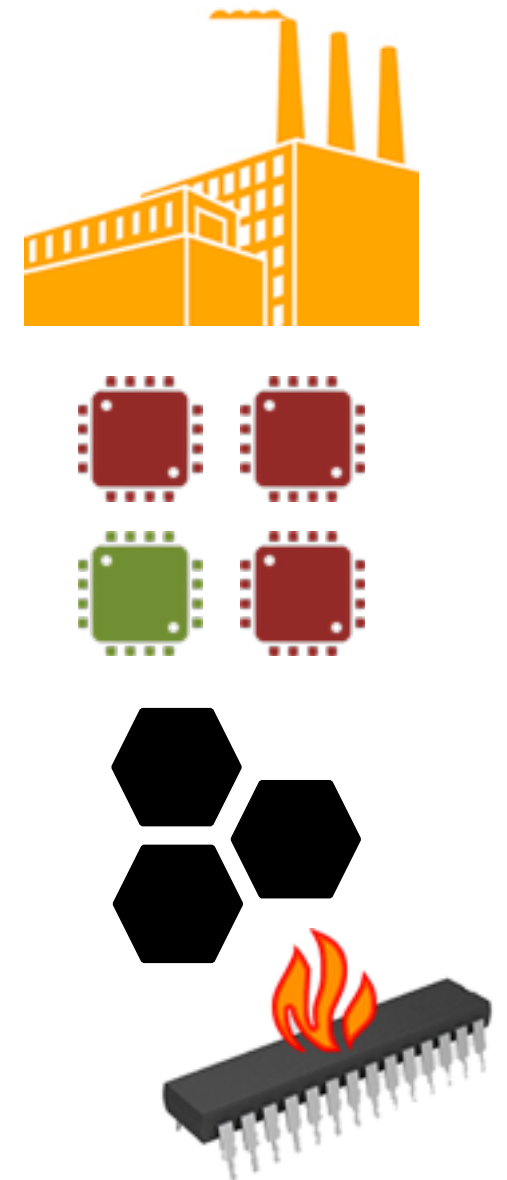


# Approximate Computation with Topaz

**Sara Achour** and Martin Rinard  
MIT EECS & CSAIL

# The world contains a lot of approximate hardware

- **What is approximate hardware?**
  - Hardware that crashes often
  - Sometimes produces wildly inaccurate results,
  - Often produces slightly inaccurate results
- **Approximate hardware exists today**
  - Manufacturing defects
  - Old machines
  - Aggressive operating conditions
  - Relaxed hardware mechanisms
  - Emerging hardware technologies
- **Approximation is a hardware design point**
  - **Energy** and **performance** savings



# Why is using approximate hardware hard?

- **Prior techniques require error model**
  - Assumptions on fault characteristics
  - Narrow applicability
- **In the wild, hardware faults can be complex**
  - Correlated with other faults
  - Dependent on hardware state
  - Yield small errors, large errors or crashes
- **Fault characteristics of future hardware unknown**
- **Need system that generalizes broadly across many approximate computing platforms.**

# Topaz

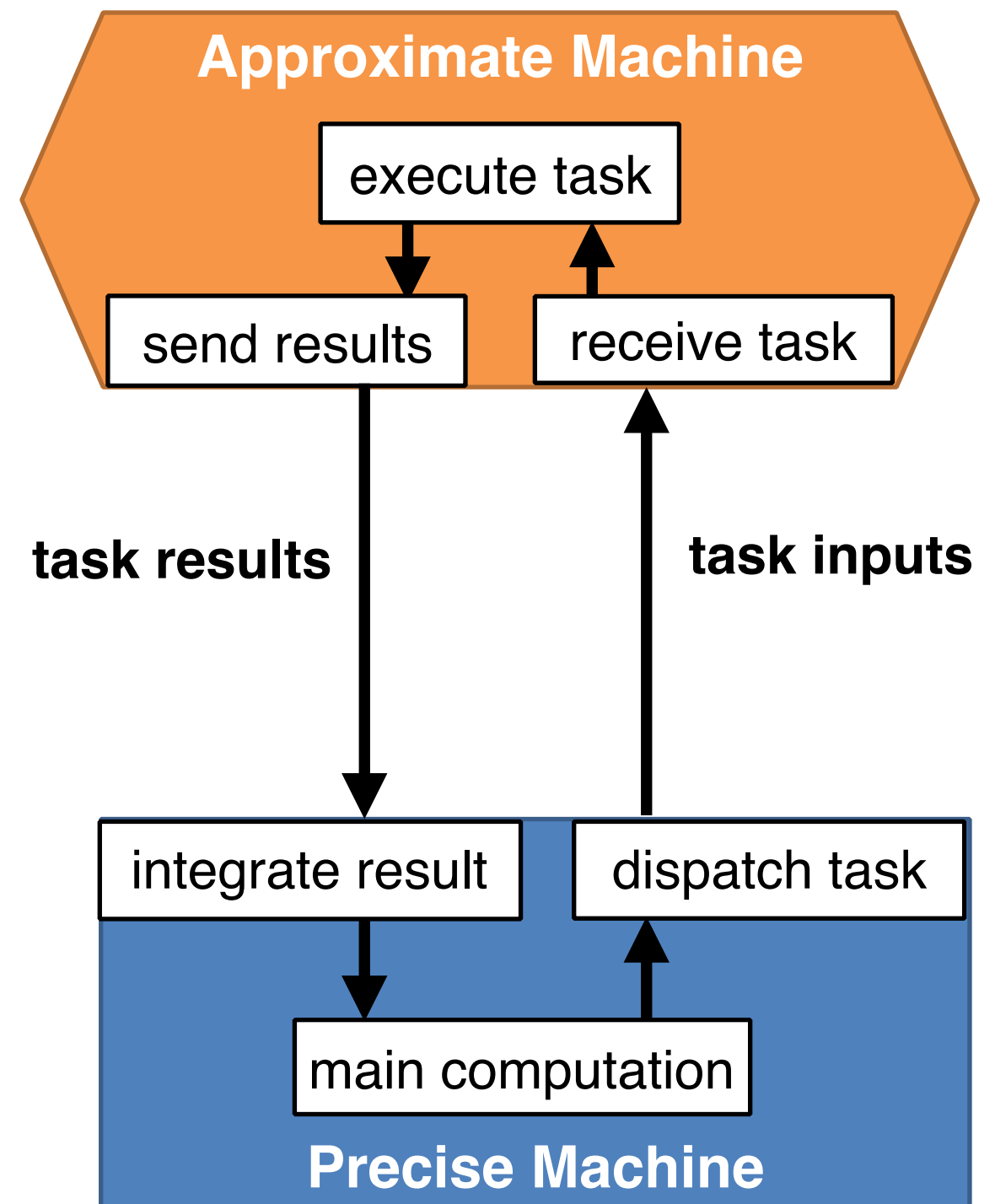
- **Topaz** is a enables the deployment of programs on approximate hardware ***with no hardware specification***
- Operates on hardware with ***complex*** fault characteristics
- Targets programs that are robust and tolerant of error

# Topaz: Key Features

- **Computational Model and Language**  
Ensures computation runs to **completion**
- **Outlier Detection**  
Computation yields an **acceptable** result
- **Optimizations**  
**Savings** from using approximate hardware

# Topaz: Computational Model

- **Task**: approximateable, self contained unit of work
- **Precise** and **approximate** machine
- **Precise** machine:
  - **Executes** main computation
  - **Dispatch** tasks
  - **Integrates** results into state
  - **Reexecutes** failed tasks
- **Approximate** machine:
  - **Performs** task computation
  - **returns** a task result



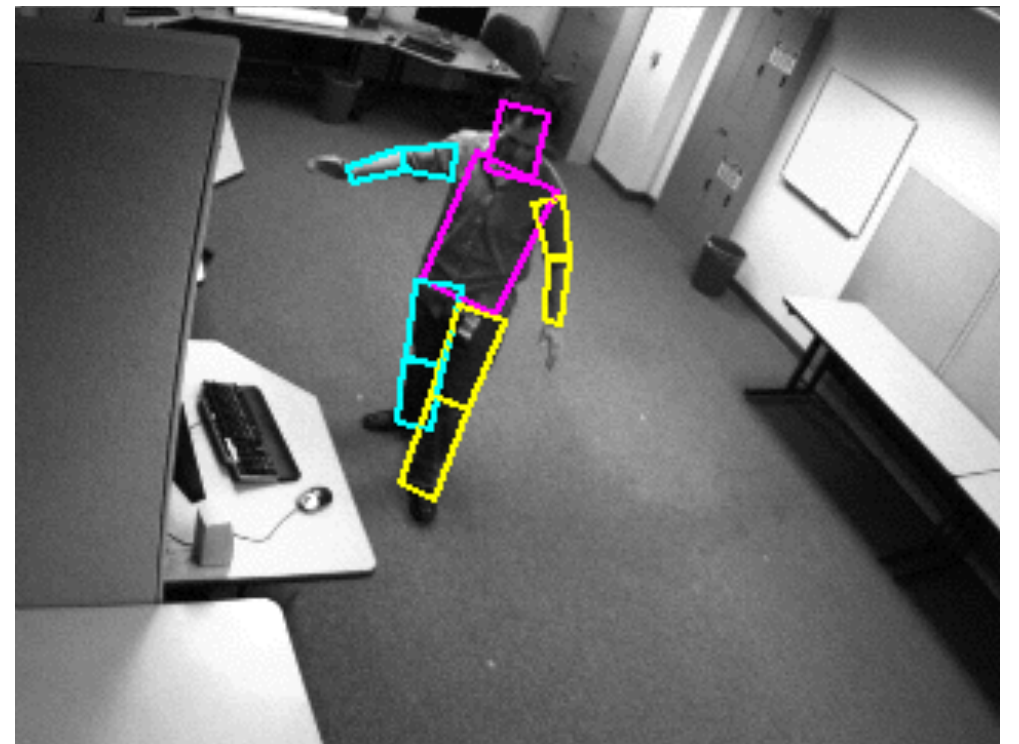
# Topaz: Taskset Construct

```
taskset  name(int i = l; i < u; i++) {  
    compute in  (d1 x1 = e1, ..., dn xn = en)  
               out (o1 y1, ..., oj yj) {  
                <task body>  
            }  
    combine { <combine body> }  
}
```

- **Taskset**: a set of  $u$  tasks, where  $i$  refers to its task
- **Compute**: the task definition. Comprised of  $n$  inputs and  $j$  outputs
  - The  $k^{\text{th}}$  input: has name  $x_k$ , type  $d_k$  and is assigned to expression  $e_k$
  - The  $k^{\text{th}}$  result: has name  $v_k$ , type  $o_k$
  - The  $\langle \text{task body} \rangle$  describes the task computation
- **Combine**: the routine for integrating the task into the main computation.

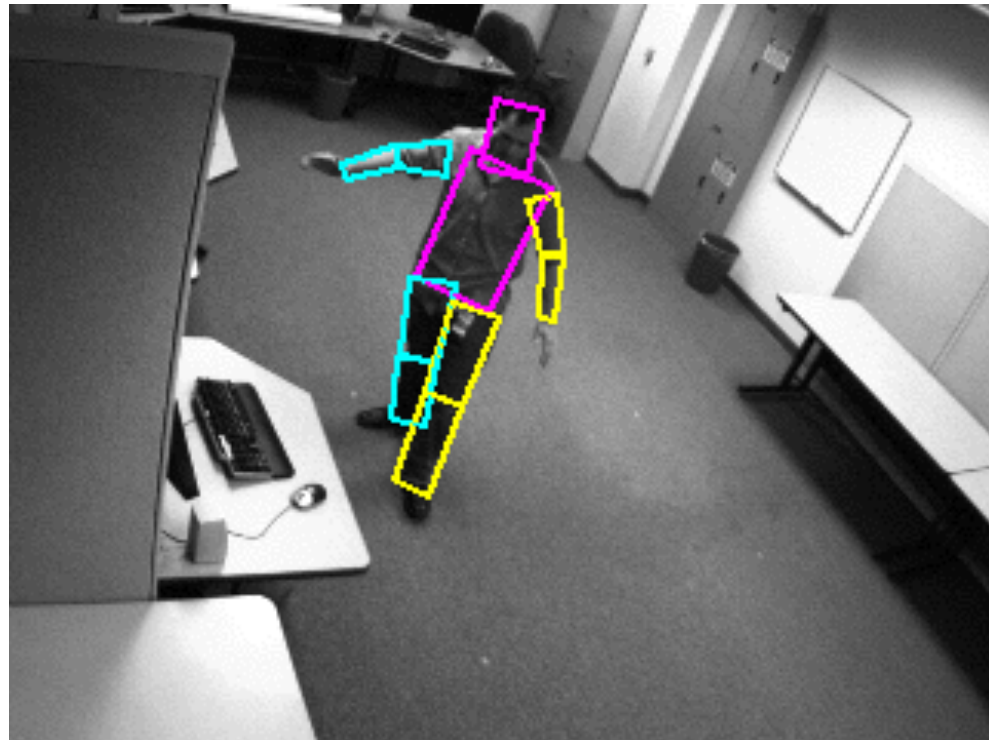
# Topaz: Bodytrack Example

- Computation: find the pose that best fits the person in each camera frame
- Target Routine: compute the weight for all poses, given an image
  - Task: compute the weight of one pose, given image
  - Integration: write computed weight to global weight array



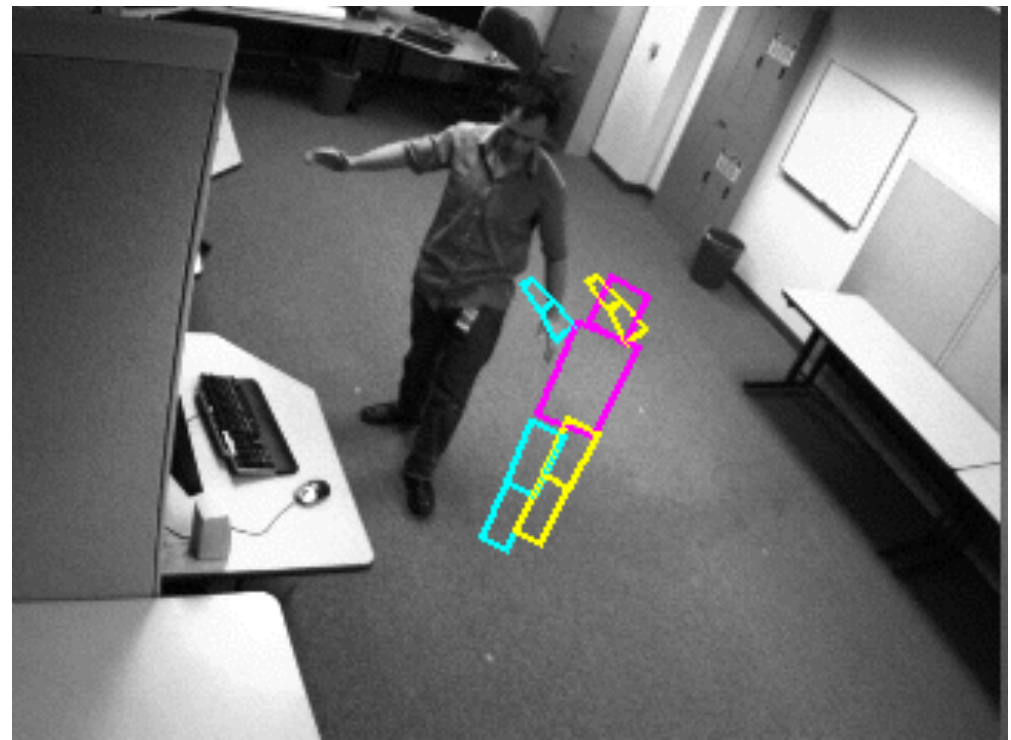


# Topaz: Bodytrack Example



**Pose is good fit**

weight = 1.03



**Pose is bad fit**

weight = 0.12

# Topaz: Bodytrack Example

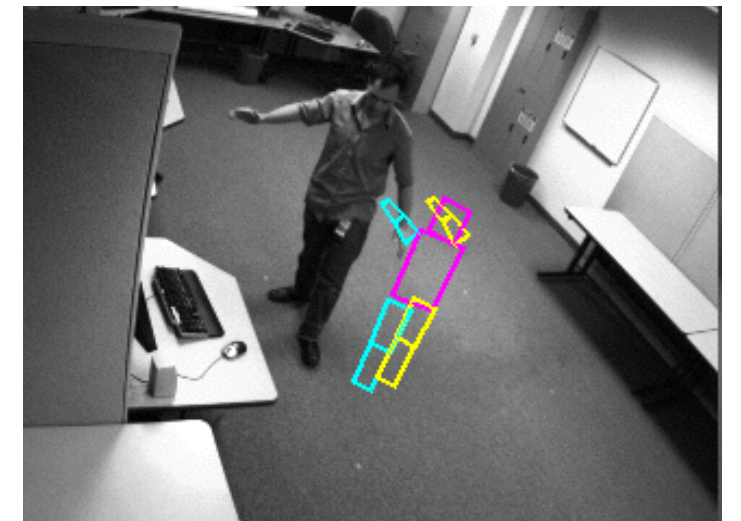
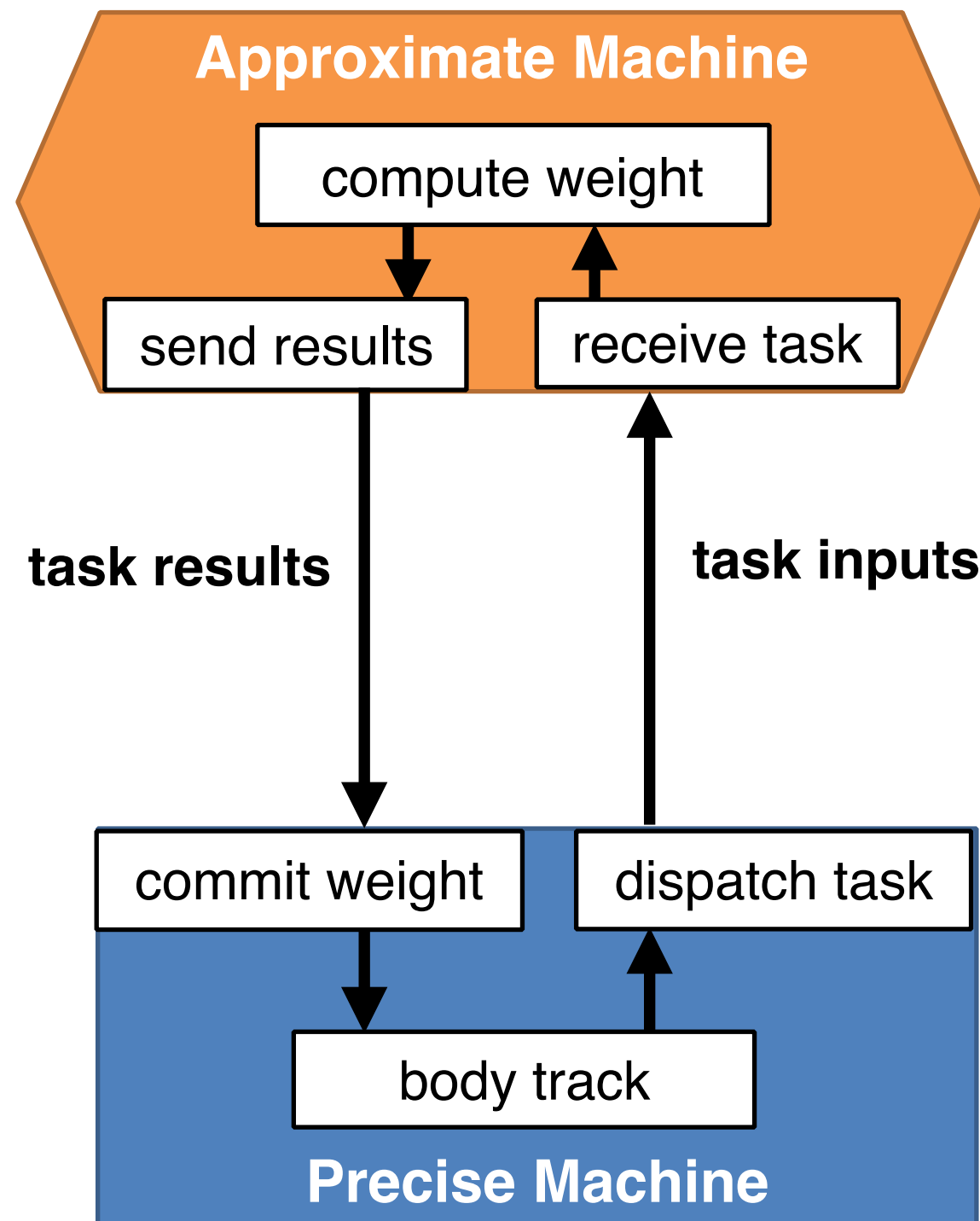
```
// computes the weights for each valid pose.
taskset calcweights(i=0; i<particles.size(); i+=1){

    compute in (
        float tpart[P_SIZE] = (float*) particles[i],
        float tmodel[M_SIZE] = (float*) mdl_prim,
        char timg[I_SIZE] = (char *) img_prim,
        int nCams = mModel->NCameras(),
        int nBits = mModel->getBytesPerPixel(),
        int width = mModel->getWidth(),
        int height =mModel->getHeight()
    ) out (float tweight) {

        tweight = CalcWeight(tpart,
                             tmodel, timg, nCams, width, height, nBits);

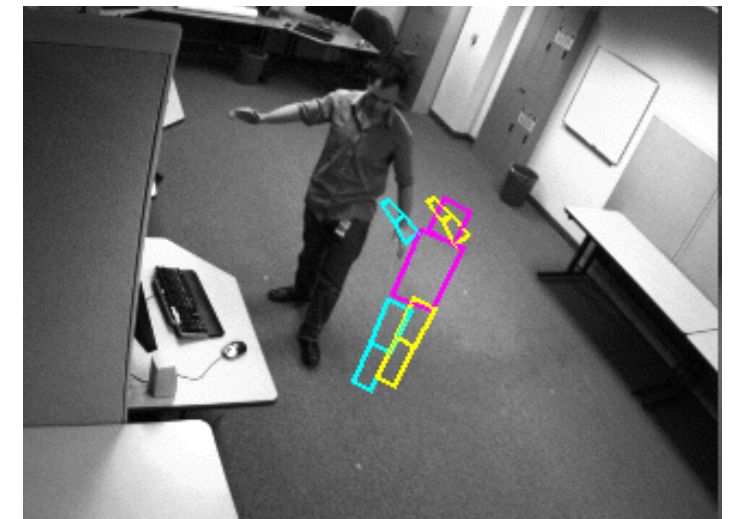
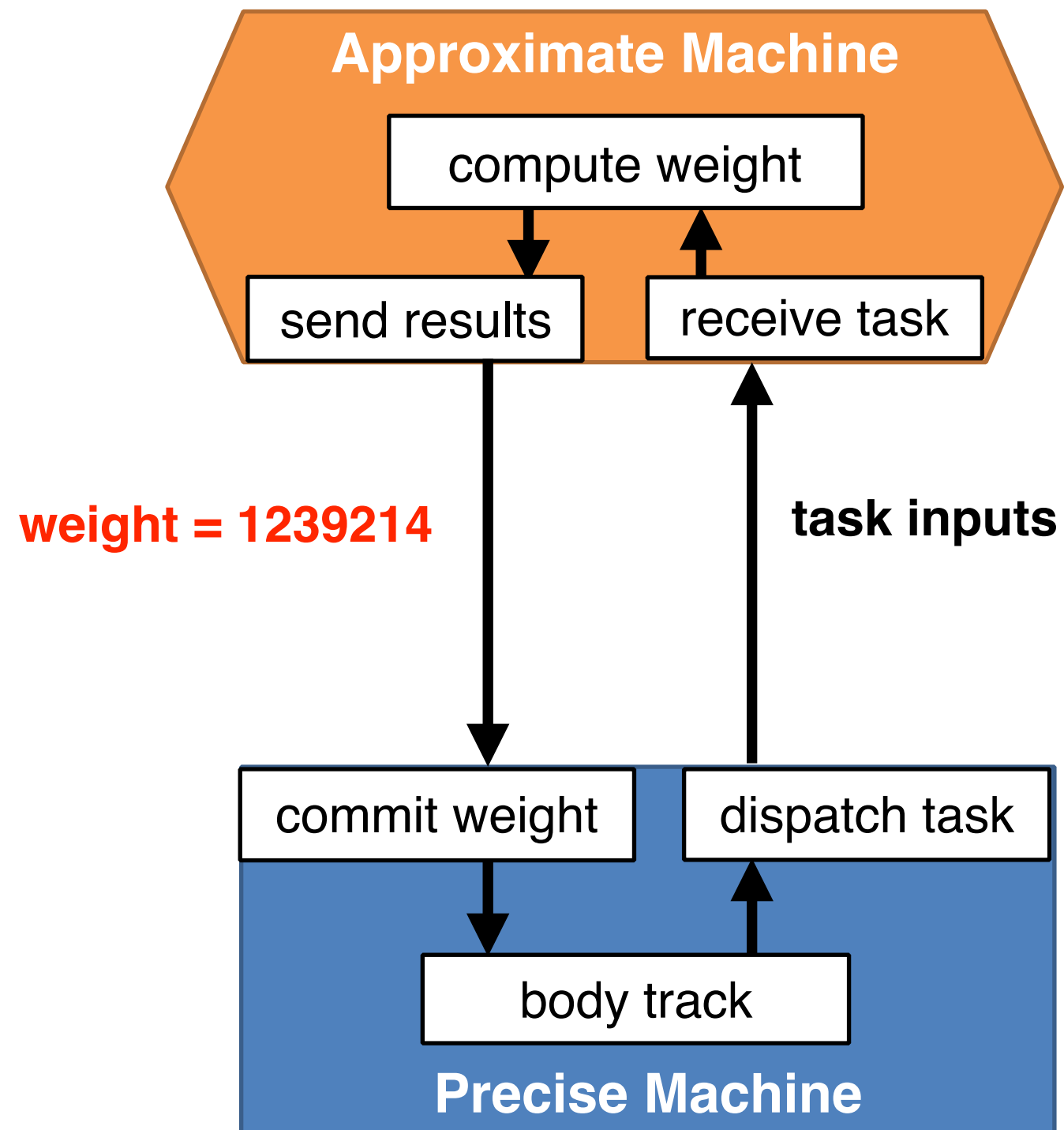
    } combine {
        mWeights[i] = tweight;
    }
}
```

# Topaz: Bodytrack Example



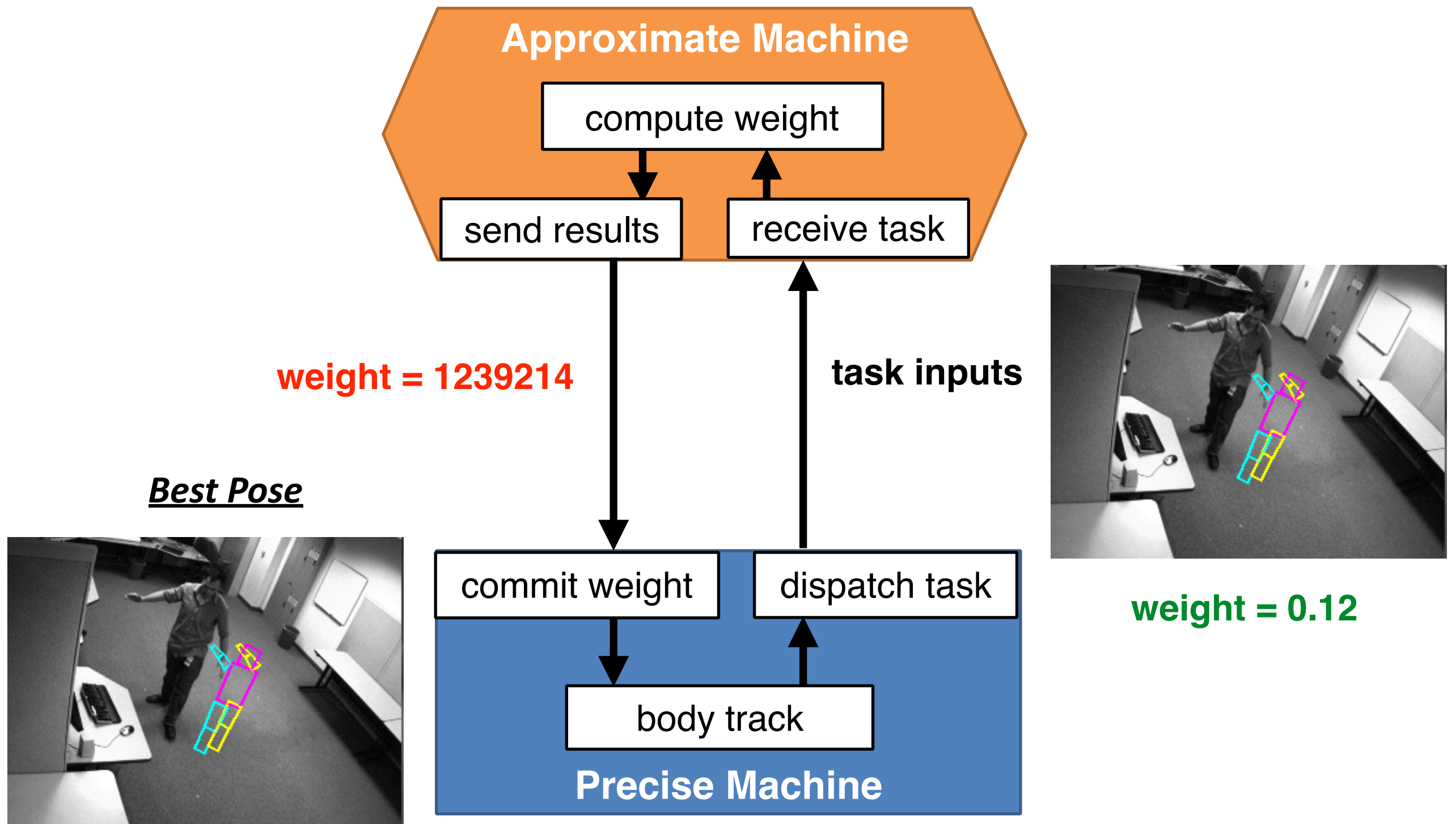
**weight = 0.12**

# Topaz: Bodytrack Example



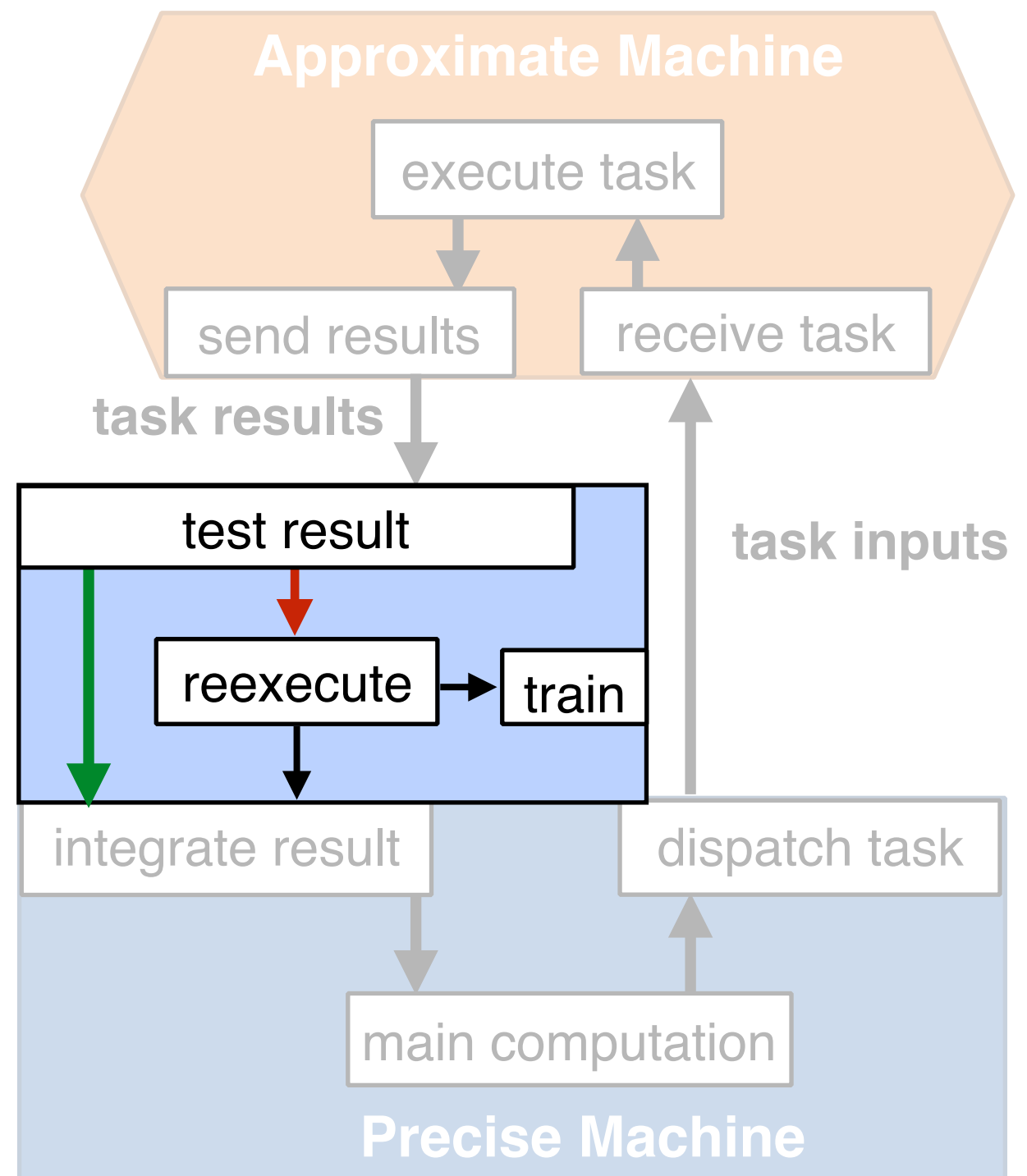
**weight = 0.12**

# Topaz: Bodytrack Example

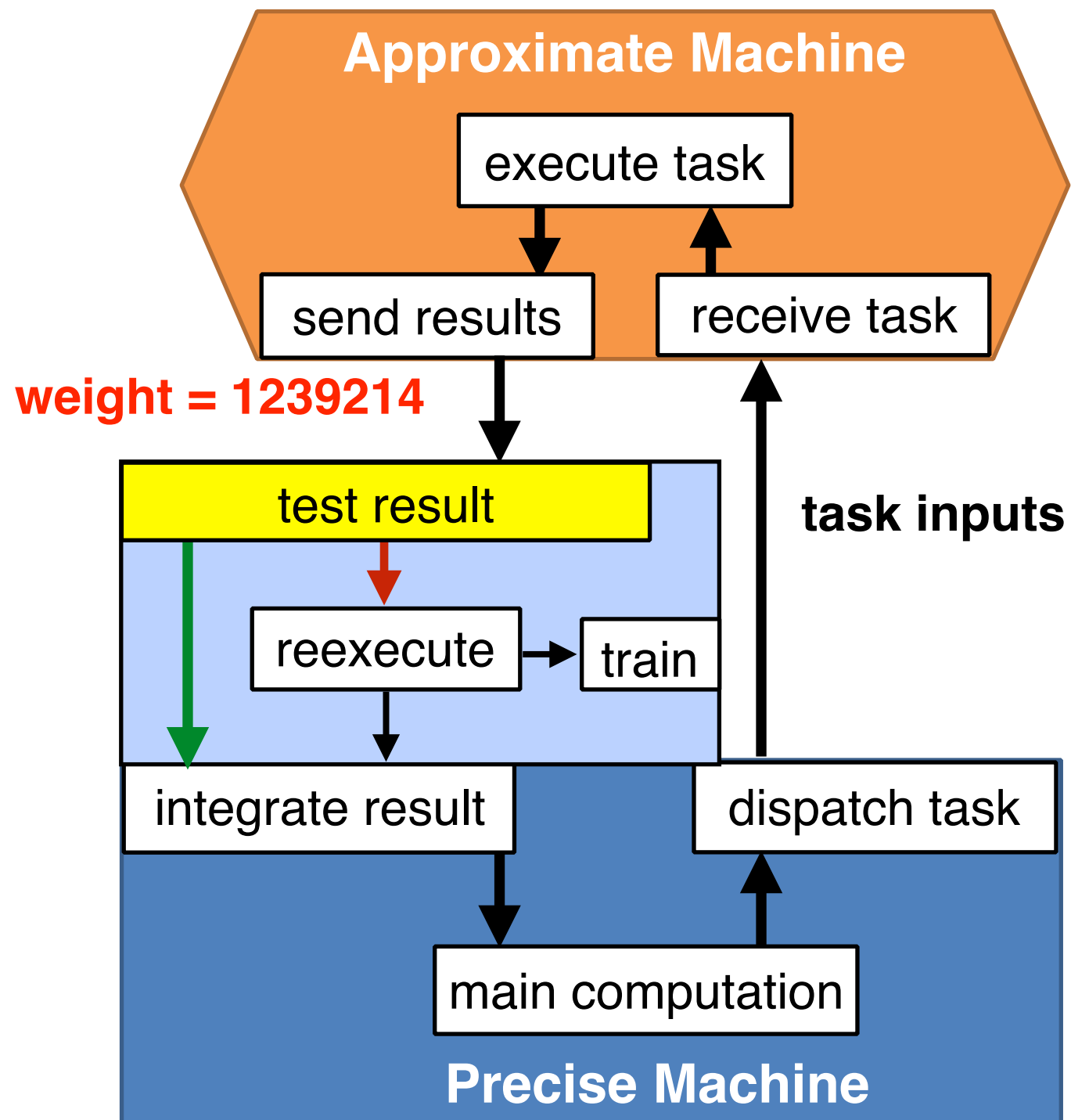


# Basic Outlier Detection: Overview

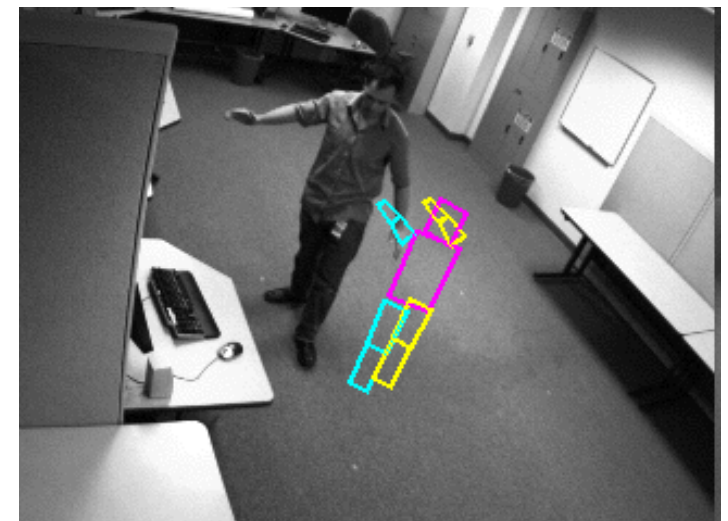
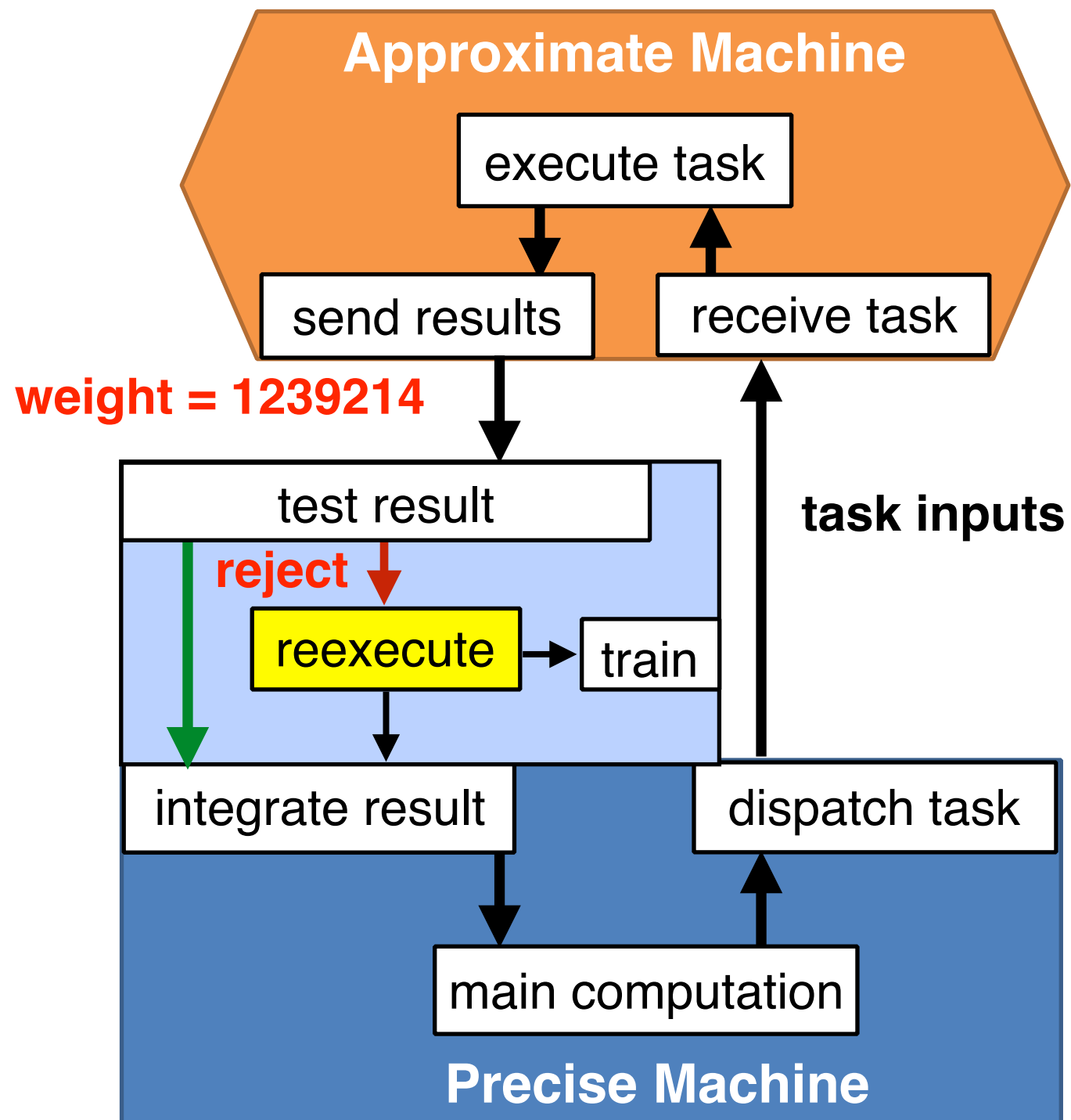
- **Outlier Detection**
- **Precise** machine performs outlier detection on result tuple
- On **accept**:
  - **Integrate** task result
- On **reject**:
  - **Reexecute** task on reliable hardware
  - **Train** outlier detector
  - **Integrate** task result



# Topaz: Bodytrack Example

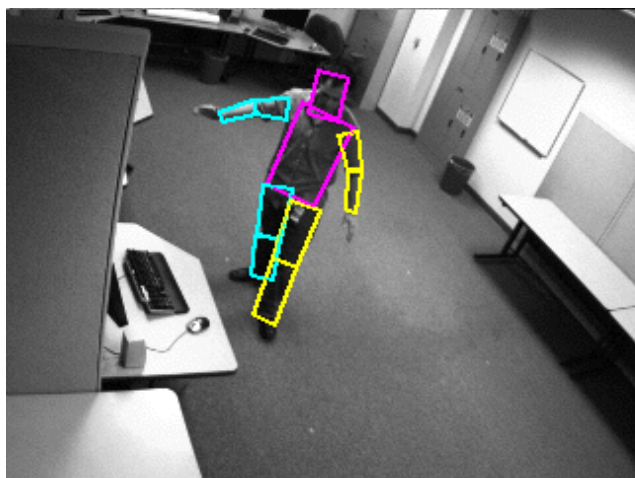
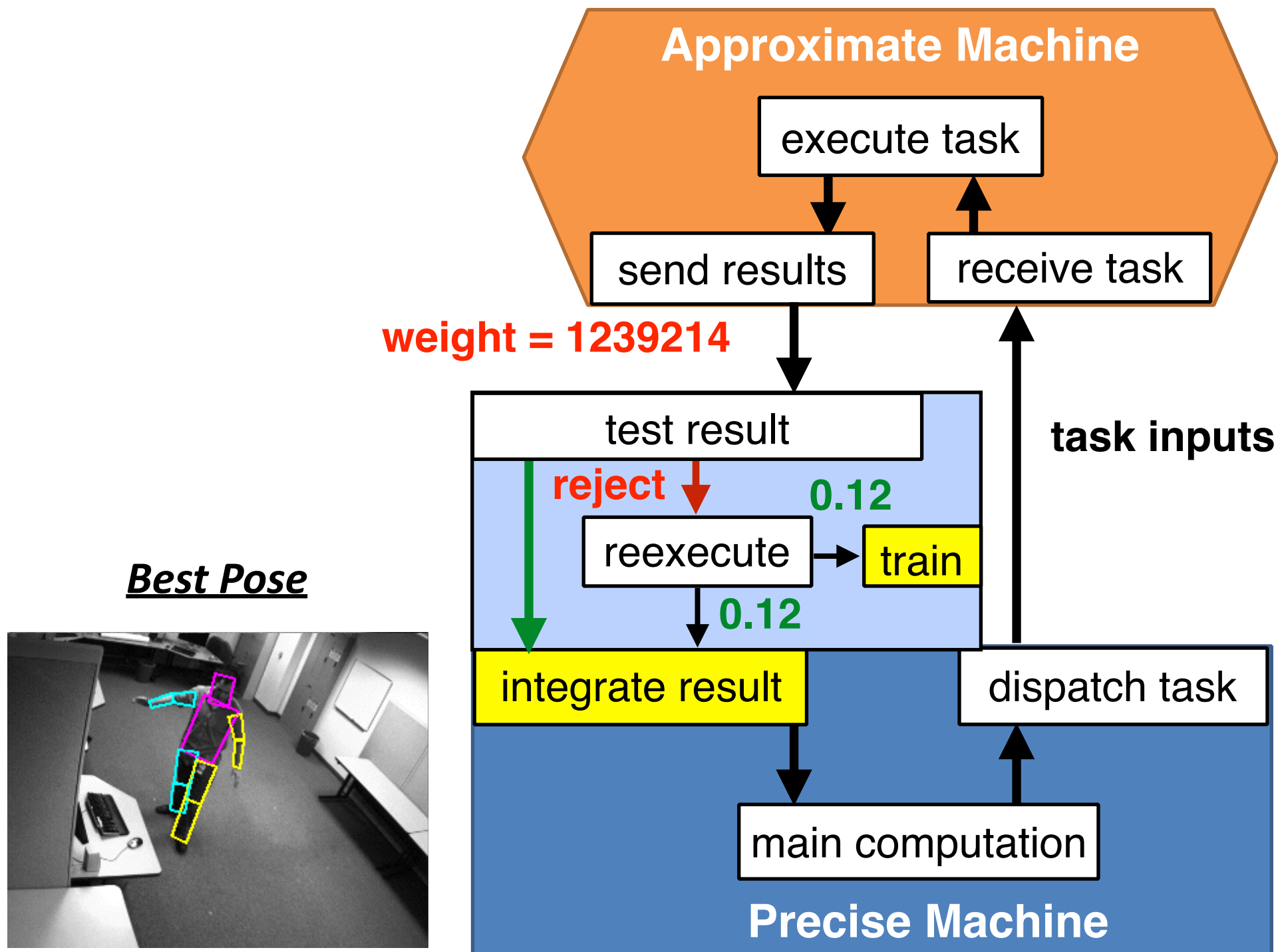


# Topaz: Bodytrack Example





# Topaz: Bodytrack Example



# Testing results using outlier detection

- **Algorithm**

Given result tuple of n elements

In acceptance region:

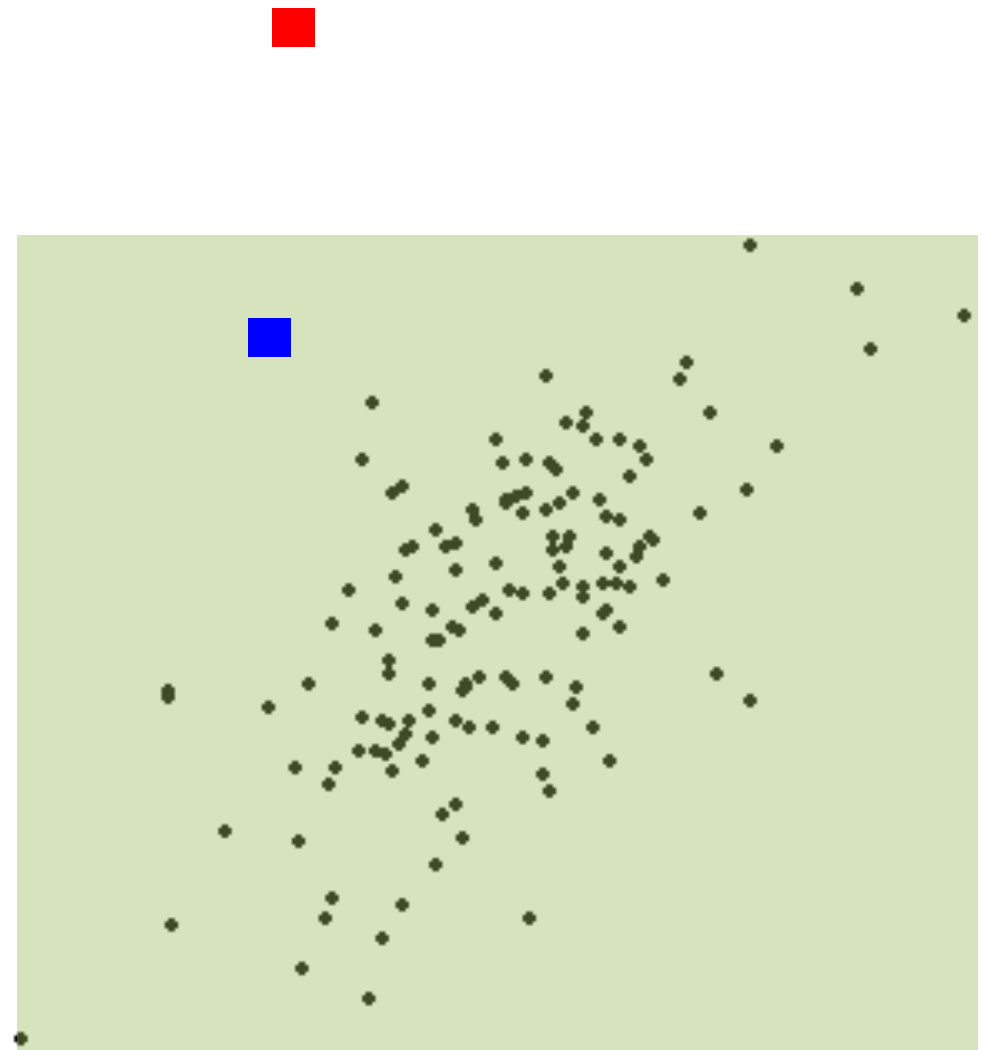
**Accept**

Otherwise:

**Reject**

- **Model: Acceptance Region**

- N-dim hyper rectangle
- Result tuple distribution
- Dim: min, max of an element



# Training the outlier detector

- **The Training Process**

- Online learning
- “Learn from failure”
- Use reexecuted task results

- **Algorithm**

**If test(x) = reject:**

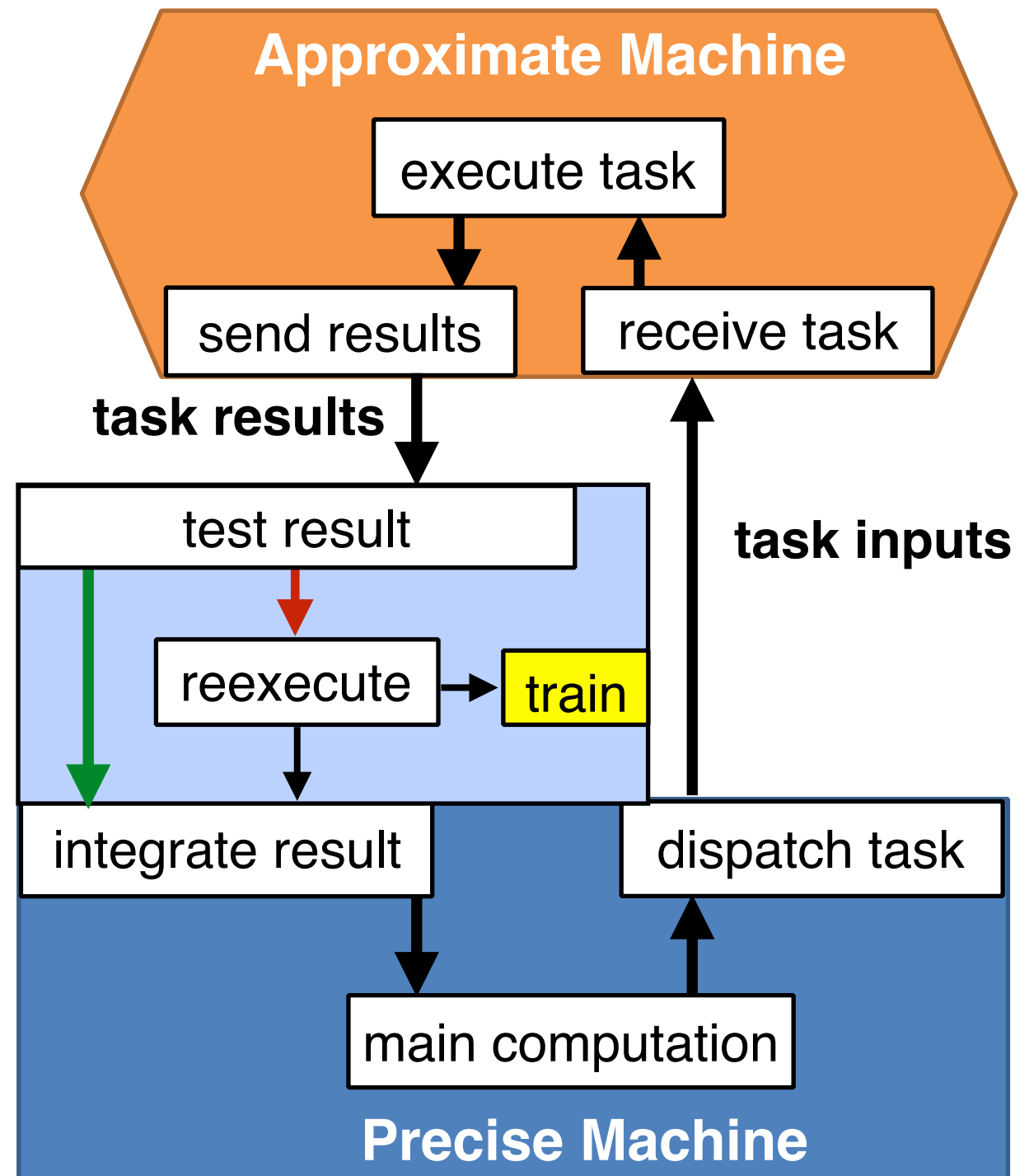
Reexecute task to obtain x'

Update r to include x'

Integrate x'

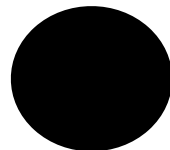
**Otherwise:**

Integrate x



# Outlier detector: an example

**Receive Task Result for Task 1**



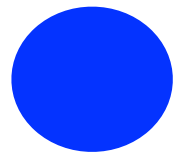
# Outlier detector: an example

**Reject task result for task 1**



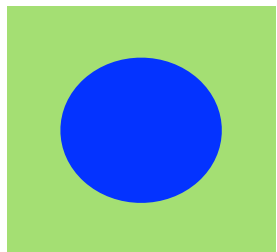
# Outlier detector: an example

Reexecute task result for task 1



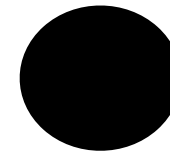
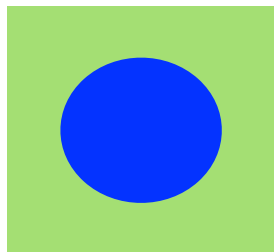
# Outlier detector: an example

**Training: Expand acceptance region to include result for task 1**



# Outlier detector: an example

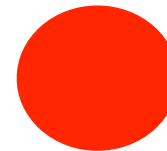
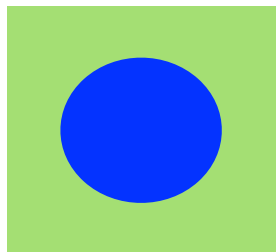
Receive task result for task 2





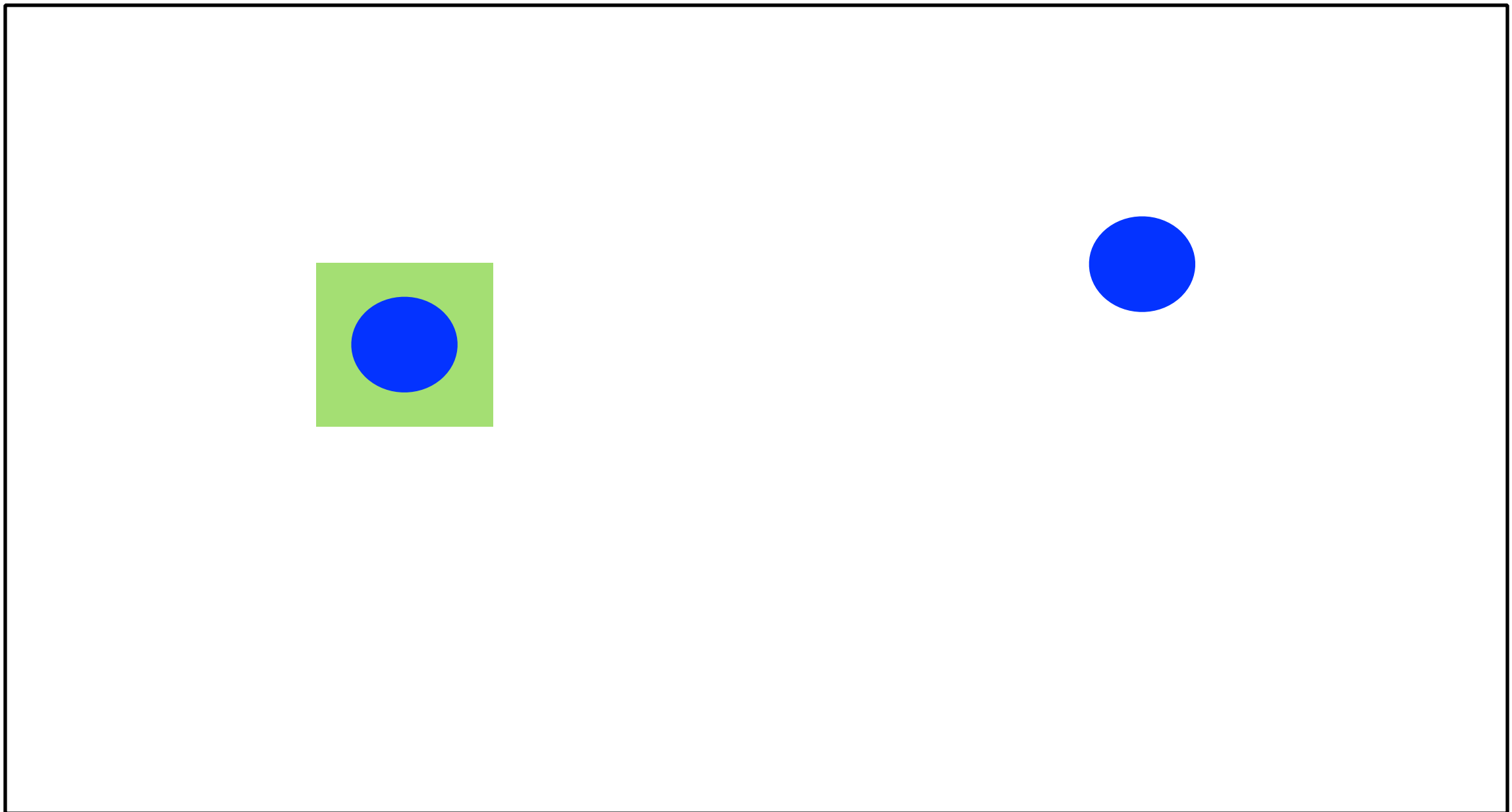
# Outlier detector: an example

**Reject task result for task 2**



# Outlier detector: an example

Reexecute task result for task 2



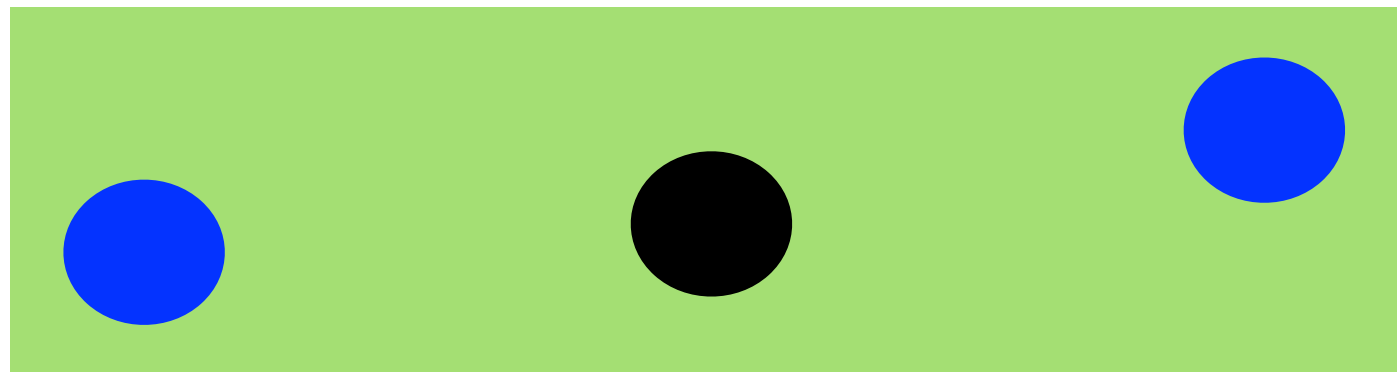
# Outlier detector: an example

**Training: expand acceptance region to include result for task 2**



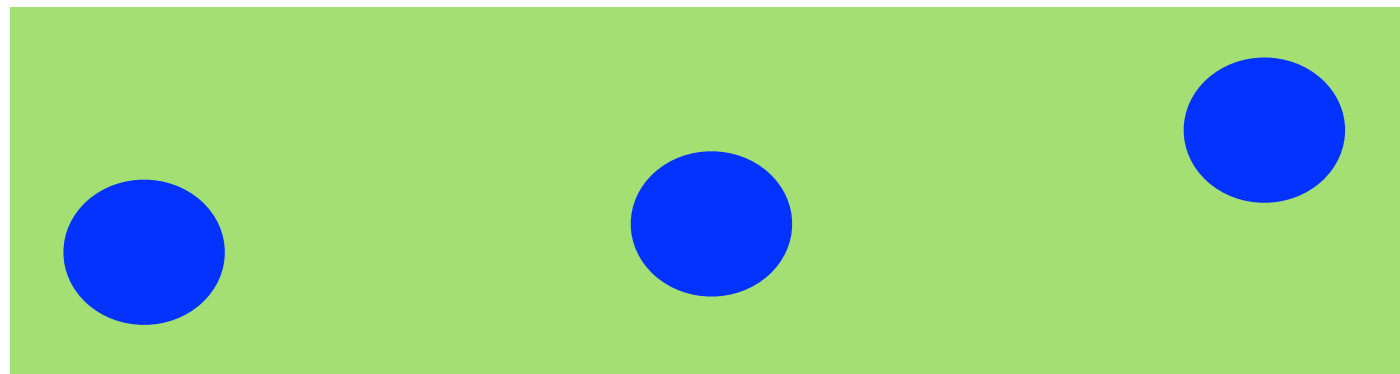
# Outlier detector: an example

Receive task result for task 3



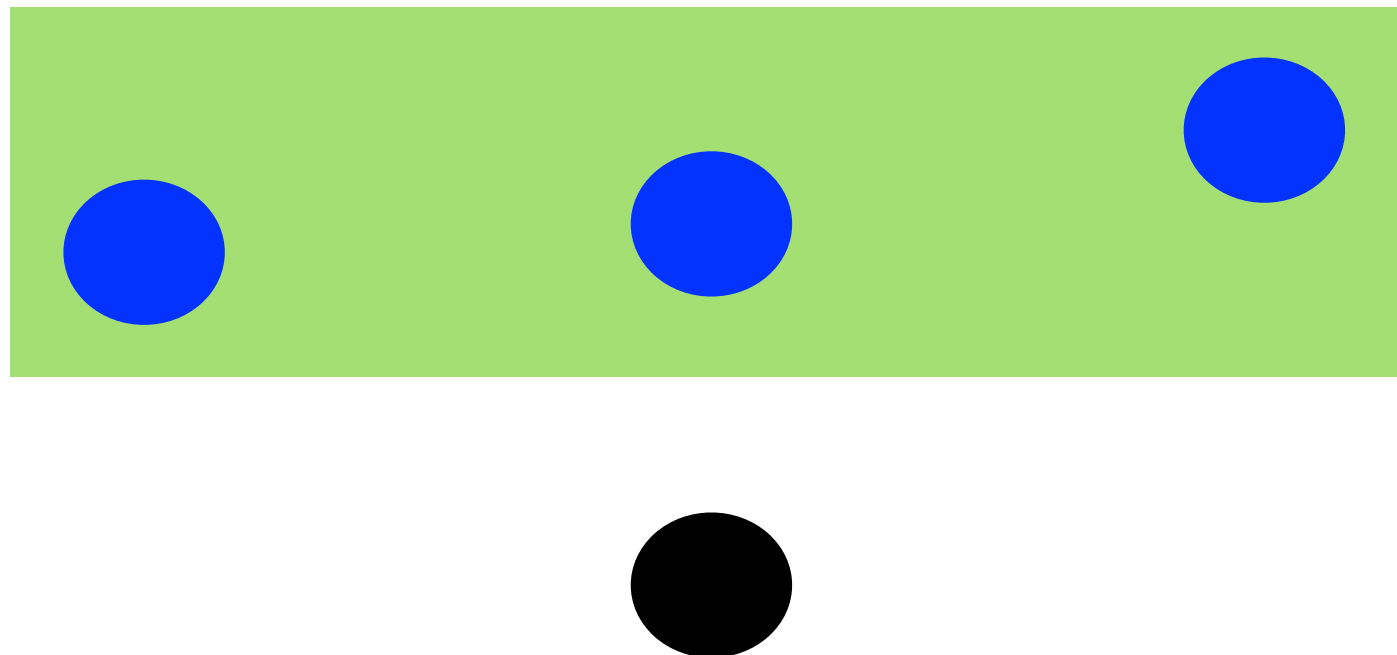
# Outlier detector: an example

Accept task result for task 3



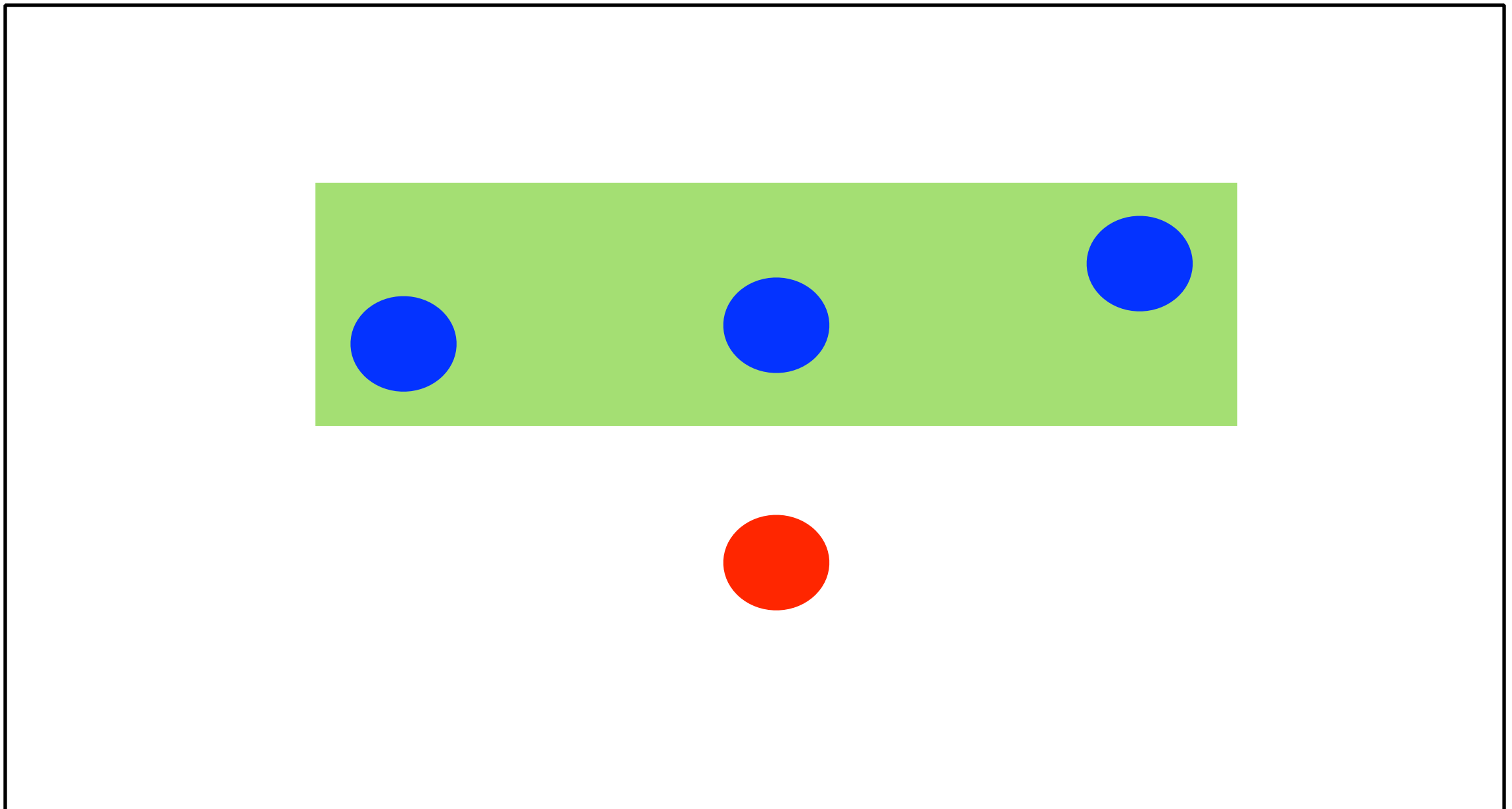
# Outlier detector: an example

Receive task result for task 4



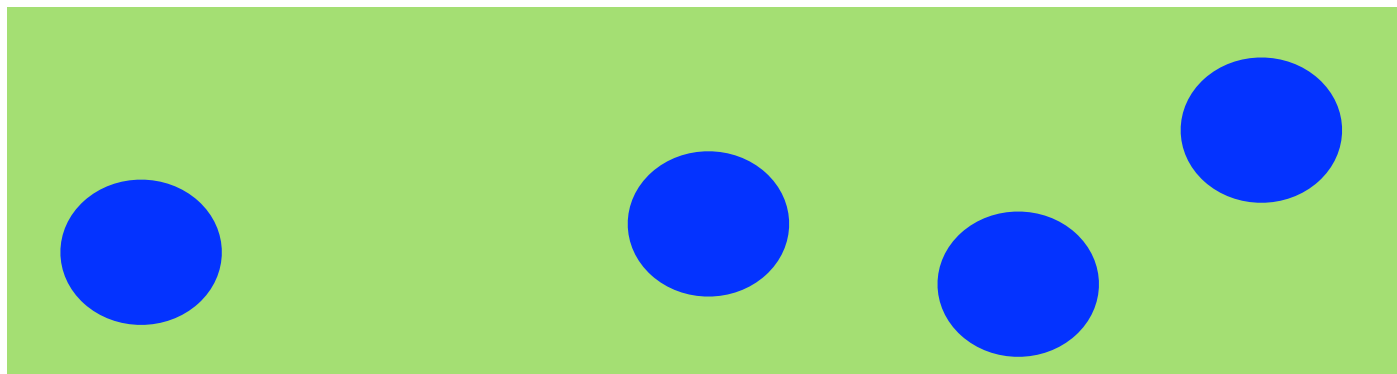
# Outlier detector: an example

**Reject task result for task 4**



# Outlier detector: an example

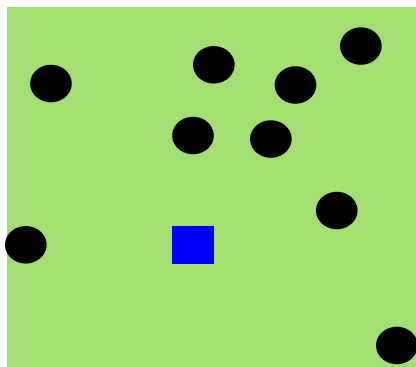
Reexecute task result for task 4



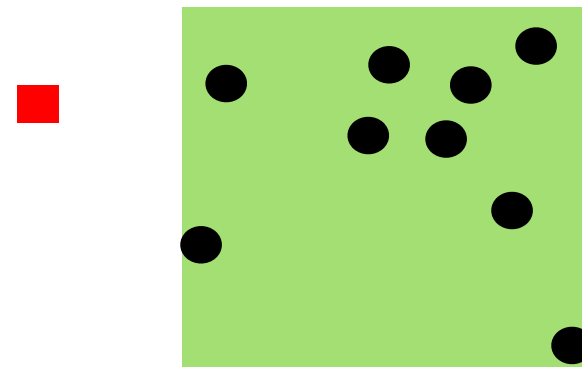


# Outlier detection: possible outcomes

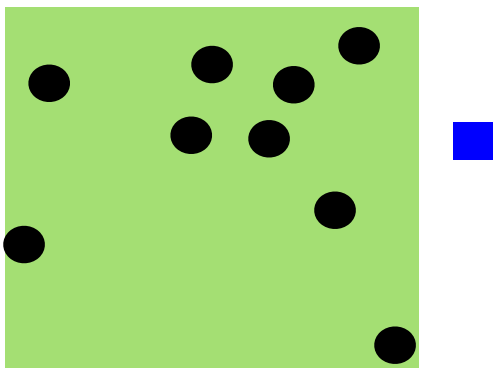
**Correct result accepted**



**Error rejected**

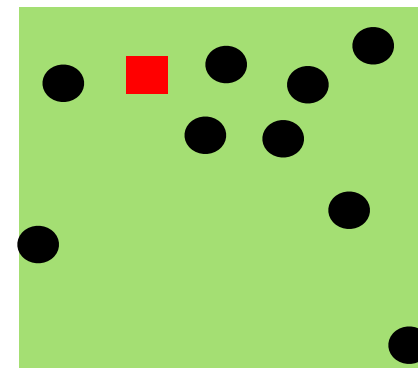


**Correct result rejected**



**Incurs overhead**

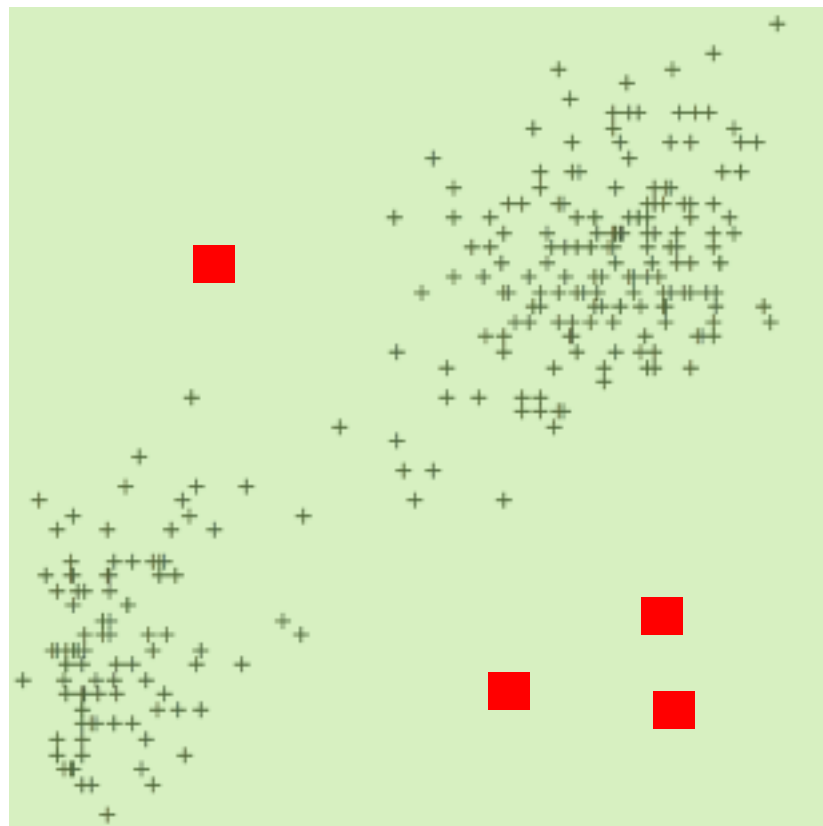
**Error accepted**



**Error integrated into  
main computation**

The basic outlier detector catches obvious task errors outside the result envelope

But, it **cannot** catch errors **between** the modes of a *multimodal result distribution*



# Multi-region outlier detection

- **Multiple regions**
  - **N**: maximum number
  - **R**: set of hyperrectangle regions.

- **Algorithm**

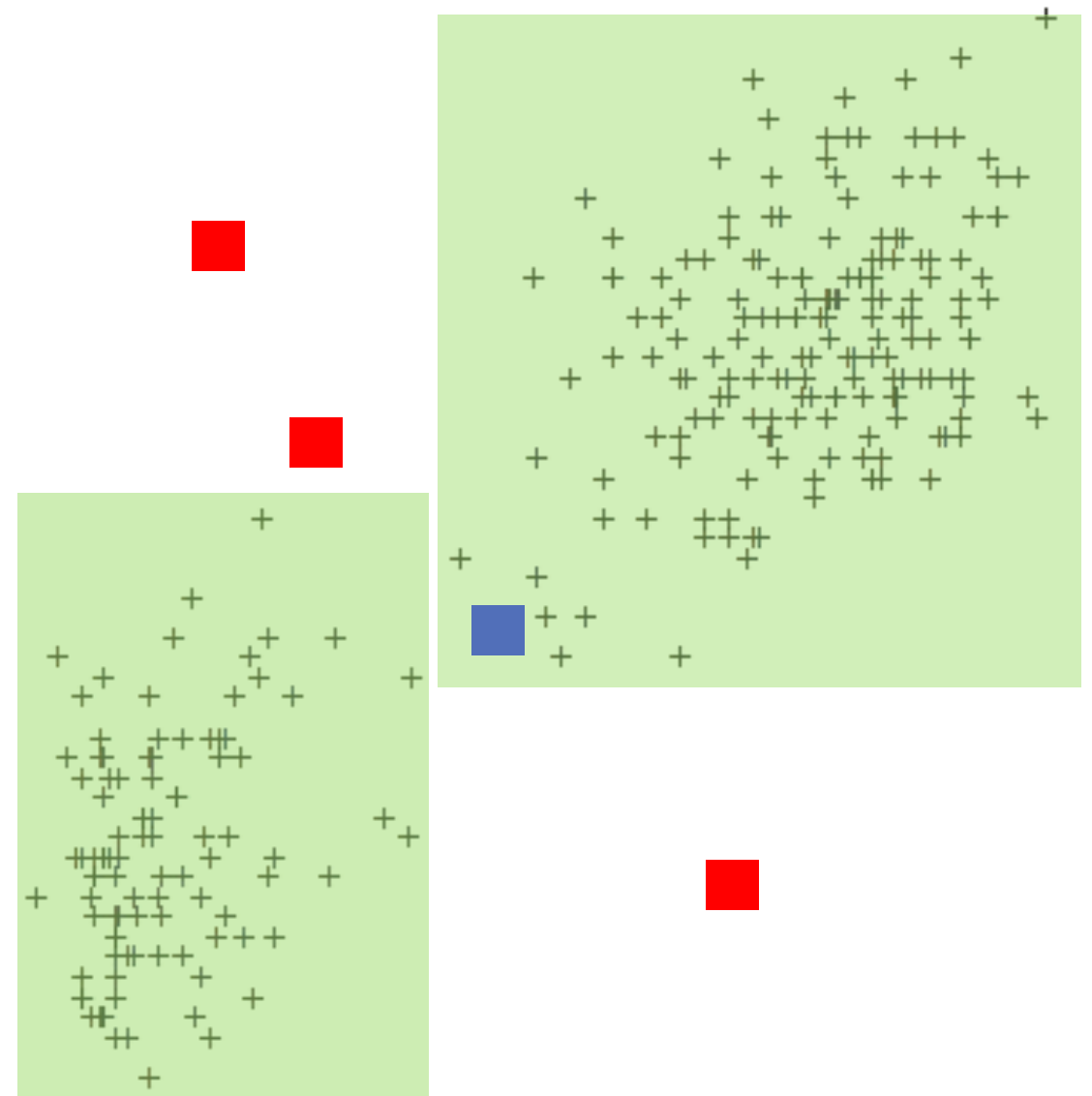
Given result tuple  $x$ :

*If exists  $r$  in  $R$  s.t.  $x$  in  $r$ :*

**Accept**

*Otherwise*

**Reject**



# Training the outlier detector

**Algorithm:** given rejected,  
reexecuted result  $x'$

**If** dne  $r$  in  $R$  s.t.  $x'$  in  $r$ :

Create region  $r'$  where  $x'$  in  $r'$

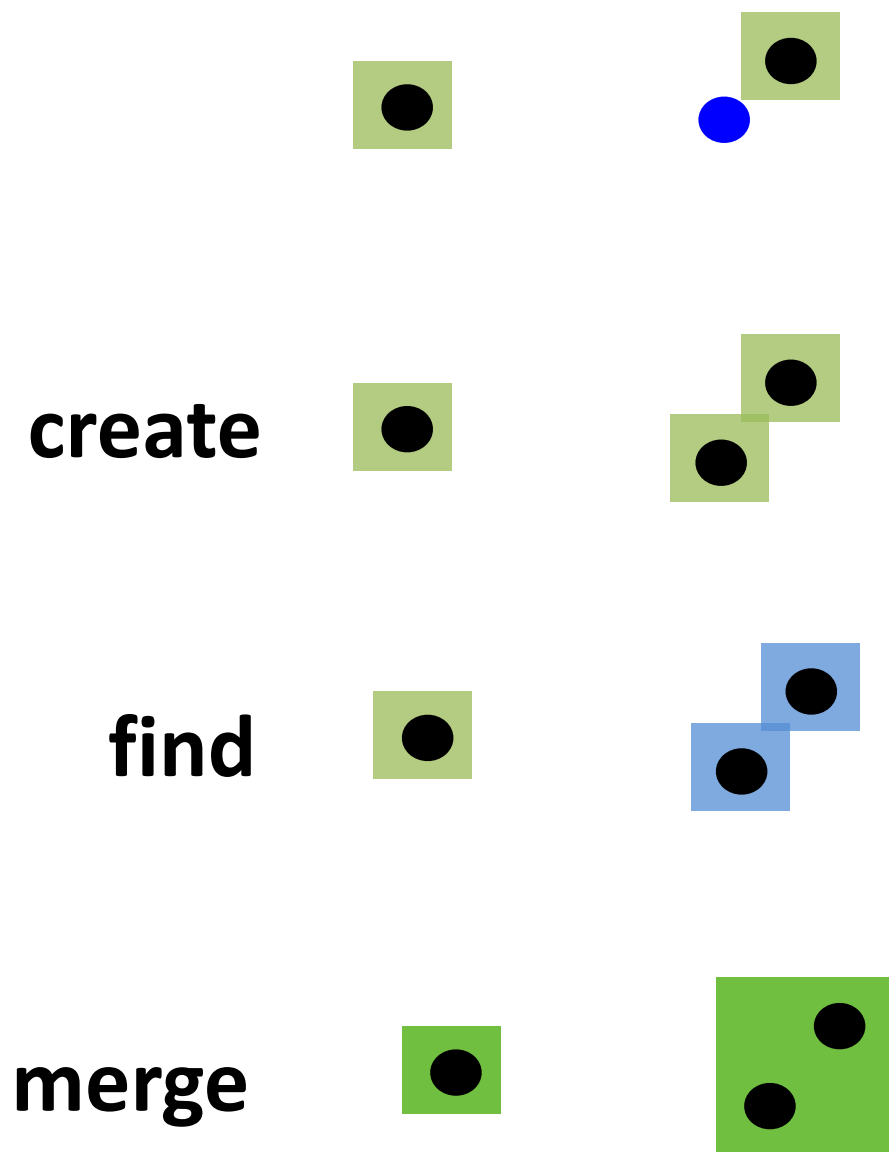
Add  $r'$  to  $R$

**If**  $IRI > N$

Find two close regions  $r_1, r_2$

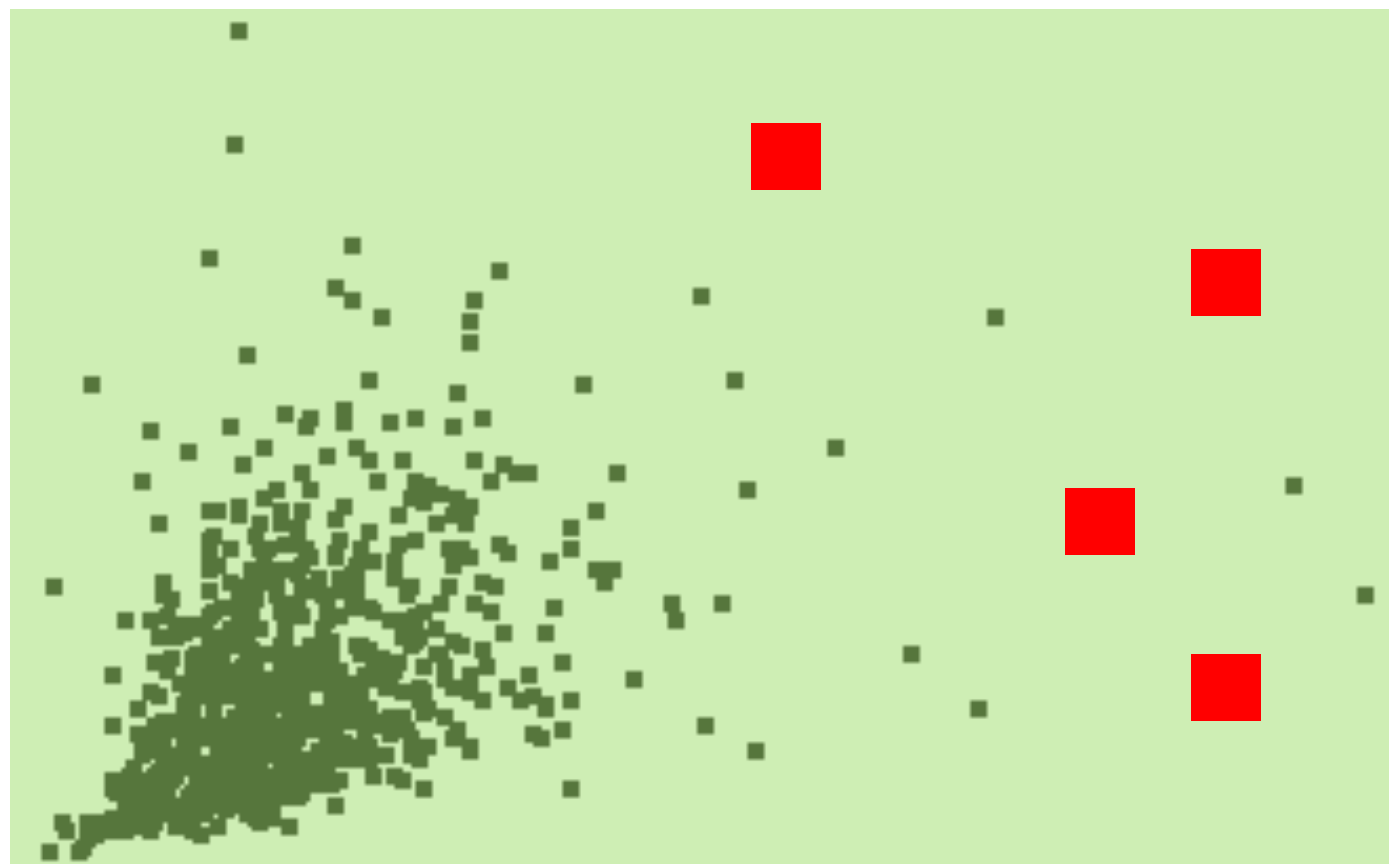
Merge  $r_1$  and  $r_2$

**Training on Reexecuted Result**  
( $N=2$ )



The multi-region outlier detector catches errors  
multimodal distributions

But, it **does not** catch errors in distributions that are  
**sparse** or **dynamic**



# Adaptive Outlier Detection

- **Adaptive Outlier Detector**

- **V<sub>t</sub>**: target rejection rate
- **V<sub>a</sub>**: actual rejection rate
- **C**: control system
- **COM(r)**: center of mass of region  $r$
- **Unlearn if we can reject more tasks**

- **Contract Algorithm**

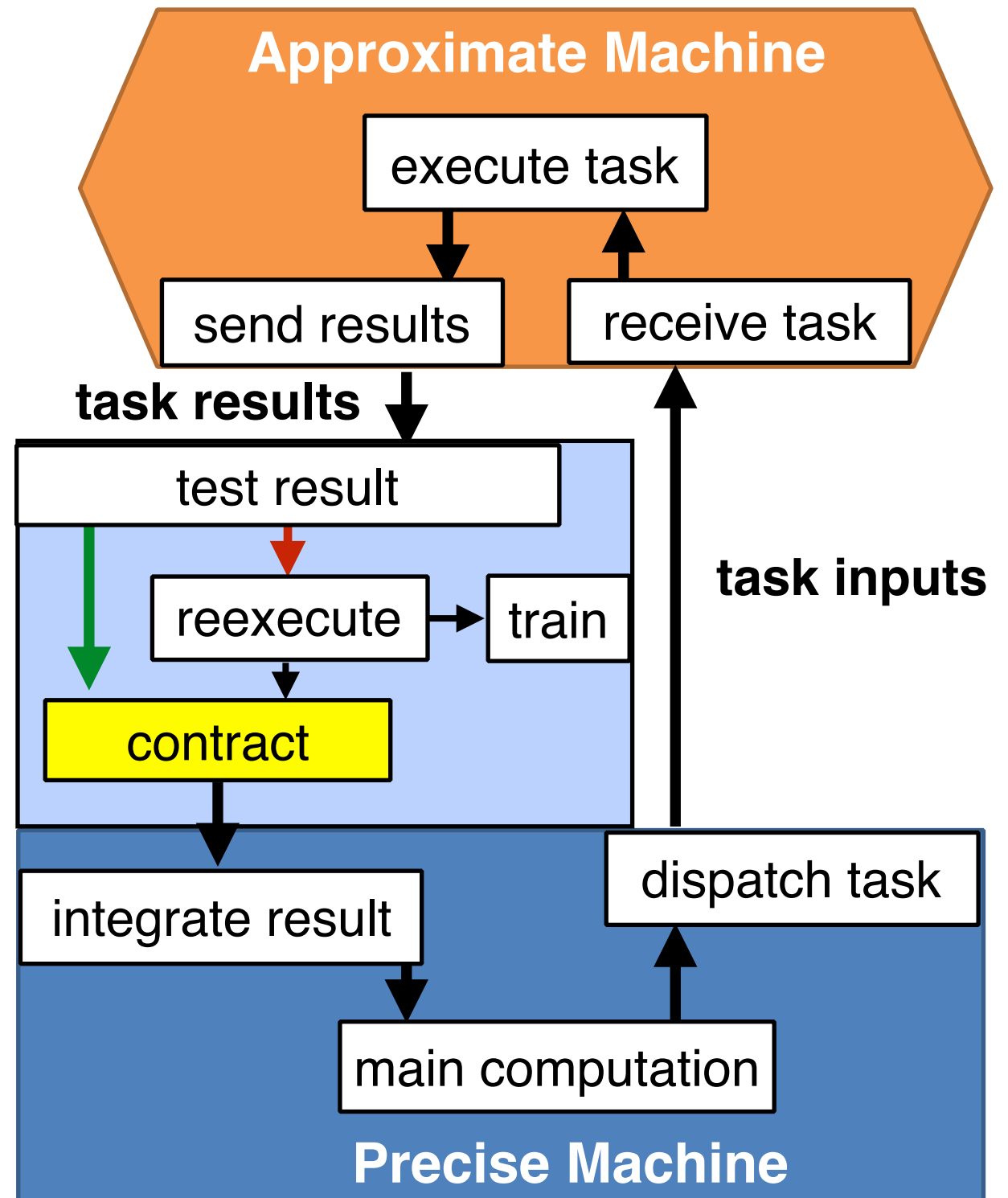
Update  $COM(r)$  where  $x$  in  $r$ ,  $r$  in  $R$

Update  $V_a$ ,  $C$

If  $V_a < V_t$ :

get factor  $f$  from  $C$

Contract all  $r$  in  $R$  by  $f$



# Adaptive Outlier Detection: *PID Control System*

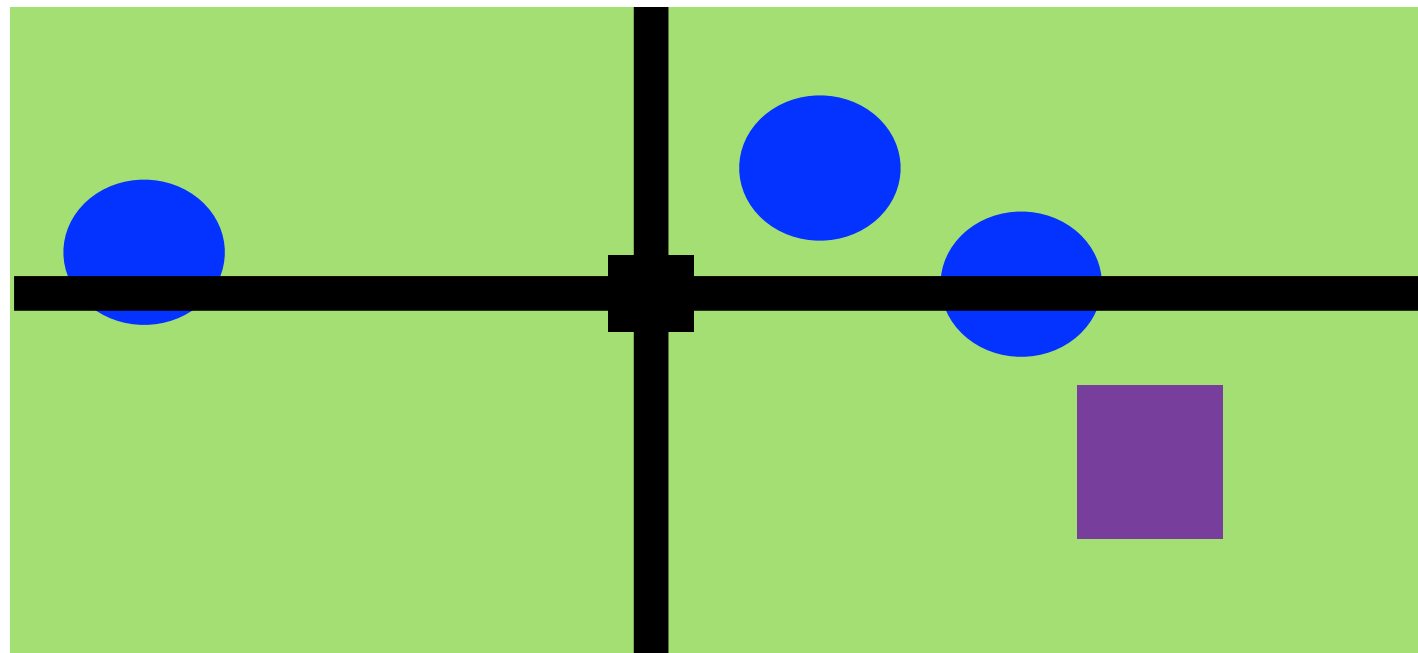
$$K_t \cdot e + K_d \cdot e' + K_i \cdot \int e$$

- ***Proportional-integral-derivative (PID)*** control
- **Error “value” (e)**: difference between measured (m), desired (d) value
  - measured value: **Va**, actual reexecution rate
  - desired value: **Vt**, target reexecution rate

# Adaptive outlier detector: an example

Pass result from **rejected** and **reexecuted** task into contraction routine

$$C(Va, Vt) = 0.23$$



**Va**: 3% tasks rejected

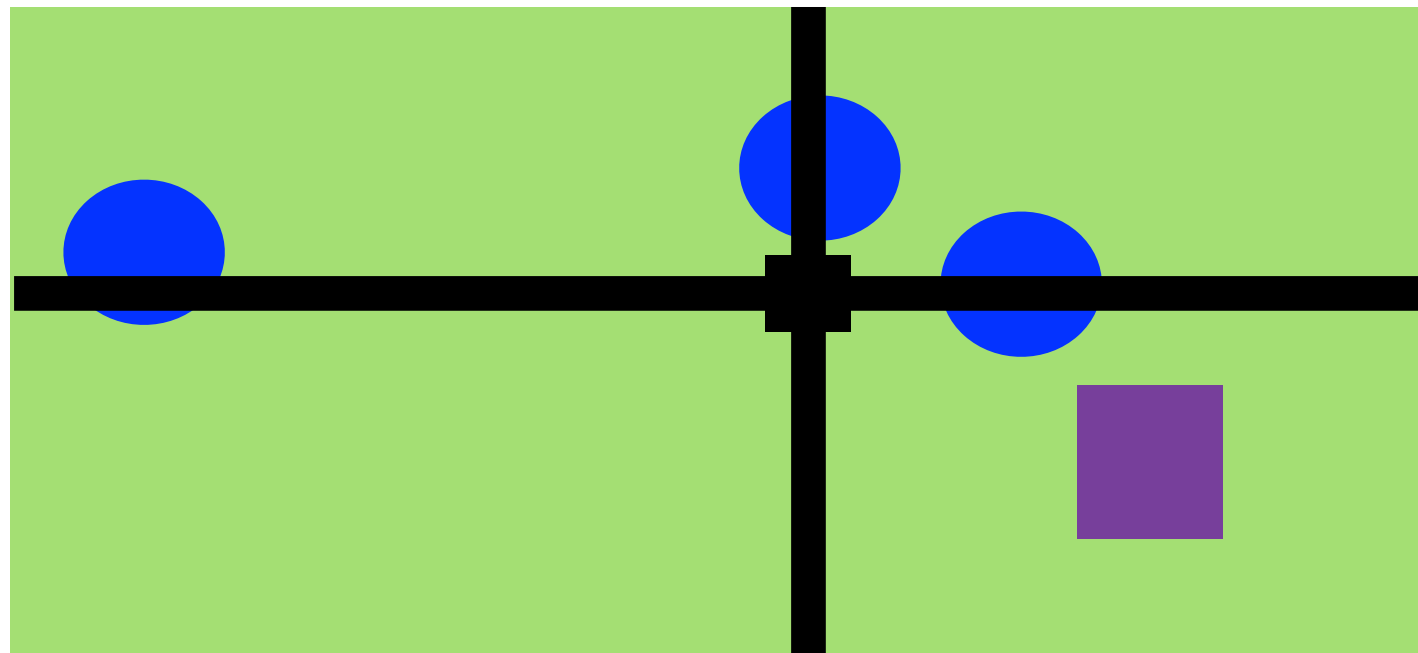
**Vt**: 7% tasks rejected



# Adaptive outlier detector: an example

**Update center of mass of region**

$$\mathbf{C}(\mathbf{V}_a, \mathbf{V}_t) = 0.23$$



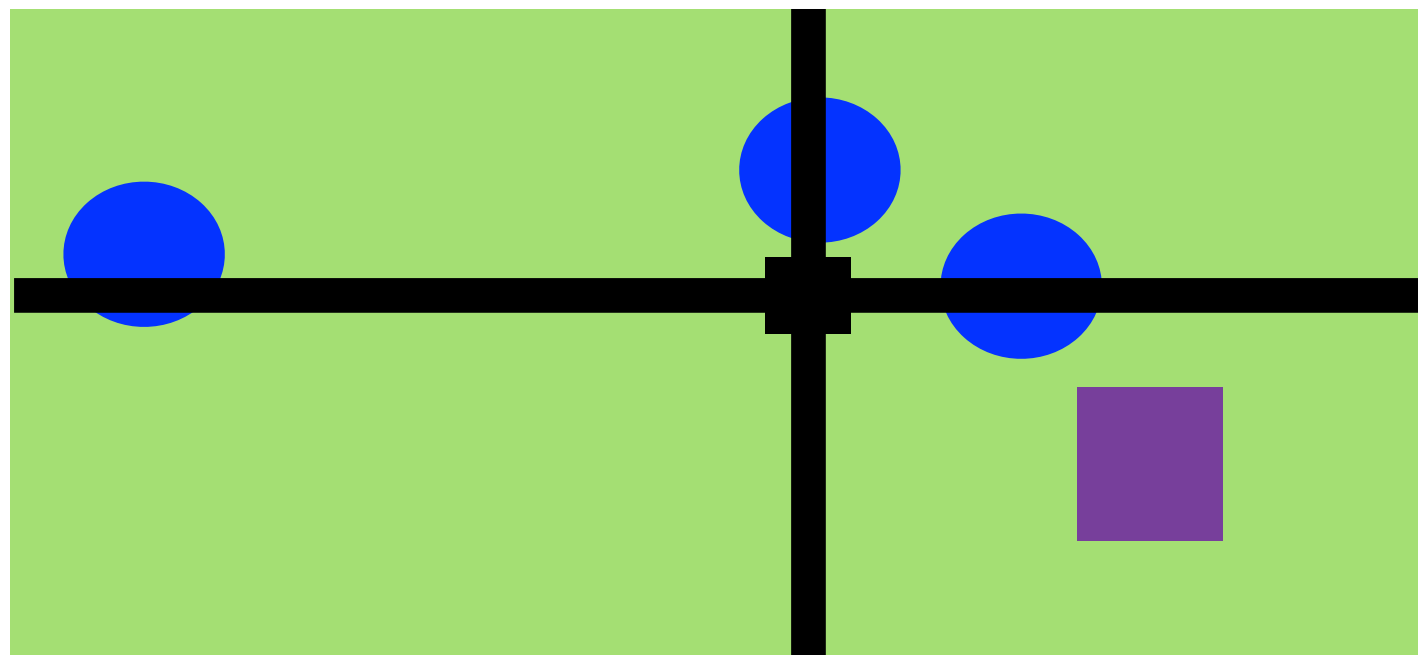
**V<sub>a</sub>**: 3% tasks rejected

**V<sub>t</sub>**: 7% tasks rejected

# Adaptive outlier detector: an example

Update  $V_a$  to reflect **rejected** task. update  $C$ .

$$C(V_a, V_t) = 0.19$$



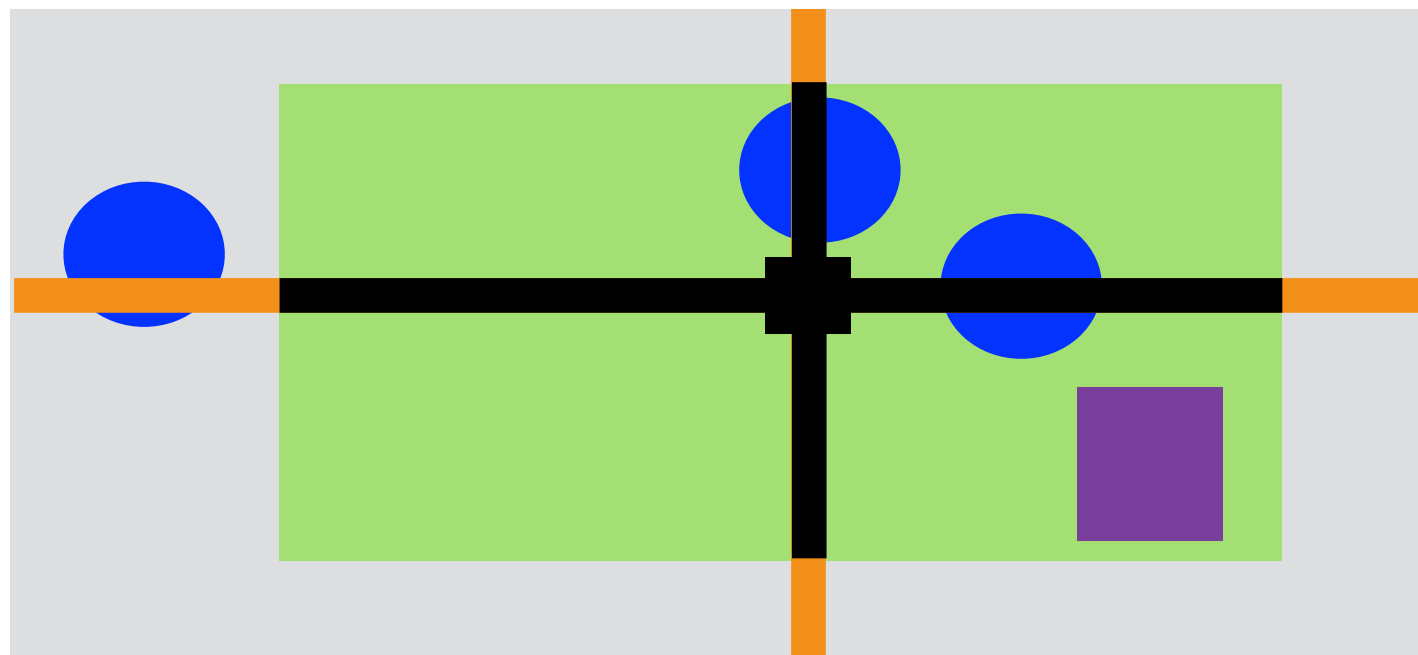
**$V_a$** : 3.8% tasks rejected

**$V_t$** : 7% tasks rejected

# Adaptive outlier detector: an example

**$V_a < V_t$ : shrink the region by 19% about center of mass**

$$C(V_a, V_t) = 0.19$$



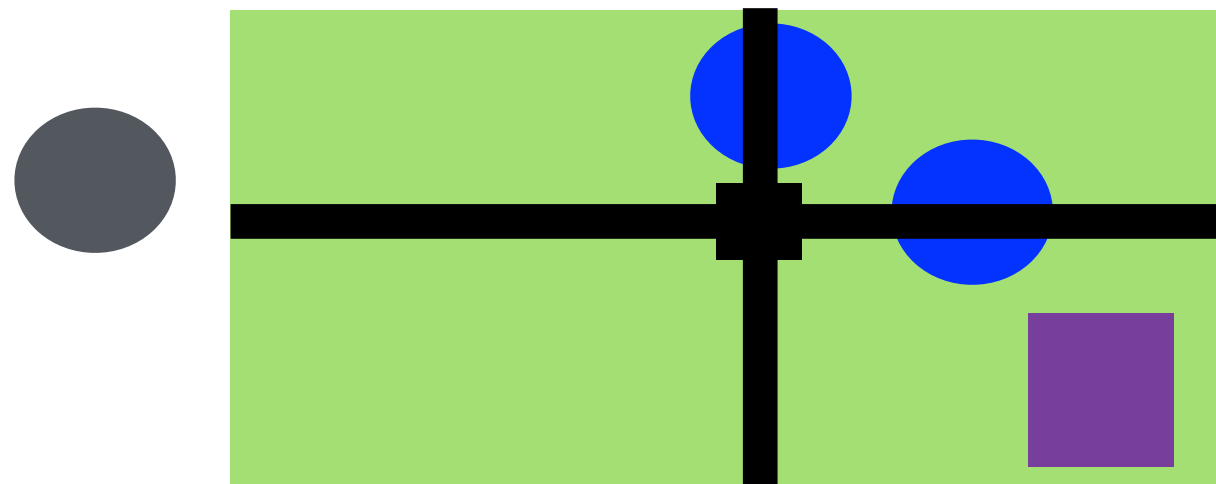
**$V_a$ :** 3.8% tasks rejected

**$V_t$ :** 7% tasks rejected

# Adaptive outlier detector: an example

**Region successfully contracted**

$$C(Va, Vt) = 0.19$$



**Va:** 3.8% tasks rejected

**Vt:** 7% tasks rejected

Topaz **mitigates crashes**  
*Language & computational model*

Topaz **corrects unacceptable** task results  
*Outlier detection*

# Optimization 1: Stable Data

- **Stable Data**
  - data that is unchanged for all tasks in taskset
- **Optimization**: selectively send stable data
- **Reduce overhead if task contains large unchanging inputs**

```
// computes the weights for each valid pose.
taskset calcweights(i=0; i<particles.size(); i+=1){
    compute in (
        float tpart[P_SIZE] = (float*) particles[i],
        const float tmodel[M_SIZE] = (float*) mdl_prim,
        const char timg[I_SIZE] = (char *) img_prim,
        const int nCams = mModel->NCameras(),
        const int nBits = mModel->getBytesPerPixel(),
        const int width = mModel->getWidth(),
        const int height =mModel->getHeight()
    ) out (float tweight) {
        tweight = CalcWeight(tpart,
                             tmodel, timg, nCams, width, height, nBits);
    }
}
```

const stable data annotation for  
inputs

# Optimization 2: Abstract Output Vector (AOV)

- **Abstract output vector (AOV)**

- Programmer defined result tuple abstraction.

- **Optimization**: perform detection on smaller AOV.

- **Aside**: handle input dependence using AOV

- **Reduces outlier detector overhead if AOV smaller than result tuple**

```
taskset  name(int i = l; i < u; i++) {  
    compute in  (d1 x1 = e1, ..., dn xn = en)  
        out (o1 y1, ..., oj yj) {  
            <task body>  
        }  
    transform out (v1, ..., vk) {  
        <output abstraction>  
    }  
    combine { <combine body> }  
}
```

transform block with AOV outputs  
v1..vk.

<output abstraction> defines the  
transformation

Does Topaz perform well in **practice**?



# Experimental Setup

- **Hardware Model**

- No-refresh DRAM  
*Protections removed*
  - Bit, time dependent errors
- Dual-voltage L1, L2 caches  
*Aggressive conditions*
  - Per-read / per-write errors
- **Benefit:** saves energy

- **Benchmarks**

- Barnes: planet simulation
- Bodytrack: machine vision
- Water: water simulation
- Blackscholes: financial analysis
- Streamcluster: k-means clustering

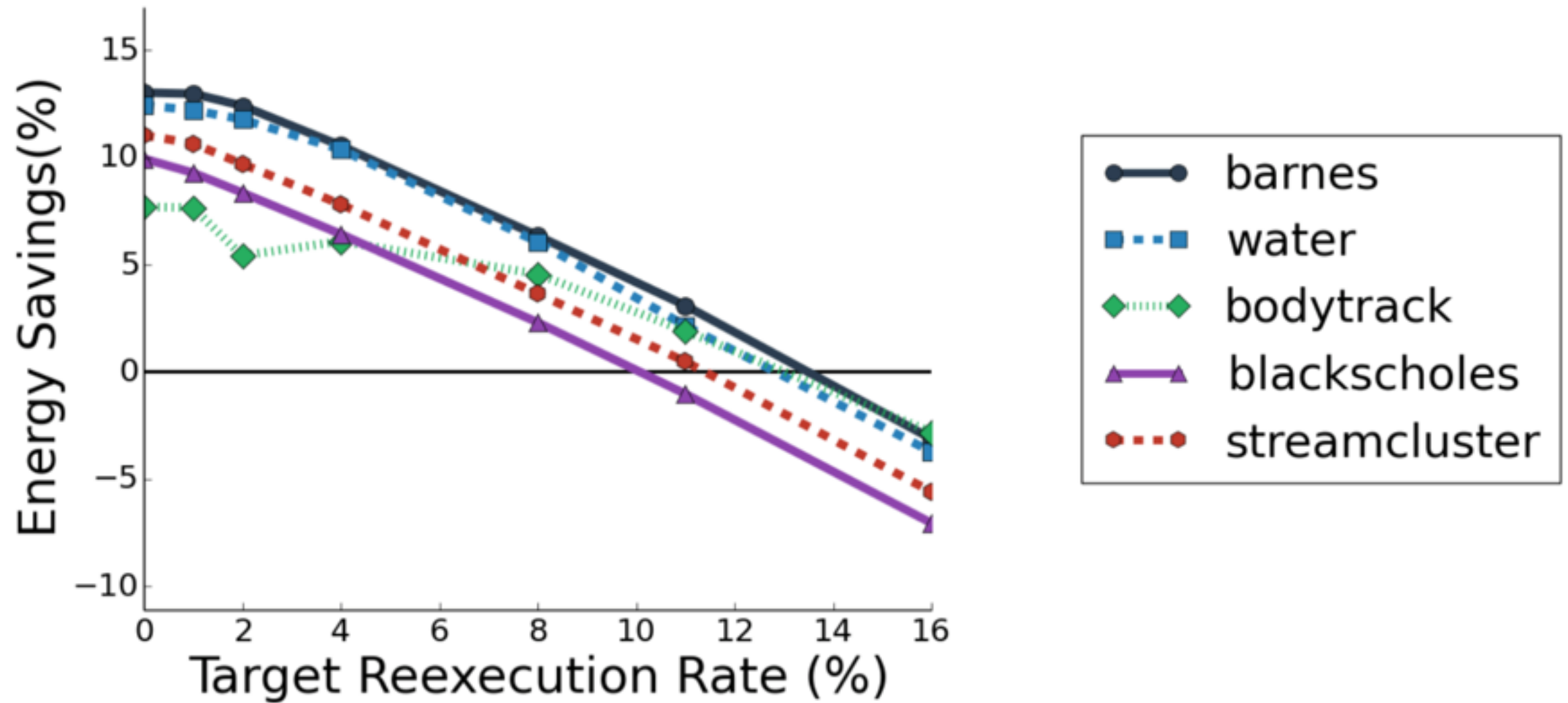
**Question:** What sorts of **energy savings** do we observe with this system

# Energy savings

Benchmarks	Model	Baseline	Detect & Reexecute	Full Topaz
barnes	basic	17.47%	14.77%	13.02%
blackscholes	basic	16.20%	14.62%	9.94%
bodytrack	basic	12.70%	8.60%	7.69%
streamcluster	basic	16.87%	15.62%	11.03%
water	basic	18.41%	15.12%	12.43%
barnes	ddep	17.47%	14.76%	13.02%
blackscholes	ddep	16.02%	14.41%	9.70%
bodytrack	ddep	12.88%	6.51%	5.02%
streamcluster	ddep	16.89%	15.58%	11.03%
water	ddep	18.41%	15.37%	12.82%

Table 4: Energy Savings, basic and ddep Hardware Models

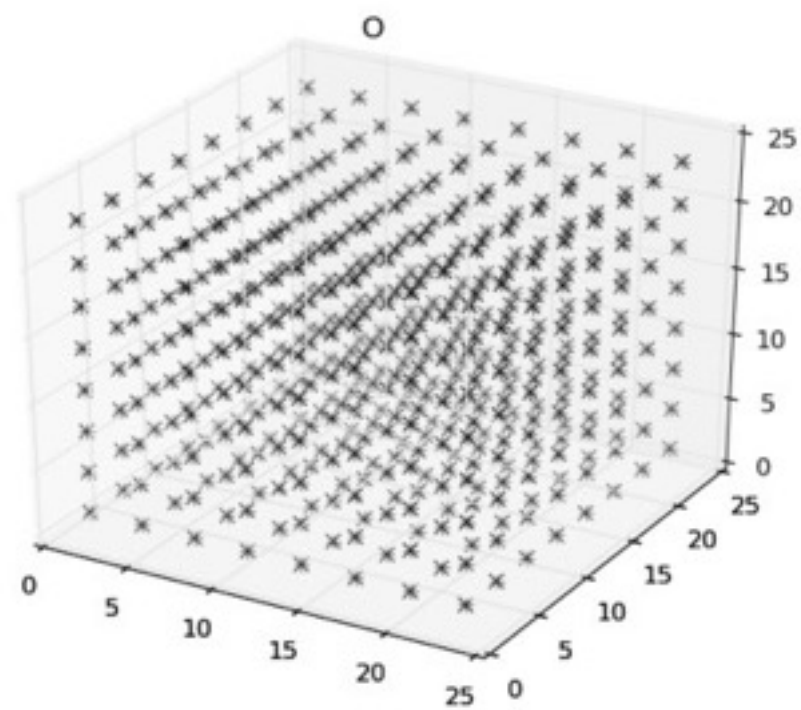
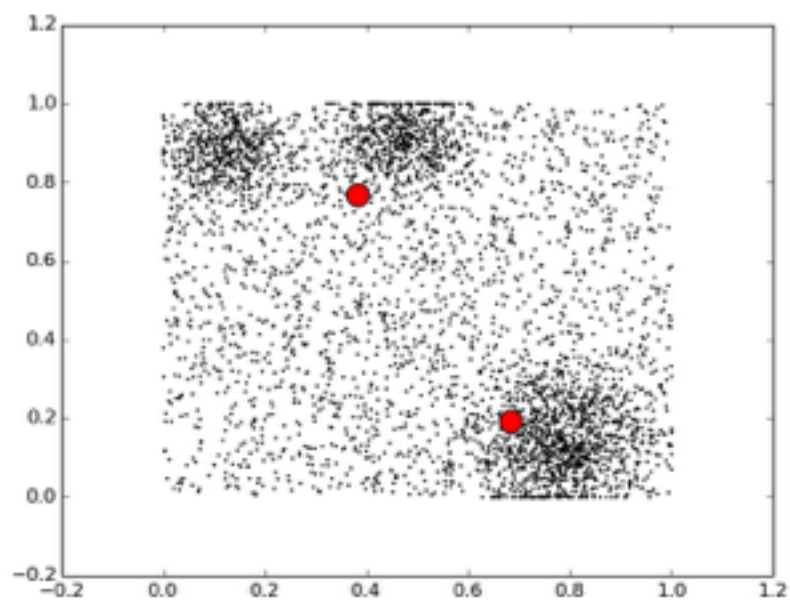
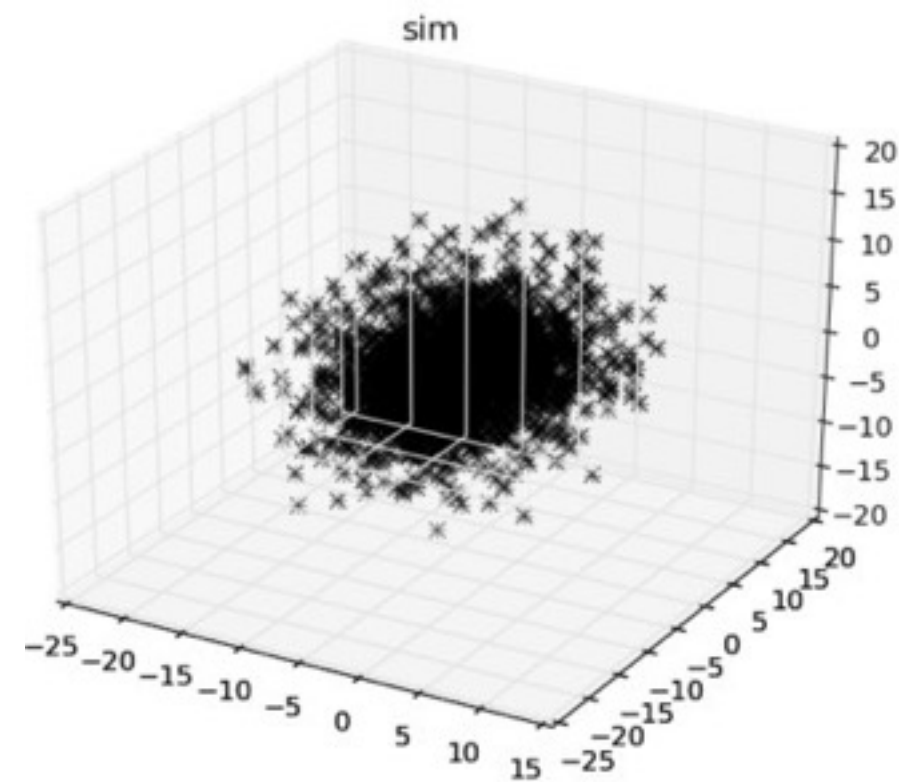
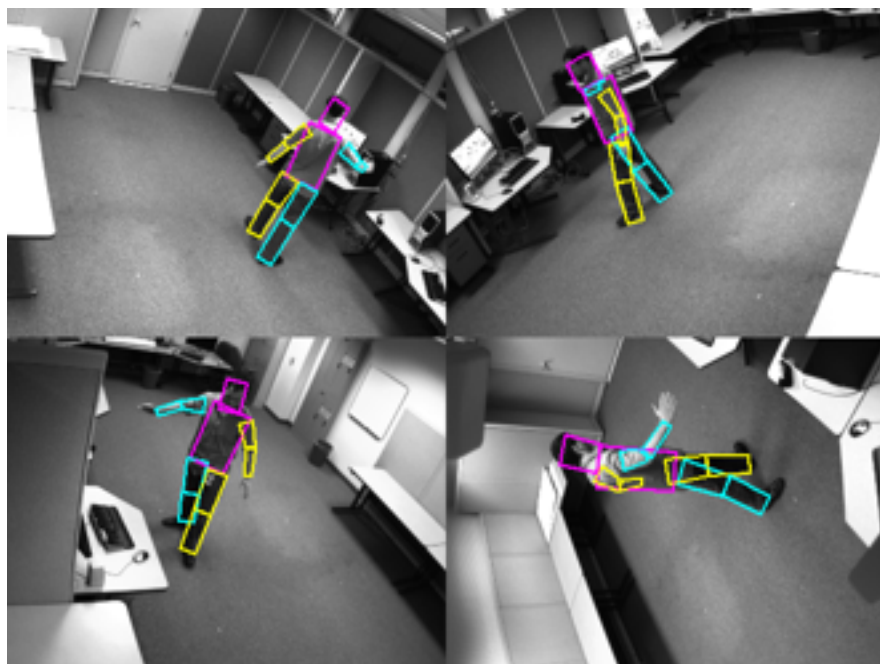
# Energy savings



(d) Energy Savings

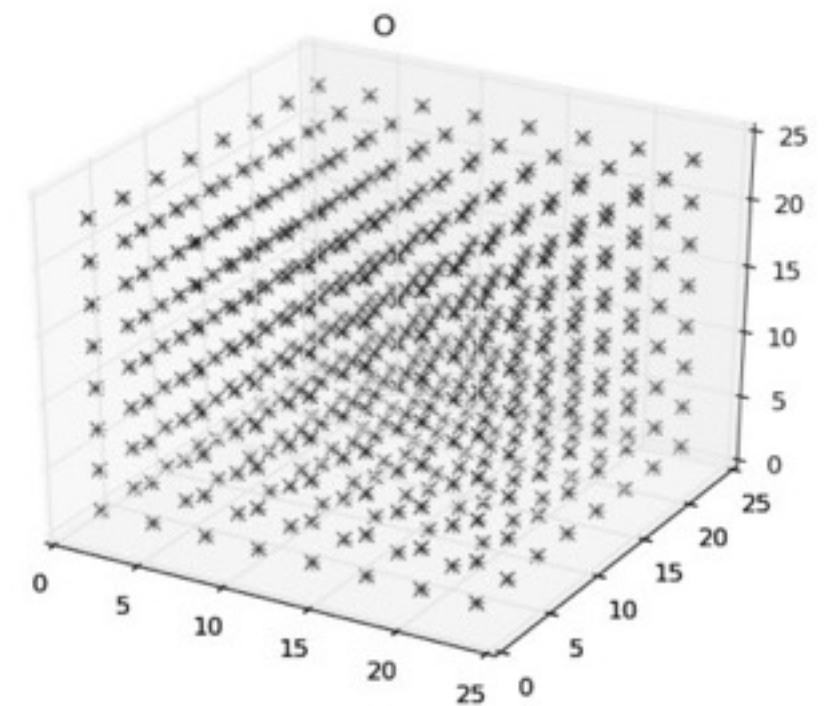
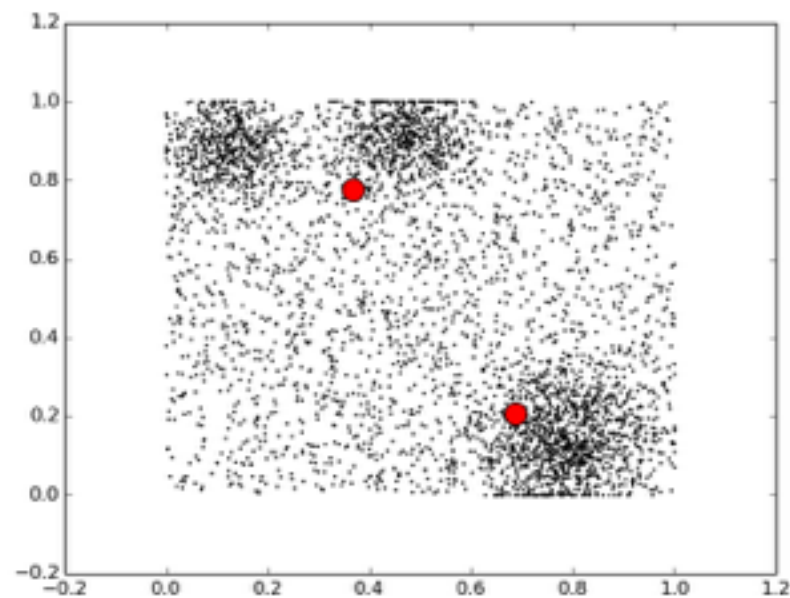
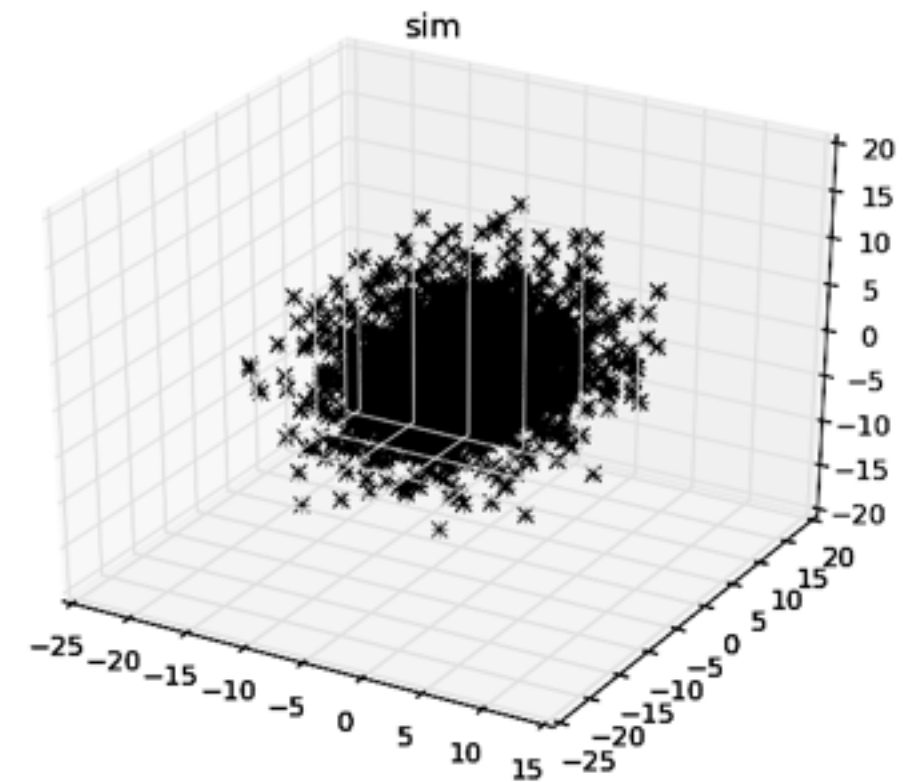
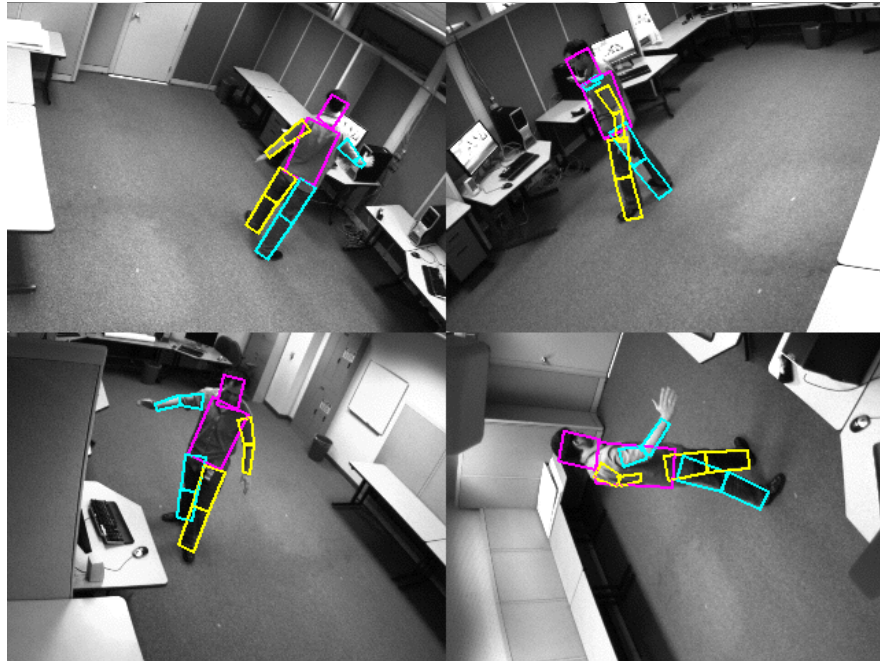
**Question:** Are the end-to-end results acceptable?

# Expected Result

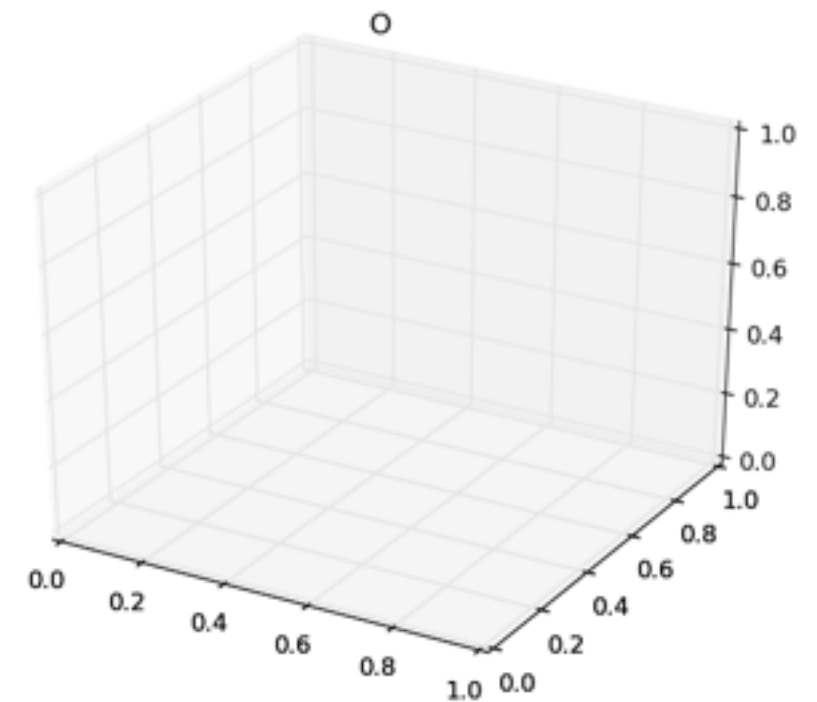
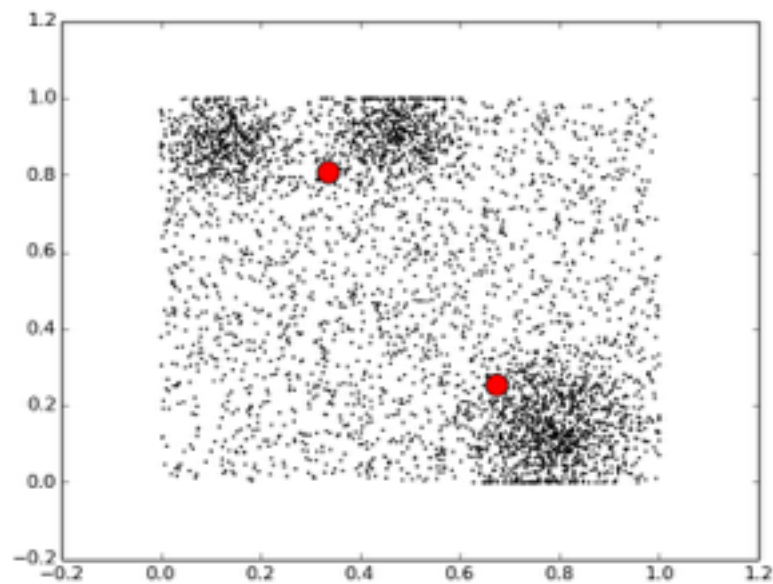
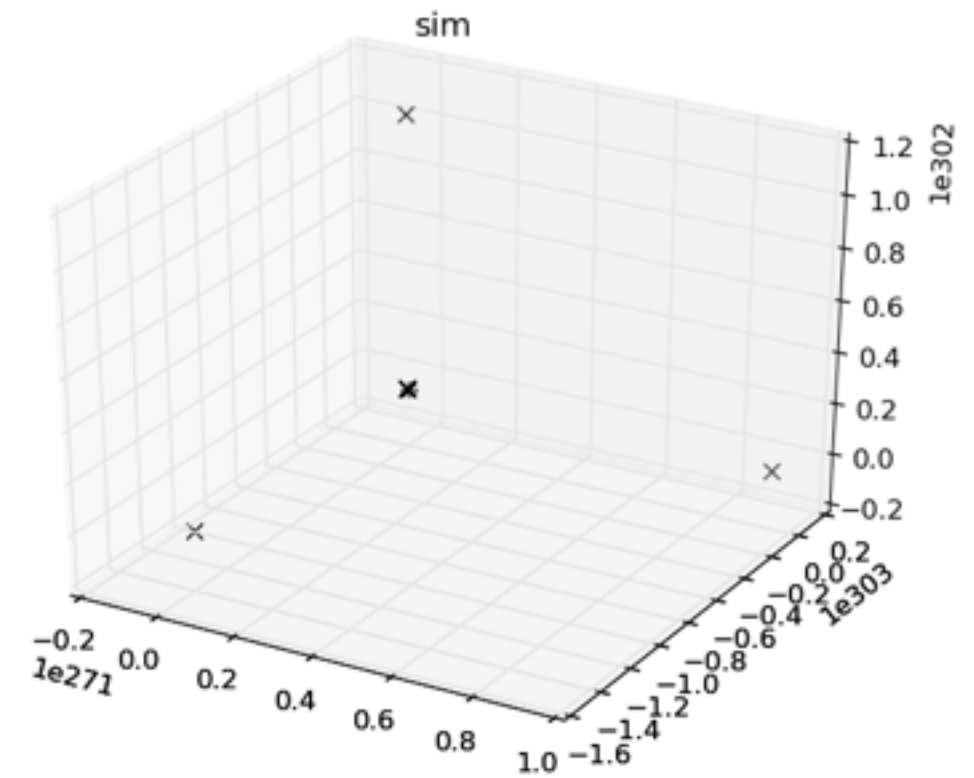




# Approximate Result **with** Outlier Detection



# Approximate Result **without** Outlier Detection





# Quantitative Analysis of Final Result Quality

Benchmark	Model	No Outlier Detector	Outlier Detector
barnes	basic	inf	0.158229%
blackscholes	basic	inf	0.135584%
bodytrack	basic	73.6327%	0.161024%
streamcluster	basic	0.6219	0.6344
water	basic	nan	0.000469%
barnes	ddep	inf	0.075927%
blackscholes	ddep	inf	0.025791%
bodytrack	ddep	73.6327%	0.317984%
streamcluster	ddep	0.6321	0.6344
water	ddep	nan	0.000383%

Table 2: End-to-End Output Quality

**Question:** Is the outlier detector adequately detecting outliers?

# Quantitative Outlier Detector Efficacy

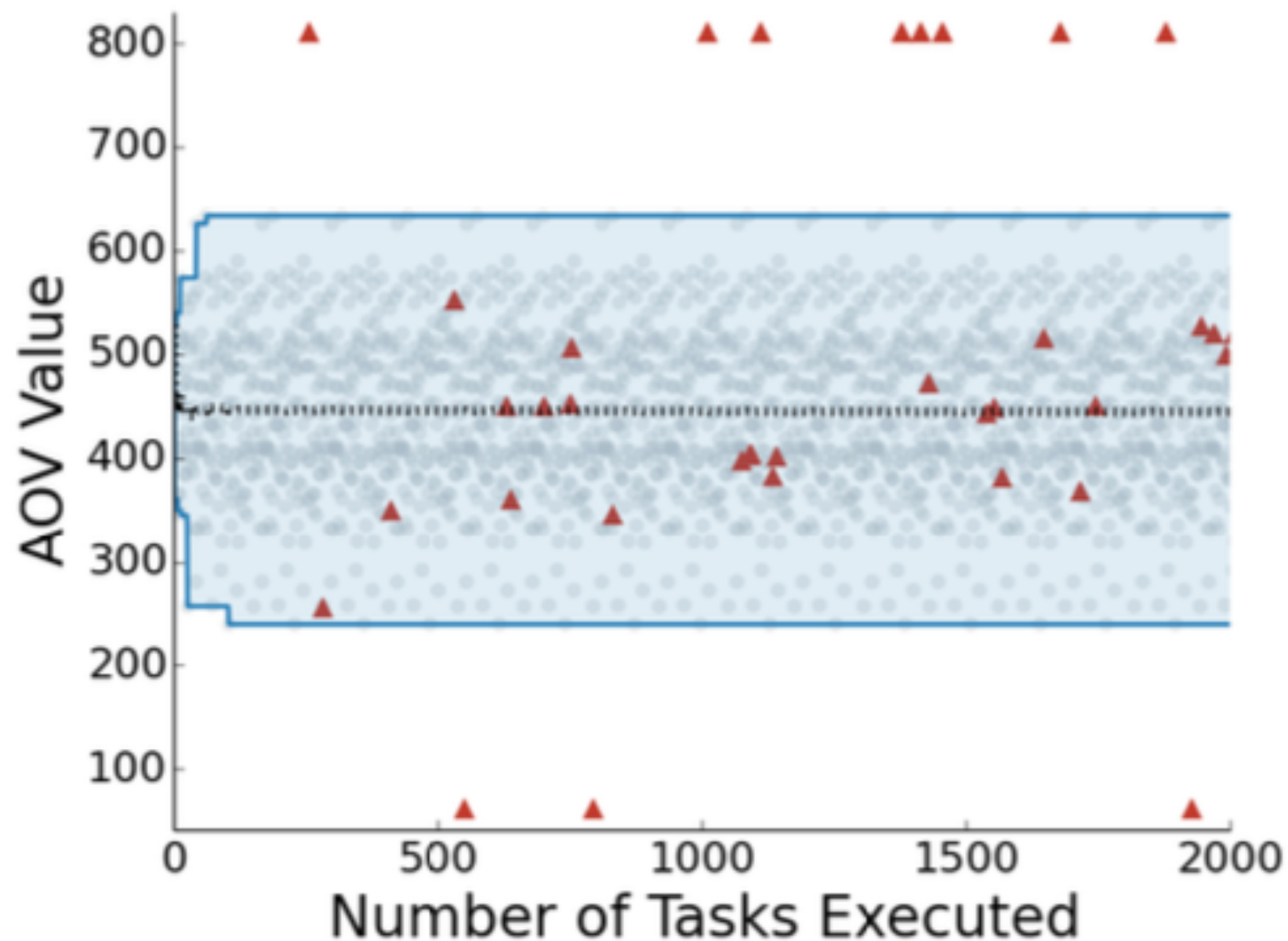
Benchmark	Hardware Model	Correct Accepted (%)	Correct Rejected (%)	Error Accepted (%)	Error Rejected (%)	Rejection Accuracy (%)	Errors Detected (%)
barnes	basic	94.48%	0.19%	2.94%	2.38%	92.58%	44.74%
bodytrack	basic	87.58%	0.16%	7.67%	4.58%	96.62%	37.39%
water-interf	basic	95.30%	0.32%	1.71%	2.67%	89.37%	60.96%
water-poteng	basic	99.51%	0.26%	0.02%	0.20%	43.59%	89.47%
blackscholes	basic	98.57%	0.04%	1.06%	0.33%	90.00%	24.06%
streamcluster	basic	98.34%	0.14%	0.37%	1.15%	89.15%	75.66%
barnes	ddep	94.22%	0.20%	3.11%	2.47%	92.59%	44.26%
bodytrack	ddep	77.34%	0.15%	16.04%	6.46%	97.67%	28.71%
water-interf	ddep	95.44%	0.33%	1.62%	2.61%	88.81%	61.71%
water-poteng	ddep	99.49%	0.26%	0.04%	0.21%	44.54%	85.48%
blackscholes	ddep	98.70%	0.04%	0.94%	0.33%	89.80%	25.88%
streamcluster	ddep	62.24%	0.11%	36.68%	0.98%	89.90%	2.59%

Table 3: Overall Outlier Detector Effectiveness

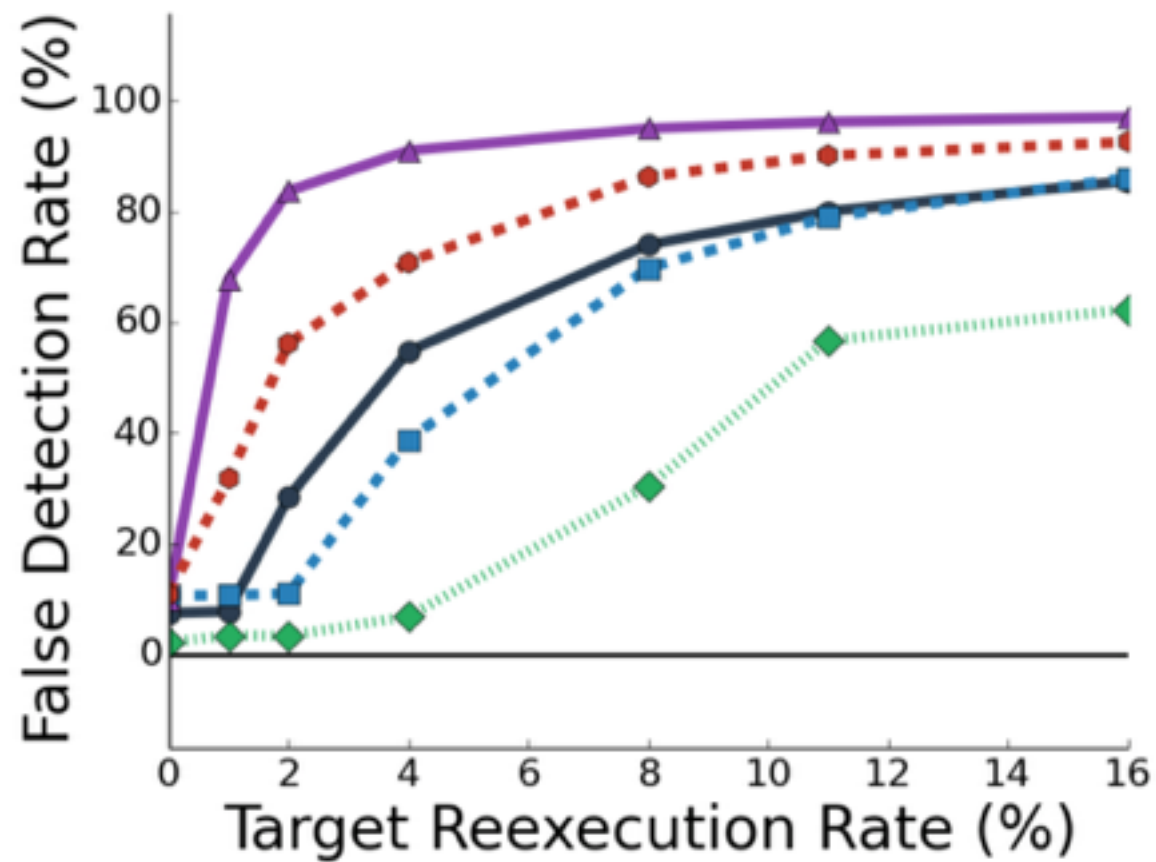
- **Some tasks are rejected:** 0.5-7% tasks rejected
- **Most rejections are errors:** 87%-98% rejections are errors
- **Some errors are undetected:** 2%-90% errors are rejected

# Qualitative Error Characteristics and Detection

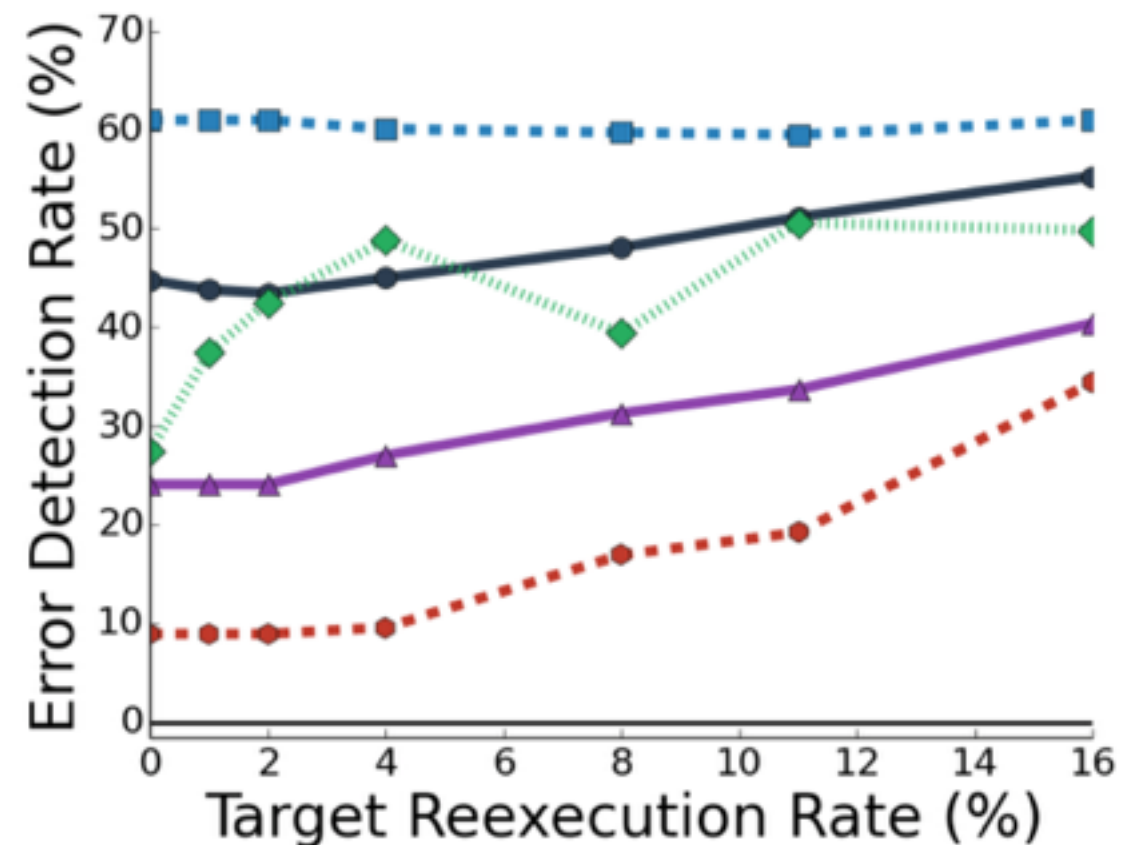
**Blackholes:** 24% of errors detected



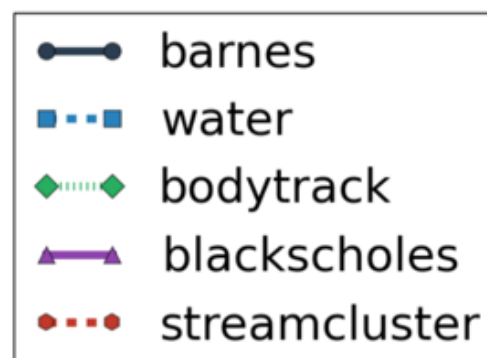
# Qualitative Error Characteristics and Detection



False Detections

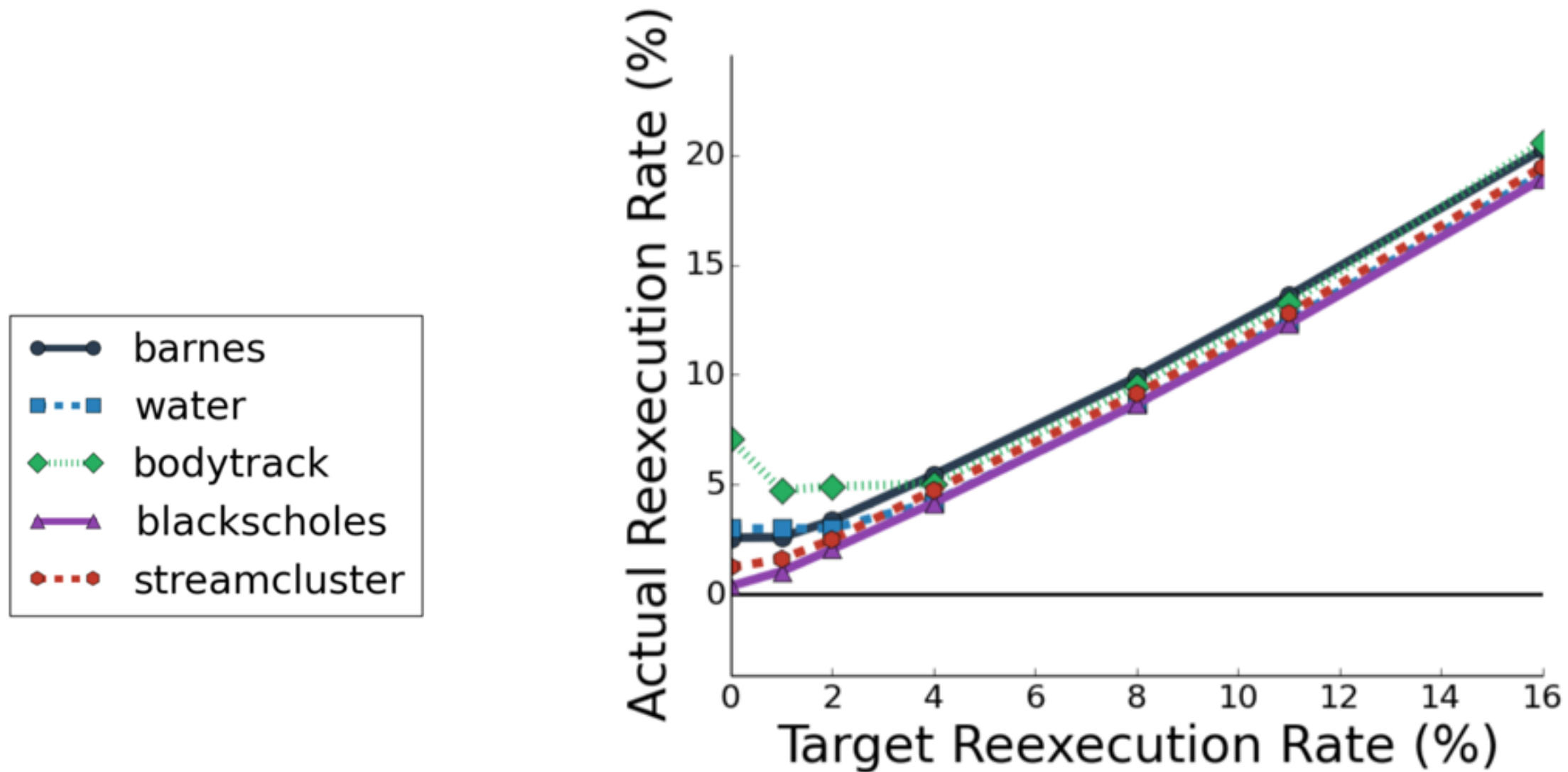


Fraction of Errors Detected



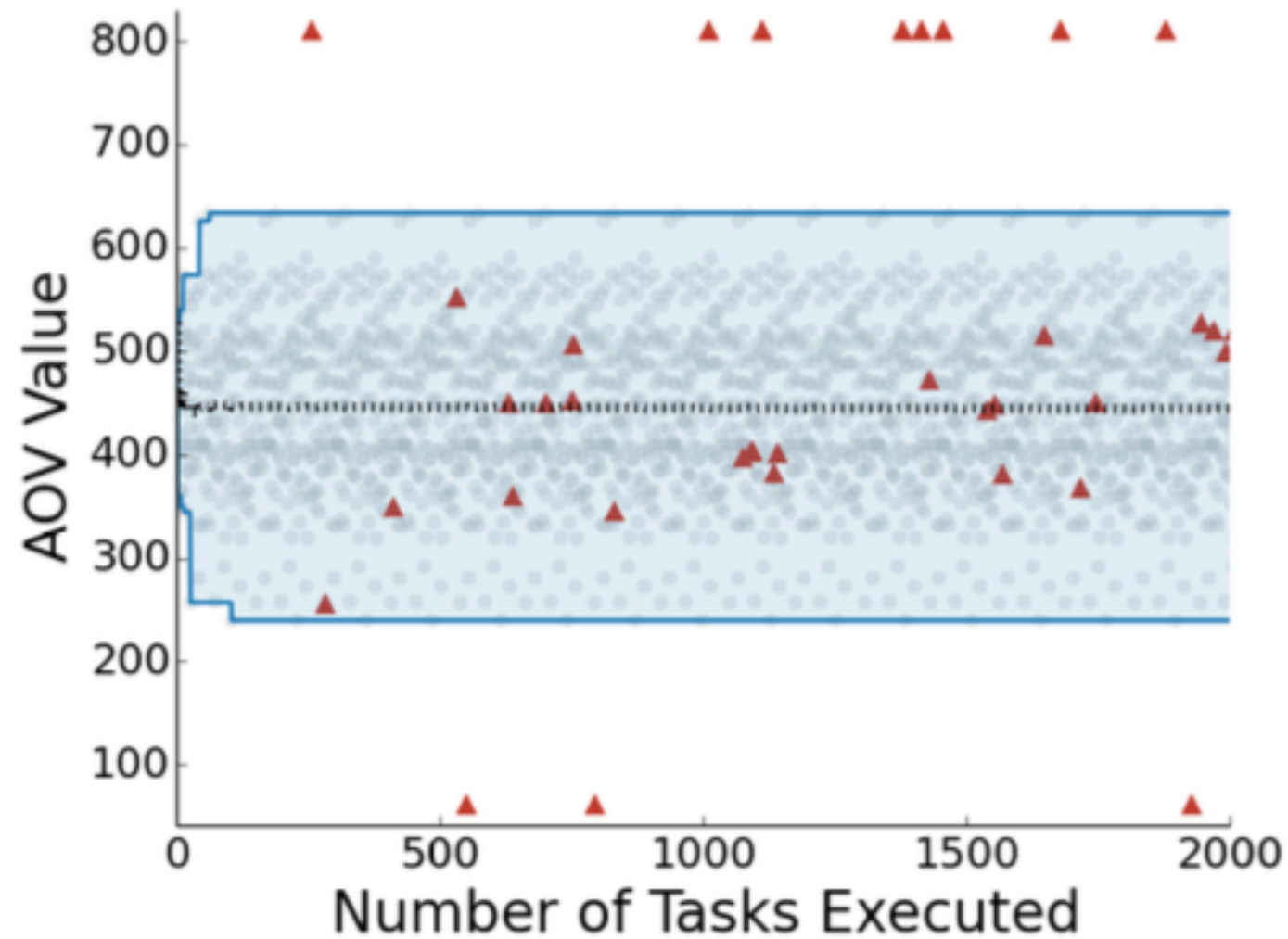
**Question:** Is the adaptive outlier detector behaving as expected?

# Quantitative Efficacy of Adaptive Outlier Detector



(a) Actual Reexecution Rate

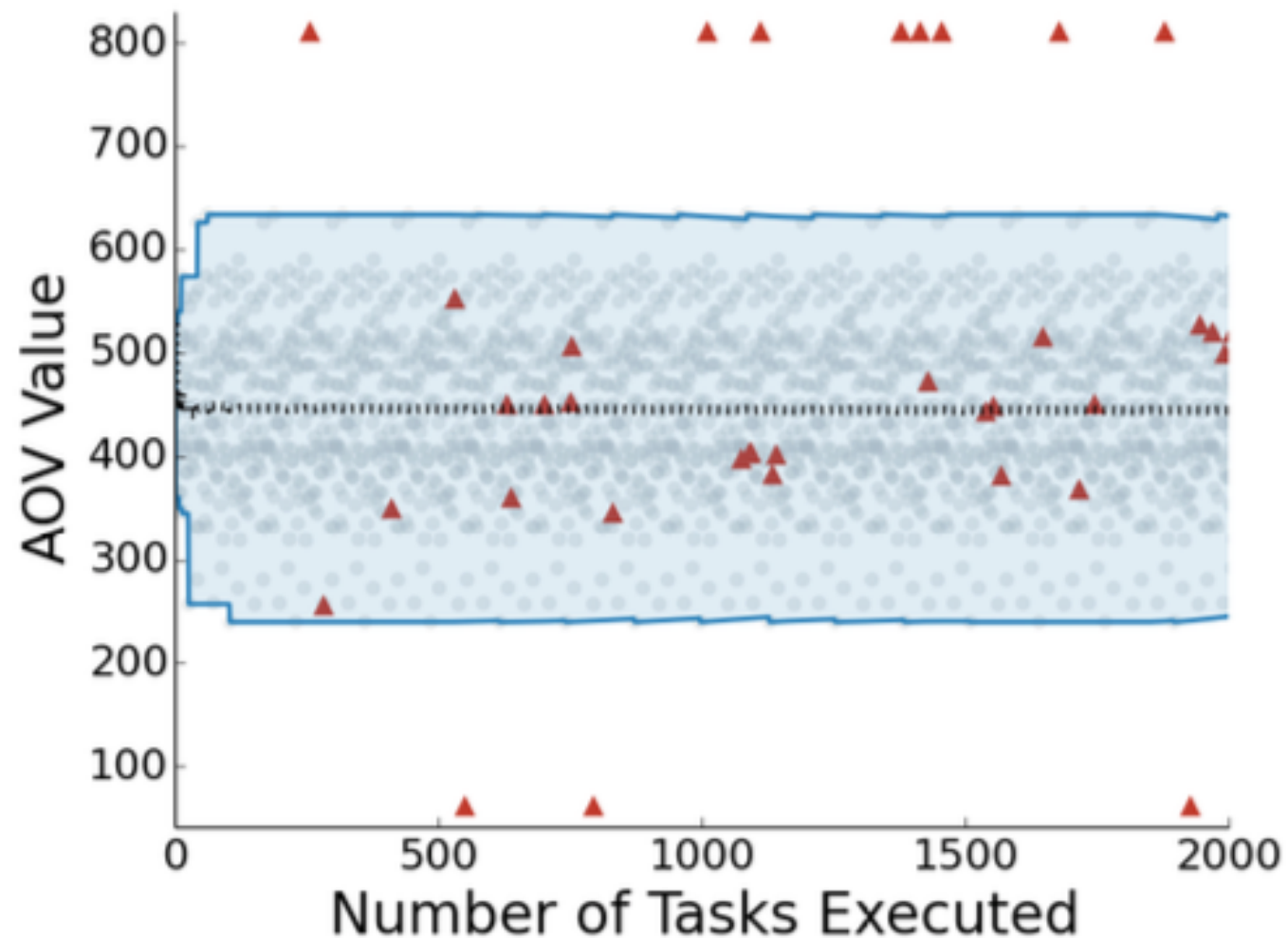
# Qualitative adaptive outlier detector efficacy



1% target reexecution rate

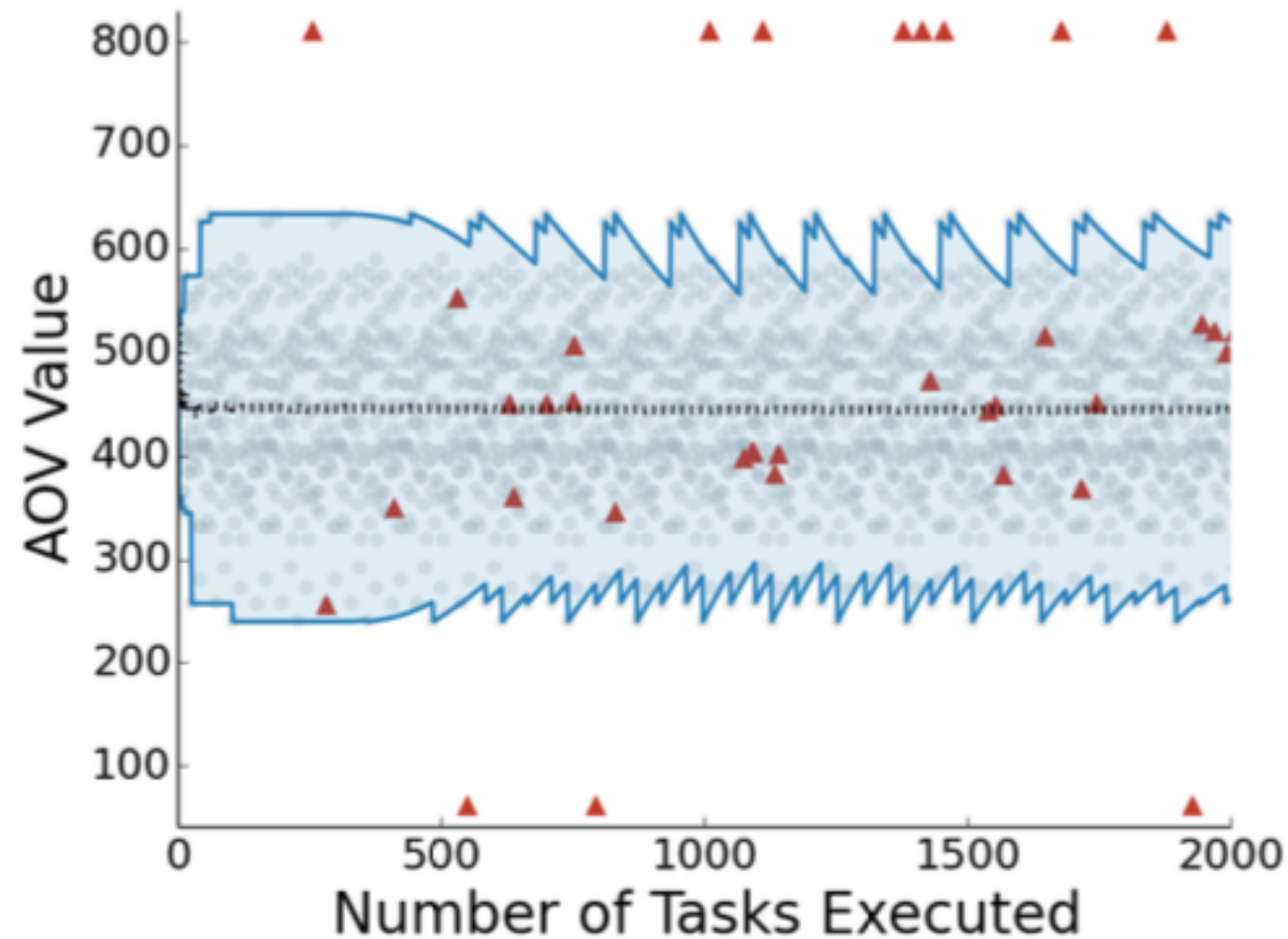


# Qualitative adaptive outlier detector efficacy



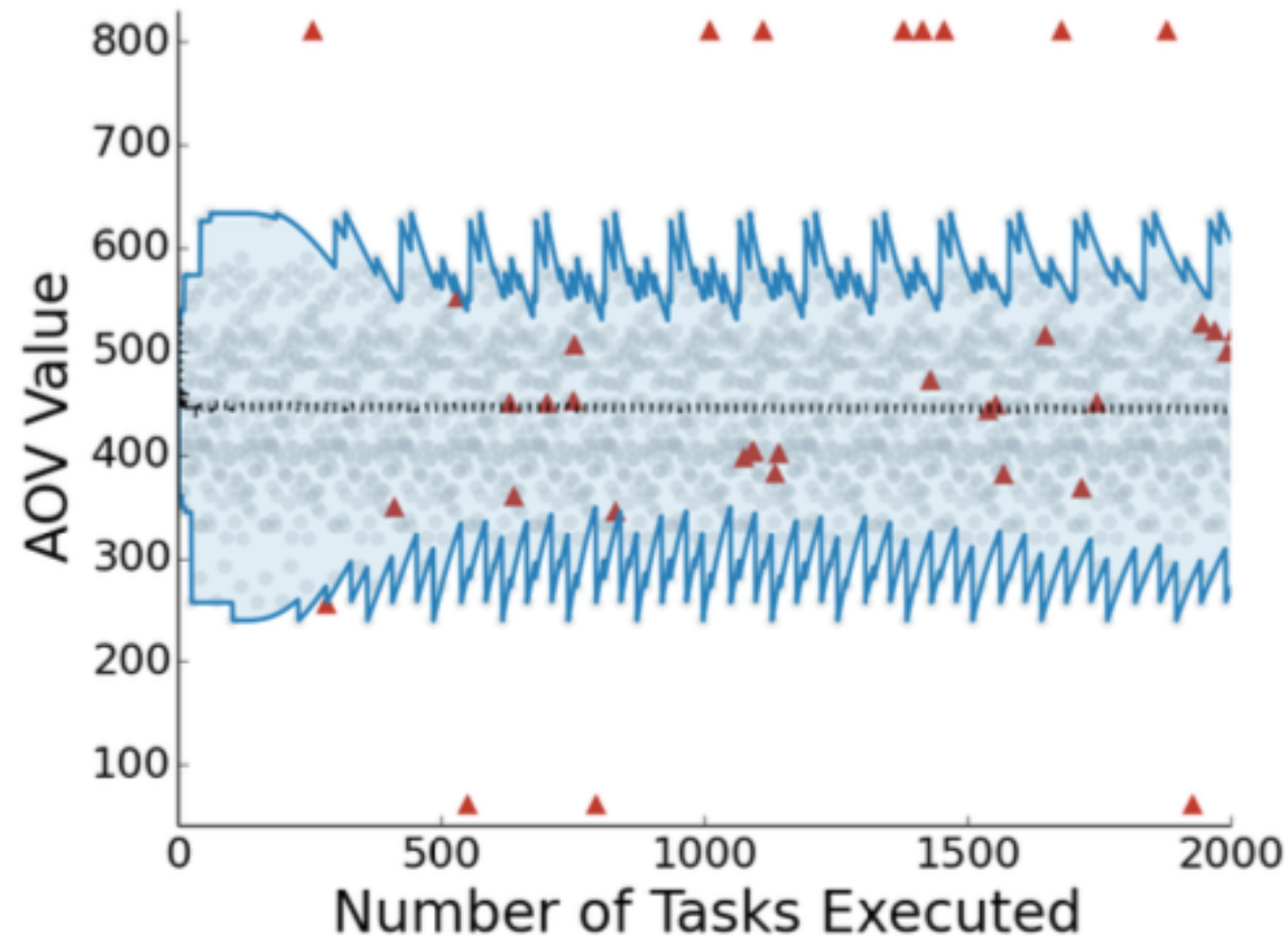
2% target reexecution rate

# Qualitative adaptive outlier detector efficacy



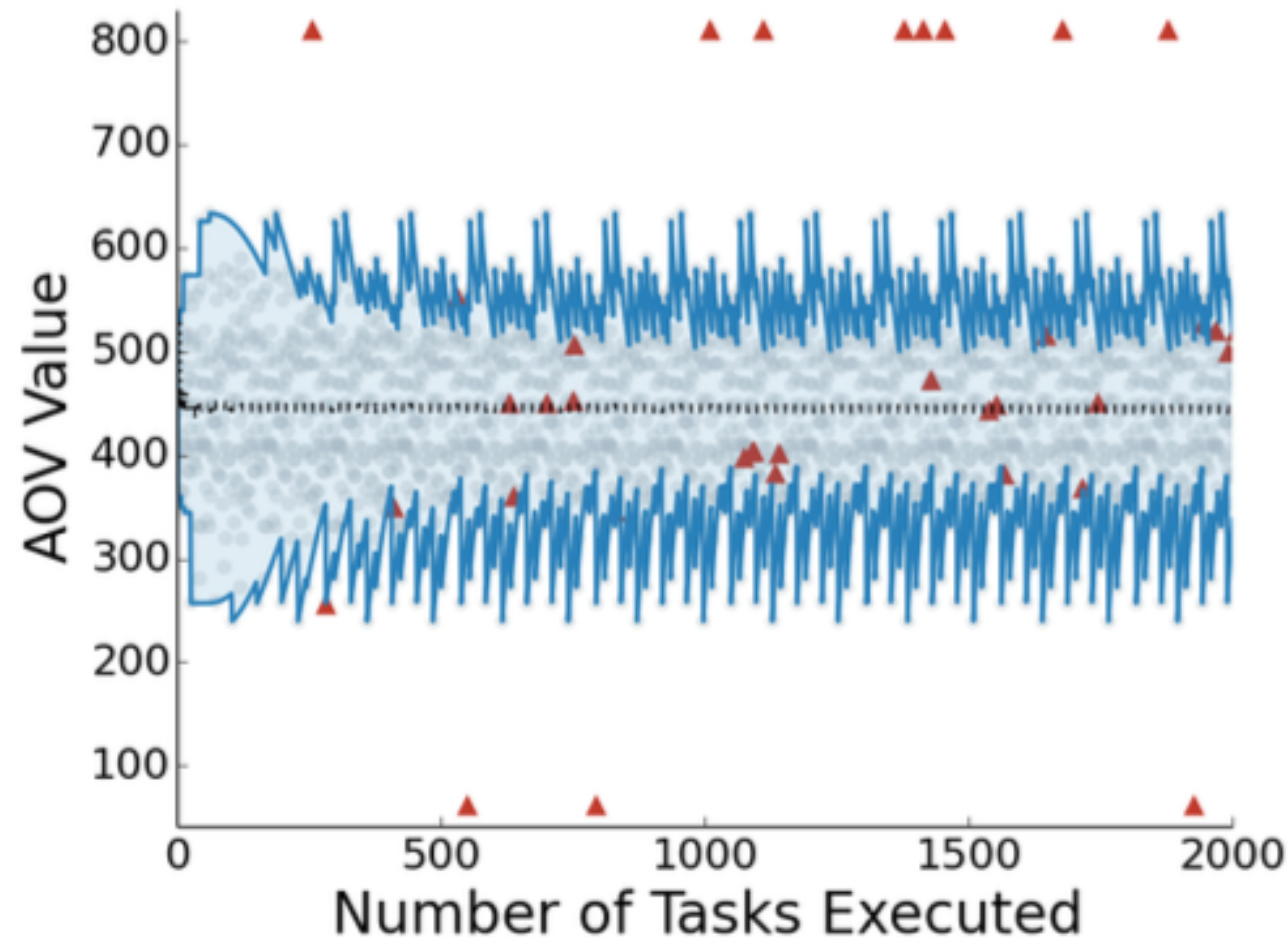
4% target reexecution rate

# Qualitative adaptive outlier detector efficacy



8% target reexecution rate

# Qualitative adaptive outlier detector efficacy



16% target reexecution rate

# Conclusion

- ***Topaz*** is an important result for approximate computing:
- Programs are not tied to a specific approximate hardware model
- Approximate hardware models are changing rapidly and specific to physical hardware

# Questions?

# Backup Slide

# Key goals

1. Computation runs to **completion**
2. Computation yields an **acceptable** result
3. **Savings** from using approximate hardware



# Optimization 1: Stable Data

- **Stable Data**: Data that is unchanged for all tasks in taskset
  - *e.g. images, data structures*
- **Optimization**: Selectively send stable data
  - with first task in taskset
  - with task following approximate machine crash
  - with task following n rejected errors
- **Reduce overhead if task contains large unchanging inputs**

```
// computes the weights for each valid pose.
taskset calcweights(i=0; i<particles.size(); i+=1){
    compute in (
        float tpart[P_SIZE] = (float*) particles[i],
        const float tmodel[M_SIZE] = (float*) mdl_prim,
        const char timg[I_SIZE] = (char *) img_prim,
        const int nCams = mModel->NCameras(),
        const int nBits = mModel->getBytesPerPixel(),
        const int width = mModel->getWidth(),
        const int height =mModel->getHeight()
    ) out (float tweight) {
        tweight = CalcWeight(tpart,
                             tmodel, timg, nCams, width, height, nBits);
    }

    const stable data annotation for
        inputs
```

# Optimization 2: Abstract Output Vector (AOV)

- **Abstract Output Vector (AOV):** Programmer defined result tuple abstraction.
- **Optimization:** Outlier detector performs detection on AOV.
  - AOV smaller than result tuple
  - lower dimensionality outlier detection
- **Aside:** Handle input dependence using AOV
- **Reduces outlier detector overhead if AOV smaller than result tuple**

```
taskset  name(int i = l; i < u; i++) {  
    compute in  (d1 x1 = e1, ..., dn xn = en)  
               out (o1 y1, ..., oj yj) {  
                <task body>  
            }  
    transform out (v1, ..., vk) {  
        <output abstraction>  
    }  
    combine { <combine body> }  
}
```

transform block with AOV outputs v1..vk.  
    <output abstraction> defines the  
        transformation

# AOV: Two Examples

```
transform out(float ea, float ev, float ephi)
{
    ephi=ev=ea=0; int k=0;
    for(int b=0; b < BATCH; b++){
        for(int d=0; d < NDIMS; d++, k++){
            ev += square(vel[k]);
            ea += square(acc[k]);
        }
        ephi += phi[b];
    }
}
```

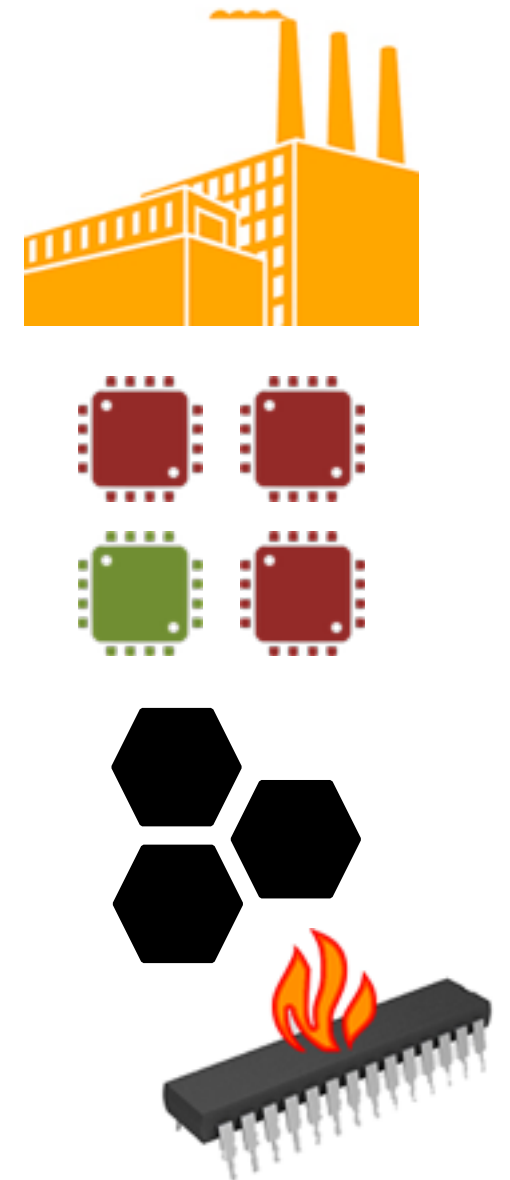
transform block for water simulation  
reduces output dimensionality

```
transform out(bweight, bp1, bp2, bp3) {
    bweight = tweight;
    bp1=tpart[3]; bp2=tpart[4]; bp3=tpart[5];
}
```

transform block for bodytrack accounts  
for input dependence

# The world contains a lot of **approximate** hardware

- Hardware with **manufacturing defects**
- **Older**, heavily used machines
- Hardware in **aggressive** conditions
- Hardware with **protections** removed
- Novel hardware created from **immature** fabrication processes
- Hardware intentionally engineered to occasionally **produce errors** for **energy** and **performance** savings



## Key Question

Can we use **approximate** hardware?

## Key Question

Can we use **approximate** hardware?

**Yes!**

# How Can we Use Approximate Hardware?

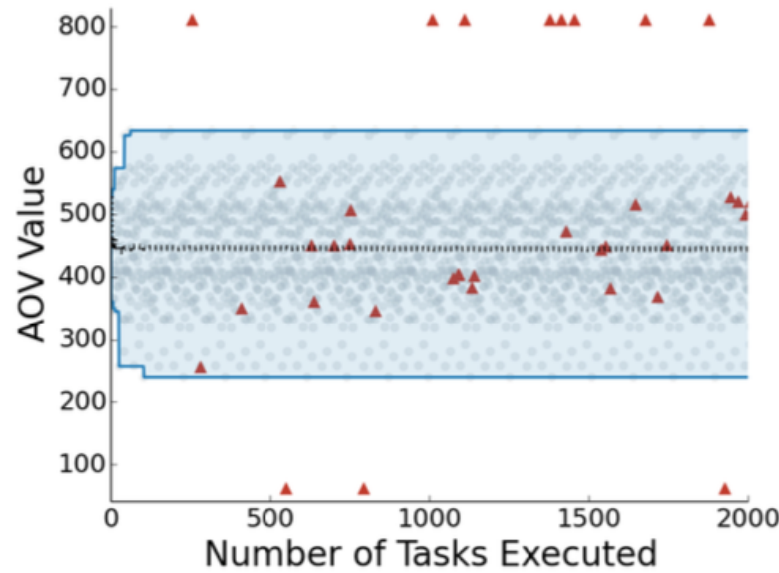
- **Just Use It:** crashes, unacceptable results
- **Static Analysis:** statically derive probabilistic error bounds
  - requires hardware specification / fault model
  - fine grain control over approximate hardware faults
  - no runtime overhead and probabilistic guarantee
- **Dynamic Systems:** adjust to faults that occur during runtime
  - weaker guarantees and runtime overhead
  - adaptive, robust

# When is it acceptable for **approximations** to occur?

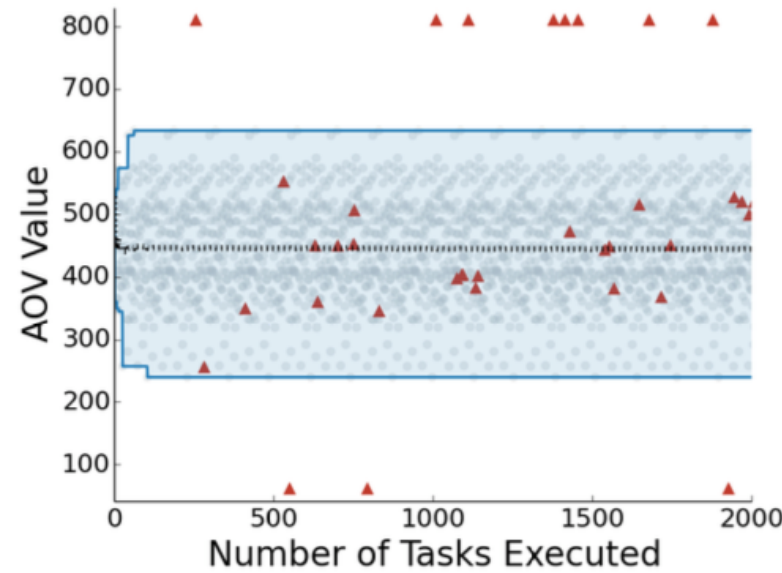
- **forgiving** and **critical** code and data
- if an error occurs in code or data:
  - **critical**: program failure
  - **forgiving**: different answer
- target programs that spend most of computation in **forgiving** regions



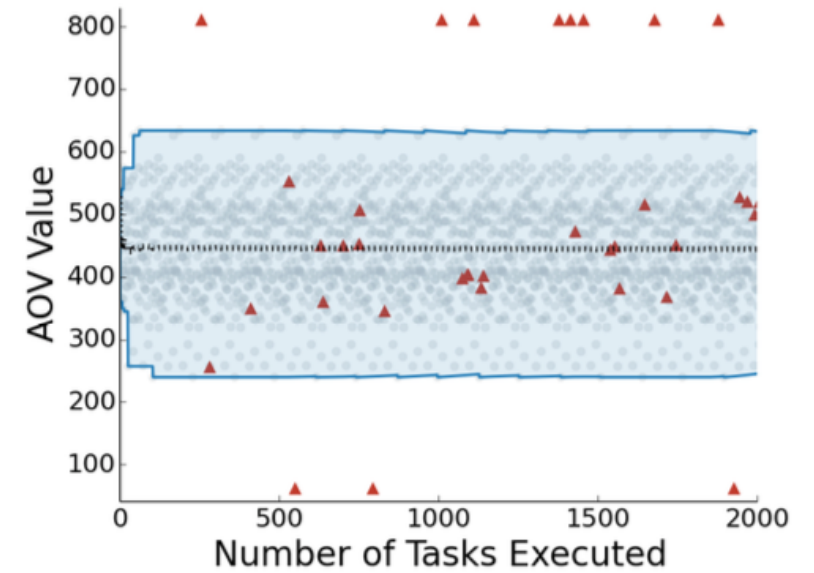
# Qualitative adaptive outlier detector efficacy



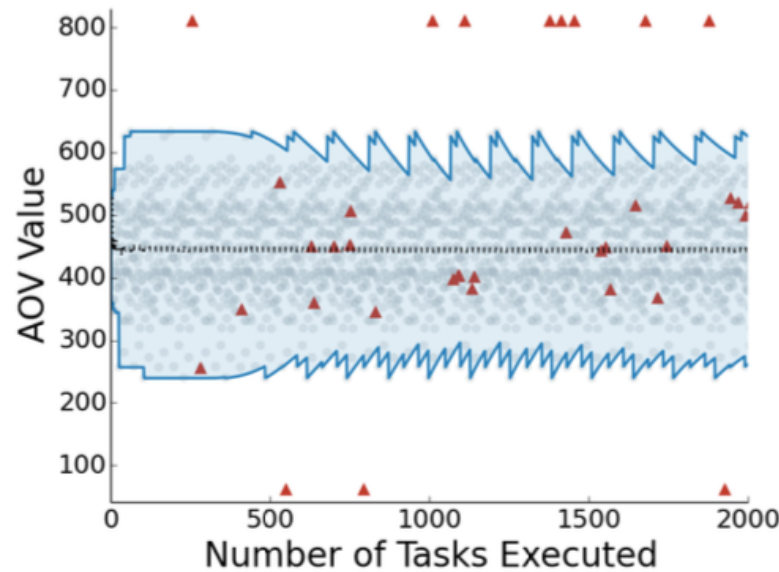
(a) 0% Target Reexecution Rate



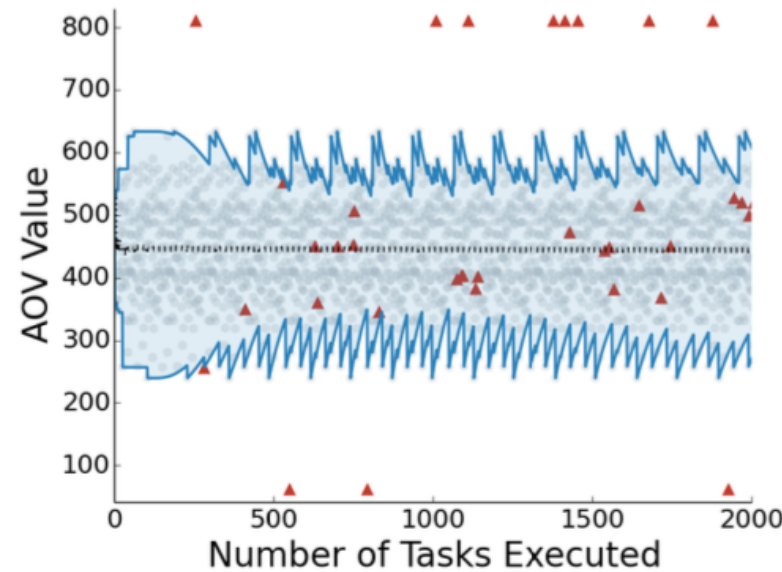
(b) 1% Target Reexecution Rate



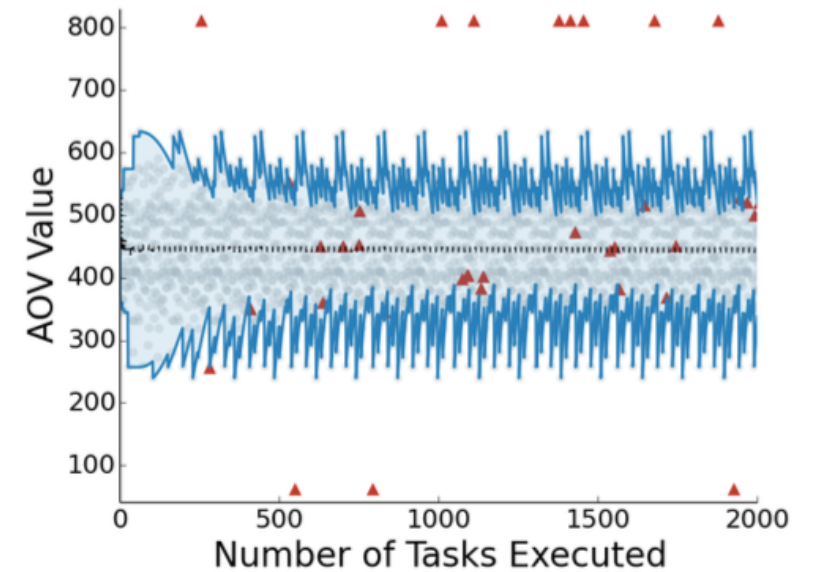
(c) 2% Target Reexecution Rate



(d) 4% Target Reexecution Rate



(e) 8% Target Reexecution Rate



(f) 16% Target Reexecution Rate