<u>Quantum Error Correction</u>

The only way to detect quantum errors is by making **measurements**, but measurement gates disruptively alter the states of measured qubits which makes things worse.

This is not a major issue in classical computers where the physical system that embodies the individual bits, the classical bits are immense in the atomic scale. The two states of a classical bit are 0 and 1. These bits are different, whereby the probability is infinitesimal for flipping from one to the other as a result of thermal fluctuations, mechanical vibrations or other extraneous interactions. Although classical systems also experience errors.

Error correction is very important in the transmission of information over large distances, because the further the signal travels, the more it attenuates. There are various ways in which errors in a quantum computer are quite different.

a) Physical qubits are individual atomic-scale systems in the form of atoms, photons, trapped ions or nuclear magnetic moments. Any coupling Any coupling to anything not under the explicit control of the computer and its program can substantially disrupt the state associated with those qubits, entangling them with computationally irrelevant features of the computer or the world outside the computer, thereby destroying the computation. For a quantum computer to work without error correction, each qubit would have to be impossibly well isolated from irrelevant interactions with other parts of the computer and anything else in its environment.
b) Measuring a qubit alters its state and more generally, destroys it's quantum correlations with other qubits with which might be entangled. Such disruptions are stochastic and introduce errors of their own.
c) Phase errors such as altering of |0> + |1> to |0> - |1> can be damaging.
d) Unlike the discrete all-or-nothing bit-flip errors suffered by classical bits, errors in the state of qubits grow continuously out of their uncorrupted state.

The simplest example of an error correction code is the three-bit repetition code, the encoder for which duplicates each bit value $0 \rightarrow 000$ and $1 \rightarrow 111$. More formally, we can define the three-bit encoder as a mapping from a 'raw' binary alphabet ***B*** to a code alphabet ***C3***

$$\boldsymbol{B} = \{0, 1\} \longrightarrow \boldsymbol{C_3} = \{000, 111\},$$

where the encoded bit-strings '000' and '111' are referred to as the logical codewords of the code C3. As an example, consider the simple case where we wish to communicate a single-bit message '0' to a recipient in a different location. Using the three bit encoding, the message that we would send would be the '000' codeword. Now, imagine that the message is subject to a single bit-flip error during transmission so that the bit-string the recipient receives is '010'. In this scenario, the recipient will be able to infer that the intended codeword is '000' via a majority vote. The same will be true for all cases where the codeword is subject to only a single error. However, if the codeword is subject to two bit-flip errors,

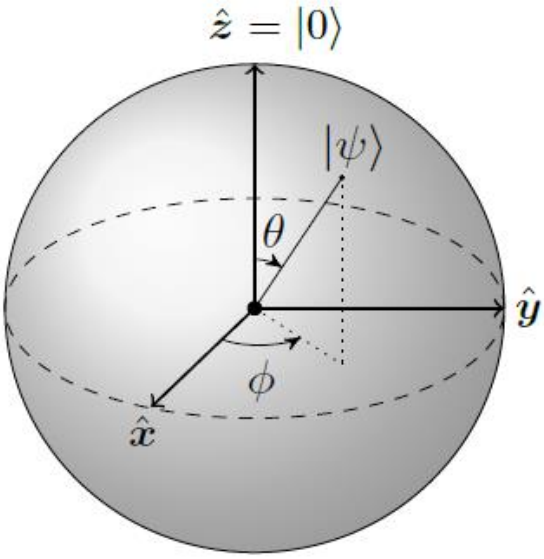


Figure 1. In the geometric representation, the state of a qubit $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$ can be represented as a point on the surface of a Bloch sphere.|

the majority vote will lead to the incorrect codeword. The final scenario to consider is when allvthree bits are flipped so that the codeword '000' becomes '111'. In this case, the corrupted message is also a codeword: the recipient will therefore have no way of knowing an error has occurred. The **distance** of a code is defined as the minimum number of errors that will change one codeword to another in this way. We can relate the distance ***d*** of a code to the number of errors it can correct as follows

$$d = 2t + 1$$

where ***t*** is the number of errors the code can correct. It is clear that the above equation is satisfied for the three-bit code where ***t*** = 1 and ***d*** = 3.

In general, error correction codes are described in terms of the [**n, k, d**] notation, where n is the total number of bits per codeword, k is the number of encoded bits (the length of the original bit-string) and d is the code distance. Under this notation, the three-bit repetition code is labelled [3, 1, 3].

In place of bits in classical systems, the fundamental unit of quantum information is the qubit. The general qubit state can be written as follows

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle\,,$$

Where α and β are complex numbers that satisfy the condition $|\alpha|^2 + |\beta|^2 = 1$. Qubits can encode information in a superposition of their basis states, meaning quantum computers have access to a computational space that scales as 2^n where n is the total number of qubits. It is by exploiting superposition, in combination with other quantum effects such as entanglement, that it is possible to construct algorithms that provide a quantum advantage. However, if such algorithms are ever to be realised on current or future quantum hardware, it will be necessary for the qubits to be error corrected.

<u>Types of Quantum Errors</u>

1. **Pauli X-type Errors**

This can be thought of as quantum bit flips that map X|0> = |1> and X|1> = |0>. The action of an X-error on the general qubit state is:

$$X|\psi\rangle = \alpha X|0\rangle + \beta X|1\rangle = \alpha|1\rangle + \beta|0\rangle$$

2. **Z-error**

Also referred to as **Phase flip**. It has no classical analogue. They map the qubit basis states Z |0> = |0> and Z |1> = |-1>, therefore have the following action on the general qubit:

$$Z\,|\psi\rangle = \alpha Z|0\rangle + \beta Z|1\rangle = \alpha|0\rangle - \beta|1\rangle$$

The digitization of the error result generalizes to arbitrary quantum error processes, including those that describe incoherent evolution of the quantum state as a result of the qubits' interaction with their environment.

For the single-error case, X-errors and Z-errors can be related to rotations on the Bloch sphere. An X-error corresponds to a π-angle rotation about the x-axis of the Bloch sphere, whilst a Z-error corresponds to a π-angle rotation about the z-axis.

Barriers to Quantum Error Correction

1. Measurement of error destroys superpositions.
2. No-cloning theorem prevents repetition.
3. Must correct multiple types of errors (e.g., bit flip and phase errors).

The three bit flip code

To protect quantum states against the effects of noise we would like to develop quantum error-correcting codes based upon similar principles. There are some important differences between classical information and quantum information that require new ideas to be introduced to make such quantum error-correcting codes possible. In particular, at a first glance we have three rather formidable difficulties to deal with:

- **No cloning**

  One might try to implement the repetition code quantum mechanically by duplicating the quantum state three or more times. This is forbidden by the no-cloning theorem. Even if cloning were possible, it would not be possible to measure and compare the three quantum states output from the channel. There is no device that will copy an unknown quantum state:

  $$|0\rangle \longrightarrow |0\rangle\,|0\rangle,\ |1\rangle \longrightarrow |1\rangle\,|1\rangle$$

  By linearity,

  $$\alpha\,|0\rangle + \beta\,|1\rangle \longrightarrow \alpha\,|0\rangle\,|0\rangle + \beta\,|1\rangle\,|1\rangle$$

  $$\neq (\alpha\,|0\rangle + \beta\,|1\rangle)\,(\alpha\,|0\rangle + \beta\,|1\rangle)$$

- **Errors are continuous**

  A continuum of different errors may occur on a single qubit. Determining which error occurred in order to correct it would appear to require infinite precision, and therefore infinite resources.

- **Measurement destroys quantum information**

In classical error-correction we observe the output from the channel, and decide what decoding procedure to adopt. Observation in quantum mechanics generally destroys the quantum state under observation, and makes recovery impossible. Let us apply the classical repetition code to a quantum state to try to correct a single bit flip error:

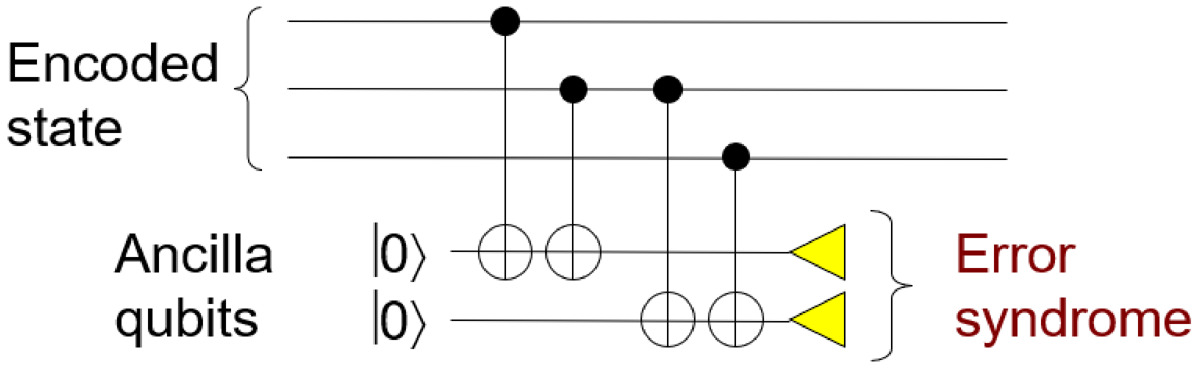
$$\alpha |0\rangle + \beta |1\rangle \rightarrow \alpha |000\rangle + \beta |111\rangle$$

Bit flip error (X) on 2nd qubit:

$$\alpha |010\rangle + \beta |101\rangle$$

The 2nd qubit is now different from 1st and 3rd. We wish to measure that it is different without finding its actual value.

Measure the Error, Not the Data. Using this circuit:



1st bit of error syndrome says whether the first two bits of the state are the same or different.

2nd bit of error syndrome says whether the second two bits of the state are the same or different.

With the information from the error syndrome, we can determine whether there is an error and where it is like: $\alpha |010\rangle + \beta |101\rangle$ which has syndrome 11. This means the second bit is different. Correct it with a X operation on the second qubit. Note that the syndrome does not depend on α and β.

<u>**Correcting Phase Errors**</u>

Hadamard transform H exchanges bit flip and phase errors:

$$H (\alpha |0\rangle + \beta |1\rangle) = \alpha |+\rangle + \beta |-\rangle$$

$$X |+\rangle = |+\rangle, X |-\rangle = - |-\rangle \text{ (acts like phase flip)}$$

$$Z |+\rangle = |-\rangle, Z |-\rangle = |+\rangle \text{ (acts like bit flip)}$$

Repetition code corrects a bit flip error, repetition code in Hadamard basis corrects a phase error.

$$\alpha |+\rangle + \beta |-\rangle \rightarrow \alpha |+++\rangle + \beta |---\rangle$$

Further Reading

References

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