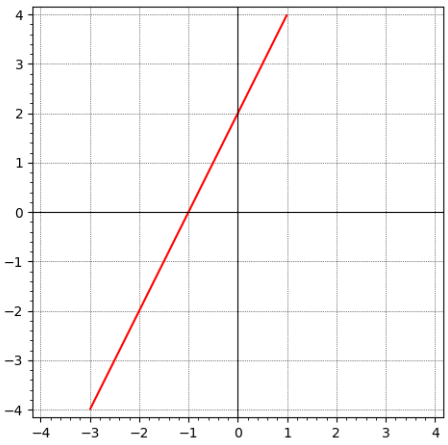
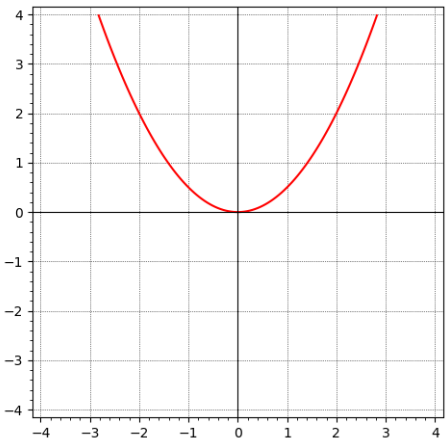
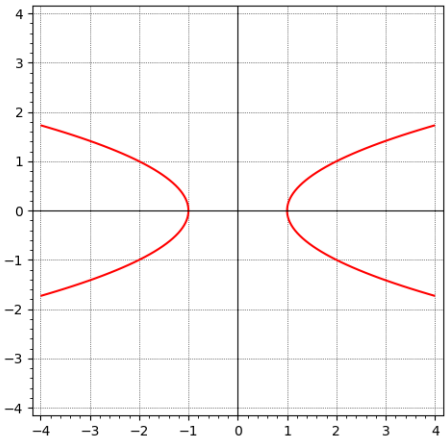
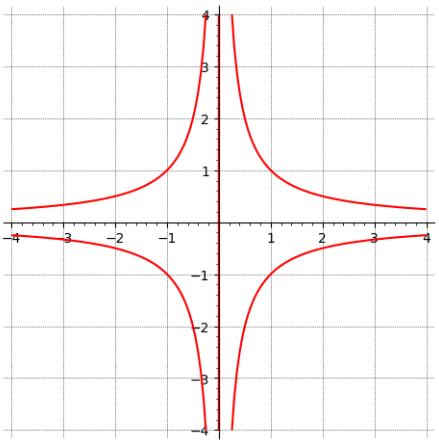
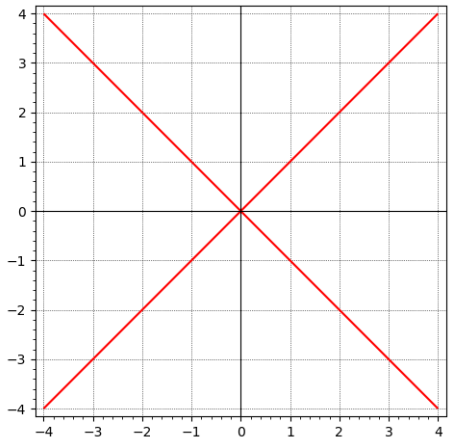
# IHS Math Seminar: January 27th, 2025

## 1 Varieties

Try these questions without Sage! You may use Sage afterward to check your answers.

1. Express each of the following graphs as varieties in $\mathbb{R}^3$.




2. What do each of the following varieties look like in $\mathbb{R}^2$? Why?
   (a) $\mathbb{V}(y - x, y - x + 1)$
   (b) $\mathbb{V}(x^2 + y^2 + 1)$
3. Sketch and describe the following varieties in $\mathbb{R}^3$ as thoroughly as possible.
   (a) $\mathbb{V}(x + 2, y - 1, z)$
   (b) $\mathbb{V}(x^2 + z^2 - 1, x^2 + y^2 + (z - 1)^2 - 1)$
   (c) $\mathbb{V}(xz, yz)$
4. Express the four-leaved rose curve $r = \sin 2\theta$ as a variety in $\mathbb{R}^2$. (Hint: Use the fact that $r^2 = x^2 + y^2$, $x = r\cos\theta$, and $y = r\sin\theta$, as well as a trigonometric identity.)
5. Prove that the set of points $\{(x, x) \in \mathbb{R}^2 \mid x \neq 1\}$ is not a variety.

## 2 Varieties in Sage

The following code may be helpful for creating 2D plots in Sage.

```
x,y = var("x,y")
plot1 = implicit_plot(x^2+y^2-1, (x,-2,2), (y,-2,2), color="red",
                      gridlines=True, axes = True)
plot2 = implicit_plot(x == y, (x,-2,2), (y,-2,2), color="blue",
                      gridlines=True, axes = True)
show( plot1 + plot2 )
```

The following code may be helpful for creating 3D plots in Sage.

```
x,y,z = var("x,y,z")
plot1 = implicit_plot3d(x^2+y^2-1, (x,-2,2), (y,-2,2), (z,-2,2),
                        color="purple", opacity=0.1, plot_points=80)
show( plot1 )
```

Use Sage to help you do the following questions.

1. Show how to factor the polynomial $x^3 - x^2y + xy^2 - y^3 - x + y$ by examining the graph of the variety $\mathbb{V}(x^3 - x^2y + xy^2 - y^3 - x + y)$.
2. Predict what you think each of the following varieties in $\mathbb{R}^3$ will look like. Then use Sage to graph them. Were the pictures what you expected?
   (a) $\mathbb{V}(x^2 + y^2 + z^2 - 1)$
   (b) $\mathbb{V}(z - x^2 - y^2)$
   (c) $\mathbb{V}(x^2 + y^2 - 1)$
   (d) $\mathbb{V}(z)$