Bayesian regularized SEM

Current capabilities and constraints

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What do we want to do?

Finding an accurate description of the phenomenon under investigation that is as parsimonious as possible.

Classical regularized SEM

Idea: add a penalty to the parameters to regularize

$$F_{regsem}\big(S, \Sigma(\theta)\big) = F\big(S, \Sigma(\theta)\big) + \boxed{\lambda P\big(\theta_{reg}\big)}$$

e.g. lasso, ridge, or elastic net.

The penalty shrinks small coefficients to zero, thereby identifying the model.

Regularized SEM: possible applications

- Cross-loadings in CFA (Jacobucci et al., 2016)
- Loadings in EFA (Jin et al., 2018; Trendafilov et al., 2017; Hirose and Yamamoto, 2014)
- Selection of covariates in MIMIC models (Jacobucci et al., 2016, 2019)
- Selection of mediators in mediation analysis (Serang et al., 2017; Zhao and Luo, 2016; van Kesteren and Oberski, 2019)
- More stable estimation of latent class models in small samples (Chen et al., 2017; Robitzsch, 2020)
- Detection of violations of MI (Magis et al., 2015; Tutz and Schauberger, 2013)
- ...

Any model with (too) many parameters where some are assumed to be zero.

Bayesian regularized SEM

Instead of a penalty function, we can use a Bayesian *shrinkage prior:*

$$p(\theta|y) \propto p(y|\theta)\boldsymbol{p}(\boldsymbol{\theta})$$

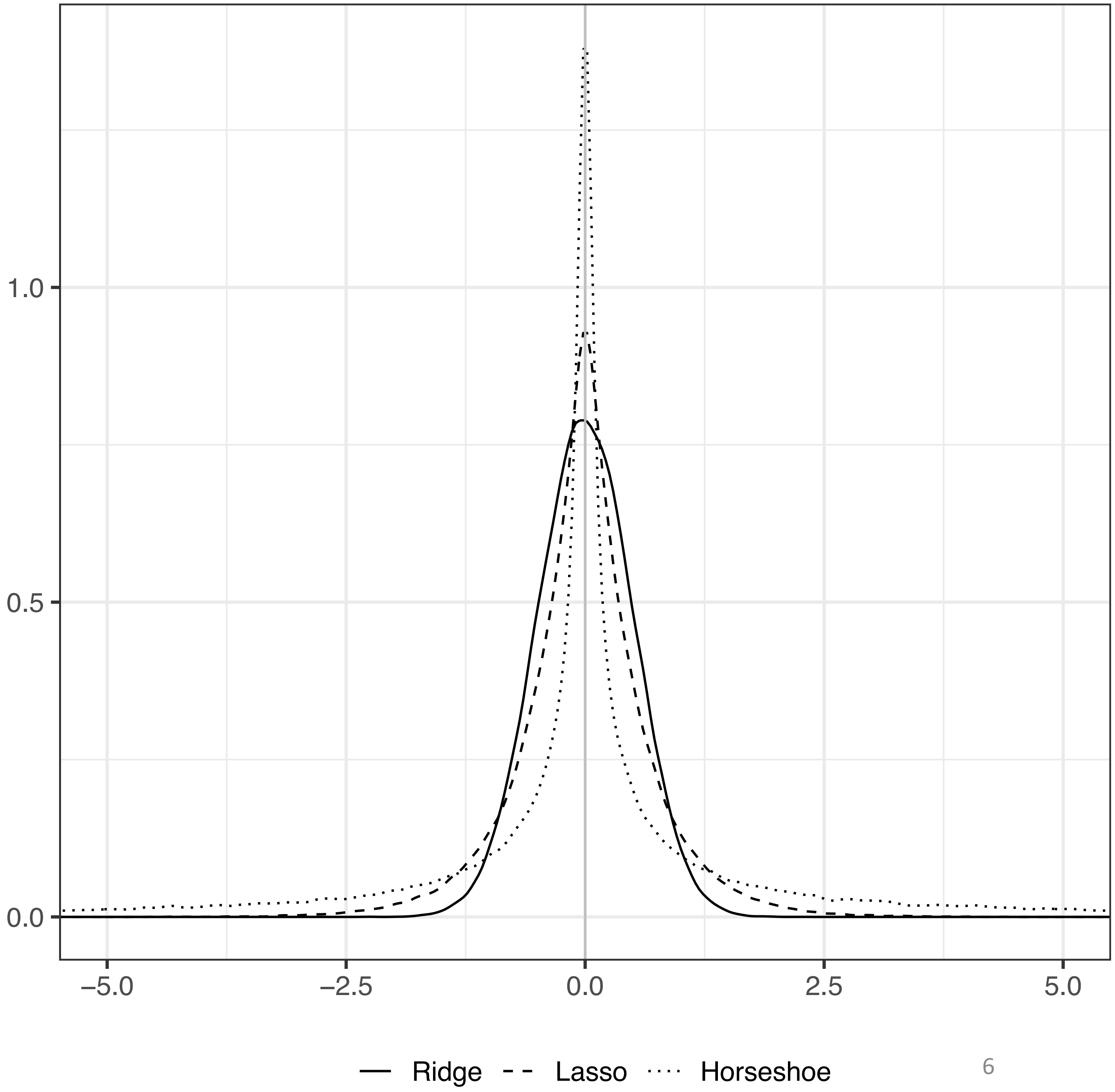
Shrinkage priors pull small coefficients towards zero, some priors in the same way as classical penalties.

See also van Erp et al. (2019)

Shrinkage priors

Properties:

1. Very peaked around zero
2. Heavy tails



Advantages Bayesian regularized SEM

- Intuitive interpretation
- Automatic uncertainty estimates
- Incorporation of prior information
- Flexibility in terms of priors considered
- Automatic estimation penalty parameter

	Ridge	*Lasso*	*Lasso extensions*	*Spike-and-slab*	*Other*
EFA				**Carvalho et al. (2008); Chen (2021);** West (2003)	Bhattacharya & Dunson (2011); Conti et al. (2014); Legramanti et al. (2020)
CFA	**Liang (2020)**; Lu et al, (2016); **Muthén & Asparouhov (2012)**; Vamvourellis et al. (2021)	**Chen, Guo et al. (2021)**; Pan et al. (2017)	**Chen, Guo et al. (2021)**	Lu et al, (2016)	
Neural drift diffusion		Kang et al. (2022)			
Item response models	Vamvourellis et al. (2021)	**Chen (2020)**			
Multiple group factor models	**Shi et al. (2017)**		Chen, Bauer et al. (2021)	Chen, Bauer et al. (2021)	
Non-linear SEM		Guo et al. (2012)	Brandt et al. (2018); Feng, Wang et al. (2015)	Brandt et al. (2018)	
General SEM		Feng, Wu et al. (2015); Feng, Wu et al. (2017)	Feng, Wu et al. (2015); Feng, Wu et al. (2017)		
Quantile SEM		Feng, Wang et al. (2017)	Feng, Wang et al. (2017)		
Latent growth curve models			Jacobucci & Grimm (2018)		

Software for Bayesian regularized SEM

User-friendly

- `Mplus`: only ridge available
- R-package `blcfa`: covariance lasso for CFA (requires `Mplus`)
- R-package `blavaan`: only ridge available
- R-package `LAWLB`: adaptive and covariance lasso for CFA
- R-package `infinitefactor`: aimed at high-dimensional EFA

Not so user-friendly:

- Description MCMC algorithm or (R) implementation online
- Code it yourself (Stan or JAGS)

Added difficulty: prior should be informative enough to identify the model

An illustration

Aim: Comparing various methods on *realistic* data.

Data (N = 1495) was generated based on an observed covariance matrix of 65 items from the ADaptive Ability Performance Test (ADAPT; Jonker et al., 2022)

EFA resulted in an eight factor solution, based on which the first three factors and 36 items were used for further analysis.

Illustration: Methods

I compared the following methods:

-`lslx`

mcp, lasso, and elastic net penalties

-`blavaan`

ridge prior with fixed sd

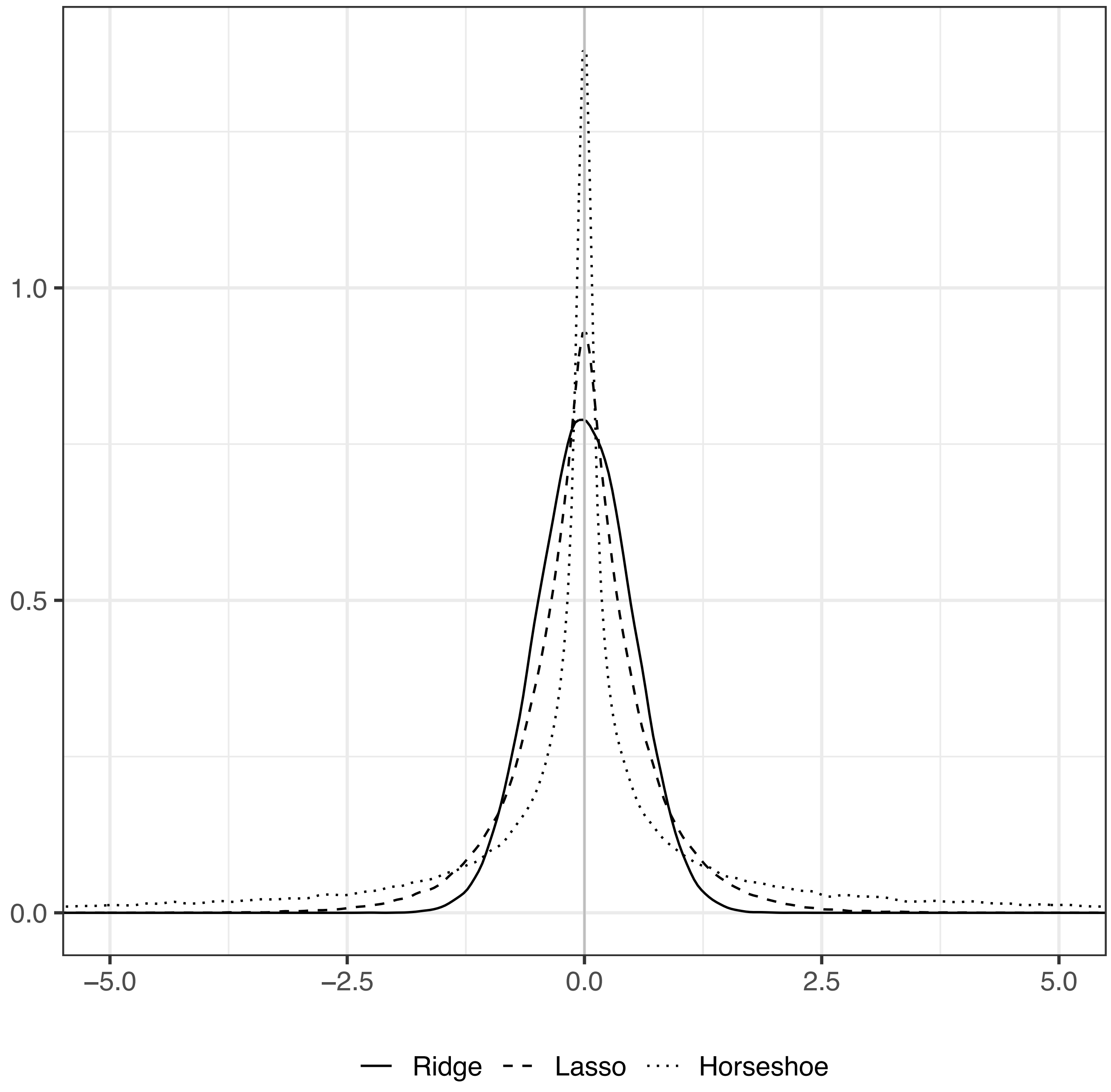
-`lawlb`

lasso and adaptive lasso

-`Stan`

ridge prior with estimated sd and regularized horseshoe prior

Shrinkage priors compared (1)



Shrinkage priors compared (2)

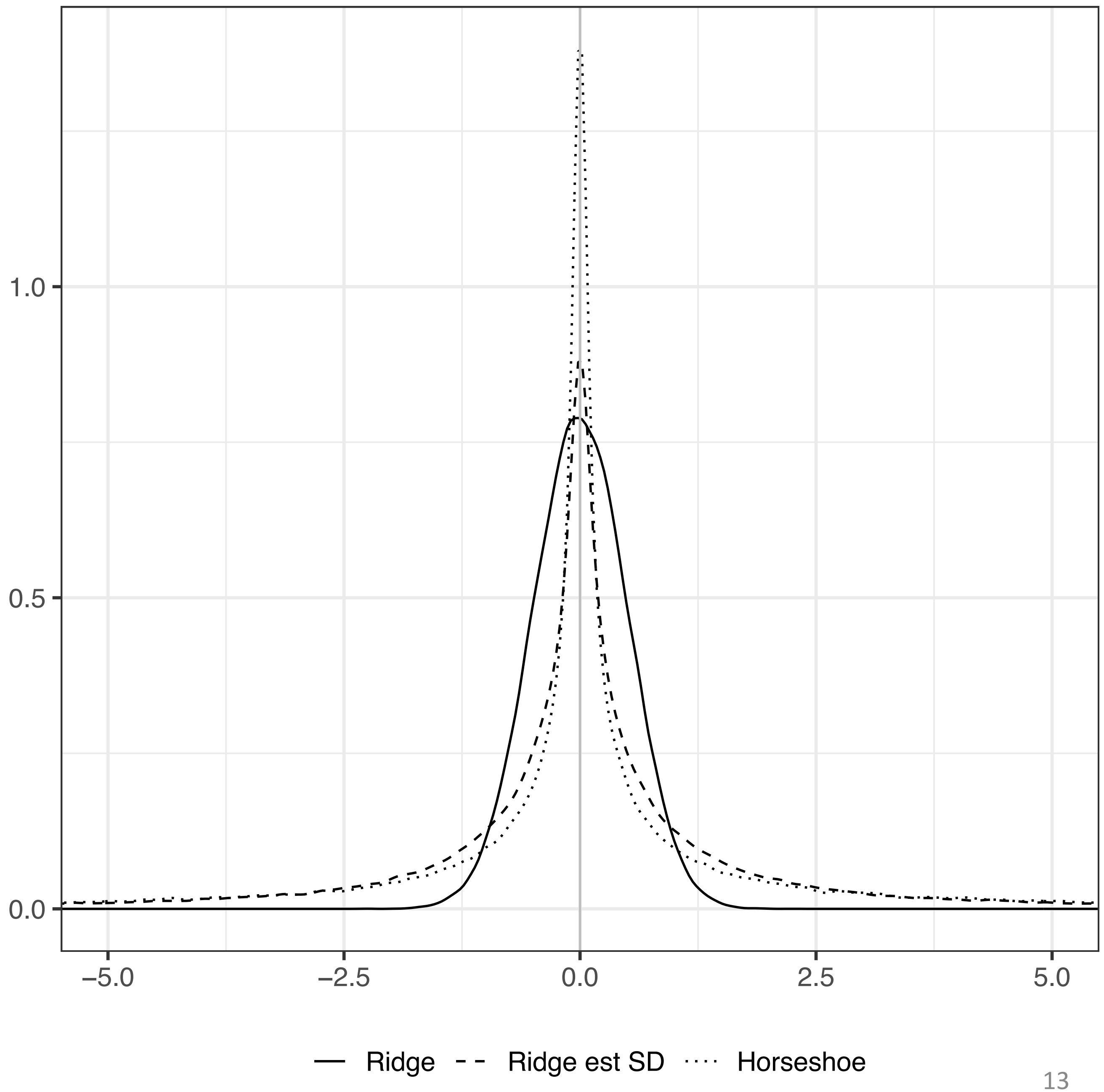


Illustration: General observations

- Model specification can be a hassle (especially for large models)
- Some packages allow simultaneous regularization of cross-loadings and residual covariances. This was not tested here.
- Some packages provide information on fit indices but these can be impacted by prior choice (see Edwards and Konold, 2023)

- Variational Bayes seems way off, especially for the ridge
- Practically no differences between the lasso and adaptive lasso
- Providing a prior guess for the number of substantial cross-loadings in the regularized horseshoe does not matter for the results

Illustration: Estimated cross-loadings

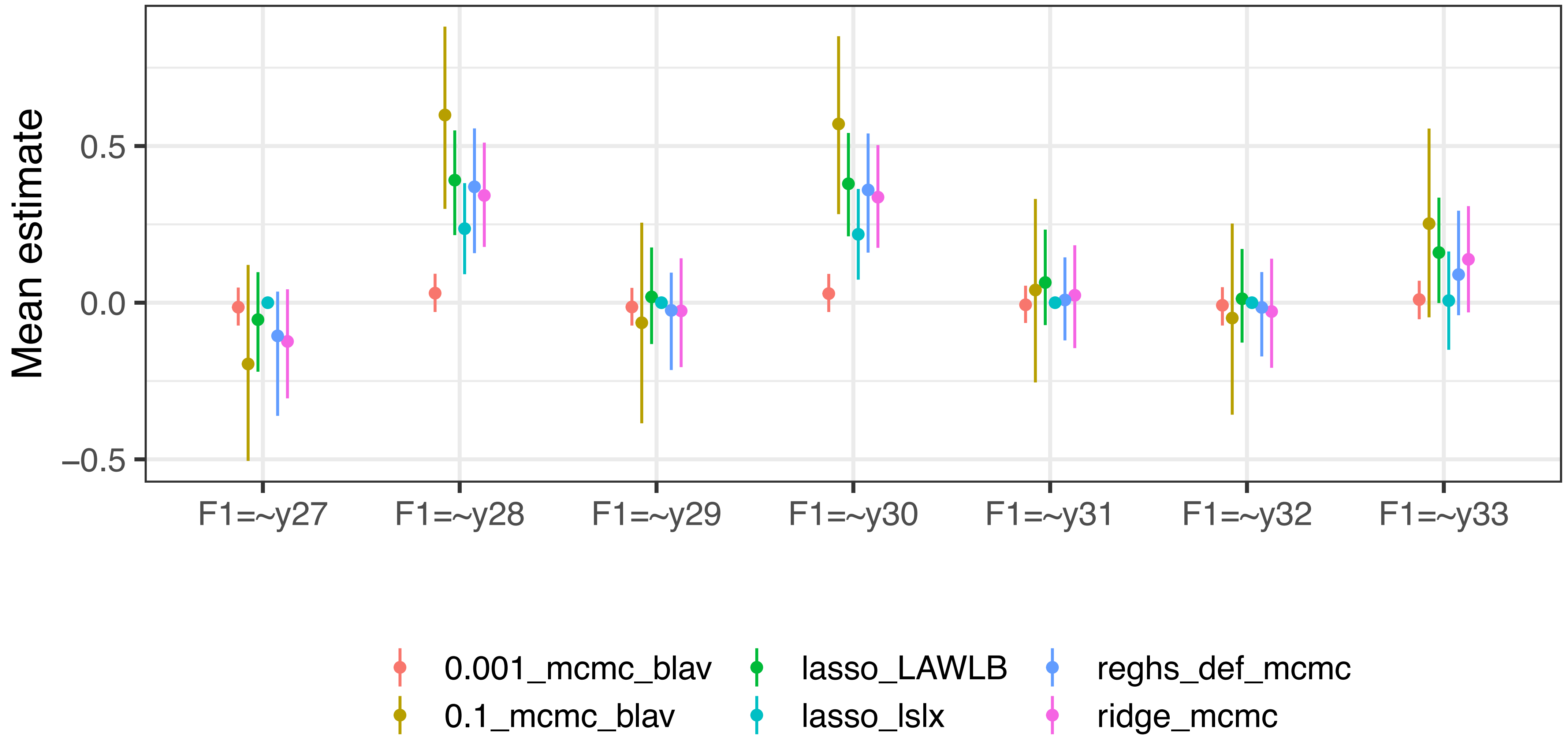
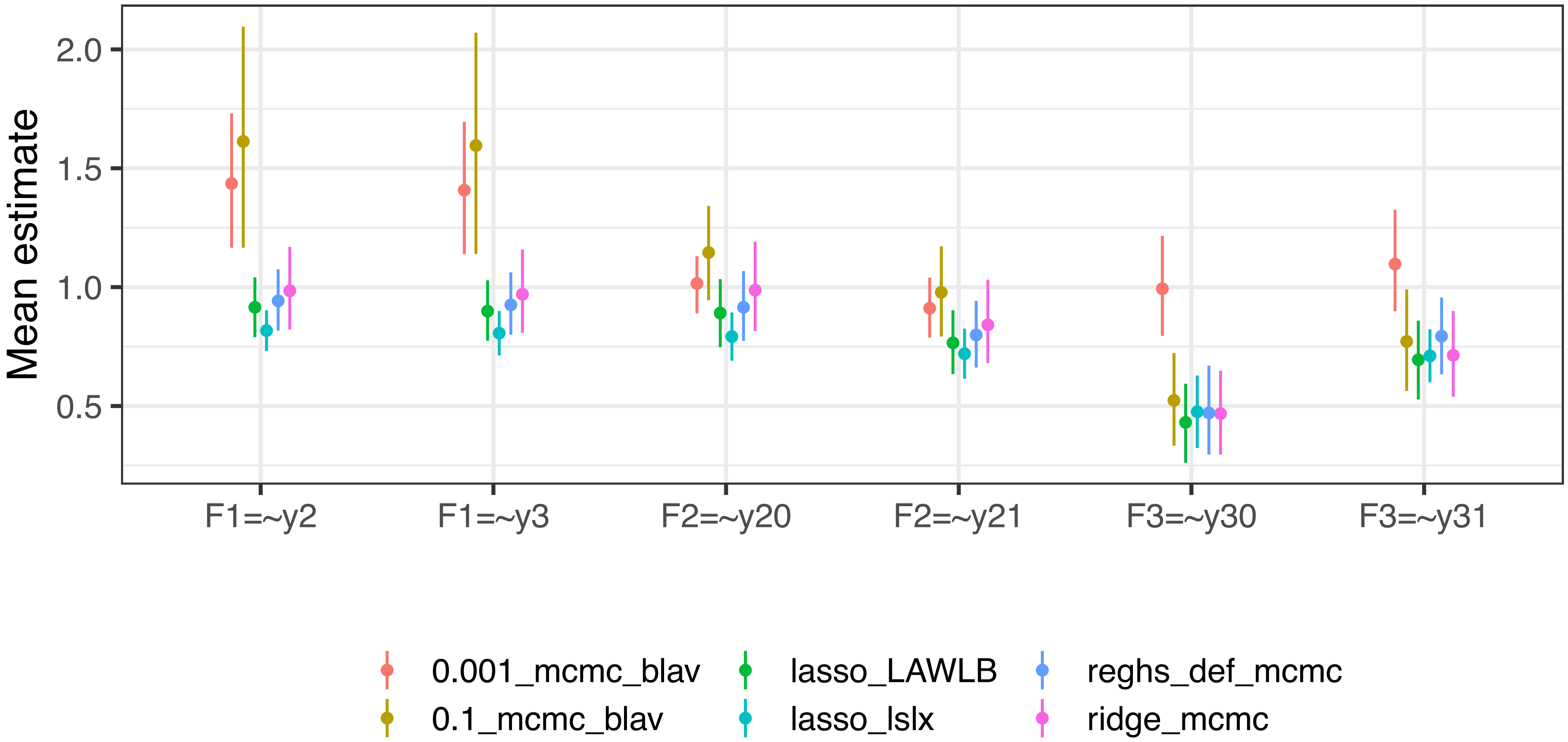


Illustration: Estimated main loadings



Current capabilities and constraints

- There are quite some possibilities for simple Bayesian regularized SEM (e.g., ridge or lasso).
- More advanced priors are possible, but not yet implemented in user-friendly software.
- Different priors can lead to different estimates, which in turn influence other parameter estimates in the model.
- Existing methods for parameter selection are arbitrary (See also Zhang, Pan & Ip, 2021)
- In addition, general guidelines based on simulation studies are lacking. Long run times and convergence issues can make simulation studies difficult.

Thank you!

Check out the preprint: *https://psyarxiv.com/92vh8*

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