

## Topics

- Canonical Forms
- Minimization of Boolean functions using the Karnaugh map method

## Preliminary Definitions

- A **literal** is a variable or the complement of a variable. Examples:  $x, y, \bar{x}$ .
- A **product term** is a single literal or a logical product of two or more literals. Examples:  $\bar{z}, x \cdot y, x \cdot \bar{y} \cdot z$ .
- A **sum term** is a single literal or a logical sum of two or more literals. Examples:  $\bar{z}, x + y, x + \bar{y} + z$ .
- A **normal term** is a product or sum term in which no variable appears more than once.
- An  $n$ -variable **minterm** is a normal product term with  $n$  literals. There are  $2^n$  such product terms.
  - A minterm  $m_i$  corresponds to row  $i$  of the truth table.
  - In minterm  $m_i$ , a particular variable appears complemented if the corresponding bit in the binary representation of  $i$  is 0; otherwise, it is uncomplemented.
- An  $n$ -variable **maxterm** is a normal sum term with  $n$  literals. There are  $2^n$  such sum terms.
  - A maxterm  $M_i$  corresponds to row  $i$  of the truth table.
  - In maxterm  $M_i$ , a particular variable appears complemented if the corresponding bit in the binary representation of  $i$  is 1; otherwise, it is uncomplemented.

Canonical Forms:

**1<sup>st</sup> canonical form:**  
**sum of products, SOP:**

$$f(x_0, x_1, \dots, x_{n-1}) = \sum_{i=0}^{2^n-1} m_i \cdot f_i$$

**2<sup>nd</sup> canonical form:**  
**Product of sums, POS:**

$$f(x_0, x_1, \dots, x_{n-1}) = \prod_{i=0}^{2^n-1} (f_i + M_i)$$

**3rd canonical form:**

$$f(x_0, x_1, \dots, x_{n-1}) = \overline{\prod_{i=0}^{2^n-1} f_i \cdot m_i}$$

**4th canonical form :**

$$f(x_0, x_1, \dots, x_{n-1}) = \overline{\sum_{i=0}^{2^n-1} f_i + M_i}$$

## Problems

1. Consider the following Boolean function  $f(x, y, z) = x' \cdot y + z' + x \cdot y' \cdot z$ .
  - a. Draw the logic circuit
  - b. Construct the truth table for the function  $f(x, y, z)$ .
  - c. From the truth table write all the canonical forms.
2. Write all the canonical forms for the Boolean functions  $f, g, h, w$  of  $(x, y, z)$  defined in the following truth table:

x	y	z	f	g	h	w
0	0	0	0	1	0	1
0	0	1	1	0	1	0
0	1	0	1	1	1	0
0	1	1	0	0	1	0
1	0	0	1	1	1	1
1	0	1	0	0	1	1
1	1	0	0	1	1	0
1	1	1	1	0	0	1

3. Find the minimal SOP and POS algebraic expressions for the functions defined by the following Karnaugh maps:

K1

ab \ cd	00	01	11	10
00	1	1		
01				
11		1	1	
10		1	1	

K2

ab \ cd	00	01	11	10
00	1	1	1	1
01				
11				
10				

K3

ab \ cd	00	01	11	10
00	1			1
01				
11				
10	1			1

K4

ab \ cd	00	01	11	10
00		1	1	
01			1	
11		1	1	
10		1	1	

4. Propose a 4-variable Boolean function, that has more than one minimal expression (in either POS or SOP form). Locate the essential prime implicants (or essential prime implicants) if they exist.
5. Let  $f(a, b, c, d) = a' \cdot c' + b' \cdot c' + a \cdot c \cdot d + a' \cdot b \cdot c'$ .
- Write the Karnaugh map directly from the expression
  - Find the minimal SOP expression for  $f(a, b, c, d)$ .
6. Let  $f(a, b, c, d) = (a + b') \cdot (c' + d) \cdot (b' + d')$
- Write the Karnaugh map directly from the expression
  - Find the minimal SOP expression for  $f(a, b, c, d)$ .

7. Find the minimal SOP expressions for the two following Boolean functions. Compare the results.

a.  $f(x_3, x_2, x_1, x_0) = \sum m_{x_3, x_2, x_1, x_0} (0, 1, 4, 5, 12, 13)$

b.  $g(x_3, x_2, x_1, x_0) = \prod M_{x_3, x_2, x_1, x_0} (2, 3, 6, 7, 8, 9, 10, 11, 14, 15)$

8. Find the minimal SOP and POS expressions for the following Boolean functions

a.  $(w, x, y, z) = \sum m_{w, x, y, z} (0, 1, 2, 4, 6, 9, 11)$

b.  $f(x_3, x_2, x_1, x_0) = \sum m_{x_3, x_2, x_1, x_0} (0, 1, 4, 5, 8, 9, 10, 11, 12, 14)$

c.  $f(x_4, x_3, x_2, x_1, x_0) = \sum m_{x_4, x_3, x_2, x_1, x_0} (0, 2, 4, 5, 6, 8, 9, 10, 12, 14, 17, 18, 20, 24, 25, 28, 30)$

9. Sometimes the output for a given combination of the input variables is not possible to define and/or is irrelevant. The existence of these don't care conditions may facilitate the minimization process. Find the minimal SOP expressions for the following Karnaugh maps:

K1

ab \ c	00	01	11	10
0		x	x	1
1	1	x	1	

K2

ab \ cd	00	01	11	10
00		1	1	
01	1	x		x
11	x		x	1
10		1	1	

10. Obtain the minimal SOP and POS expressions for the following functions with don't care conditions:

a.  $f(x_3, x_2, x_1, x_0) = \sum m_{x_3, x_2, x_1, x_0} (4, 5, 6, 8, 9, 10, 13) + \sum d_{x_3, x_2, x_1, x_0} (0, 7, 15)$

b.  $f(x_3, x_2, x_1, x_0) = \sum m_{x_3, x_2, x_1, x_0} (1, 3, 5, 7, 9) + \sum d_{x_3, x_2, x_1, x_0} (6, 12, 13)$