IHS Math Seminar: January 14th, 2025

1 Setting up SageMath

- Go to cocalc.com
- Sign up by using the **Sign Up** button at the top right of the screen. Use **Sign In** later.
- Click ⊕**New** at the top left of the page to create a new worksheet. Choose a name for your worksheet, and then click **Sage worksheet**.

2 Using Sage

Run each of the following blocks of code. What does each one do?

When typing out blocks of code, pressing **Enter** goes to the next line without running any code. Pressing **Shift+Enter** runs all of the lines of code within your current block, and displays the output of the last line.)

- ```
  2 + 2
  ```
- ```
  3^5 + 4*(2-3)
  ```
- ```
  a = 5
  b = 2
  a + b
  ```
- ```
  (n(sin(pi/3)))
  ```
- ```
  (n(sin(pi/3), digits = 5))
  ```
- ```
  a + 7
  b
  ```
- ```
  print("Algebraic Geometry!")
  ```
- ```
  print(10/3, 10/3.0, 10//3, 10 % 3)
  ```
- ```
  print(2 == 2, 2 == 3, 2 < 2, 2 <= 2)
  ```
- ```
  factor(-2024)
  ```
- ```
  x = var("x")
  solve(x^2 + 3*x + 2 == 0, x)
  ```
- ```
  x, a, b, c = var("x, a, b, c")
  solve(a*x^2 + b*x + c == 0, x)
  ```
- ```
  x, y = var("x, y")
  solve([x + y == 6, x - y == 4], x, y)
  ```
  ```

3 Practice with Sage

Using what you've learned about code on the previous page, answer at least two of the following questions:

1. Simplify as much as possible: $10 + 3\left((2(30 - \sqrt{12})\right) + \left(\frac{2+\sqrt{3}}{3}\right)^2$
2. Calculate $\frac{23-2i}{3+2i}$
3. Is $2^{11} - 1$ prime? How about $2^{13} - 1$?
4. Find the solution (x, y, z) for the following system of equations. Give your answer as exact fractions, as well as approximations to the nearest thousandth.

   $12x + 13y + 7z = 8$
   $4x - 11y - 4z = 17$
   $-5x + 9z = 19$

4 Polynomials

1. How can we write the integer 0 as a monomial in the variables $x_1, x_2, \ldots, x_n$? What about writing 1 as a polynomial?
2. Suppose f and g are polynomials of degree m and n, respectively.
   (a) What can you say about the degree of fg? Explain.
   (b) What can you say about the degree of $f + g$? Explain.
3. We say that a is a **zero** of the polynomial $f(x)$ if $f(x)$ is equal to zero when evaluated at $x = a$. It's true that a is a zero of $f(x)$ if and only if $(x - a)$ is a factor of $f(x)$.
   (a) Prove the fundamental theorem of algebra: A nonzero polynomial $f(x)$ of degree n can have at most n zeros.
4. For any polynomial $f(x)$, we call $f(a)$ the evaluation of $f(x)$ at a. To compute the evaluation, substitute $x = a$ in the expression of $f(x)$.
   (a) Show that a polynomial $f(x)$ is zero if and only if the evaluation of $f(x)$ at all integers a is zero. (You'll need to use 3a!).
   (b) Show that two polynomials f and g are equal if and only if the evaluation of $f(x)$ is equal to the evaluation of $g(x)$ at all integers a.