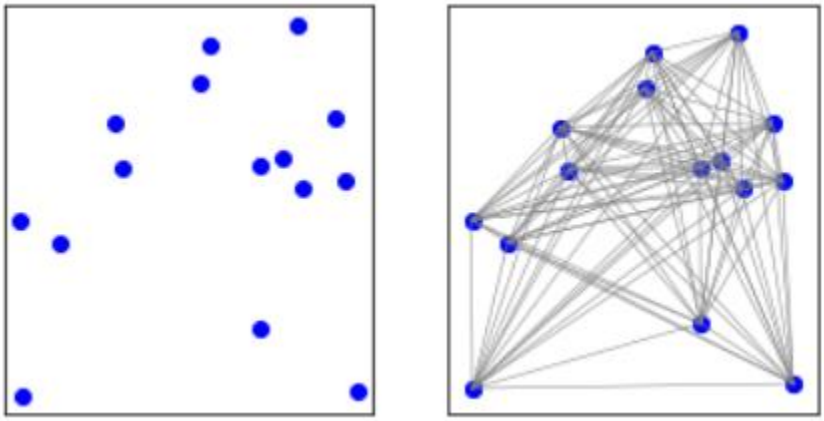
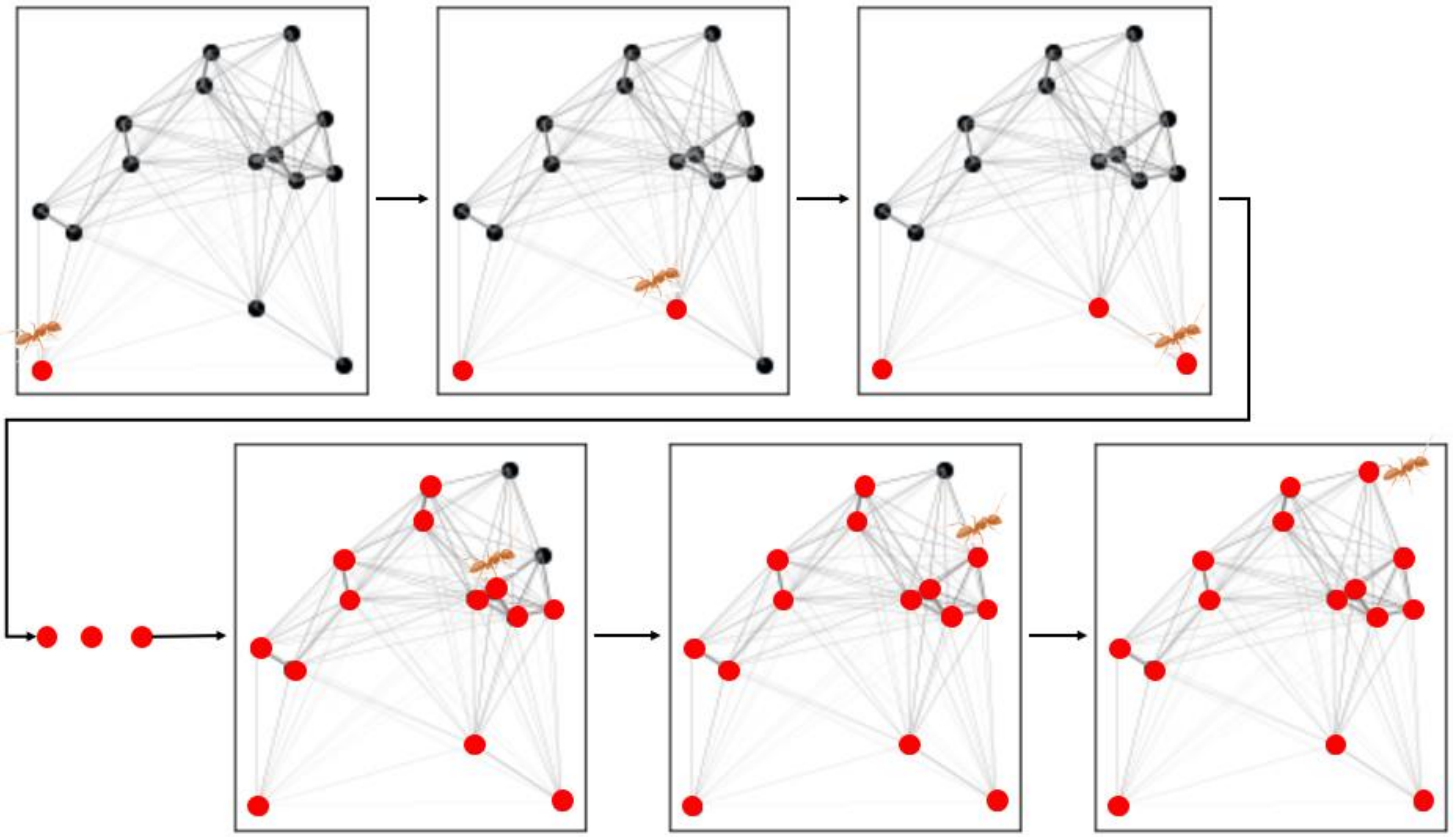
Ant Colony Optimization (ACO)

Ant Colony Optimization (ACO) solves problems of finding paths in graphs. It is inspired by the ants' behavior when searching for food. The ants leave pheromones that allow other ants to follow the path to food. The more ants go for a specific path, the more pheromones.

The following example aims to find the shortest path to visit all nodes without repetition.



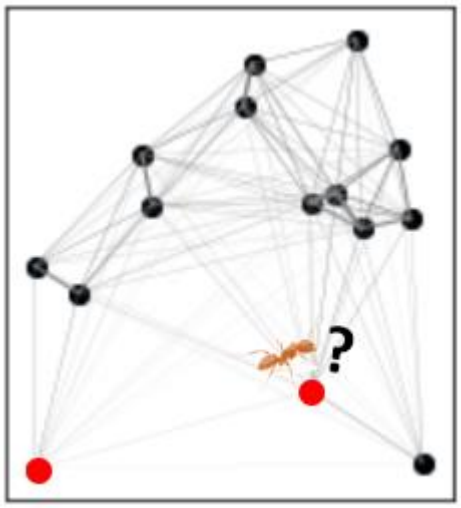
In this algorithm, an artificial ant must simulate a path starting at a specific point. It moves node by node, choosing based on the pheromones of each path.



First, we start with the abbreviations:

- c_{ij}: path from the node i to the node j
- τ_{ij}: pheromones in the path from the node i to the node j
- η_{ij}: heuristic in the path from the node i to the node j
- ρ: evaporation rate, between 0 and 1

Selecting the following node:



$$P(c_{ik}) = \frac{\tau_{ik}^{\alpha} * \eta_{ik}^{\beta}}{\sum_{j \in N_i} \tau_{ij}^{\alpha} * \eta_{ij}^{\beta}}$$

Where N_i represents all the nodes where the ant can move in the next step. In the image, it represents all the black nodes. α and β are parameters that control the relative importance of the pheromone versus the heuristic information. $\eta_{ik} = 1/d_{ik}$, where d_{ik} is the length of component c_{ik}.

At the beginning of the algorithm:

- All the pheromones can be initialized with a small value.

Move the ants one by one:

- Ants start to move around the graph node by node using the previous equation.
- All the ants must move to the graph.

When all the ants record the graph:

- The pheromones must be updated.
- Ants deposit pheromones to their path proportional to its distance.
- The pheromones evaporate

The ant a update its path based on the following equation:

$$\Delta\tau_{ij}^{a} = \begin{cases} \frac{1}{L_a} & if\ ant\ a\ used\ c_{ij} \\ 0 & otherwise \end{cases}$$

where, L_a is the total length of the path used by the ant a.

All the pheromones evaporate based on the following equation:

$$\tau_{ij} = (1 - \rho) * \tau_{ij}$$

Example: