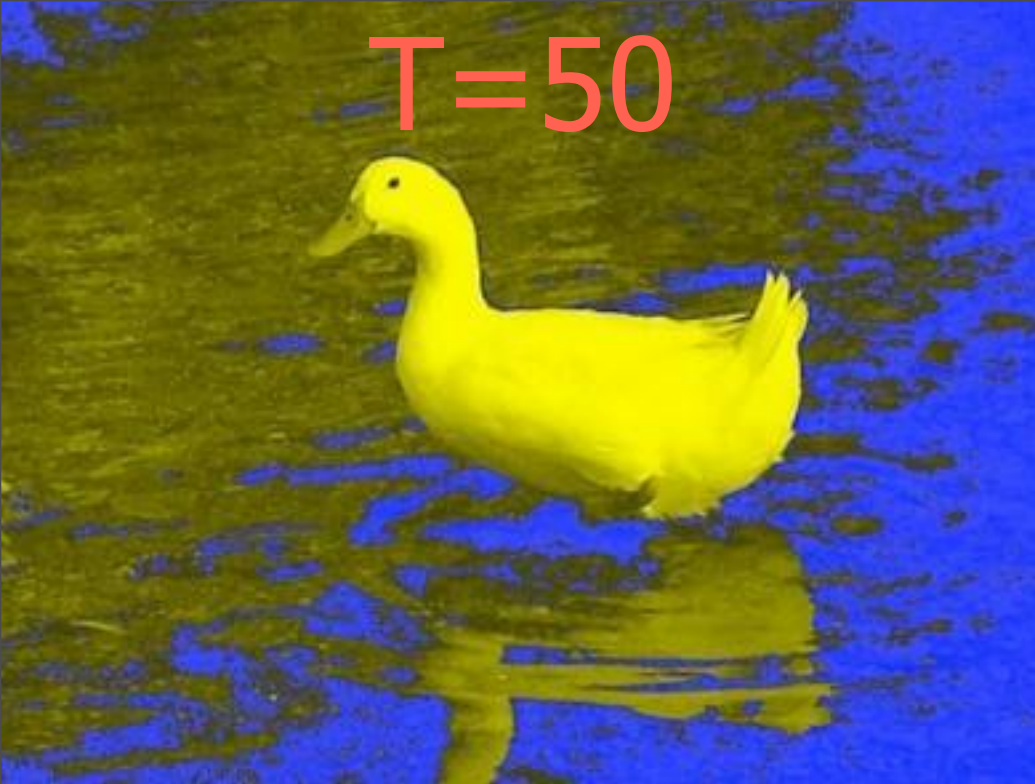
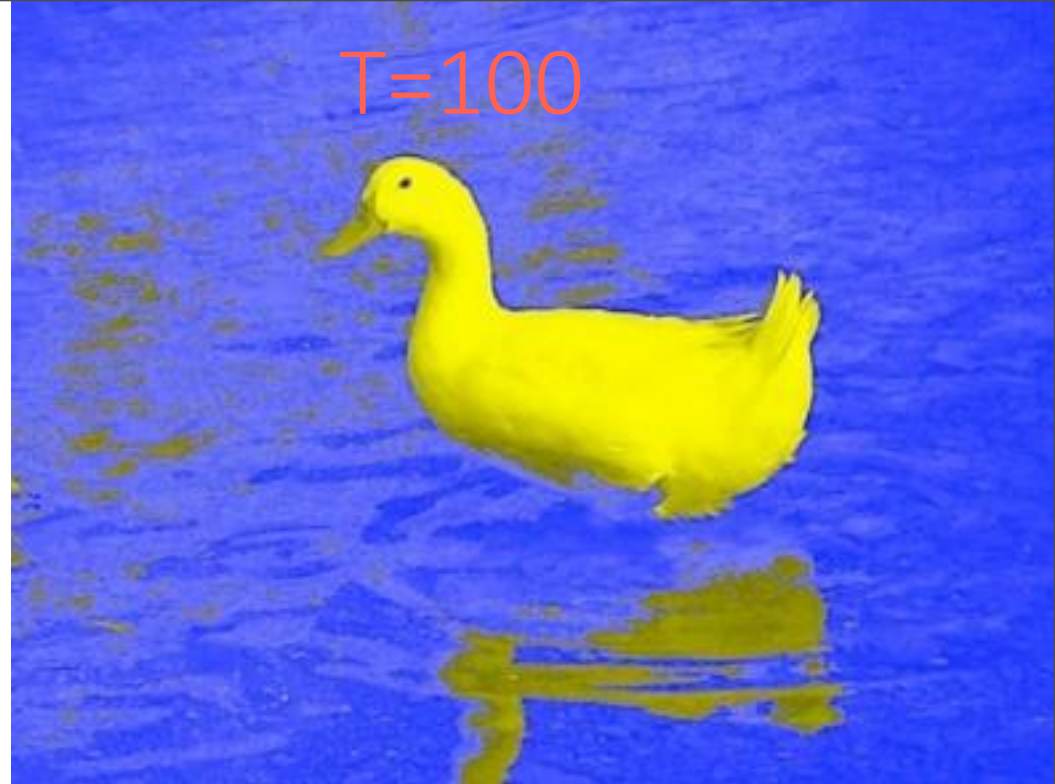




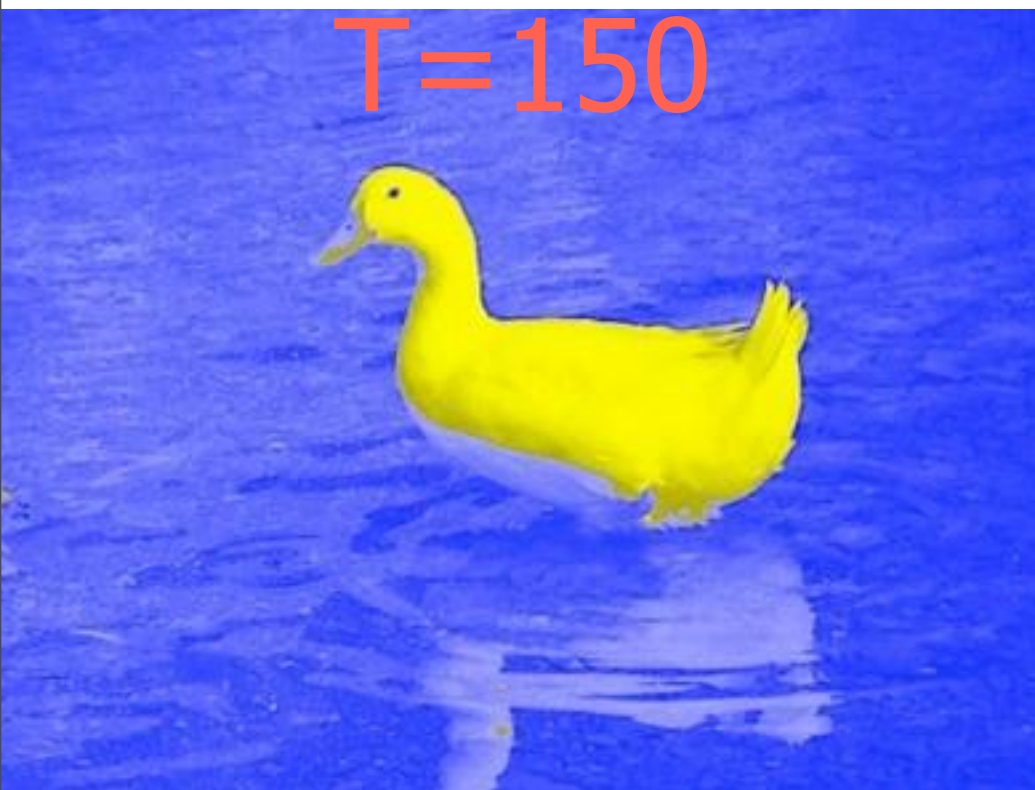
$T=50$



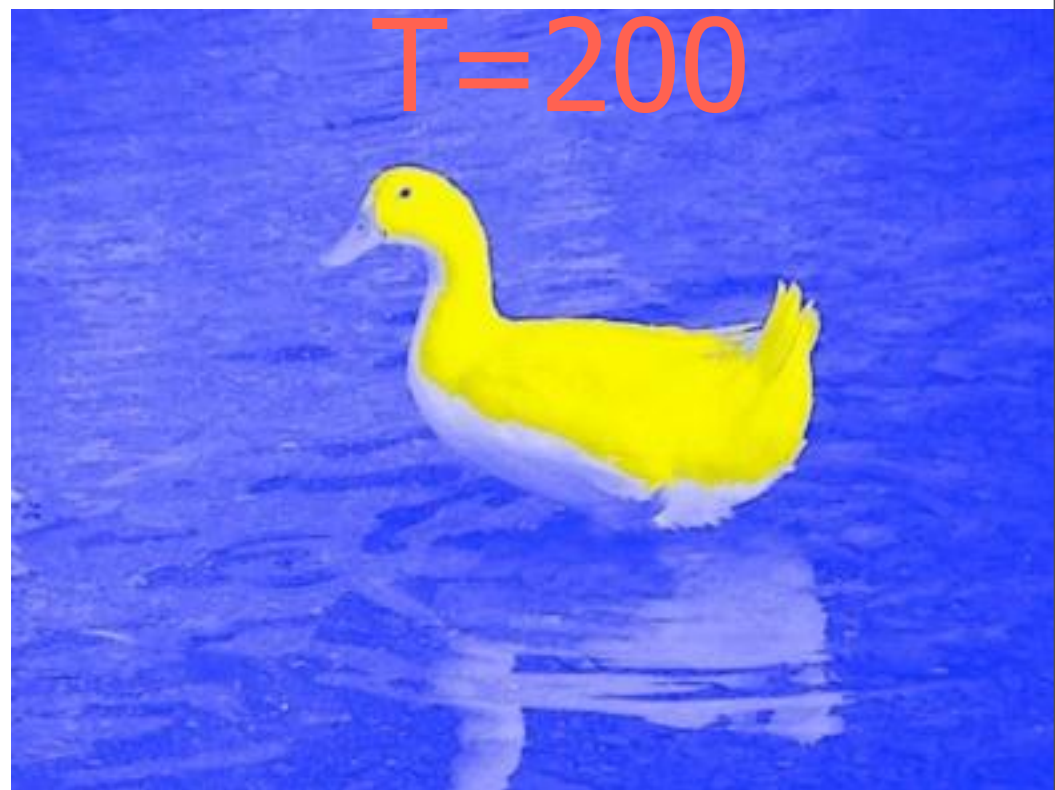
$T=100$



$T=150$

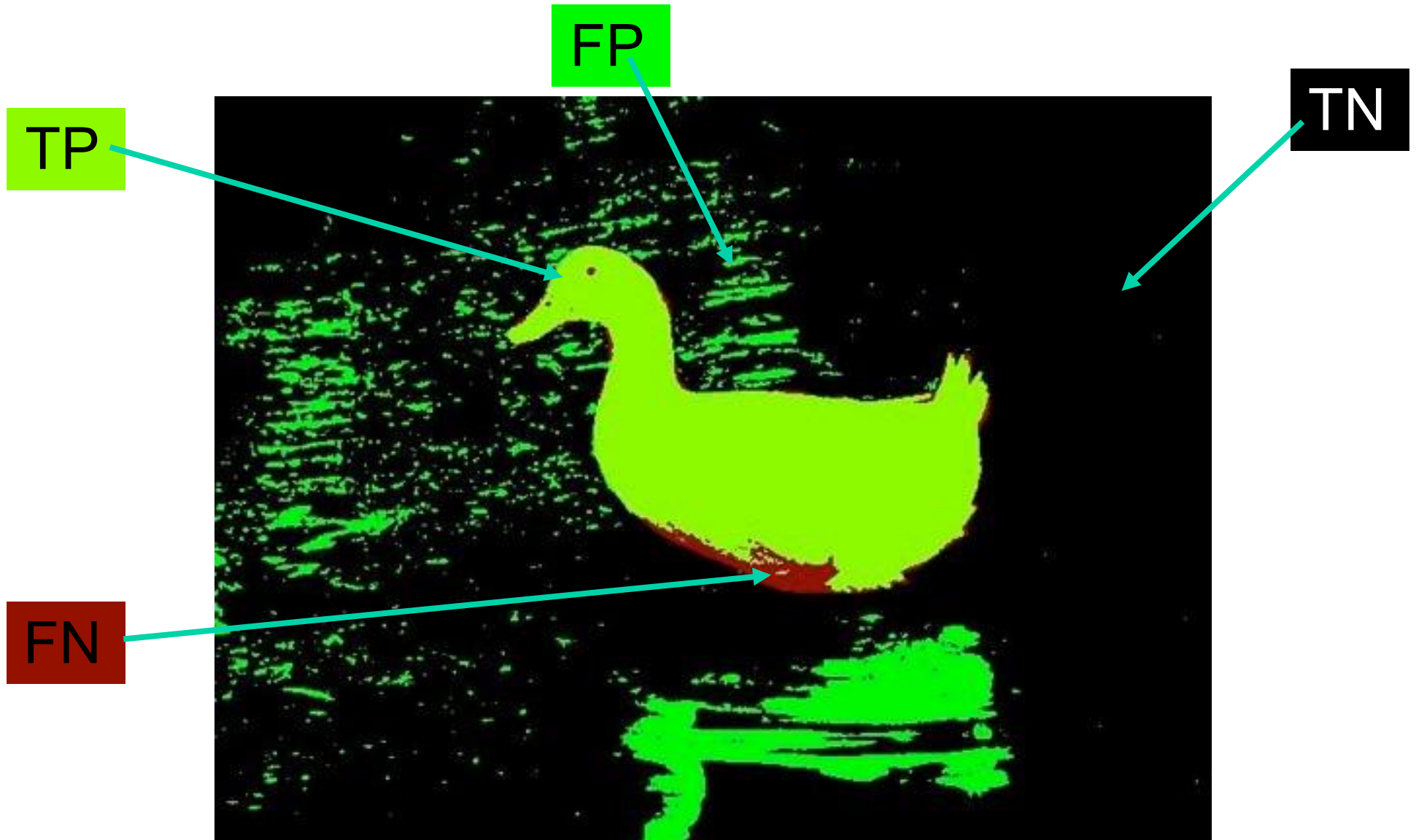


$T=200$





# Classification outcomes

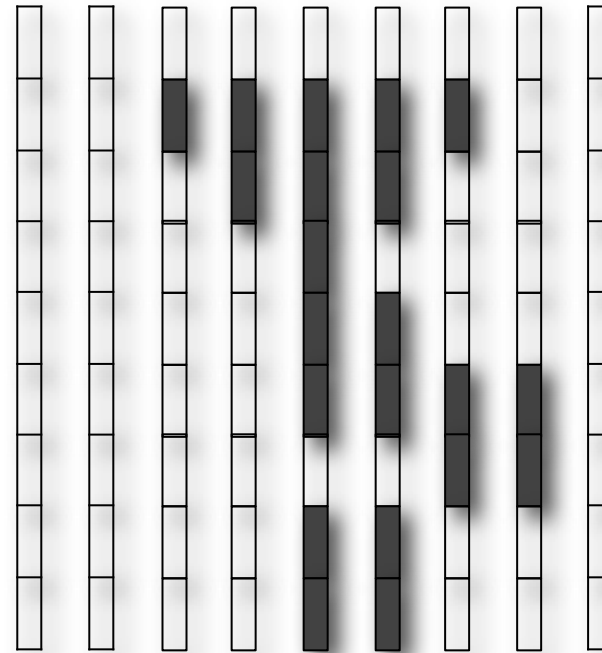


# Limitations of Thresholding

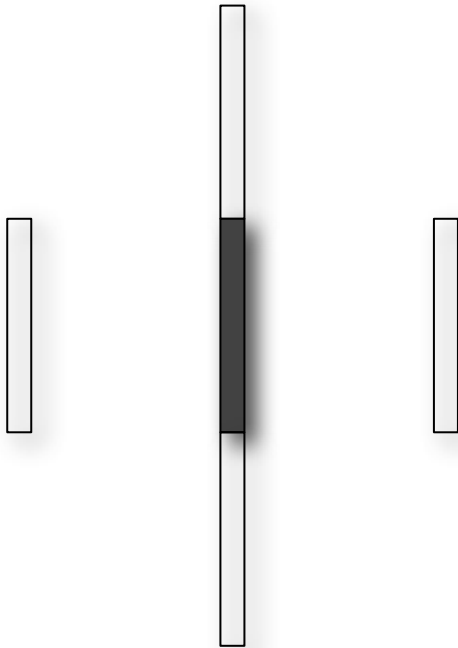
- Why can we segment images much better by eye than through thresholding processes?
- We might improve results by considering image **context**: Surface Coherence

# Pixel Connectivity

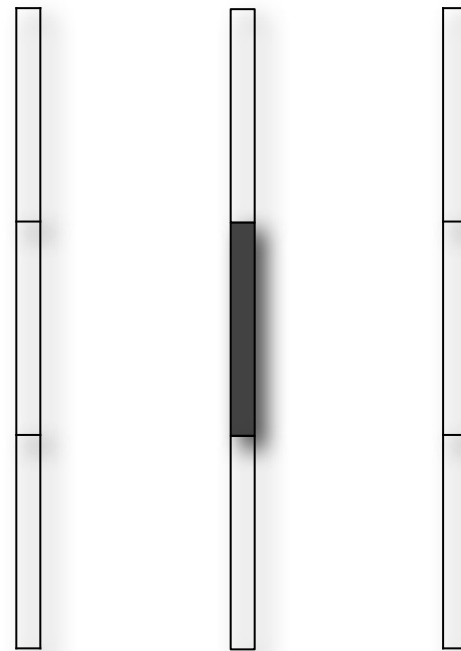
- We need to define which pixels are neighbours.
- Are the dark pixels in this array connected?



# Pixel Neighborhoods



4-neighbourhood



8-neighbourhood

# Pixel Paths

- A 4-connected path between pixels  $p_1$  and  $p_n$  is a set of pixels  $\{p_1, p_2, \dots, p_n\}$  such that  $p_i$  is a 4-neighbour of  $p_{i+1}$ ,  $i=1, \dots, n-1$ .
- In an 8-connected path,  $p_i$  is an 8-neighbour of  $p_{i+1}$ .

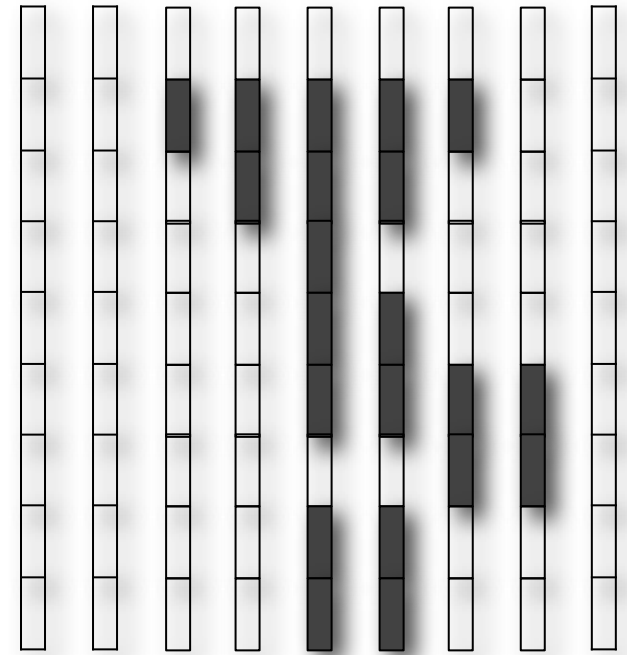
# Connected Regions

- A region is 4-connected if it contains a 4-connected path between every pair of its pixels.
- A region is 8-connected if it contains an 8-connected path between every pair of its pixels.



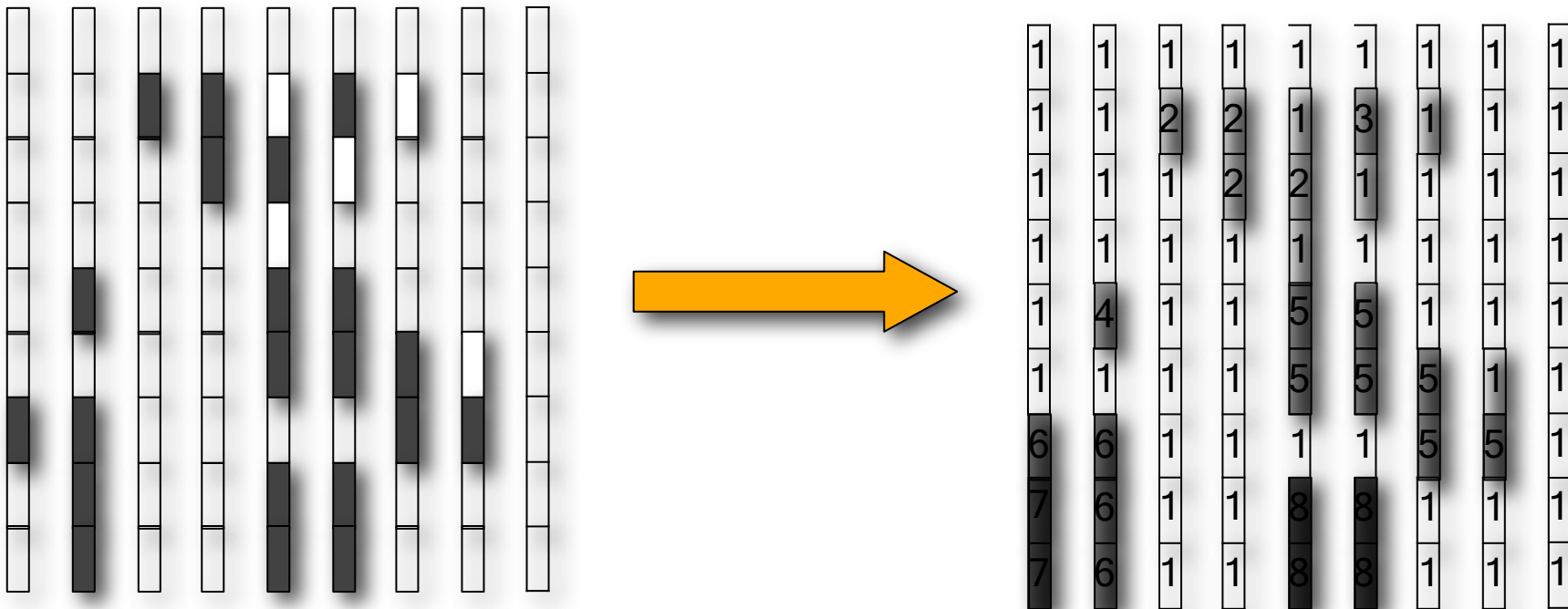
# Connected Regions

- Now what can we say about the dark pixels in this array?
- What about the light pixels?



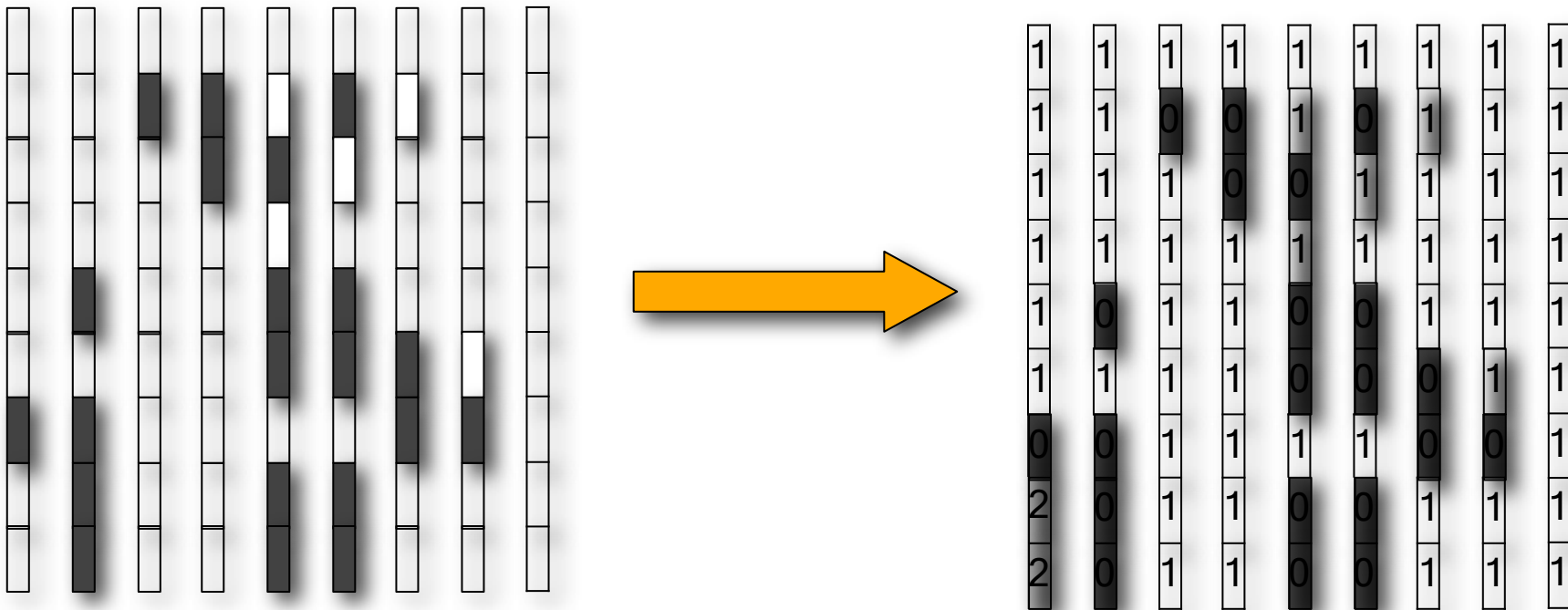
# Connected Components Labelling

- Labels each connected component of a binary image with a separate number.



# Foreground Labelling

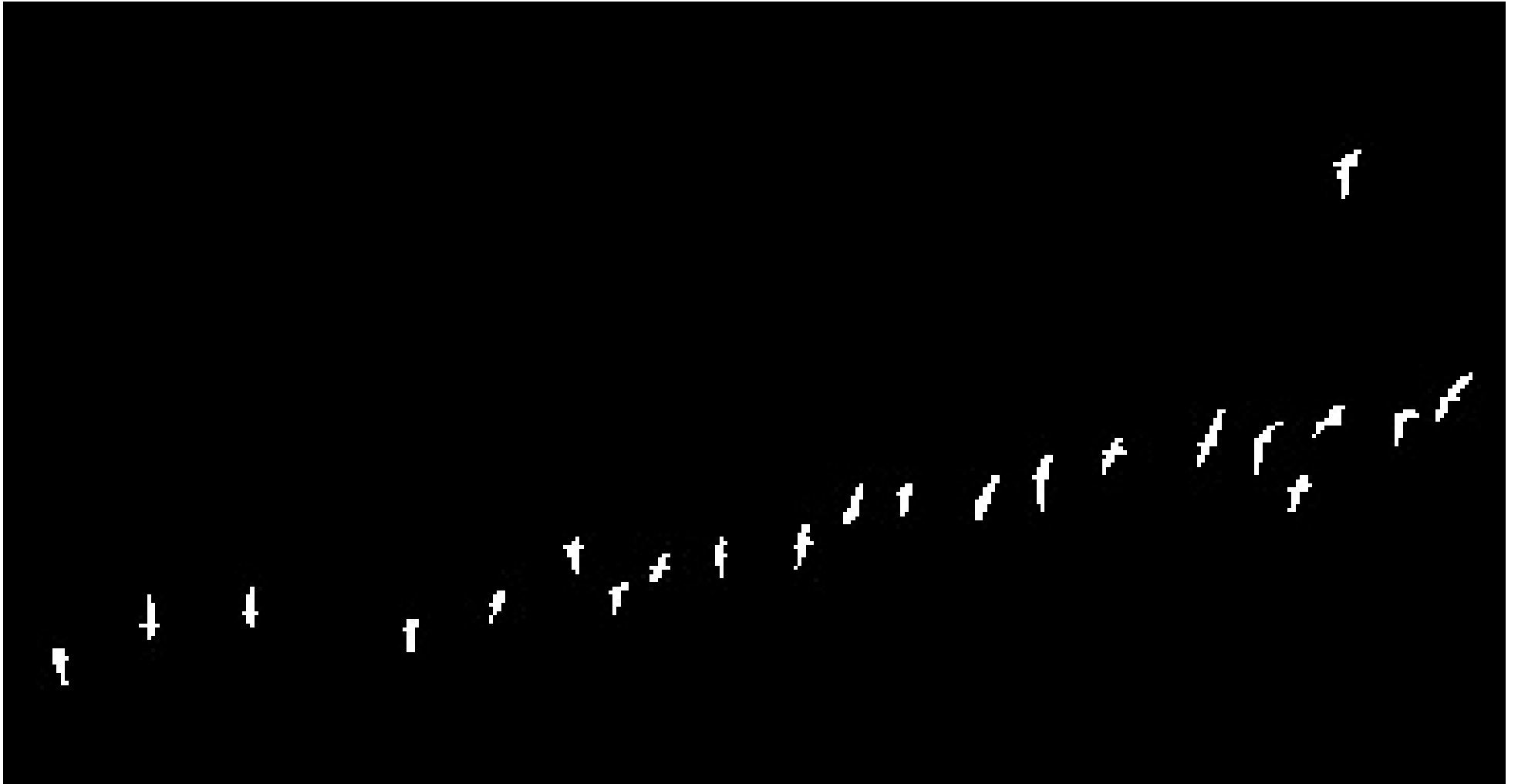
- Only extract the connected components of the foreground



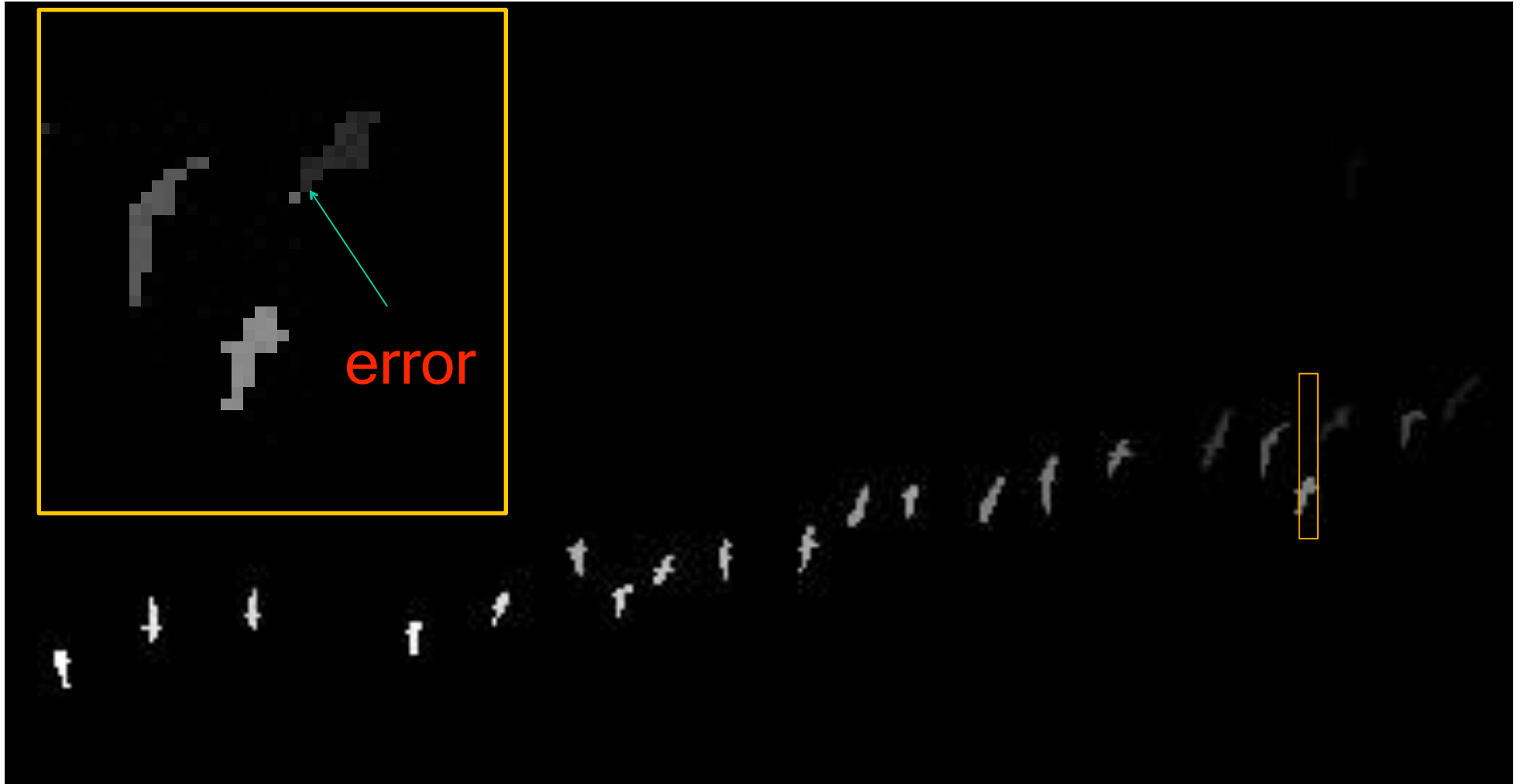
# Goose Detector



# Goose Detector

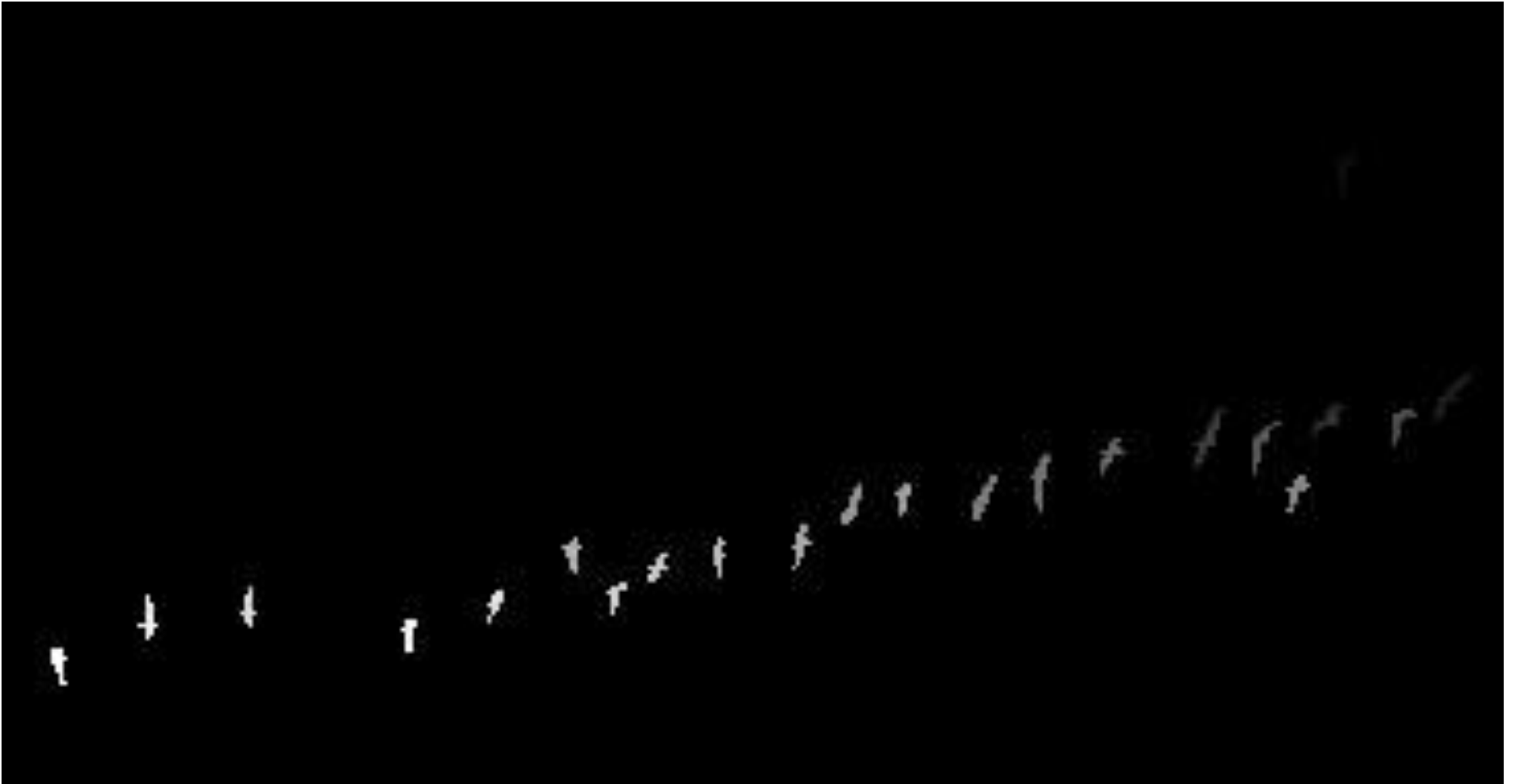


# Goose 4--components (26)



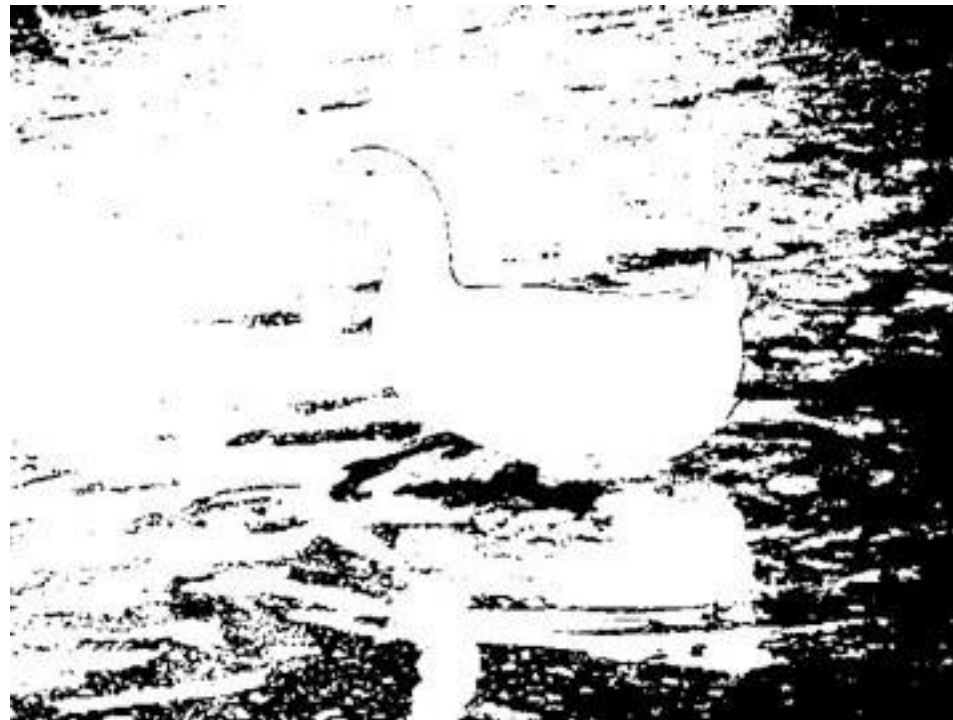


# Goose 8-components (22)



# Connected Components

- What happens if we use the connected components algorithm on this image?



- How might we improve our implementation?

# Region Growing

- Start from a seed point or region.
- Add neighbouring pixels that satisfy the criteria defining a region.
- Repeat until we can include no more pixels.

# Region Growing example

- Pick a single seed pixel.
- Inclusion test is a simple grey level threshold:

```
function test = include(p)  
    test = (p >= T);
```

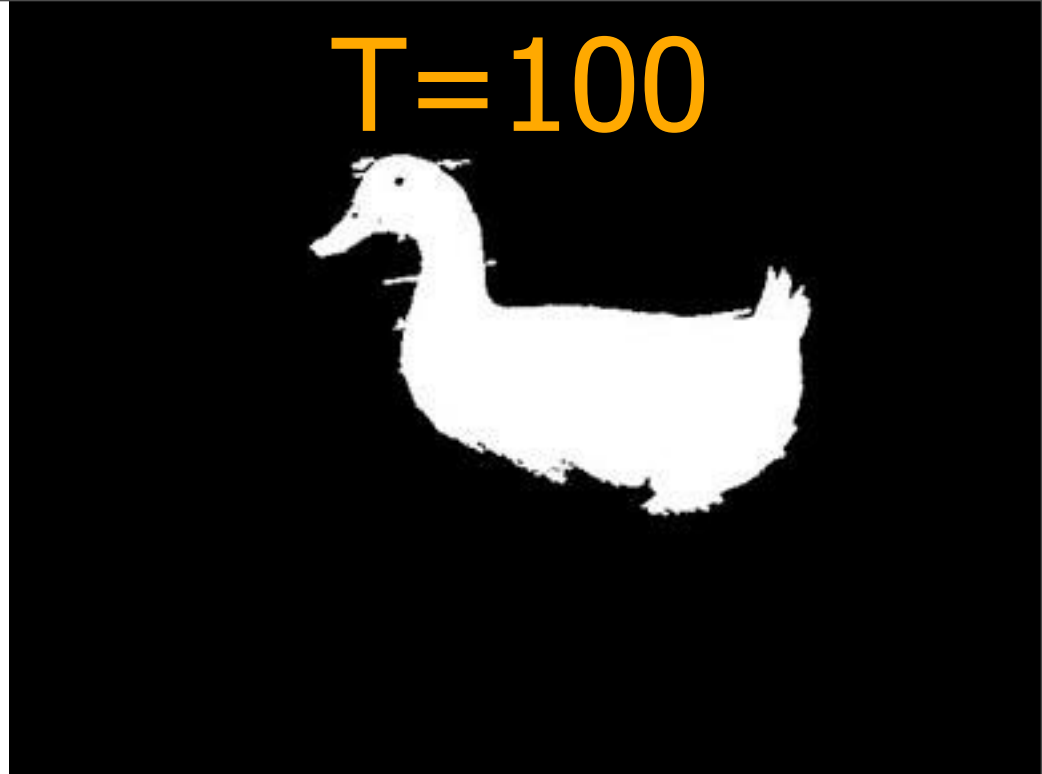


Seed  
pixel

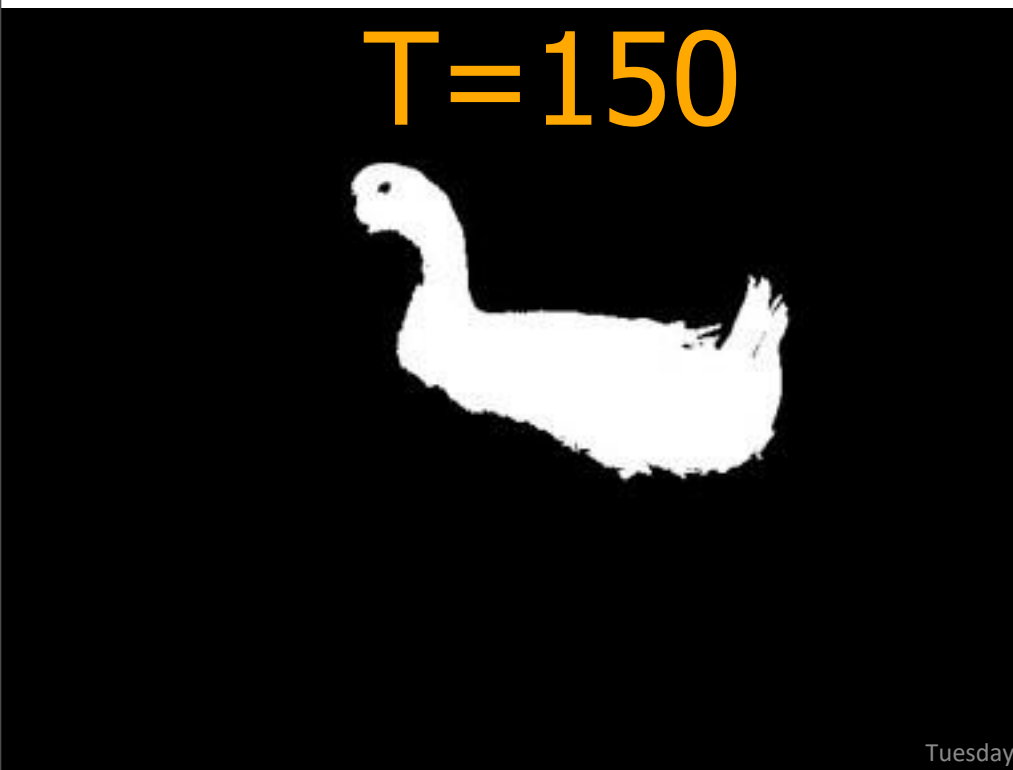
$T=50$



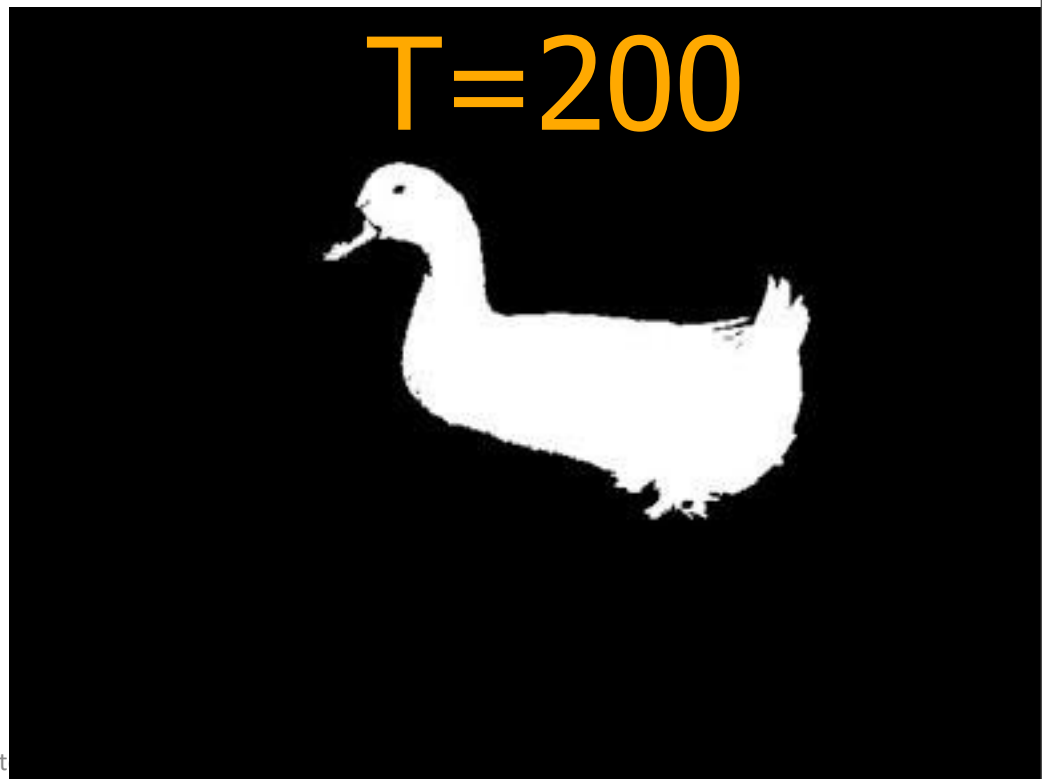
$T=100$



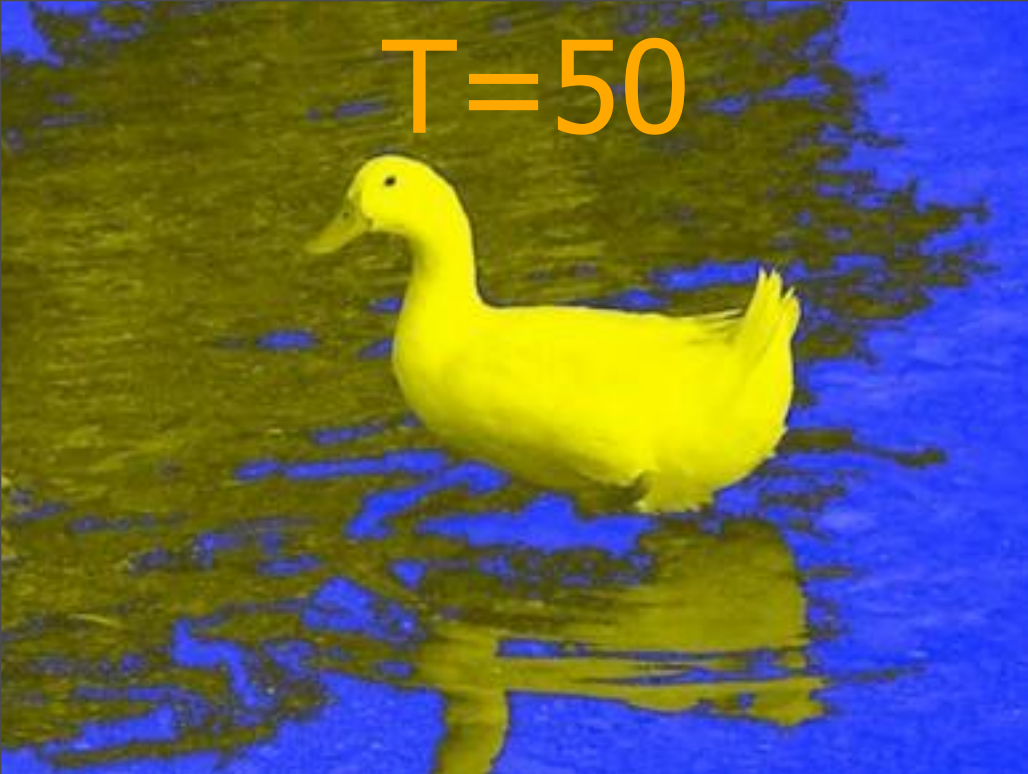
$T=150$



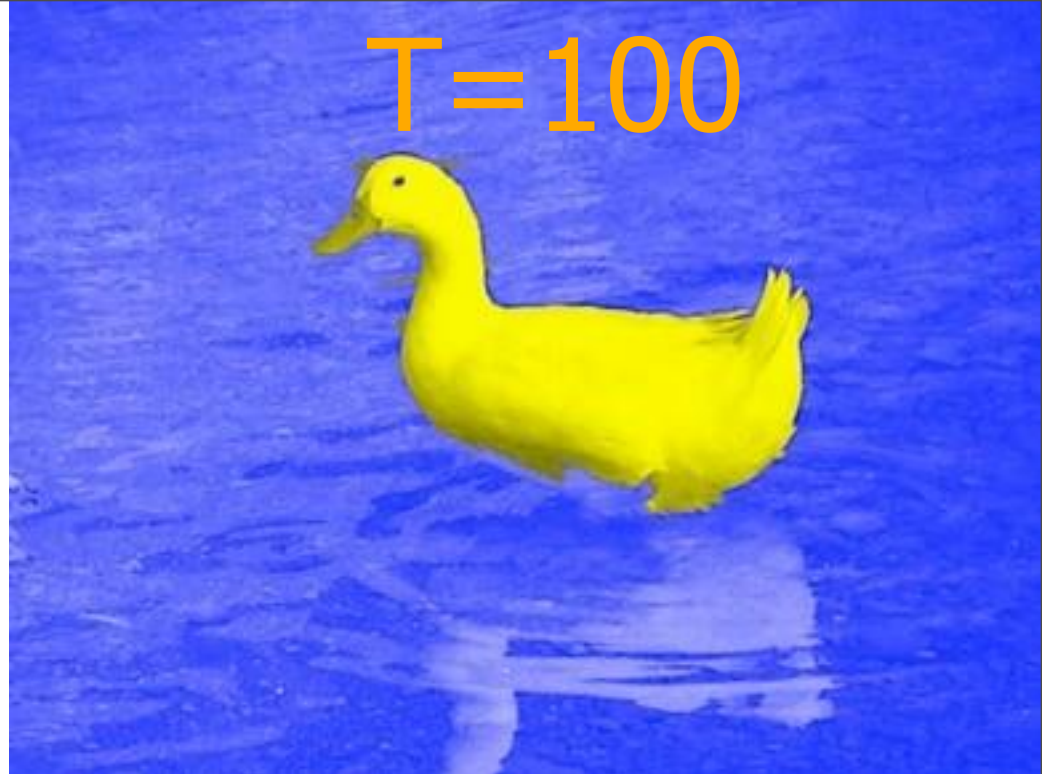
$T=200$



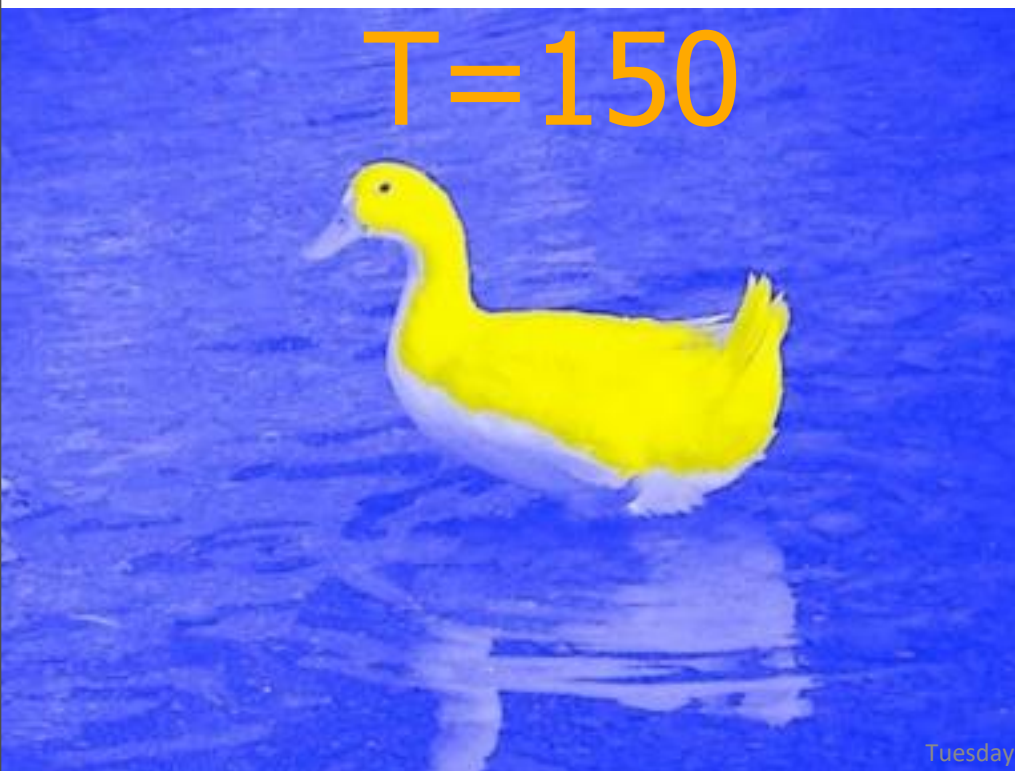
$T=50$



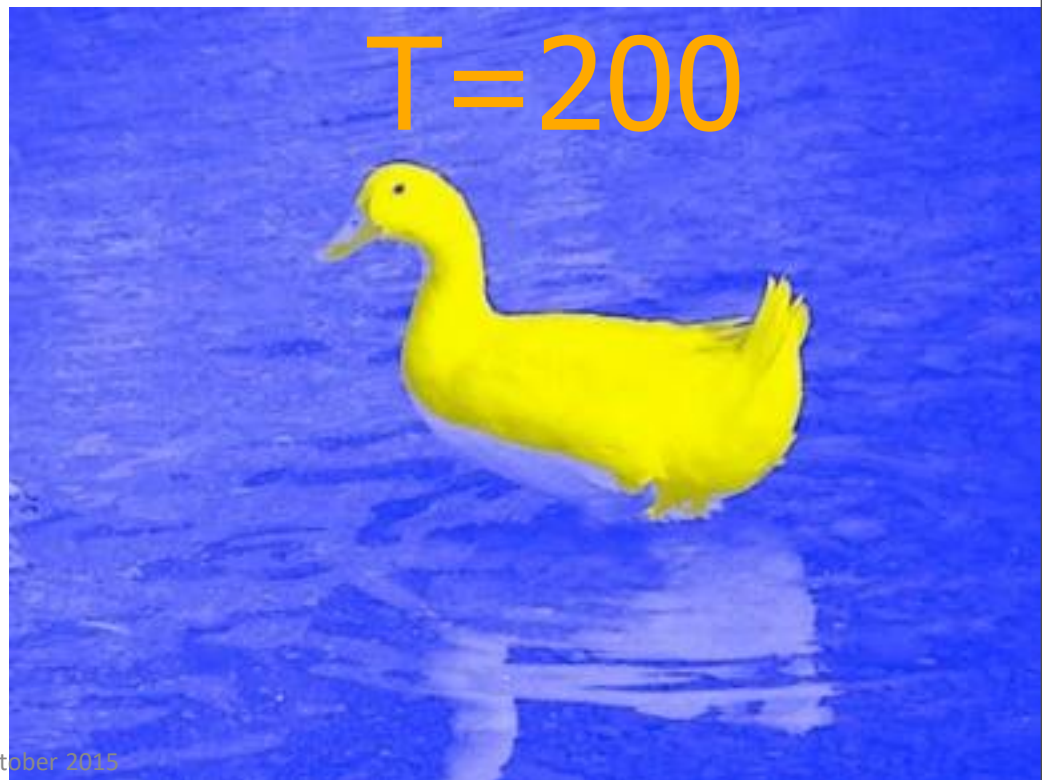
$T=100$



$T=150$



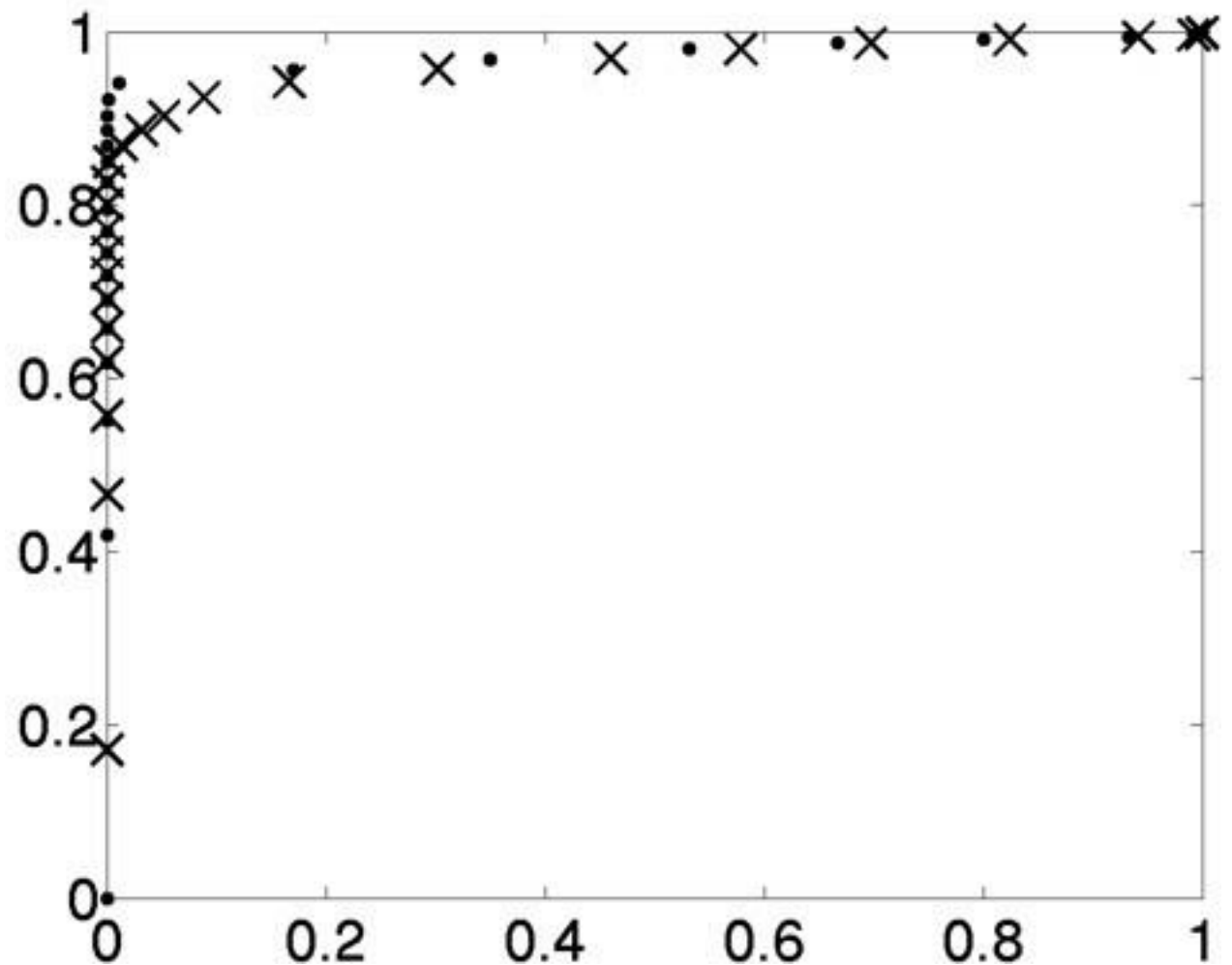
$T=200$





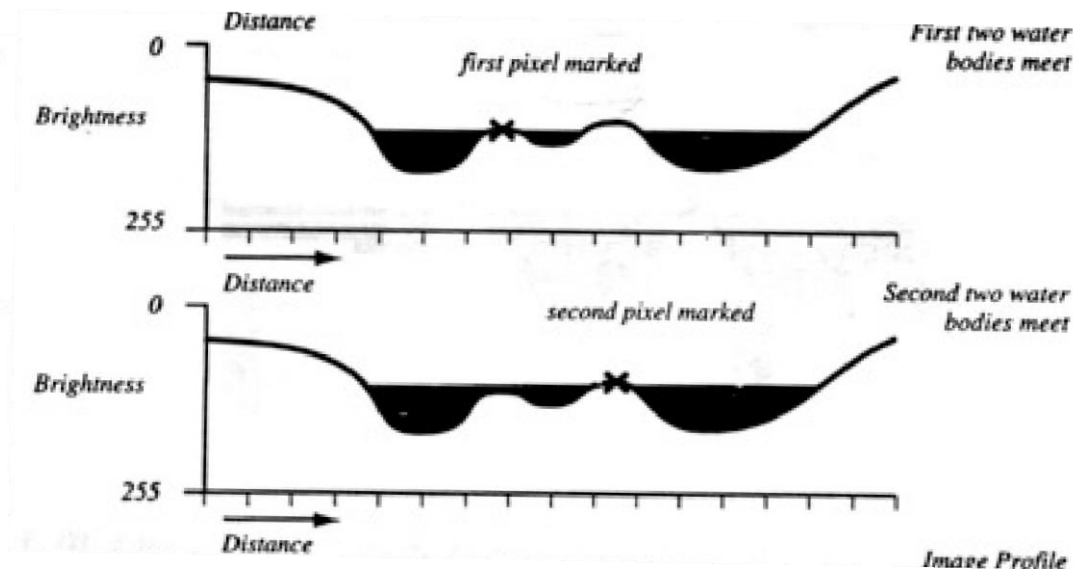
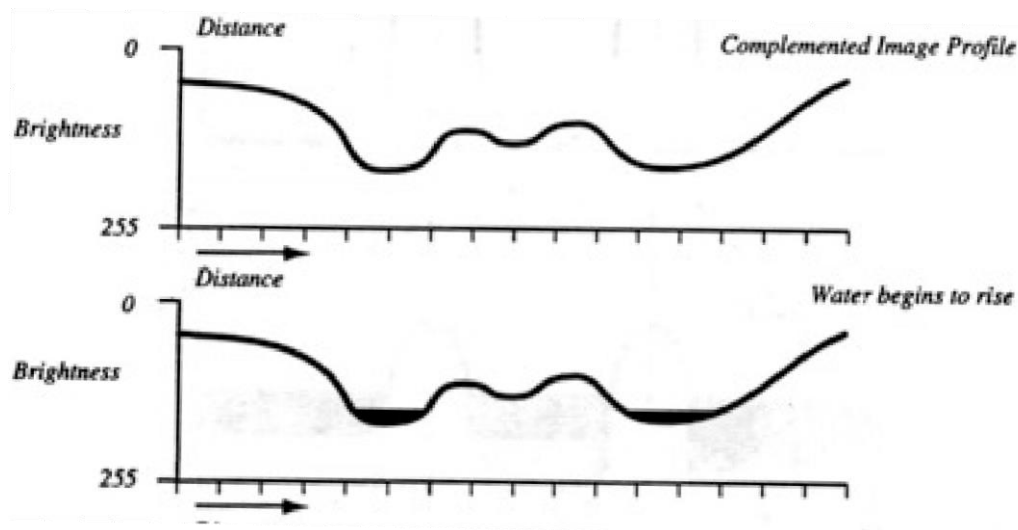
# ROC Curve

- Region growing
- × Thresholding

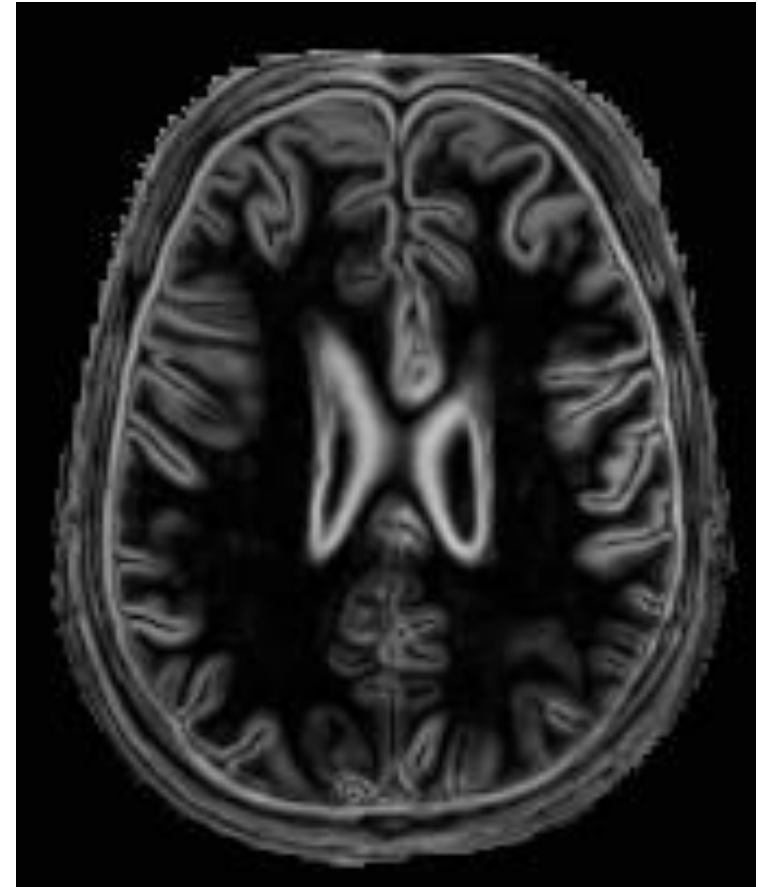
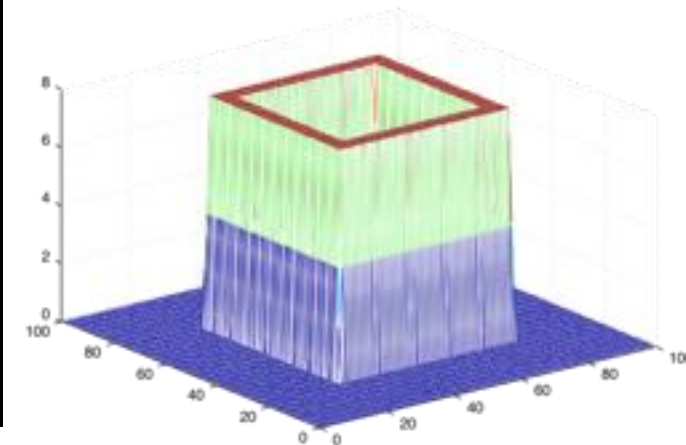
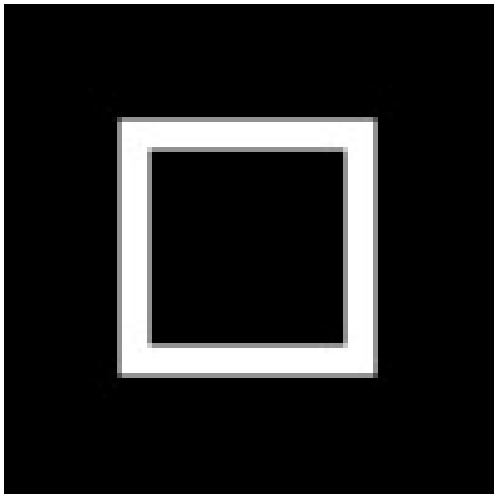
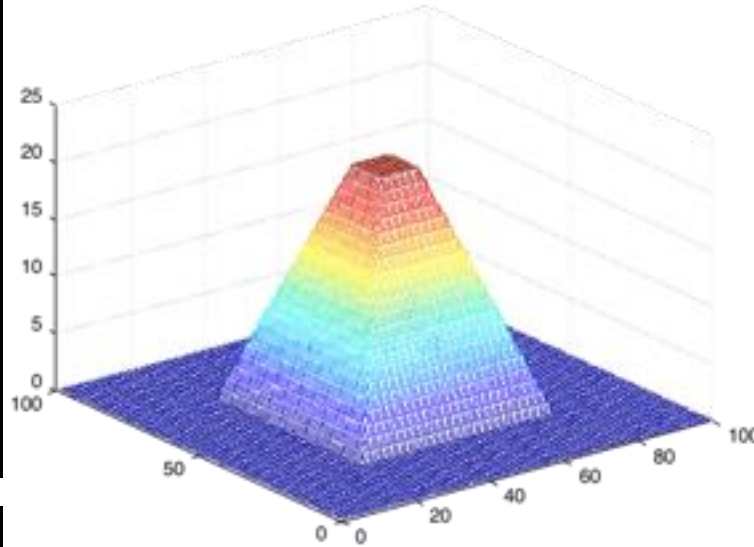
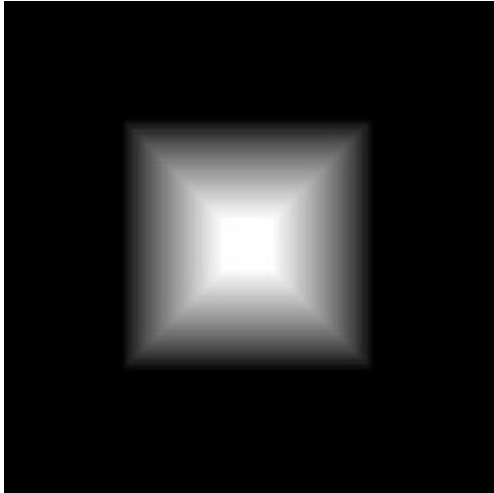


# Watershed Algorithm

- The objective is to find watershed lines.
- Suppose that a hole is punched in each regional minimum and that the entire topography is flooded from below by letting water rise through the holes at a uniform rate.
- When rising water in distinct catchment basins is about to merge, a dam is built to prevent merging. These dam boundaries correspond to the watershed lines.



# Watershed Segmentation

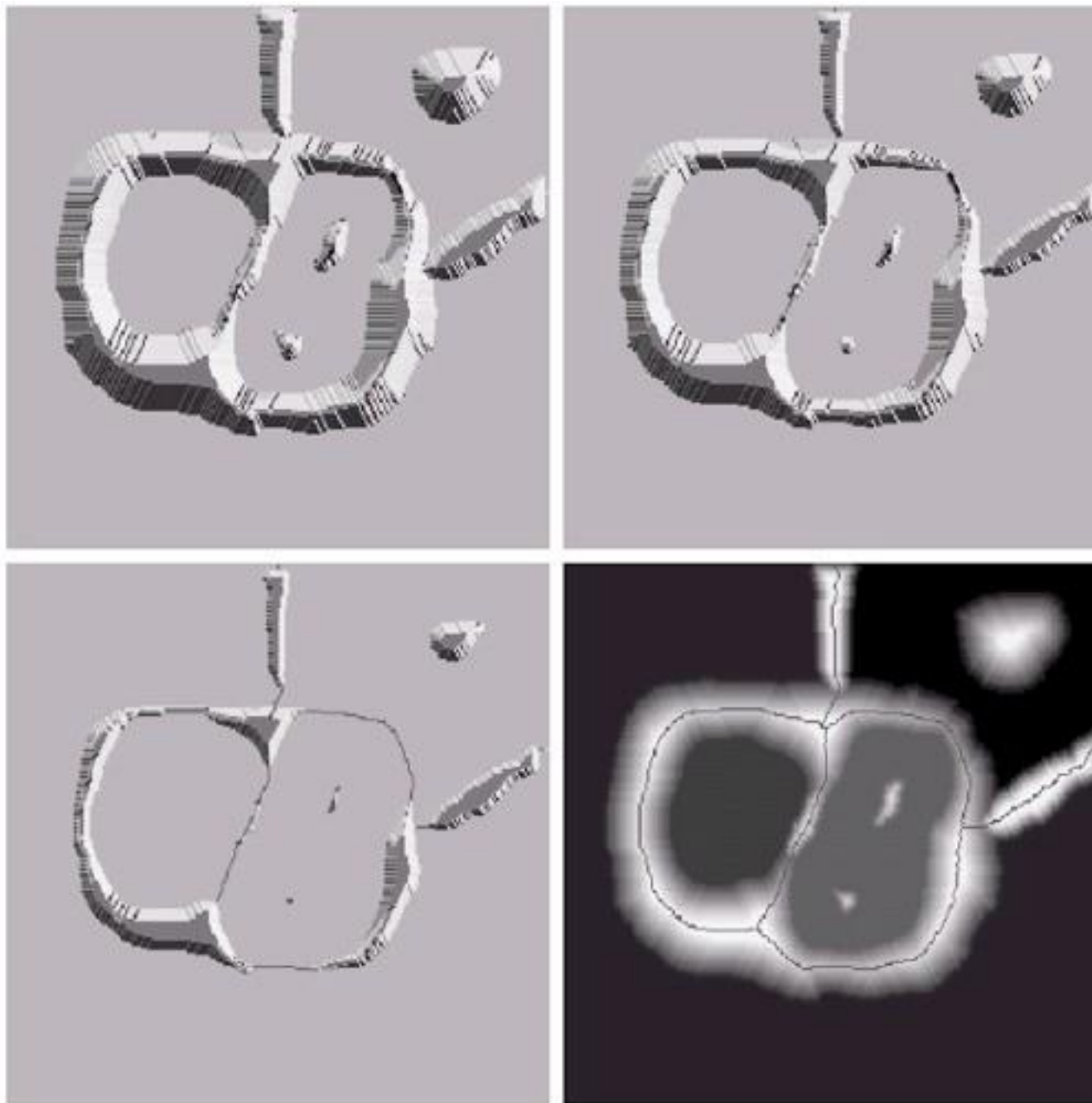


In this case, each object is distinguished from the background by its raised edges

# Basic Steps

1. Piercing holes in each regional minimum of  $I$
2. The 3D topography is flooded from below gradually
3. When the rising water in distinct catchment basins is about to merge, a dam is built to prevent the merging





e f  
g h

**FIGURE 10.44**

*(Continued)*

(e) Result of further flooding. (f) Beginning of merging of water from two catchment basins (a short dam was built between them). (g) Longer dams. (h) Final watershed (segmentation) lines. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

4. The dam boundaries correspond to the watershed lines to be extracted by a watershed segmentation algorithm

# Watershed Algorithm

Set label at every pixel to -1

For each seed point  $u$

For each neighbour  $v$  of  $u$

Label[ $v$ ] = label [ $u$ ]

Add  $v$  to priority queue: priority by gray level

while(elements in queue)

$u = \text{popMinimum}(\text{Queue});$

For each neighbour  $v$  of  $u$

If (label[ $v$ ]  $\neq$  -1)

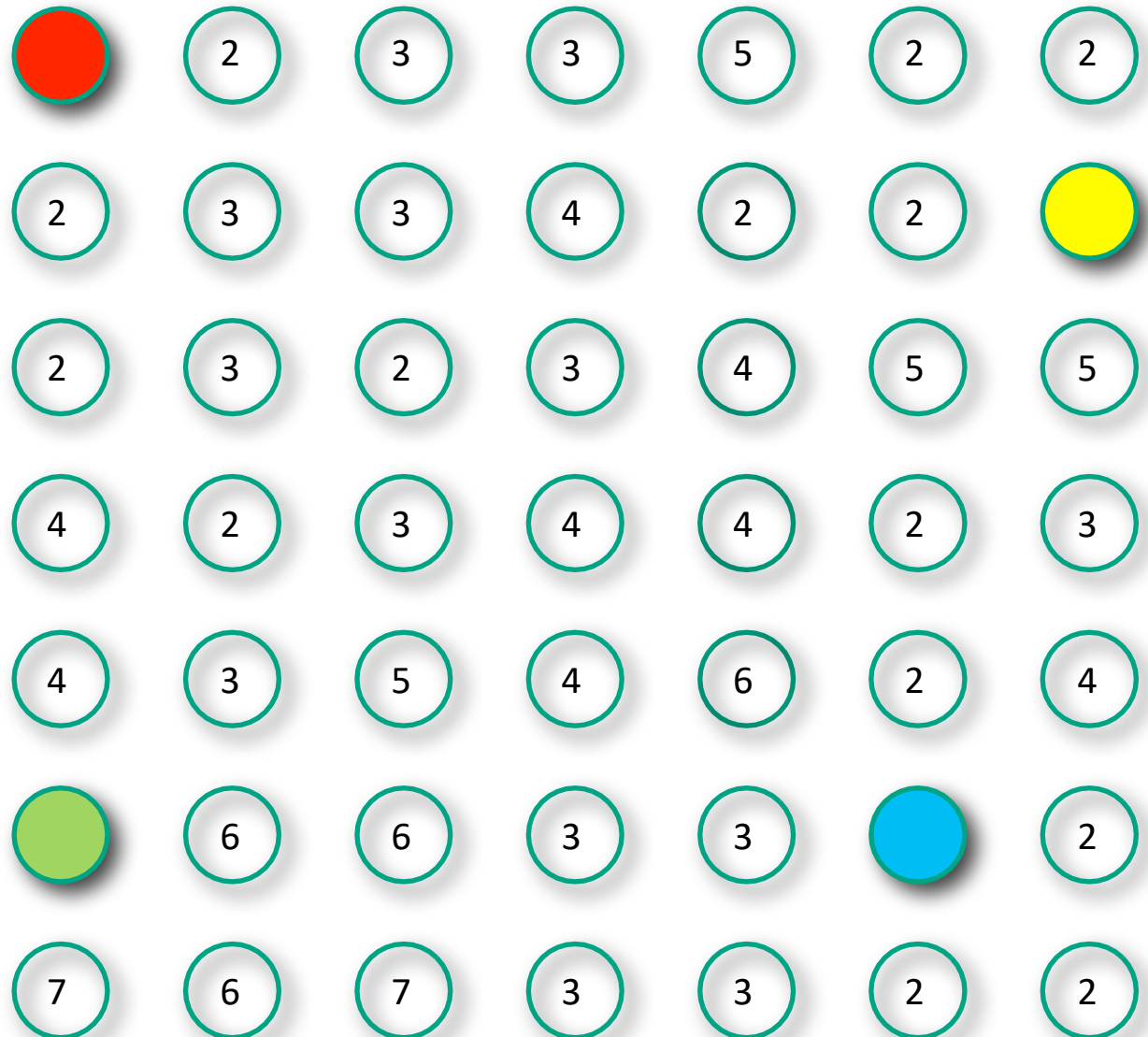
Label[ $v$ ] = label [ $u$ ]

Add  $v$  to priority queue: priority by gray level



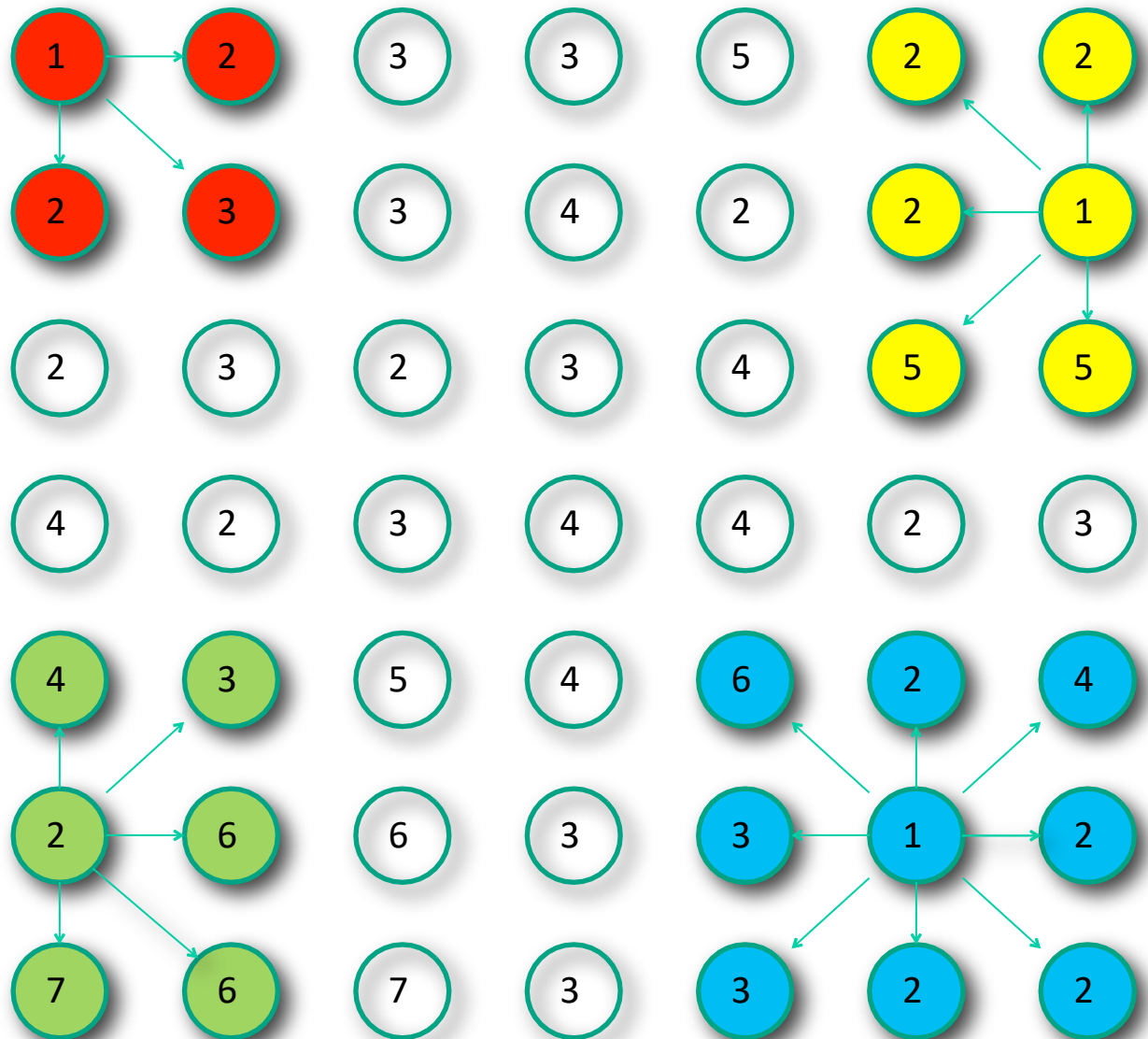
Watershed  
algorithm. Search  
simultaneously from  
seeds in order  
determined by gray  
level.

Direction Order:  
N,NE,E,SE,  
S,SW,W,NW



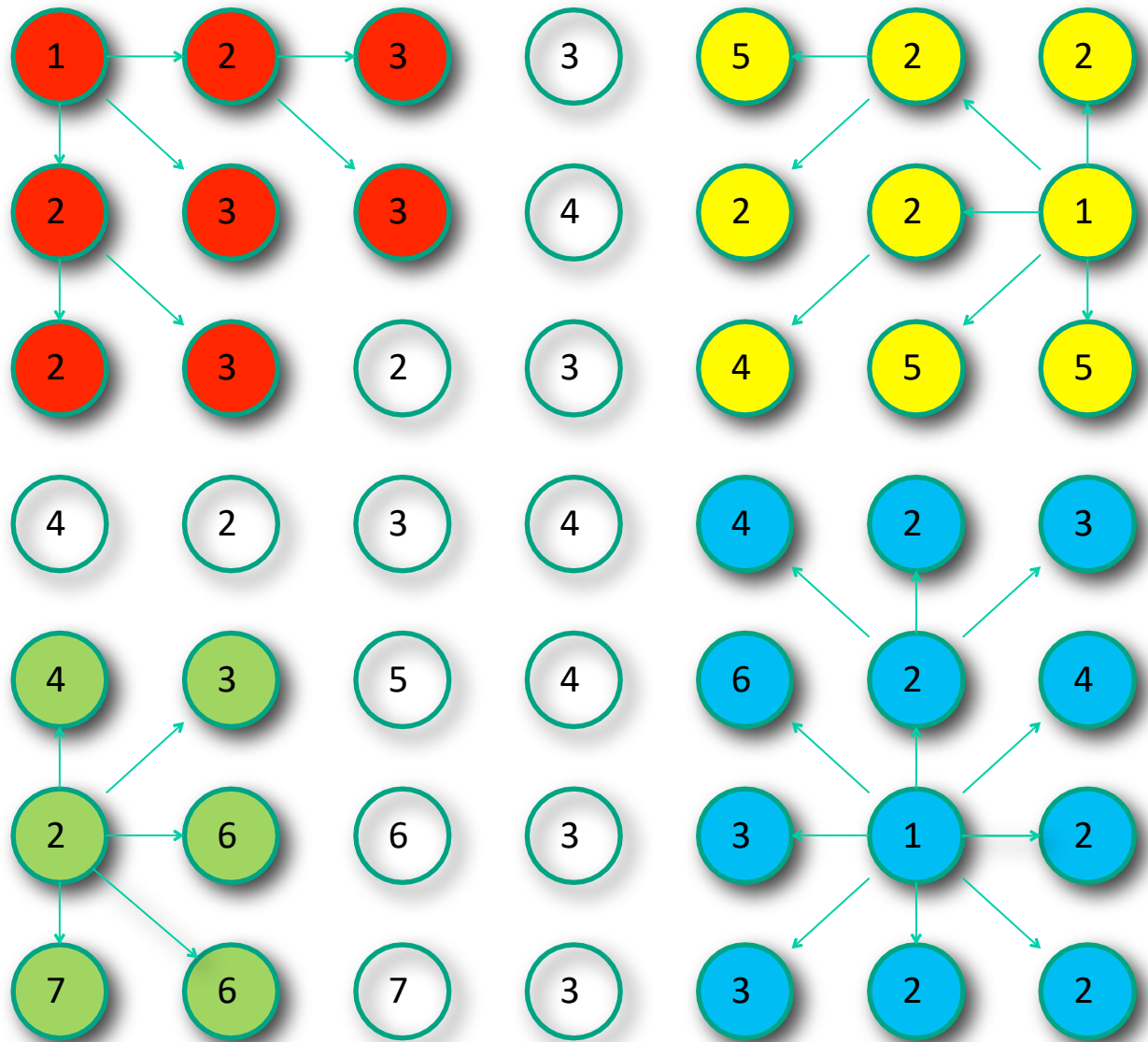
Watershed  
algorithm. Search  
simultaneously from  
seeds in order  
determined by gray  
level.

Direction Order:  
N,NE,E,SE,  
S,SW,W,NW



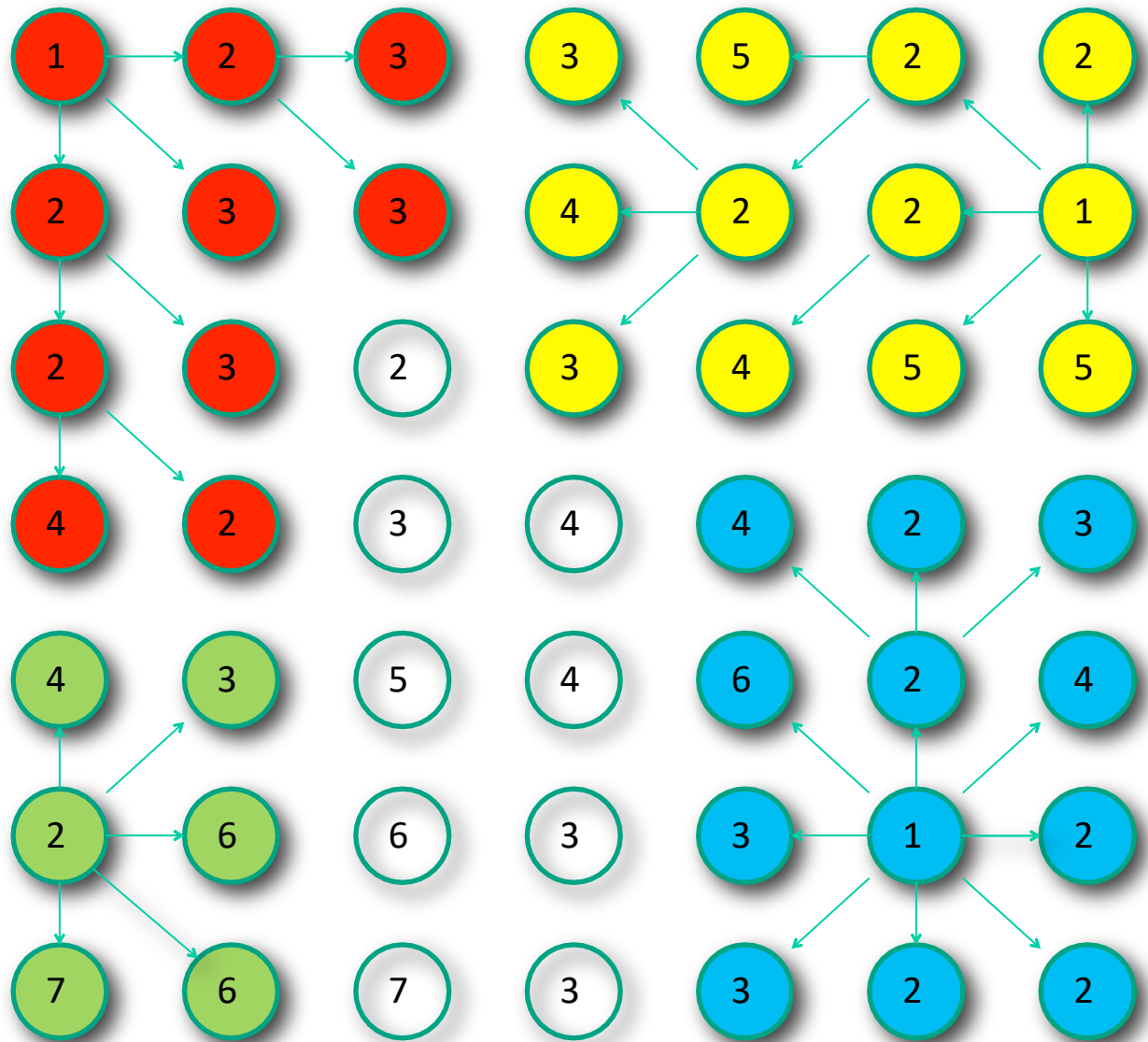
Watershed  
algorithm. Search  
simultaneously from  
seeds in order  
determined by gray  
level.

Direction Order:  
N,NE,E,SE,  
S,SW,W,NW



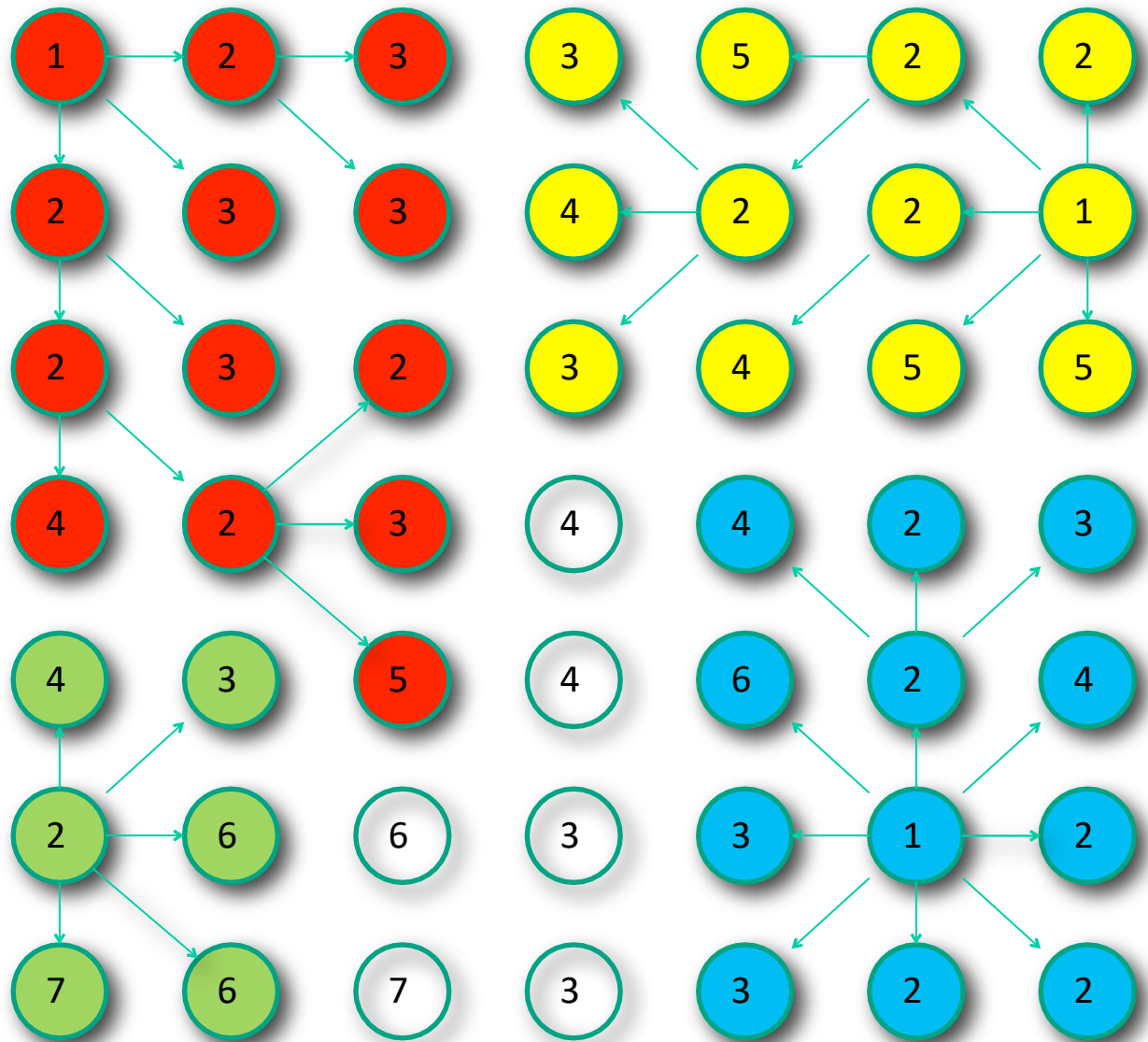
Watershed  
algorithm. Search  
simultaneously from  
seeds in order  
determined by gray  
level.

Direction Order:  
N,NE,E,SE,  
S,SW,W,NW



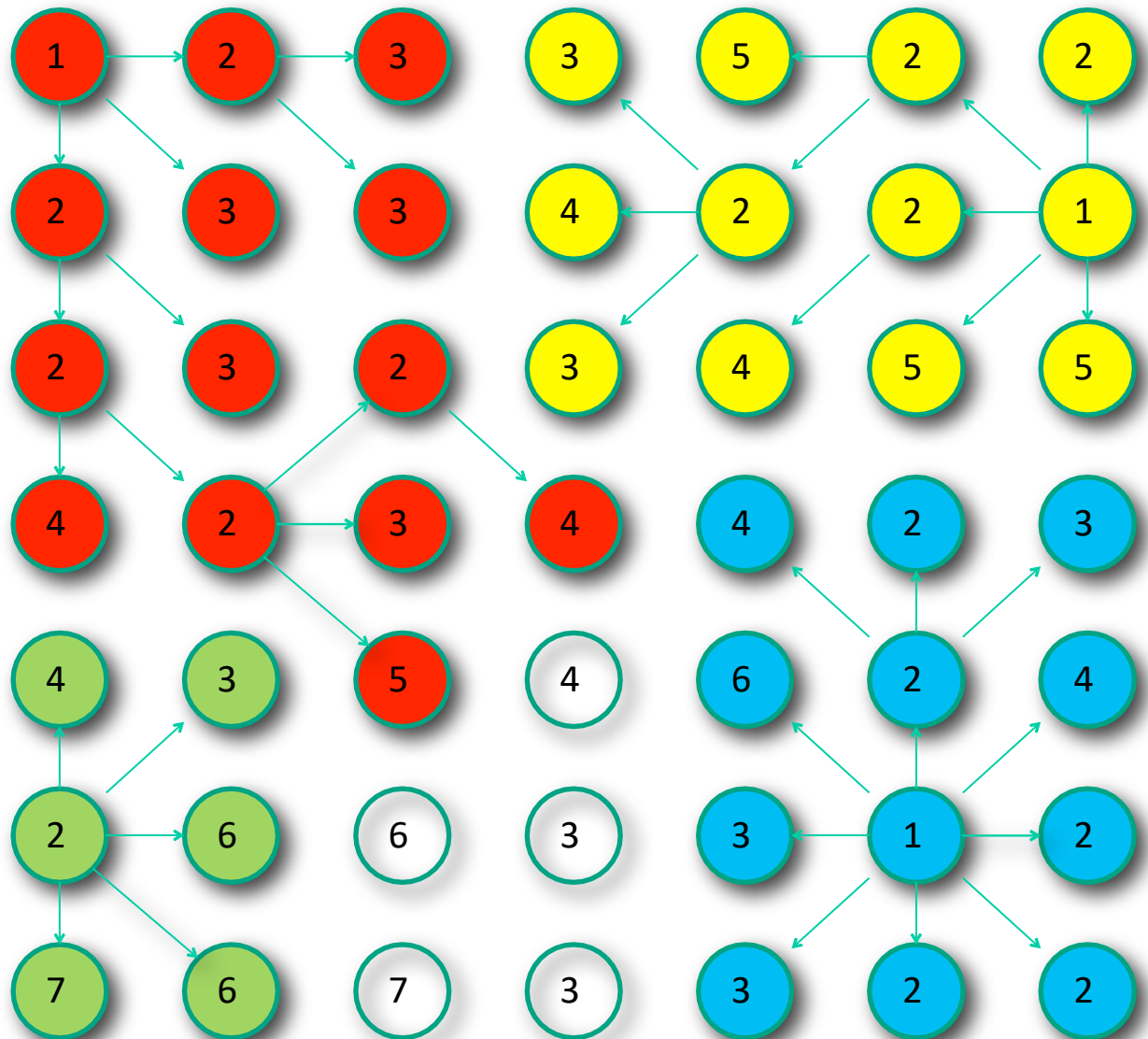
Watershed  
algorithm. Search  
simultaneously from  
seeds in order  
determined by gray  
level.

Direction Order:  
N,NE,E,SE,  
S,SW,W,NW



Watershed  
algorithm. Search  
simultaneously from  
seeds in order  
determined by gray  
level.

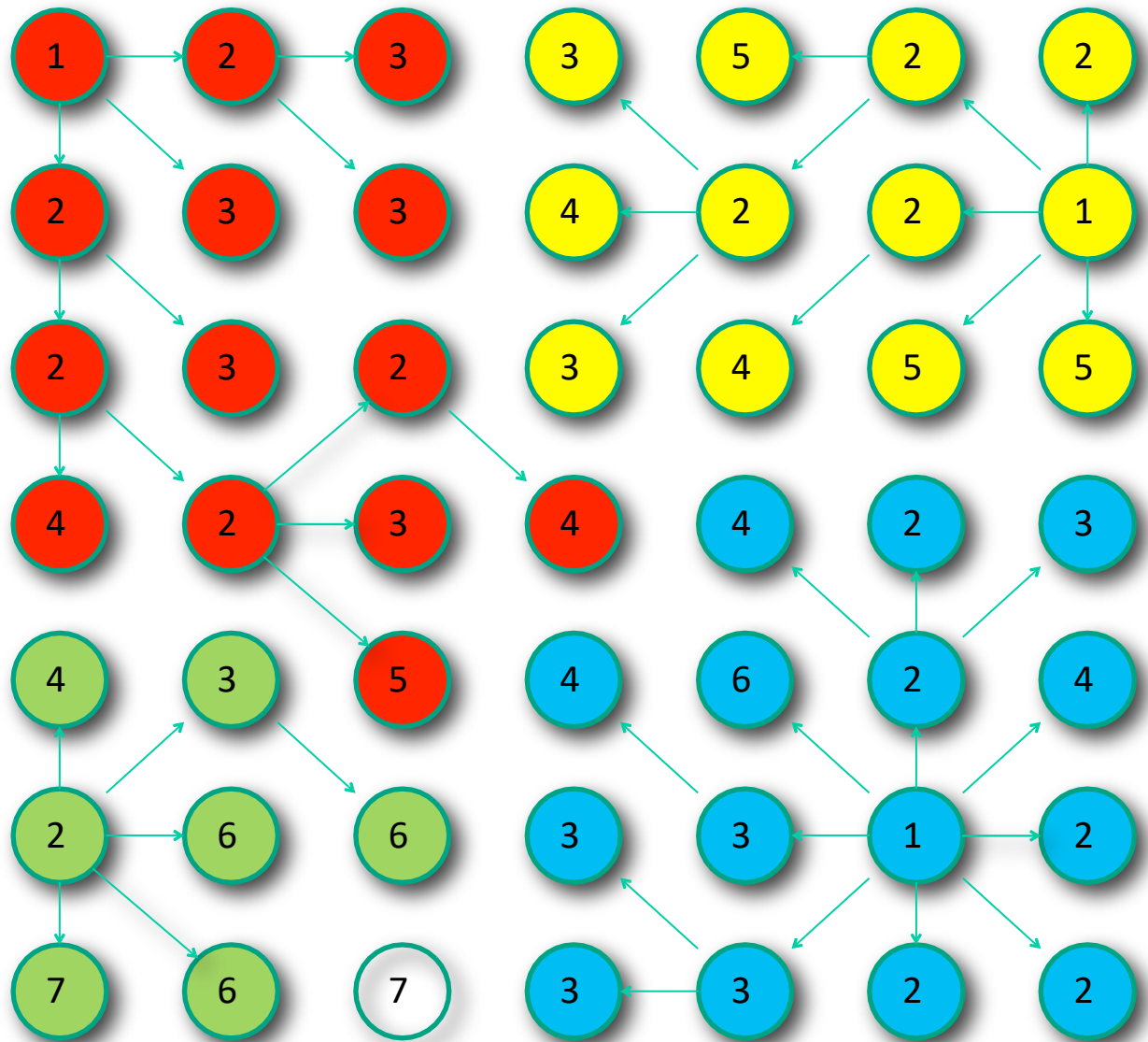
Direction Order:  
N,NE,E,SE,  
S,SW,W,NW





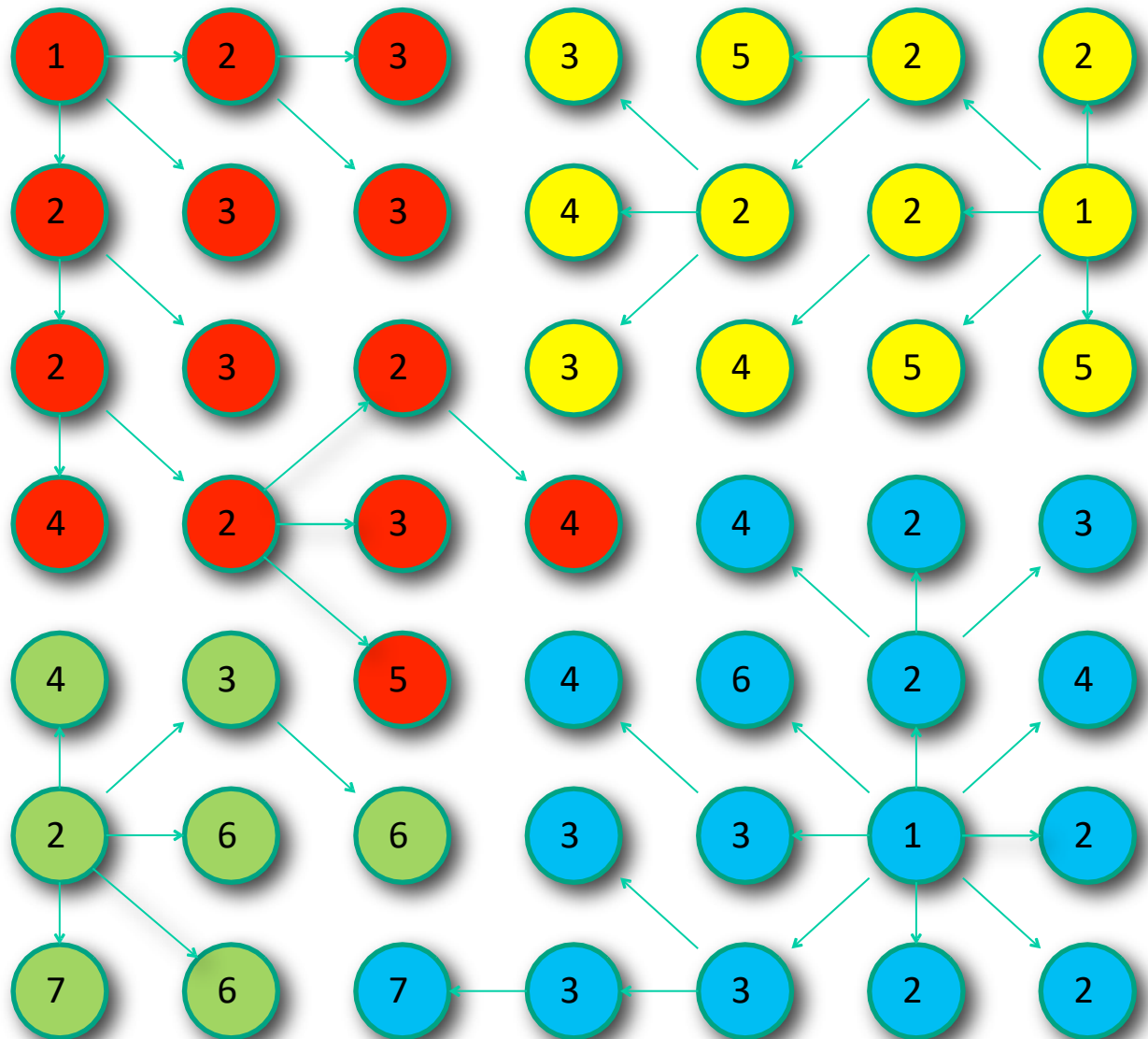
Watershed  
algorithm. Search  
simultaneously from  
seeds in order  
determined by gray  
level.

Direction Order:  
N,NE,E,SE,  
S,SW,W,NW



Watershed  
algorithm. Search  
simultaneously from  
seeds in order  
determined by gray  
level.

Direction Order:  
N,NE,E,SE,  
S,SW,W,NW



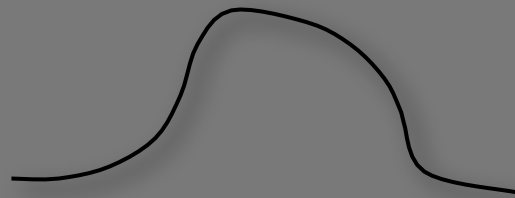
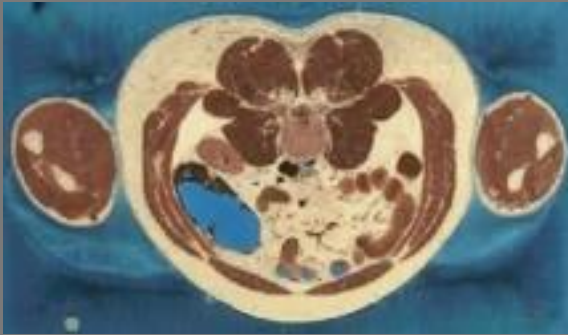
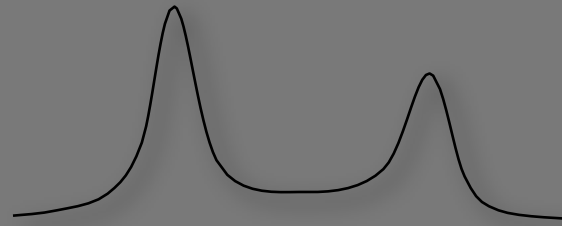
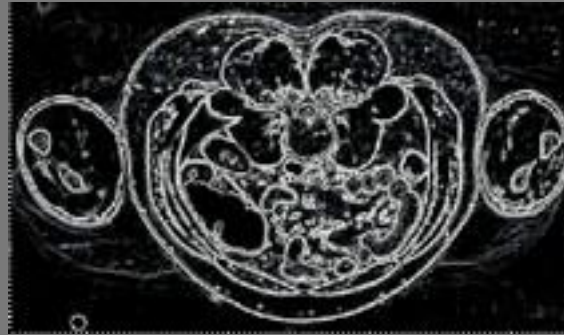
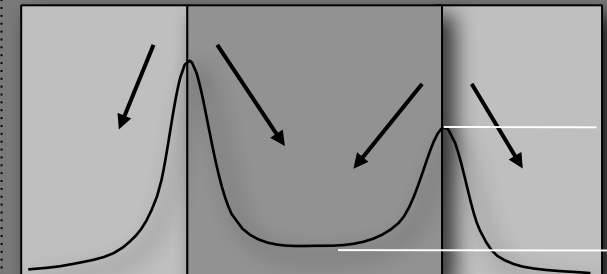


Image (filtered)

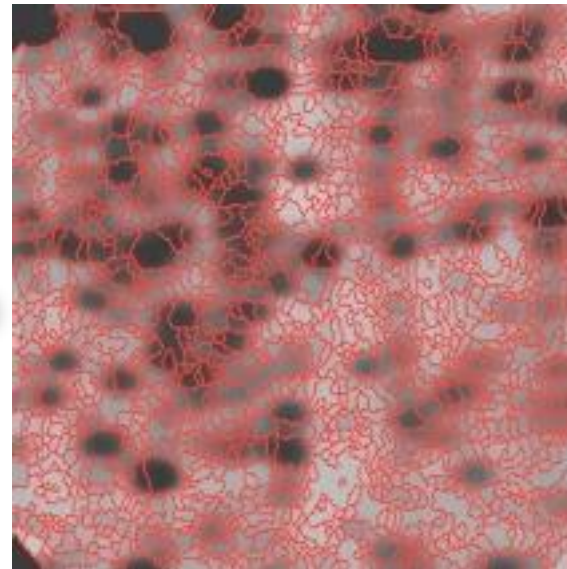
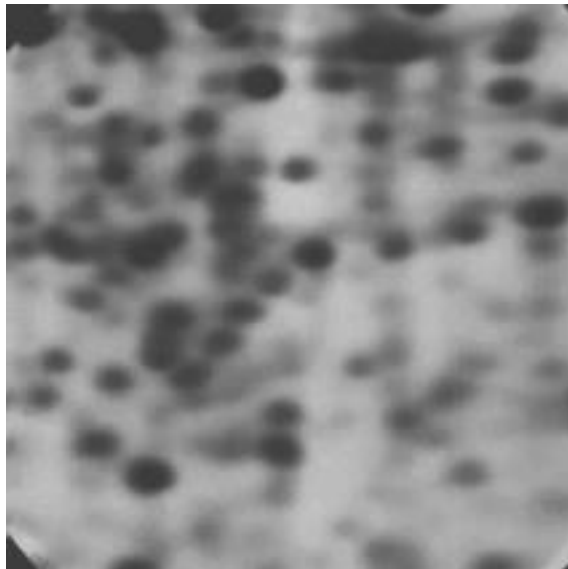


Feature Extraction  
"Edge Map"



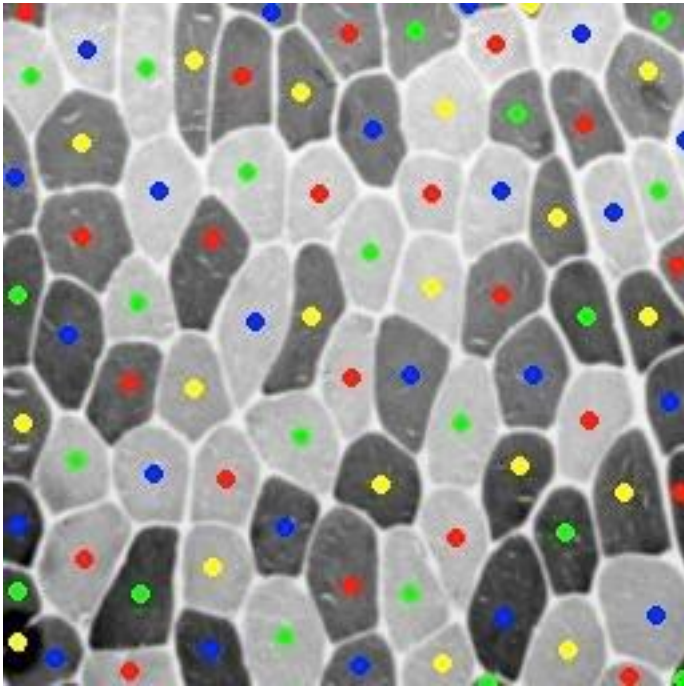
Watershed Transform

Watershed Depth

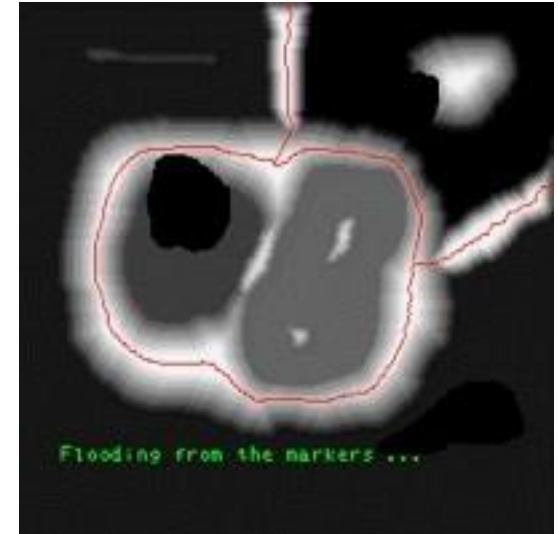


Problem:  
Watershed  
algorithm  
typically over-  
segments

# Initialization



IDEA: Initialize regions using other information (cell nuclei) or by hand--marking. Pre-defines number of segments



# Analysis (pros & cons)

## Thresholding

- Relies on a single global threshold
- Produces separate regions (may prefer connected components)

## Region growing

- Supervised, requires 1+ seed point
- Relies on a single threshold
- Produces one region

## Watershed

- May be supervised or unsupervised
- No need for threshold parameter
- Partitions image into many regions
- To get good results best to specify number and positions of seeds

# Overview

What is image segmentation?

Types of segmentation algorithms

1. Thresholding, region labelling and growing algorithms
  - (connected components, region growing, watershed)
2. Statistical Segmentation
  - (k-means, mean shift)
3. Graph based methods
  - (Merging algorithms, splitting algorithms, split/merge)
4. Edge based methods
  - (Intelligent Scissors, Snakes)



# Overview

What is image segmentation?

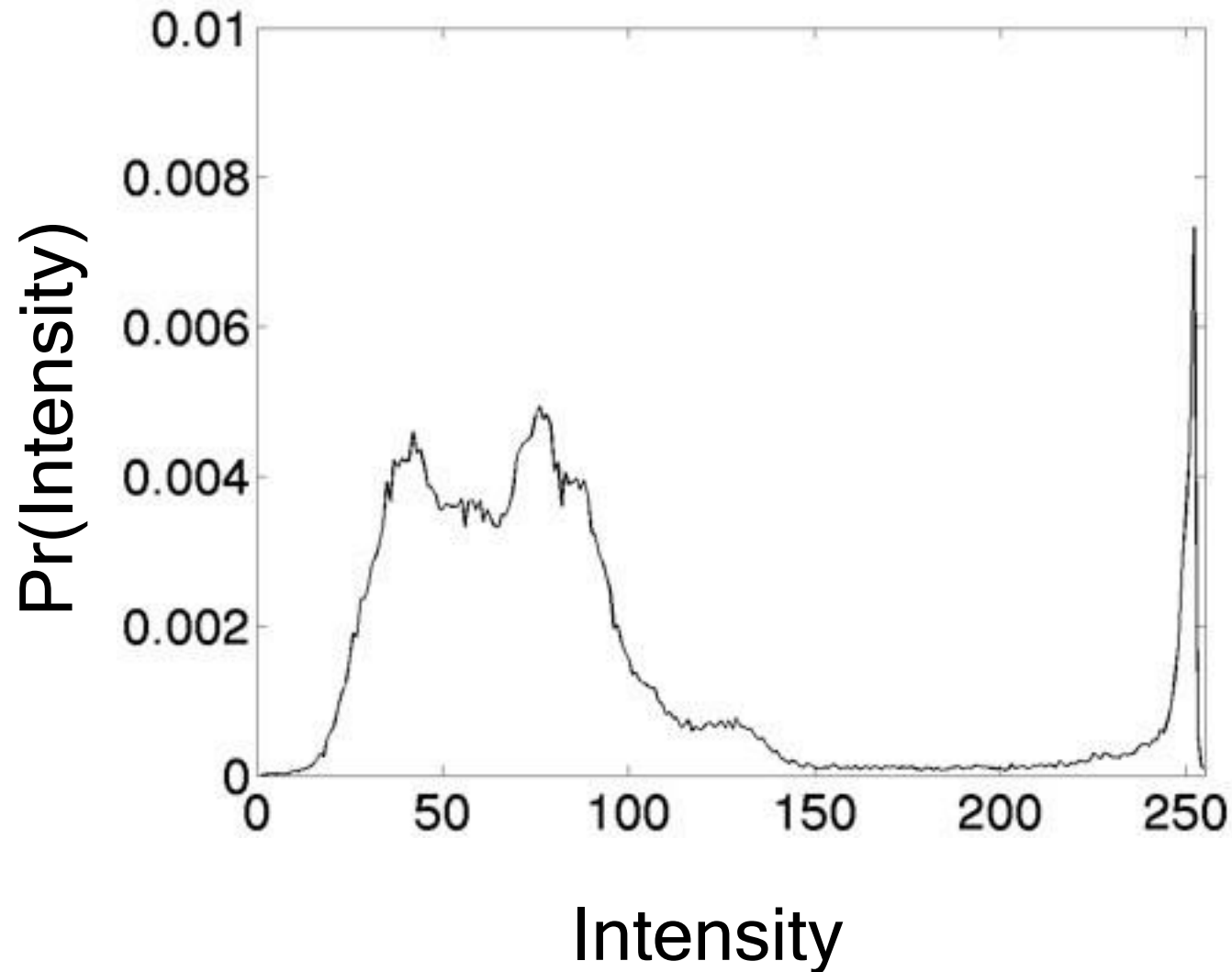
Types of segmentation algorithms

1. Thresholding, region labelling and growing algorithms
  - (connected components, region growing, watershed)
2. Statistical Segmentation
  - (k-means, mean shift)
3. Graph based methods
  - (Merging algorithms, splitting algorithms, split/merge)
4. Edge based methods
  - (Intelligent Scissors, Snakes)

# Choosing Threshold Automatically



Q. For binary thresholding, how can we choose the threshold automatically?

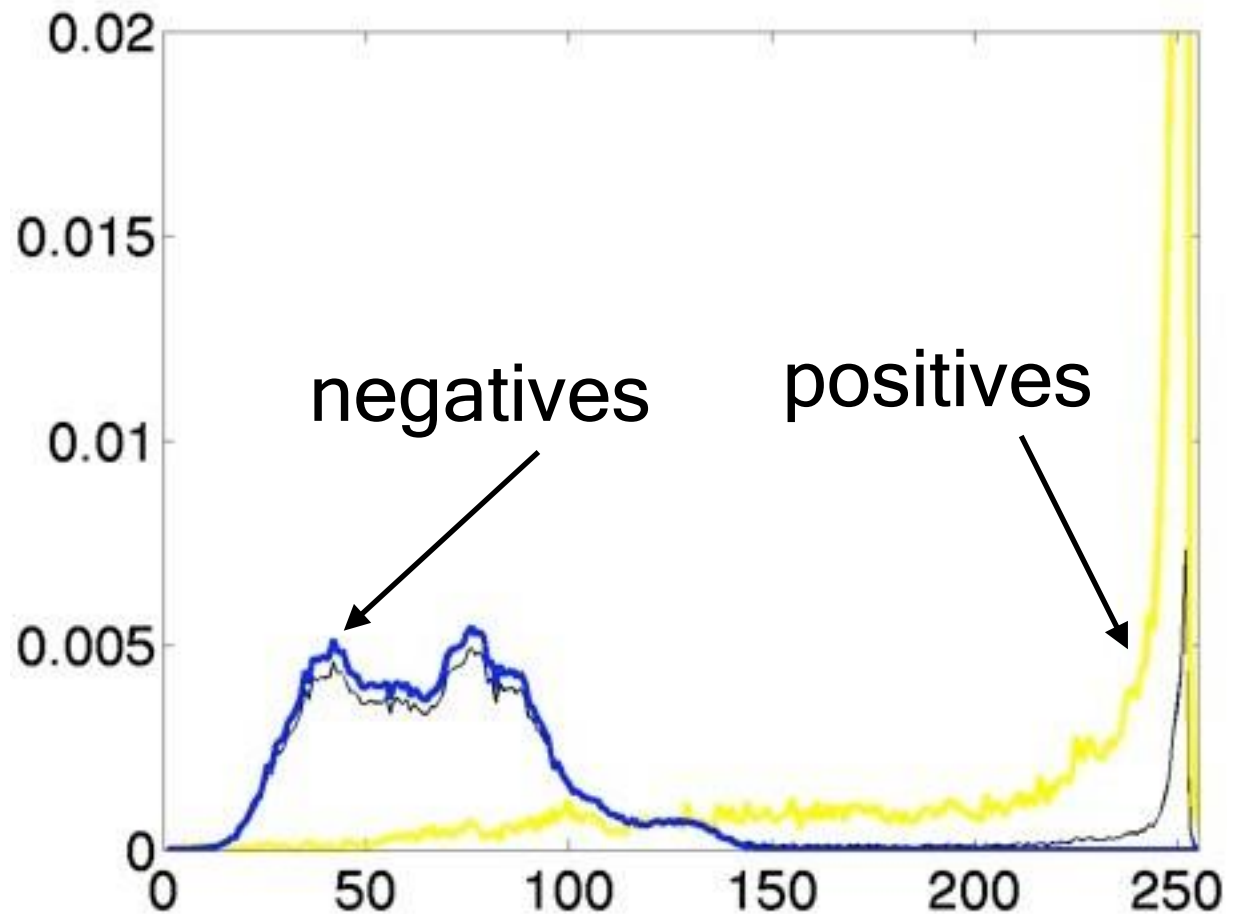




# Choosing Threshold Automatically

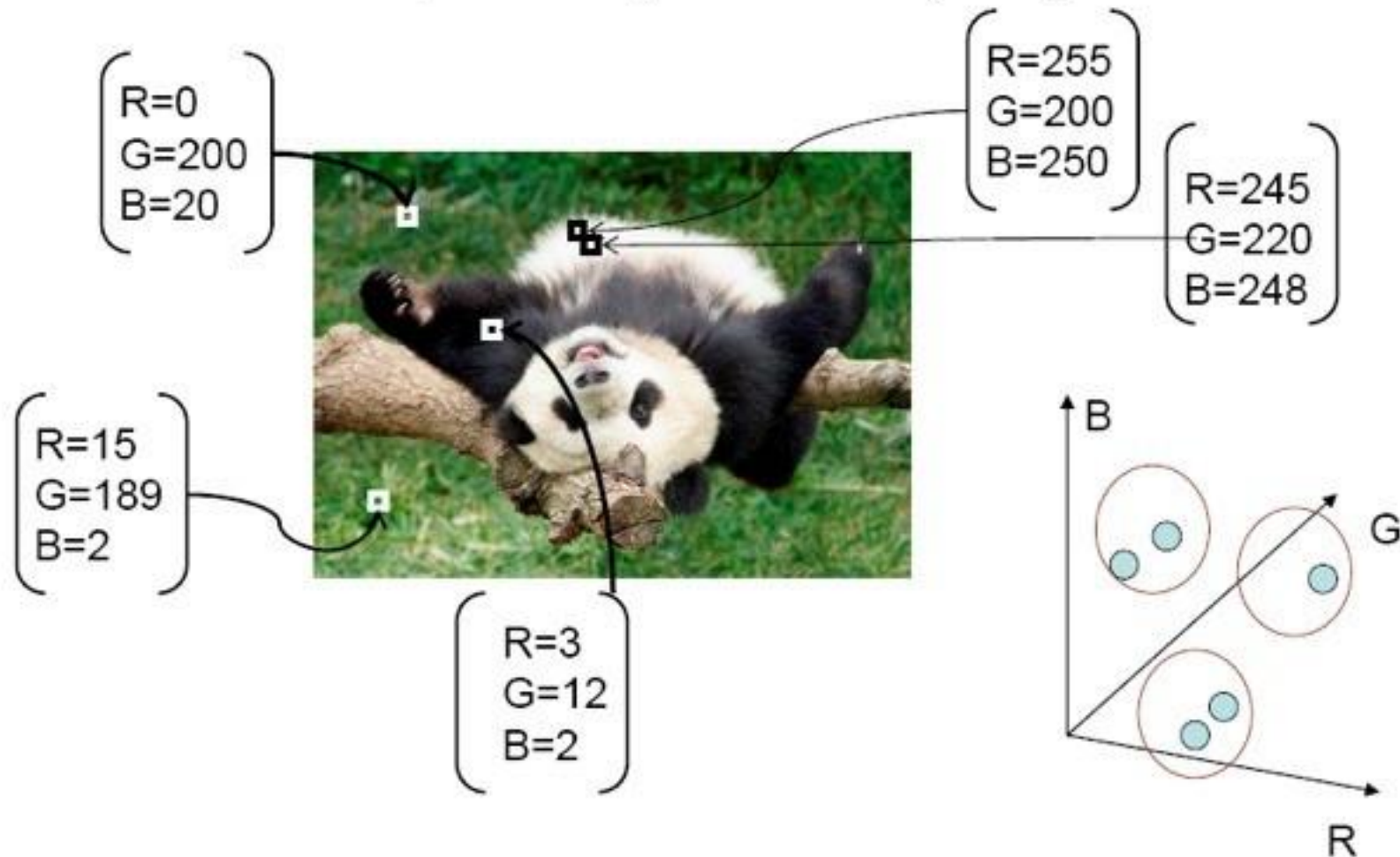


Q. For binary thresholding, how can we choose the threshold automatically?



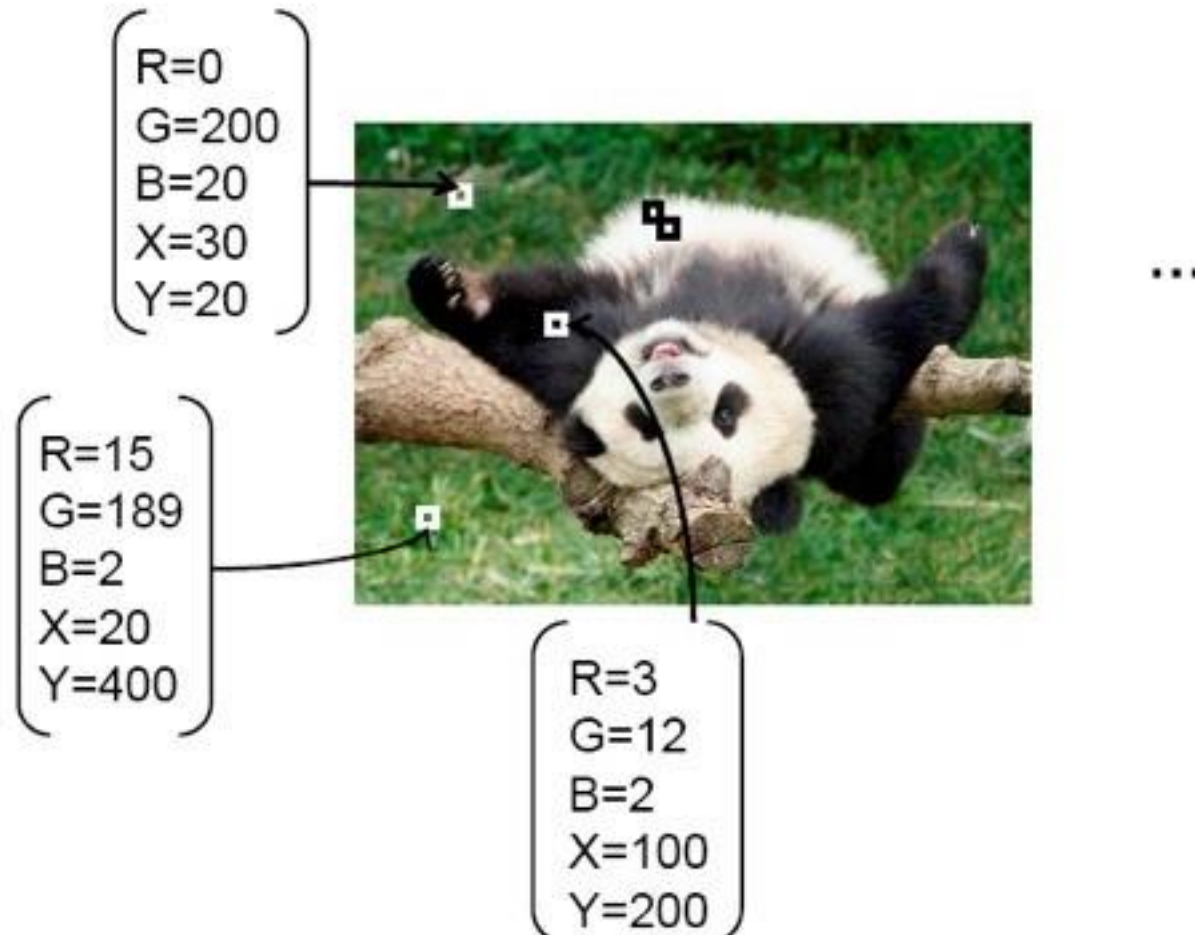
# Segmentation as clustering

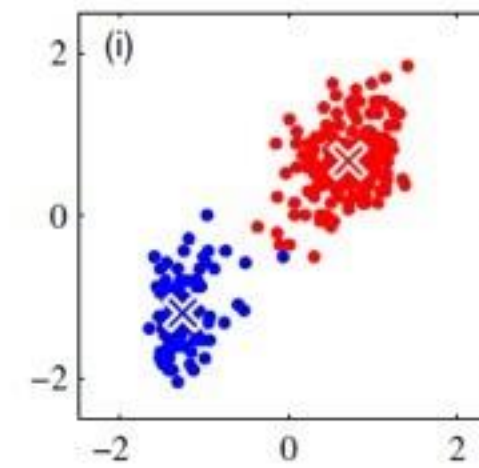
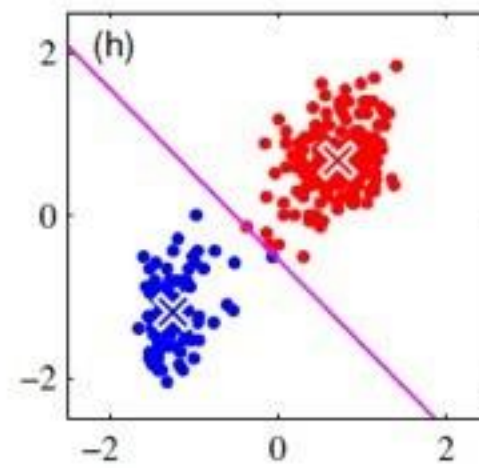
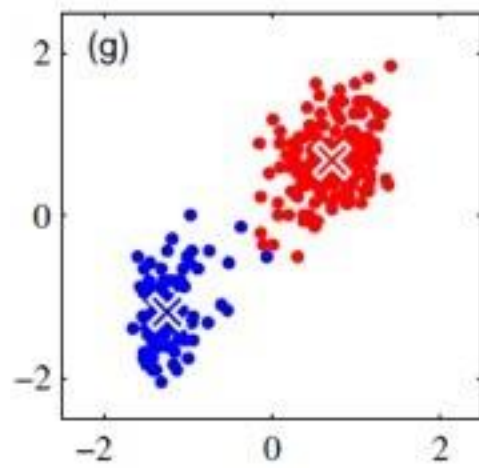
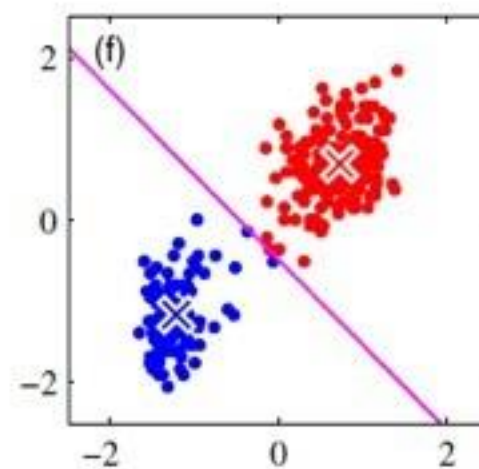
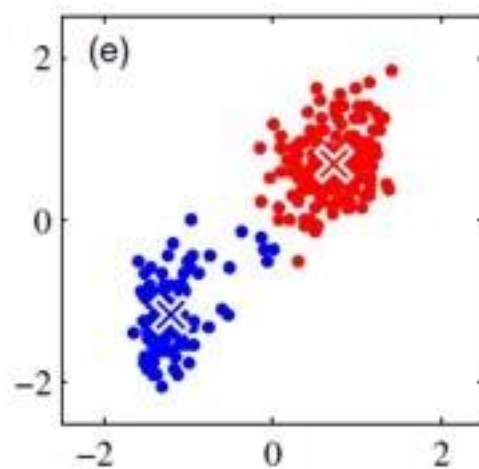
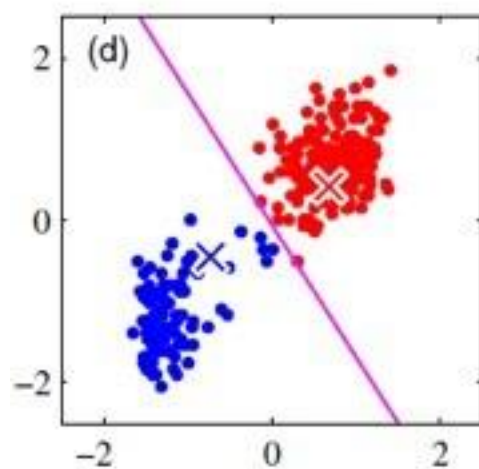
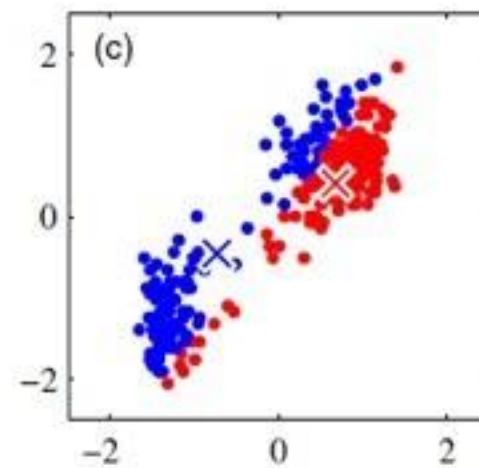
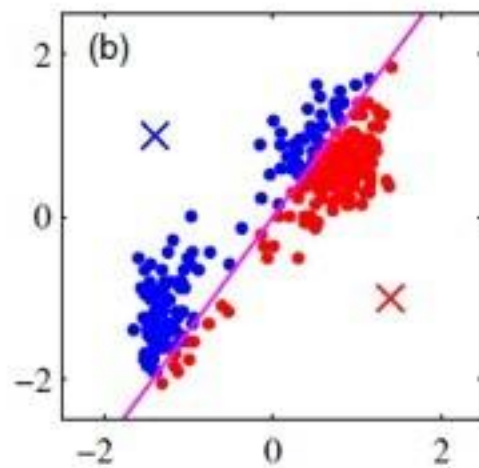
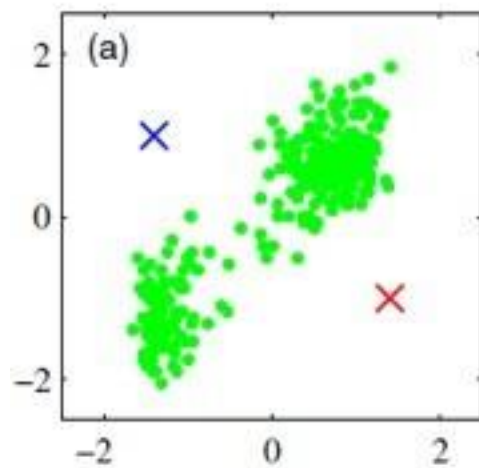
- Cluster similar pixels (features) together



# Segmentation as clustering

- Cluster similar pixels (features) together







$K = 2$  $K = 3$  $K = 10$ 

Original image

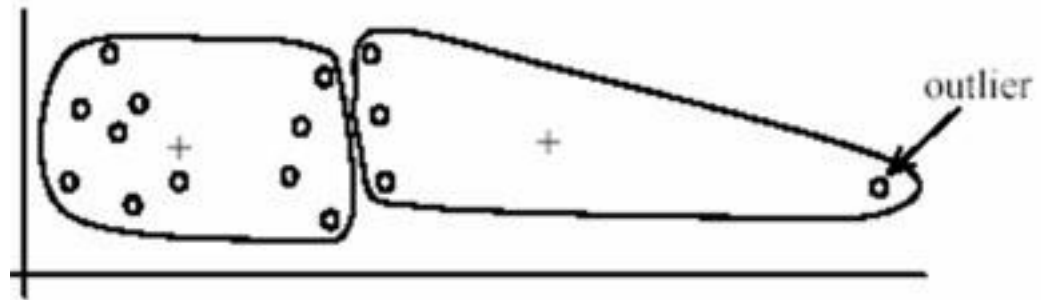


**Figure 9.3** Two examples of the application of the  $K$ -means clustering algorithm to image segmentation showing the initial images together with their  $K$ -means segmentations obtained using various values of  $K$ . This also illustrates the use of vector quantization for data compression, in which smaller values of  $K$  give higher compression at the expense of poorer image quality.

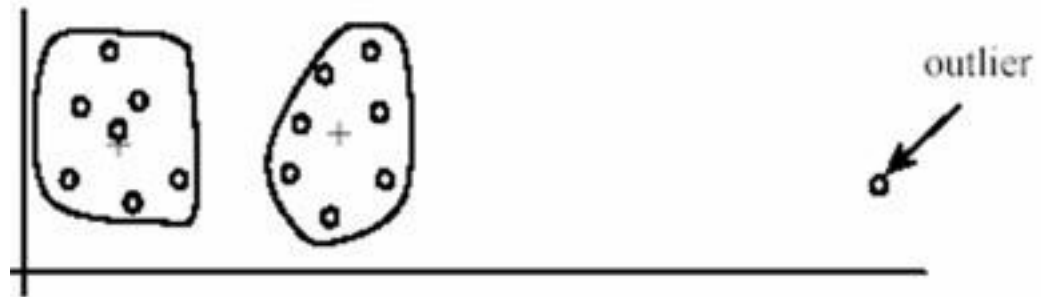
From Bishop (2006)

# K-Means

- Pros
  - Simple and fast
  - Converges to a local minimum of the distance function
- Cons
  - Need to pick K
  - Sensitive to initialization
  - Sensitive to outliers
  - Only finds “spherical” clusters



(A): Undesirable clusters



(B): Ideal clusters

# Comparison

## Watershed

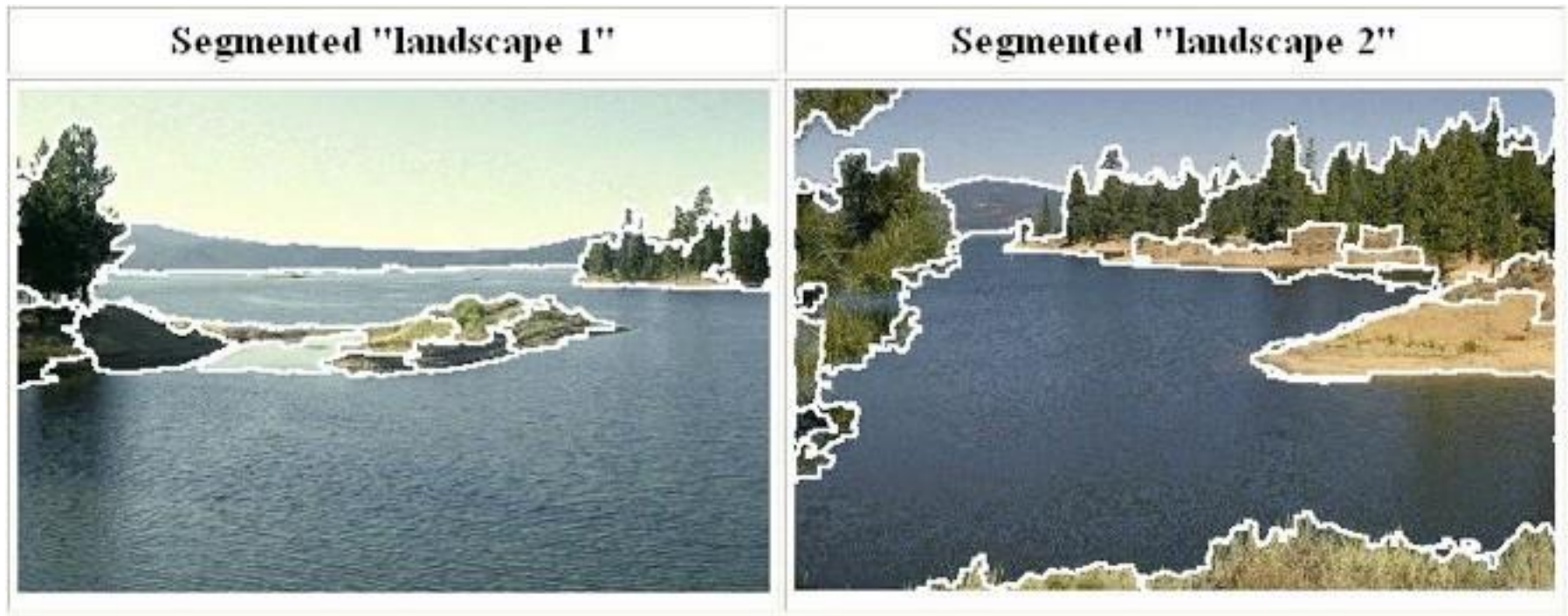
- May be supervised or unsupervised
- No need for threshold parameter
- Partitions image into many regions
- To get good results best to specify number and positions of seeds

## K-Means

- Unsupervised
- Works in high--dimensions
- May produce disjoint segments
- Must specify number of clusters but not position



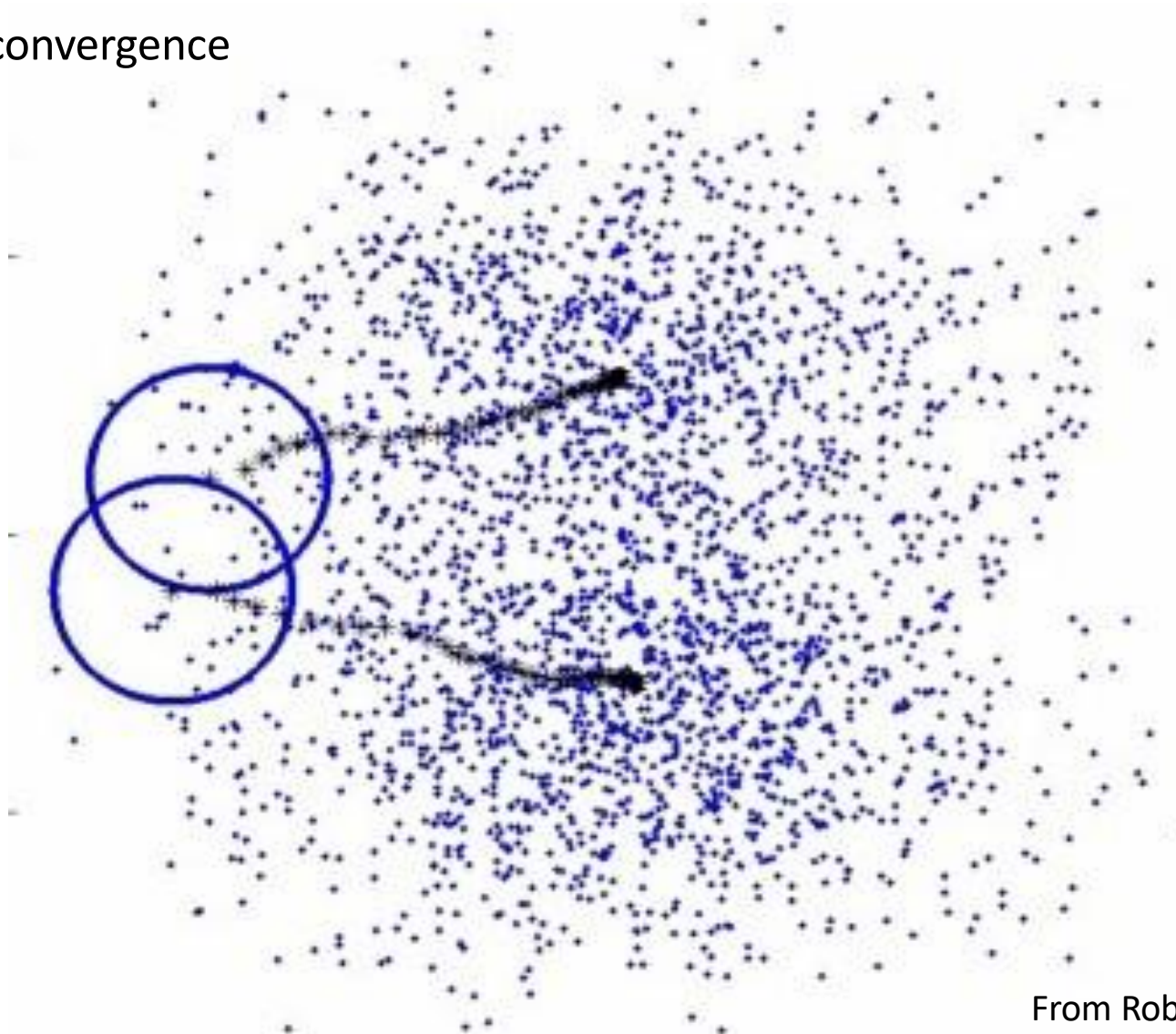
# Mean shift Segmentation

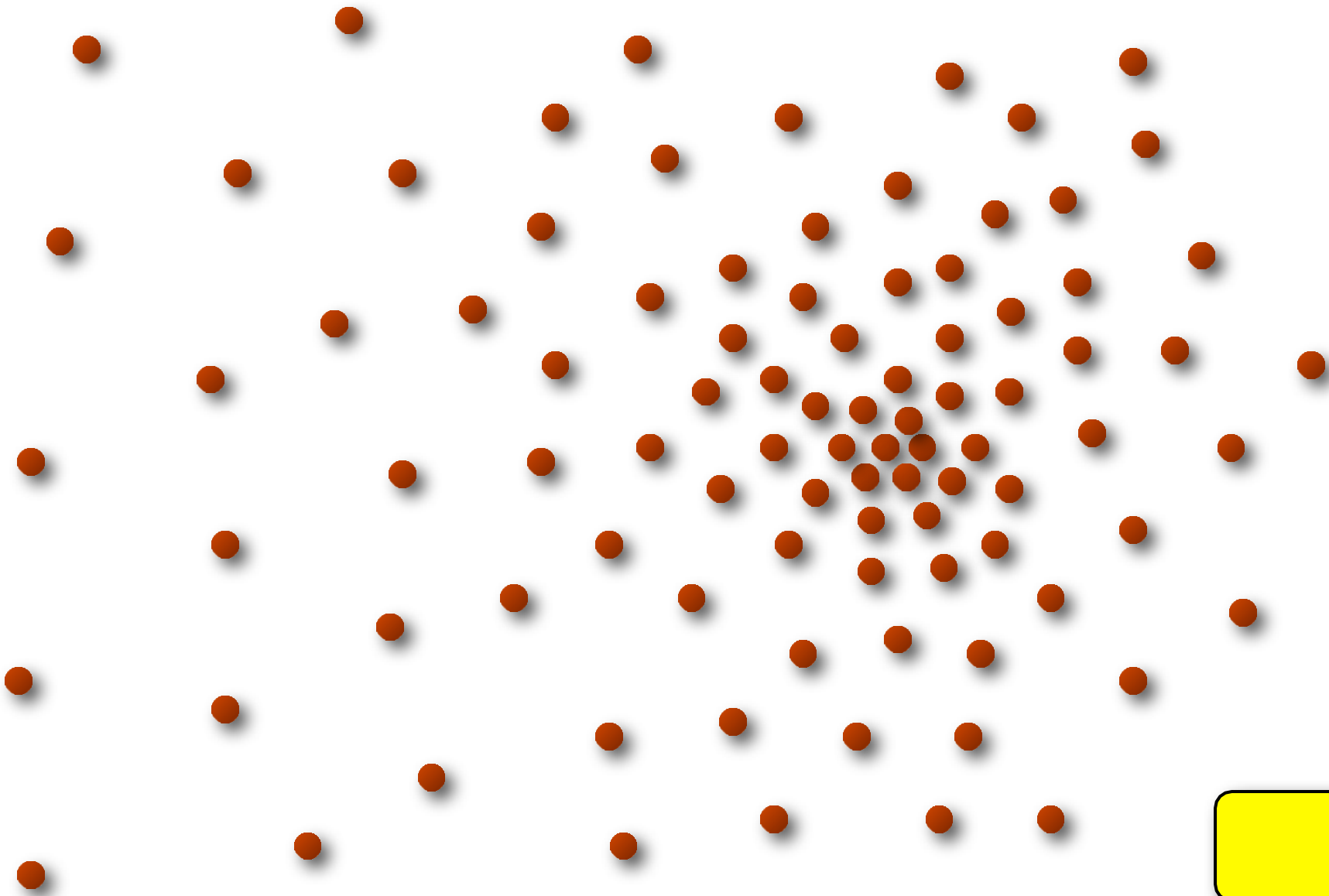


Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.  
D. Comaniciu and P. Meer

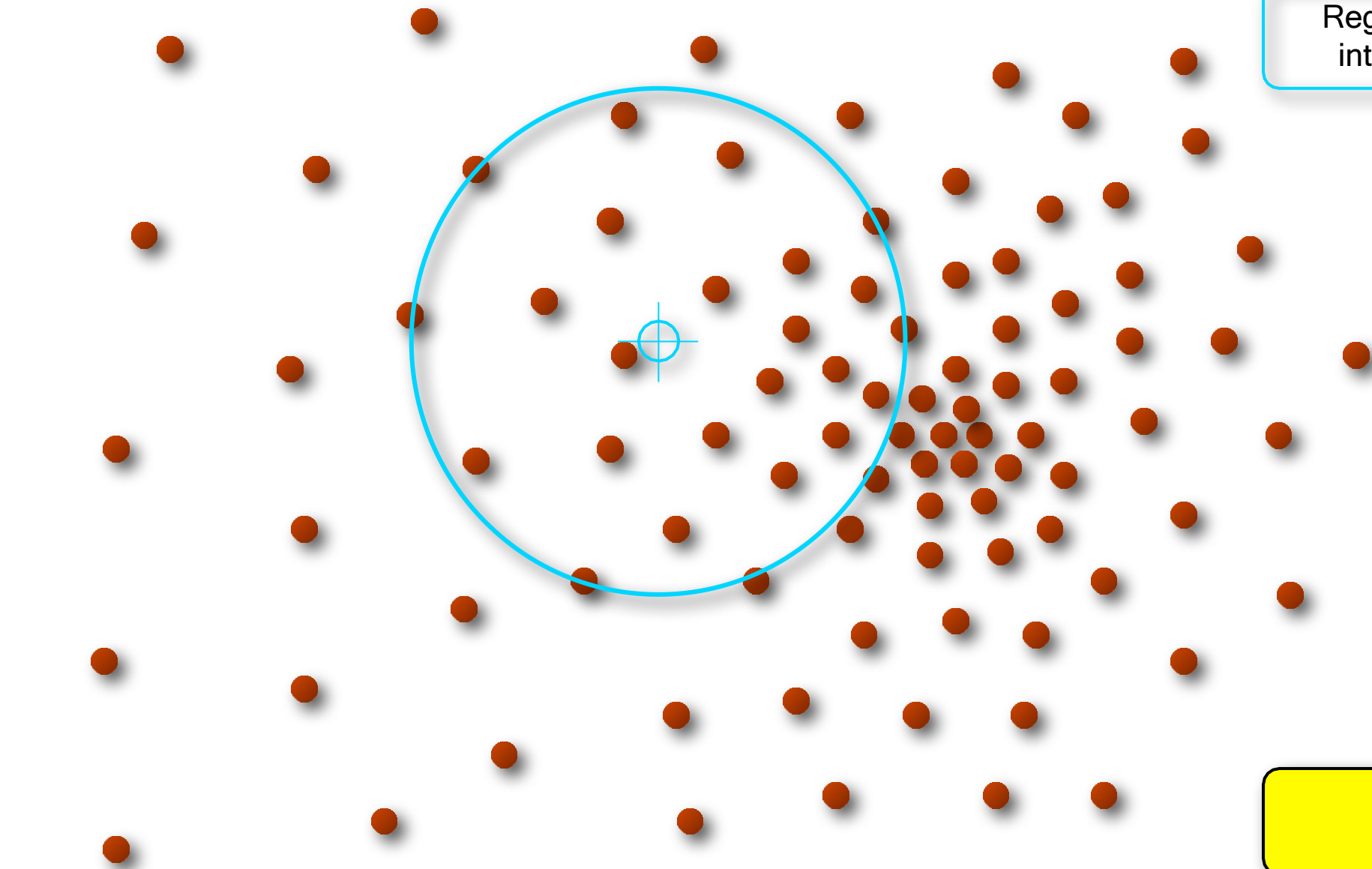


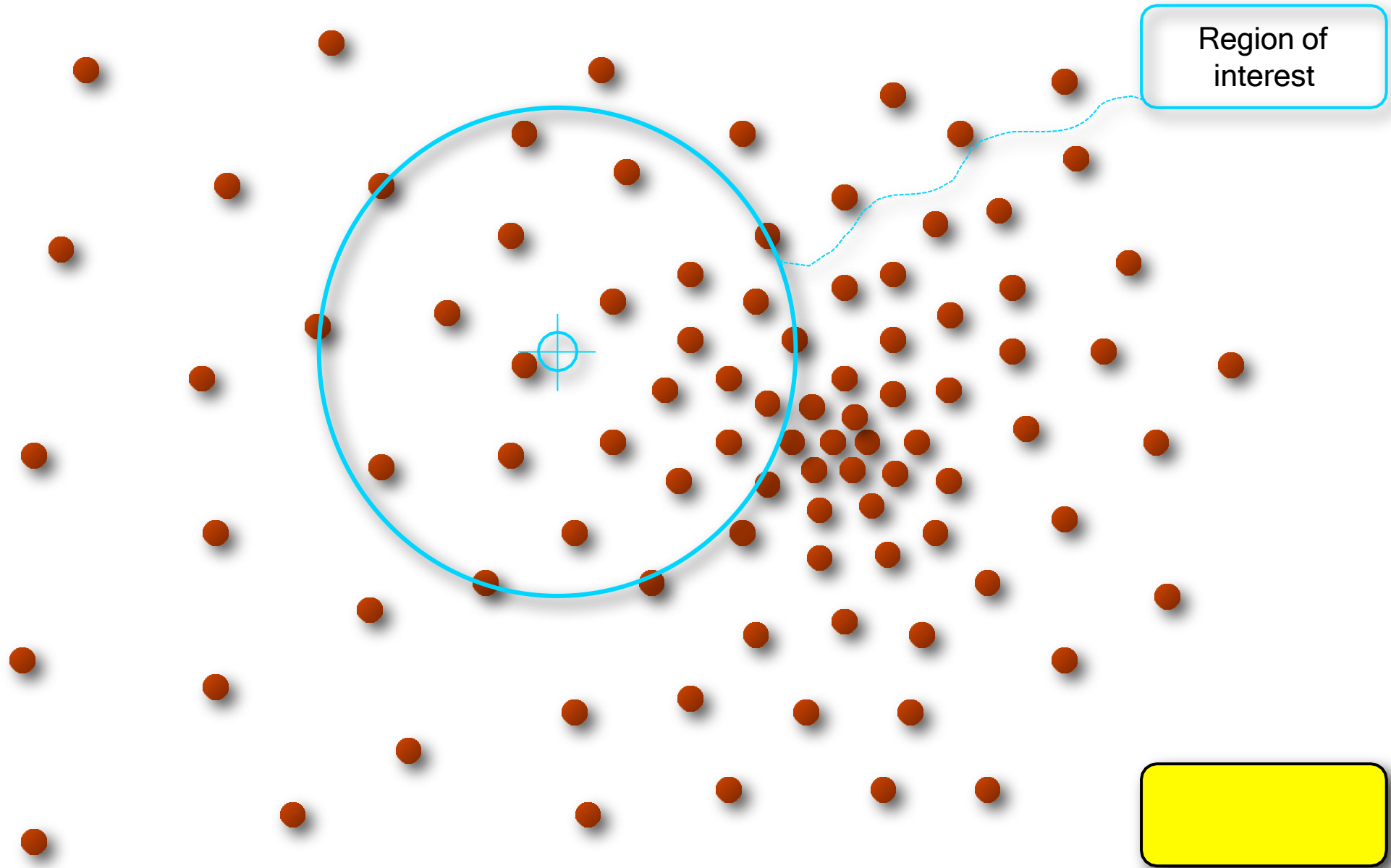
- The mean shift algorithm seeks a *mode* or local maximum of density of a given distribution
  - Choose a search window (width and location)
  - Compute the mean of the data in the search window
  - Center the search window at the new mean location
  - Repeat until convergence

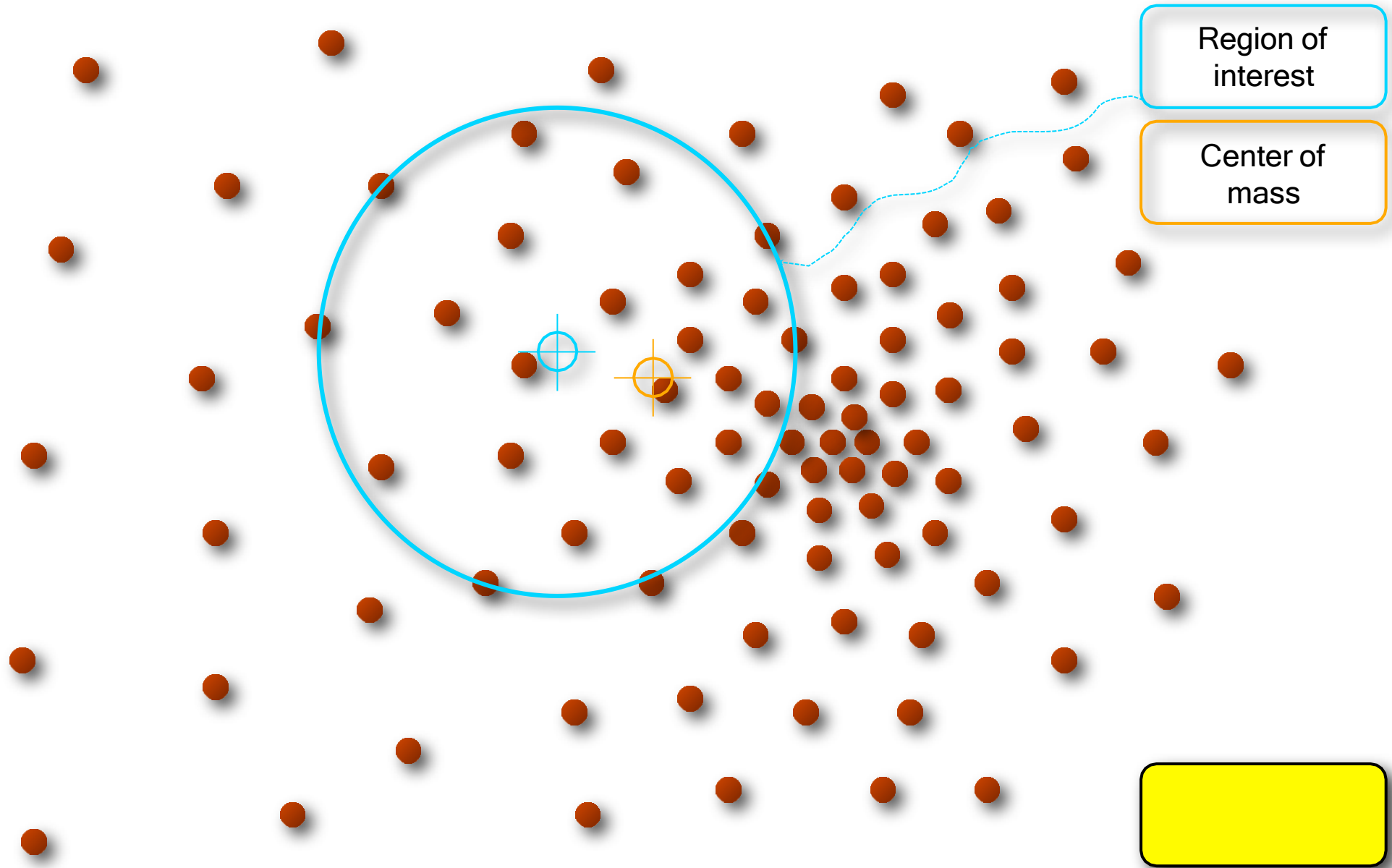


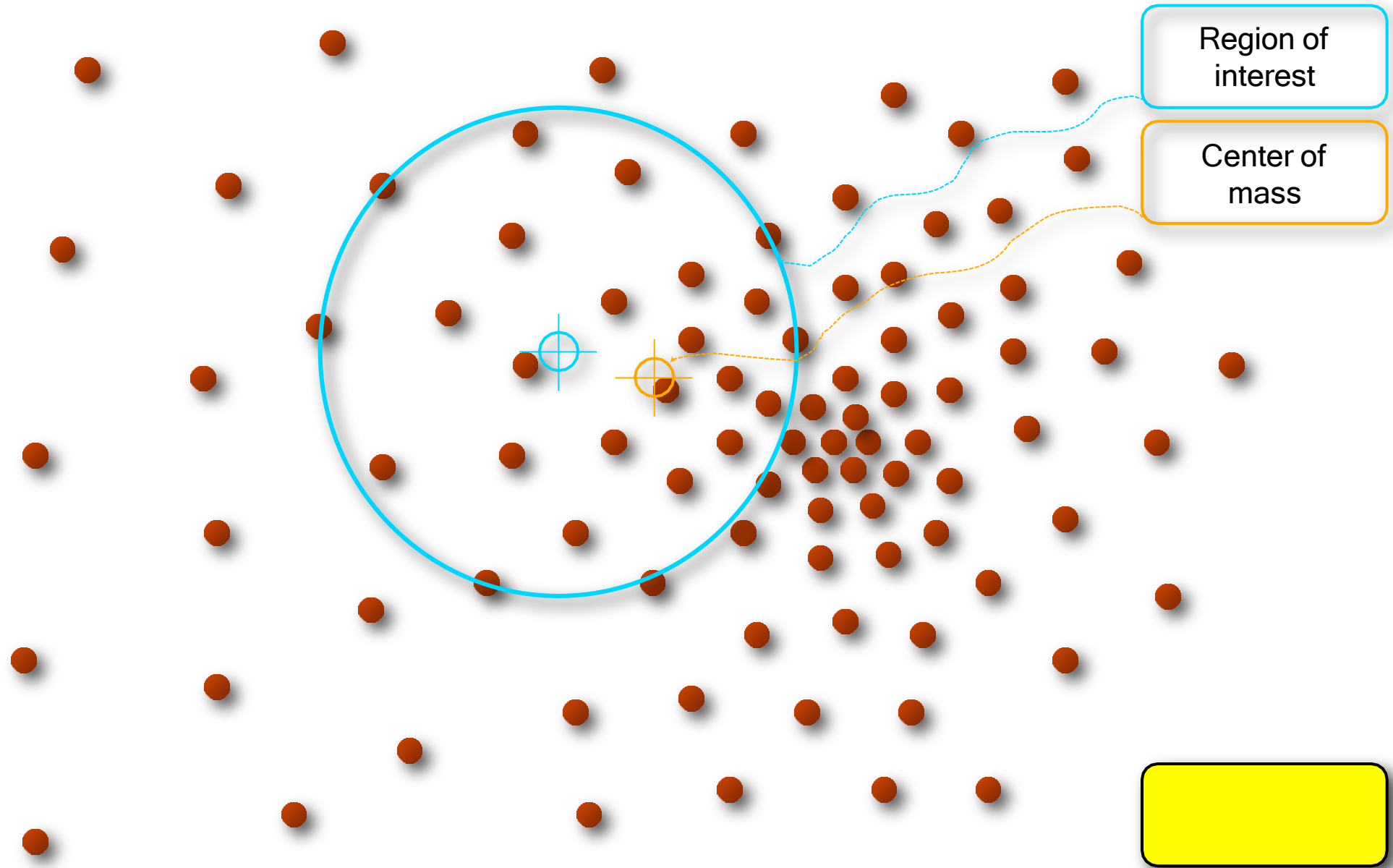


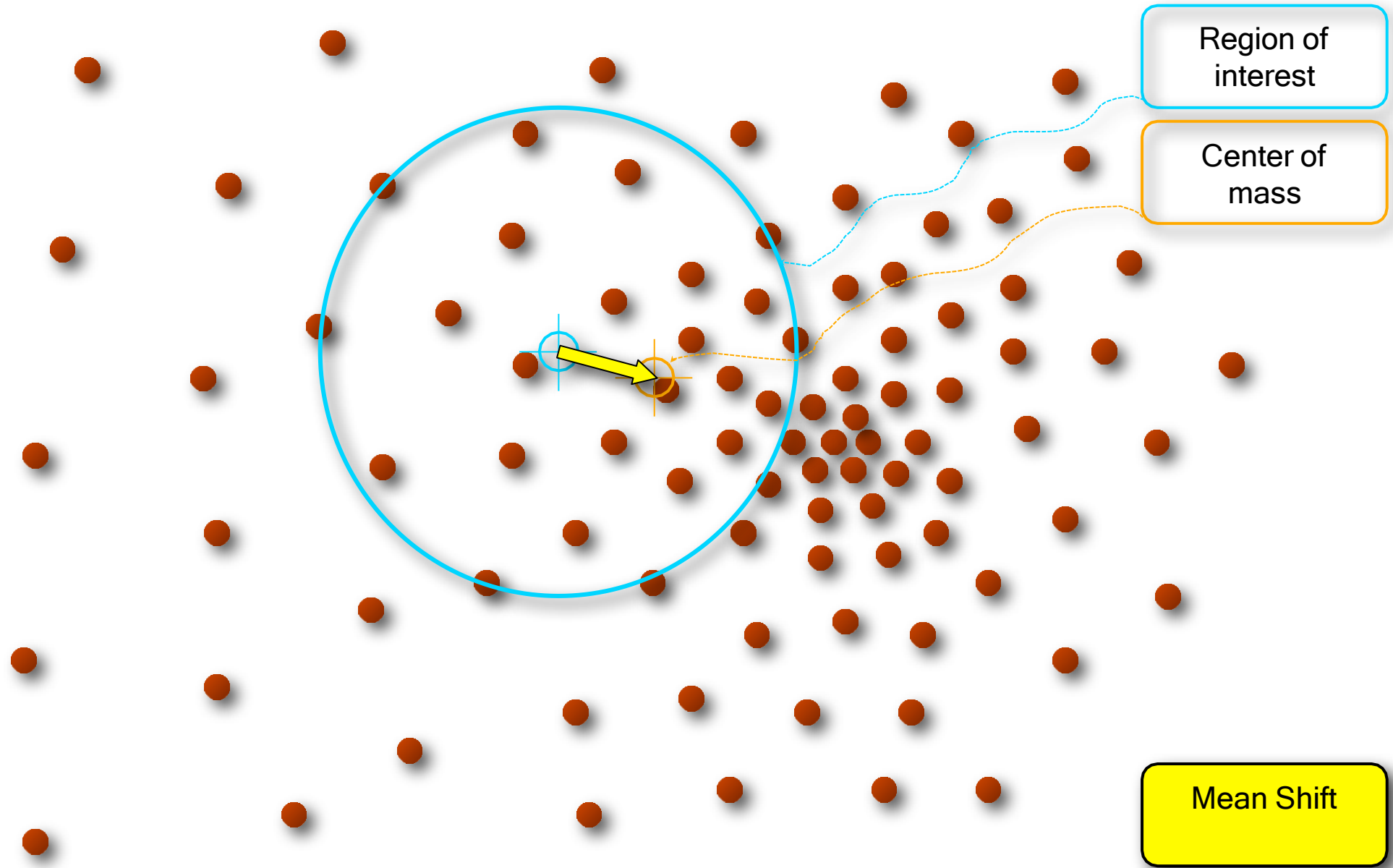
Region of  
interest

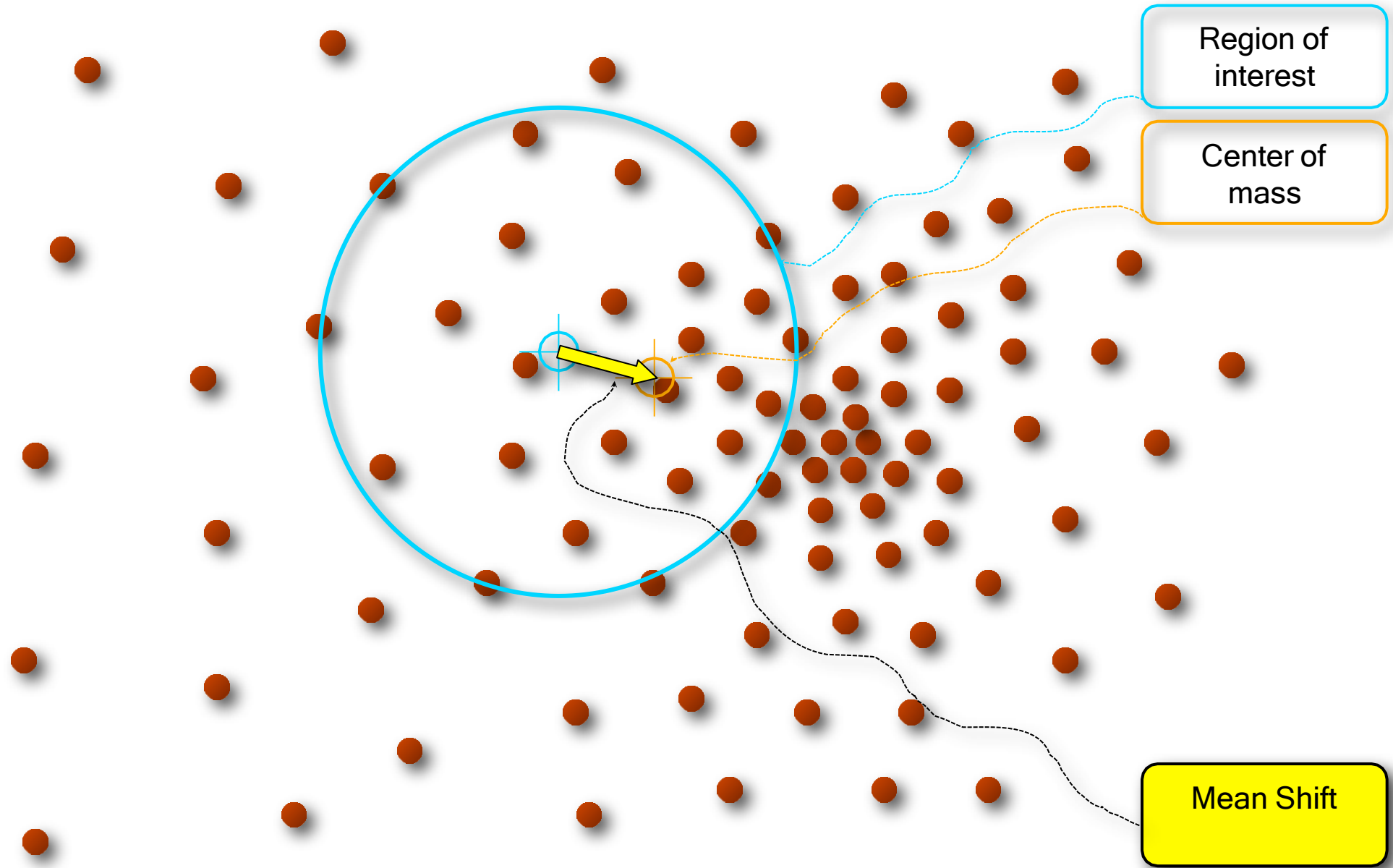




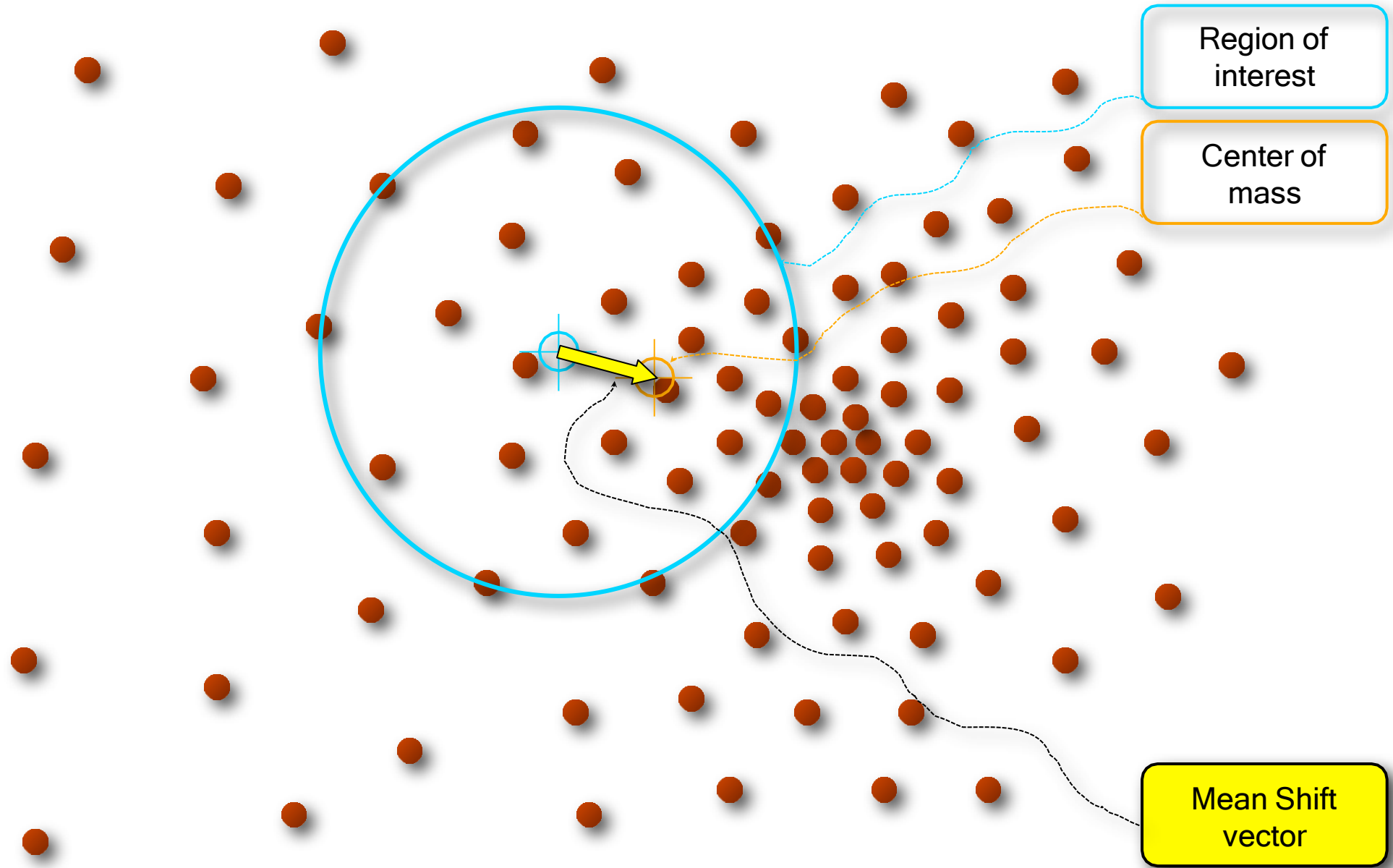


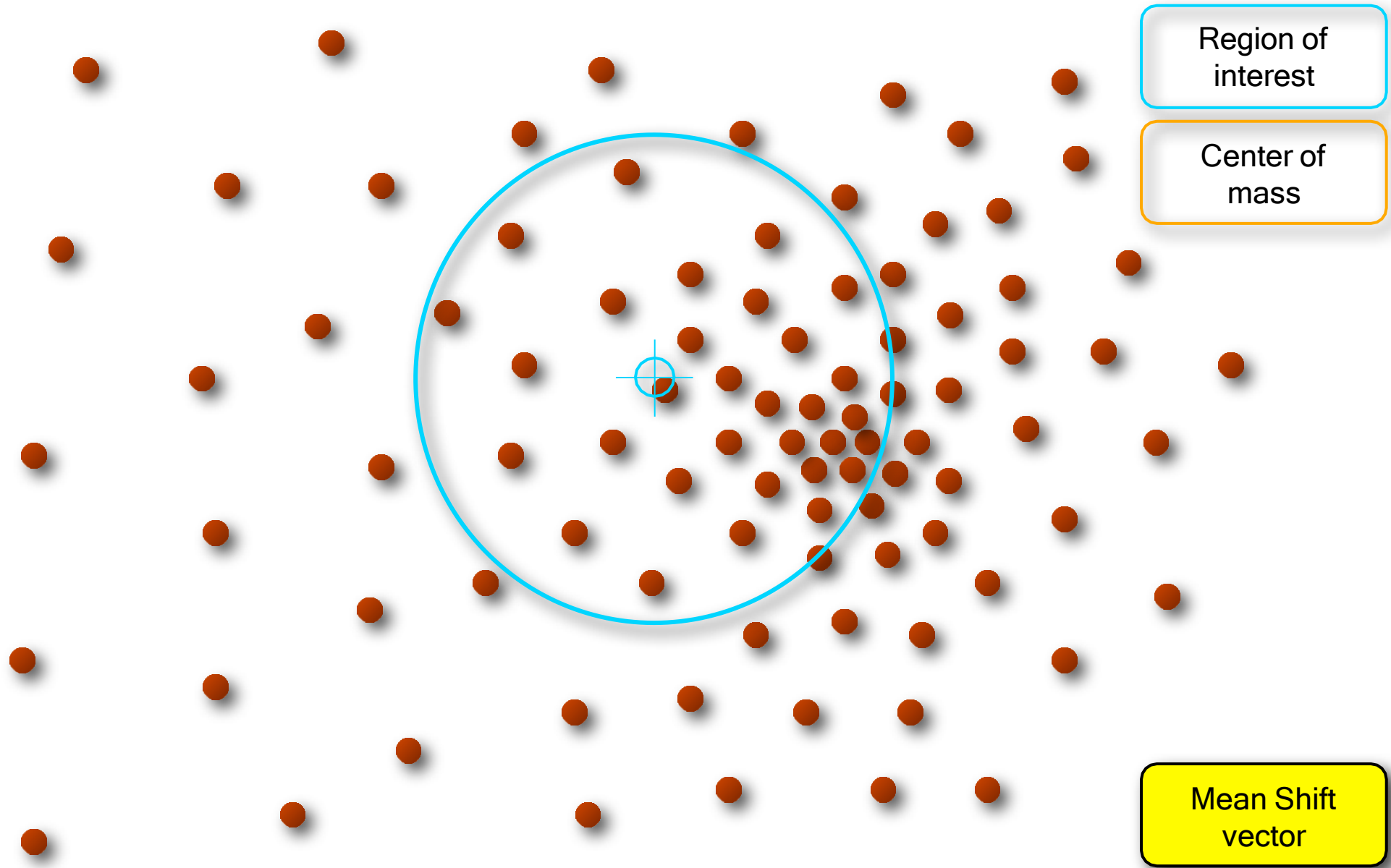


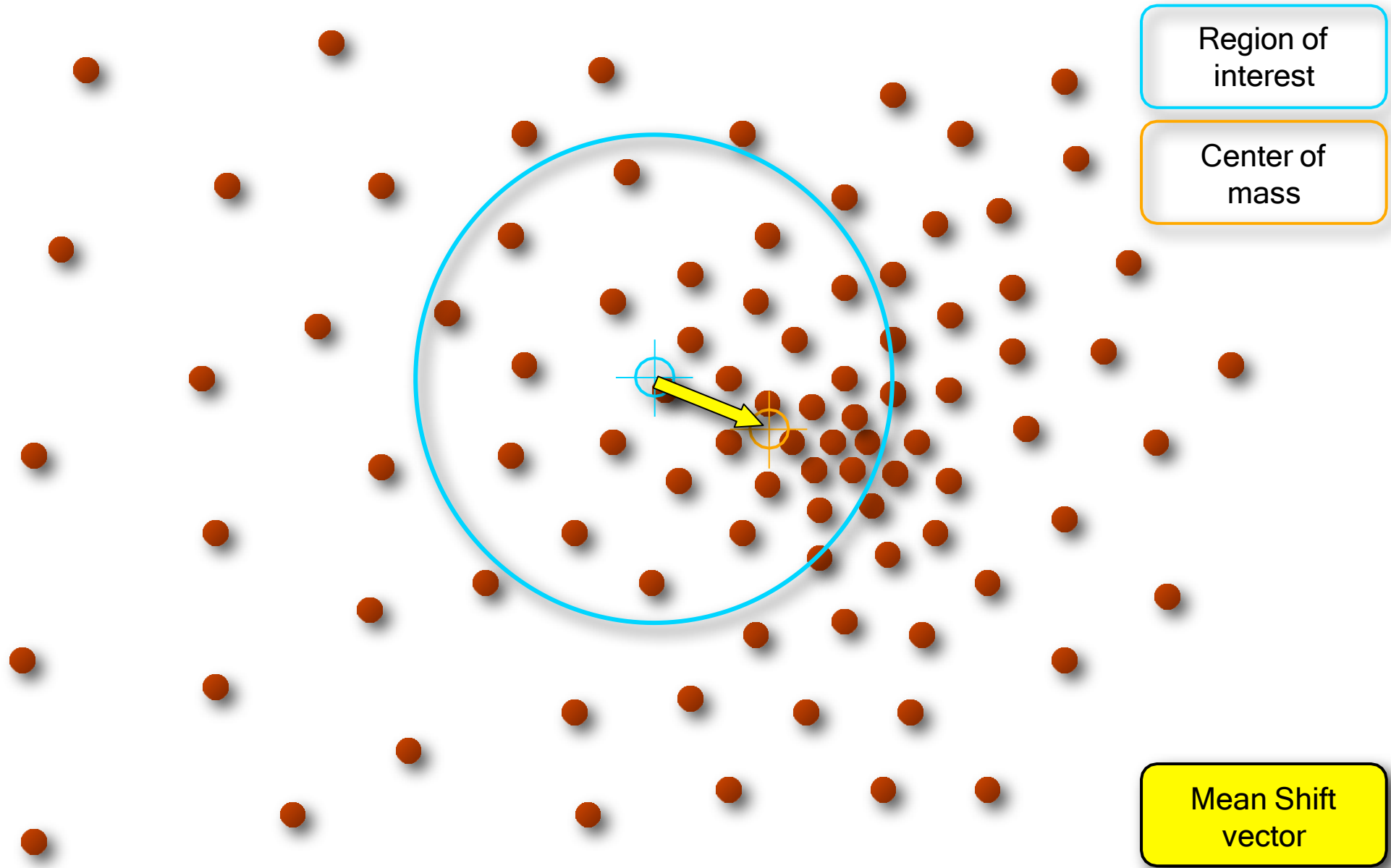


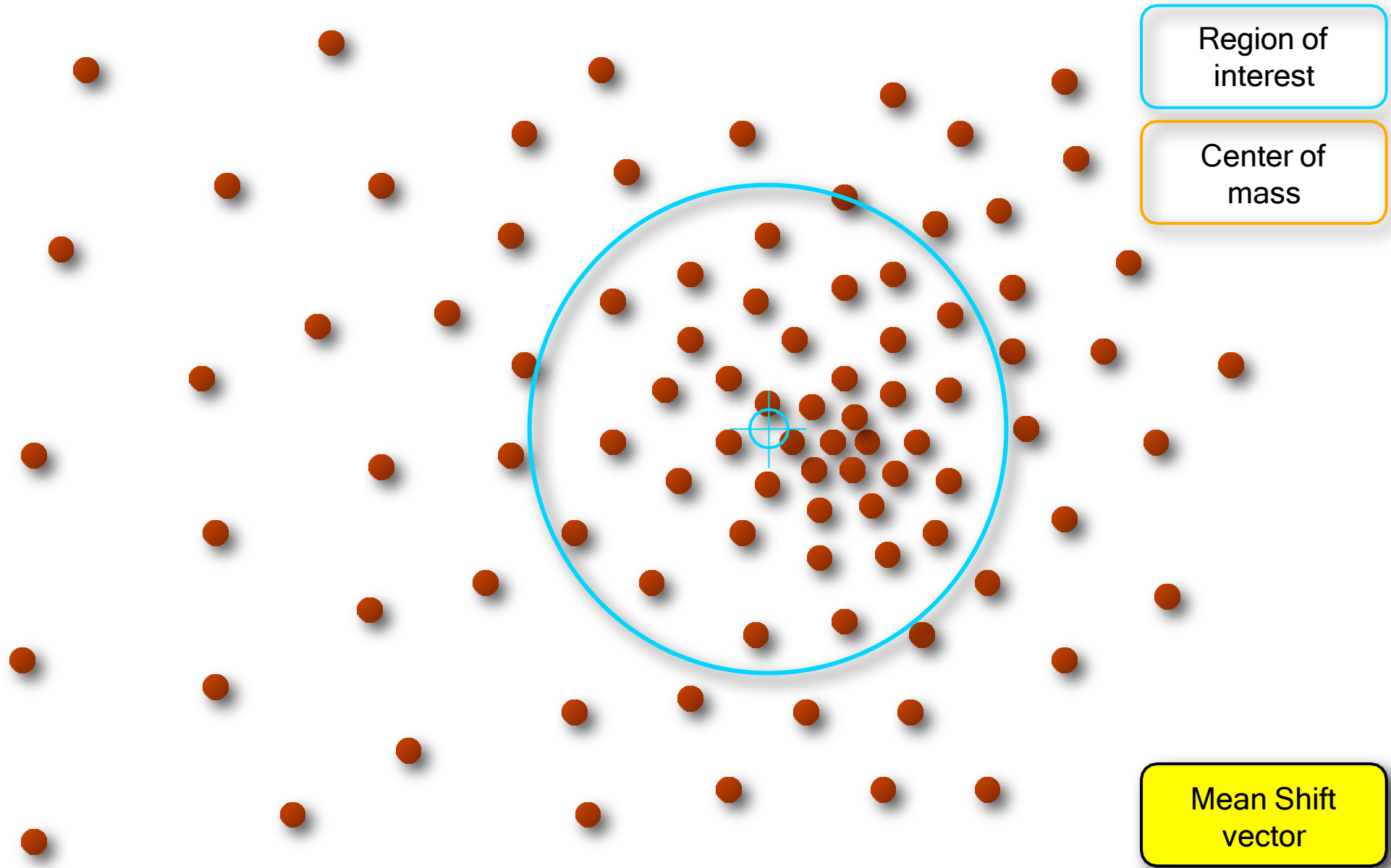


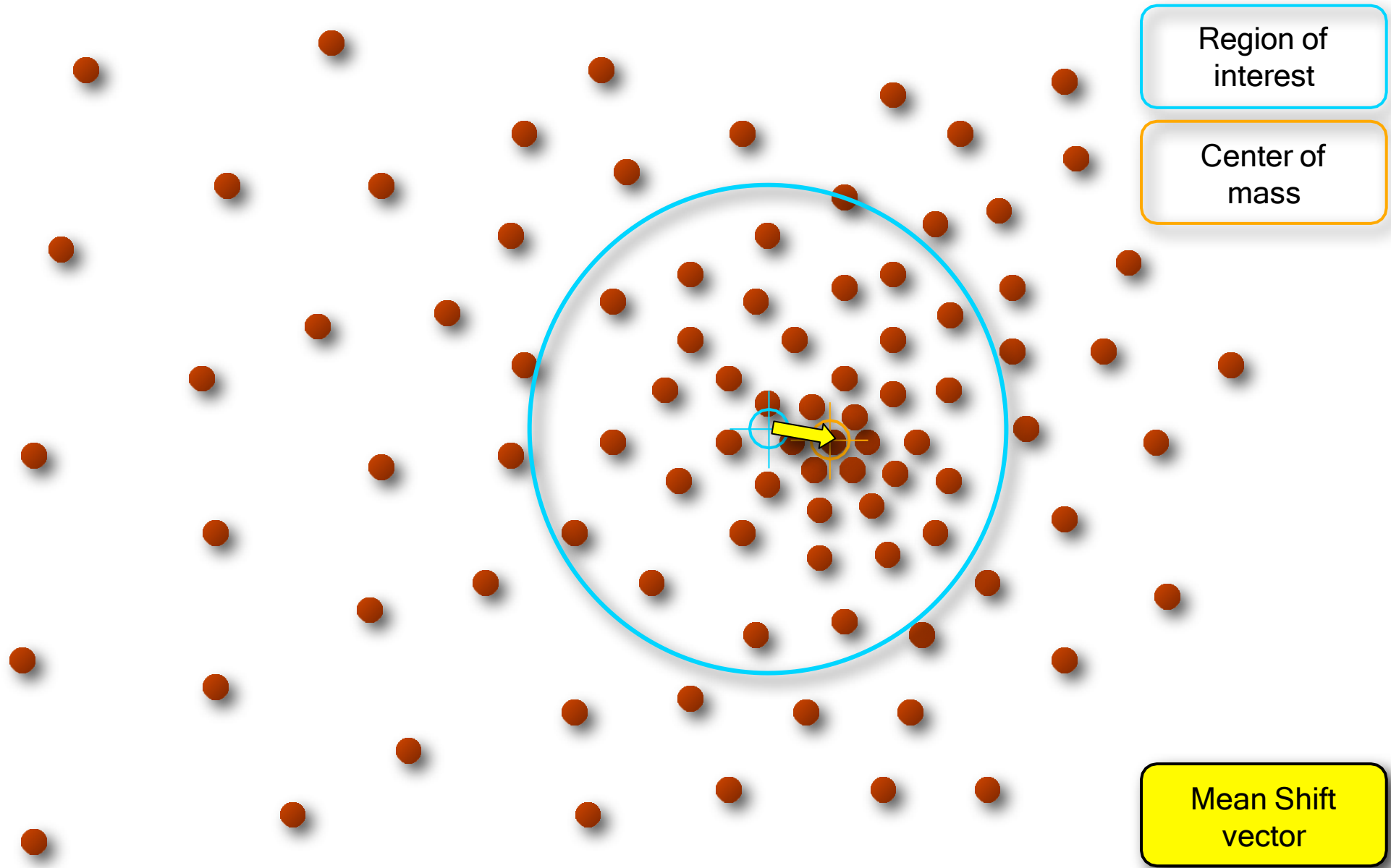


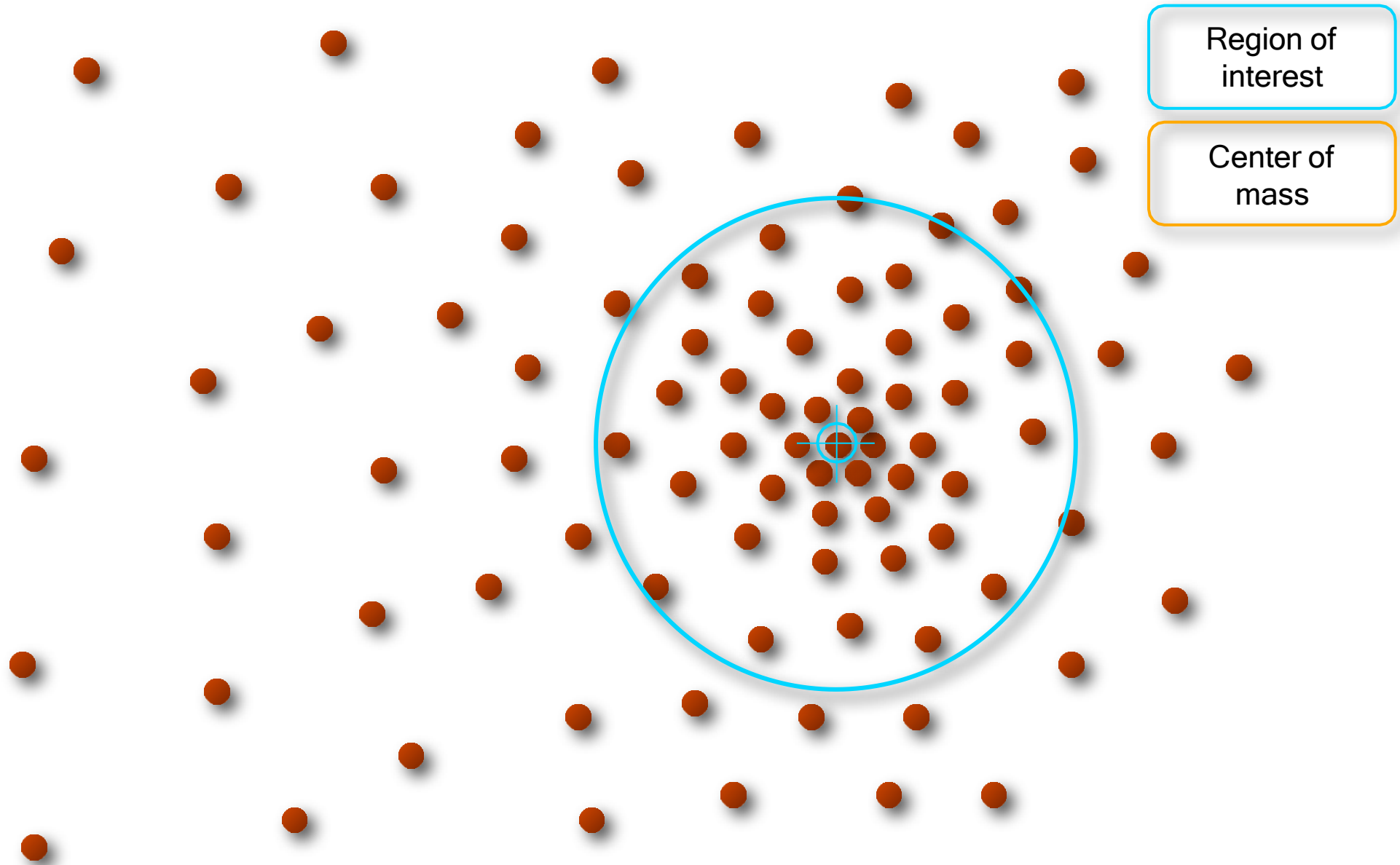


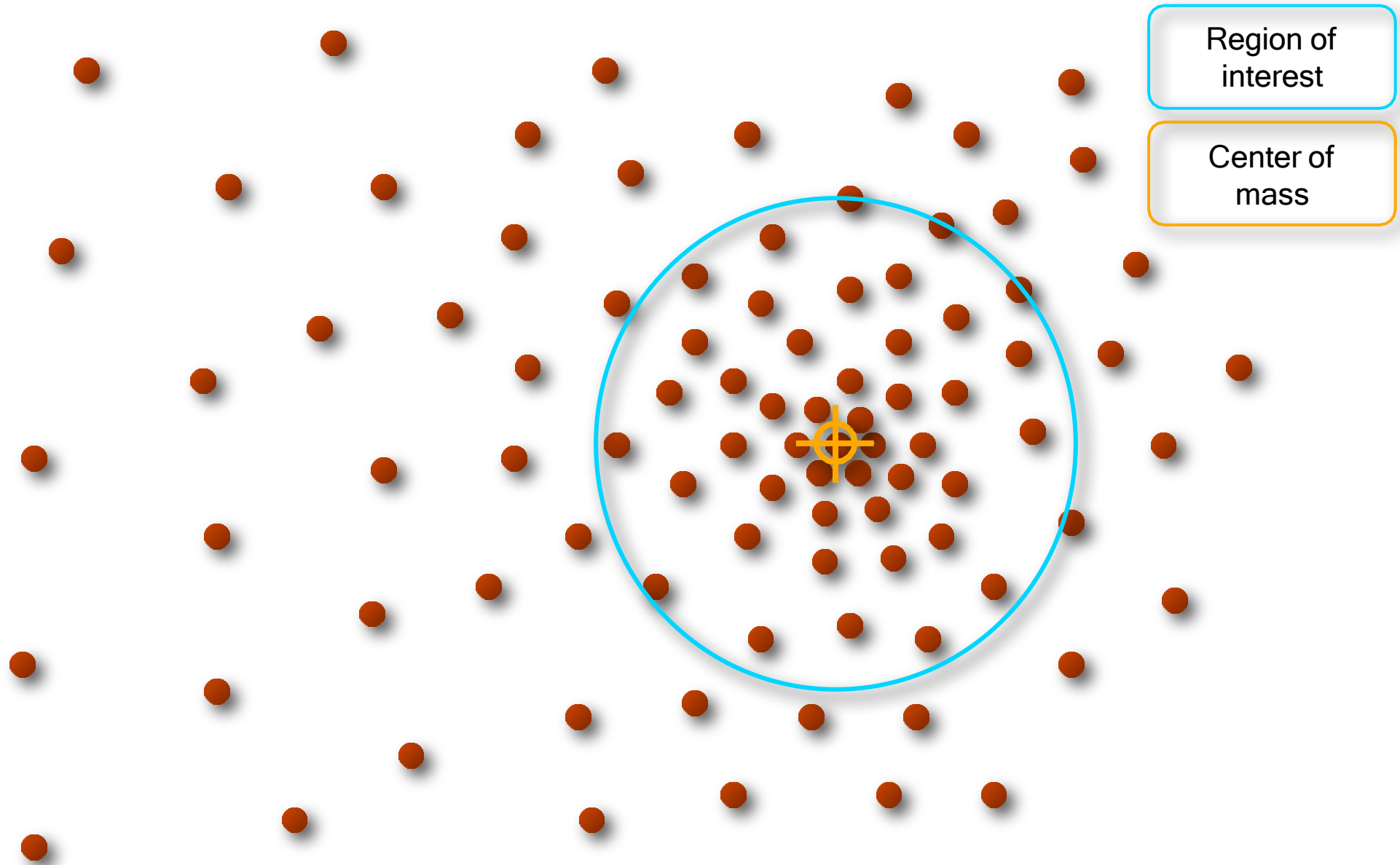






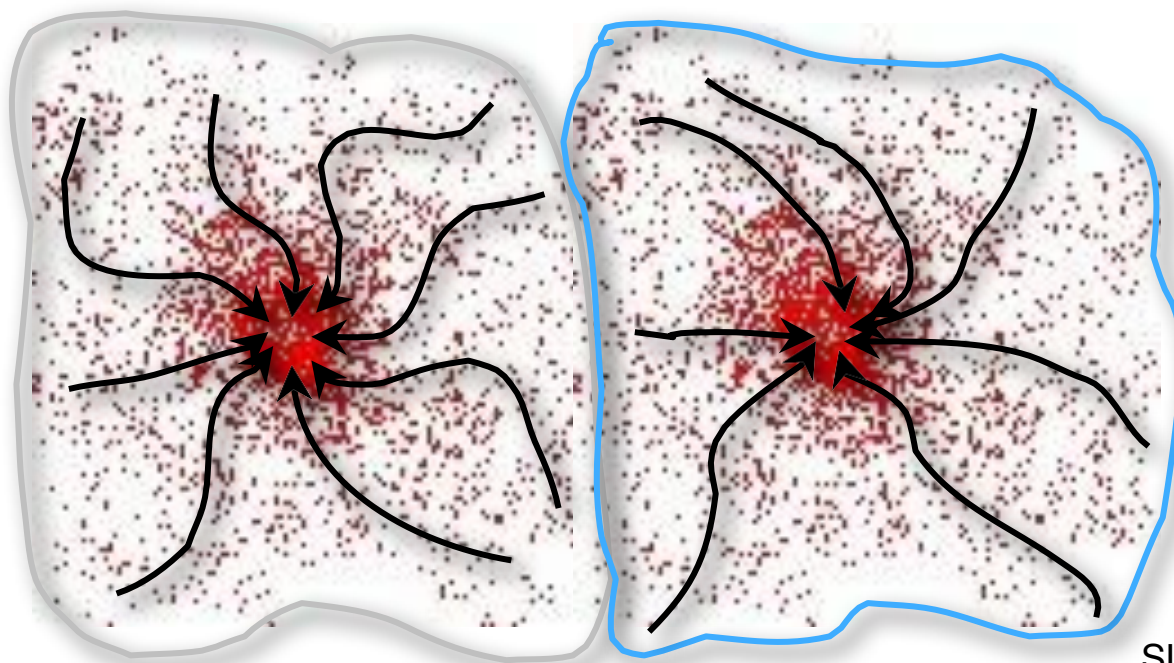






# Mean shift Clustering

- Cluster: all data points in the attraction basin of a mode
- Attraction basin: the region for which all trajectories lead to the same mode

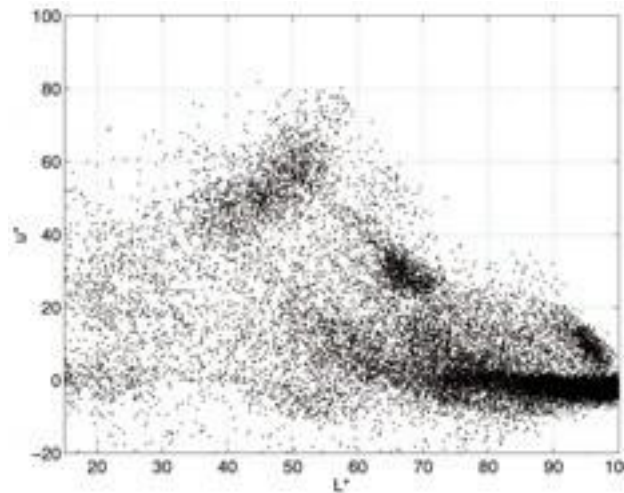


Slide by Y. Ukrainitz & B. Sarel

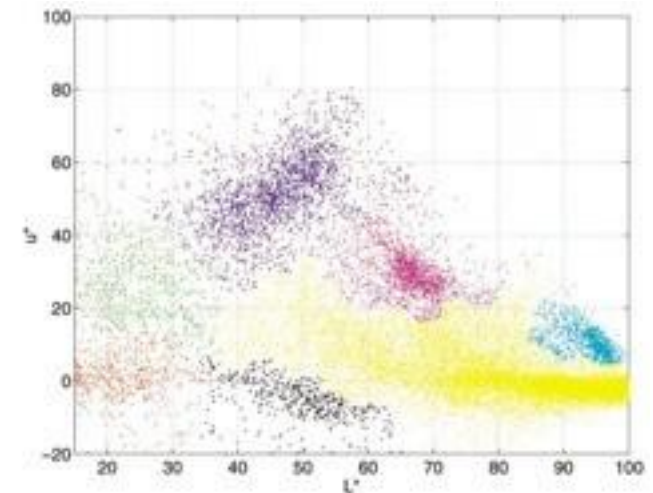


# Mean shift Clustering

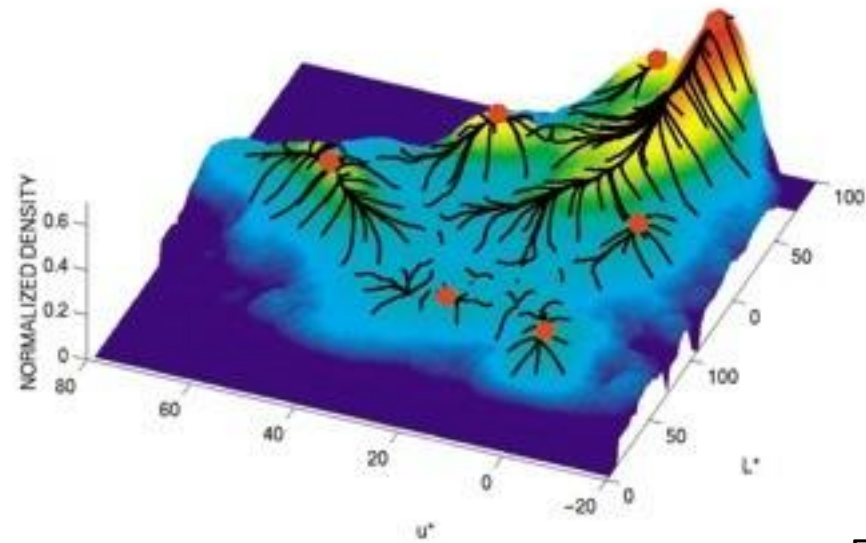
- Find features (color, gradients, texture, etc)
- Initialize windows at individual pixel locations
- Perform mean shift for each window until convergence
- Merge windows that end up near the same “peak” or mode



(a)



(b)



(c)

From Rob Fergus

# Mean-shift Clustering Result







# More Results



# Mean shift Clustering

- Pros
  - Does not assume spherical clusters
  - Just a single parameter (window size)
  - Finds variable number of modes
  - Robust to outliers
- Cons
  - Output depends on window size
  - Computationally expensive
  - Does not scale well with dimension of feature space

# Comparison

## Watershed

- May be supervised or unsupervised
- No need for threshold parameter
- Partitions image into many regions
- To get good results best to specify number and positions of seeds

## K-Means

- Unsupervised
- Works in high--dimensions
- May produce disjoint segments
- Must specify number of clusters but not position

## Mean-Shift

- Unsupervised
- Works in high--dimensions (kind of)
- May produce disjoint segments
- “Only” specify windows size

# Overview

What is image segmentation?

Types of segmentation algorithms

1. Thresholding, region labelling and growing algorithms
  - (connected components, region growing, watershed)
2. Statistical Segmentation
  - (k-means, mean shift)
3. Graph based methods
  - (Merging algorithms, splitting algorithms, split/merge)
4. Edge based methods
  - (Intelligent Scissors, Snakes)



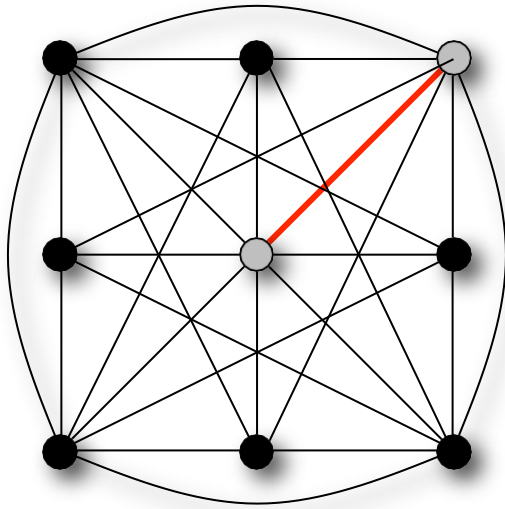
# Overview

What is image segmentation?

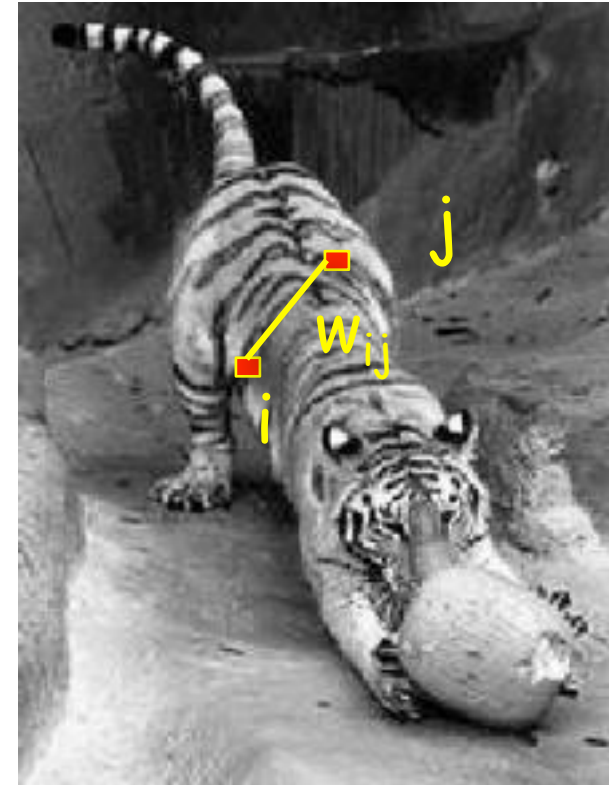
Types of segmentation algorithms

1. Thresholding, region labelling and growing algorithms
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2. Statistical Segmentation
  - (k-means, mean shift)
3. Graph based methods
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4. Edge based methods
  - (Intelligent Scissors, Snakes)

# Images as Graphs



Similar pixels

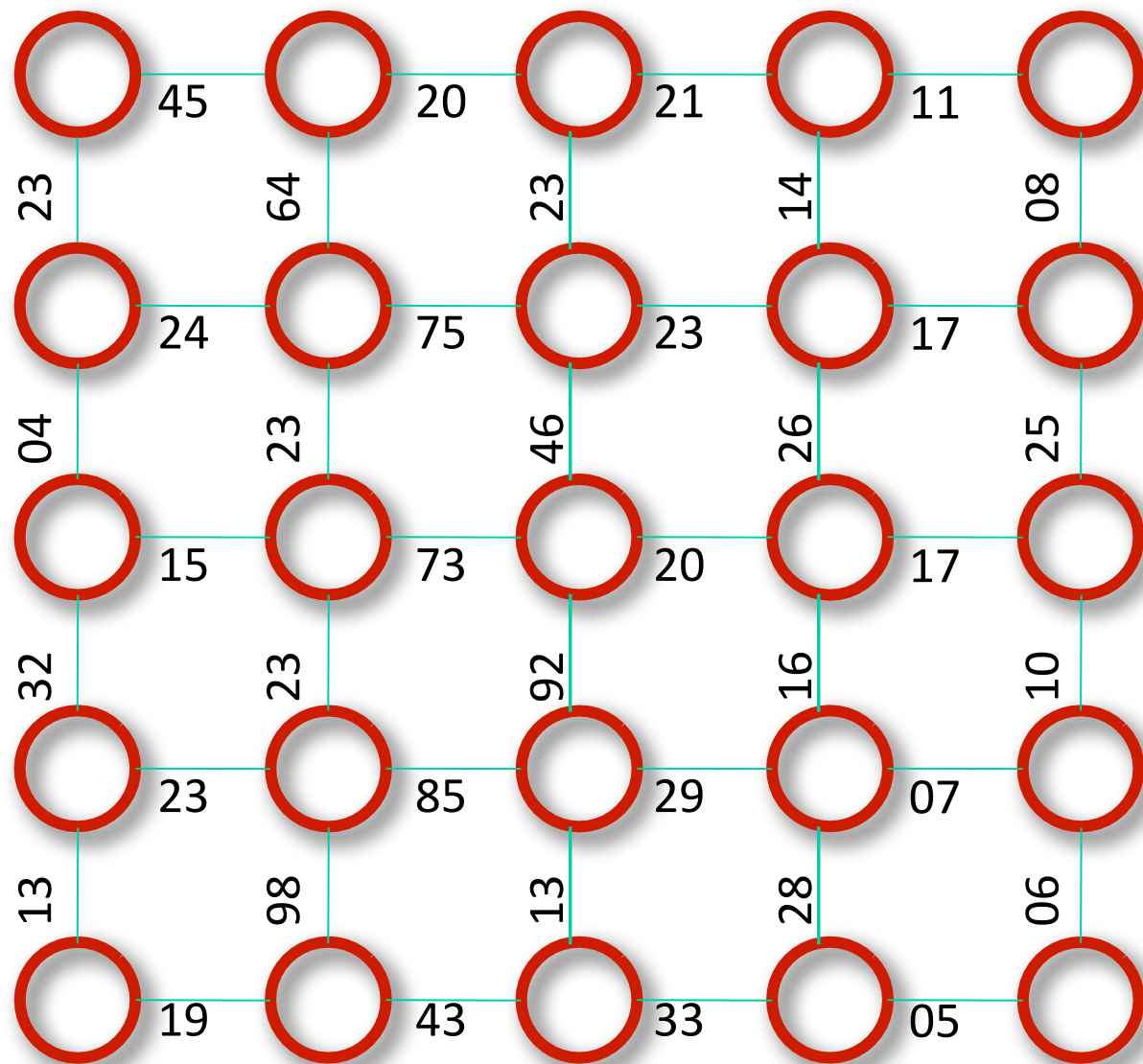


- Node for every pixel
- Edge between every pair of pixels (or every pair of “sufficiently close” pixels)
- Each edge is weighted by the **affinity** or similarity of the two nodes

Source: S. Seitz

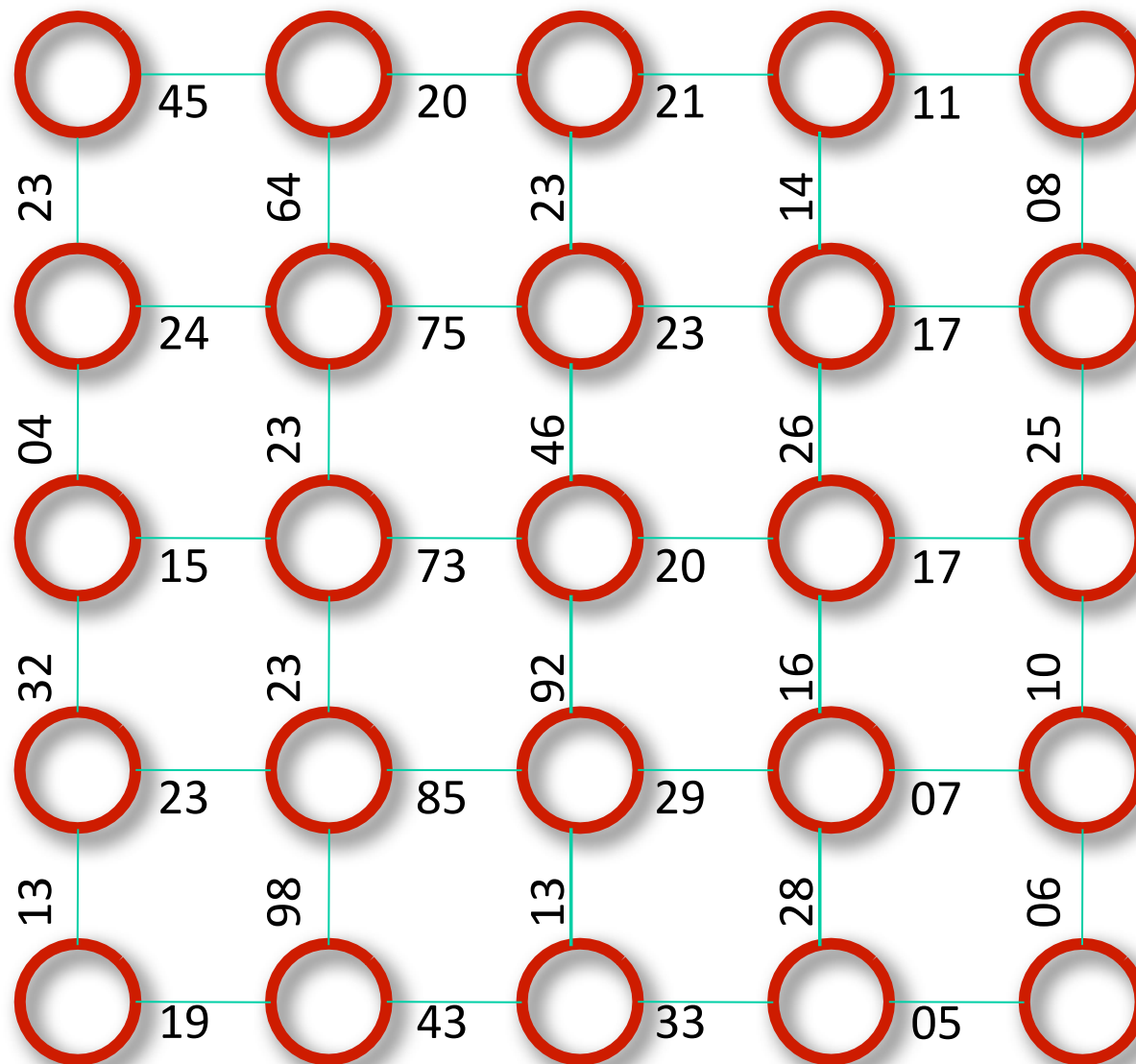
# Greedy Merging Algorithm

1. Initialize each pixel to a separate segment
2. Assign a cost,  $c_{ij}$  to each edge  
(possibly based on pixel difference)
3. Sort all edges by edge strength
4. Connect edges in order of strength, merging segments as we go
5. Stop when we have reached some threshold edge strength



Extract all edge strengths

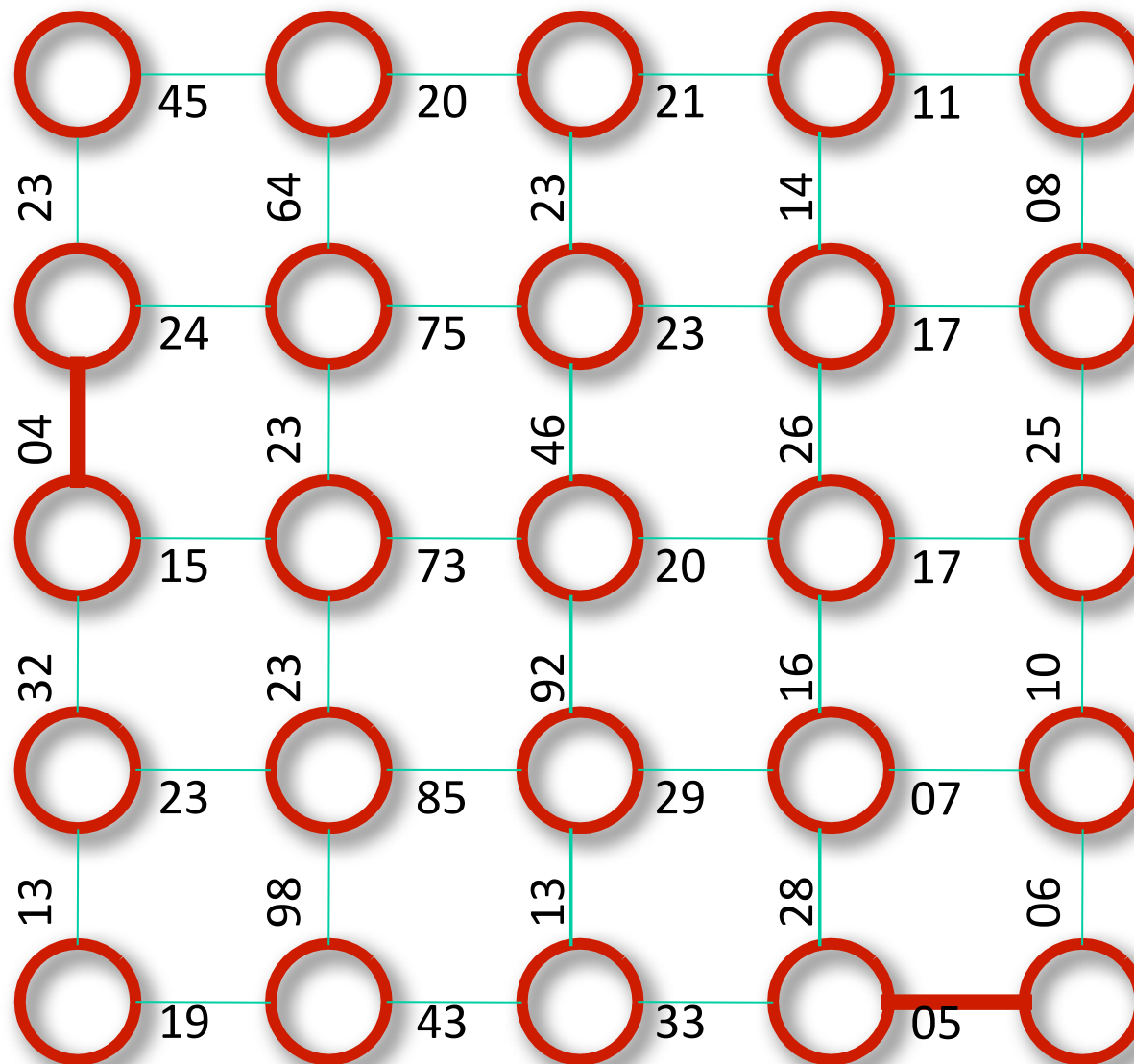
45, 20, 21, 11 ,24, 75, 23, 17, 15, 73, 20 ,17, 23, 85, 29, 07, 19,43,33, 05, 13, 22,  
04,23, 98, 23, 23, 64, 13, 92, 46, 23, 28, 16, 26, 14, 06, 10, 25, 08



Sort edge strengths (remembering where they came from)

04, 05, 06, 07, 08, 10, 11, 13, 13, 14, 15, 16, 17, 17, 19, 20, 20, 21, 22, 23, 23, 23, 23,  
23, 23, 24, 25, 26, 28, 29, 33, 43, 45, 46, 64, 73, 75, 85, 92, 98

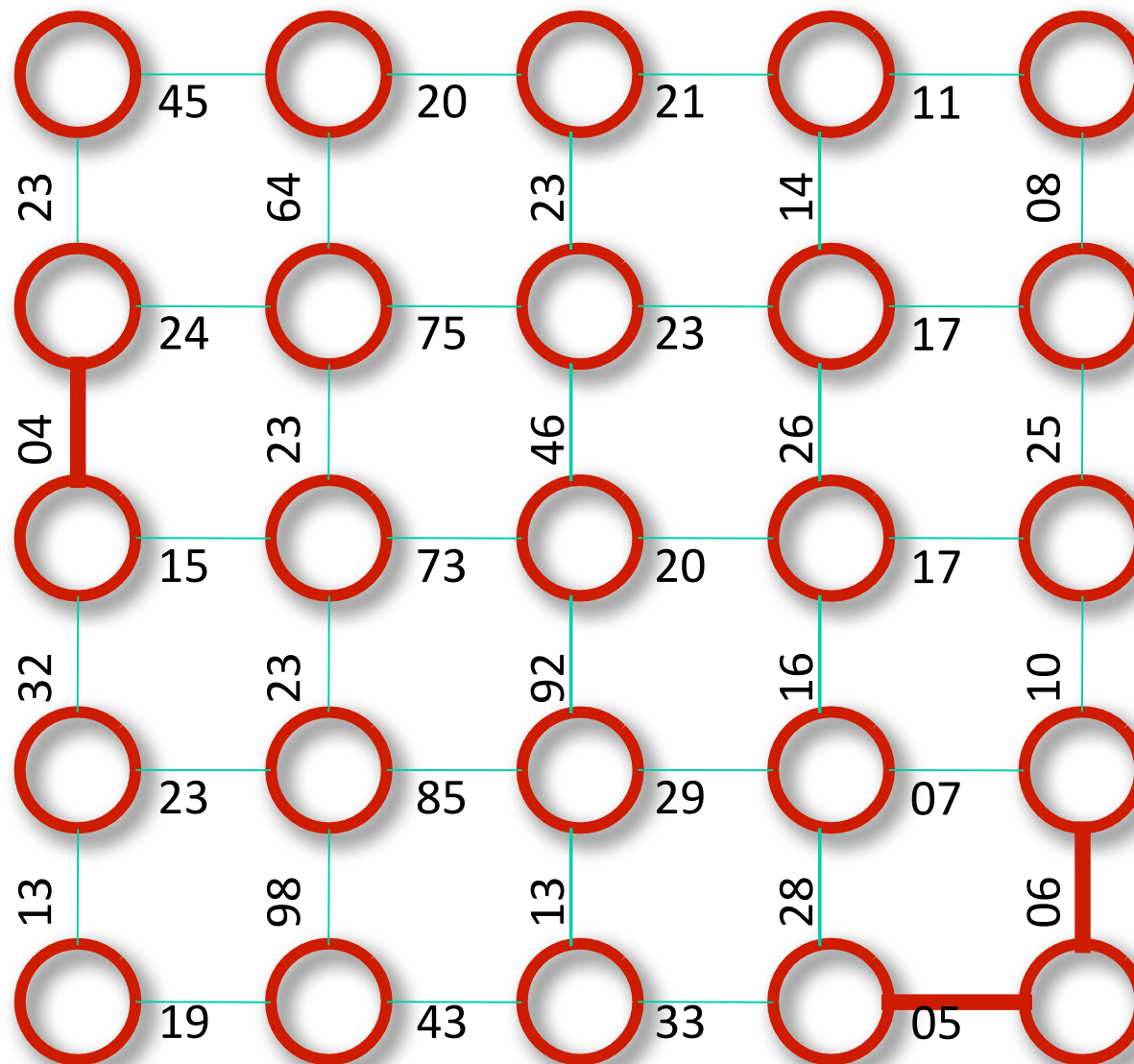
04, 05, 06, 07, 08, 10, 11, 13, 13, 14, 15, 16, 17, 17, 19, 20, 20, 21, 22, 23, 23, 23, 23,  
23, 23, 24, 25, 26, 28, 29, 33, 43, 45, 46, 64, 73, 75, 85, 92, 98



Connect together pixels, joining weakest first (until reach some threshold)

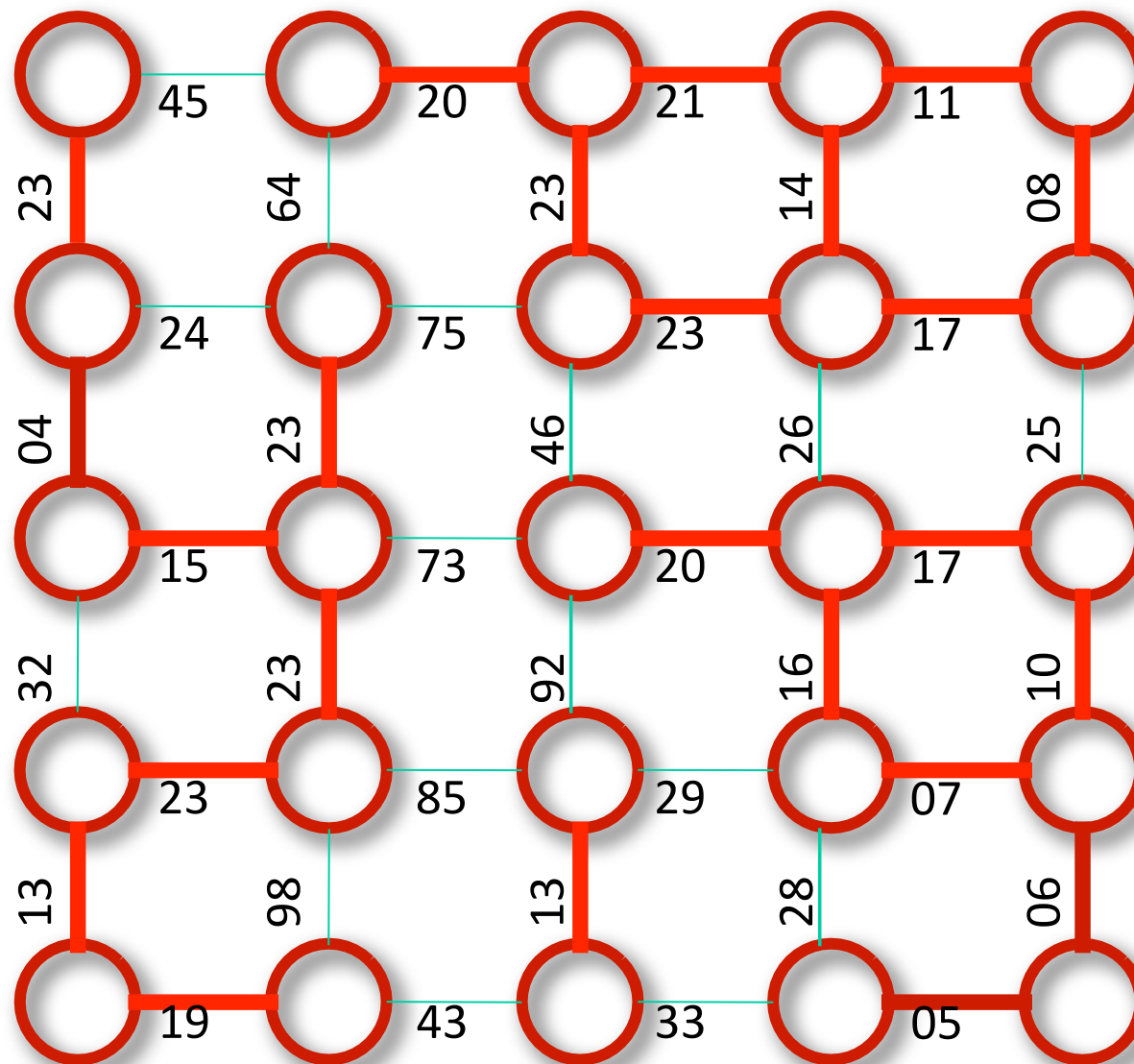
04, 05, 06, 07, 08, 10, 11, 13, 13, 14, 15, 16, 17, 17, 19, 20, 20, 21, 22, 23, 23, 23, 23, 23, 23, 24, 25, 26, 28, 29, 33, 43, 45, 46, 64, 73, 75, 85, 92, 98





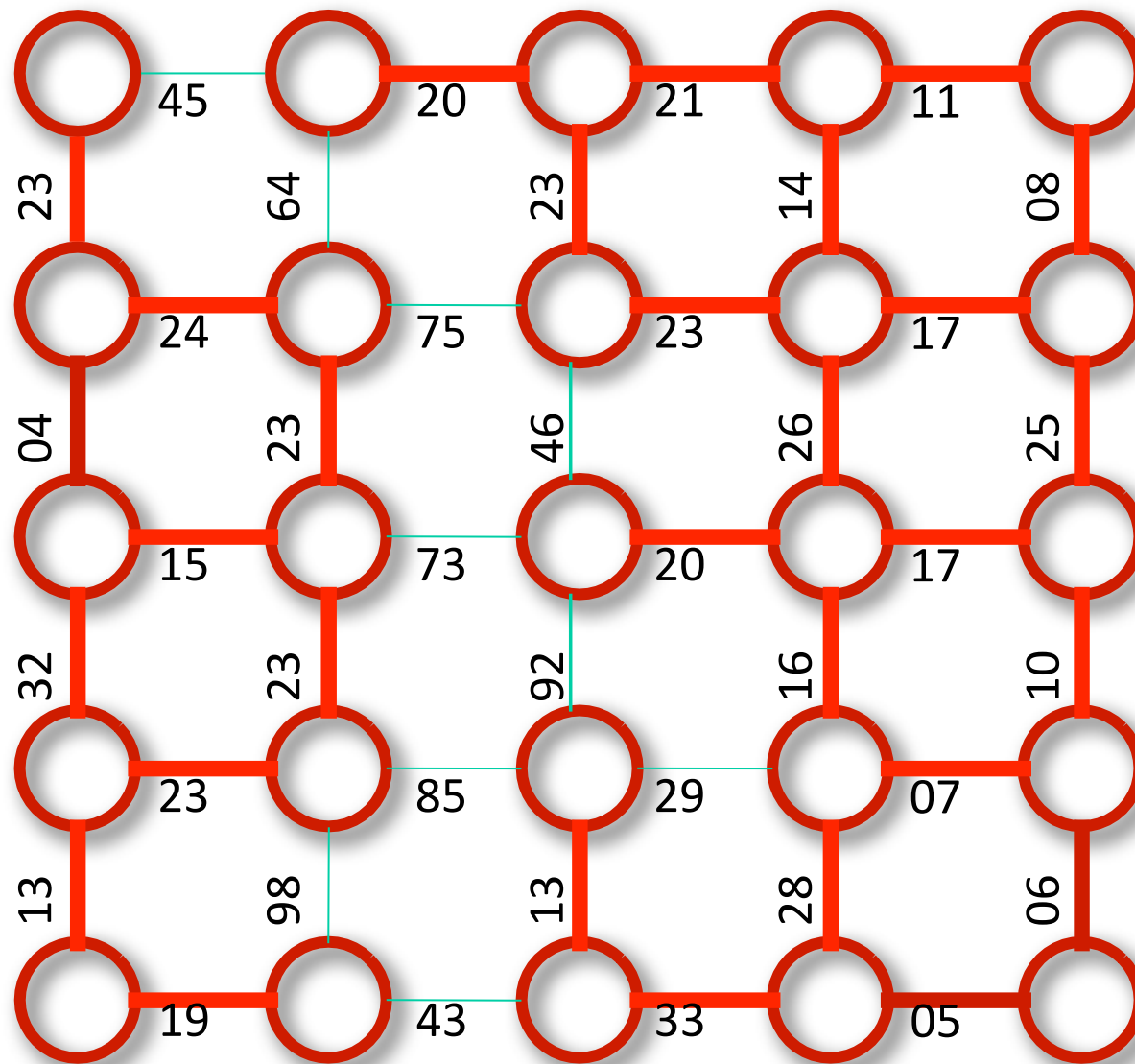
Connect together pixels, joining weakest first (until reach some threshold)

04, 05, 06, 07, 08, 10, 11, 13, 13, 14, 15, 16, 17, 17, 19, 20, 20, 21, 22, 23, 23, 23, 23,  
23, 23, 24, 25, 26, 28, 29, 33, 43, 45, 46, 64, 73, 75, 85, 92, 98



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04, 05, 06, 07, 08, 10, 11, 13, 13, 14, 15, 16, 17, 17, 19, 20, 20, 21, 22, 23, 23, 23, 23,  
23, 23, 24, 25, 26, 28, 29, 33, 43, 45, 46, 64, 73, 75, 85, 92, 98



Connect together pixels, joining weakest first (until reach some threshold)

04, 05, 06, 07, 08, 10, 11, 13, 13, 14, 15, 16, 17, 17, 19, 20, 20, 21, 22, 23, 23, 23, 23,  
23, 23, 24, 25, 26, 28, 29, 33, 43, 45, 46, 64, 73, 75, 85, 92, 98

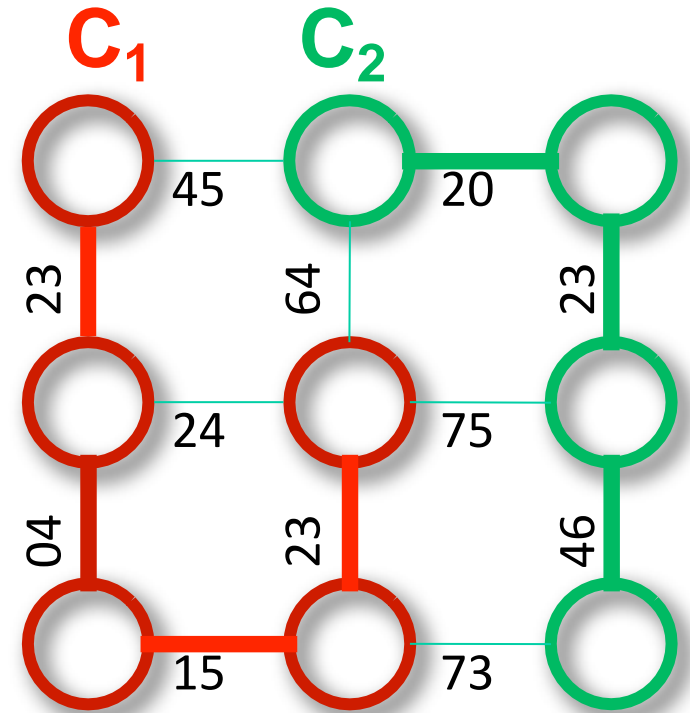
# A Better Merging Algorithm

Efficient Graph-Based Image Segmentation (Felzenswalb, Huttenlocher [IJCV 2004])

- Assign weight  $w(e_{ij})$  to each edge  $e_{ij}$
- Sort edges by weight and run through
- Define  $\text{Int}(C)$  to be the maximum cost weight in the minimum spanning tree of cluster  $C$
- Define  $\text{Dif}(C_1, C_2)$  to be lowest cost link joining components
- Merge components if

$$\text{Dif}(C_1, C_2) < \min(\text{Int}(C_1) + \tau(C_1), \text{Int}(C_2) + \tau(C_2))$$

$$\text{where } \tau(C) = k / |C|$$



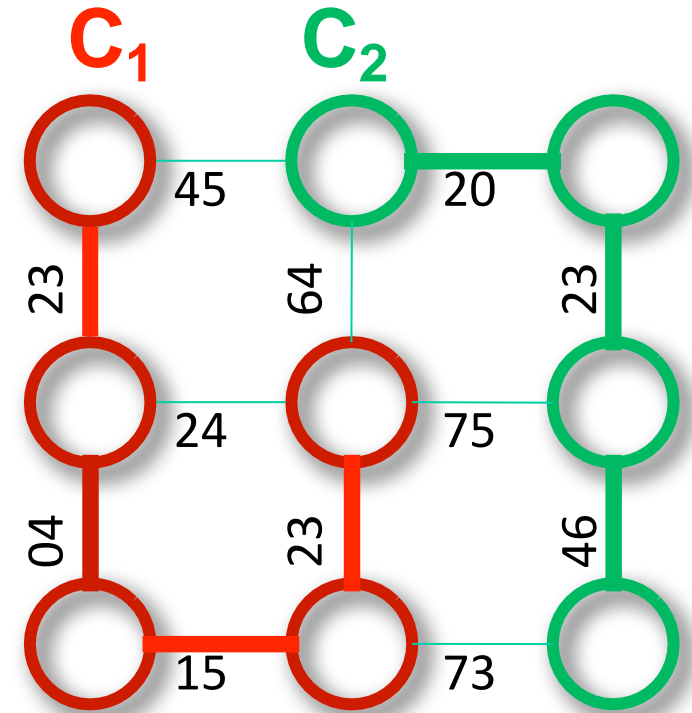
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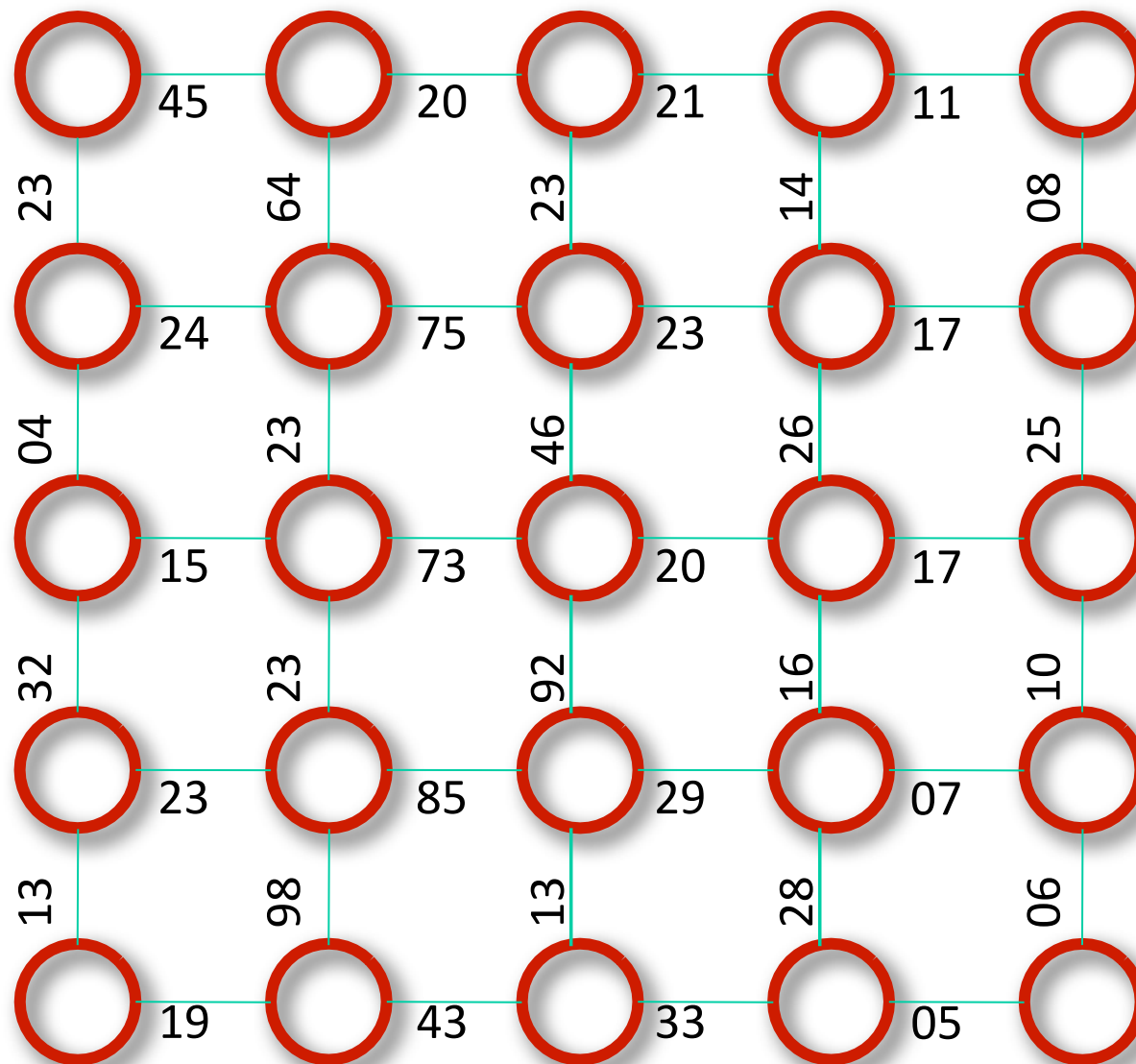
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$$\text{Dif}(C_1, C_2) < \min(\text{Int}(C_1) + \tau(C_1), \text{Int}(C_2) + \tau(C_2))$$

$$\text{where } \tau(C) = k / |C|$$



Here:  $\text{Int}(C_1) = 23$   
 $\text{Int}(C_2) = 46$   
 $\text{Dif}(C_1, C_2) = \min(45, 64, 73, 75) = 45$



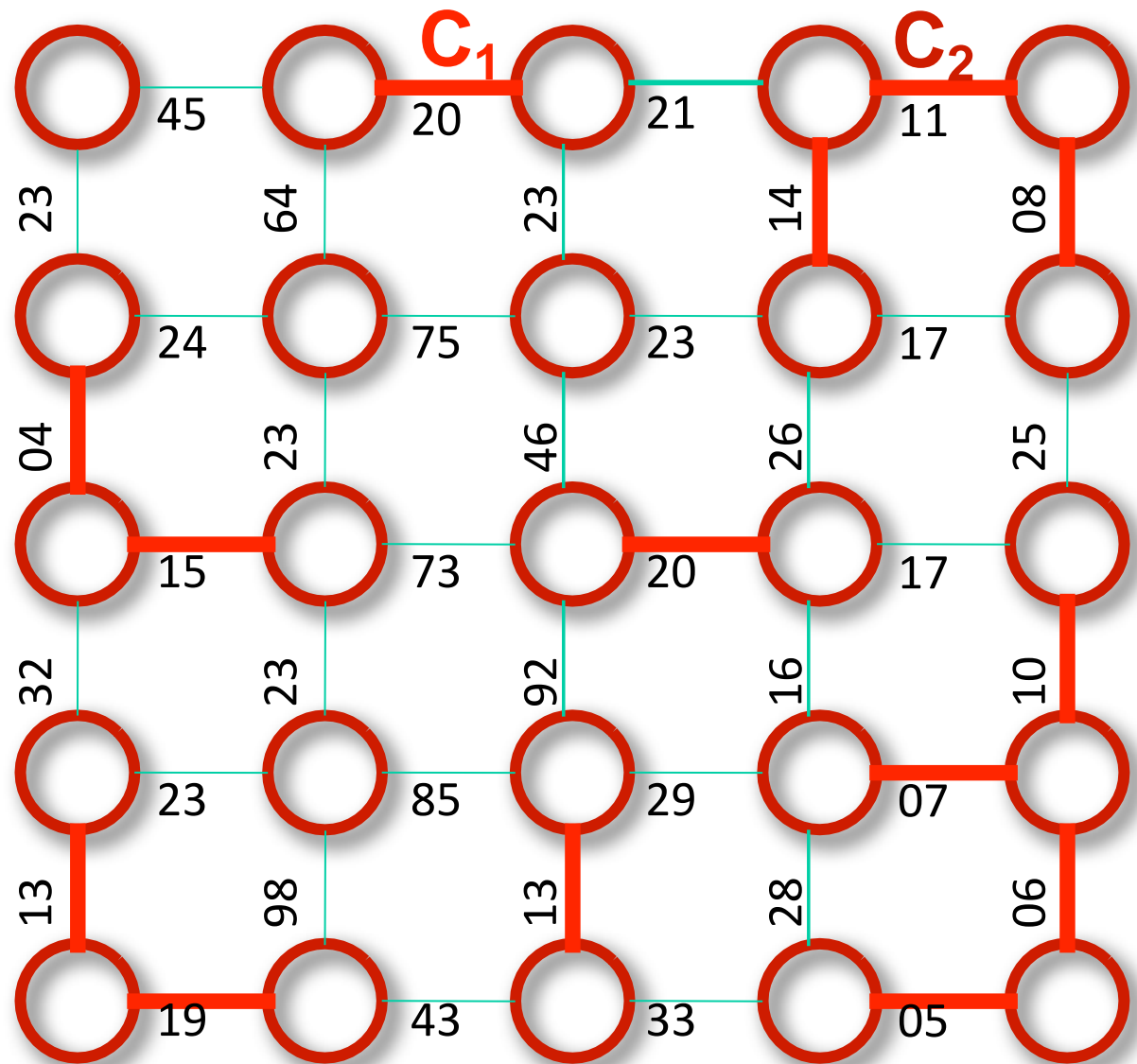
Sort edge strengths (remembering where they came from)

04, 05, 06, 07, 08, 10, 11, 13, 13, 14, 15, 16, 17, 17, 19, 20, 20, 21, 22, 23, 23, 23, 23,  
23, 23, 24, 25, 26, 28, 29, 33, 43, 45, 46, 64, 73, 75, 85, 92, 98

Let  $k = 50$   
 $\text{Int}(C_1) = 20$   
 $\text{Int}(C_2) = 14$   
 $\text{Dif}(C_1, C_2) = 21$

$$\text{Mint} = \min(20 + 50/2, 14 + 50/4) = 26.5$$

$\text{Dif}(C_1, C_2) < \text{Mint}$   
 so merge



Sorted Edge Strengths: 04, 05, 06, 07, 08, 10, 11, 13, 13, 14, 15, 16, 17, 17, 19, 20, 20, 21, 22, 23, 23, 23, 23, 23, 23, 23, 24, 25, 26, 28, 29, 33, 43, 45, 46, 64, 73, 75, 85, 92, 98



Let  $k = 50$

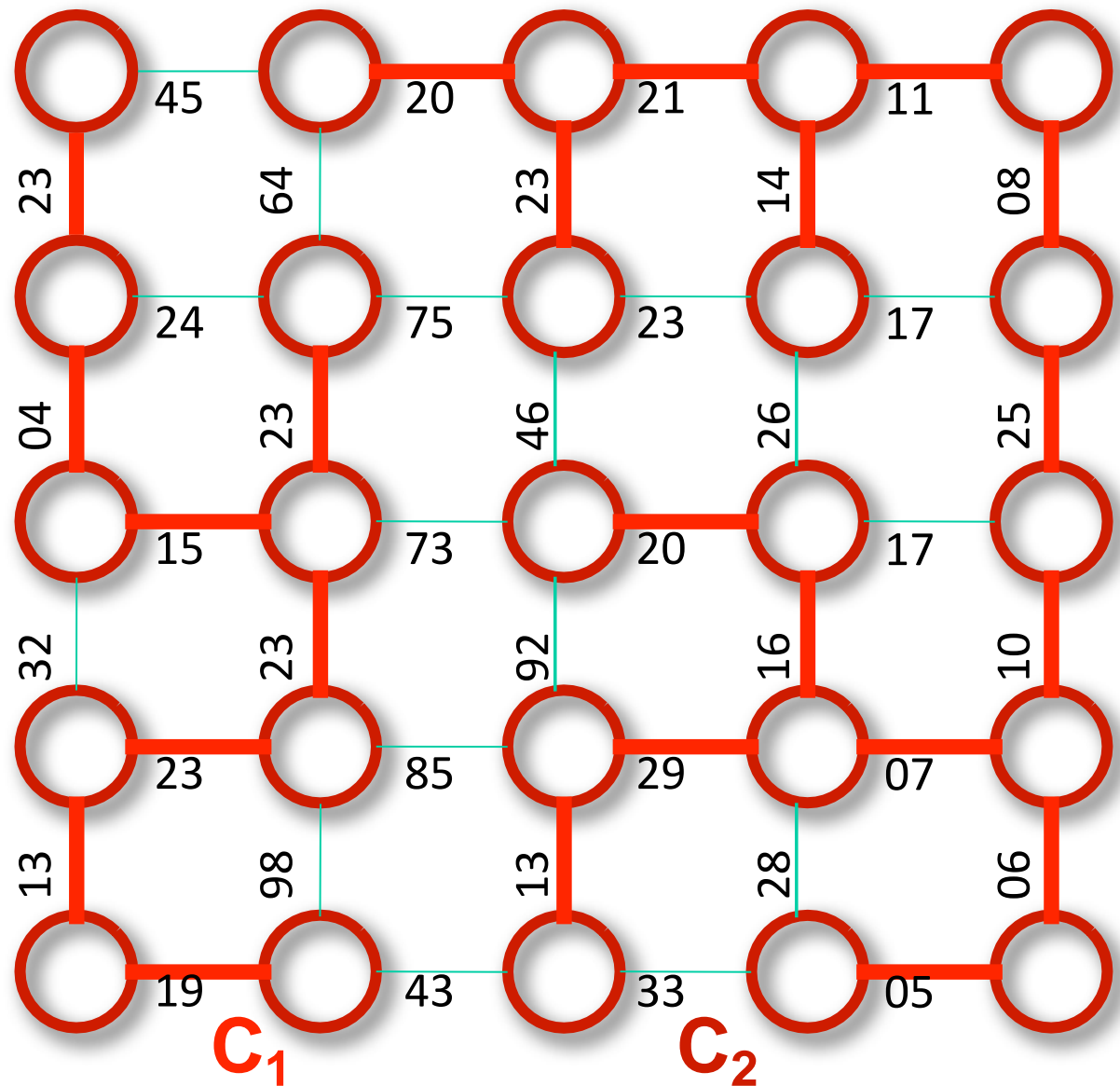
$\text{Int}(C_1) = 23$

$\text{Int}(C_2) = 29$

$\text{Dif}(C_1, C_2) = 43$

$\text{Mint} = \min(23+50/9, 19+50/16)$   
 $= 28.566$

$\text{Dif}(C_1, C_2) > \text{Mint}$   
 so don't merge



Sorted Edge Strengths: 04, 05, 06, 07, 08, 10, 11, 13, 13, 14, 15, 16, 17, 17, 19, 20, 20, 21, 22, 23, 23, 23, 23, 23, 23, 23, 24, 25, 26, 28, 29, 33, 43, 45, 46, 64, 73, 75, 85, 92, 98

# Example Results

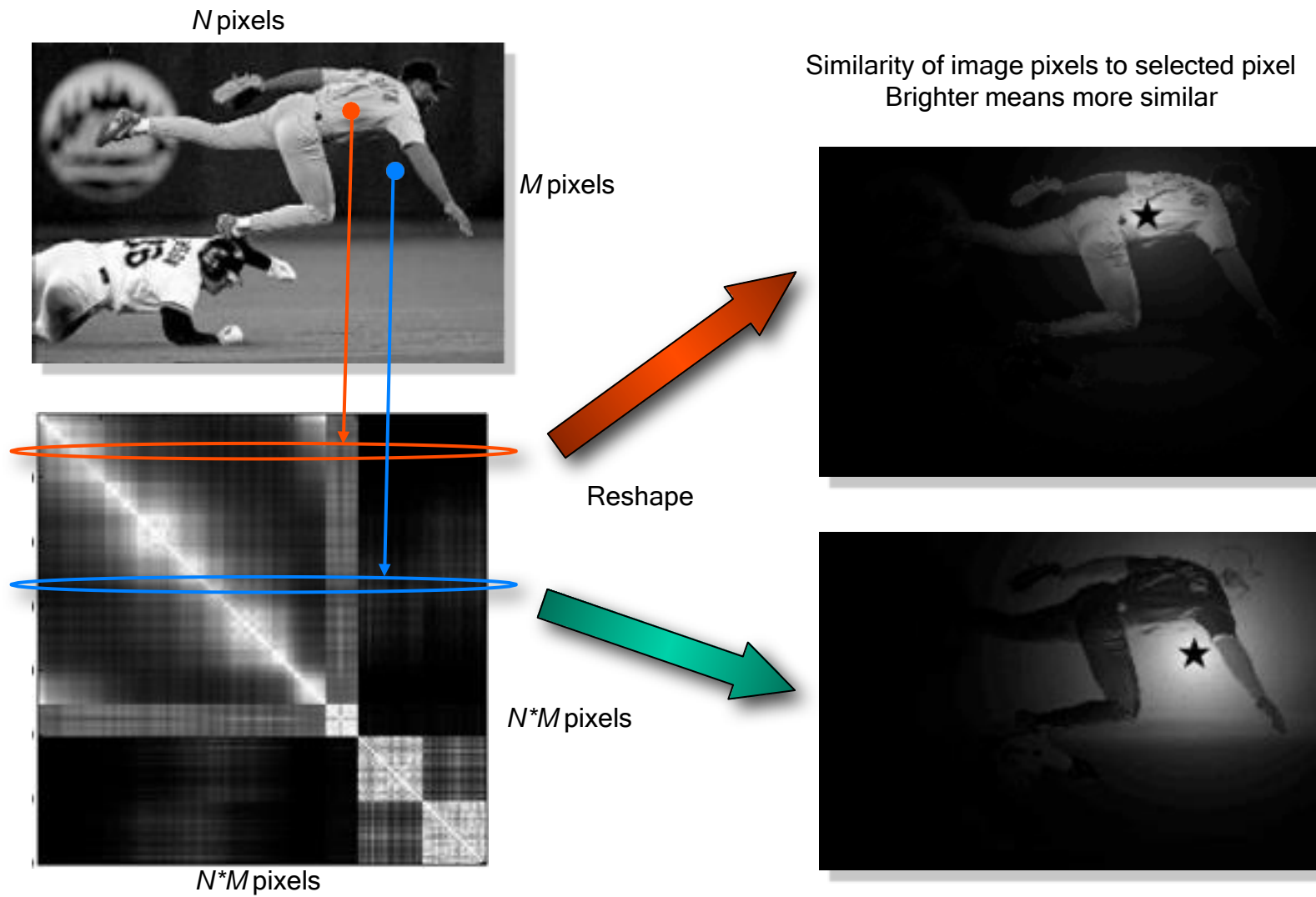


# Example Results



- Very fast
- But sensitive to noise
- Greedily chooses (too) large regions

# Affinity Matrix



# Pixel Similarity Functions

Intensity

$$W(i, j) = e^{\frac{-\|I_{(i)} - I_{(j)}\|_2^2}{\sigma_I^2}}$$

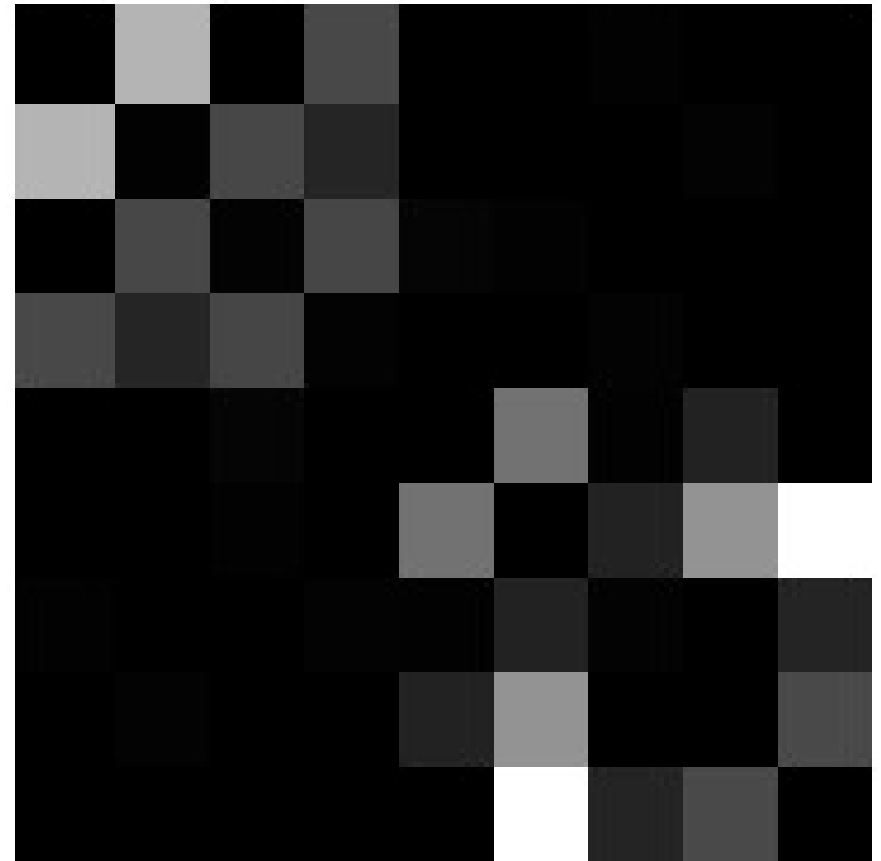
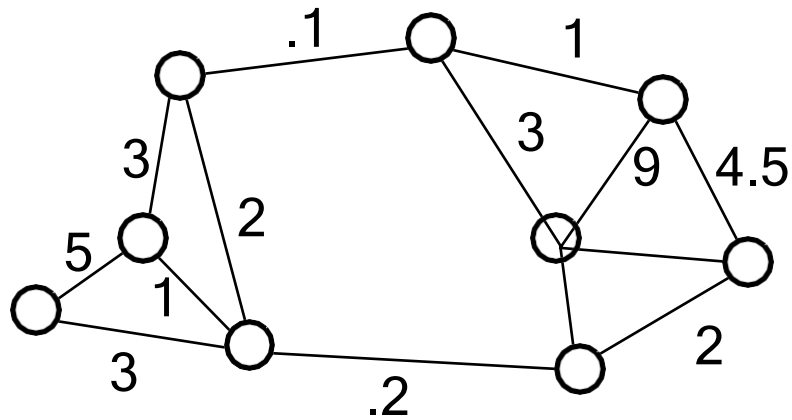
Distance

$$W(i, j) = e^{\frac{-\|X_{(i)} - X_{(j)}\|_2^2}{\sigma_X^2}}$$

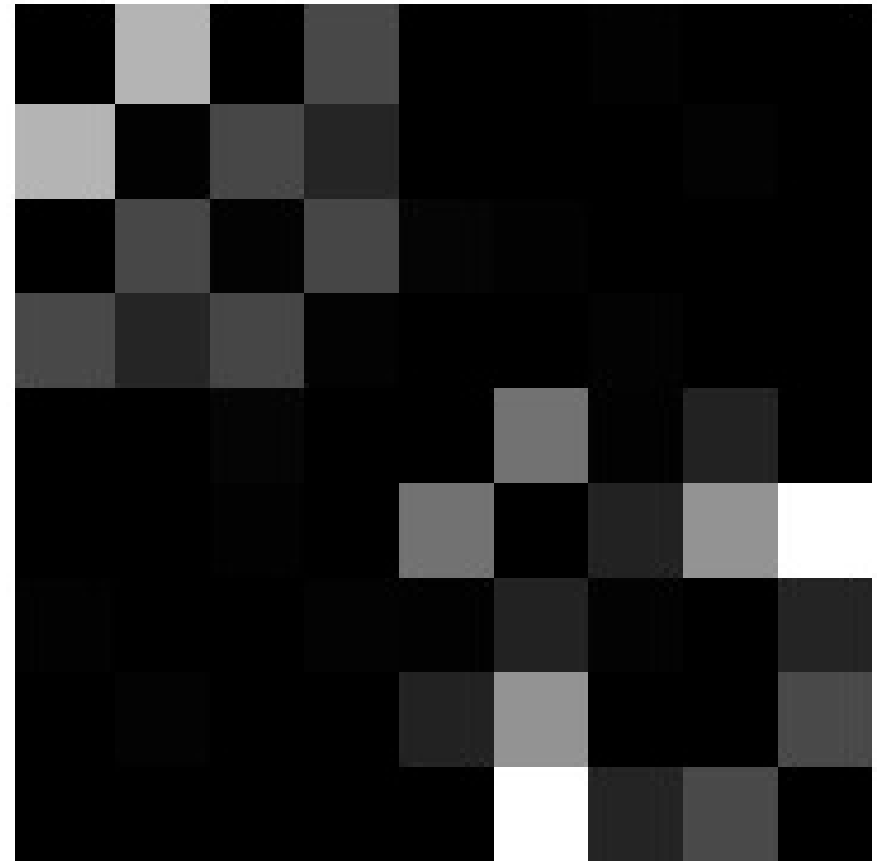
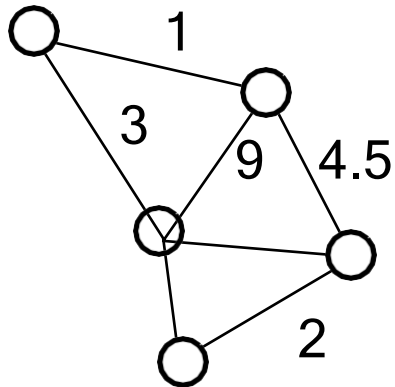
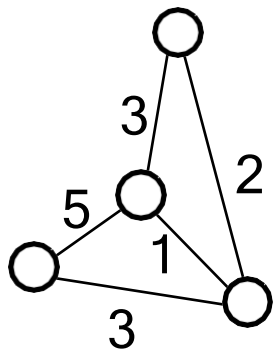
Texture

$$W(i, j) = e^{\frac{-\|c_{(i)} - c_{(j)}\|_2^2}{\sigma_c^2}}$$

# Minimum Cut and Clustering



# Minimum Cut and Clustering





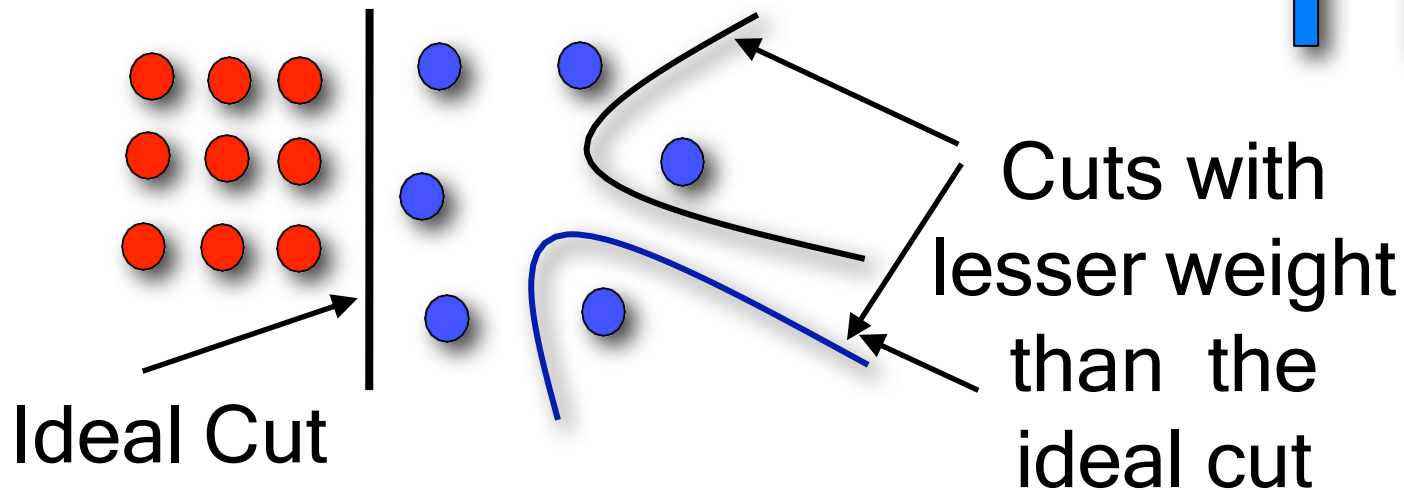
# Minimum cut

Criterion for partition:

$$\min cut(A, B) = \min_{A, B} \sum_{u \in A, v \in B} w(u, v)$$

## Problem

Weight of cut is directly proportional to the number of edges in the cut.

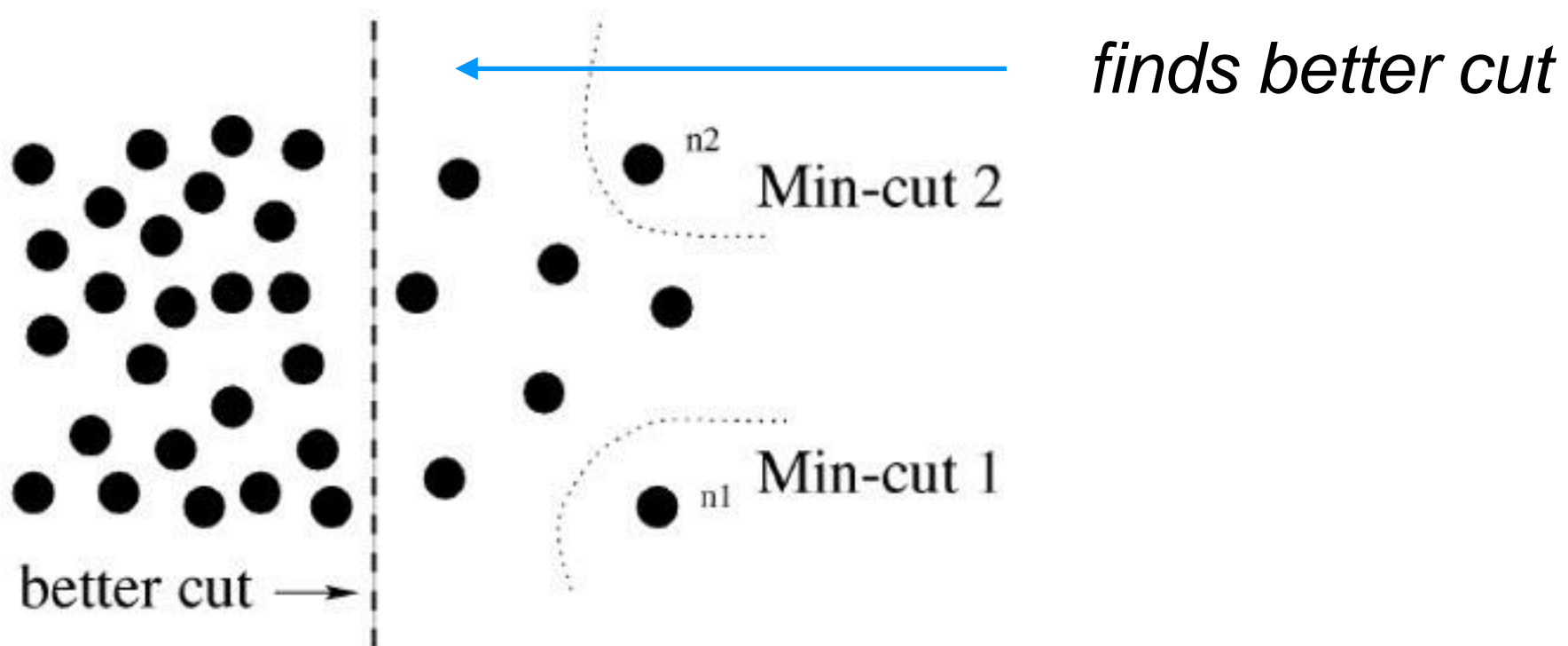


*First proposed by Wu and Leahy*

# Normalized Cut

Normalized cut or balanced cut:

$$Ncut(A, B) := cut(A, B) \frac{1}{vol(A)} + \frac{1}{vol(B)}$$



# Analyzing Normalized Cuts

- State of the art results
- Huge storage requirement and time complexity
- Bias towards partitioning into equal segments
- Has problems with textured backgrounds