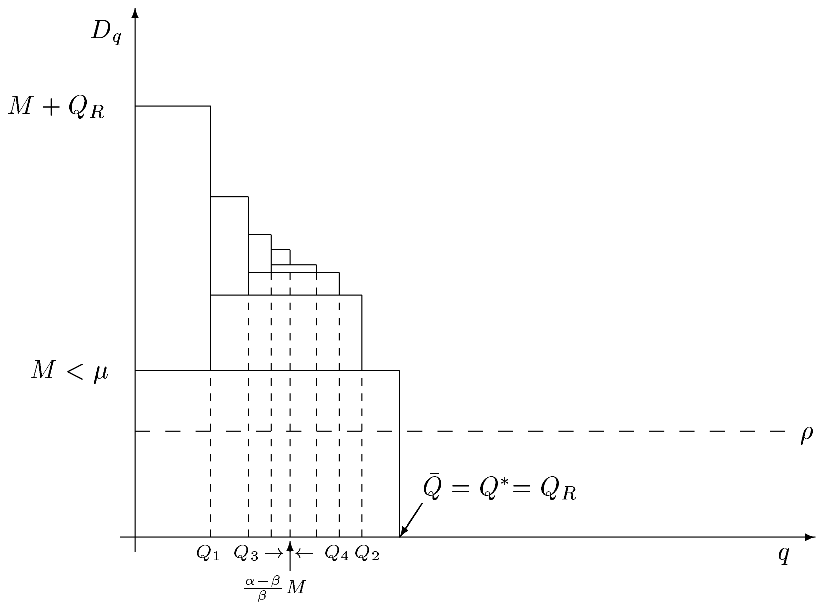

Figure 4. $M < \mu$

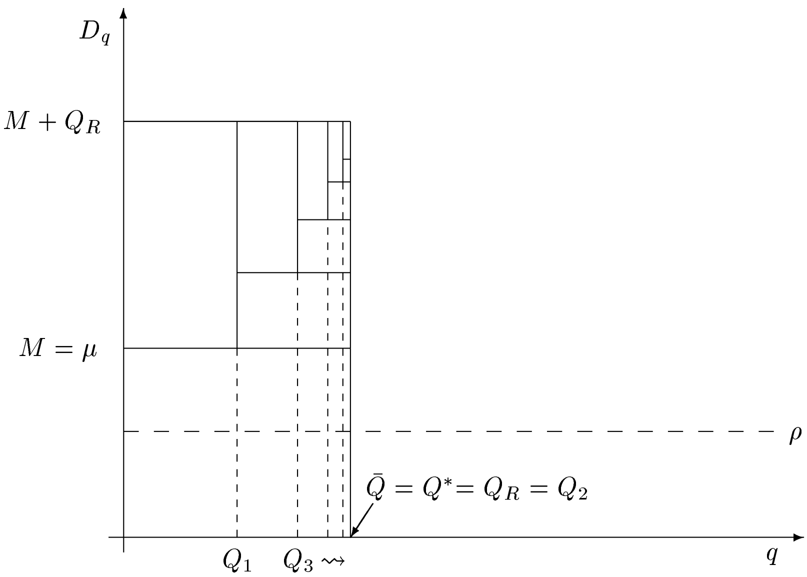

Figure 5. $M = \mu$