Black-Box Terminal Characterization Modeling of DC-to-DC Converters

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Abstract-**This paper proposes a hardware-oriented modeling methodology for dc-dc converters when the information about their internal structure and parameters is not available. The proposed model is a non-terminated two-port hybrid $G_{ij}(s)$ parameters network built from experimental frequency response functions measured at the input and output terminals, and post-processed using system identification methods to implement the final model. The two-port nature of the model enables its simple interconnection to simulate larger distributed power systems. The paper includes the complete derivation—providing insight into the experimental measurement and model construction—and experimental validation for an open- and closed-loop buck dc-dc converter rated at 25 W, 20-to-5 Vdc. The excellent results obtained verify the proposed modeling methodology.**

I. Introduction

To improve the design process of power electronics converters, engineers employ hierarchical modeling with an increasing level of detail on what is called a top down modeling approach. The most detailed models are used to study specific phenomena, e.g., turn-on and turn-off semiconductor transients, ripple current or voltages, etc.; simpler ones with ideal switches for instance are used for filter design, average models for behavioral studies, and small-signal models for control system design. What is common to all these models is that the designer has complete knowledge of the power converter structure and parameters [1-3].

When designing power electronics conversion systems feeding multiple power converters, there is a need to employ simpler behavioral models—from the higher levels of the modeling hierarchy—in order to study and assess all possible system interactions of interest, such as power quality, stability, reliability, safety, thermal management, weight, size and cost of the system [4]. However, system engineers will rarely have access to all the information required to model all the converters, and in fact in most cases they will have no knowledge at all besides the input and output voltages and power rating of the system components. This is common in systems built by integrating power converters, filters, protection devices, and loads designed and manufactured by different companies, which normally provide little or no information about the design and internal structure of their products. Consequently, the design and construction of these systems finally relies entirely on the experience of engineers, and the testing of the built system under multiple operating modes.

This paper presents the development of 'black-box' models capturing the input-output dynamic of power converters with unknown structure, i.e., converters with only input and output terminals accessible. The model is derived directly from hardware using a network analyzer to measure input-output transfer functions of interest, which are then processed and used to build a two-port network equivalent circuit. The models developed are suitable for system level design, and specifically for the study of dynamic interactions between cascaded and paralleled converter systems. This differs from previous black box converter models that were oriented to obtain plant models for control design [5-7]. The paper includes the complete derivation—providing insight into the experimental measurement and model construction—and validation for an open- and closed-loop buck dc-dc converter power supply (25 W, 20V−5V).

II. Black Box Terminal Characterization

Black-box terminal characterization (BBTC) refers to a hardware oriented modeling approach that does not need a priori information of the converter parameters or a relationship between its internal state variables. The structure of the model is based on a non-terminated two port network known as a hybrid model or G parameters model where the input port is represented by a Norton equivalent circuit and the output by a Thevenin equivalent circuit as shown in Fig.1 [8-10].

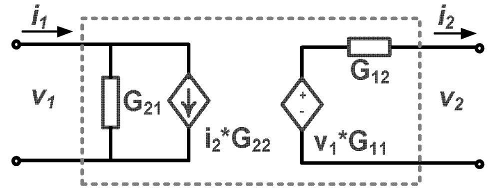

Fig. 1 BBTC Model structure

The input variables of the model are the output current I_2 and input voltage V_1, and the output variables are the output voltage V_2 and input current I_1. This modeling approach is based on the assumptions that the converter is mildly non-linear and its dynamic, regardless of its topology or control system, may be captured by four transfer functions referred to hereinafter as the G_{ij} parameters. These four transfer functions measured directly from the hardware capture the frequency response and internal dynamic between these terminals. The G_{ij} parameters are hence defined as:

$$G_{11} = \left.\frac{v_2}{v_1}\right|_{i_2=0} \qquad G_{12} = \left.\frac{v_2}{i_2}\right|_{v_1=0} \qquad G_{21} = \left.\frac{i_1}{v_1}\right|_{i_2=0} \qquad G_{22} = \left.\frac{i_1}{i_2}\right|_{v_1=0} \tag{1}$$

The following equation represents the terminal input-output relationship of the variables for a converter in open-loop.

$$\begin{bmatrix} v_2 \\ i_1 \end{bmatrix} = \begin{bmatrix} G_{11OL} & G_{12OL} \\ G_{21OL} & G_{22OL} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} \quad (2)$$

Assuming the internal parameters of the converter are known, a closed form of the equation for the closed loop case can be formulated in terms of the open loop transfer functions, the control loop gain (T_{loop}), and control to input current (Y_c) transfer function as shown in equations (2). From this analysis it is clear that frequency responses measured at the converter power terminal will capture the power stage dynamics together with the control dynamics.

$$\begin{bmatrix} v_2 \\ i_1 \end{bmatrix} = \begin{bmatrix} \dfrac{G_{11OL}}{1+T_{loop}} & \dfrac{G_{12OL} \cdot Y_c}{1+T_{loop}} \\ G_{21OL} + \dfrac{G_{11OL}}{1+T_{loop}} & G_{22OL} + \dfrac{G_{12OL} \cdot Y_c}{1+T_{loop}} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} \quad (3)$$

The methodology used to obtain BBTC models is summarized with the flowchart illustrated in Fig. 2. This methodology is presented in detail in the following sections using a converter with known parameters and topology, in this way the results obtained using BBTC models can be compared to a white box approach, i.e., when all the converter parameters are known. The converter schematic is shown in Fig.3

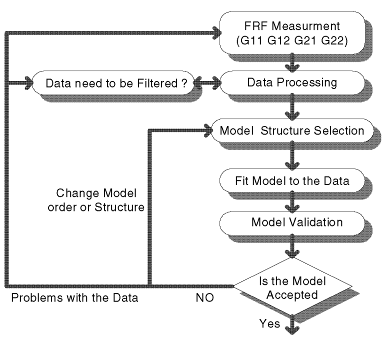

Fig. 2 BBTC Models flowchart

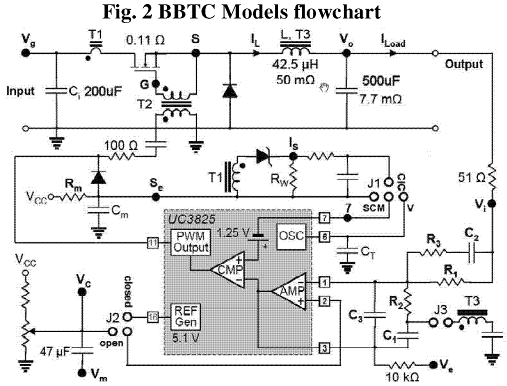

Fig. 3 Converter schematic

III. MEASUREMENT OF FREQUENCY RESPONSE FUNCTIONS

A frequency response function (FRF) is defined for a linear system as the Fourier transformation of its impulse response. Experimentally, FRF's are obtained using a network analyzer such as the Agilent 4395A. The network analyzer produces a broad-band sinusoidal excitation signal that is amplified by a linear amplifier. The disturbance is injected into the circuit using a transformer. Current probes and voltage probes connected to the input channels of the network analyzer sense the currents and voltages of interest to calculate the magnitude and phase of the input disturbance, the magnitude and phase of the output response and compute the corresponding ratio.

Since real data is used to build models and to validate them it is important to understand how to properly measure FRF's and its limitations. Fig.4 shows the experimental setups used to measure G22 and G12. For instance when measuring G11 and G22 the disturbance signal is chopped by the switch and reconstructed at the converter output filter, which can be considered as a sample-and-hold effect; thus, from the sampling theorem these FRF's can be measured up to half the switching frequency. The lower limit is 10 Hz, which is imposed by the network analyzer. To ensure a good measurement at low frequencies the IF bandwidth should be set to one fifth of the starting frequency. For instance, if the start frequency is 10 Hz, the IF bandwidth should be set to 2 Hz. A similar setup is used to measure G11 and G21 as illustrated in Fig.5

It is important to mention that disturbance sources such as current ripples, voltage ripples and switching noise, can affect the frequency range in which FRF's can be measured accurately. For instance, depending on the magnitude of these disturbances the FRF range could be reduced to one quarter of the switching frequency. Fig.6 shows the ripple of the input and output variables measured at the terminals using AC coupling.

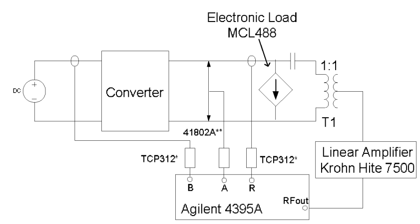

Fig. 4 Measurement setup for G22 and G12

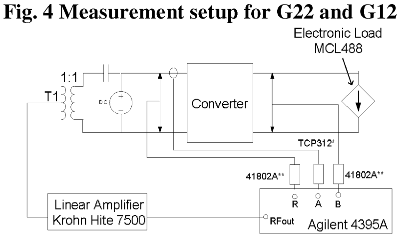

Fig. 5 Measurement setup for G11 and G21

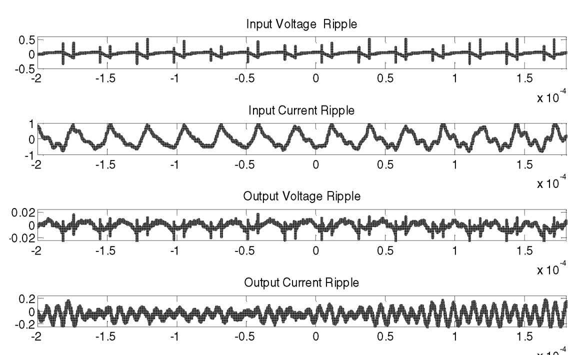

Fig. 6 Voltages and current ripples

A. FRF'S MEASUREMENT OF OPEN LOOP CONVERTER

Two G22 FRF's of an open loop buck converter with know parameters are presented in Fig.7. One is measured using a network analyzer and the other is obtained using average models. The two FRF are similar at low frequencies but as the frequency increases the average models fails to predict the degradation of the phase that occurs before half the switching frequency [11]. After half the switching frequency it is difficult to ensure a good measurement because aliasing phenomena become an important issue [12]; however, this is not a problem for BBTC models since the intention is to capture the slow dynamic of the converter; a measurement up to half the switching frequency is hence deemed sufficient.

For the G21 and G12 FRF's, measurements beyond half the switching frequency are possible because in this frequency range the dynamics of these FRF's are governed by the input and output converter filters respectively. Thus, the most important factor that affects the measurements would be the switching noise, and the voltage and current ripple.

Fig.8 shows the G12 FRF obtained from measurements and from simulation using an average model, where the differences between them are apparent. For instance, at low frequencies the average model does not account for the effect of PCB and interconnections parasitic resistances. Something similar occurs at high frequencies, however in this case the discrepancy is attributed to parasitic inductances and capacitances neglected in the average model. As shown, average models can only be as good as the information available and considered when building the model.

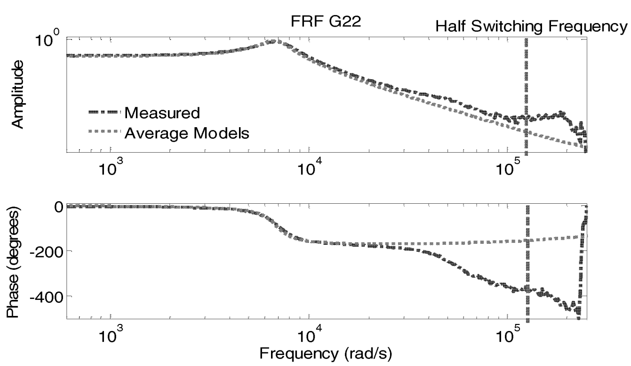

Fig. 7 FRF G22 from measurement and average model

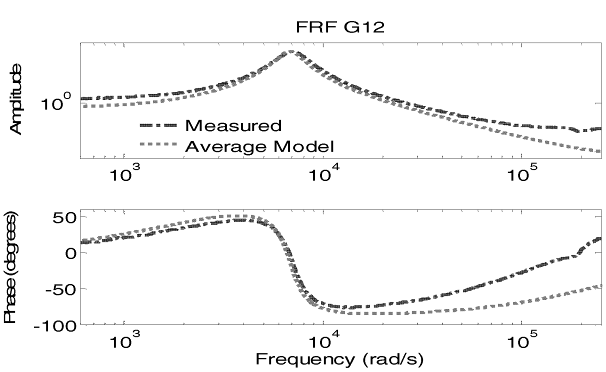

Fig. 8 FRF G12 from measurement and average model

B. FRF'S MEASUREMENT OF CLOSED LOOP CONVERTER

The measurement setup used for closed loop converter is the same used for open loop. The difference in this case when compared with open loop converter measurements resides in the lower frequency bound. For instance, with open loop converters it is possible to start measuring from 10Hz using a relatively low disturbance magnitude, whereas for closed loop converters low frequency FRF measurements depend on the disturbance rejection characteristics of the feedback control.

Closed loop converters are designed to have a good disturbance rejection at low frequencies. Therefore, any low frequency disturbance injected would be compensated by the feedback network and the network analyzer would not have any information at the output to compute the FRF. Also some times the response is so small that it is over powered by noise in the system. To deal with this limitation it is recommended to use a higher disturbance magnitude at low frequencies paying attention not to saturate the control signals, and in addition narrowing the network analyzer bandwidth (2 Hz IF Bandwith). Using the settings described above measurements starting as low as 100 Hz gave reliable results for the converter used in these experiments.

IV. SYSTEM IDENTIFICATION

System identification deals with the problem of estimating models of dynamic systems, from input and output measurements. The system identification algorithm implemented to identify models from the measured FRF's can be described in terms of three steps, data pre-processing, model structure selection, and setting of algorithms options. This process is iterative therefore is indispensable to use any available software package for system identification. In this work the system identification toolbox from Matlab was used.

Data pre-processing refers to the creation of a data object or variable that stores all frequencies in a column vector and magnitude and phase is stored in a complex variable, the object is formed by

Frfdata=idfrd(Mag,freq,Ts)

where the variable Mag is a column vector that stores the magnitude and phase in complex form, the column vector freq stores all frequencies, and Ts is the sampling time, which for continuous system is set to zero.

Ones the data is stored in an IDFRD object, another aspect of data processing such as filtering the data before the estimation can be implemented. The data is filtered and compressed using a local smoothing technique as.

Sfrfdata= spafdr(frfdata,Resol)

The argument Resol defines the frequency resolution over the frequency interval of the data. The wider the resolution the smoother the estimate would be.

One of the most important steps in system identification is to select a model structure from a predetermined set of models that best describes the data. The general-linear black-box model structure provides flexibility for both the system dynamics and stochastic dynamics; however, the general linear model requires intensive computation with no guarantee of global convergence. Therefore, simpler models that are a subset of the general model are preferred, such as ARX, ARMAX, output error and Box-Jenkins [13].

Another model structure known as state-space model is also widely used [14]. Especially in complex high order systems. Equation (4) describes a state-space model,

$$\begin{aligned} \dot{x}_{(t)} &= Ax_{(t)} + Bu_{(t)} + Ke_{(t)} \\ y_{(t)} &= Cx_{(t)} + Du_{(t)} + e_{(t)} \end{aligned} \qquad (4)$$

where x(t) is the state vector, y(t) is the system output, u(t) the system input and e(t) is the stochastic error. A, B, C, D, and K are the system matrices. The dimension of the state vector x(t) is the only setting needed for the state-space model. In general, the state-space model provides a more complete representation of the system than polynomial models because state-space models are similar to first principle models. Also, the identification procedure does not involve nonlinear optimization; therefore, the estimation reaches a solution regardless of the initial guess. Because of the advantages showed by state-space models (later confirmed in practice), this structure was chosen for the identification algorithm used in this work. The estimation using a state space model structure is obtained by the following function.

pem(Sfrfdata,order)

The last but not less important step during the identification process is to set the algorithm properties. These properties help to find the models that best fit the data. For instance, focus is one of the most used properties in this work. It is used to apply a weighting to the fit between the model and the data. For example,

m11=pem(Sfrfdata,4,'Foc',[10^3,10^5])

will fit the data into a fourth order state space model concentrating the effort on the data between 10^3 and 10^5 rad/s.

A. Identification of Open Loop FRFs

The models obtained through system identification are compared with the measured FRF's; the results are illustrated in Fig. 9. As shown, the resulting models overlap the measured FRF's in the frequency range of interest. It should be mentioned that a perfect match is not required, since as long as the dc gains and dominant modes are captured in the model there will not be any significant difference when comparing the time-domain response of the models and corresponding hardware. The G_{ij} models obtained are given in the appendix.

B. Identification of Closed Loop FRFs

Closed loop is an integral part of power converters therefore, it is important to extend the proposed methodology to converters that have any form of feedback network. In this section a BBTC model is obtained for the converter shown in Fig.3. The converter is operated in closed loop using a three poles two zeroes error amplifier to regulate the output voltage. System identification of closed loop is not an issue as long as a linear feedback network is used to implement the control [15-16]. The identification process can be done in two ways. One is called indirect identification where knowledge of the control or the plant is needed to build the model. The other one is called direct identification which basically treats the data as if were obtained from an open loop system. In this work direct identification is used since the main assumption is that no information about the control structure of the converter would be available. Fig.10 Shows the FRF's measured from the hardware and FRF's of the models obtained using system identification. The close match between these Bode plots is apparent. The G_{ij} models obtained are given in the appendix.

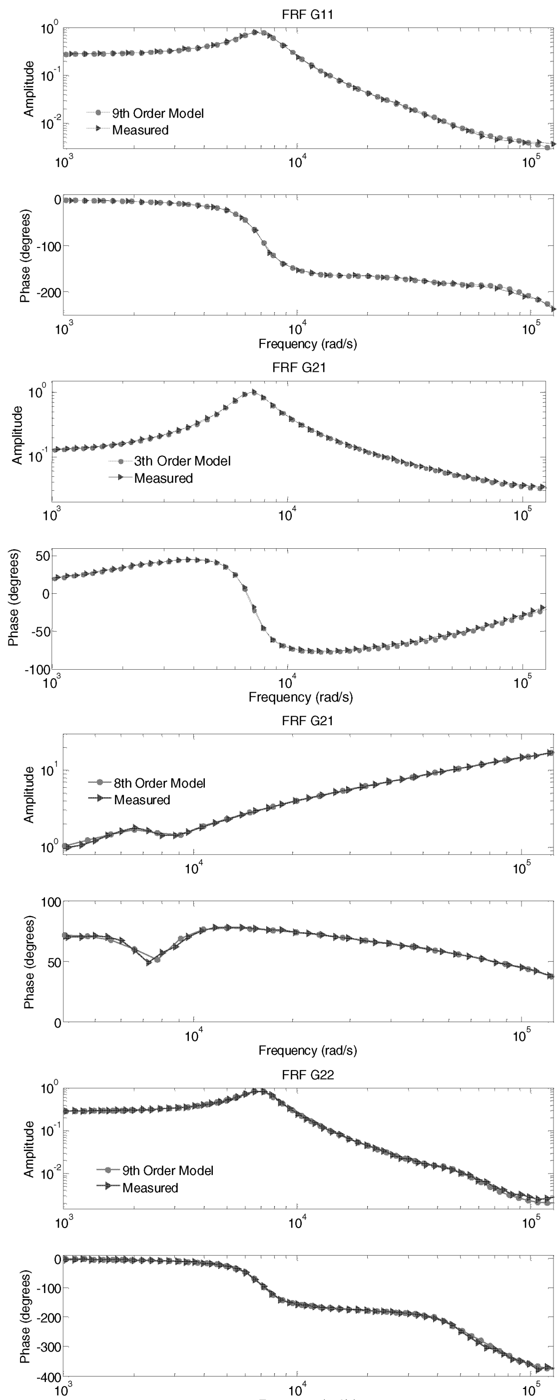

Fig. 9 Open loop FRFs from measurements and identified models

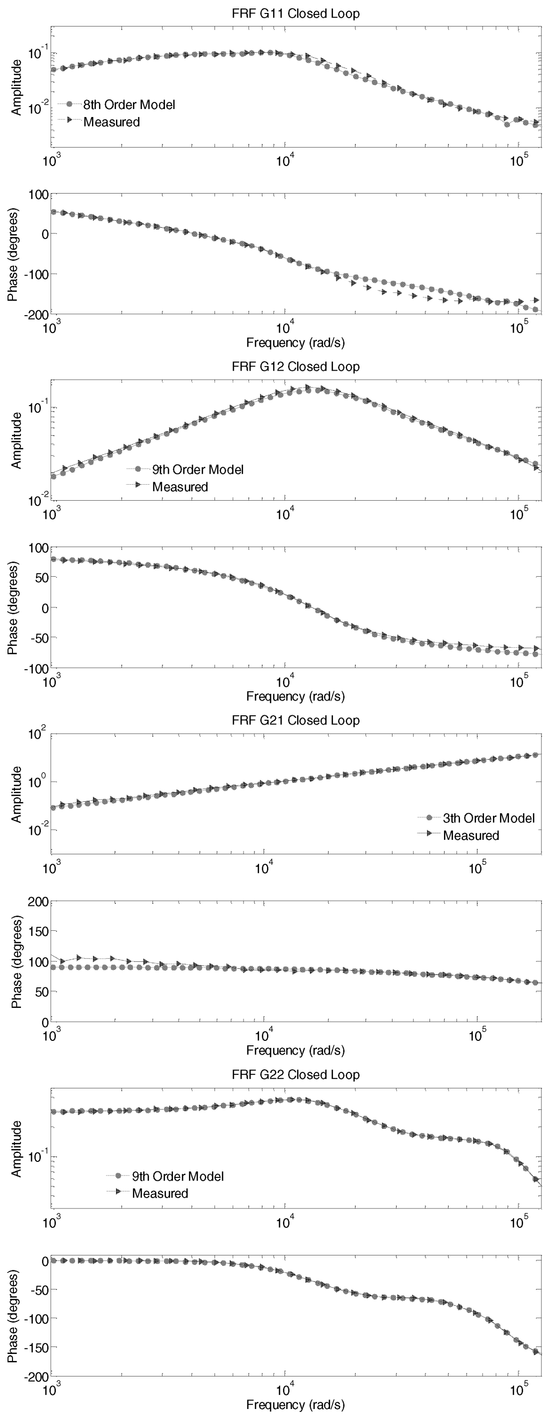

Fig. 10 Closed loop FRFs from measurements and identified models

V. MODEL VALIDATION

The main objective in this section is to decide whether or not the model is acceptable for system level simulations. To make this decision, validation data (data not used to estimate the model) is compared with data produced by the models. The criteria to decide how close the model should follow the validation data depends on the application. In this work the interest is to investigate how close BBTC models are able to predict input-output variables; therefore, the criterion used is to obtain a model that describes the hardware as close as possible.

The model validation is carried out in the time domain, where measured input-output variables are compared against the ones predicted by the model when applying a load disturbance. The load disturbance is implemented with an electronic load working as a constant current sink. The current step goes from 2.5 Amps to 5 amp and back to 2.5 amps, which represents a step load from 50% to 100%. To have a fair comparison between the model and the hardware the current step produced by the electronic load is recorded and then used in the simulation as the load step for the model.

A. VALIDATION BBTC MODEL FOR OPEN LOOP CONVERTER

The output voltage and input current transient response is compared with the ones obtained from the BBTC model. Fig.11 shows a very good matching during transients and steady state. Thus, the proposed modeling methodology effectively captures the internal converter dynamics.

B. VALIDATION BBTC MODELS FOR CLOSED LOOP CONVERTERS

The black box model structure used for the closed loop case is linear therefore fail to predict inherent nonlinear behavior of regulated converters such as varying dynamics. To address this limitation, an analysis was performed to determine how do the FRF's change when the input voltage varies ±25% around its nominal point. The results indicated that the FRF's present slight changes except for G22, which showed considerable changes on the FRF dc gain; thus G22 cannot be represented by one FRF but for a family of FRF's as shown in Fig.12.

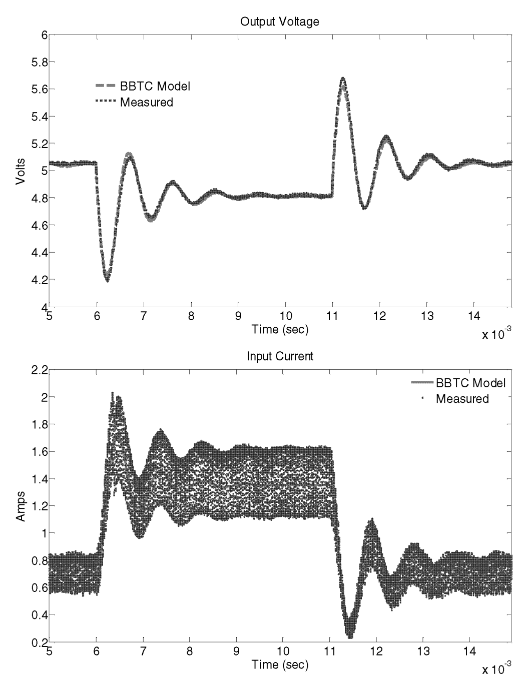

Fig. 11 Open loop transient response

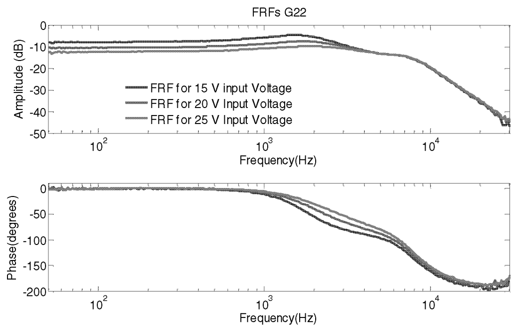

Fig. 12 Effect of changes of the operating point on FRF G22

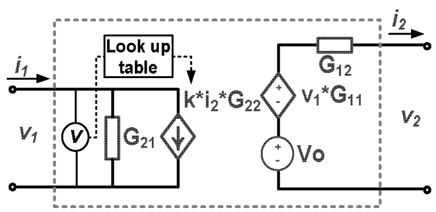

Fig. 13 Nonlinear BBTC model

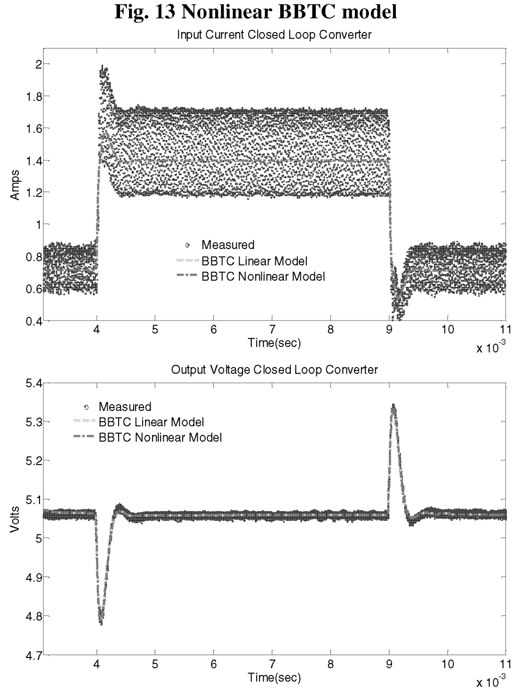

Fig. 14 Close loop transient response with constant input voltage

Since the changes of FRF G22 are mostly on the dc gain a simple approach that uses a lookup table and a multiplier is used to model this family of FRF's, avoiding the use of more elaborated approaches like polytopic modeling [17]. The model for G22 is implemented using the input voltage as a command variable for a look up table that pulls out a multiplier factor for the FRF G22 measured at nominal voltage.

In addition to the lookup table the model for FRF G11 assumes perfect disturbance rejection at low frequencies; therefore there is no information on the dc output voltage operating point. This information is introduced in the model using the variable Vo. For the open loop case this is not necessary because the dc gain of G11 acts as the transfer ratio from input to output. The black box model structure for closed loop converters is illustrated in Fig. 13. The modified BBTC model structure is compared with the linear BBTC model illustrated in Fig.1 for two different cases. The first one shows the case when the input voltage of the converter is regulated, thus remaining constant through the transient. Fig. 14 shows the results using both models. Since the input voltage is kept constant during the transient both the linear and nonlinear BBTC models overlap and predict very well the transient and steady state behavior of the converter.For the second case a 0.65 Ω resistor is placed in series with the dc source. When the load step occurs there is a voltage drop at the converter terminals, this experiment emulates the cases when a converter is fed from an unregulated bus converter. The simulated results using a linear and a nonlinear BBTC models are compared against the output voltage and input current measured at the converter terminals. Fig.16 shows that the linear model is able to predict very closely the transient and steady state output voltage; however, the linear BBTC model fails to predict the steady state input voltage and input current conditions.

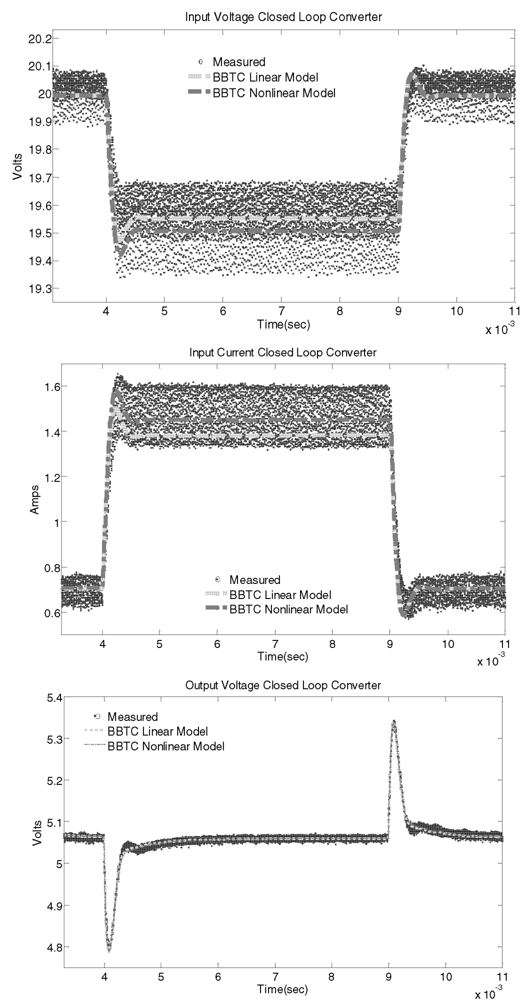

Fig. 15 Closed loop transient response with unregulated input voltage

VI. CONCLUSION

This paper has presented the derivation of BBTC models for dc-dc converters using G parameters and system identification methods. The results obtained with the model showed a very good agreement with experimental results in both frequency and time domains, for both open loop and closed loop cases, thus validating the proposed methodology. The advantage of this modeling approach is that no knowledge of the converter internal structure nor any of its parameters are required. The latter however are not neglected as its is customary when constructing average models for simplification purposes, but on the contrary, all transfer functions measured and used to build the proposed models intrinsically include accurate values of all parasitic inductances, capacitances and resistances present the circuit. Furthermore, the two terminal port nature of this model makes it appropriate for building cascaded and paralleled converter structures, allowing the simple and straightforward simulation of distributed power electronics systems comprised of converters from multiple different vendors and having different functionalities.

ACKNOWLEDGMENT

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REFERENCES

[1] B.M Gordon,. R.A McMahon, Zhong-Min Li; P.A Mawby,. A.N Githiari, "A comparison of IGBT models for use in circuit design", IEEE Transactions on Power Electronics July 1999 Page(s):607 - 614

[2] C.E.C Ortiz "Circuit oriented average modeling of switching power converters". 2005 European Conference on Power Electronics and Applications, 11-14 Sept. 2005 Page(s):10 pp.

[3] Jian Sun; "Characterization and performance comparison of ripple-based control for voltage regulator modules**".** IEEE Transactions on Power Electronics, Volume 21, Issue 2, March 2006 Page(s):346 - 353.

[4] R. Jayabalan,; B.Fahimi,; A.Koenig,; S. Pekarek "Applications of power electronics-based systems in vehicular technology: state-of-the-art and future trends" ,; IEEE 35th Annual Power Electronics Specialists Conference, 2004. PESC 04. 2004 Volume 3, 20-25 June 2004 Page(s):1887 - 1894 Vol. 3

[5] Ju-Yeop Choi; Cho, B.H.; VanLandingham, H.F.; Hyung-soo Mok; Joong-Ho Song "System identification of power converters based on a black-box approach" IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications Volume 45, Issue 11, Nov. 1998 Page(s):1148 – 1158

[6] Johansson, B.; Lenells, M.; "Possibilities of obtaining small-signal models of DC-to-DC power converters by means of system identification" Twenty-second Internationa Telecommunications Energy Conference INTELEC 10-14 Sept. 2000 Page(s):65-75

[7] Chau, K.T.; Chan, C.C." Nonlinear identification of power electronic systems" Proceedings International Conference on Power Electronics and Drive Systems, 1995., 21-24 Feb. 1995 Page(s):329 - 334 vol.1

[8] Altowati, A., Suntio, T, Zenger, K Input filter interactions in multi-module parallel switching-mode power supplies IEEE International Conference on Industrial Technology, Dec. 2005 Page(s):851 - 856

[9] Bo Hyung Cho "Modeling and Analysis of Spacecraft Power System" PhD Dissertation Virginia Polytechnic Institute and State University. Pag 11. 1985..

[10] P.G. Tavazzi, V. Varoli, V Maranesi, **"**Two-port characterization of PWM voltage regulators at low frequencies**".** IEEE Transactions on Industrial Electronics, Volume 35, No. 3, Aug. 1988 Page(s):444 - 450

[11] A. R. Brown and R. D. Middlebrook, "Sampled-data modeling of switching regulators," in Proc. IEEE PESC'81, 1981, pp. 349–369.

[12] G. C. Verghese and V. J. Thottuvelil, "Aliasing effects in PWM power converters," in *Proc. IEEE PESC'99*, 1999, pp. 1043–1049.

[13] Black-box models from input-output measurementsLjung, L.; Instrumentation and Measurement Technology Conference, 2001. IMTC 2001. Proceedings of the 18th IEEEVolume 1, 21-23 May 2001 Page(s):138 - 146 vol.1

[14] Ljung*:* "System Identification - Theory For the User, 2nd ed", Prentice Hall 1999

[15] L Ljung, I Gustavsson,. T Soderstrom, **"**Identification of linear, multivariable systems operating under linear feedback control" IEEE Transactions onAutomatic Control, Volume 19, Issue 6, Dec 1974 Page(s):836 - 840

[16] Wei Xing Zheng Improved direct identification of linear systems in closed-loop operation; Proceedings of the 41st IEEE Conference on Decision and Control, Dec. 2002 Page(s):275 - 280 vol.1

[17] A Fujimori, L Ljung, **"**A polytopic modeling of aircraft by using system identification".; International Conference on Control and Automation, ICCA '05. Volume 1, 26-29 June 2005 Page(s):107 - 112 Vol. 1

APPENDIX

Models identified for open loop FRF's:

$$G11OL = \frac{-260.7s^8 + 2.309e007s^7 - 9.719e012s^6 + 5.134e017s^5 + 3.124e022s^4 + 5.882e027s^3 + 1.576e032s^2 + 2.551e036s + 1.678e040}{s^9 + 144500s^8 + 5.264e010s^7 + 5.101e015s^6 + 3.992e020s^5 + 1.3e025s^4 + 2.255e029s^3 + 2.293e033s^2 + 1.199e037s + 6.3e040}$$

$$G12OL = \frac{0.1479s^3 + 6.772e004s^2 + 5.24e009s + 1.275e013}{s\text{^}3 + 2.362e006s^2 + 5.744e009s + 1.173e014}$$

$$G21OL = \frac{7.175e006s^7 + 6.1e012s^6 + 9.205e017s^5 + 2.512e023s^4 + 1.486e028s^3 + 5.176e031s^2 + 9.554e035s}{s^8 + 910255s^7 + 3.97e011s^6 + 9.986e016s^5 + 1.719e022s^4 + 2.141e027s^3 + 7.463e031s^2 + 3.114e035s + 3.915e039}$$

$$G22OL = \frac{171s^8 + 1.838e007s^7 + 6.107e012s^6 + 7.004e016s^5 + 2.944e022s^4 + 1.691e027s^3 + 5.974e031s^2 + 2.599e036s + 4.619e038}{s^9 + 104400s^8 + 3.875e010s^7 + 2.426e015s^6 + 1.957e020s^5 + 5.076e024s^4 + 2.052e029s^3 + 7.02e032\,s^2 + 9.28e036s + 1.616e039}$$

Models identified for closed loop FRF's

$$G11CL = \frac{-554.1s^7 + 4.466e007s^6 - 3.294e012s^5 + 3.893e017s^4 + 7.561e021s^3 + 3.216e025s^2 + 2.484e029s}{s^8 + 117261s^7 + 1.044e010s^6 + 9.33e014s^5 + 1.352e019s^4 + 1.496e023s^3 + 7.569e026s^2 + 3.508e030s + 4.392e033}$$

$$G12CL = \frac{3029s^8 + 2.563e008s^7 + 8.516e012s^6 + 2.441e017s^5 + 3.488e021s^4 + 4.684e025s^3 + 2.519e029s^2 + 2.215e032\,s}{s^9 + 1.125e005s^8 + 4.909e009s^7 + 1.599e014s^6 + 3.378e018s^5 + 5.468e022s^4 + 6.067e026s^3 + 4.571e030s^2 + 1.61e034s + 1.18e037}$$

$$G21CL = \frac{8.651e007s^2 + 7.822e012s}{s^3 + 2.651e006s^2 + 1.451e012s + 9.381e016}$$

$$G22CL = \frac{2727\,s\text{^}8 + 4.954e008\,s\text{^}7 + 1.741e014\,s\text{^}6 + 2.711e019s^5 + 3.669e024s^4 + 1.883e029s^3 + 6.342e033s^2 + 9.462e037s + 5.041e041}{s^9 + 200600s^8 + 6.35e010s^7 + 7.502e015s^6 + 7.622e020s^5 + 4.176e025s^4 + 1.561e030s^3 + 3.109e034s2 + 3.29e038s + 1.753e042}$$