

SAR: Stroke Authorship Recognition

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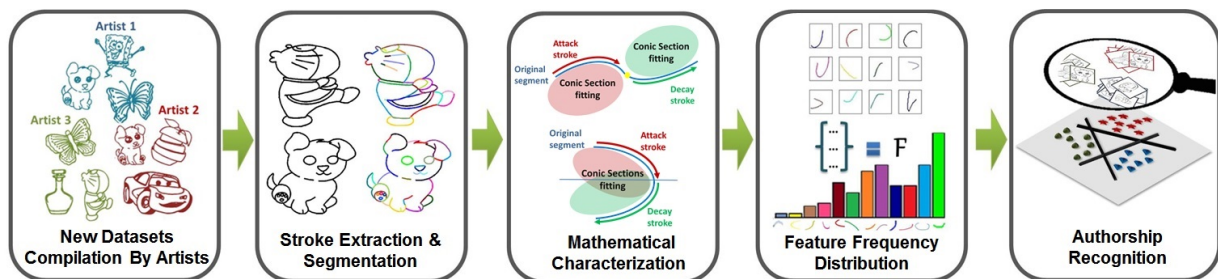


Figure 1: The SAR pipeline: 1. compile sketches from different artists; 2. extract strokes and split them into segments; 3. characterize each stroke segment mathematically 4. represent each sketch as a distribution of feature frequency; 5. Train a machine to recognize authorship based on 4.

Abstract

Are simple strokes unique to the artist or designer who renders them? If so, can this idea be used to identify authorship or to classify the drawings? Also, could training methods be devised to develop particular styles? To answer these questions, we present the Stroke Authorship Recognition (SAR) approach, a novel approach to distinguish 2D digitized drawings. The SAR method converts a drawing into a histogram of stroke attributes that differentiate it from drawings of other artists. We describe extensive classifications on a large variety of datasets, which validate SAR's ability to distinguish unique authorship of artists and designers. Derivative applications also demonstrated include: the detecting fraudulent sketches, a training artists to learn a particular style, monitoring artist's training progress, and the first quantitative way to measure the quality of automatic sketch synthesis tools.

1. Introduction

Research in voice and face recognition have well-developed methods to recognize and distinguish individuals based on data cues [TEBEH06]. Similarly, we investigate whether individuals (specifically artists) are distinguishable by the way they draw, and if so, the extent to which this uniqueness can identify their drawings, to help train them, or to detect sketch fraud. To our knowledge, this work is the first on authorship recognition from drawings. Our new method, called sketch authorship recognition, or SAR assumes that an artist's style can be recognized by the frequency distribution of certain mathematical descriptors, which are deduced from basic sketching strokes. A fundamental assumption is that the artist style manifests itself mathematically in the artist's basic strokes. For example, we have seen that Disney's Mickey Mouse tends to exhibit rounded strokes using many, nearly elliptic curves, whereas Looney Tunes Daffy Duck consists of a combination of straighter and tightly curved strokes. By

extracting enough strokes from a digitized drawing, a characteristic histogram is created. This histogram captures an inherent style of the overall drawing. As illustrated in Figure 1, SAR consists of five steps: (1) obtain sketches drawn, (2), extract strokes from each sketch, (3) split strokes into simple segments, (4) collect pertinent mathematical attributes of strokes into a frequency distribution, and (5) use (4) to train a machine to recognize the author.

SAR is useful in a variety of applications such as fraud detection (i.e. fraudulent versus original sketches), where there is a need to examine fine-grained stroke-level features to discriminate sketches that *look* similar. This problem cannot be approached using existing shape matching techniques, which are unable to detect local and fine differences. Also, these methods are heavily dependent on the content of the sketch and not its style. Moreover, SAR can play a major role in artistic style training and reproduction, since it can be used to quantitatively assess how the sketching style

of an artist in-training is becoming similar to a desired target style throughout the training process. For example, newly recruited artists at Disney are required to undergo a six month training procedure to familiarize themselves with the company's sketch techniques. This is done to ensure that new and already existing characters can be created with the same Disney *look*. Clearly, there is a need for an automated technique to examine how an artist progresses during his/her sketch style training. Other areas that can make use of SAR include handwriting verification, design patent litigation and brand marking.

Contributions: In this work, (1) we propose a novel method of sketch authorship recognition (SAR) based on the hypothesis that the collection of an artist's strokes in a sketch are unique to that artist and they can be used to define his/her style which can be used to detect sketch fraud. (2) We compile 2 new sketch datasets collected from a number of experienced artists. These datasets are designed to expose SAR to different levels of challenges and sketch variations. They will be made publicly available, along with SAR source code, to allow for further research on this topic. (3) We develop two SAR-enabled sketch applications. The first provides artists in-training with immediate feedback on how close their sketching style is to a particular target style and monitors their progress throughout the training period. The other provides the first quantitative and automatic measure to evaluate the quality of automatic sketch synthesis tools.

2. Related Work

In this section, we survey previous work that is most related to our approach, grouped into four main categories.

Shape Matching. Determining stroke authorship seems akin to shape matching, but has different requirements as further inspection is needed. Numerous methods have been developed for shape matching and classification in the past [MM92, BMP01, JHY03, BBM05], as well as, recently [MOA11, IAP*11]. Shape matching searches for similar shapes between two images, where one is usually considered the query image. Our system differs from shape matching in two main aspects. First, shape matching focuses on global information of contours such as zero crossings of curvature, while SAR segments the contour and studies detailed local features from each curve segment e.g. the eccentricity of a conic fit. Second, shape matching is usually applied to low resolution images for computational reasons, where these images tend to be classified by their content, e.g., a set of different mice silhouettes/shapes from different artists tend to belong to the same object class. Our SAR technique can be employed efficiently on images of various sizes, especially those at high resolution. Unlike shape matching, SAR is less dependent on sketch content and more focused on sketch style.

Sketch and Artistic Style Analysis. [BSM*13] provides a data driven approach to analyze style in portrait sketches. This technique is also used for portrait sketch synthesis.

While their focus is to mimic and synthesize a particular sketching style of only human portraits sketches, our focus is to discriminate sketches based on their authorship (i.e. artistic style). In their study, they analyze sketches at the level of strokes and shapes. However, they handle stroke analysis differently, since they focus on global features. SAR on the other hand exploits local stroke features that are necessary in discriminating authorship of sketches that look similar as is the case in sketch fraud detection. Moreover, [BSM*13] digitally collect portrait sketches using the Wacom pen to build a library of strokes while we allow artists to express their style freely and to erase or redraw their sketches using any medium they prefer. Interestingly, we use the sketch synthesis dataset of [BSM*13] to demonstrate how SAR can quantitatively evaluate sketch synthesis. SAR results are on par with those reported by the authors after an online study with human participants.

[LFTC13] collected 13,000 drawings of faces. Unlike our work, they do not study the problem of authorship as they focus on auto-correction of strokes for novice artists. [LYFD12] mimicked a particular artistic style through matching that was based on filtered velocities and shape context. Concurrently, [KNBH12] synthesized new drawings using hatching styles of sketching that are learned by example, while [FTP03] provided an example-based method to modify line drawings for the purpose of reproducing different artistic styles. Also, [CGL*08] studied where artists draw lines in sketches of objects such as tools, automobile parts and bones. They concluded that artists tend to draw similar lines in consistent locations. In our experiments, we use the dataset of this work to show interesting new results regarding the uniqueness of sketch style despite strict sketching constraints imposed on the artists. Finally, a large body of work focuses on generating artistically stylized rendering using 2D input images or videos of non-photorealistic rendering (NPR). We refer the reader to the survey of [KCWI13] for details. Although this work targets the analysis of sketches and artistic styles of different sketches, it does not address the important problem of how authorship can be determined based on stroke cues manifesting themselves in sketches.

Sketch Recognition and Retrieval. [EHA12] developed an automated data-driven method to explore a large collection of hand-drawn sketches using drawings collected by many non-experts. Their primary goal was to represent sketch content to perform object recognition from a sketch and *not* sketch style. Concurrently, [SWZZ12] proposed a system that provides real-time recognition and retrieval of semantically meaningful attributes of hand-drawn sketches. Their work is not limited to pre-defined object classes. Other sketch retrieval methods are based only on geometric similarity between sketches [SMGE11, EHBA11]. Unlike our work, the field of sketch recognition and retrieval is based on classifying sketch content among a discrete number of semantic categories. They do not provide comparisons among

similar sketches or across artistic styles and do not address the problem of authorship recognition.

Forensic Handwriting Analysis. Forensic handwriting analysis is a well studied problem [SL03] as is signature analysis, and more precisely, off-line feature based signature verification [IP08, KC13]. Handwriting analysis tools tend to be very specific to the problem domain, so they use features centric to handwriting such as letter height, pitch, baselines, crossings, etc. They do not generalize well to sketch analysis [SS04]. Their focus is on the uniqueness of letter and punctuation formation, flow and structure [FK01], whereas we investigate the uniqueness of hand-drawn strokes in a much more general context.

Graphical Based User Authentication. A number of graphical based user authentication methods such as doodles sketches were proposed as an alternative to conventional authentication methods. As an example, PassDoodle is a graphical based authentication mechanism, which attempts to identify users by their handwritten designs (doodles). For authentication, the query doodle is represented on a regular grid and matched to training doodles in order to determine user authenticity [VKR04, GM07]. The effectiveness of using PassDoodles for user authentication is demonstrated here [Ren09]. [OKX*08], on the other hand, provided a sketch based authentication method by extracting edge orientation pattern features from a user sketch input and then using a feature similarity measure to find the closest sketch in a training database. Unlike SAR, these authentication methods neither build a classification model nor provide an analysis of different artistic styles. Moreover, they do not build an intermediate sketch representation of the user input which makes them similar to shape matching techniques. [Gan10] is an exposition of the shortcomings of graphical based authentication methods.

3. Stroke Authorship Recognition

In this section, we give a detailed description of our proposed SAR approach. The overall pipeline of SAR is depicted in Figure 1. Given a sketch image, the first stage in SAR is to extract major stroke contours in the image and segment these strokes into stroke segments. These strokes occur both at the silhouette and the interior of the sketch. Stroke segments from many sketches across multiple artists are grouped (using low-level image features) to form a universal dictionary of stroke segments. This dictionary is employed in a hierarchical bag-of-words model, which is used to represent a sketch image \mathbf{I} as a histogram of stroke segments. We claim that this stroke histogram encodes some of the characteristics of an artist's unique style and thus can be used to discriminate this artist's sketches from others, no matter what the sketch is about. Discrimination is performed in a supervised manner using multi-class classification, where each class designates an artist. In what follows, we elaborate on the individual stages of SAR.

3.1. Stroke Extraction and Segmentation

A sketch image \mathbf{I} is decomposed into strokes which are further split into stroke segments that can expose authorship. We believe that representing sketches at such a local scale can identify stroke segments that play an essential role in authorship discrimination.

Stroke Extraction. We use Adobe Live Trace, an off-the-shelf digitization technique, to decompose a sketch into a set of paths, each of which comprises a number of Bezier curves depicting the artistic strokes [ADO10]. There exists a number of other commercial and non-commercial digitization techniques, such as the recent work by Noris et al. [NHS*13]. However, we choose to use Adobe Live Trace in SAR, since it stays faithful to the original sketch and it is widely accessible and easy to use. With Adobe Live trace, we can apply different levels of digitization to the original sketch and evaluate the effect of this digitization on sketch style as we discuss later in Section 6.2. Extracted strokes are then traced and sampled as pixels in the image.

Stroke Segmentation. Strokes extracted from \mathbf{I} can be quite long and contain a rich amount of geometric information. Modeling such a stroke as a whole entity is quite a difficult task in itself, since it should encode all possible variations that a particular stroke can take on. Instead, we resort to breaking each stroke into smaller units (called segments) that are represented in a more straightforward and conventional manner. We fit a b-spline curve to each stroke and identify break points as locations of local maximum curvature in the b-spline. We explicitly handle linear strokes by placing break points at its two ends to avoid over-segmentation. The pixels between two consecutive breakpoints (or the beginning/end of the stroke) are grouped together and denoted as a stroke segment. Figure 2 shows the stroke segments extracted from two sketches taken from one of our sketch datasets.

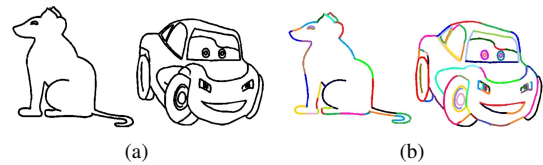


Figure 2: Stroke segmentation. (a) shows two sketches after digitization and (b) shows their extracted stroke segments (color-coded)

3.2. Mathematical Characterization

After \mathbf{I} is decomposed into stroke segments, we represent it according to its stroke content. We aim to describe a stroke segment's structure and the manner in which it is drawn. We focus on local features that are simple to extract, representative, and invariant to various deformations (e.g. rotation, translation, and scale). In this paper, each segment is described by four simple features that encode eccentricity, symmetry, local consistency, and inflection. The first three features describe the stroke segment itself, while the

last one describe its relationship to its adjacent neighbors. These features are both empirically validated and biologically inspired. When considering the process needed to analyze drawing traits at the stroke level, one fundamental factor arises, namely the neuro-geometry of hand-arm movement, i.e. the physiology in drawing strokes. In the foundational paper of [Mor81], it was shown that different subjects produce hand/arm movements that are very similar to or coincide with simple curve strokes. Although low curvature change was generally maintained across different subjects and experiments, it was also found that individuals exhibited unique stroke characteristics. This finding was also confirmed in the work of [ABM82]. In [FH85], a mathematical model based on minimizing jerk (change of curvature) and analyzing stroke velocity was verified empirically. In summary, this and other work (see references in [FH85]) strongly suggest that (1) gesturing and strokes carry unique and identifiable aspects for an individual, and (2) that curvature and its change are key features for analyzing simple strokes. Inspired by the above physiological findings, we characterize stroke segments by focusing on curvature and its change in a drawn curve.

Moreover, in analyzing a large number of simple strokes, we see that a conic fit to a stroke segment is faithful to its original geometry and representative enough for the purpose of authorship recognition. From each conic, we extract eccentricity, symmetry, local consistency, and inflection features that are conveniently translation, orientation and scale invariant. Needless to say, this invariance also holds when the entire sketch is represented using stroke segment features. We give a description of these four features next.

Eccentricity: This feature represents the change of curvature in a stroke segment. It measures how the segment deviates from being circular, i.e. how rounded or sharp it is. By fitting a conic section to the stroke segment [Tau91], eccentricity (ϵ) is computed as a function of the ratio between the conic's major and minor axis lengths as in (1).

$$\epsilon = \begin{cases} \sqrt{1 - \frac{b^2}{a^2}} & \text{if the conic is an ellipse} \\ \sqrt{1 + \frac{b^2}{a^2}} & \text{if the conic is a hyperbola} \end{cases} \quad (1)$$

Symmetry: To crudely evaluate how symmetric a stroke segment is, we simply take the absolute difference in eccentricity of the segment's two halves (refer to Figure 3(a)). The segment is divided into two equal length parts (denoted as *attack* and *decay* following typical sketch nomenclature) and the eccentricity of each part is computed as described above.

Local Consistency: This feature measures the extent of local variation within a stroke segment. It is computed as the average absolute difference between the eccentricity of the entire segment and the eccentricity of distinct overlapping parts of this segment. To generate these distinct parts, we employ a sliding window approach across the segment, where the window size is one third the size of the segment itself and the

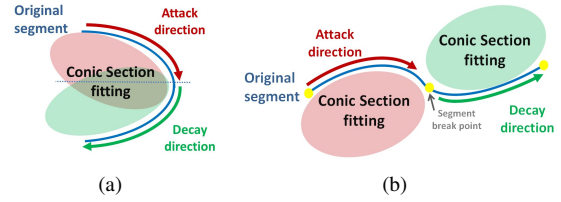


Figure 3: Attack and decay parts used to construct stroke segments. (a) shows a stroke segment divided into equal length parts enabling the computation of the symmetry feature. (b) shows two adjacent stroke segments connected by an inflection breakpoint. The inflection feature is computed by comparing these two segments.

step size is half the window size. This feature captures subtle changes in shape and curvature.

Inflection: The previous three features describe the stroke segment itself. However, characteristics of artistic style are encoded in how stroke segments are sequenced. Of special interest are locations of inflection, where the sign of curvature changes. An inflection point will be detected by SAR as a breakpoint between two stroke segments that together form a stroke (refer to Figure 3(b) for an example). To encode such relational information at an inflection point, we compute the absolute difference in eccentricity between each pair of stroke segments that share this inflection point. For stroke segments whose breakpoints are not inflection points, we set their inflection feature to a nominal value (-1).

3.3. Feature Frequency Distribution

Having each stroke segment s_i of sketch image **I** characterized by the 4D feature vector of the features described above, this stage represent **I** as distribution of frequency of those features.

Building a Universal Stroke Segment Dictionary. Our assumption is that the frequency in which an artist uses particular types of strokes is a suitable indicator of his/her authorship. To formalize this observation, we compile a dictionary of stroke segments that tends to be universal among different artists. The dictionary is learned on the stroke segments of the sketches that are used in training only. To construct a dictionary of n elements, we apply hierarchical k-means clustering on a large set of stroke segments. In our experiments, we set $n = 60$. We denote this dictionary as the universal stroke segment dictionary as it tends to capture the most commonly used stroke segments among artists. This clustering step is first stage of the bag-of-words (BoW) model that is popularly used to represent natural images for image classification [SZ03]. We use hierarchical k-means, since it is more robust and less sensitive to the choice of n than traditional k-means clustering. And it generates a tree of n cluster centers (and not only a set of centers as in the case of k-means), which encodes the membership of any stroke segment at all levels of the tree and not just the leaves. To encode a stroke segment, one has to *traverse* this tree starting from the root

and recursively select among its children the nearest cluster center in 4D feature space. As a result, each stroke segment is encoded as a binary membership vector of length n , where each value reflects whether the feature traversed the corresponding tree node or not. In Figure 4, we show image examples of cluster centers in the universal stroke segment dictionary, as well as, the color-coded membership of each stroke segment in a sketch. Such assignment is independent of where a stroke segment exists in a sketch.

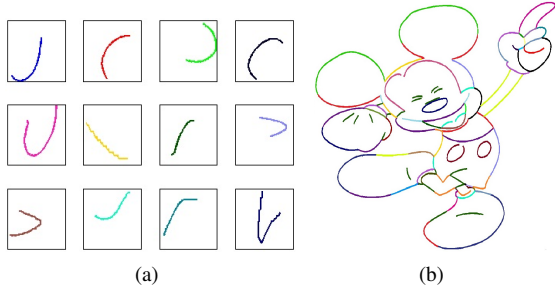


Figure 4: (a) Some elements of the universal stroke segment dictionary constructed from the segments of the training sketches. (b) Color-coded nearest neighbor assignment of segments to dictionary elements.

Bag-of-Words Features. After constructing the universal dictionary, we represent a sketch as a histogram of stroke segments following the traditional Bag-of-Words (BoW) framework. Sketch image I containing m stroke segments produces m binary membership vectors each of length n . These membership vectors are pooled together to produce the BoW feature vector \mathbf{f}_I . We use Gaussian weighted mean pooling to encode the frequency in which an artist uses each element of the universal dictionary. As we will see, coupling \mathbf{f}_I with a discriminative model enables authorship recognition. Since the original 4 stroke segment features are invariant to rotation, translation, and scale, the BoW feature vector is invariant to these deformations [LW07].

3.4. Authorship Recognition

So far, a sketch image is represented by a sparse n -dimensional histogram depicting the frequency in which each type of stroke segment is used by the artist in the sketch. As such, we expect the BoW feature to possess enough discriminative power to determine authorship based on simple strokes alone. We validate this assumption empirically in Section 6.1. Given a training set of sketches labelled according to the artist who drew them, we build a discriminative model using the training BoW features. In order to reduce testing time and to pinpoint the most discriminative portions of the BoW feature, we employ a forward feature selection procedure, which greedily appends a single feature at a time *only* if this addition improves classification accuracy [JZ97]. Based on our experiments, only a small subset of the n features is actually used for discrimination. Using these subsampled BoW histograms, we build a multi-class classifier to assign authorship to an unseen sketch image. For sketch

fraud detection, we use a binary classifier (original vs. fraudulent) instead. We experimented with a variety of classifiers and found that either an RBF (Radial Basis Function) kernel SVM or a kNN classifier can be used for this purpose. However, we decided on the kNN classifier because of its better generalizability properties and its minimal training time. We use two experimental setups to evaluate the accuracy of our classifier: 2-fold cross validation and leave-one-out. The first is a popular setup in image classification, while the latter sheds light on how dependent the classifier performance is to the amount of training data.

4. Sketch Datasets

To evaluate the performance of SAR, we compile two new sketch datasets collected from experienced artists. Participating artists were screened, so as to guarantee high levels of sketching capability. The chosen artists were graphical or interior designers by profession, each with 7-10 years of sketching experience. We gave them the freedom to draw using a pen, pencil or digitally at any scale. Moreover, they were allowed to correct their strokes or redraw the entire sketch with no time limit. Since we had direct access to these professionals, we were able to administer different sketching scenarios. This allowed us to control the *content variety* (i.e. what the artist draws in a sketch) and the extent of *sketching constraints* (i.e. how the artist should sketch). These two factors impact the strokes an artist chooses and ultimately his/her style. To our knowledge, this work is the first to compile such a diverse set of sketch data for the purpose of studying authorship from strokes. To allow for further research in this area, we will make all these datasets (images and annotations) publicly available. Next, we describe the datasets used in this paper (refer to Table 1 for a summary).

Free Style Dataset. We collected 10 images of objects from diverse semantic categories and then asked 7 artists to sketch them. The objects were chosen to be detailed enough to adequately reflect artistic style and to be relatively easy to replicate by an experienced artist in a reasonable amount of time. The 70 sketches (10 from each artist) were collected in three weeks. No constraints or specific instructions were given to the artists, thus, giving them complete sketching freedom.

Fraud Dataset. Here, we simulate a sketch fraud scenario. Using the same images in the free style dataset, we chose the sketches of one of the artists as *original* drawings. We asked the other 6 artists to draw all the original sketches. We provided them with an instruction sheet, where they were requested to draw the original sketches in such a way that it would be very hard to distinguish their *own* drawings from the originals, thus, simulating sketch fraud. The 6 artists were prohibited from using methods or supplies (e.g. translucent tracing paper) that could produce copies of the original sketches. Over a period of 2 weeks, we collected a total of 60 sketches. (refer to Figure 5 for examples). Since the artists were requested to simulate sketch fraud, this dataset is highly constrained from an artistic style perspec-

Dataset	# Artists	# Sketches	# Strokes	Content Variety	Sketching Constraints Level
Free Style	7	70	4000	10 different sketching objects	No constraints (Draw this sketch)
Fraud	7	70	4000	10 different sketching objects	High constraints (Draw and simulate fraud)
Line Study	11	107	6000	Up to 12 different objects	Very high constraints (Re-draw over the original Image)

Table 1: A summary of datasets used in this paper. We compiled the first two and reused the line study dataset of [CGL*08].

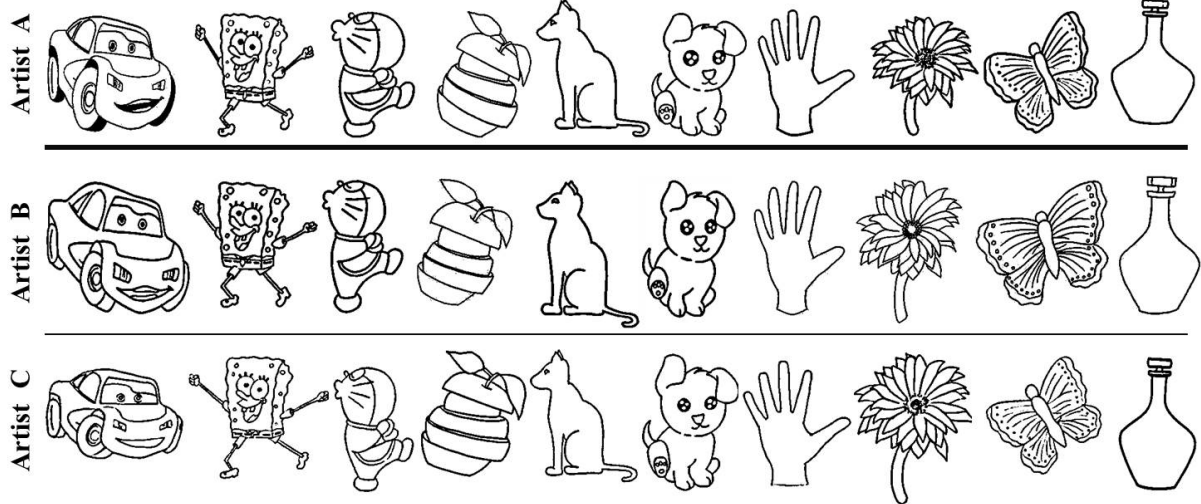


Figure 5: Samples of sketches from the fraud dataset. The first row shows the original sketches drawn by Artist A. The other three rows show the sketches of 2 other artists (Artists B and C), who attempted sketch fraud. Obviously, the *fraudulent* sketches look extremely similar to the originals, making authorship recognition (manual or automatic) quite difficult.

tive. Therefore, it poses a substantial challenge to any authorship recognition system, albeit manual or automatic.

Line Study Dataset. We also use the line study dataset generated in [CGL*08]. This dataset is a collection of sketches of various objects (e.g. mechanical tools, automobile parts, and bones) drawn by multiple artists. The sketching tasks were highly constrained, since artists were asked to draw their sketched lines over a faded copy of the original image. Clearly, the task of recognizing sketch authorship in this dataset is more difficult than the previous two. We selected all sketches from artists, who drew at least 6 sketches, thus, leading to a dataset of 107 sketches from 11 artists.

5. Human Sketch Authorship Recognition

For comparison and as a baseline, we study the inherent difficulty of the authorship recognition problem for humans. The human visual system (HVS) is renowned for its effectiveness in successfully completing many high-level recognition tasks (e.g. object and action recognition), so much so, that it remains the *gold standard* to which automated recognition systems aspire. Despite significant advances in computer vision, the HVS almost always outperforms automated methods in such tasks. However, there *do exist* recognition tasks, in which the HVS underperforms. These tasks usually deal with *fine-grained* recognition of objects (e.g. faces),

where the inter-class variation is minimal and on par with the intra-class variation. We motivate this fact with an example. Although the HVS can easily discriminate between a 'chair' and a 'dog', it does not do so well in recognizing a person's face in a large dataset of people (e.g. the entire population of a country) from the same cultural and racial background. The differences between people's faces are so subtle that the minute details discriminating them cannot be uncovered by the HVS. However, it is exactly these details that automated recognition methods focus on. This enables them to outperform the HVS in these tasks (e.g. robust face recognition [WYG*09]). In this section, we provide extensive empirical evidence from two user studies that highlight the sheer difficulty people and experienced artists face when trying to recognize authorship from 2D sketches.

Authorship Recognition by Non-artists. The aim of this study is to shed light on how accurately people can recognize sketch authorship. As an online quiz, participants are first shown sample sketches from the free style dataset labeled with artists who drew them. We showed 5 randomly selected sketches from each of the 7 artists. Next, we administered for each online participant a total of 5 questions, each of which asked him/her to assign an *unseen* sketch to one of the 7 artists that he/she thought drew it. We chose to show users only 5 of the 10 images per artist so as not to

overwhelm them. We allowed participants to zoom-in to the images when needed. To reach a large number of participants, we published our user study on Amazon Mechanical Turk (AMT). To validate the quality and seriousness of each Turker’s answers (as usually done in practise), we administer a control question at the beginning of the quiz. Moreover, Turkers are randomly directed to one of the many versions of the online study and answers from unique workers are recorded.

After two months of activity and more than 2000 unique participants, the accumulated results of this study show that people can successfully recognize the authorship of a sketch (among 7 different artists) with an average accuracy of 36% and with standard deviation of 10%. Participants took an average of 4 minutes to complete the assigned quiz. From this result, we see that average human performance is only moderately better than a random choice classifier (i.e. choose one of the 7 artists at random irrespective of what the sketch is), which has an average accuracy of 14% in this case. Obviously, this establishes that sketch authorship recognition is quite difficult for people. More importantly, we show later in this paper that our automated SAR approach achieves an average accuracy of 60% under the same conditions.

Sketch Fraud Detection by Artists. Similar to the previous study, but targeting artists specifically, since it is conceivable that the average person might find this task difficult due to his/her lack of sketching experience and/or artistic talent. The aim here is to evaluate the performance of artists in discriminating fraudulent sketches from original ones. A total of 25 experienced artists were given an online quiz, where 5 original and 5 fraudulent sketches were made known to each user. The sketches were taken from the fraud dataset described earlier. Each artist is then given a set of 5 recognition questions, whereby he/she needs to label the unseen sketch as original or fraud. Moreover, artists were given the chance to view their quiz results and to email us feedback. Again, not all the images in the dataset are shown to the online users so as not to overwhelm them.

The quiz results show that the artists could distinguish original sketches from fraudulent ones only 52% of the time, as compared to 50% random chance. Each artist took an average of 5 minutes to complete the task. As expected, most of the feedback we received elaborated on how truly difficult the task was. Later, we show that SAR significantly outperforms the artists’ recognition accuracy, thus, motivating the plausibility of using SAR in an automated fraud detection system for sketches (e.g. patent drawings and cartoon sketches). We include links to both online quizzes in the **supplementary material**.

6. Experimental Results and Evaluation

This section presents a number of experimental results we obtained to assess multiple facets of the proposed SAR approach. (1) To evaluate the effect of sketching constraints on SAR, as well as, its overall effectiveness in recognizing

sketch authorship and detecting sketch fraud, we test SAR on the datasets described in Section 4. (2) We also evaluate the sensitivity of SAR to a number of algorithmic variations. These results, in turn, are used to design the most representative and discriminative variant of SAR.

6.1. Computational Authorship Recognition

To answer the questions: Are simple strokes unique to the artist who draws them? If they are, then to what extent can they identify the author who drew them? To our knowledge, these questions are not adequately addressed in prior work. To answer them, we conduct several experiments on three separate datasets, which provide a rich diversity of sketch content and sketching constraints. In what follows, we report SAR classification accuracy for each dataset and provide cross-dataset analysis. We also evaluate SAR’s ability to detect sketch fraud using the fraud dataset.

Free Style Dataset. On this constraint-free dataset, SAR accuracy on test data is 60% and 57% using leave-one out and 2-fold validation respectively with 8% standard deviation. For comparison, random chance in this case is 14%.

Fraud Dataset. Is artistic style among different artists still preserved when they are consciously attempting to commit fraud? To answer this, we use our fraud dataset to build a 7-class SAR classifier to discriminate among the 7 artists. Using this dataset, random chance is 14%. Here, SAR reaches a significantly higher average accuracy of 51% (leave-one-out) and 45% (2-fold cross validation) with 9% standard deviation. This result suggests that an artist’s sketching style is still distinguishable (to a certain extent) even when attempting sketching fraud. To visualize the discrimination power of SAR, we show the BoW features of three sketches (same object) drawn by 3 artists from this dataset in Figure 6. One of the sketches is an original and the rest are fraudulent. Although all three sketches look very similar, their underlying features have obvious differences, which SAR capitalizes on to determine authorship.

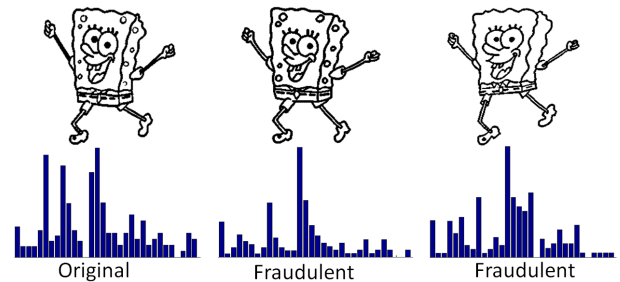


Figure 6: BoW features of the same sketch object drawn by 3 artists (one original and two fraudulent). Although they look very similar, their features are different.

Line Study Dataset. The sketching task in this dataset (compiled by [CGL*08]) is highly constrained as discussed in Section 4. As it is similar in spirit to the fraud dataset, we expect similar results. We train and test SAR in the same way

as before but with a total of 11 artists and 107 sketches. With 11 classes, random chance is 9%, which is significantly lower than SAR’s average accuracy of 33% (leave-one out) and 30% (2-fold) with 7% standard deviation. This noteworthy discrepancy in performance indicates that artists still maintain some uniqueness in their sketching style, even though they are being forced to copy a different style altogether.

We give a performance summary on the three datasets in Table 2. The slight difference in accuracy between leave-one out (i.e. all but one sketch are used for training) and 2-fold cross validation (i.e. only 80% of the sketches are used for training) indicates that SAR is reasonably insensitive to the amount of training data used. This also suggests that SAR is generalizable, a promising property for a classifier in the presence of unseen data.

Table 2: A summary of SAR performance on three datasets. Average accuracy % is reported for leave-one-out and 2-fold setups. Random chance accuracy % based on the number of classes is also.

	Rand Choice	Leave-one out	2-Fold	std
Free Style	14	60	57	8
Fraud	14	51	45	9
Line Study	9	33	30	7

Cross-Dataset Analysis. In the datasets above, the contributing artists were subject to varying levels of sketching constraints, ranging from unconstrained (free style dataset) to highly constrained (line study dataset). To investigate the sensitivity of SAR performance with sketching constraints, we compare its accuracy across all datasets in Figure 7(a). The datasets are ordered in increasing levels of constraint. To normalize the effect of different numbers of classes across datasets, we plot the ratio of SAR accuracy to random chance. As expected, recognizing sketch authorship becomes harder as more constraints are imposed on the artist. However, on all the datasets, SAR performance is significantly (at least 2.5 times) higher than random chance. This suggests that artists do *not* lose all characteristics of their unique style even under the strictest of constraints.

Fraud Recognition Experiment. Here, we setup a fraud recognition experiment (original vs. fraudulent) that predicts SAR performance in a real-world scenario. It is unrealistic to assume that the SAR classifier will have access to fraudulent training examples from *all* artists. However, we expect that when more fraudulent samples are used in training, SAR’s test performance on fraudulent sketches from artists, who were *not* included in training, will improve. In other words, knowing more about how fraud *looks* like will help SAR better detect sketch fraud, even for fraudulent artists whose sketches are not trained on. To validate this expectation, we train the SAR binary (fraud vs. original) classifier with fraudulent sketches from increasingly more fraudulent artists (i.e. from 1 to 5 artists). In each case, 80% of the original sketches are used for training (2-fold validation). Then,

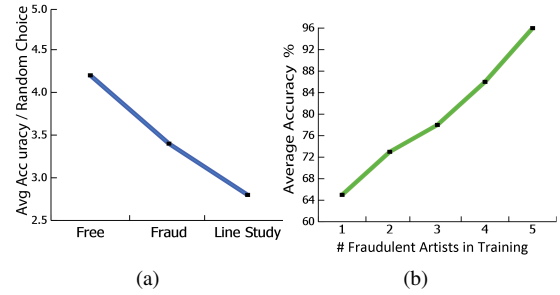


Figure 7: (a) The ratio between SAR accuracy and random chance on three datasets organized in increasing order of sketching constraints. Clearly, the more constraints imposed on artists, the less discriminative their strokes become. (b) Fraud recognition performance improves as more fraudulent artists are used in training, especially on sketches from artists who were not included in training.

each SAR classifier is tested on the remaining original and fraudulent sketches. This means that SAR will always be tested on fraudulent sketches from artists, whose style has not been seen during training. In Figure 7(b), we plot the average test accuracy of SAR as the number of fraudulent artists used in training increases. Clearly, this result endorses our aforementioned expectation. In fact, SAR’s fraud detection performance reaches 96% when 5 of the 6 fraudulent artists are used for training and the 6th for testing. Despite the extremely high similarity between fraudulent and original sketches, we conclude that SAR is able to effectively spot artistic fraud. In fact, this result suggests that SAR can be useful in an online learning setup, where test samples are sequentially classified and in turn weighted and then used in re-training the classifier.

6.2. Algorithmic Details

Since SAR comprises multiple computational modules (refer to Figure 1), varying the details of any module can lead to a variant of SAR. In this section, we investigate many of these variations and provide algorithmic details to reproduce the SAR classifier.

Feature Contribution. Stroke segments are represented by 4 biologically inspired features, as discussed in Section 3.3. Here, we study the contribution of each feature to SAR’s discriminative power. To do this, we employ a conventional forward feature selection method that greedily determines which features (along with their importance weights) should be incorporated into the SAR classifier. This is a data-driven process, so different training-test splits of the dataset can lead to different feature selections and weights. By training SAR (with 2-fold validation) multiple times on the free style dataset, we accumulate an average selection weight for each feature (refer to Figure 8(a)). We see that eccentricity and local consistency are the most discriminative features followed by inflection and symmetry, where no one feature seems to dominate.

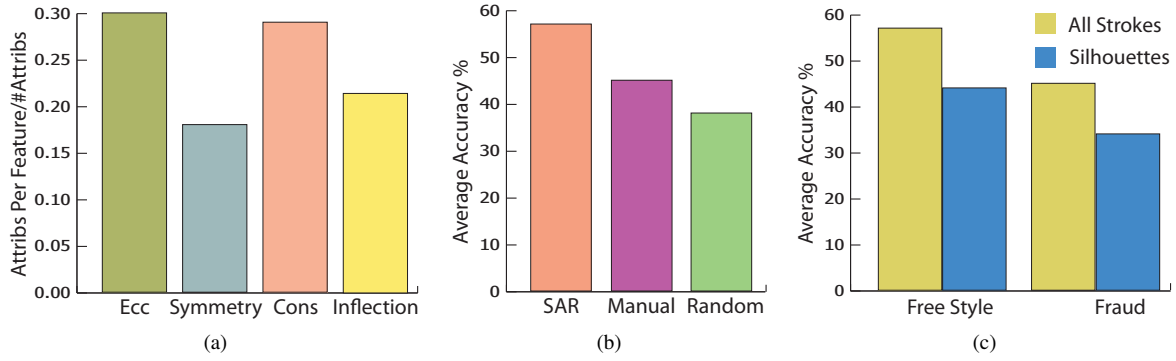


Figure 8: (a) Contribution of each feature type to SAR recognition. (b) SAR accuracy on the free style dataset when its segmentation method is replaced with a manual or random one. SAR outperforms manual segmentation by around 15% and random segmentation by around 20%. (c) SAR accuracy on the free style dataset, where stroke segments come from the silhouette only or from the entire sketch. Adding internal stroke segments *does* improve accuracy but the improvement is only about 10%.

Variations in Stroke Segmentation. To justify our stroke segmentation method (refer to Section ??, we compare it against two baseline methods: manual and random stroke segmentation. For manual segmentation, we asked 10 artists to manually segment sketches into strokes as they see fit, while in the random case, breakpoints are selected uniformly at random within each extracted stroke. We set a minimum length for each stroke segment to prevent singularities. SAR accuracy (2-fold) using each of the three segmentation methods on the free style dataset is summarized in Figure 8(b). Clearly, our automated segmentation method outperforms the other two. Similar to the human recognition result in Section 5, SAR improvement over manual segmentation suggests that human performance on a fine-grained classification task such as authorship recognition does not seem optimal.

Silhouettes vs. All Strokes. In many cases (e.g. the Mickey Mouse cartoon character), the silhouette of a sketch can be very discriminative, so much so, that silhouettes have been used as primary features in other recognition tasks (e.g. action recognition [LZL08]). Here, we study how discriminative silhouette stroke segments are on their own within the SAR pipeline. In Figure 8(c), we use two datasets to compare SAR accuracy when stroke segments are taken from the silhouette alone or from the entire sketch. Interestingly, silhouettes seem to be quite discriminative in their own right, as SAR performance remains high even when only silhouettes are used. In fact, including stroke segments from the sketch interior only improves accuracy by about 10%.

Effects of Digitization. As mentioned in Section 3.1, stroke segments are extracted from a sketch using the well-known Adobe Live Trace digitization technique. The level of digitization is controlled by the user to tradeoff faithfulness to the original sketch with compactness of representation. For instance, high levels of digitization tend to smooth out strokes, thus, masking aspects of the artist’s intrinsic style and possibly affecting SAR performance. To investigate the effect of

digitization on SAR, it is applied to the fraud dataset after applying three distinct levels of digitization on its sketches. We report the average accuracies in Table 3. As expected, the less faithful the digitization is to the original sketch (i.e. the higher the digitization level), the lower the SAR accuracy is. In fact, the accuracy reduces significantly and reaches near random chance when the highest level of digitization is used. This result suggests another application that could benefit from SAR, namely the quantitative assessment of various digitization techniques. SAR can conceivably be used to evaluate how much of the unique sketching style is preserved after digitization.

Table 3: Effects of digitization on SAR accuracy %. Sketches from the free style dataset are processed using three levels of digitization in Adobe Live Trace. SAR accuracy decreases significantly with higher levels of digitization.

level of digitization	leave-one-out	2-fold cross validation
Low	51	45
Medium	42	39
High	28	23

Style not Content. SAR is designed to represent and classify artistic sketch style, irrespective of sketch content in general. This is a primary factor that distinguishes SAR from existing shape matching techniques discussed earlier in Section 2. In fact, our experiments show that sketches of different content (objects) drawn by the same artist tend to have more similar BoW features than sketches of the same content drawn by different artists. For instance, the BoW feature of a flower drawn by one artist is more similar to a butterfly drawn by that artist than the feature of the same flower drawn by another artist, as illustrated in Figure 9. To quantify this observation, we use a normalized Gaussian similarity measure $s(\mathbf{I}_1, \mathbf{I}_2)$ based on the Euclidean distance between two BoW features. From the free style dataset, we randomly select a sketch \mathbf{I}_i^A drawn by artist A and the same sketch \mathbf{I}_i^B drawn by another artist B to compute $s(\mathbf{I}_i^A, \mathbf{I}_i^B)$. We then randomly select a sketch \mathbf{I}_j^A drawn by A with $i \neq j$

and compute $s(\mathbf{I}_i^A, \mathbf{I}_j^A)$. By performing this operation across the whole dataset, we compute the average inter-artist similarity $s(\mathbf{I}_i^A, \mathbf{I}_j^B) = 68\%$ and the average intra-artist similarity $s(\mathbf{I}_i^A, \mathbf{I}_j^A) = 89\%$. This result shows that SAR targets an artist’s style by abstracting it from sketch content.

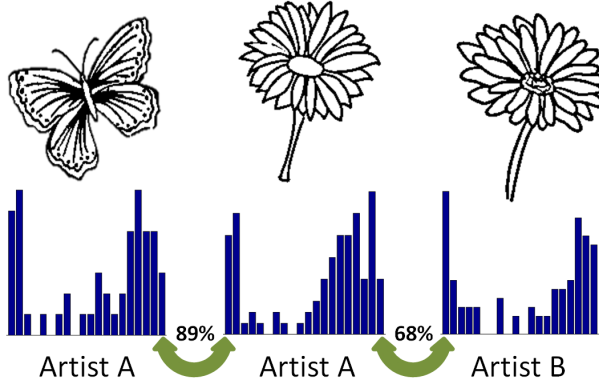


Figure 9: A visual comparison between the BoW features of three sketches from the free-style dataset: a flower drawn by Artist A (middle), a butterfly drawn by Artist A (left), and the same flower object drawn by Artist B (right). From the features themselves and their average pairwise similarity scores, we see that SAR discrimination is based on artistic style and not sketching content.

7. Applications

In this section, we show SAR’s feasibility in two useful applications other than sketch fraud detection, namely sketch style training and sketch synthesis evaluation.

7.1. SAR for Sketch Style Training

Knowing that an artist’s sketching style is dynamic (i.e. it evolves over time), how persistent is it when artists undergo extensive training for the explicit purpose of altering their style and adopting another? For example, this type of training is administered in major cartoon companies (e.g. Walt Disney), usually for months on end. For this purpose, we develop a SAR-based application that allows artists, designers, and animators in-training to quantify their progress in adopting the *target* style of a particular artist, whose sketches have been encoded in SAR. Trainees are able to draw and/or upload sketches to examine how close their artistic style has become to the target. We demonstrate this application next.

Style Training Setup We first determine a target style by finding a reasonably known artist, who has published his work online along with specific instructions on how to follow his drawing style. The target artist also provides YouTube videos showing how his sketches can be drawn step-by-step. We select five sketches from his portfolio, specifically those with video instructions. Then, we identify three artists with varying levels of sketching and artistic experience (novice, intermediate, and advanced) to undergo the

style training process. The advanced level artist is a professional with 10 years of experience and the intermediate artist has 3, while the novice artist only sketches as a hobby.

In the first stage of training, we ask all three artists to draw the five target sketches and then use the SAR-based tool to give them quantitative feedback on how similar their sketching style is to the target. The measure based on BoW features defined earlier in Section 6.2 is used to compute style similarity. In the second stage, we ask the artists to draw the sketches again after consulting a set of textual instructions on how to draw the target sketches. These instructions are obtained from the target artist’s website. In the third stage, we provide the artists with their new SAR similarity scores along with instructional online videos. After submitting their newest sketches, we again compute their SAR similarity scores. Sample sketches of both novice and intermediate level artists at the first and last stages of training are shown in Figure 10. It is obvious from these results that the novice artist has improved more significantly than the intermediate artist in adopting the target style.

Style Training Results Figure 11 plots the evolution of the SAR similarity score throughout the 3-stage training process for each artist. As expected, each trainee’s score increases with training. The novice artist exhibits an overall improvement of 10%, which can be attributed to the explicit use of textual and video instruction for target training. The initial similarity scores for the intermediate and advanced level artists are much higher than the novice one, but they also exhibit an improvement of 4% and 2% respectively. The slight improvement by the advanced artist conveys how difficult it is for an artist with a well-defined style and extensive experience to adopt a new style, as well as, how much easier it is for a novice artist to do the same. Moreover, we expect that repeating these training stages (especially the last one) will further improve SAR similarity to the target style. Based on our results, we conclude that SAR can be successfully incorporated in the sketch training process (e.g. in major cartoon companies) to quantitatively monitor training progress. A training executable is available in the **supplementary material**. We will also release the training source code to extend this application to a larger set of sketches from more participating artists and to allow users to build their own training applications using their own sketch data.

7.2. SAR for Evaluating Sketch Synthesis Results

As mentioned in Section 2, there is a number of recent methods that focus on sketch synthesis and artistic style analysis. Their aim is to develop automatic sketch synthesis tools that mimic a particular artistic style. Most of these tools do not provide a quantitative assessment of how close the synthesized style is to the target one, thus, making it difficult to compare synthesis methods. A few of them however have validated their synthesis quality through extensive user studies, which are cumbersome to compile and require careful analysis. Since SAR is fully automatic, easy to use, and has proven style discrimination power (refer to Section 6.1), we



Figure 10: Samples sketches drawn by artists at the first and final stages of training along with the target style sketches. Sketches on the left correspond to the intermediate artist while ones on the right to the novice artist. The progress in their sketching styles is noticeable for both artists but more so for the novice one.

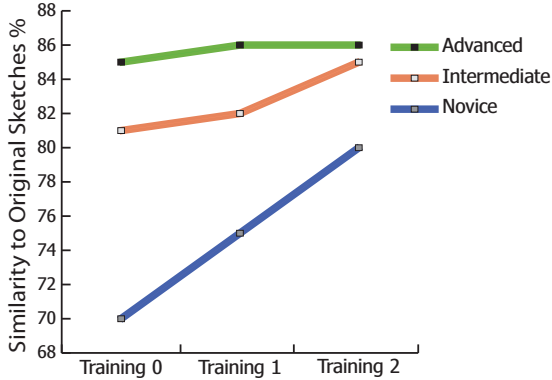


Figure 11: SAR similarity-to-target scores for 3 novice, intermediate, and advanced level artists after 3 stages of training. The novice artist has the least score in the initial stage but progresses the most with training.

propose to use it in quantitatively evaluating sketch synthesis methods without the need to conduct tedious user studies. To exemplify this application, we use SAR to analyze the synthesis results of Berger et. al. [BSM*13]. In their work, portrait sketches are synthesized from artistic styles of 7 artists, each of which drew 24 portrait sketches. The real and synthesized sketches are publicly available.

We first use SAR to classify artistic style among the real portrait sketches only. In this case, SAR accuracy is 52% (leave-one-out) and 50% (2-fold), as compared to random chance accuracy of 14%. When classifying only synthesized sketches, we obtain 54% accuracy (leave-one out) and 51% accuracy (2-fold) respectively. The similarity in performance between the two scenarios suggests that the synthesized portraits are as hard to distinguish as the real ones and that the synthesis tool of [BSM*13] maintains a very similar amount of style diversity as in the real sketches. Next, we use SAR to classify the real sketches using the synthesized ones only as training and vice versa. In this case, SAR accuracy dropped to 26% (leave-one-out) and 25% (2-fold). This performance drop can be used as a quantitative measure of synthesis quality, as it allows for comparison between different synthesis methods. In fact, an ideal synthesis tool should produce a

SAR accuracy drop close to zero, since the style of the synthesized sketches should be undistinguishable from that of the real sketches.

Our results are on par with those of the three user (perceptual) studies conducted by Berger et. al. [BSM*13]. In fact, we setup the SAR classification experiments above to follow the same setup used in these user studies, where the only difference between the two is that SAR performs the classification automatically without any human feedback. The participants in the user studies registered a very similar accuracy when classifying the real and synthesized portraits separately (as did SAR). More importantly, the drop in classification by human participants is around 20%, which is comparable to the 25% drop reported by SAR. This result validates that SAR can be used to automatically and quantitatively evaluate sketch synthesis tools, without the need for user studies.

8. Conclusions and Future Work

In this paper, we shed light on a new direction in sketch analysis, namely authorship recognition through stroke analysis. We propose a stroke authorship recognition (SAR) approach that discriminates between artistic sketch styles based on the choice and frequency of use of basic strokes. From our extensive experiments and user studies, we provide empirical evidence regarding four interesting conclusions related to sketch analysis.

Uniqueness of Sketch Style. Based on results in Section 6.1, we conclude that SAR *does* encode unique and consistent characteristics of an artist's sketching style, which are in turn used to discriminate one artist's sketches from others. This result validates SAR's applicability in important real-world tasks, such as sketch fraud detection (e.g. for design patents and cartoon characters) and training/teaching artistic style.

Style and Sketch Constraints. The extent to which SAR can be used for discrimination is dependent on the sketching constraints imposed on the artist. Although overall SAR accuracy decreases with more constraints, unique elements of artistic style still persist even under the strictest of constraints (fraud).

Silhouettes. We identify that a sketch's silhouette is a rich-

er source of discriminative information than internal strokes in general.

Human Performance. Our extensive user studies empirically validate the difficulty of this fine-grained recognition problem and indicate that SAR can improve upon human performance. All the compiled data (datasets and user studies) and source code will be made publicly available to enable further research on this exciting topic and to allow for quantitative comparison with future methods.

Future Work We aim to improve the discriminative nature of SAR by improving the quality of the learned universal stroke segment dictionary. One way to do this is to investigate how strokes are actually generated by the artists. This can be done by tracking their hand movements and how they hold the pencil/pen while sketching. We believe this will provide us with more information about an artist's style of sketching. Although the segmentation process is currently viewed as a pre-processing step in SAR, we aim to investigate how supervised information (artist labels) can be incorporated in this process (possibly through supervised dictionary learning) in order to produce stroke segments that are inherently discriminative. Furthermore, we will extend our compiled datasets to more sketches and contributing artists.

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