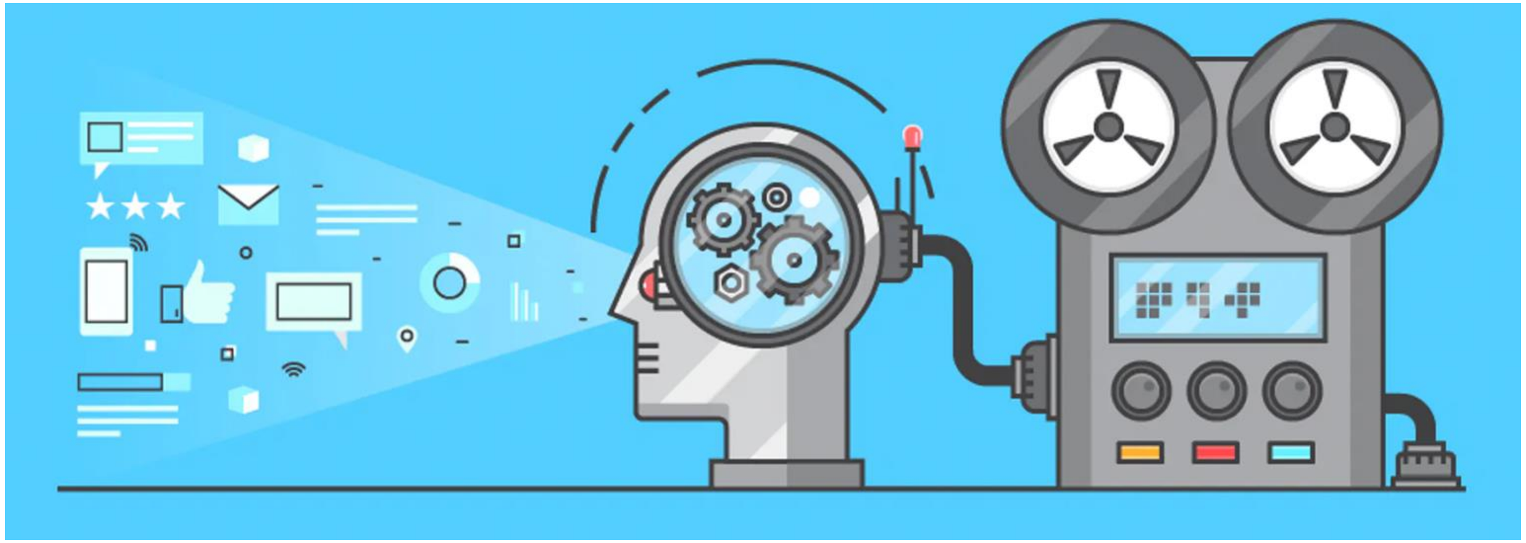
Deep Learning and Neural Networks

Topic 2:

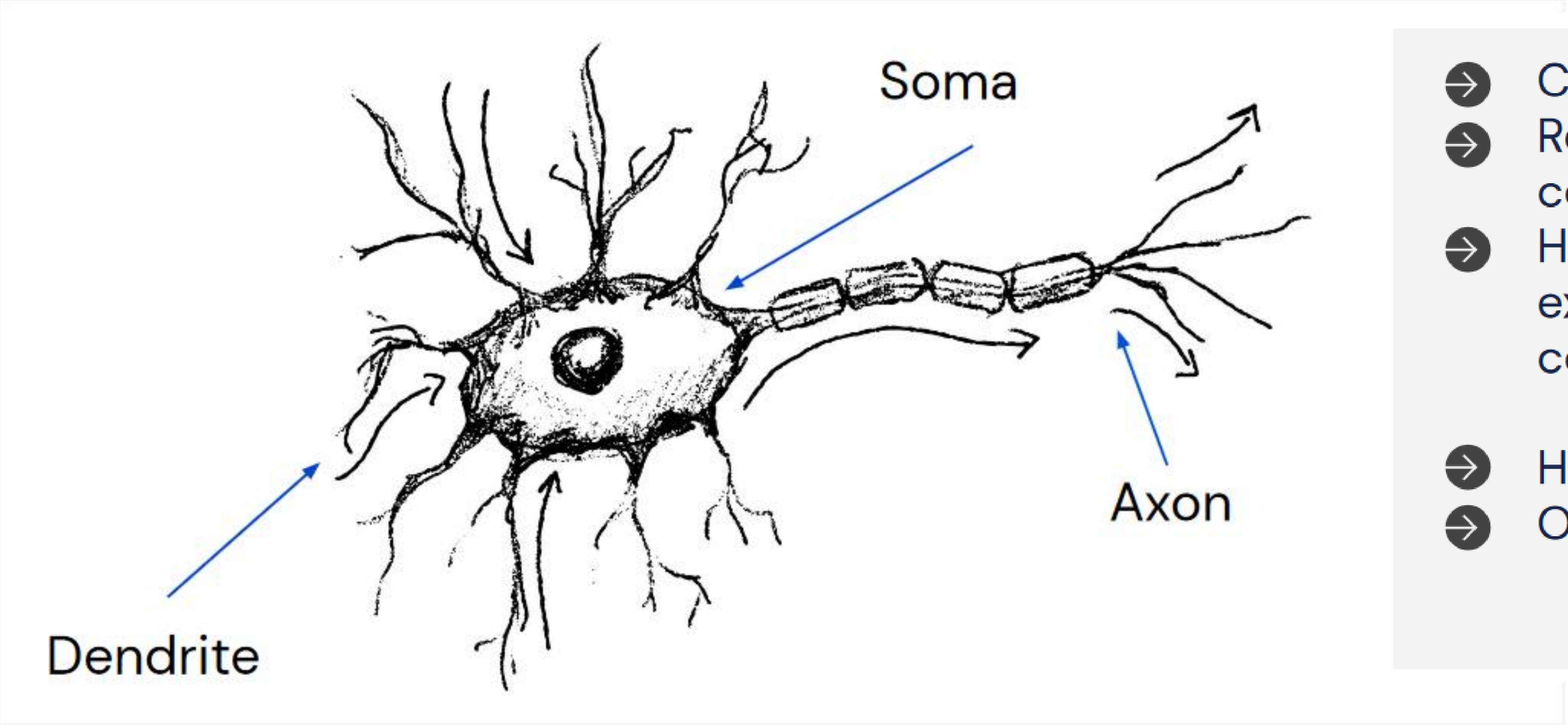
Shallow Networks



Ricardo Abel Espinosa Loera, McS

Researcher in DL & Computer Vision

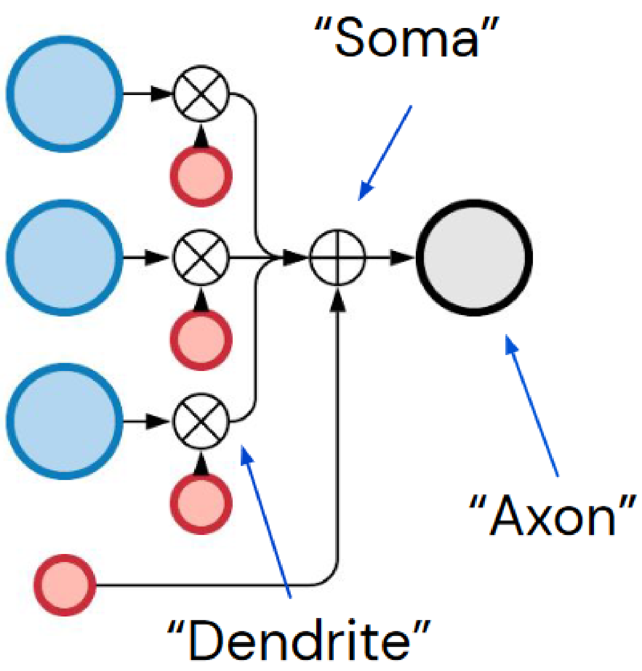
NEURAL NETWORKS
A model of a real neuorn



- Connected to others
- Represents simple computation
- Has inhibition and excitation connections
- Has a state
- Outputs spikes

NEURAL NETWORKS

A model of an artificial neuron



$$\sum_{i=1}^{d} \mathbf{w}_i \mathbf{x}_i + b$$

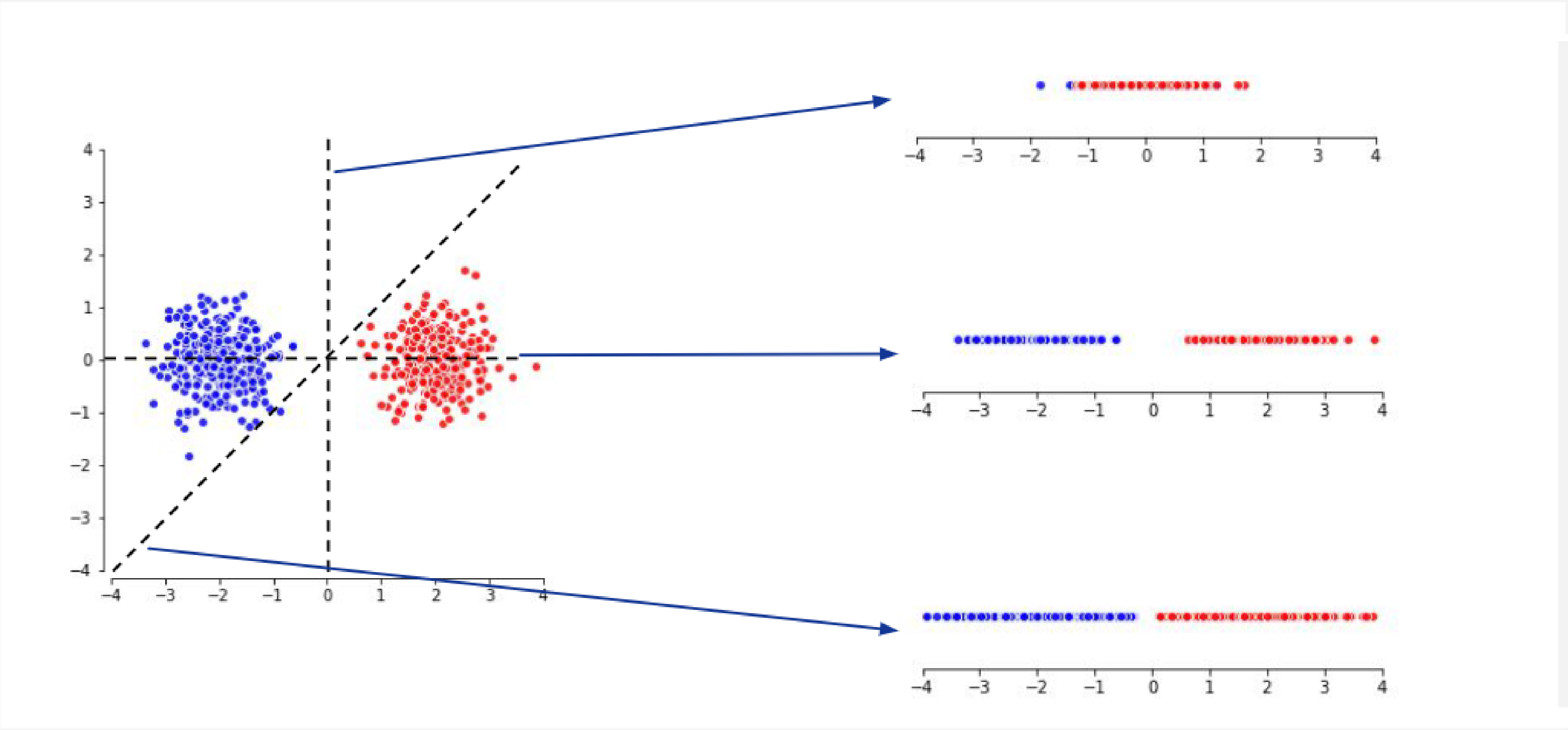
$$\sum_{i=0}^{d} \mathbf{w}_i \mathbf{x}_i \qquad \mathbf{x}_0 := 1$$

- Easy to compose
- Represents simple computation
- Has inhibition and excitation connections

- Is stateless wrt. time
- Outputs real values

NEURAL NETWORKS

A model of an artificial neuron

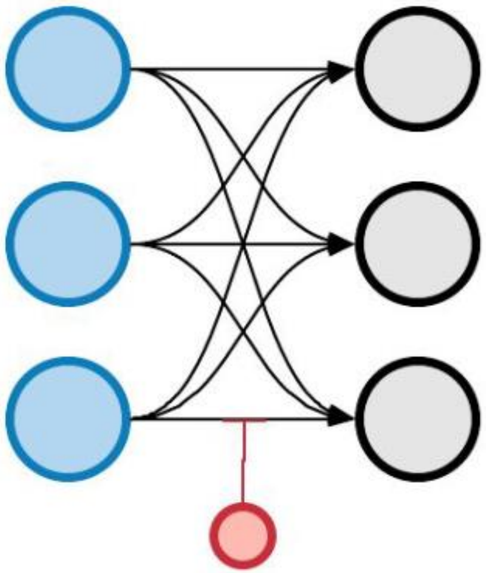


- Easy to compose
- Represents simple computation
- Has inhibition and excitation connections

- Is stateless wrt. time
- Outputs real values

NEURAL NETWORKS

A linear layer □ Single layer neural networks



$$h(\mathbf{x}, \mathbf{w}, b) = \langle \mathbf{w}, \mathbf{x} \rangle + b$$

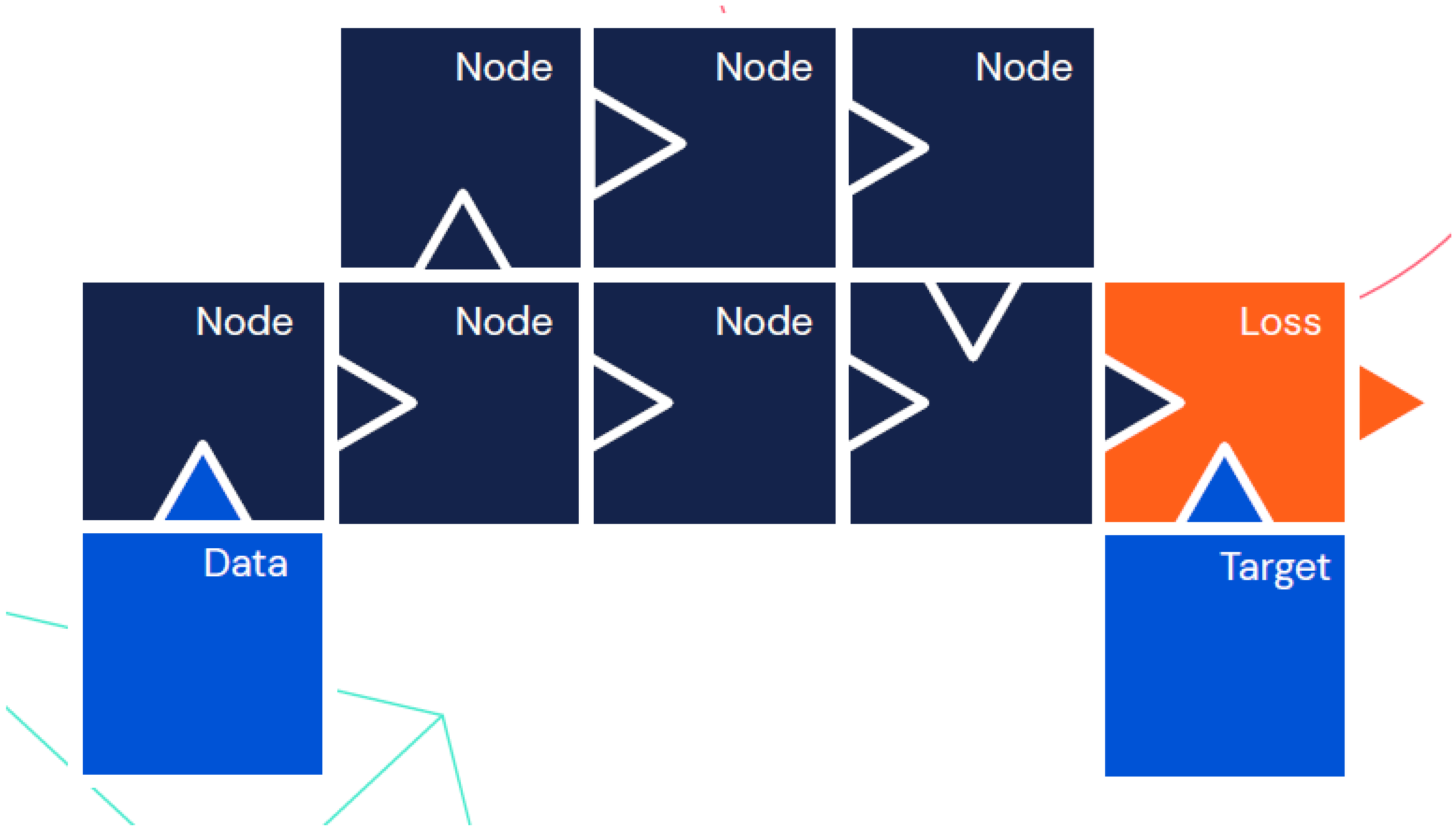
$$f_{\text{linear}}(\mathbf{x}, \mathbf{W}, \mathbf{b}) = \mathbf{W}\mathbf{x} + \mathbf{b}$$

- Easy to compose
- Collection of artificial neurons
- Can be efficiently vectorised
- Fits highly optimised hardware (GPU/TPU)

In Machine Learning linear really means affine.
Neurons in a layer are often called units.
Parameters are often called weights.

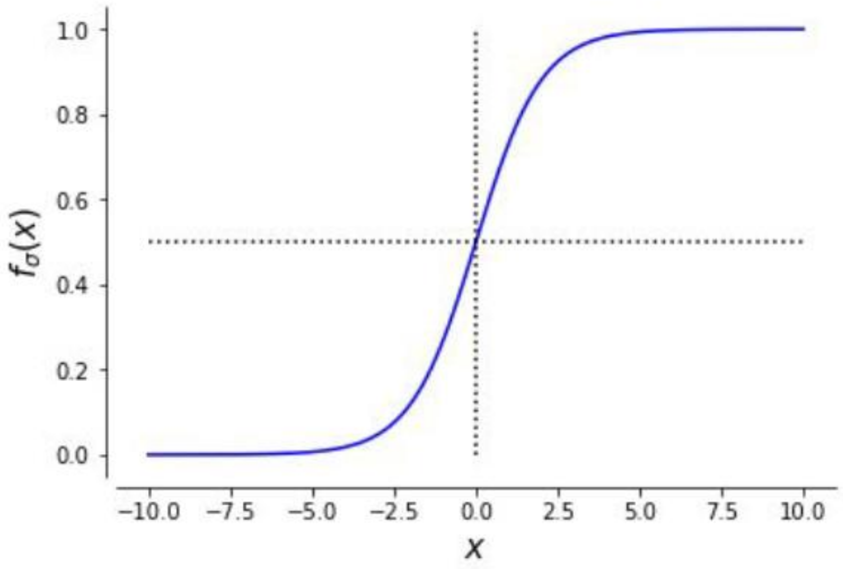
NEURAL NETWORKS

A linear layer Single layer neural networks



NEURAL NETWORKS

Sigmoid activation function



$$f_\sigma(\mathbf{x}) = \frac{1}{1 + e^{-\mathbf{x}}}$$

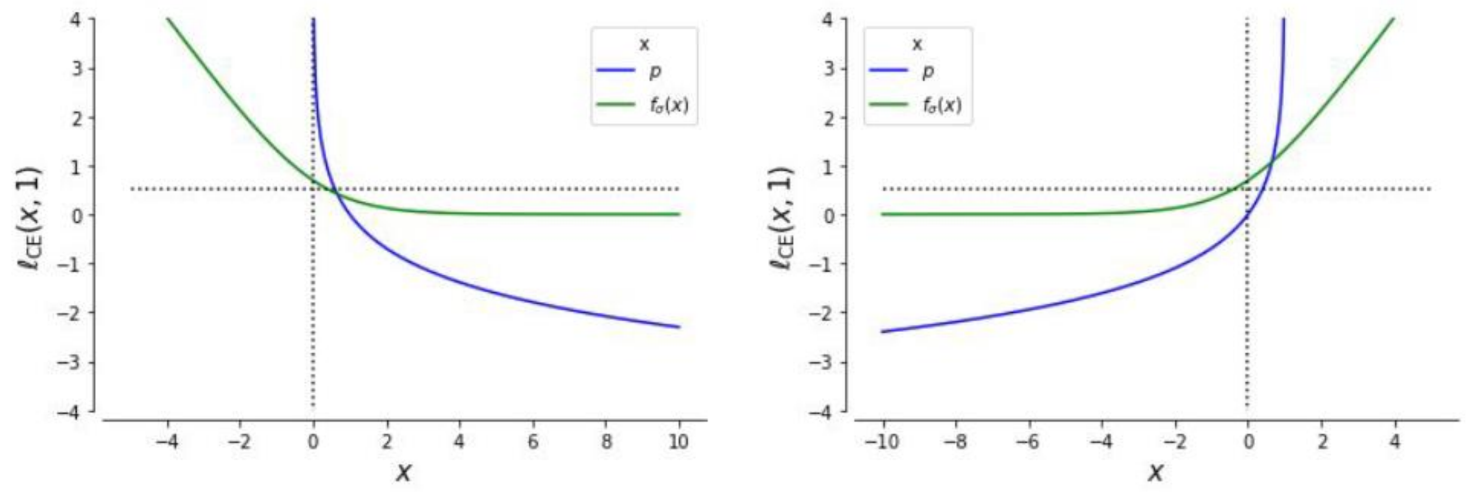
$$f_\sigma(\mathbf{x}) = \frac{e^{\mathbf{x}}}{e^{\mathbf{x}} + 1}$$

- Introduces non-linear behaviour
- Produces probability estimate
- Has simple derivatives

- Saturates
- Derivatives vanish

Activation functions are often called non-linearities.
Activation functions are applied point-wise

NEURAL NETWORKS
Cross Entropy Loss Function



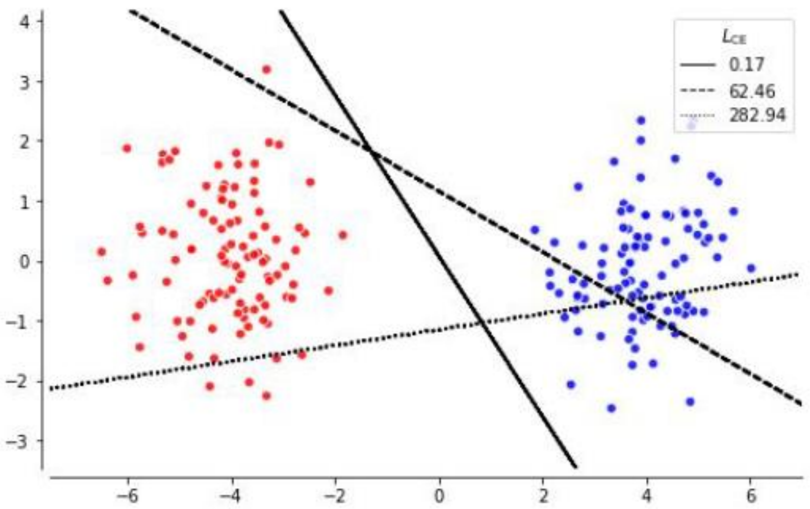
$$\ell_{CE}(\mathbf{p}, \mathbf{t}) = -[\mathbf{t} \log \mathbf{p} + (1 - \mathbf{t}) \log(1 - \mathbf{p})]$$

- Encodes negation of logarithm of probability of correct classification
- Composable with sigmoid
- Numerically unstable

NEURAL NETWORKS

The simplest “neural” network classifier

Cross entropy loss is also called
negative log likelihood or logistic loss.
Being additive over samples allows for efficient learning.



$$L_{\mathrm{CE}}(\mathbf{p}, \mathbf{t}) = -\sum_{i=1}^{n} [\mathbf{t}^{(i)} \log \mathbf{p}^{(i)} + (1 - \mathbf{t}^{(i)}) \log(1 - \mathbf{p}^{(i)})]$$

- Encodes negation of logarithm of probability of **entirely correct** classification
- Equivalent to logistic regression model
- Numerically unstable

NEURAL NETWORKS

Softmax

$$f_{\mathrm{sm}}(\mathbf{x}) = \frac{e^{\mathbf{x}}}{\sum_{j=1}^{k} e^{\mathbf{x}_j}}$$

$$f_{\mathrm{sm}}([x, 0]) = \left[\frac{e^x}{e^x + e^0}, \frac{e^0}{e^x + e^0}\right]$$

$$= [f_\sigma(x), 1 - f_\sigma(x)]$$

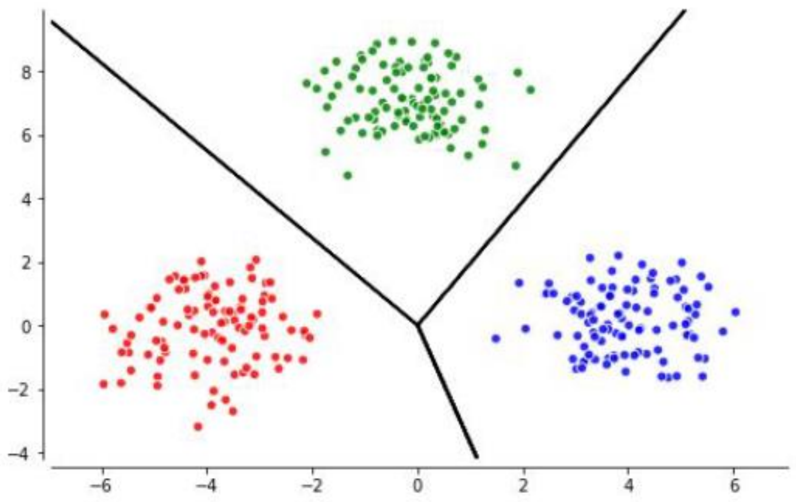
- Multi-dimensional generalisation of sigmoid
- Produces probability estimate
- Has simple derivatives

- Saturates
- Derivatives vanish

Softmax is the most commonly used final activation in classification.
It can also be used to have a smooth version of maximum.

NEURAL NETWORKS

Softmax + Cross-entropy



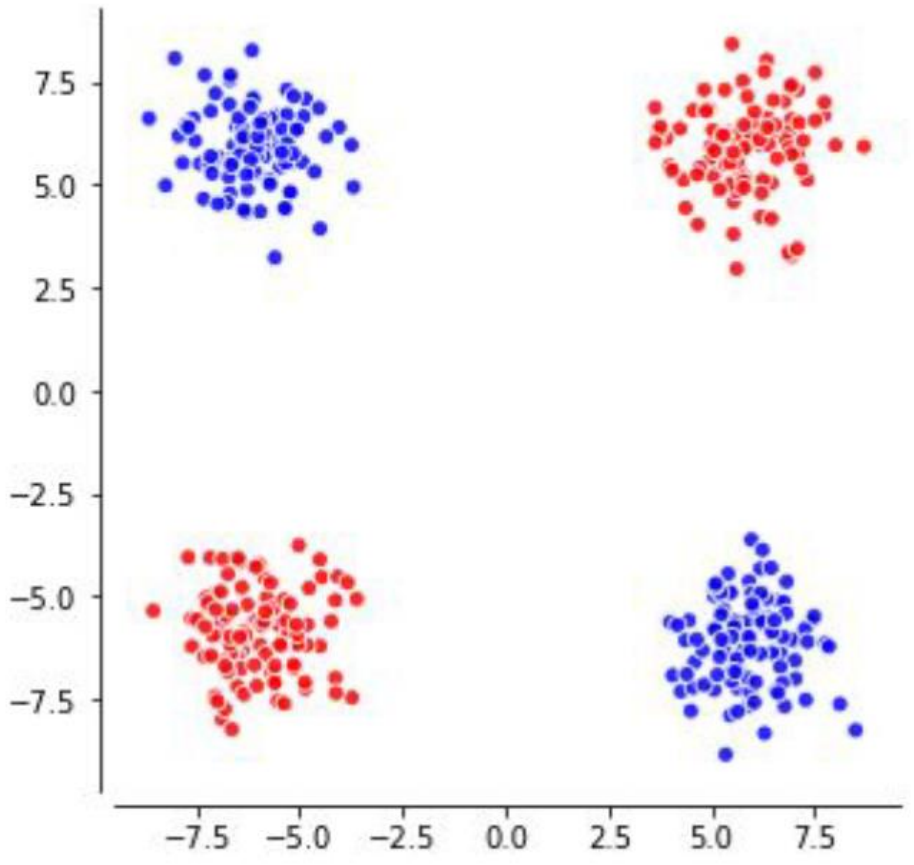
$$\ell_{\mathrm{CE}}(f_{\mathrm{sm}}(\mathbf{x}), \mathbf{t}) = -\sum_{j=1}^{k} \mathbf{t}_j \log[f_{\mathrm{sm}}(\mathbf{x}_j)] = -\sum_{j=1}^{k} \mathbf{t}_j [\mathbf{x}_j - \log \sum_{l=1}^{k} e^{\mathbf{x}_l}]$$

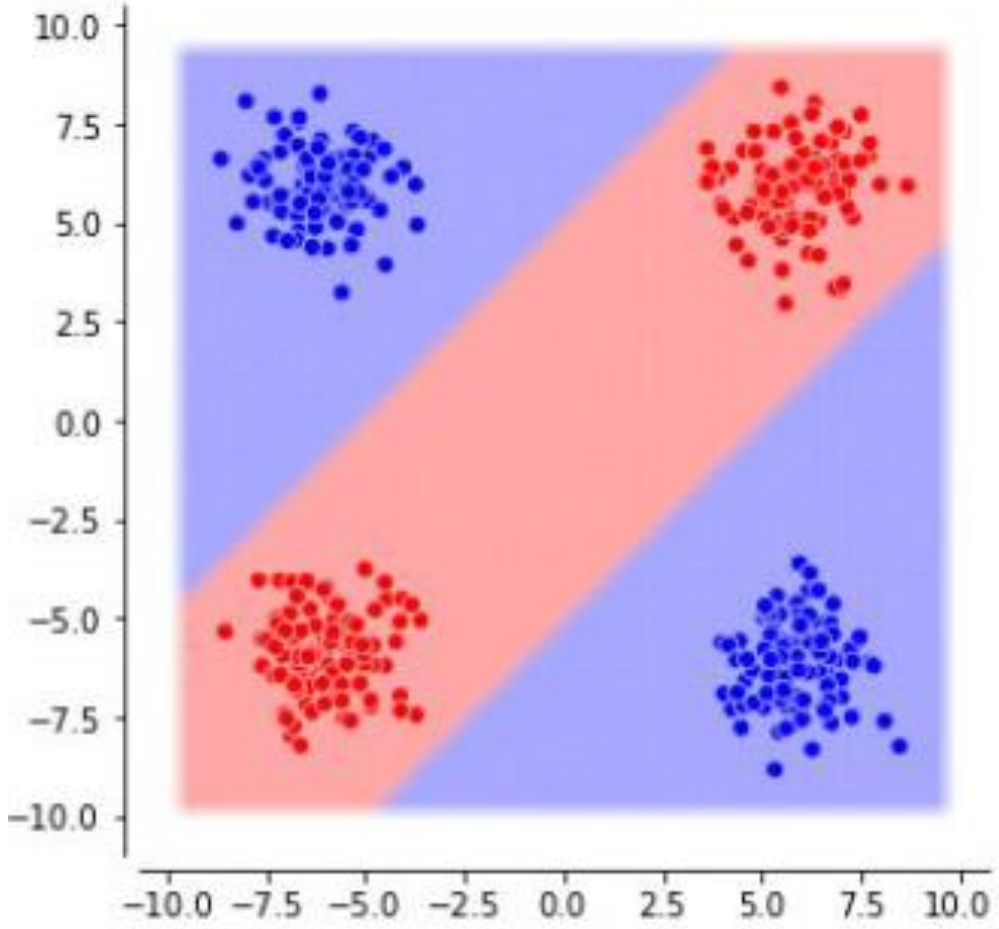
- Encodes negation of logarithm of probability of entirely correct classification
- Equivalent to multinomial logistic regression model
- Numerically stable combination

Widely used not only in classification but also in RL.
Cannot represent sparse outputs (sparsemax)

NEURAL NETWORKS

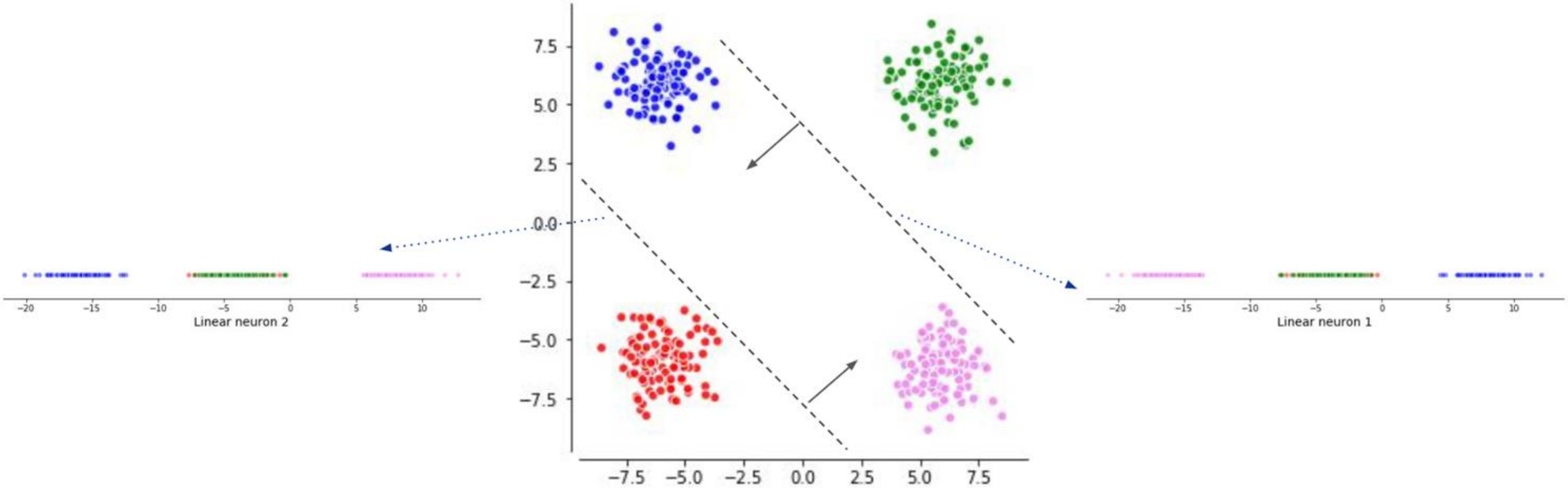
Limitations –> The XOR problem





NEURAL NETWORKS

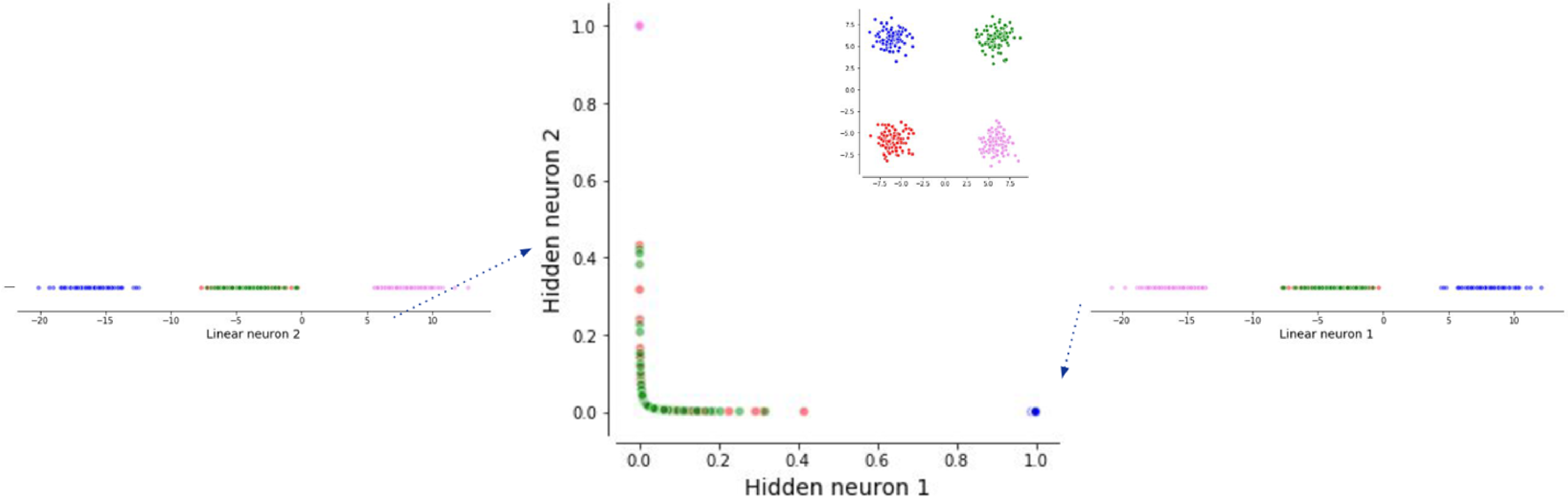
Limitations –> The XOR problem



$$\mathbf{W} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -4 \\ -4 \end{bmatrix}$$

NEURAL NETWORKS

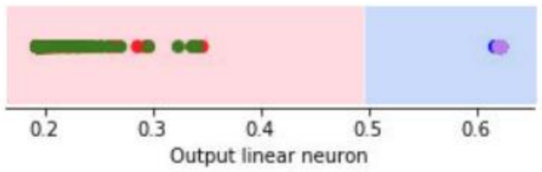
Limitations –> The XOR problem

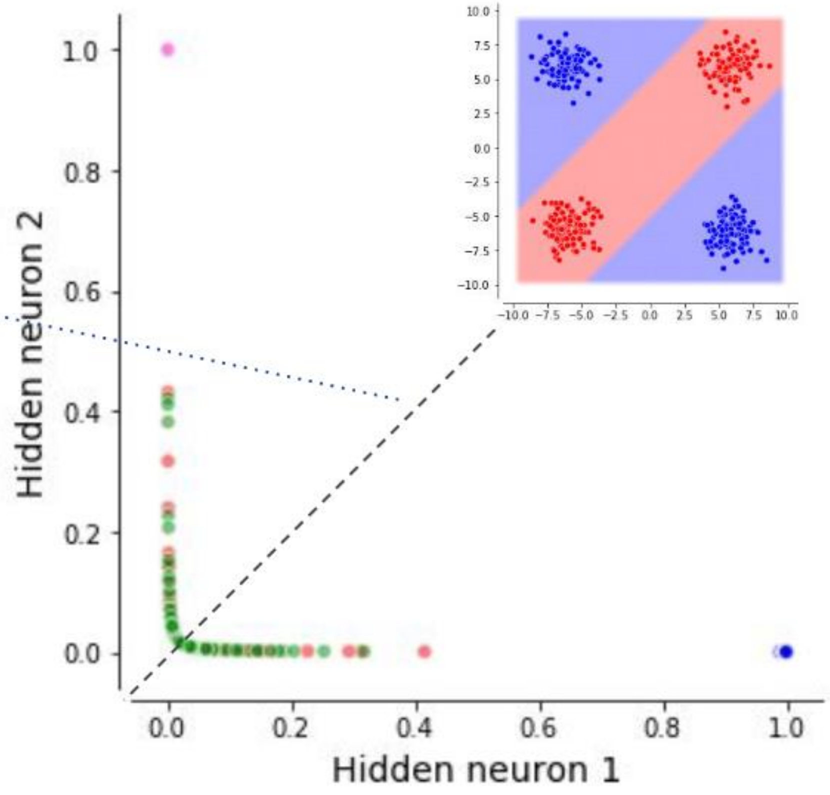


$$\mathbf{W} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -4 \\ -4 \end{bmatrix}$$

NEURAL NETWORKS

1 Hidden Layer Neural Network vs the XOR



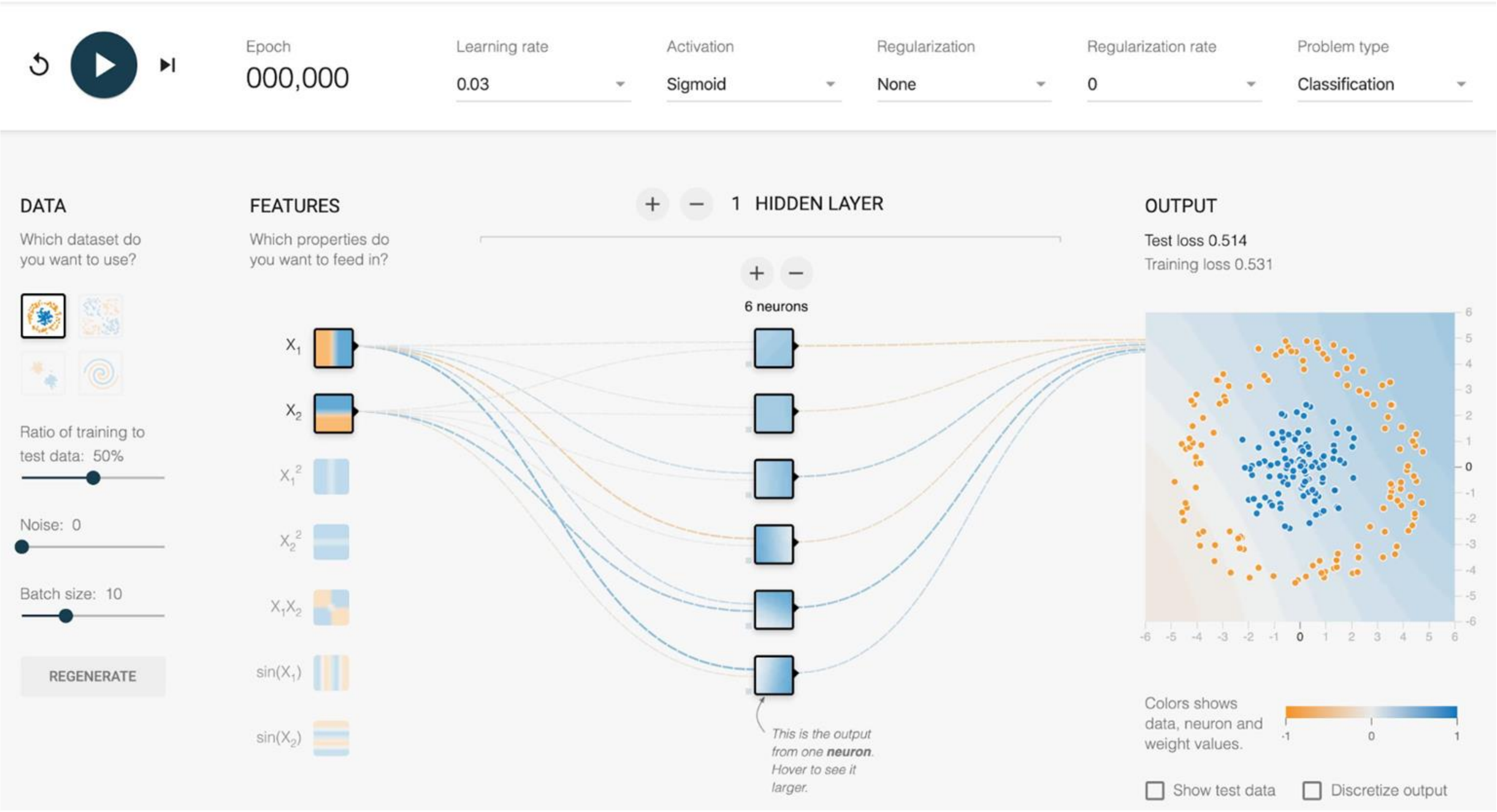


- With just 2 hidden neurons we solve XOR
- Hidden layer allows us to bent and twist input space
- We use linear model on top, to do the classification

$$\mathbf{W} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -4 \\ -4 \end{bmatrix}$$

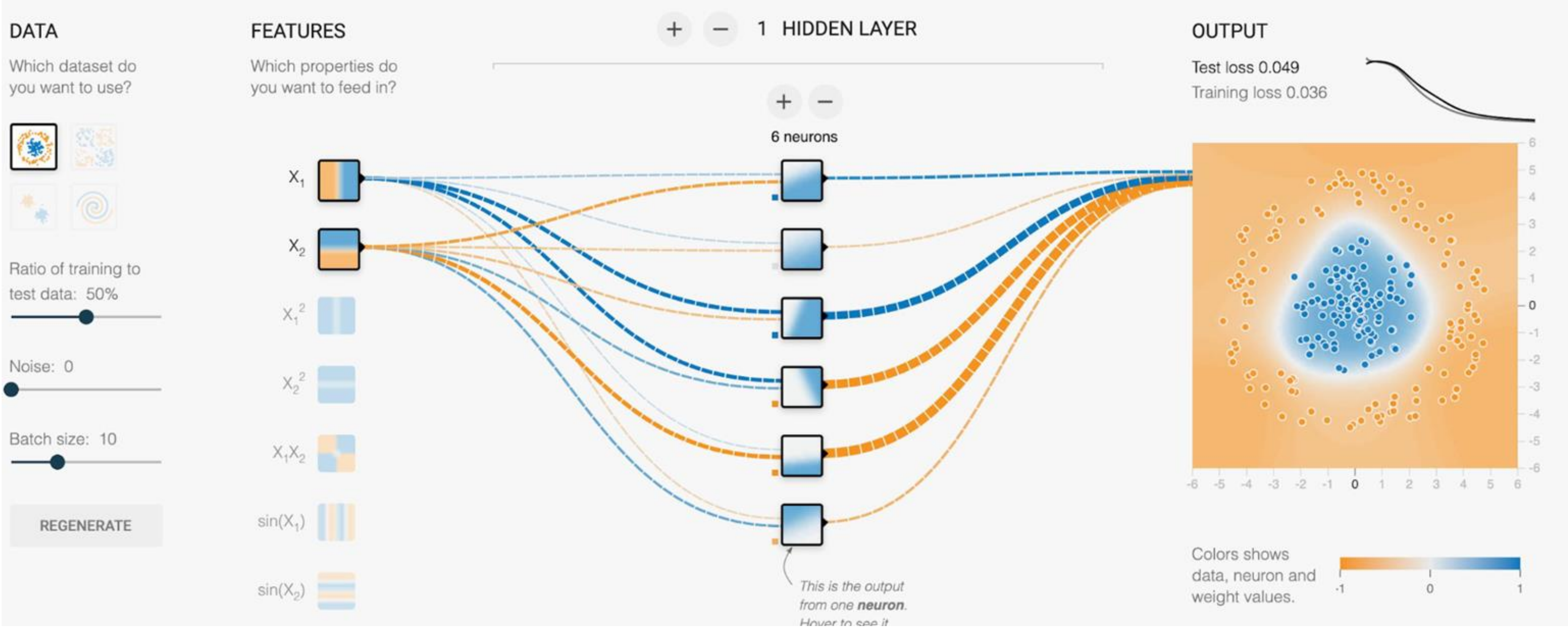
NEURAL NETWORKS
Neural Network Playground

http://playground.tensorflow.org/ by **Daniel Smilkov** and **Shan Carter**



NEURAL NETWORKS

Neural Network Playground



NEURAL NETWORKS
Representation Learning

- Methods that allow a machine to be fed with raw data to automatically discover representations needed for detection or classification

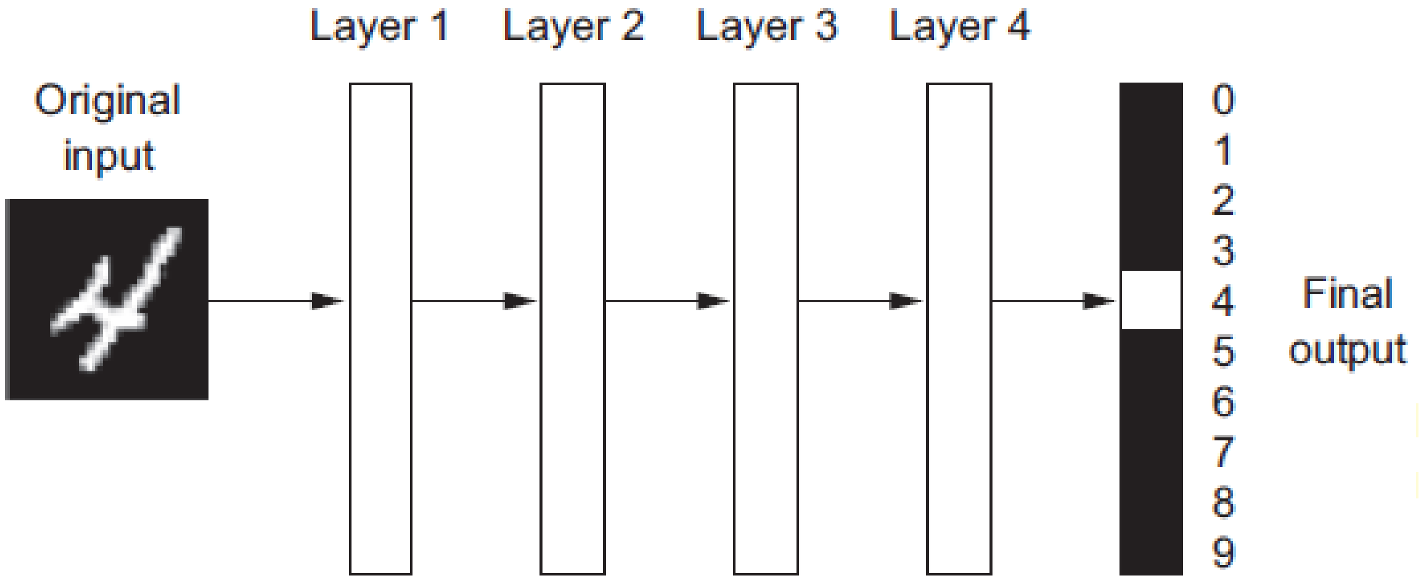
- Deep Learning methods are Representation Learning Methods

- Use multiple levels of representation

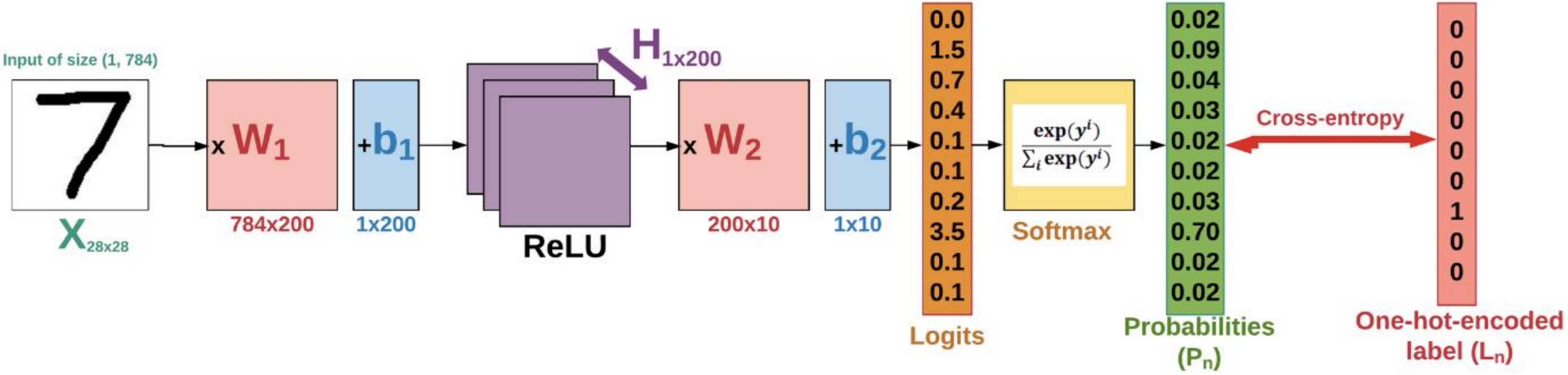
- Composing simple but non-linear modules that transform representation at one level (starting with raw input) into a representation at a higher slightly more abstract level

- Complex functions can be learned

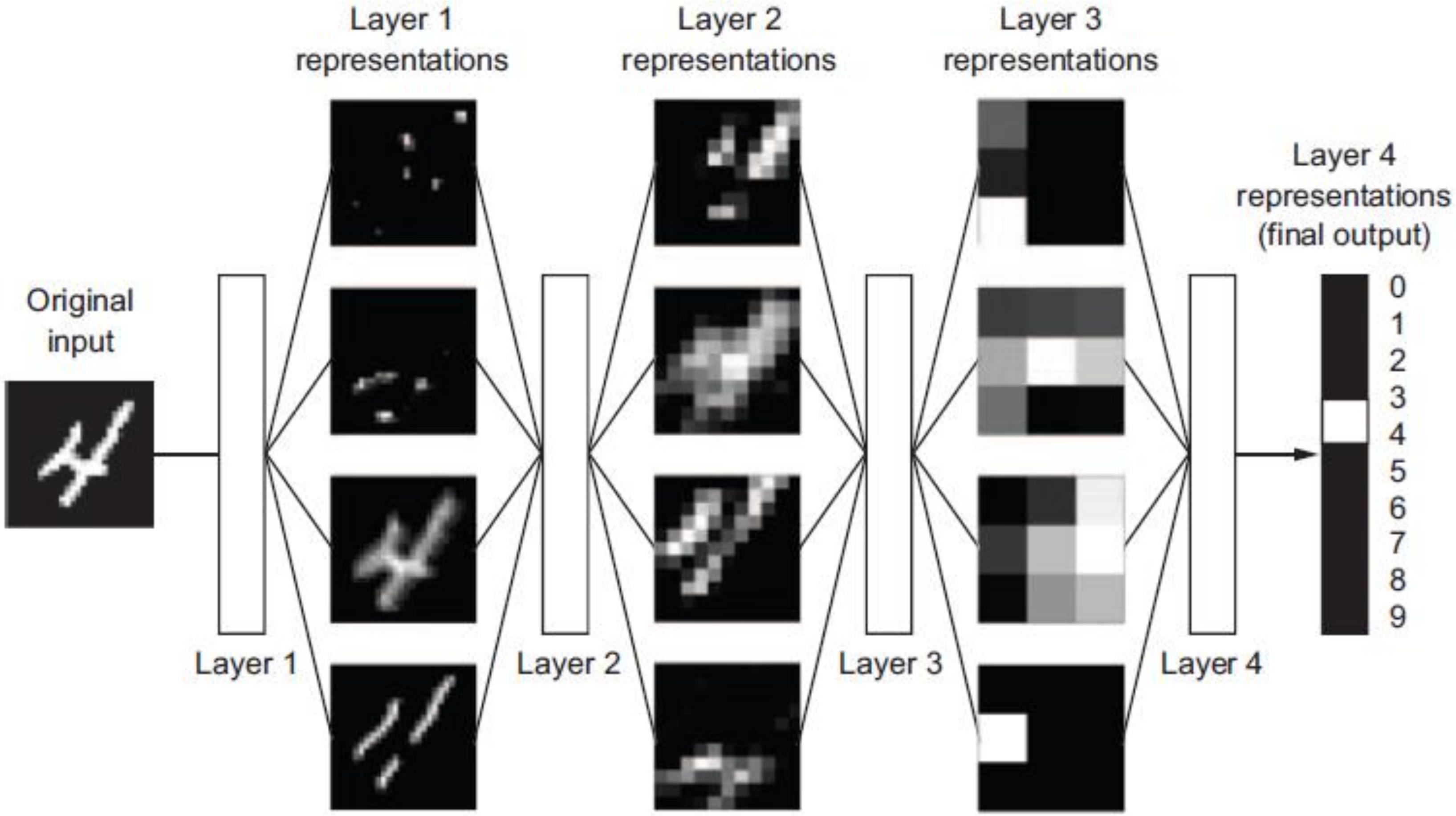
NEURAL NETWORKS
what are they?



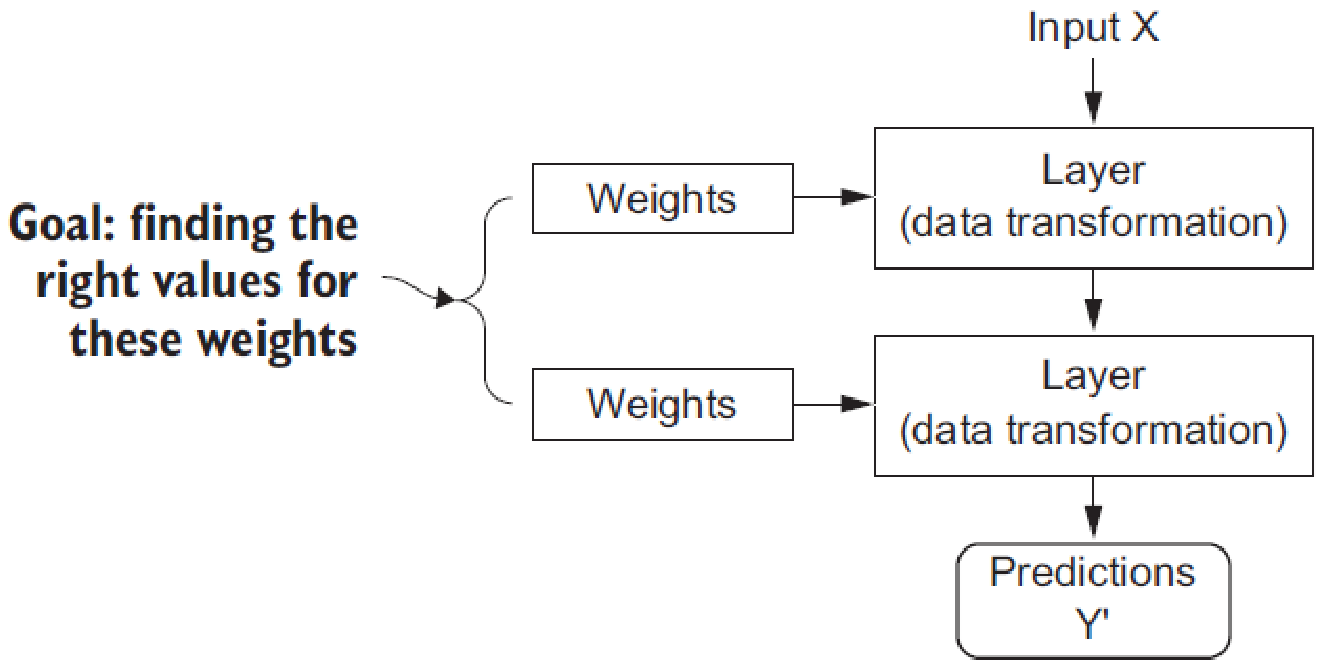
NEURAL NETWORKS
what are they?

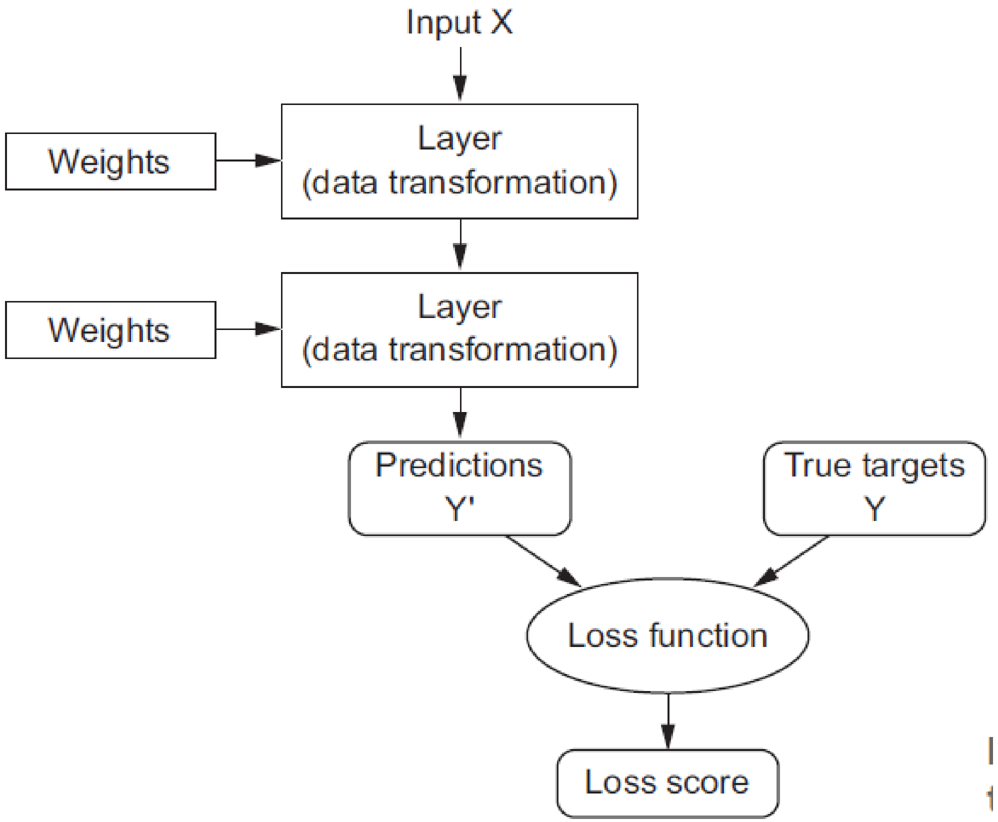


NEURAL NETWORKS
what are they?



NEURAL NETWORKS
How are they trained?





NEURAL NETWORKS

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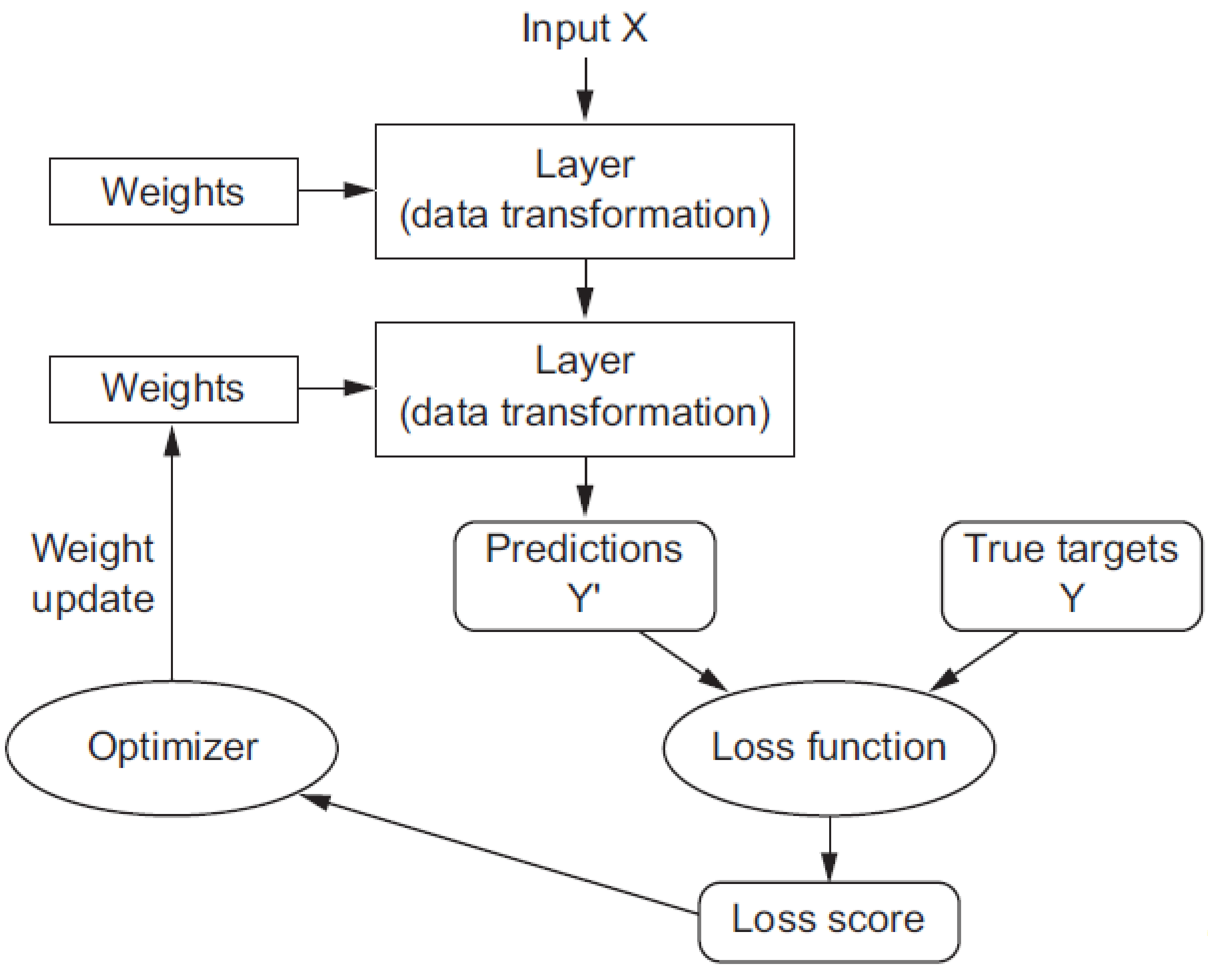

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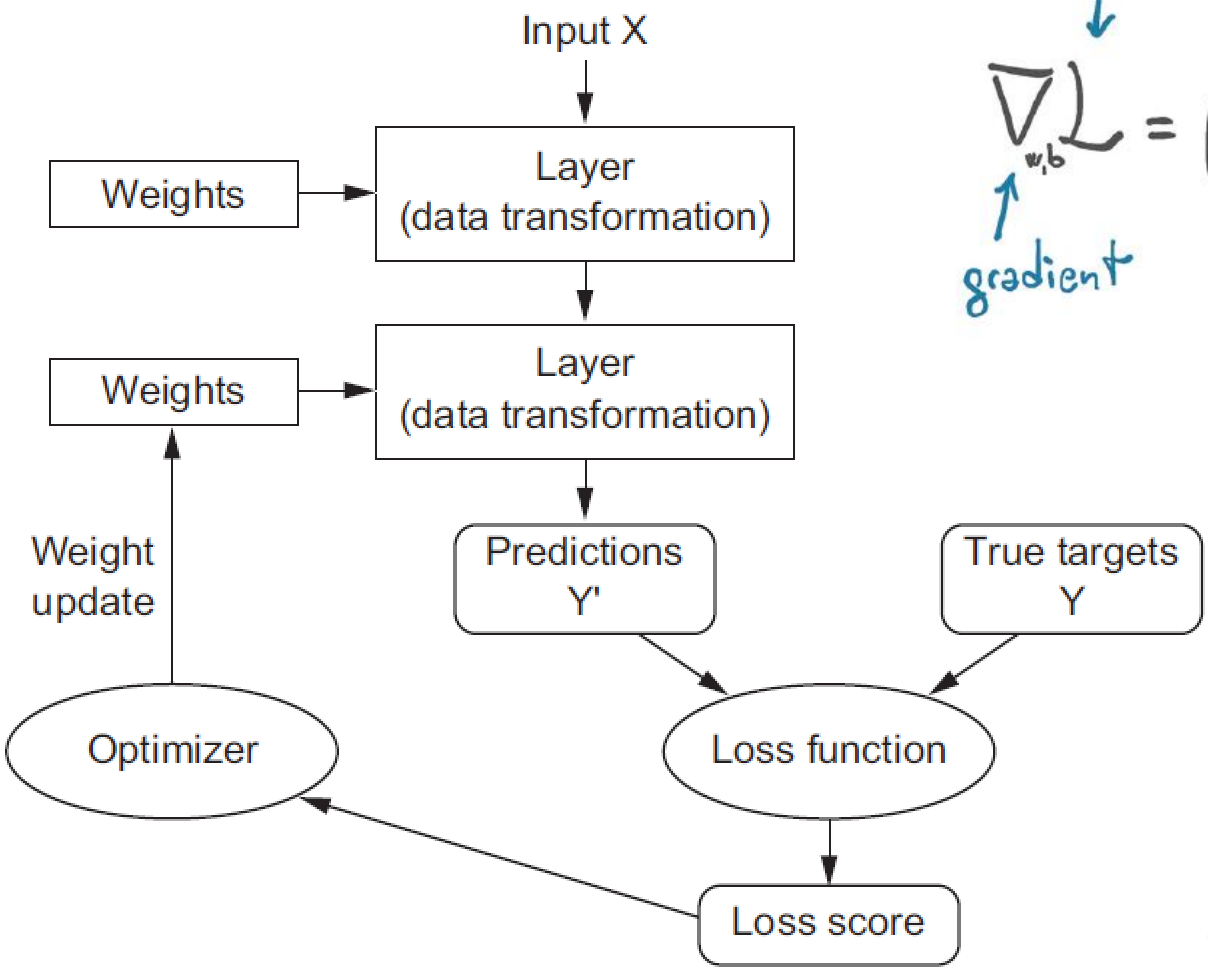

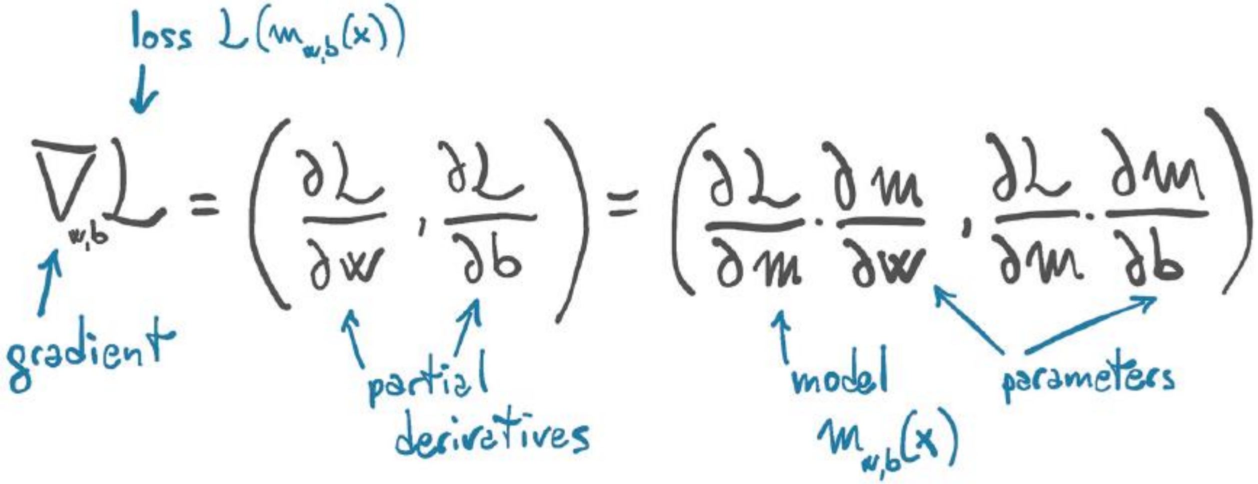

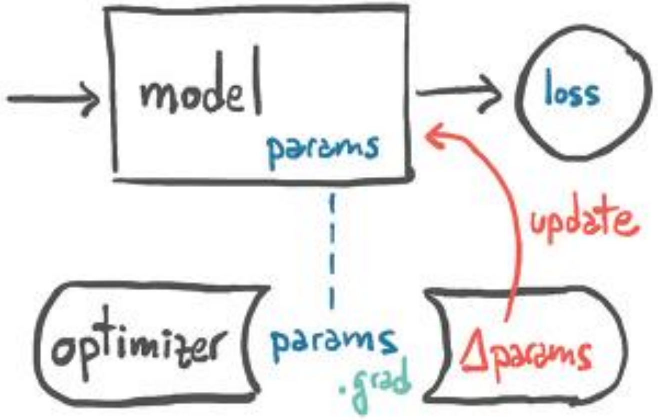

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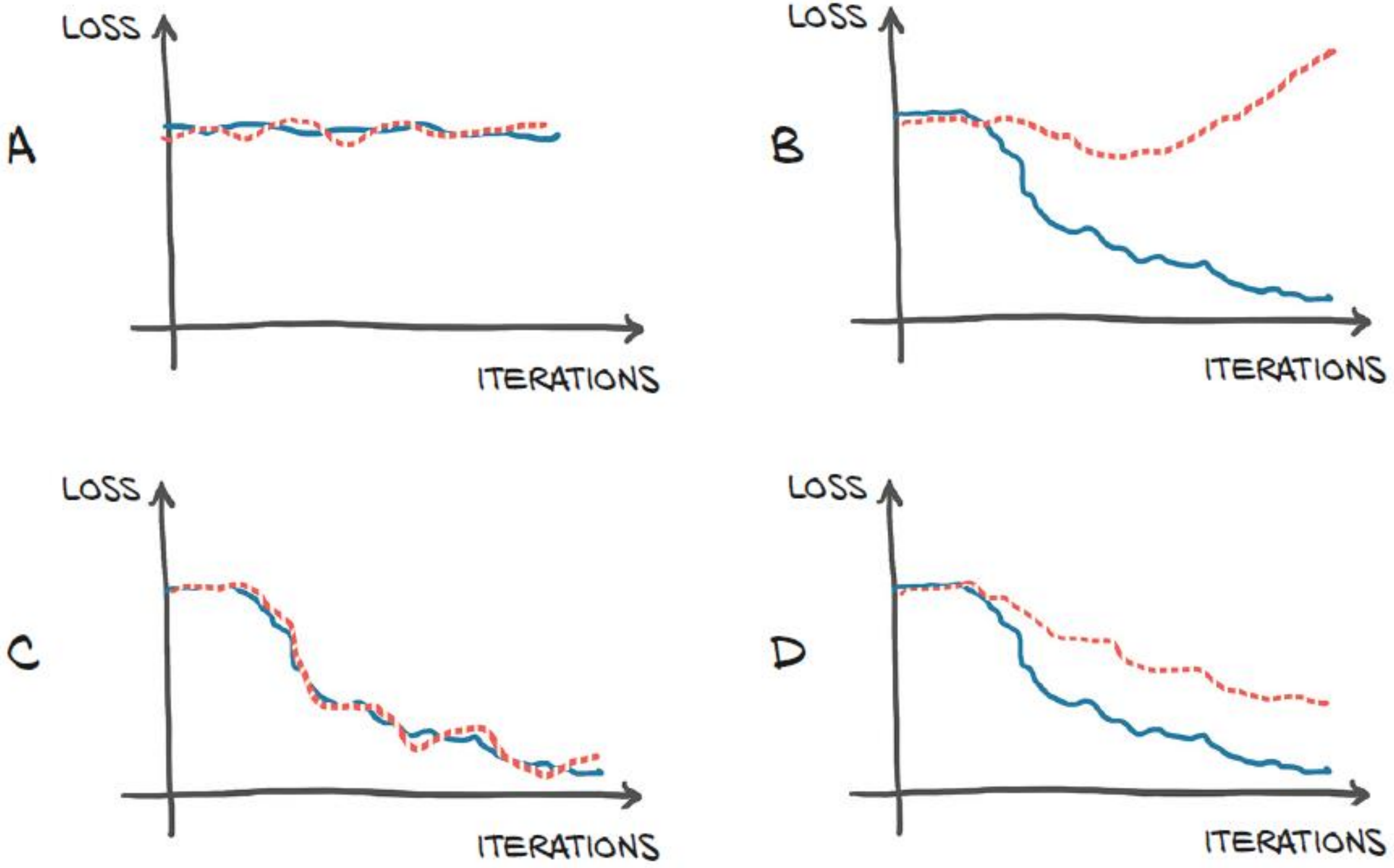

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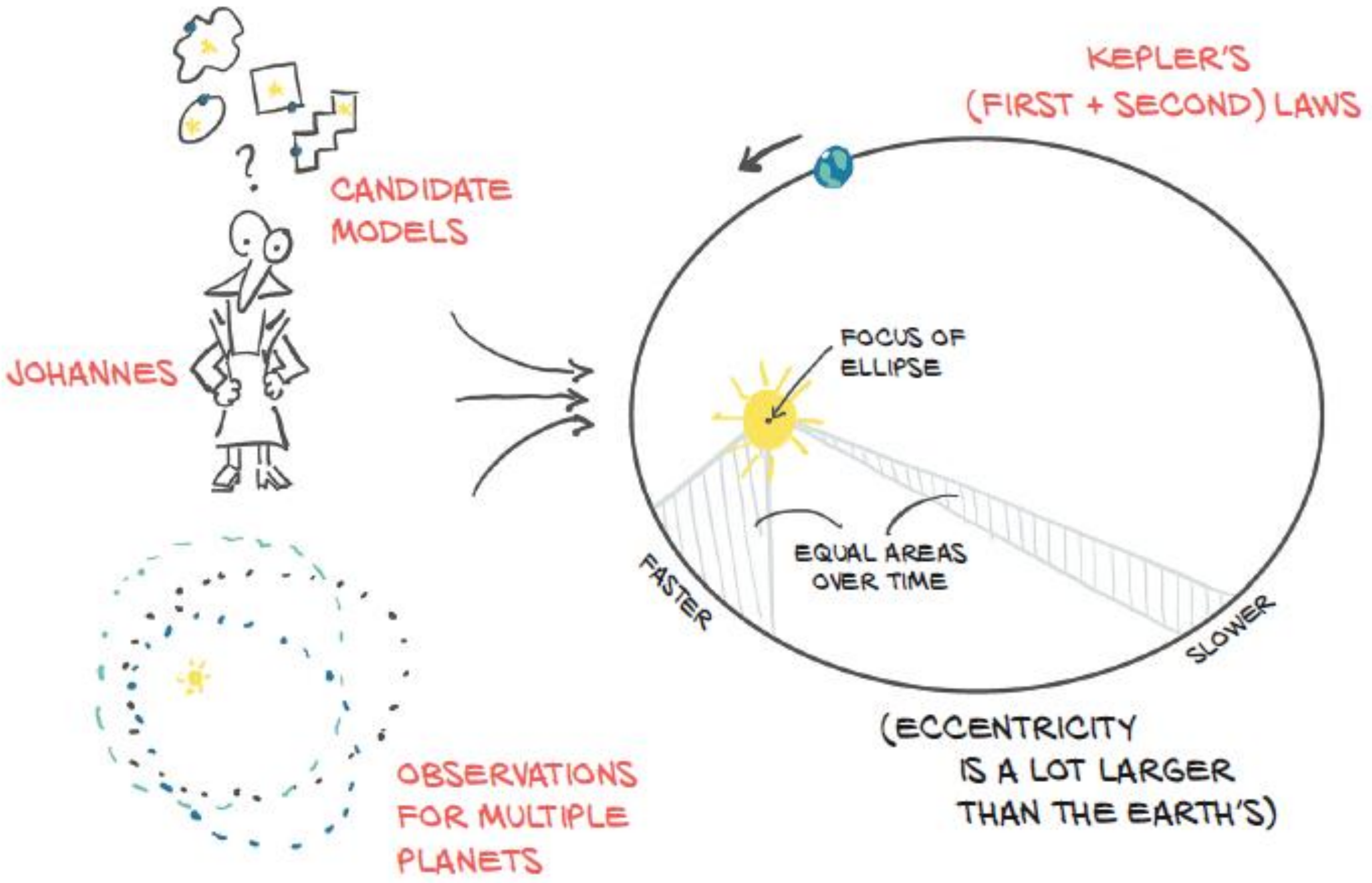

NEURAL NETWORKS

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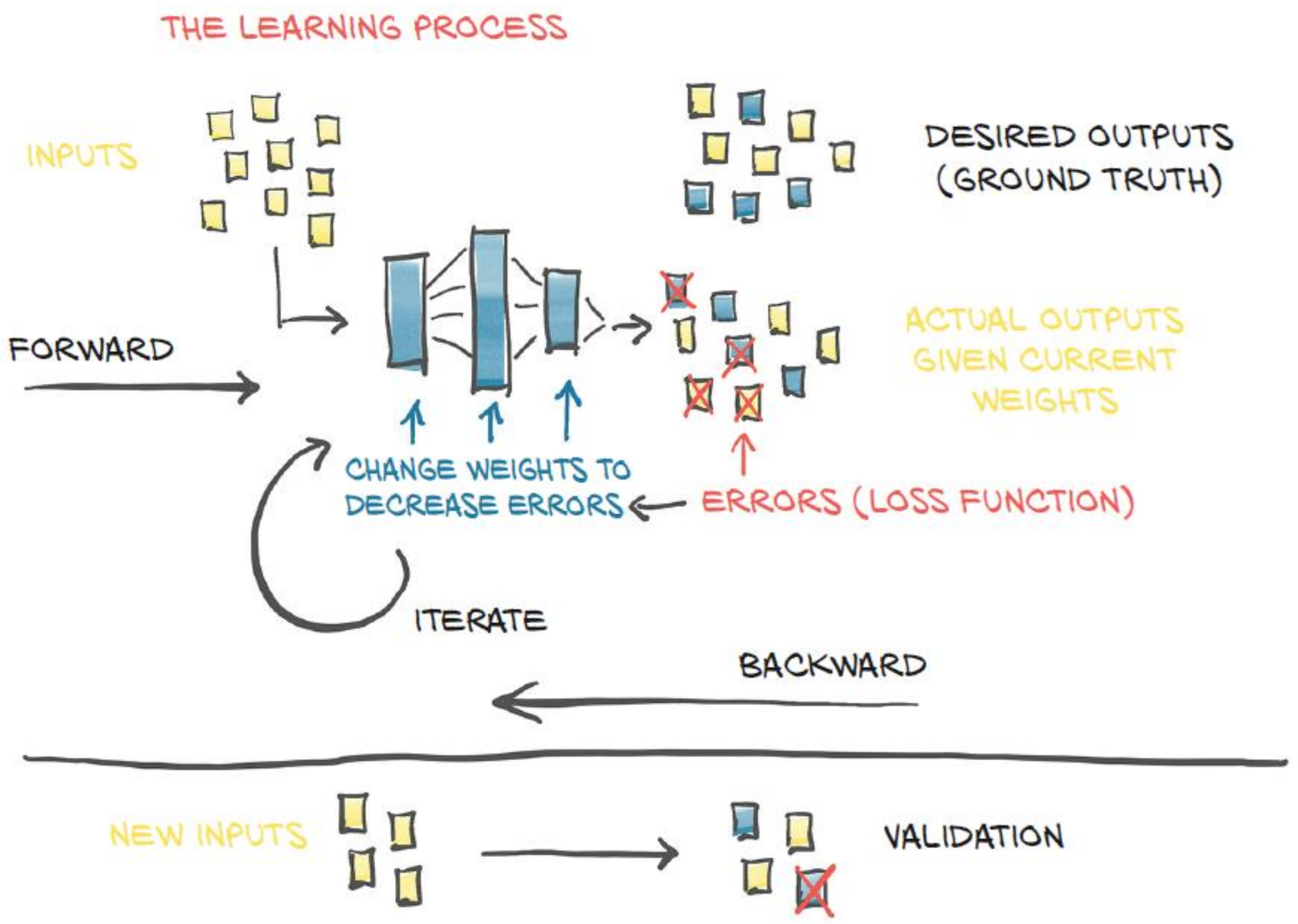


NEURAL N

Mathematical Intuitions

THE "NEURON"

LEARNED PARAMETERS

$$o = \tanh(\underbrace{wx + b}_{\text{LINEAR TRANSFORMATION}})$$

OUTPUT

INPUT

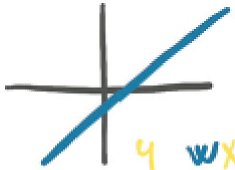
NONLINEAR FUNCTION (ACTIVATION)

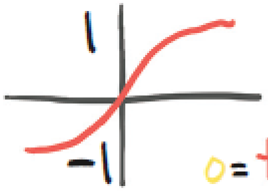
LEARNED

↓

$w = 2$

$b = 6$





$o = \tanh(y)$

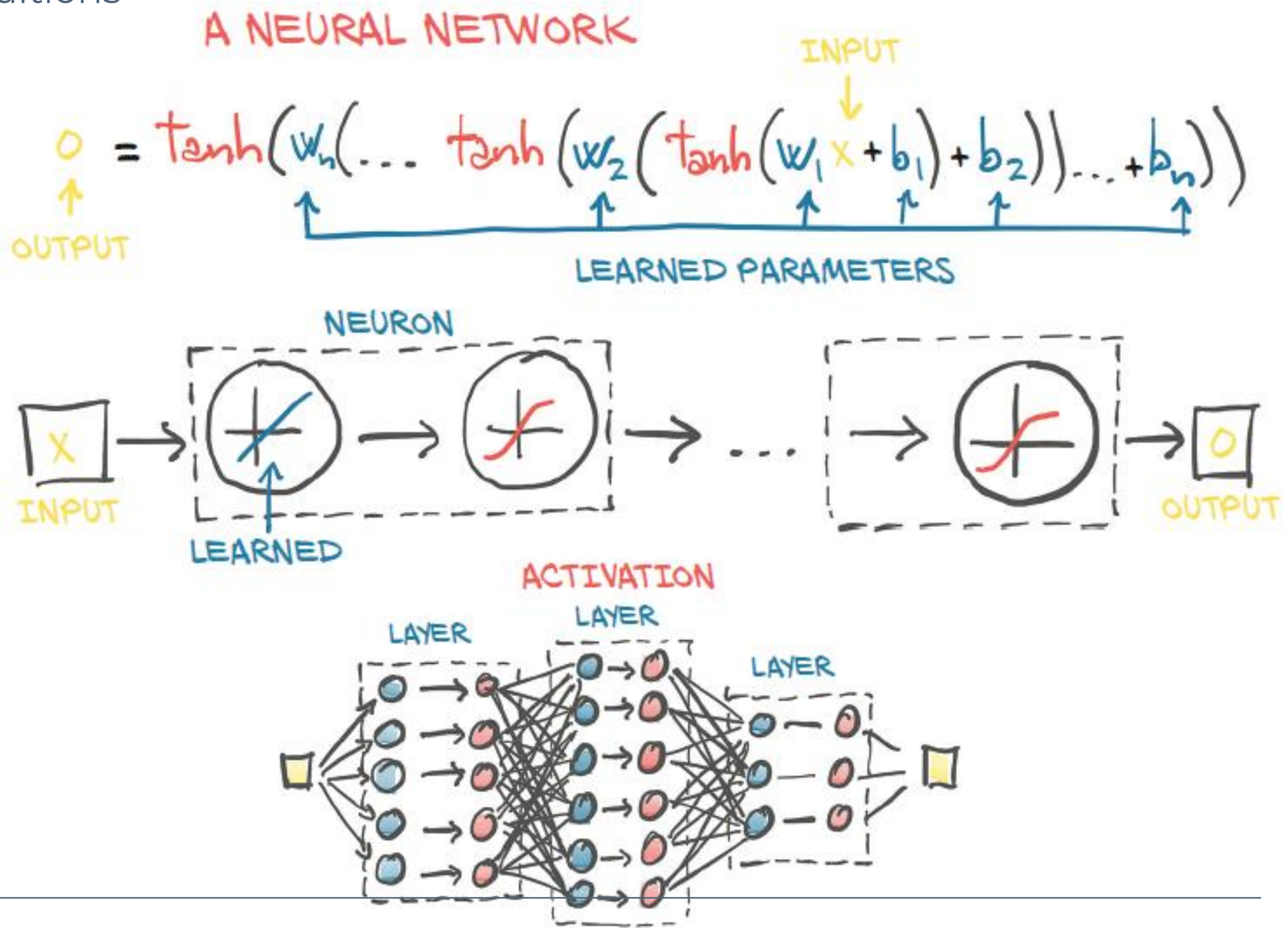
$18 \longrightarrow 2 \times 18 + 6 = 42 \rightarrow \tanh(42) = 1$

$-2.79 \longrightarrow 2 \times (-2.79) + 6 = .042 \rightarrow \tanh(.042) = 0.3969$

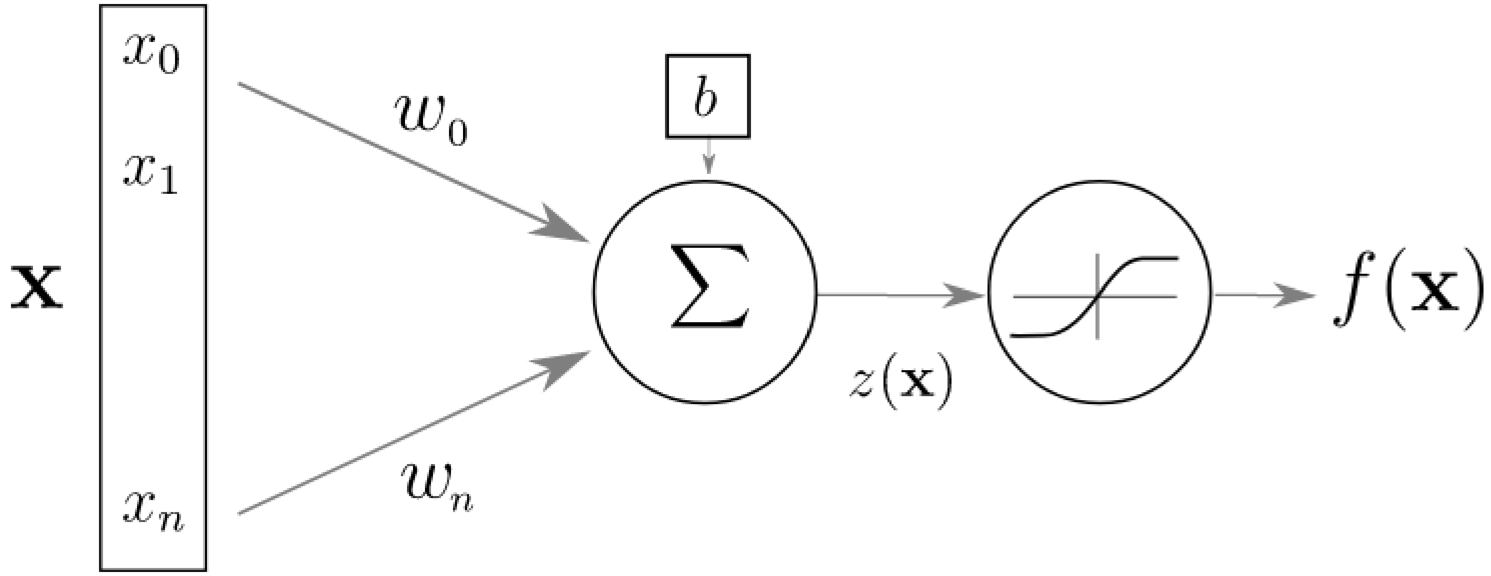
$-10 \longrightarrow 2 \times (-10) + 6 = -14 \rightarrow \tanh(-14) = -1$

NEURAL NETWORKS

Mathematical Intuitions



NEURAL NETWORKS
Mathematical Intuitions



$$z(\mathbf{x}) = \mathbf{w}^T\mathbf{x} + b$$

$$f(\mathbf{x}) = g(\mathbf{w}^T\mathbf{x} + b)$$

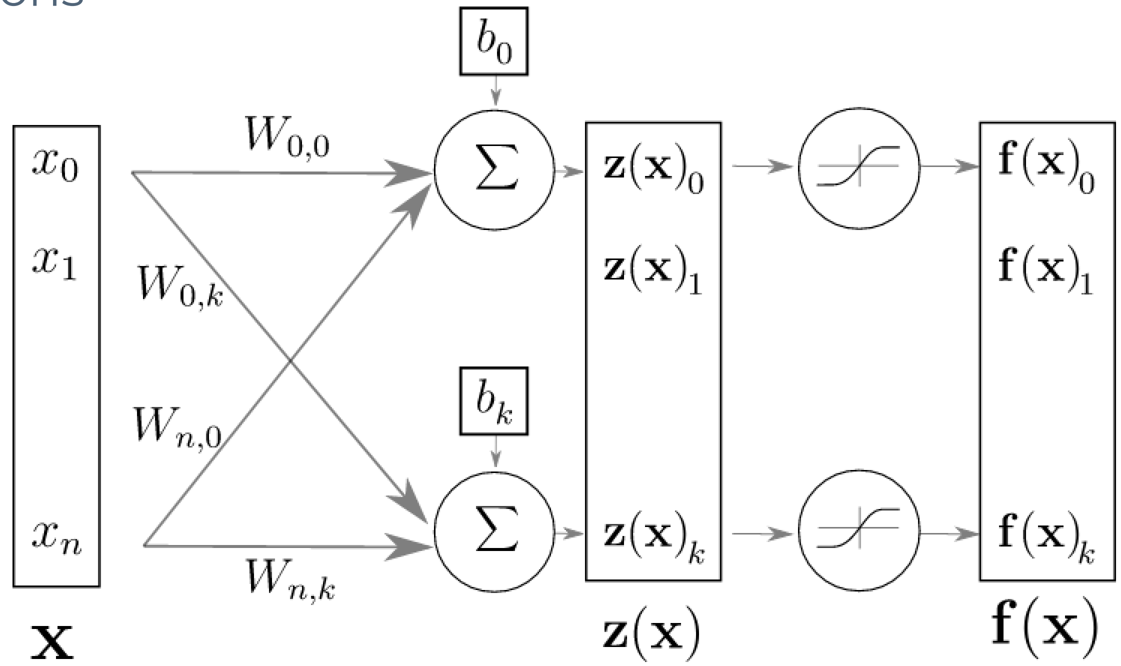
$\mathbf{x}$,$f(\mathbf{x})$ input and output

$z(\mathbf{x})$ pre-activation

$\mathbf{w}$,b weights and bias

g activation function

NEURAL NETWORKS
Mathematical Intuitions

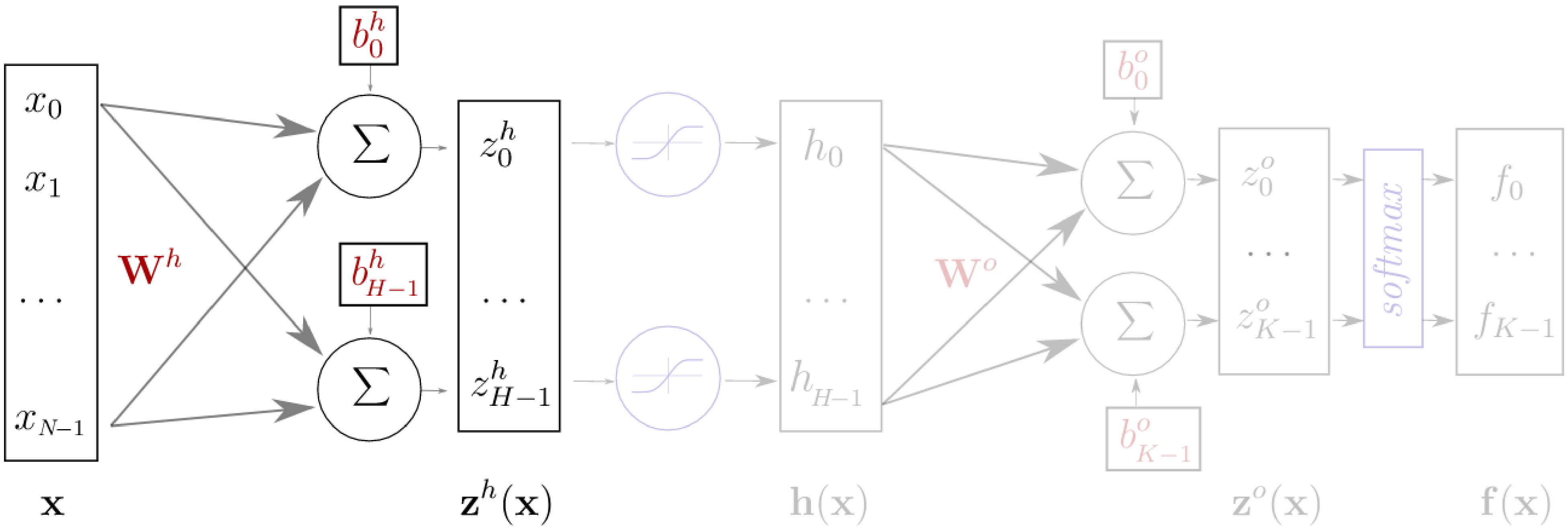


$$\mathbf{f}(\mathbf{x}) = g(\mathbf{z}(\mathbf{x})) = g(\mathbf{W}\mathbf{x} + \mathbf{b})$$

W,**b** now matrix and vector

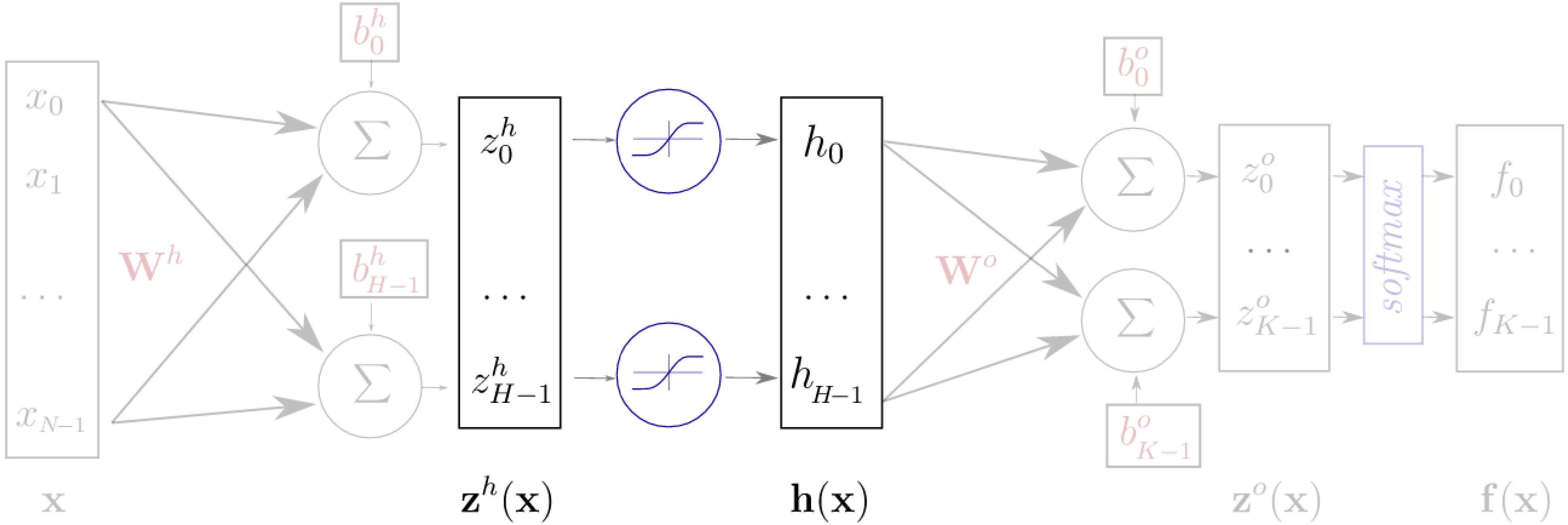
NEURAL NETWORKS

Mathematical Intuitions



- $\mathbf{z}^h(\mathbf{x}) = \mathbf{W}^h\mathbf{x} + \mathbf{b}^h$
- $\mathbf{h}(\mathbf{x}) = g(\mathbf{z}^h(\mathbf{x})) = g(\mathbf{W}^h\mathbf{x} + \mathbf{b}^h)$
- $\mathbf{z}^o(\mathbf{x}) = \mathbf{W}^o\mathbf{h}(\mathbf{x}) + \mathbf{b}^o$
- $\mathbf{f}(\mathbf{x}) = softmax(\mathbf{z}^o) = softmax(\mathbf{W}^o\mathbf{h}(\mathbf{x}) + \mathbf{b}^o)$

NEURAL NETWORKS
Mathematical Intuitions



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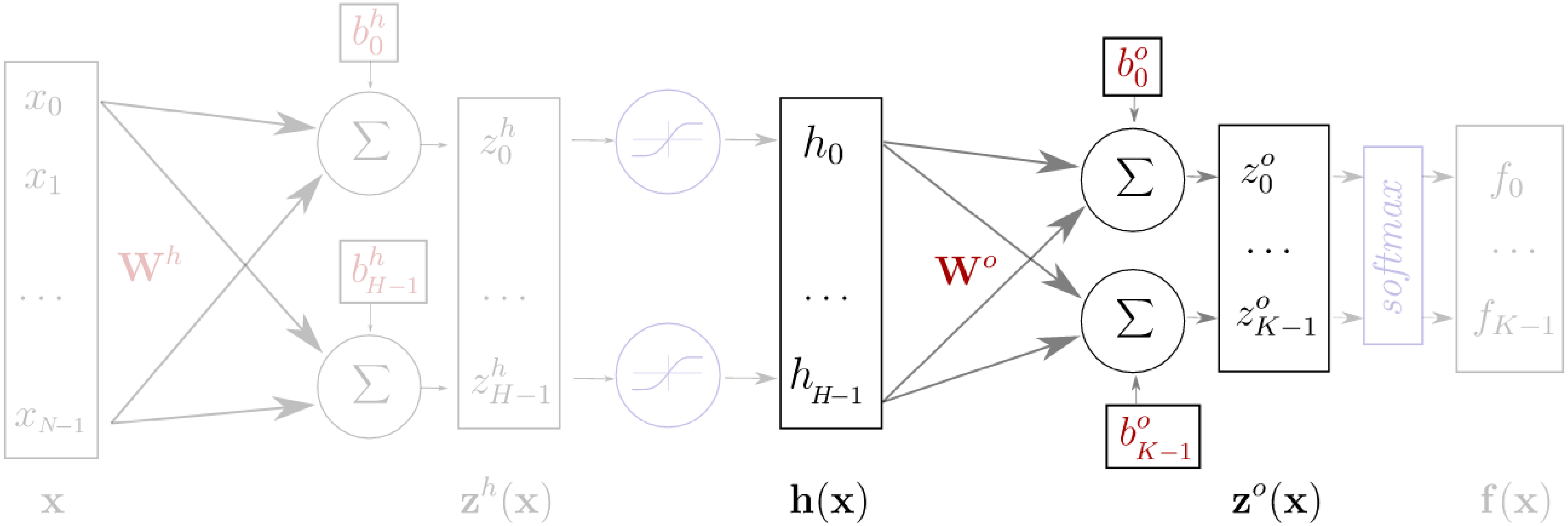
NEURAL NETWORKS

Mathematical Intuitions



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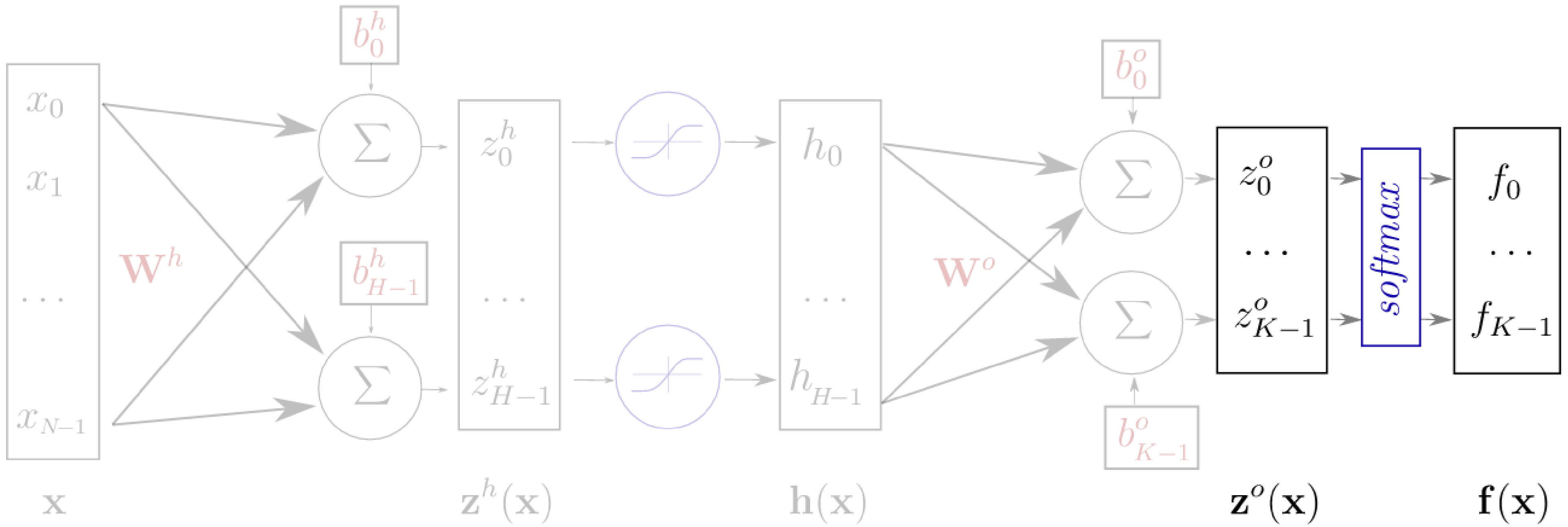
NEURAL NETWORKS
Mathematical Intuitions



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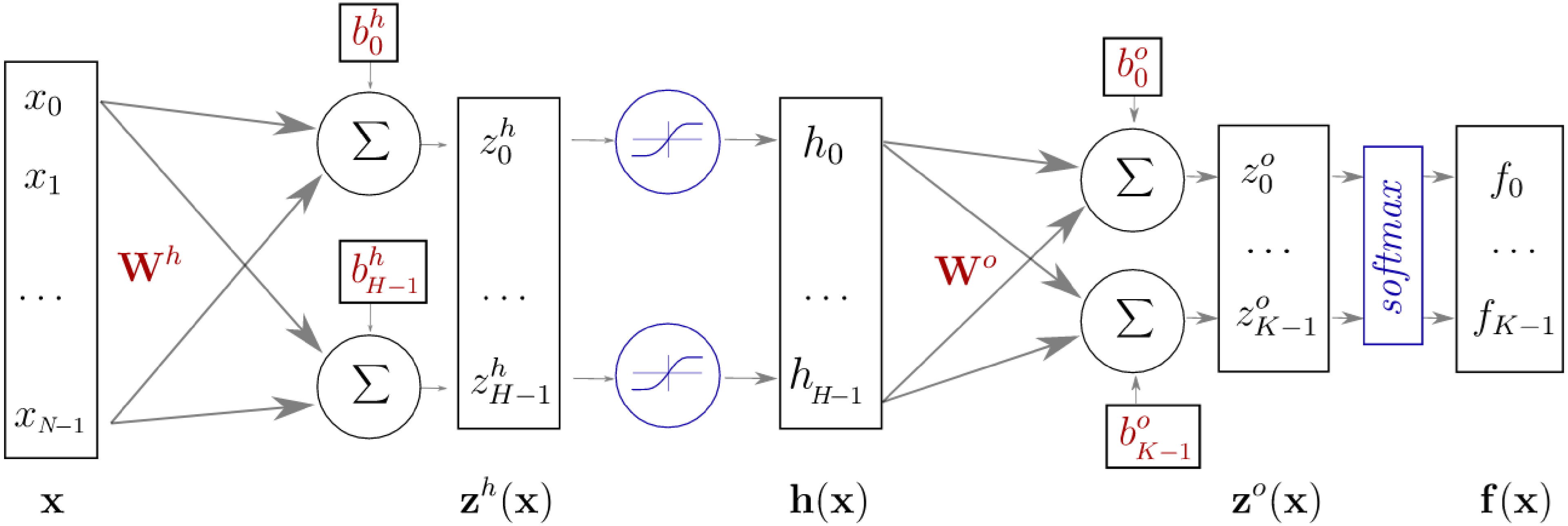
NEURAL NETWORKS
Mathematical Intuitions



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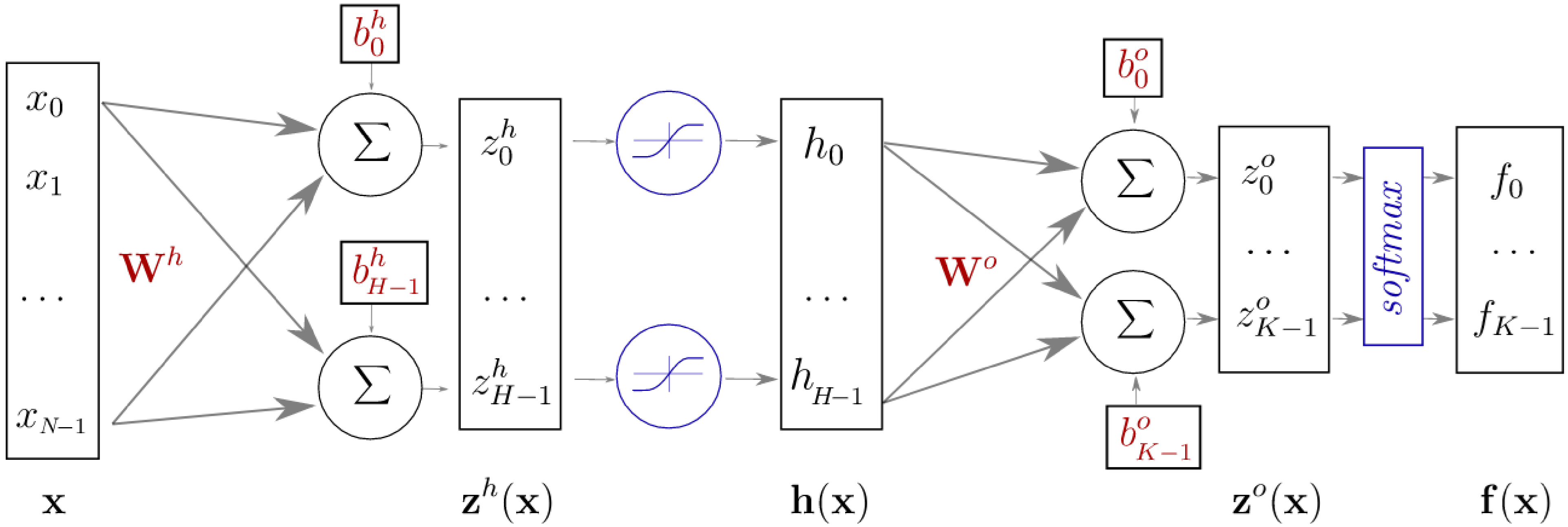
NEURAL NETWORKS

Mathematical Intuitions

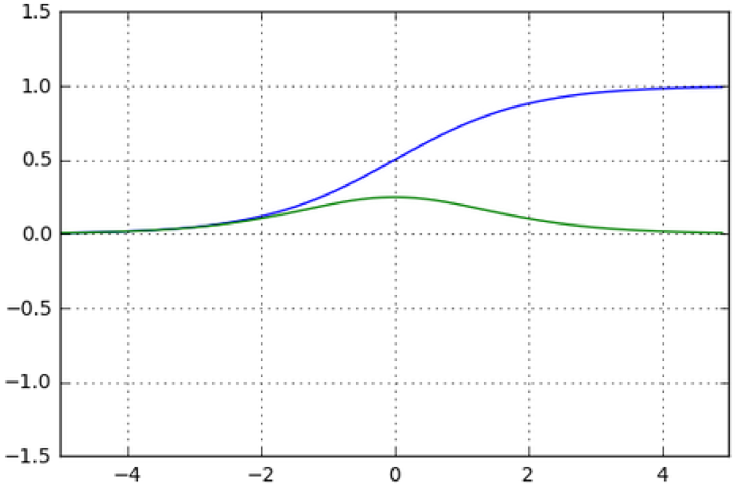


Keras Implementation

```
model = Sequential()
model.add(Dense(H, input_dim=N))  # weight matrix dim [N * H]
model.add(Activation("tanh"))
model.add(Dense(K))               # weight matrix dim [H x K]
model.add(Activation("softmax"))
```

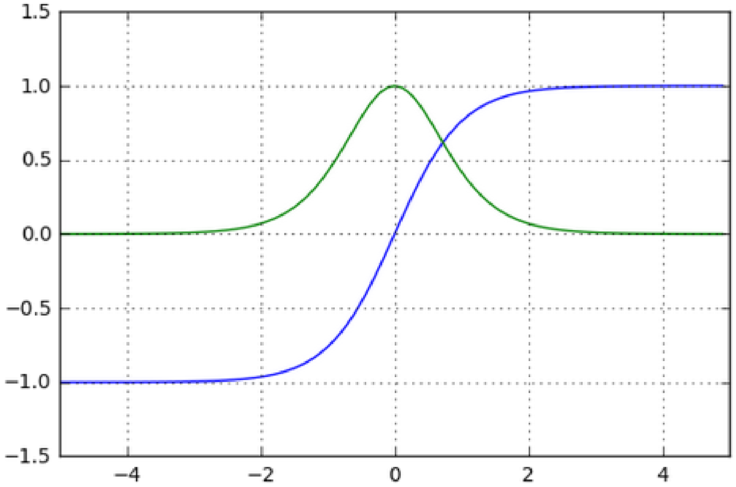
NEURAL NETWORKS

Activation Functions



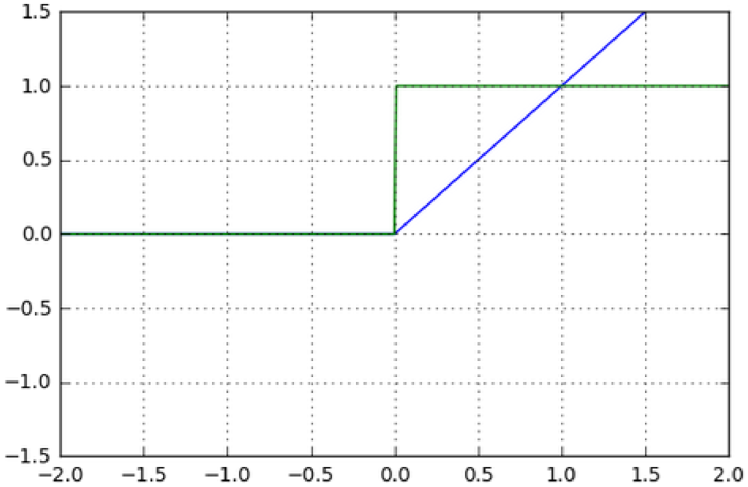
$$\mathrm{sigm}(x) = \frac{1}{1+e^{-x}}$$

$$\mathrm{sigm}'(x) = \mathrm{sigm}(x)(1 - \mathrm{sigm}(x))$$



$$\tanh(x) = \frac{e^{2x}-1}{e^{2x}+1}$$

$$\tanh'(x) = 1 - \tanh(x)^2$$

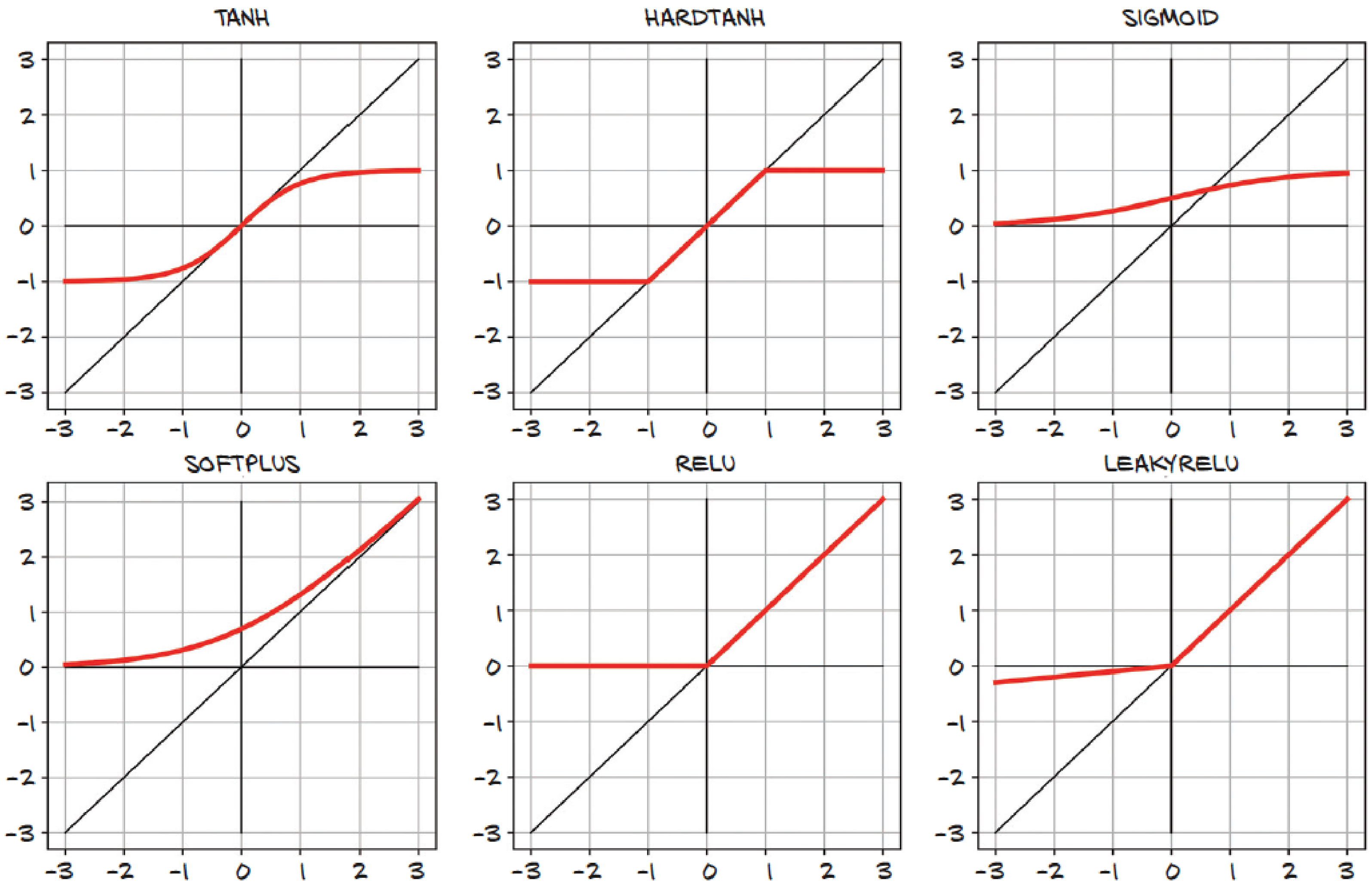


$$\mathrm{relu}(x) = \max(0, x)$$

$$\mathrm{relu}'(x) = 1_{x>0}$$

NEURAL NETWORKS

Activation Functions



NEURAL NETWORKS
Activation Functions

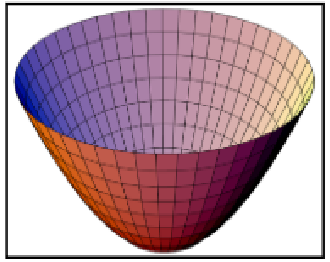
Largest difference between simple ML models and neural networks is:

– Nonlinearity of neural network causes interesting loss functions to be non-convex

Logistic Regression Loss:

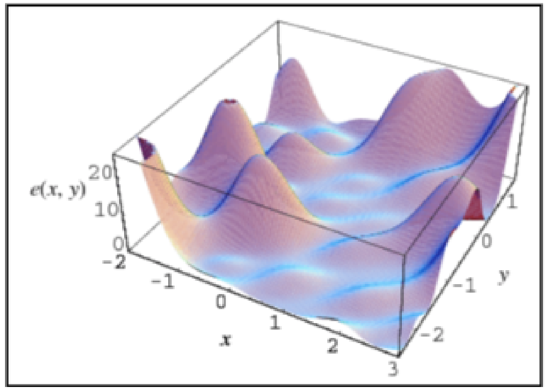
Linear Regression with Basis Functions:

$$E_D(\mathrm{w}) = \frac{1}{2}\sum_{n=1}^{N}\{ t_n - \mathrm{w}^T \varphi(x_n) \}^2$$



Neural Network Loss:

$$J(\boldsymbol{\theta}) = -E_{\boldsymbol{x},\boldsymbol{y}\sim\hat{p}_{data}} \log p_{\text{model}}(\boldsymbol{y} \mid \boldsymbol{x})$$



Use iterative gradient-based optimizers that merely drives cost to low value, rather than

- Exact linear equation solvers used for linear regression or
- convex optimization algorithms used for logistic regression or SVMs