Configuration Synthesis for Programmable Analog Devices with Arco

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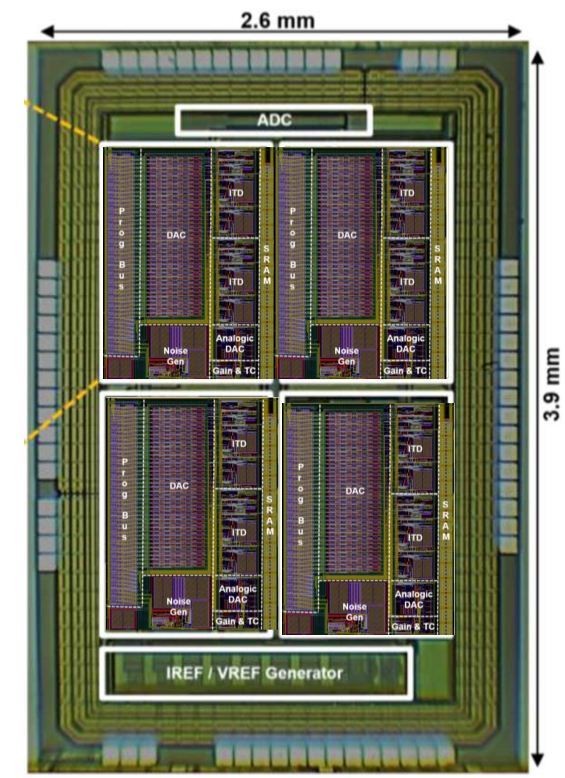
$$E = E_{tot} - ES$$

$$S = S_{tot} - ES$$

$$\partial ES/\partial t = (Q + k_0) \cdot E \cdot S - k_r \cdot ES$$

$$ES(0) = 0.423$$

Arco Compiler



Dynamical Systems

Programmable Analog Devices

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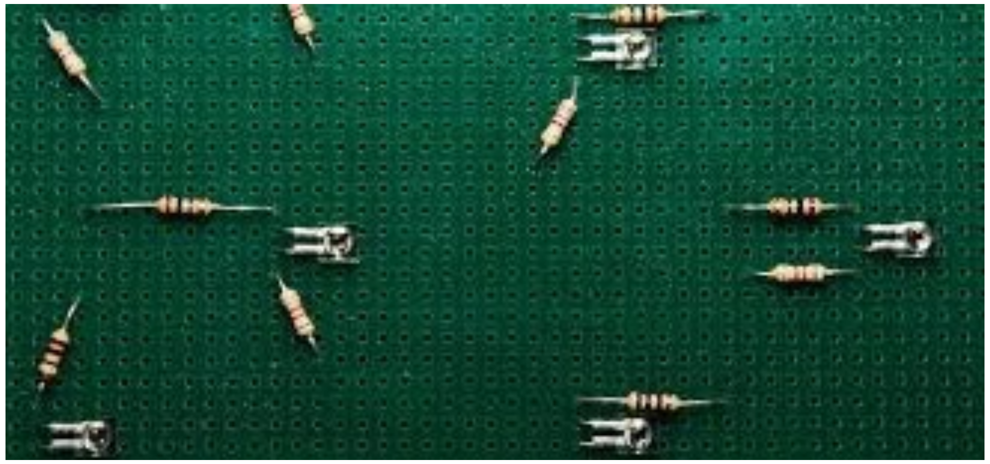
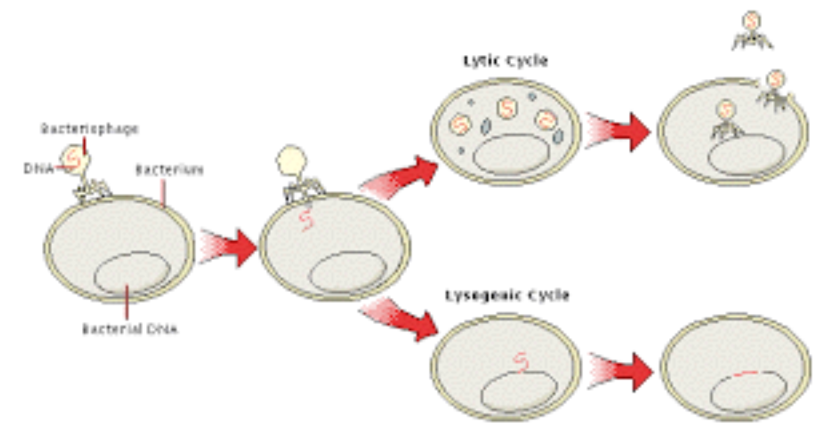
- **state variables** that model physical quantities (E, S, ES)
- **differential equations** that specify continuous dynamics of *state variables* over time

Modeling the Physical World



Dynamical Systems Model Biological Processes



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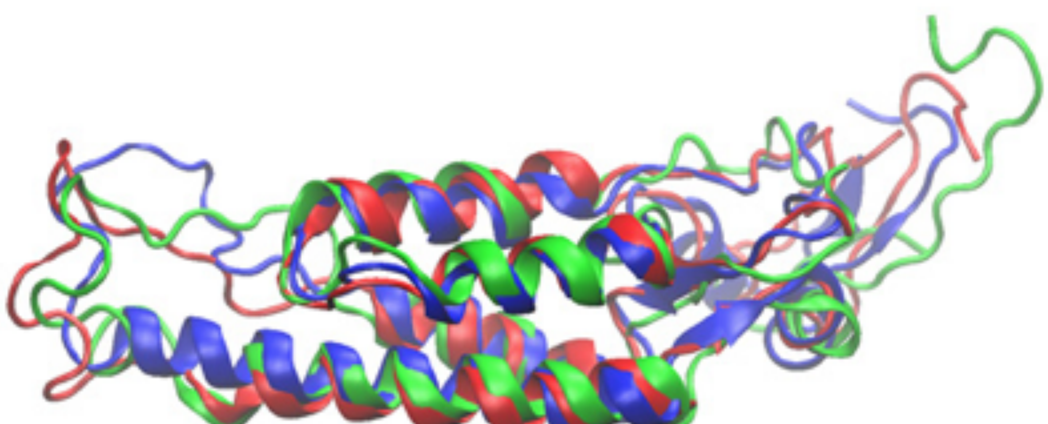

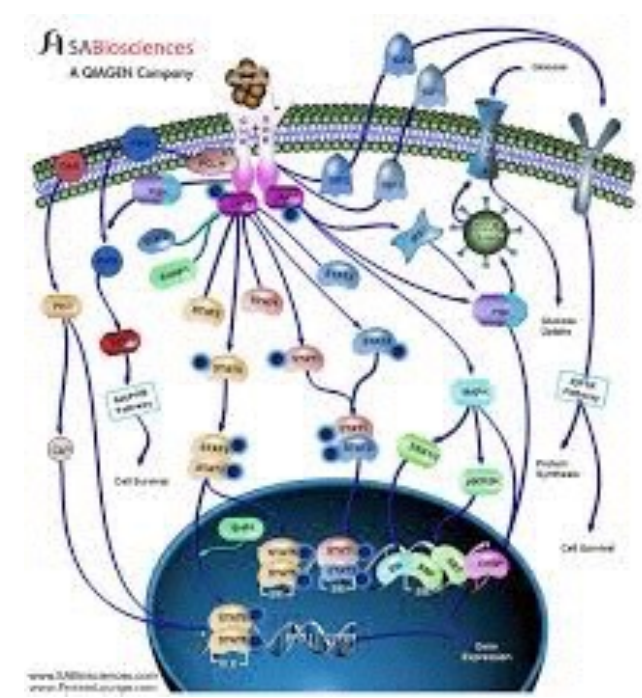

Goal: Simulate Dynamical System

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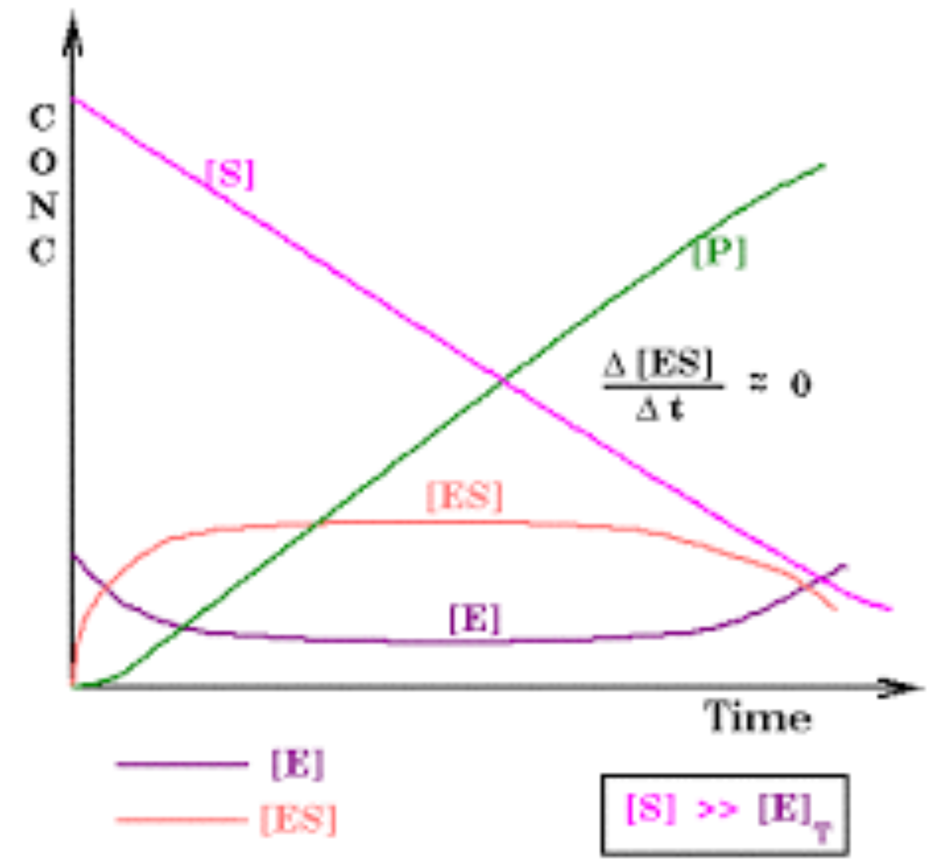
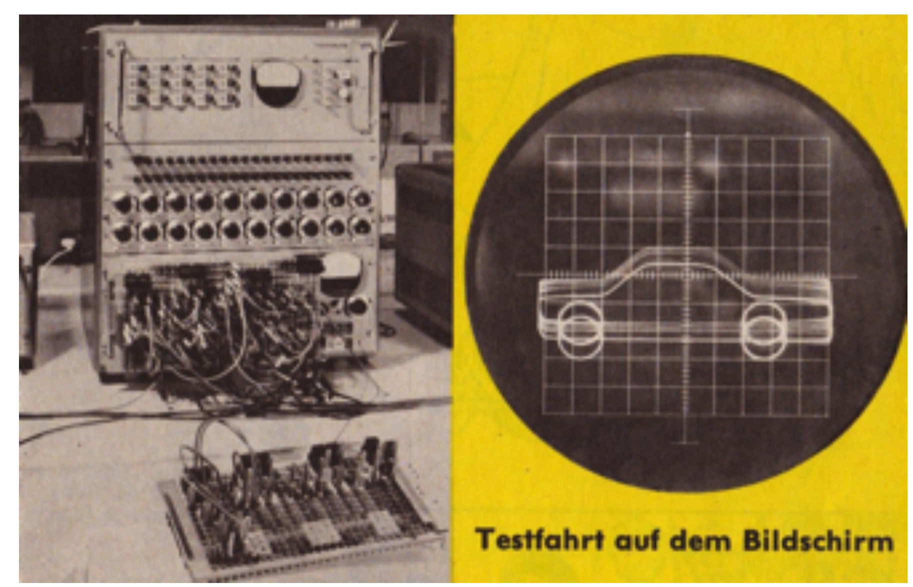


- Compute dynamics of state variables over time
- Show trajectory as function of time

Analog Computing circa 1950

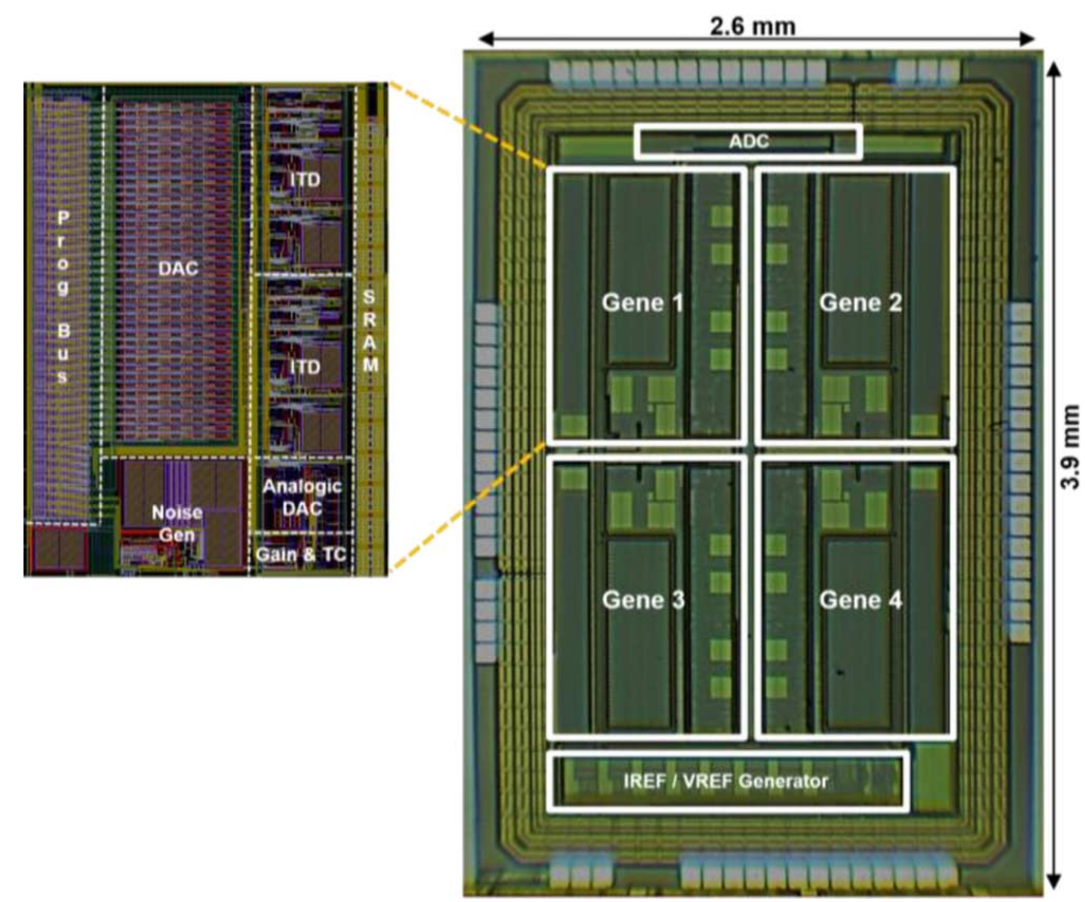
- **<u>direct mapping</u>**
  - variables → properties of wires
  - properties: voltage, current
  - dynamics → circuit dynamics
- **<u>straightforward simulation</u>**:
  - power up circuit
  - measure circuit properties over time
- **<u>1970-2010</u>**: *Age of Digital Computers*
  - Analog computers out of fashion

Telefunken RAT 700



Analog Computers Are Back: Programmable Analog Devices

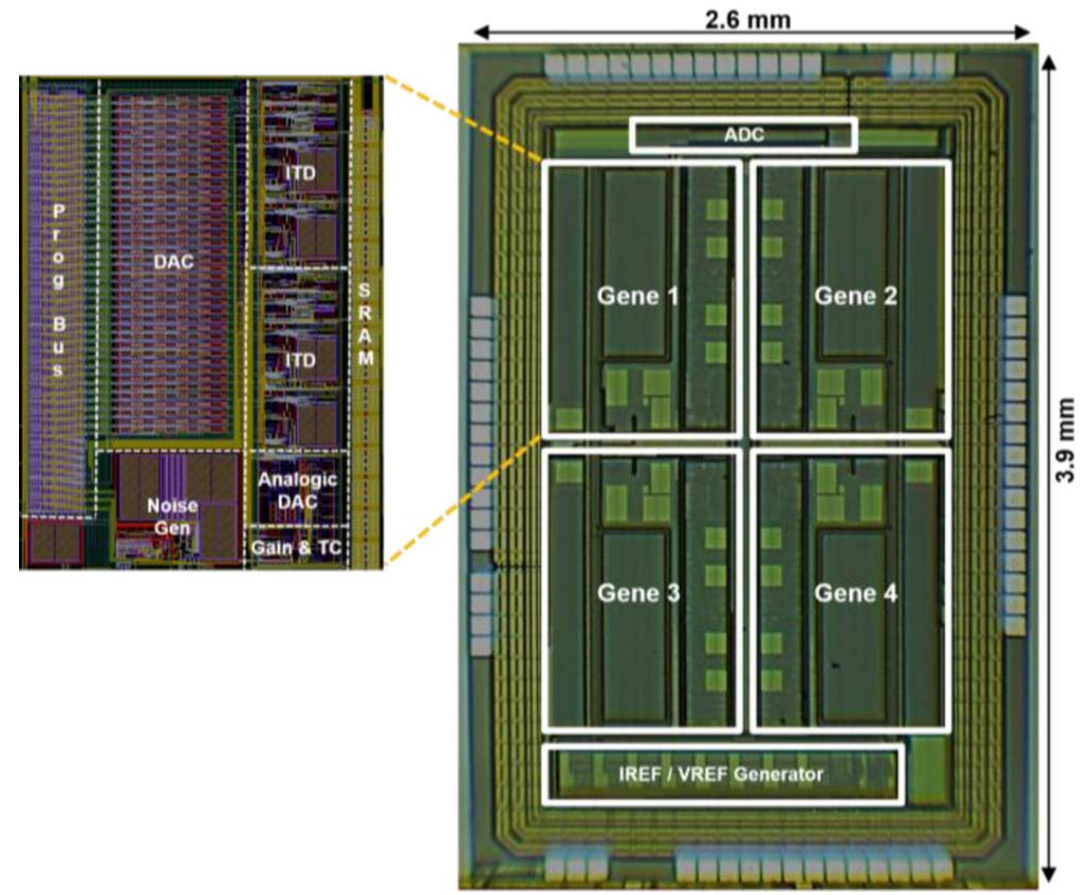
- modernized hardware
  - solid state devices
  - modern semiconductor technology
- new capabilities
  - powerful, heavily optimized analog components
  - digital reprogrammability
  - exploit analog noise



S. S. Woo, J. Kim, and R. Sarpeshkar. A cytomorphic chip for quantitative modeling of fundamental bio-molecular circuits. IEEE Trans. Biomed. Circuits and Systems, 9(4):527–542, 2015.

Analog Computers Are Back: Programmable Analog Devices

- **<u>good abstraction for differential equations</u>**
  - continuous (no discretization)
  - direct mapping of dynamics
  - circuit noise maps to reaction stochastics
- **<u>benefits</u>**:
  - energy efficient
  - fast



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What Does It Take To Manually Program These Devices?

- **domain expertise, hardware expertise, AND ability to manage details**
  - understand system you want to simulate
  - Map parts of differential equations onto analog components
  - Choose constants to specialize components for specific computation
  - Connect analog components to implement system
  - Produce a binary file to implement configuration
- Net Result:
  4 to 8 hours for a single system of 4 differential equations

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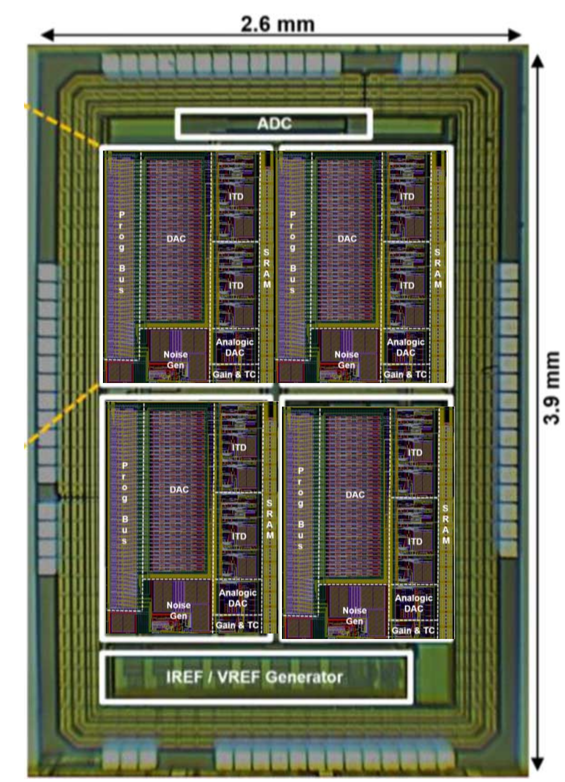

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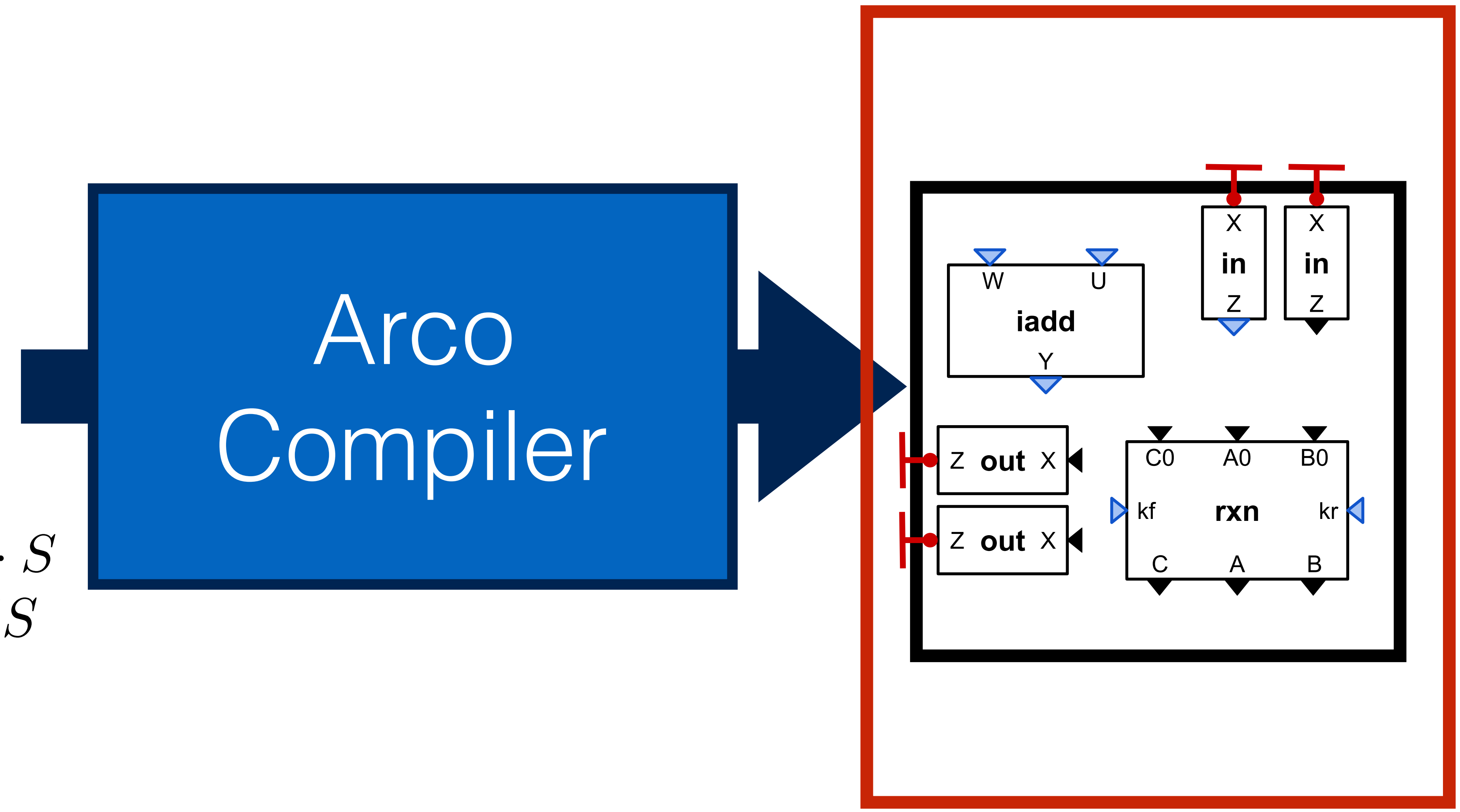
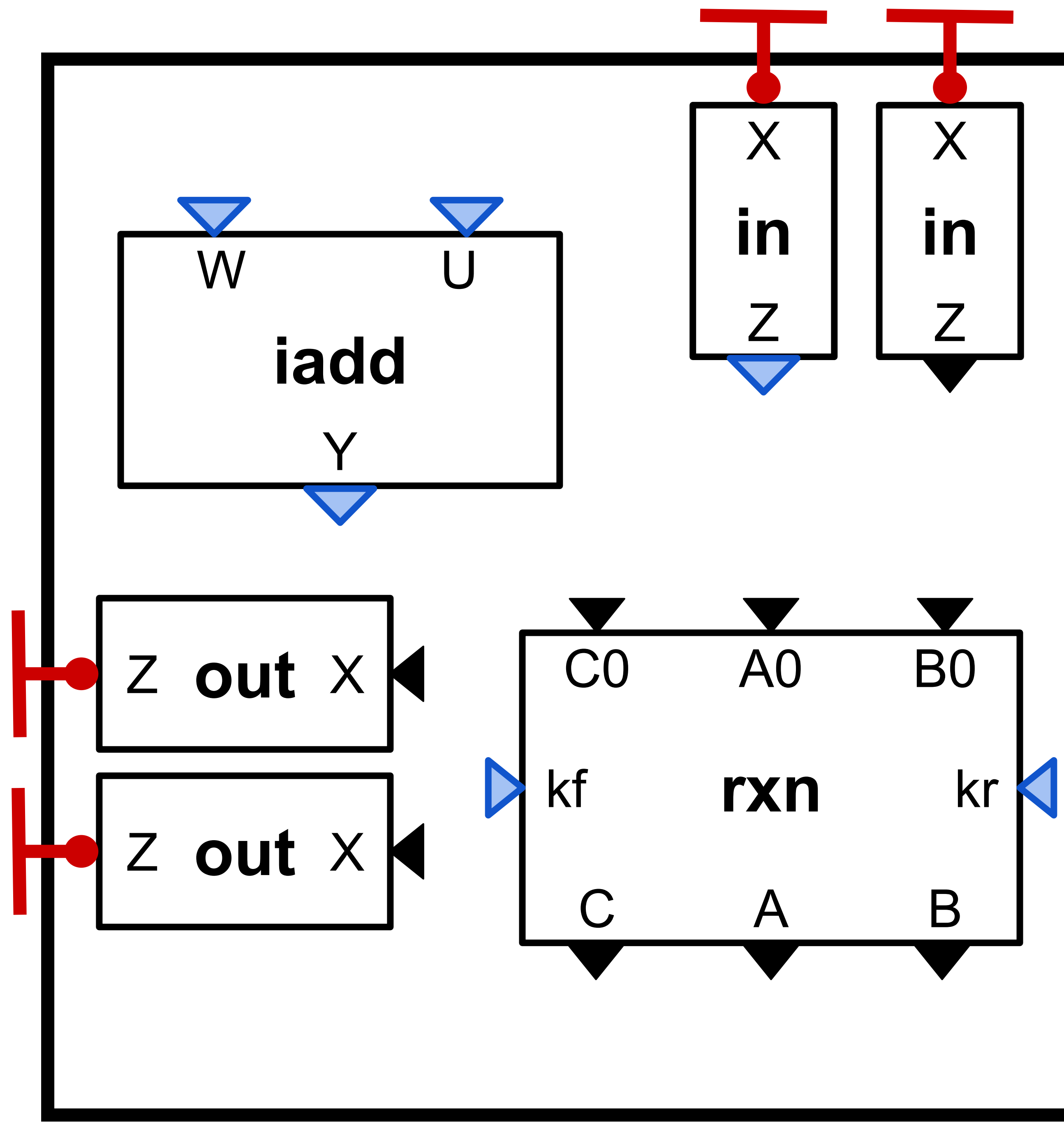


Dynamical Systems

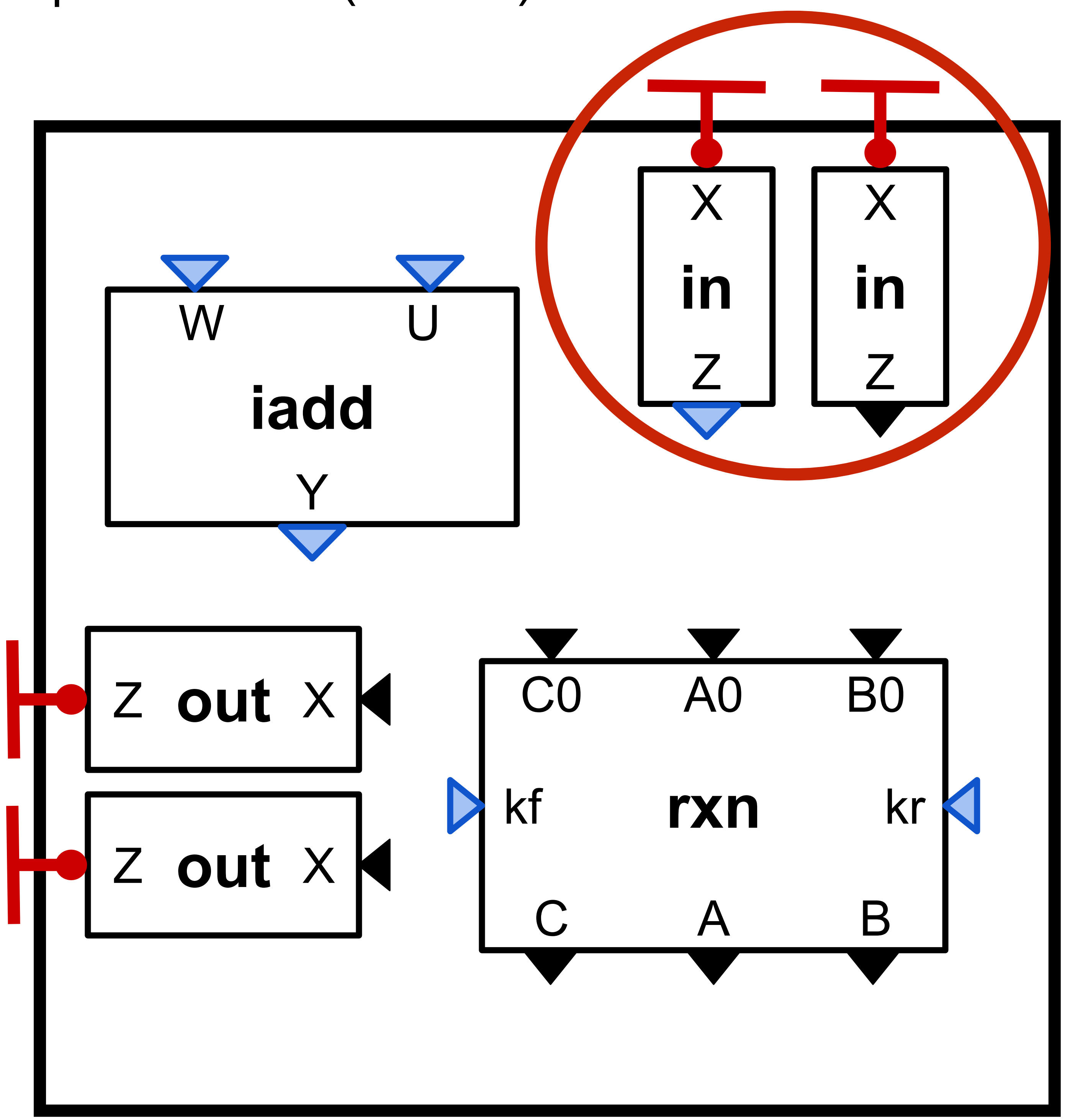
Programmable Analog Devices

Programmable Analog Device



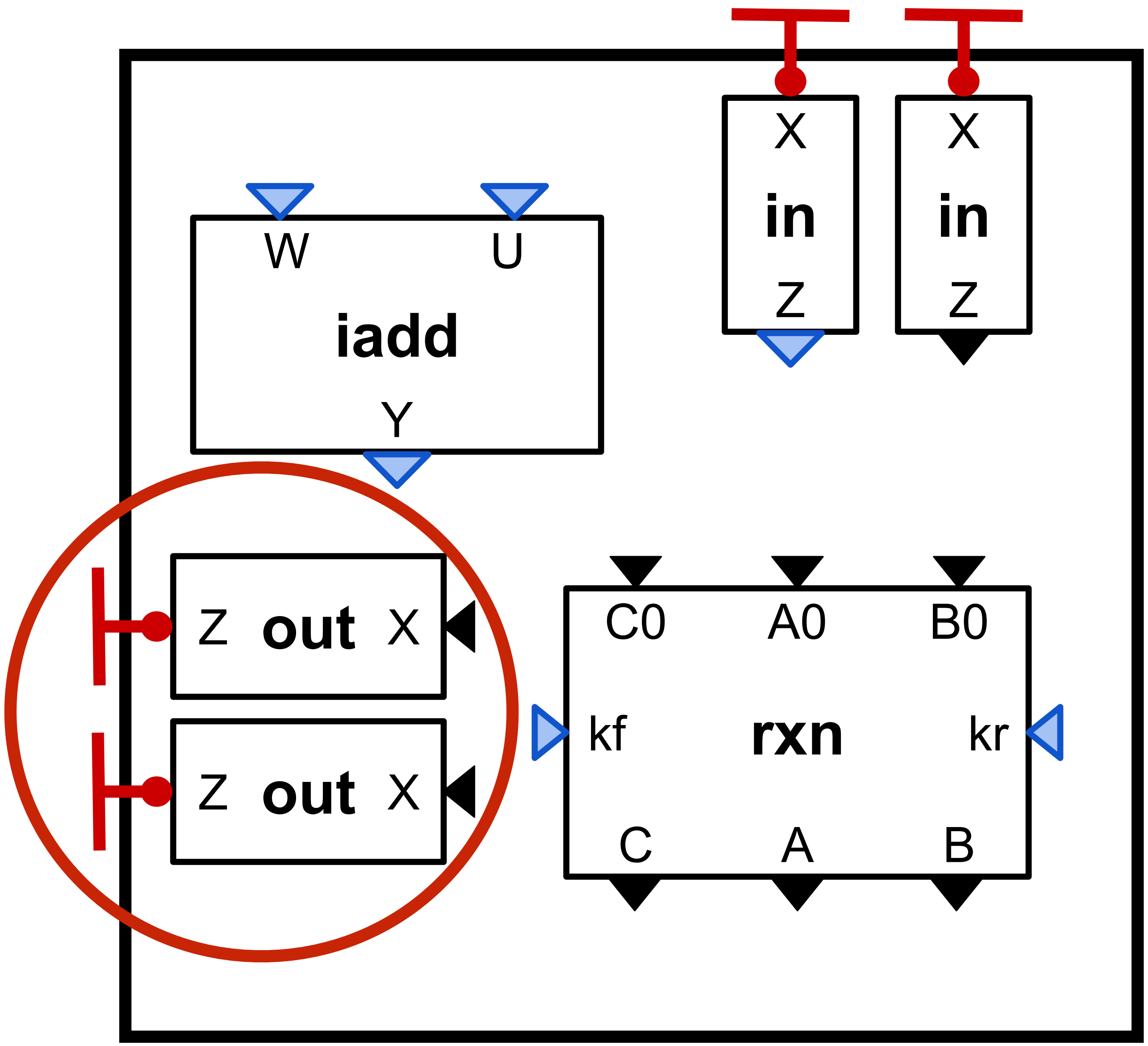
Programmable Analog Device

input components (DAC)



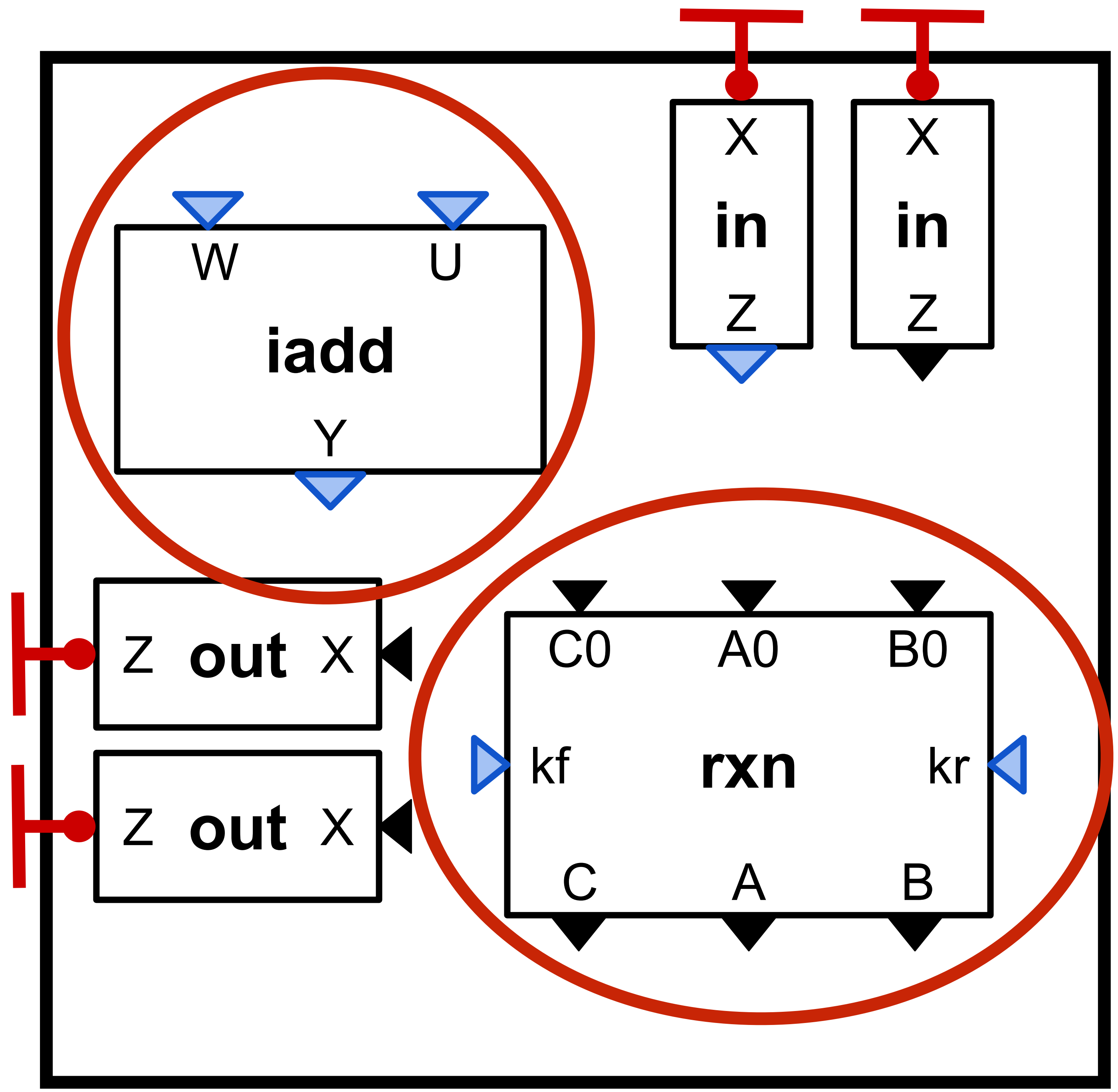
Programmable Analog Device

output components (ADC)

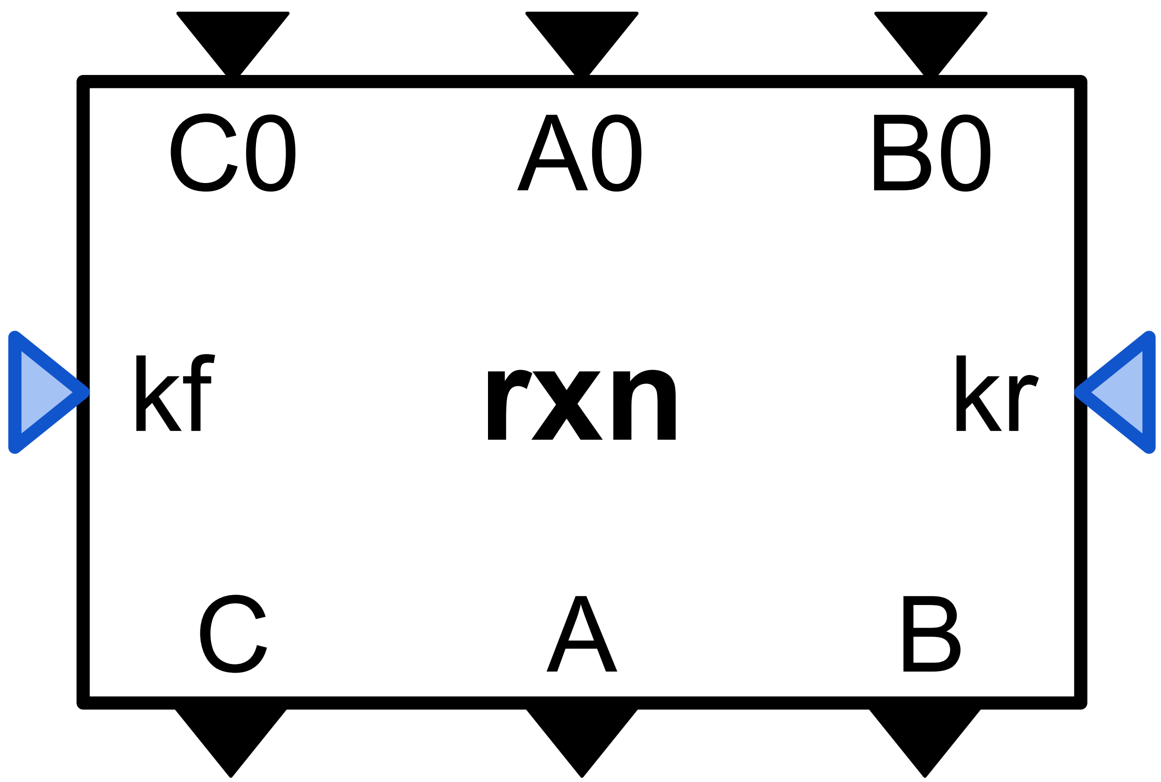


Programmable Analog Device

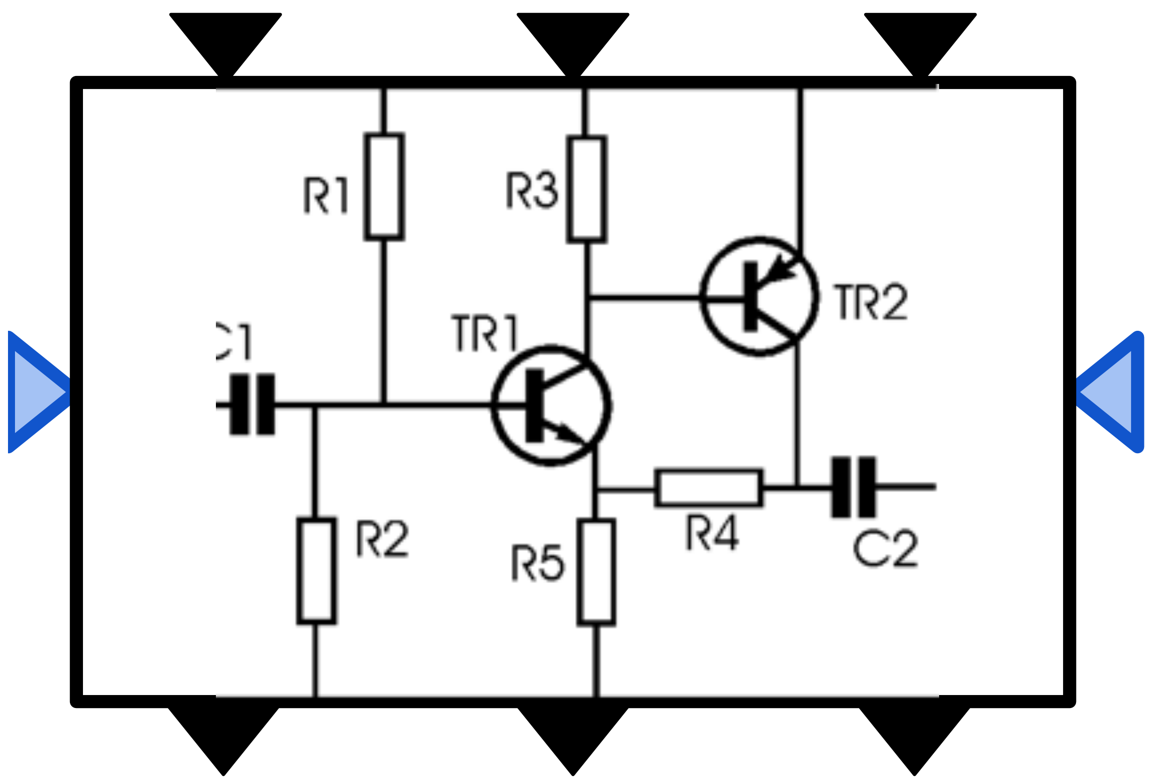
analog computation components



Analog Computation Component

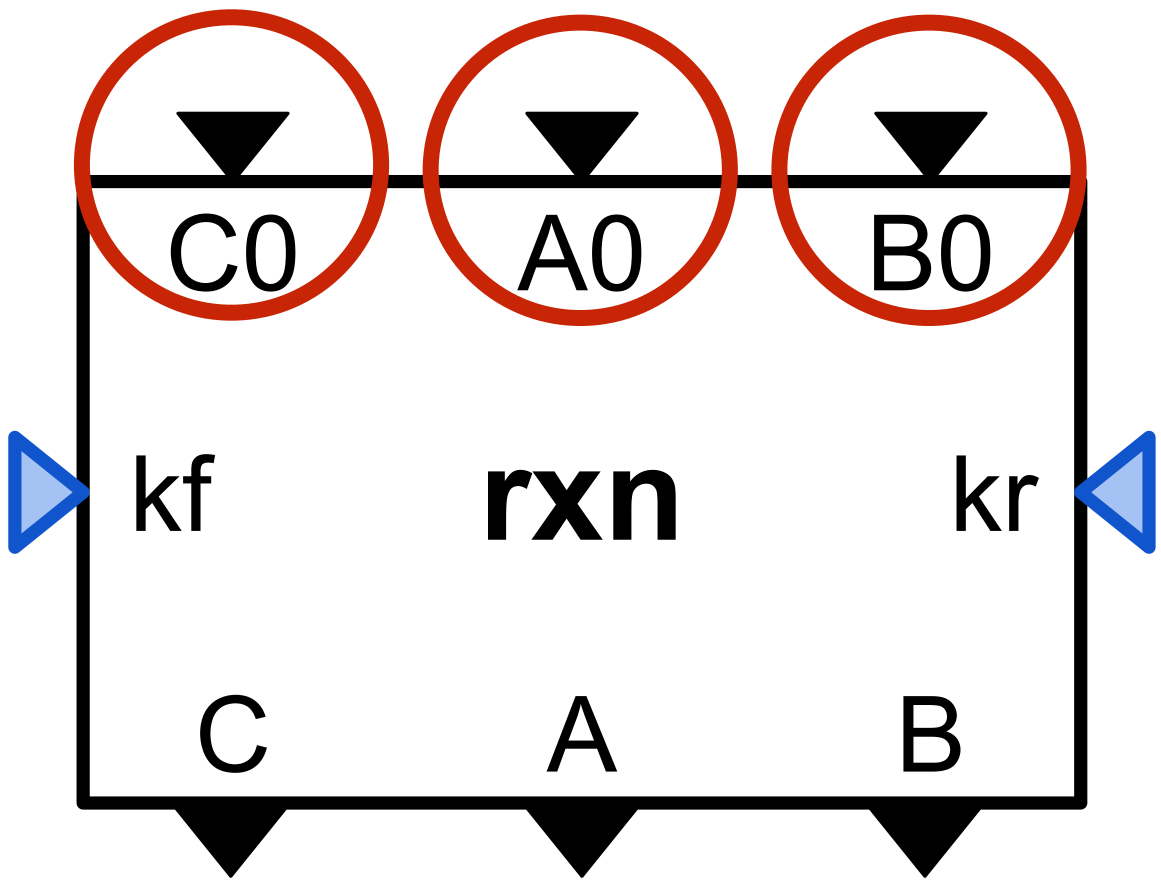


Analog Computational Component



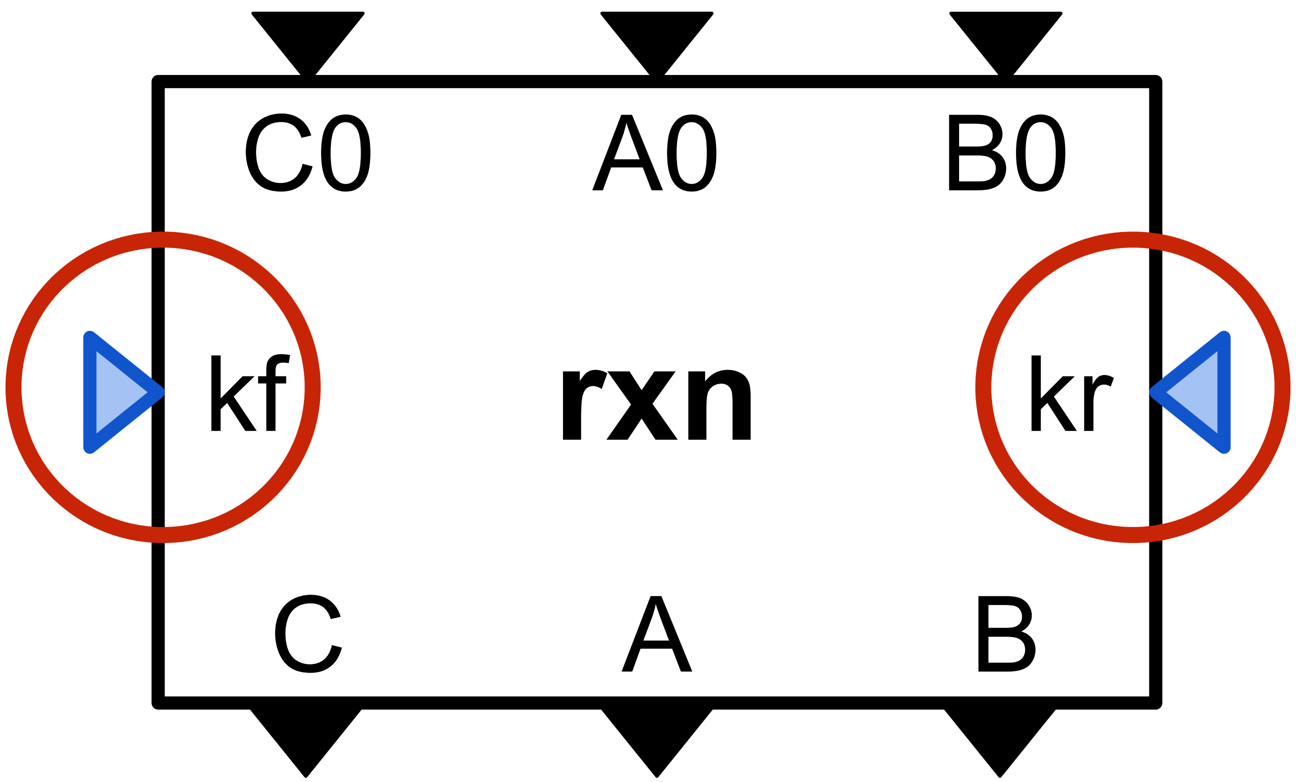
Analog Computational Component

voltage analog inputs



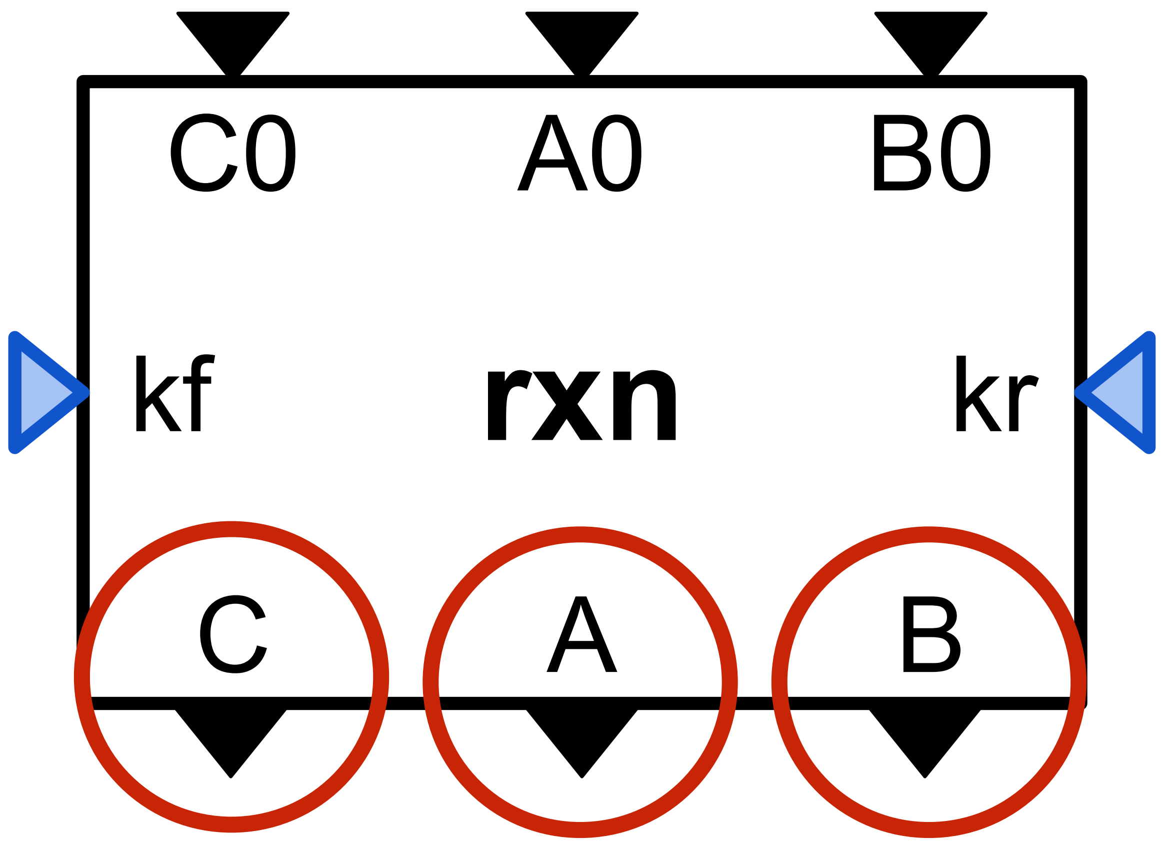
Analog Computational Component

current analog inputs



Analog Computational Component

voltage analog outputs



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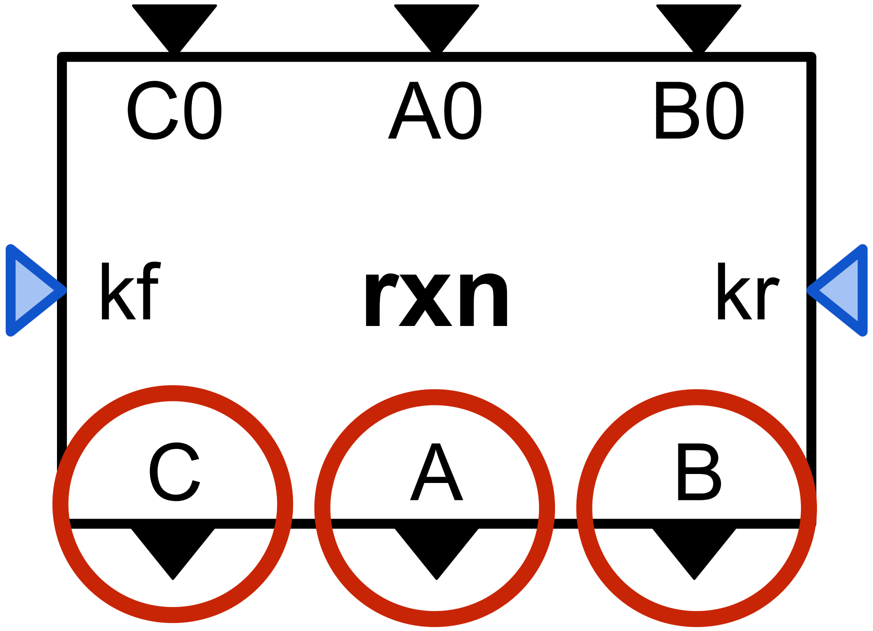

$$A_V = A0_V - C_V$$

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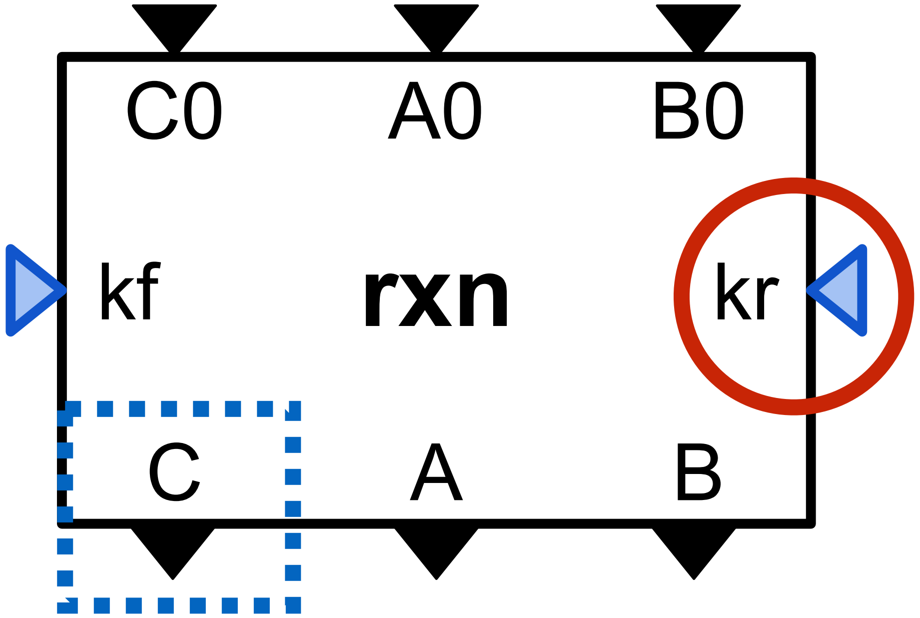
$$C_V(0) = C0_V$$

$$\partial C_V / \partial t = kf_I \cdot A_V \cdot B_V - kr_I \cdot C_V$$

Hardware Component Abstraction



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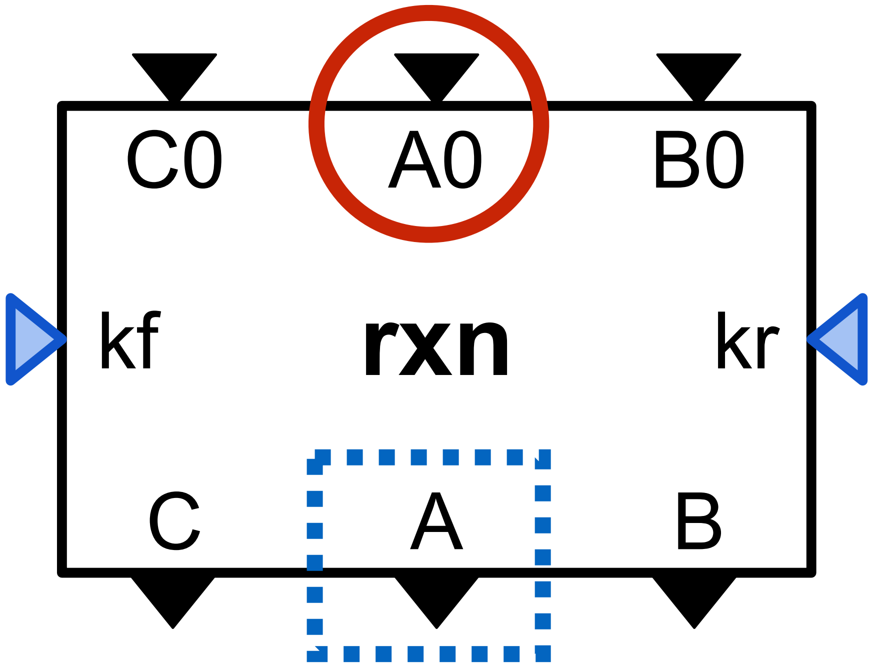

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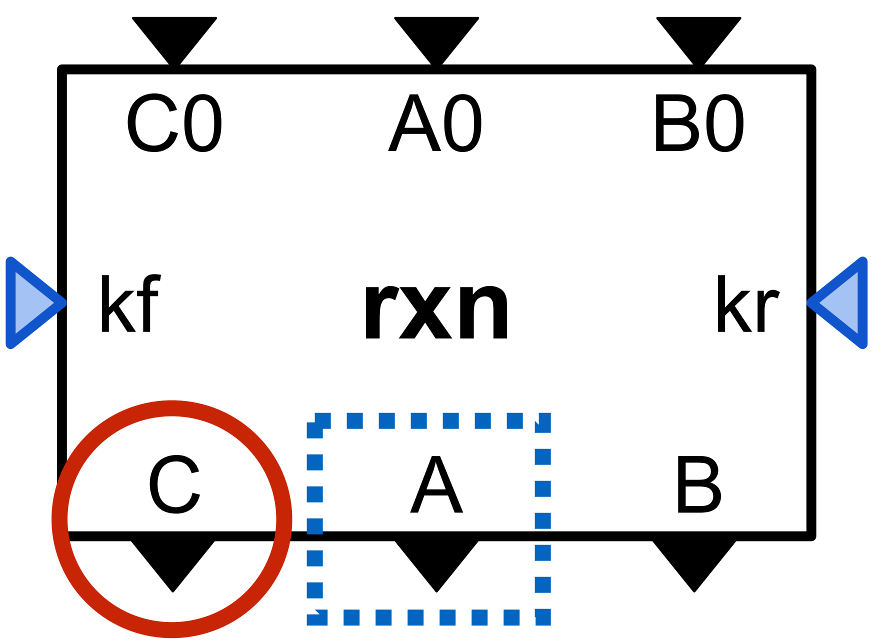

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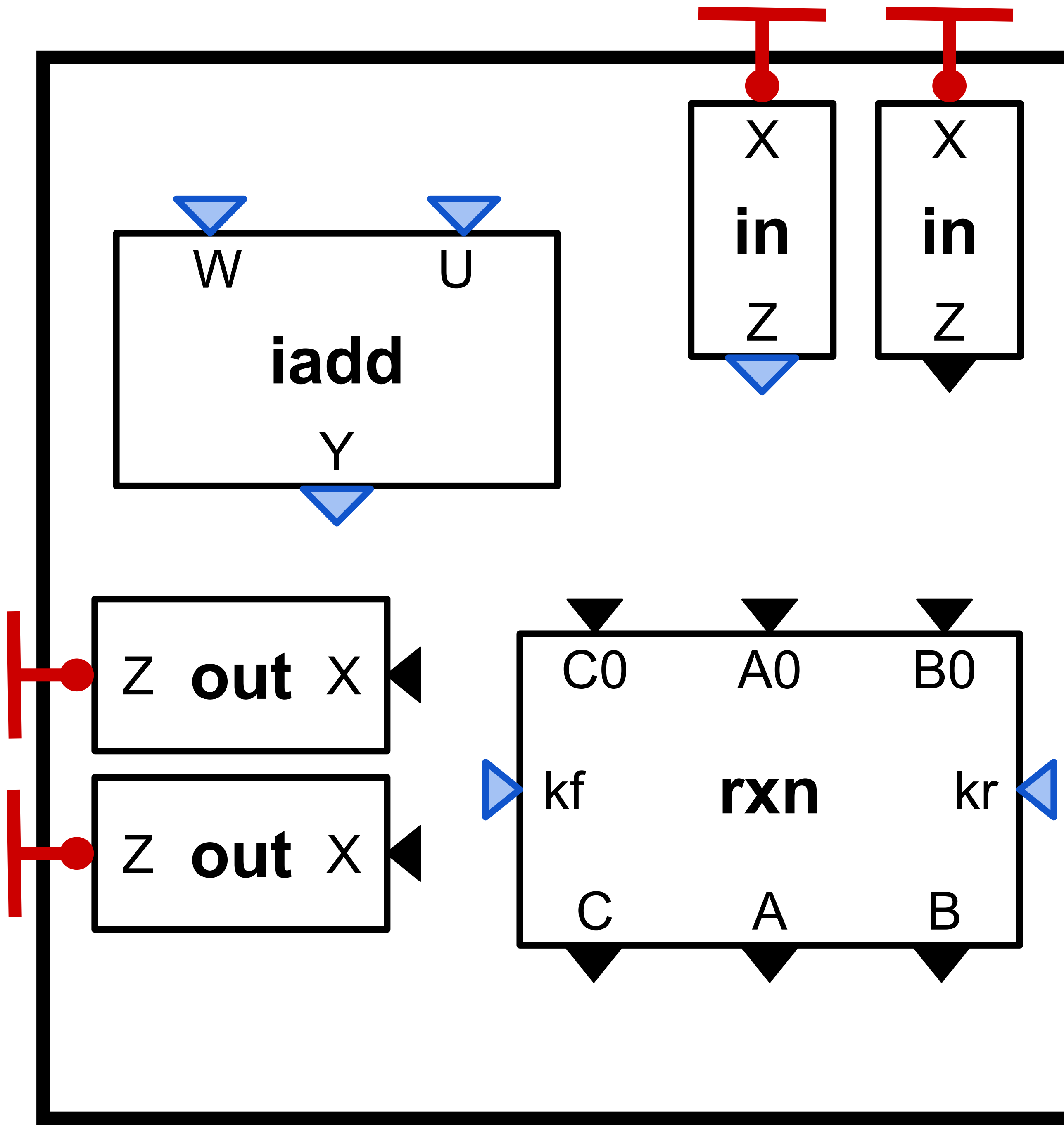
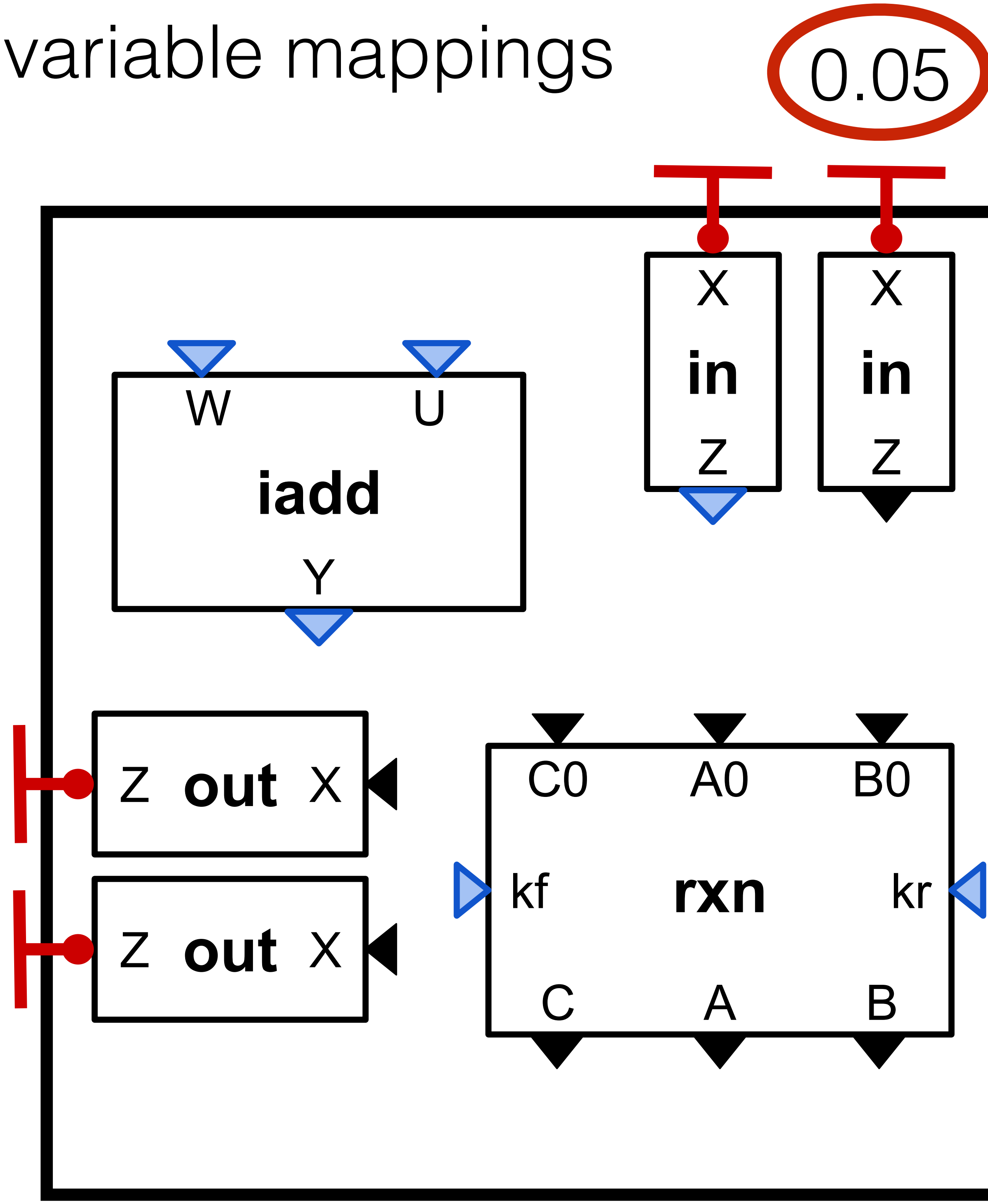


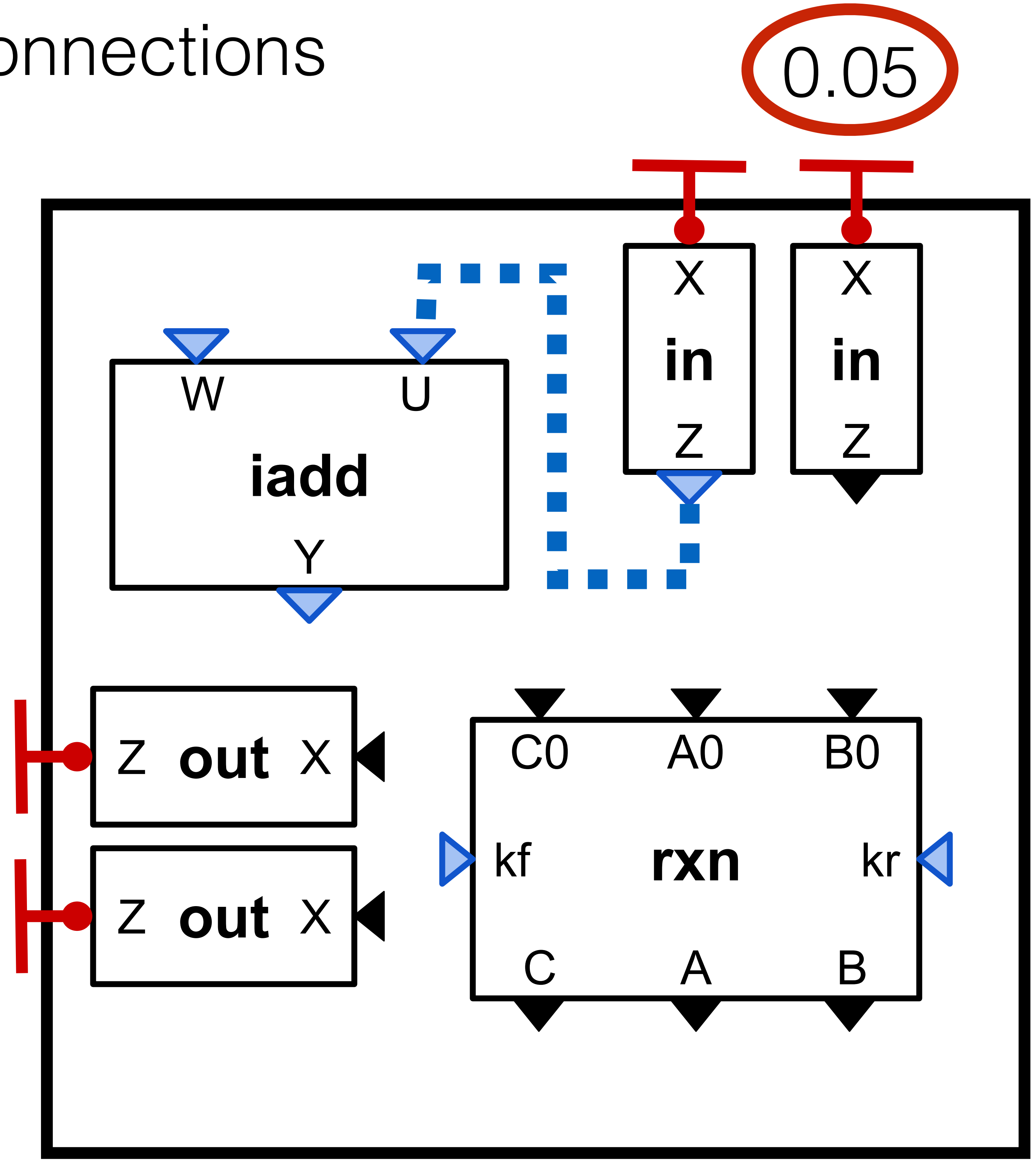
Programmable Analog Device

value and variable mappings



Programmable Analog Device

connections



map dynamical system specification to analog device

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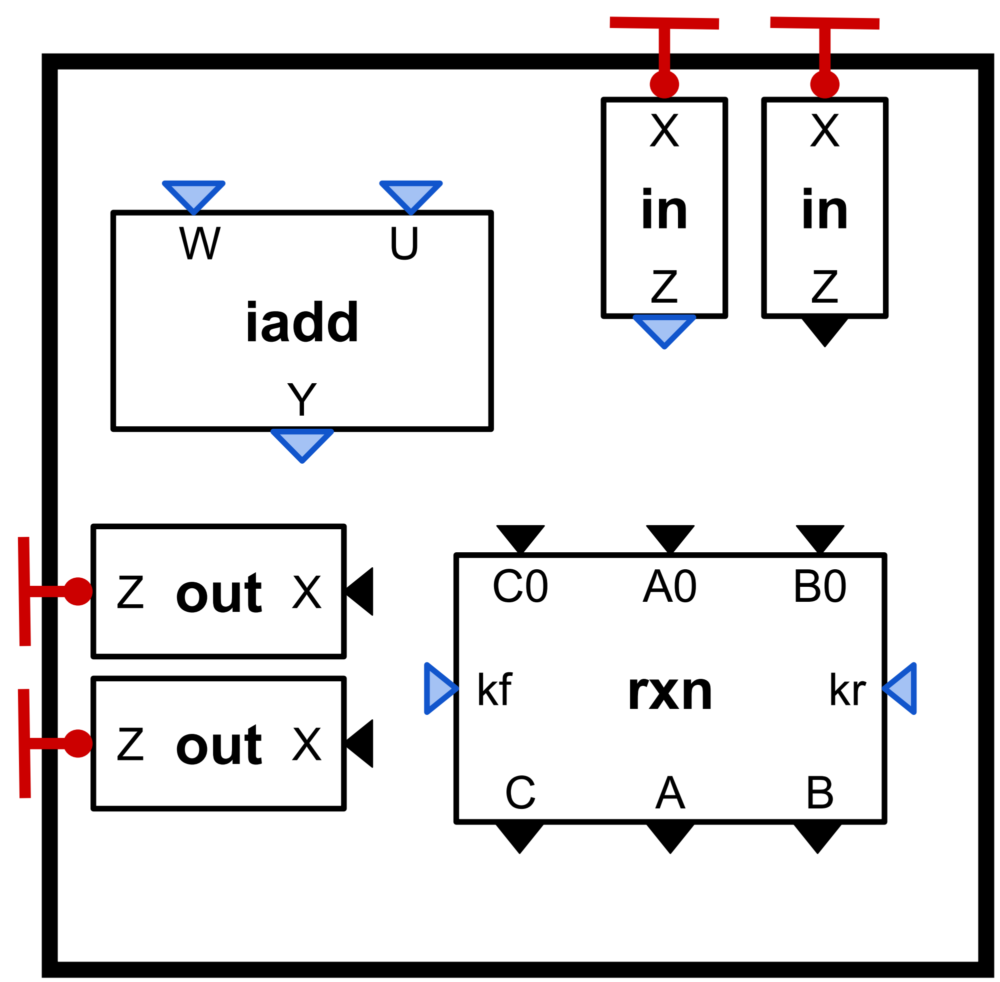

Arco Compiler

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2. Hardware Selection
3. Unification
4. Relation Entanglement
5. Input & Output Components

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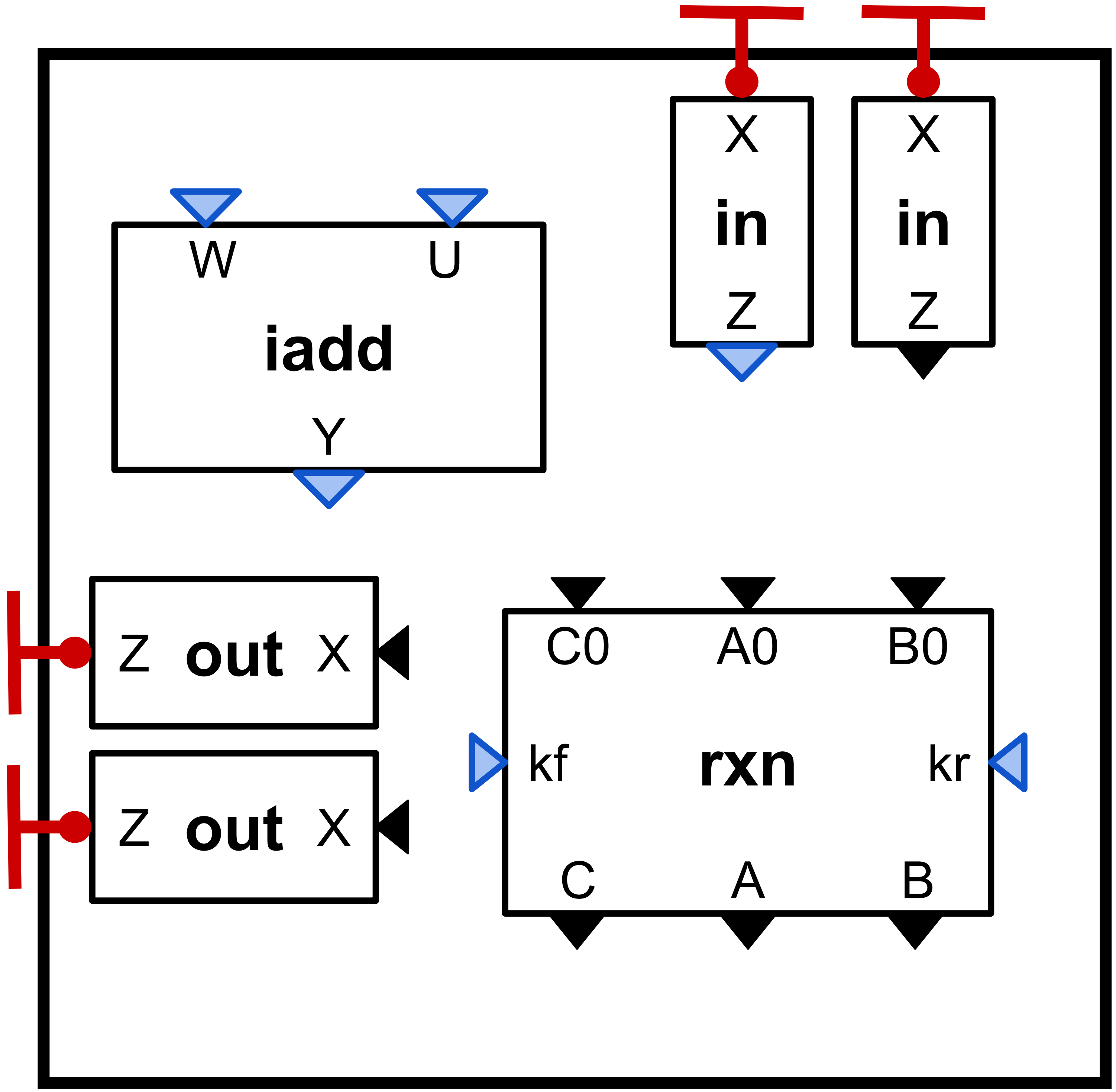

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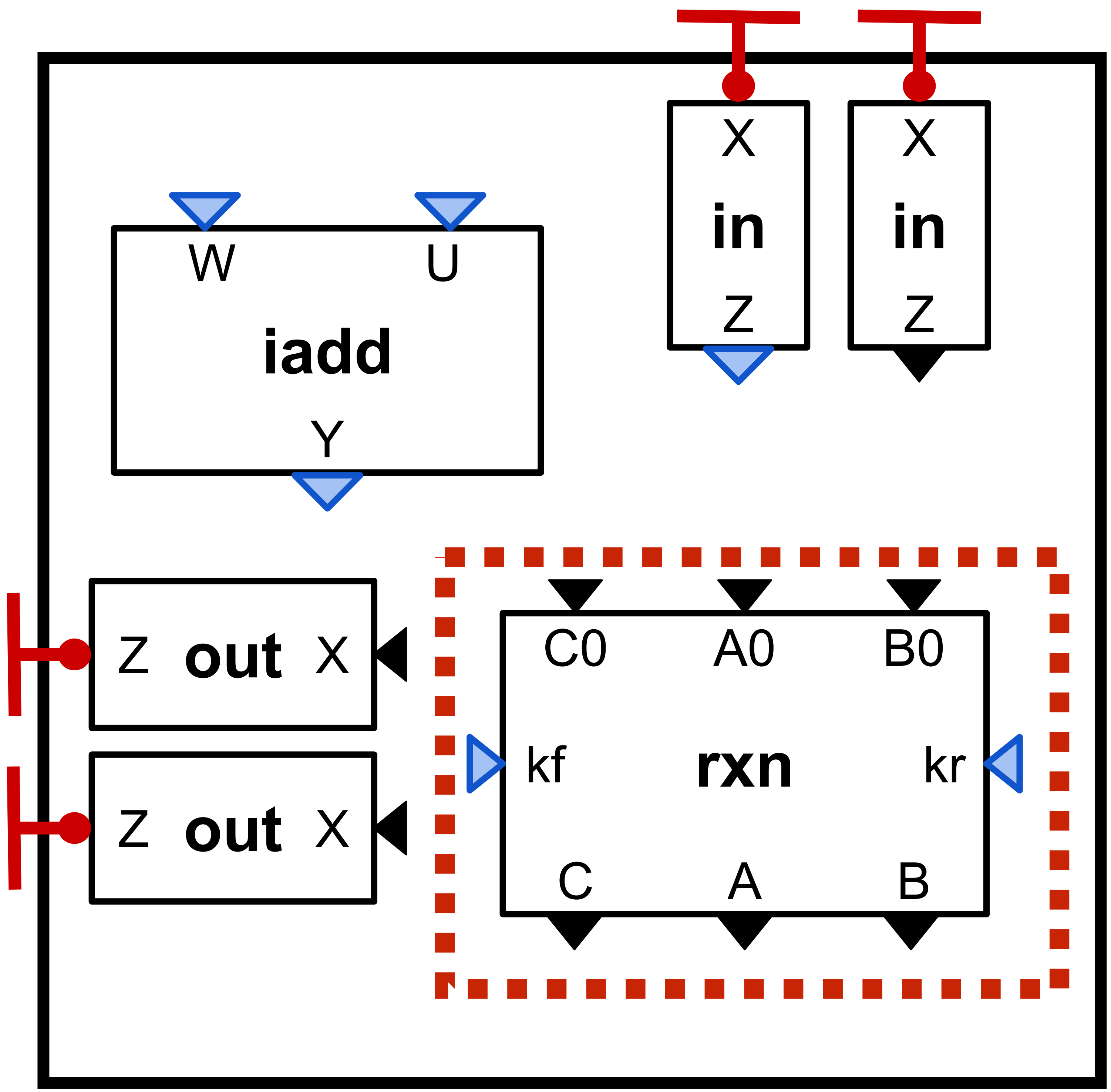

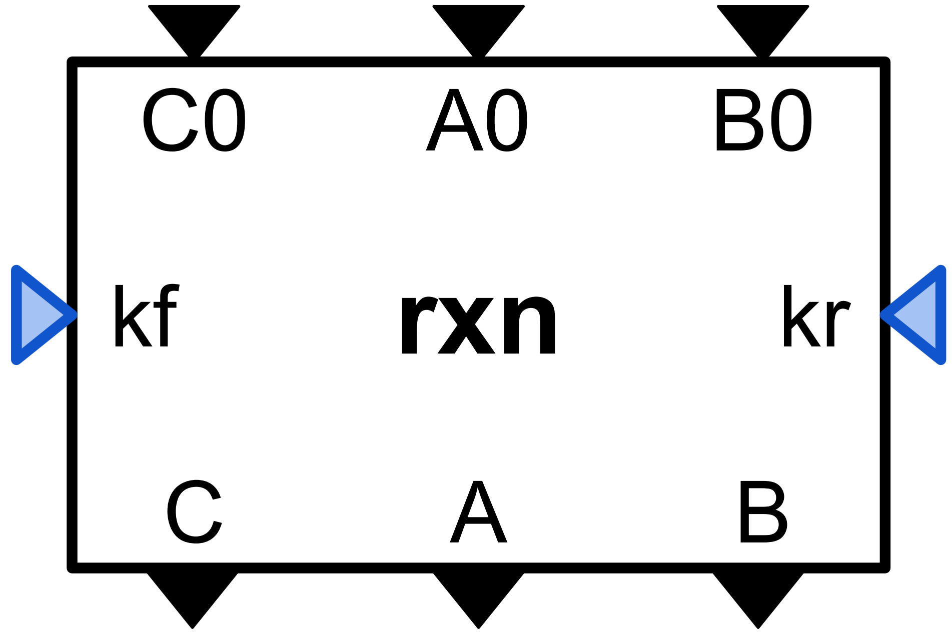

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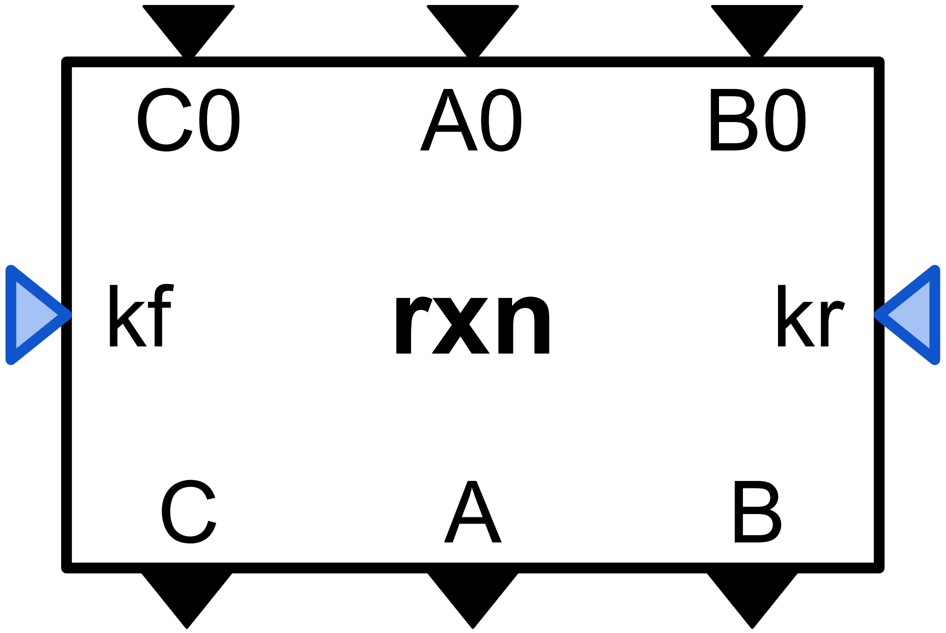

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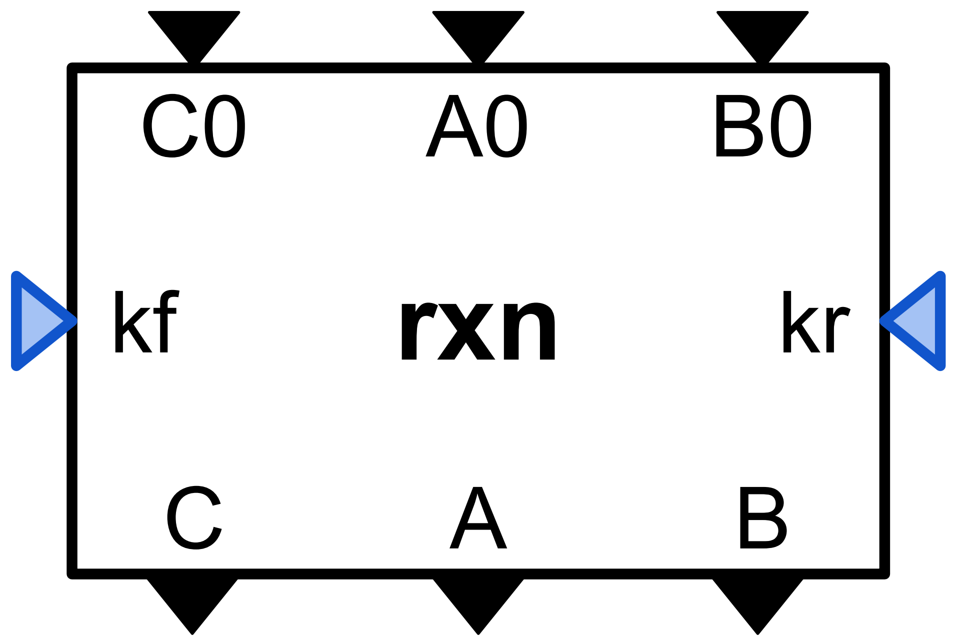

$$S = S_{tot} - ES \longmapsto A_V = A0_V - C_V$$

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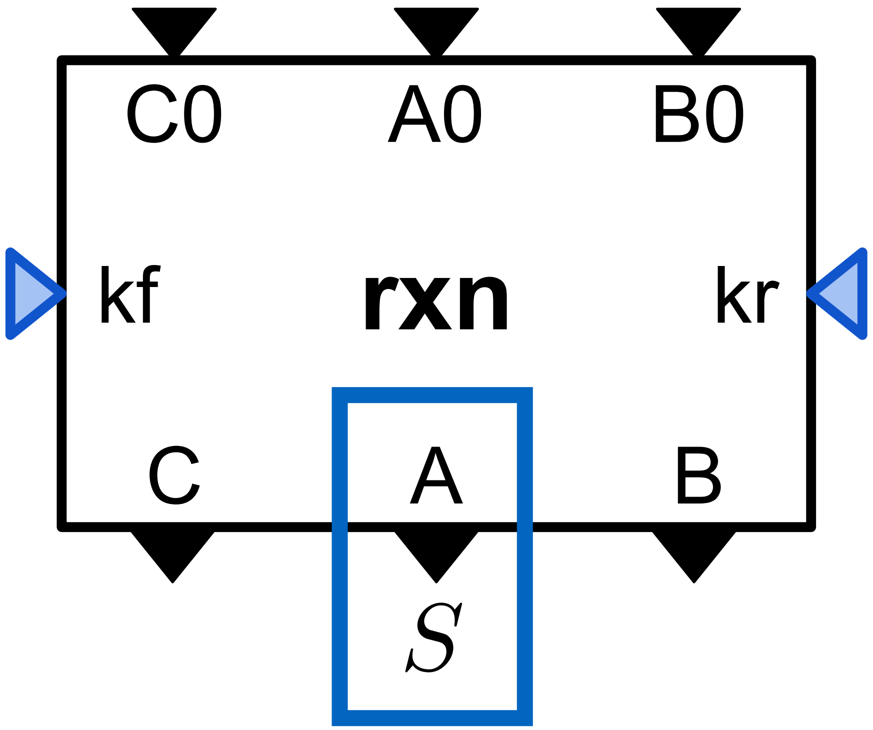
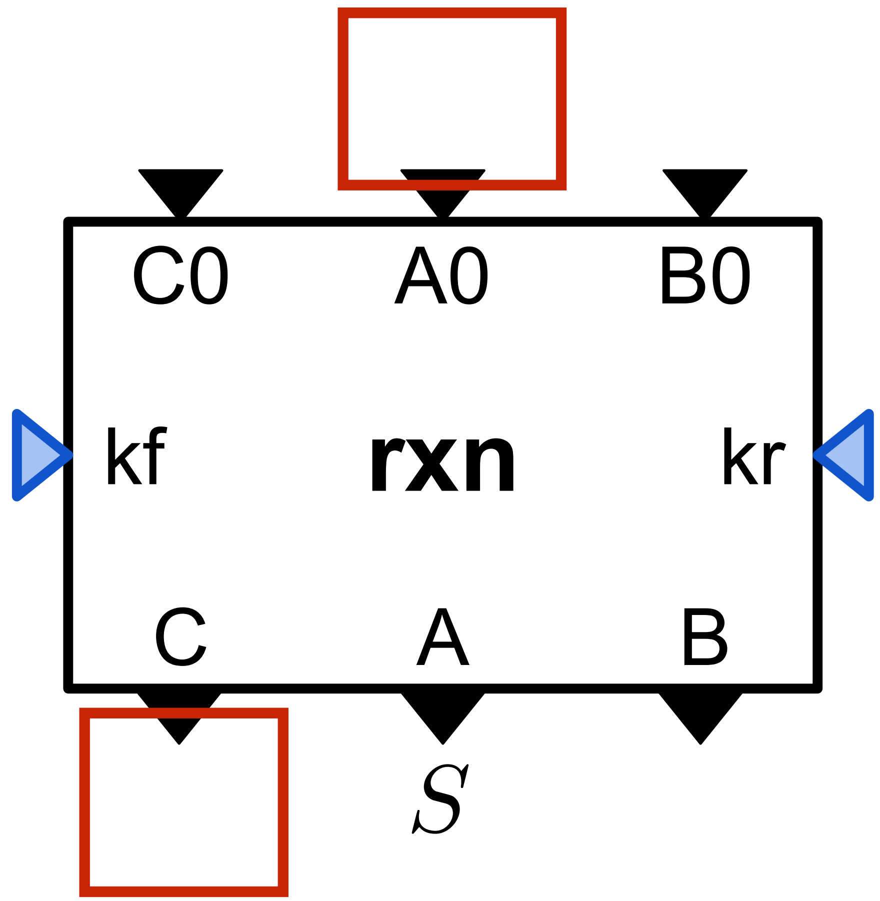


$$S \mapsto A_V$$

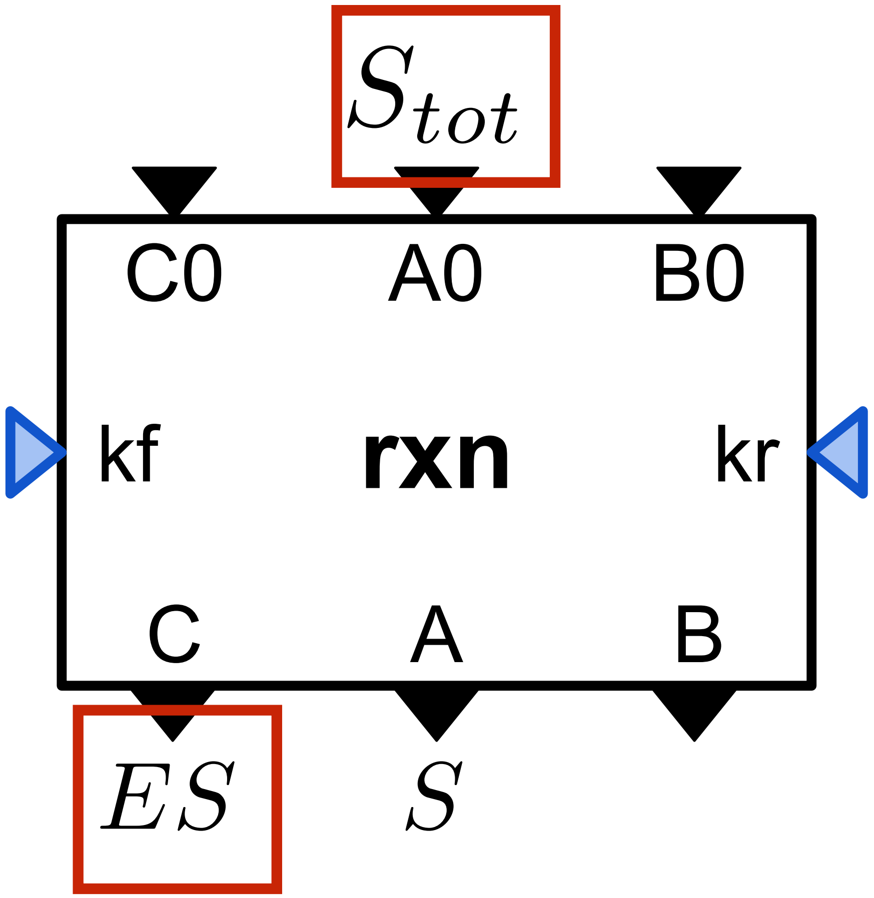
$$S = S_{tot} - ES \qquad A_V = A0_V - C_V$$

$$S = A_V$$



$$S_{tot} - ES \mapsto \underline{A0_V} - \underline{C_V}$$

Unification: Maps variables, expressions onto hardware variables s.t. the relations are algebraically equivalent



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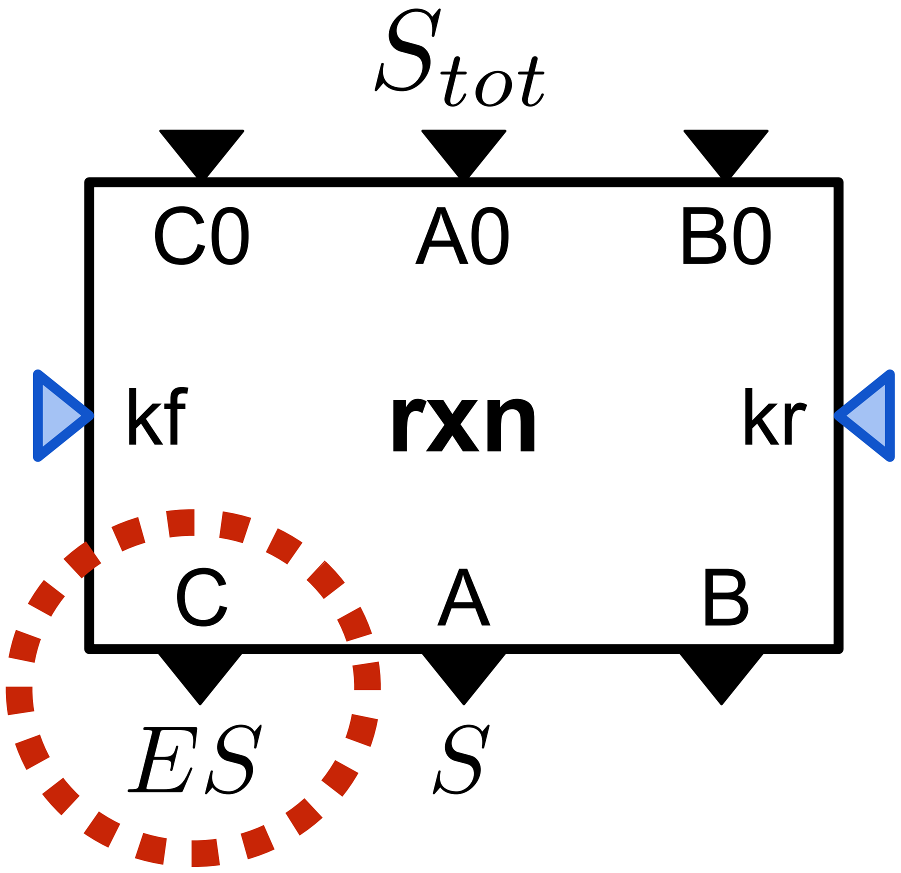

C_V and ES are entangled

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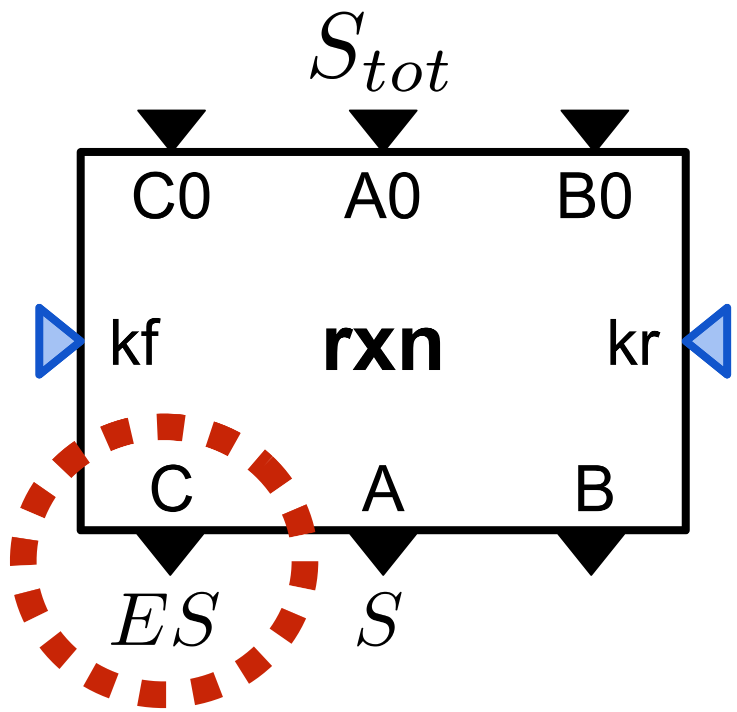

Problem: C_V and ES are entangled

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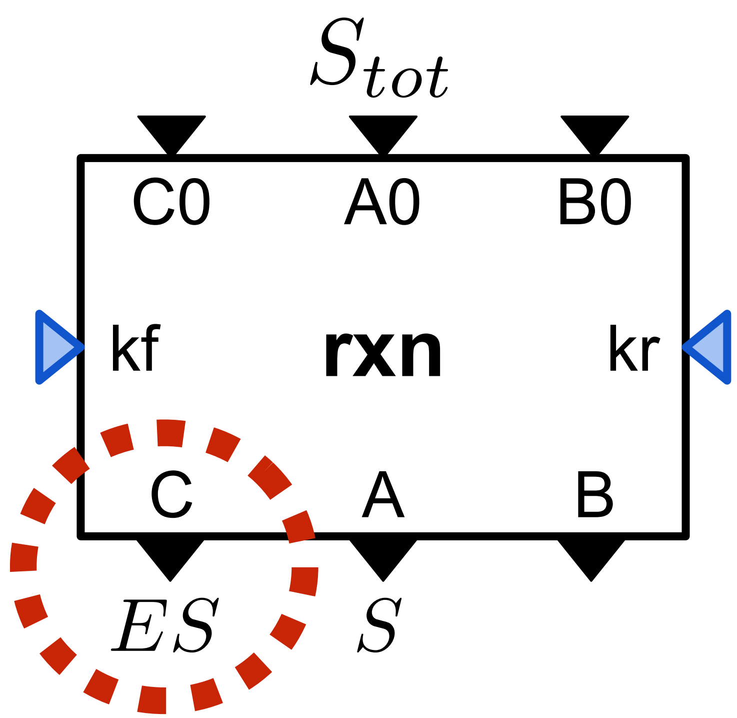

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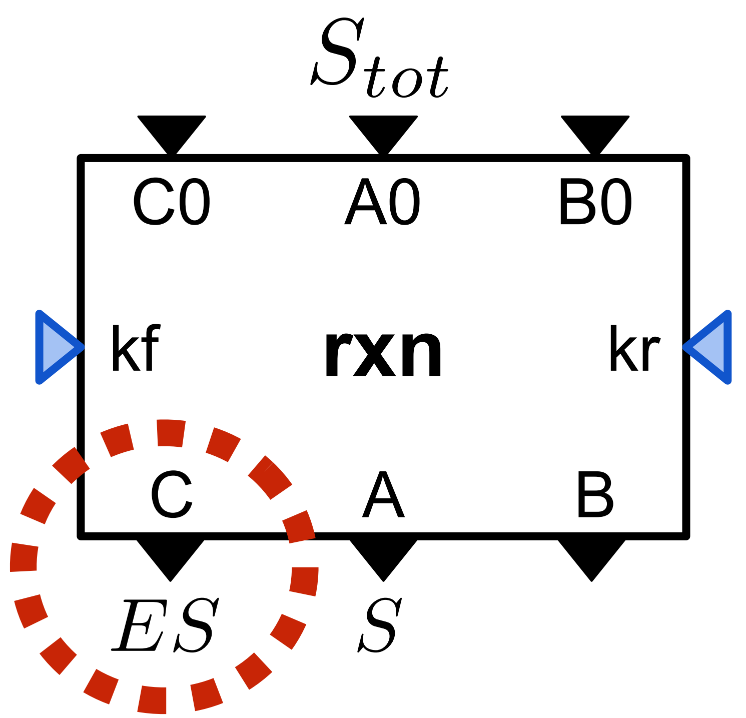

Problem: C_V and ES are entangled

$$\partial C_V / \partial t = kf_I \cdot S \cdot B_V - kr_I \cdot ES \quad \bigwedge$$
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$$C_V(0) = C0_V \quad \bigwedge$$
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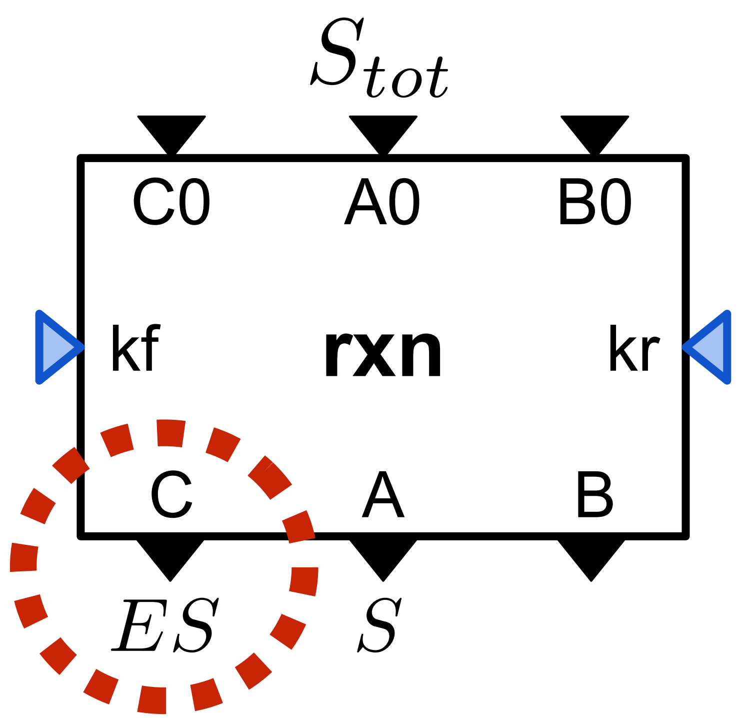
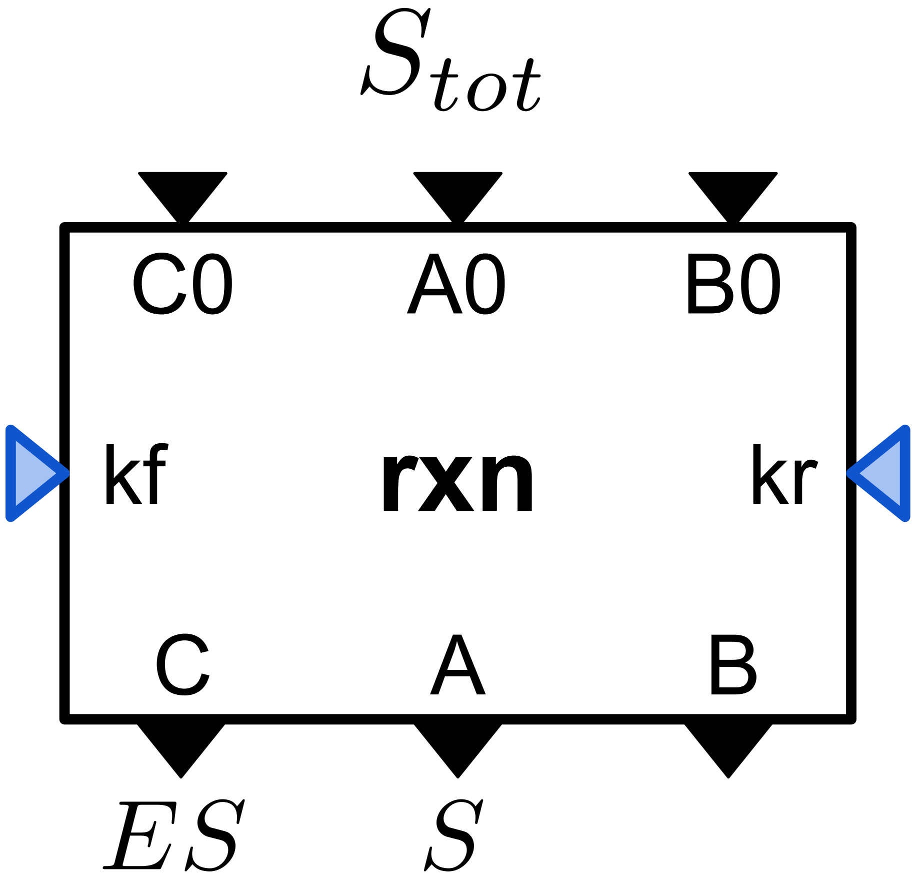


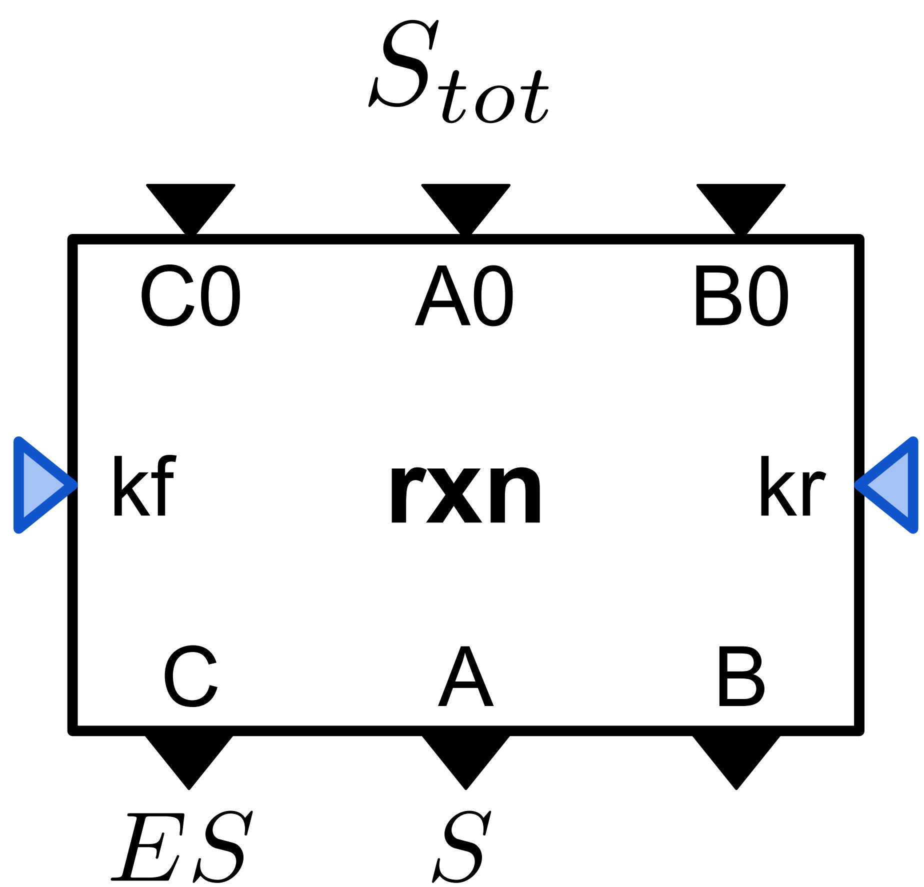
Solution: ***detangle*** C_V and ES through Unification

$$(Q + k_0) \cdot E \cdot S - k_r \cdot ES \mapsto kf_I \cdot S \cdot B_V - kr_I \cdot ES$$

$$0.423 \mapsto C0_V$$

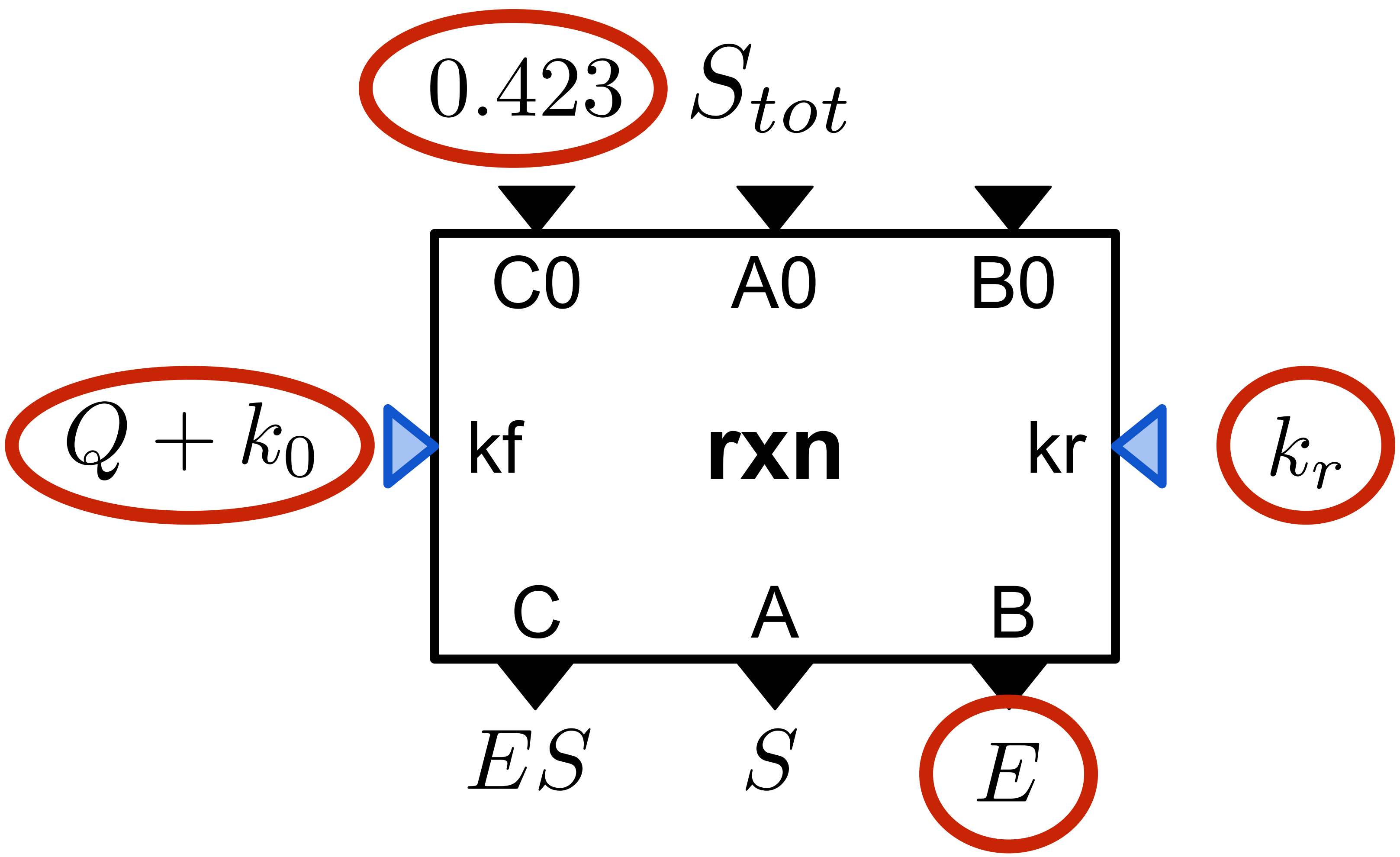


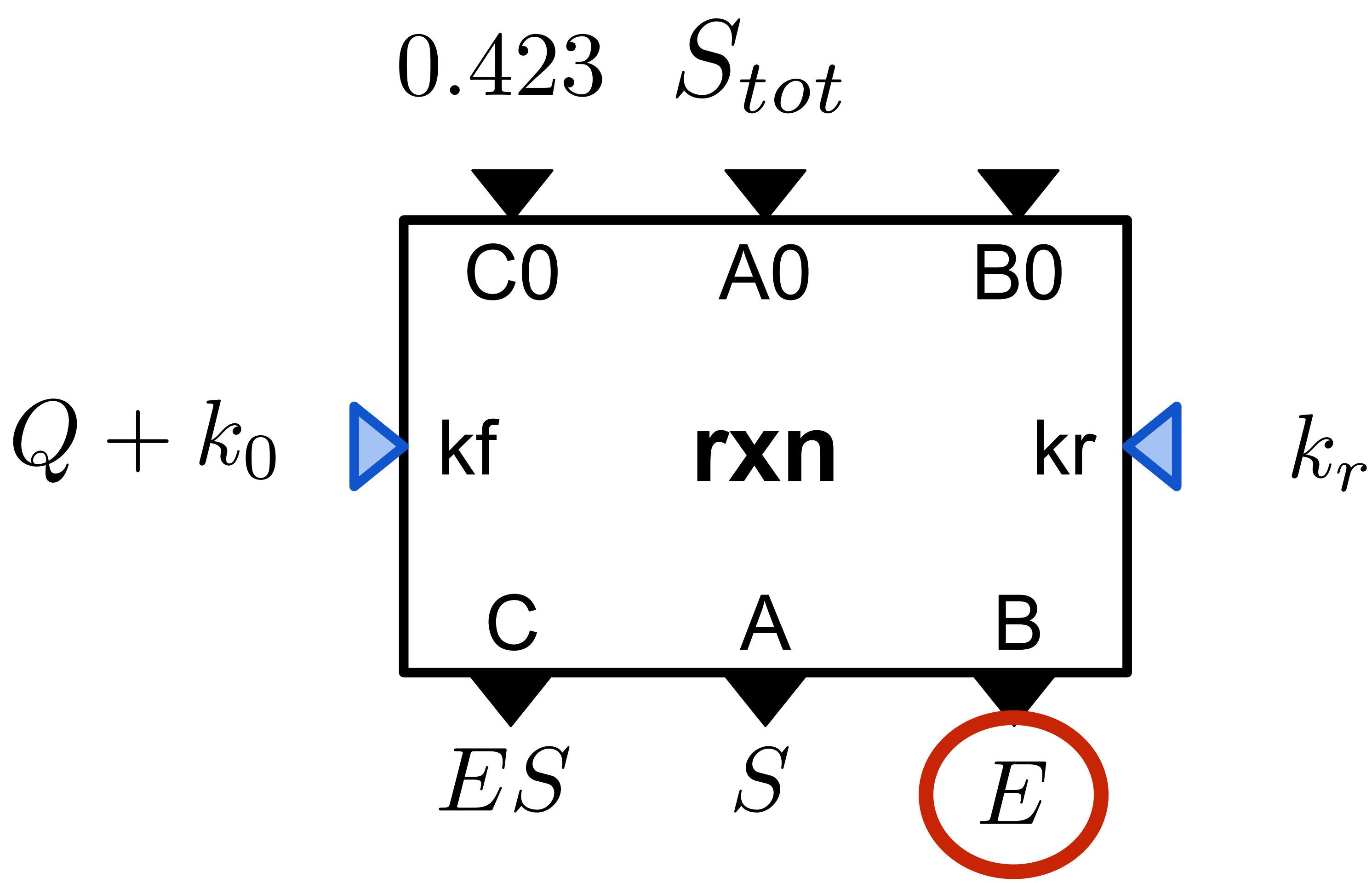
S_{tot}
C0
A0
B0
kf
rxn
kr
C
A
B
ES
S



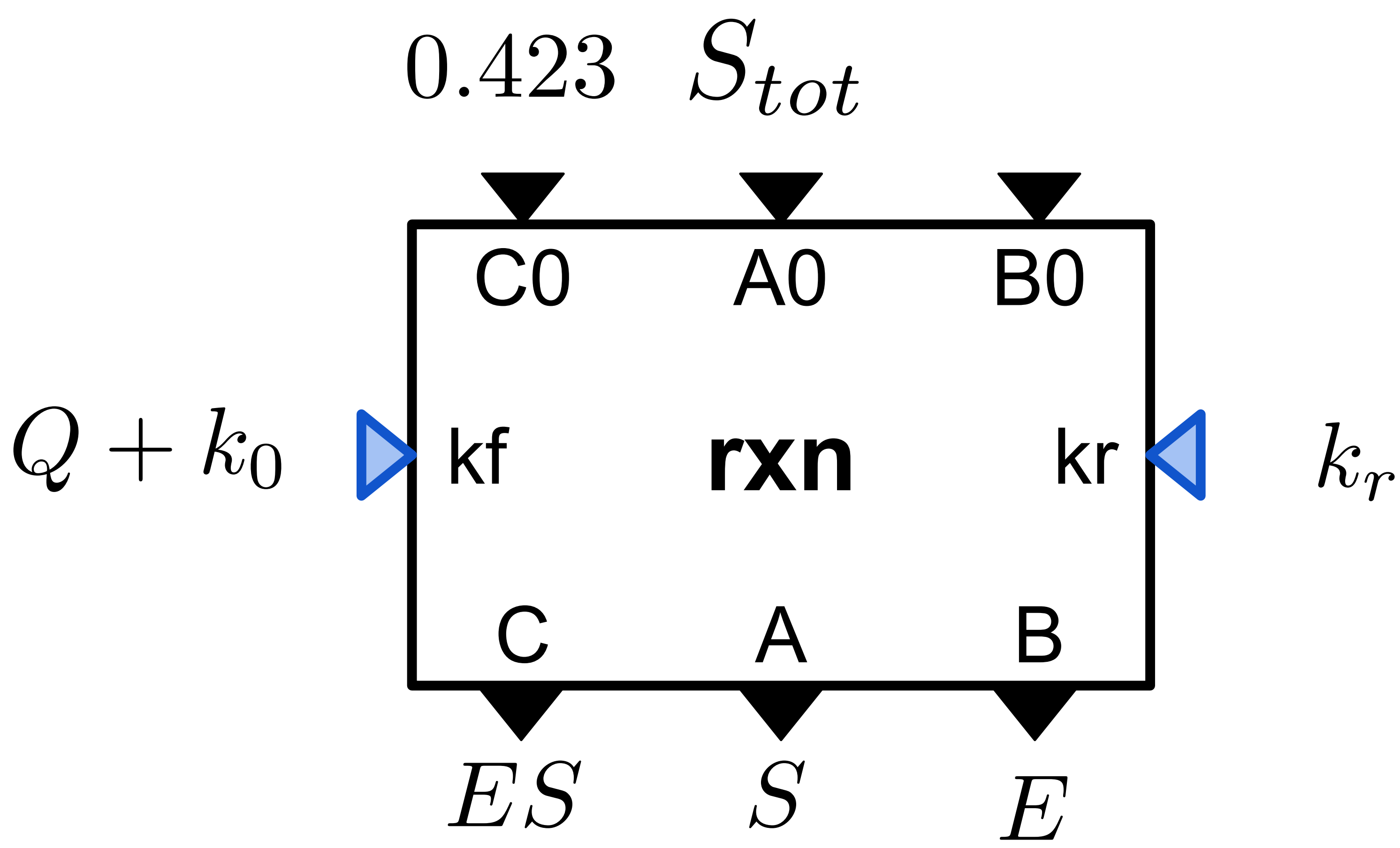
$$(Q + k_0) \cdot E \cdot S - k_r \cdot ES \mapsto kf_I \cdot S \cdot B_V - kr_I \cdot ES$$

$$0.423 \mapsto C0_V$$

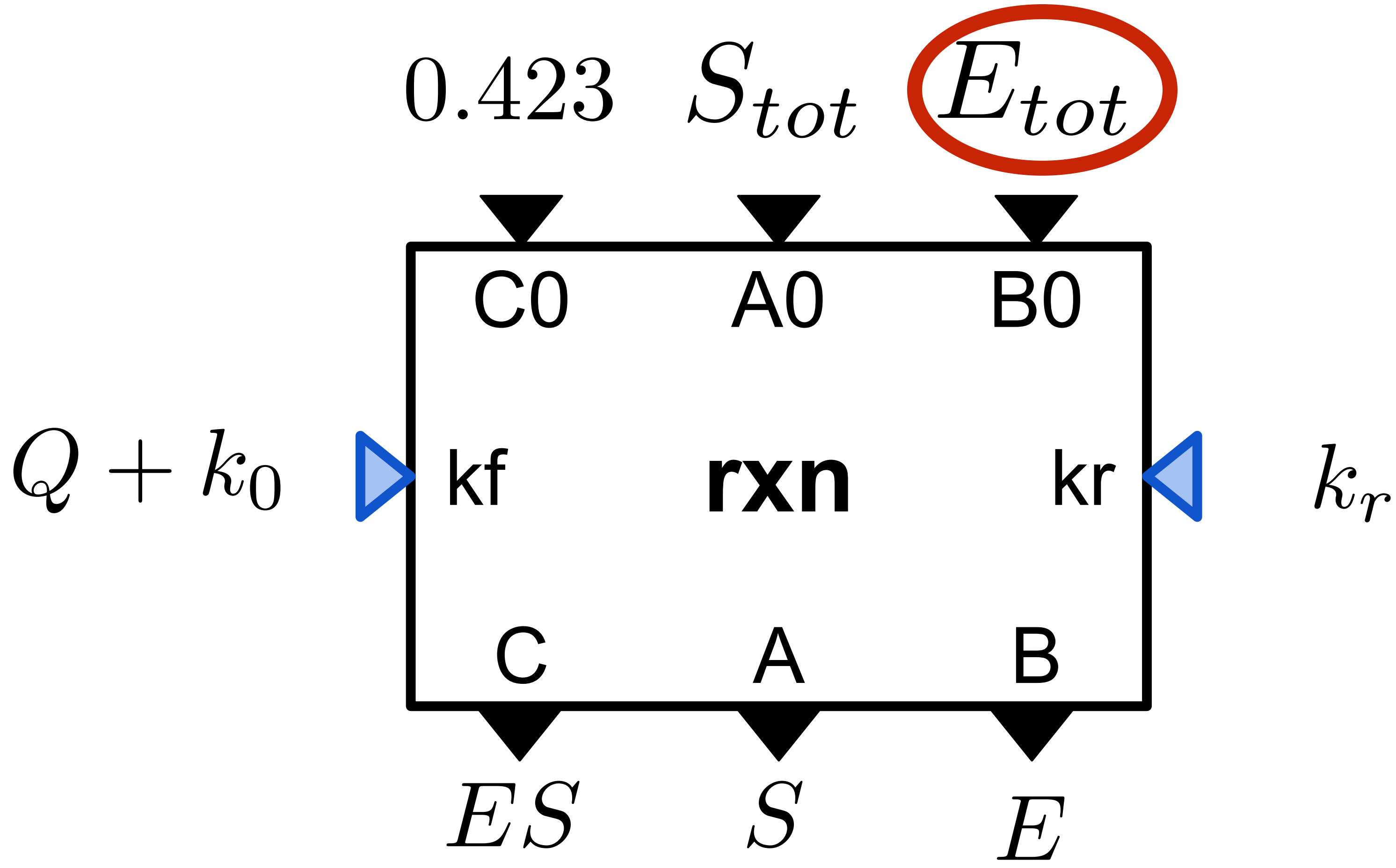

0.423
S_{tot}
C0
A0
B0
$Q + k_0$
kf
rxn
kr
k_r
C
A
B
ES
S
E



B_V and E are entangled



$$E_{tot} - ES \mapsto B0_V - ES$$


0.423
S_{tot}
E_{tot}
C0
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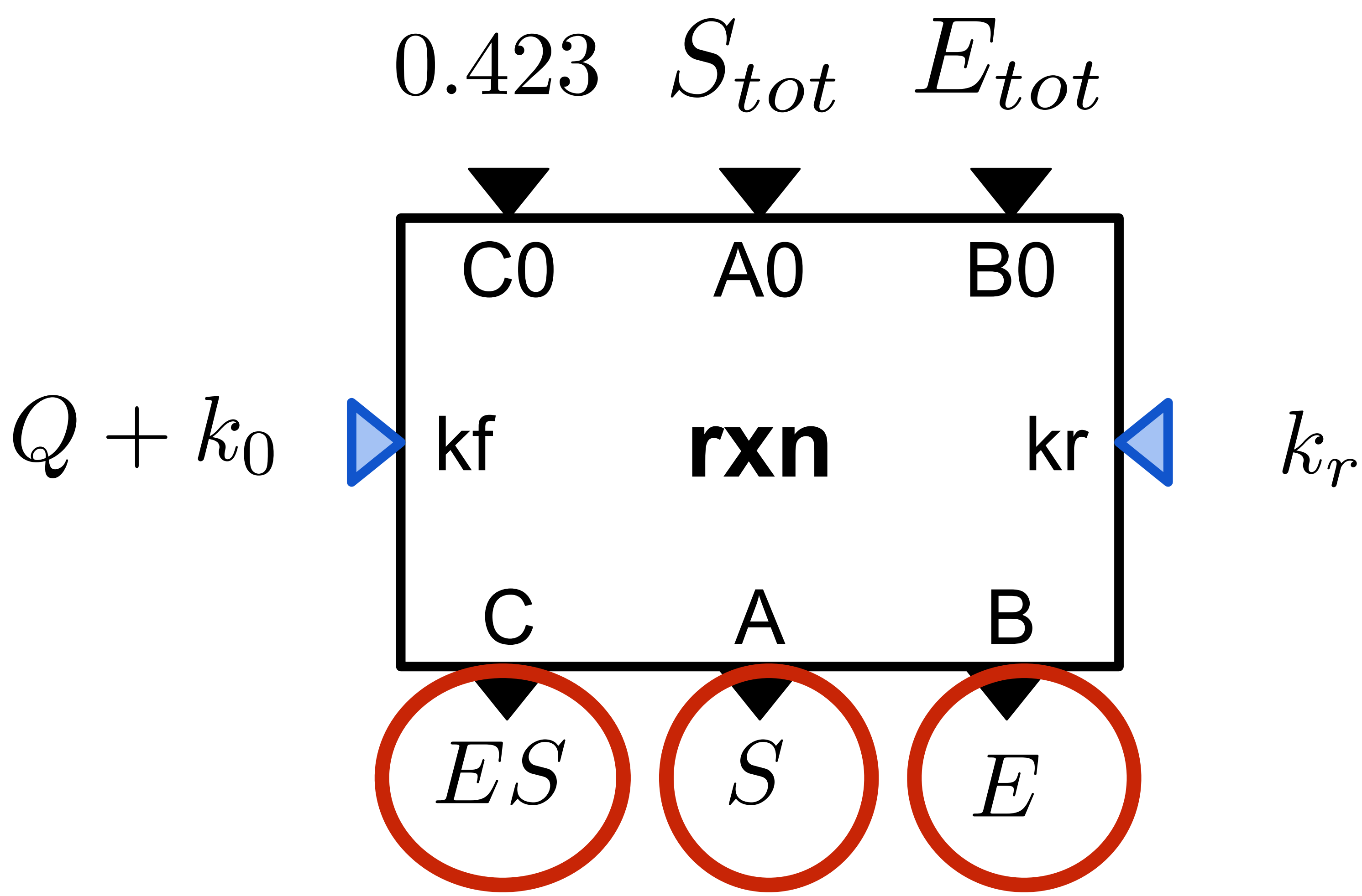
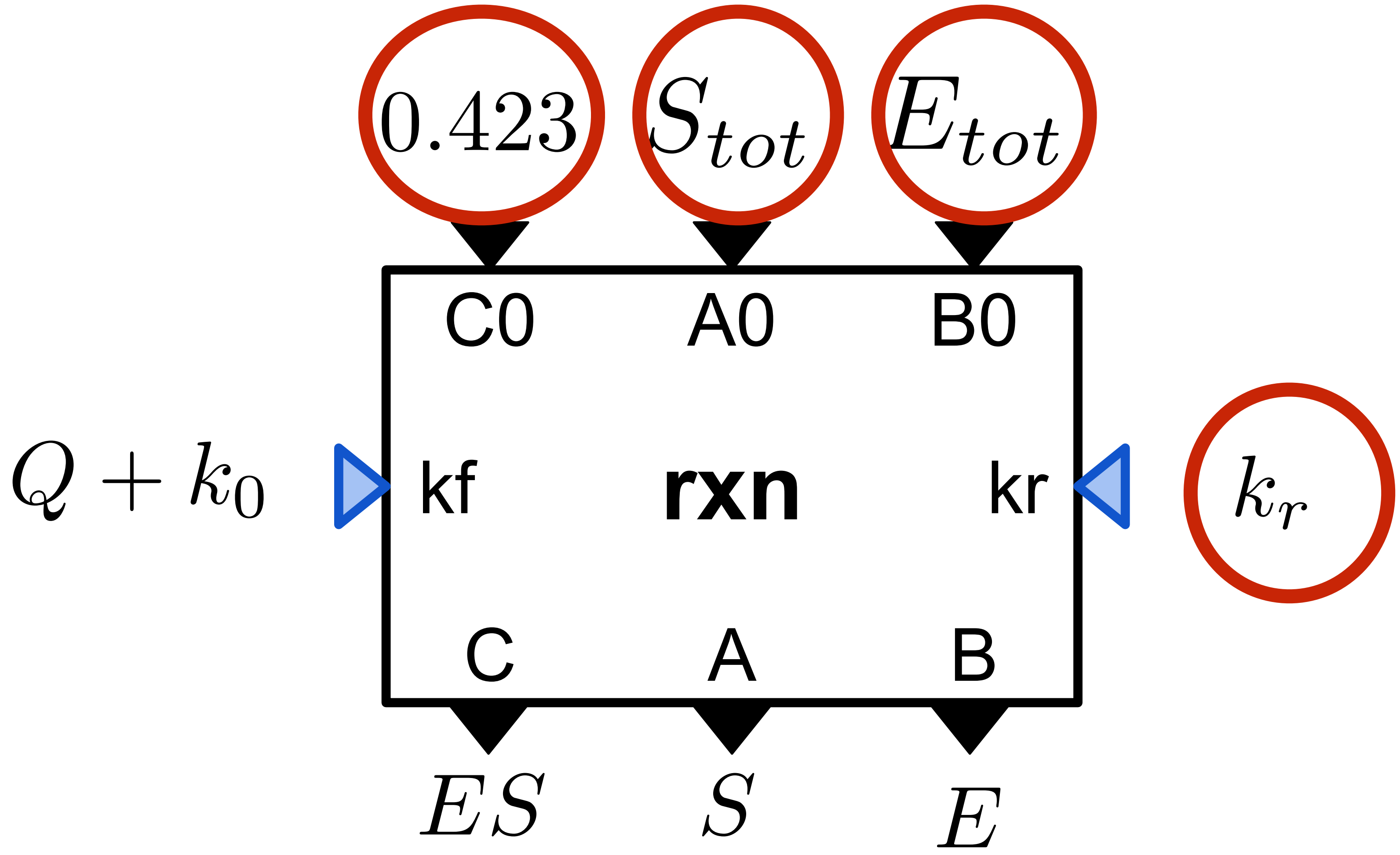
C
A
B
ES
S
E



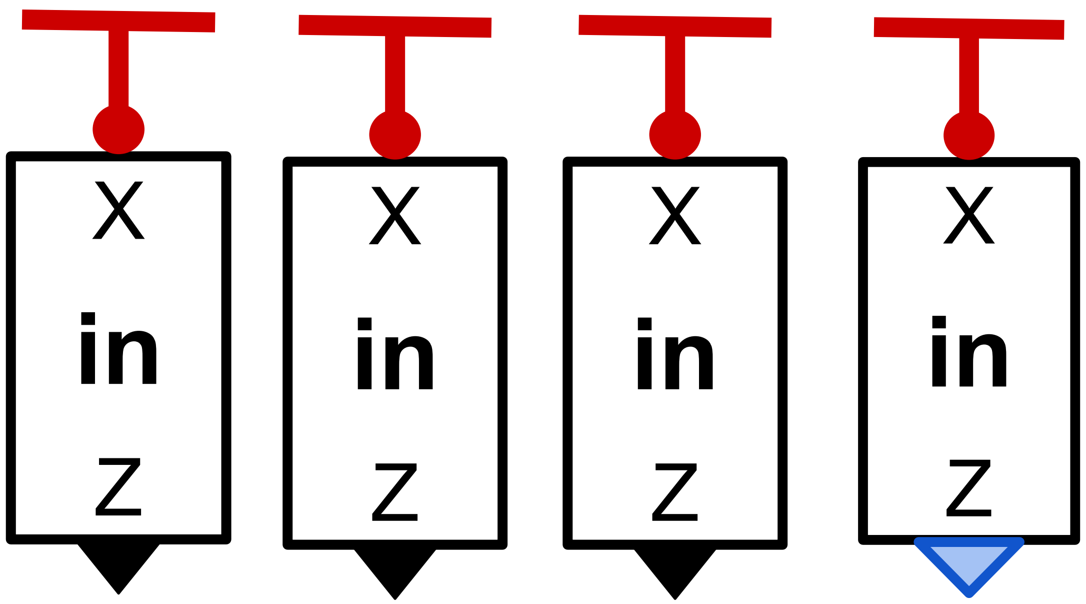
specialized to model ES, S, E


0.423
S_{tot}
E_{tot}
C0
A0
B0
$Q + k_0$
kf
rxn
kr
k_r
C
A
B
ES
S
E

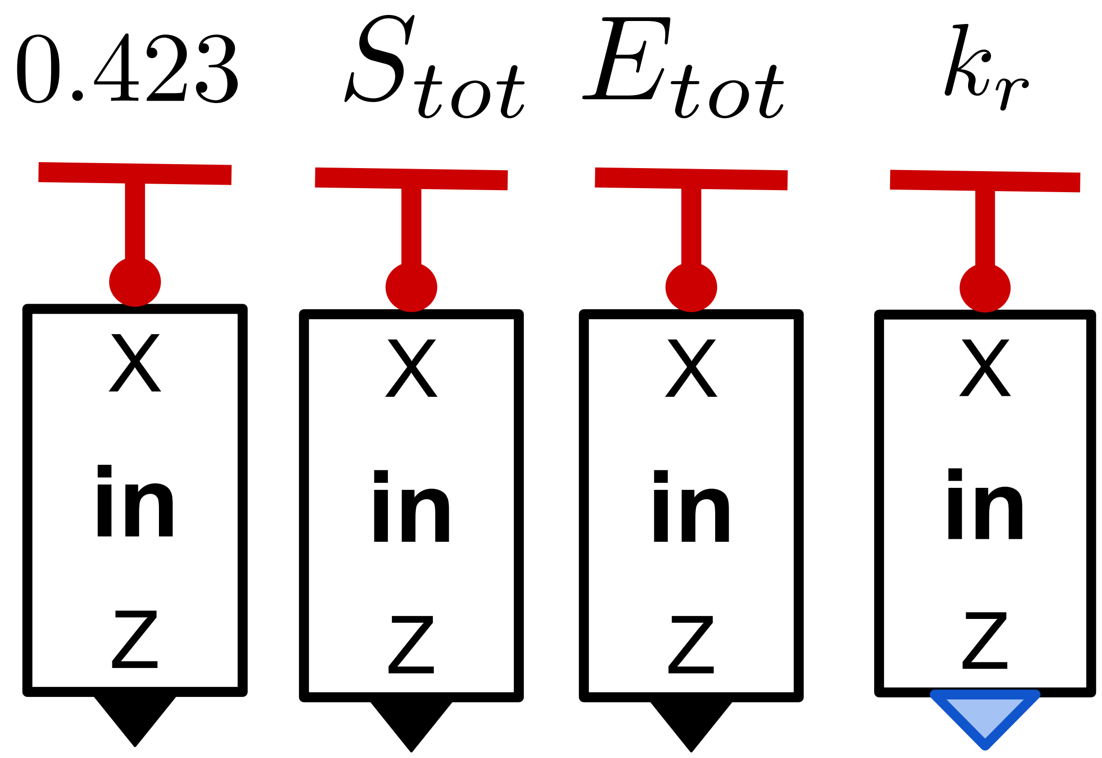
Arco Compiler

1. Equation Selection
2. Hardware Selection
3. Unification
4. Relation Entanglement
5. Input & Output Components

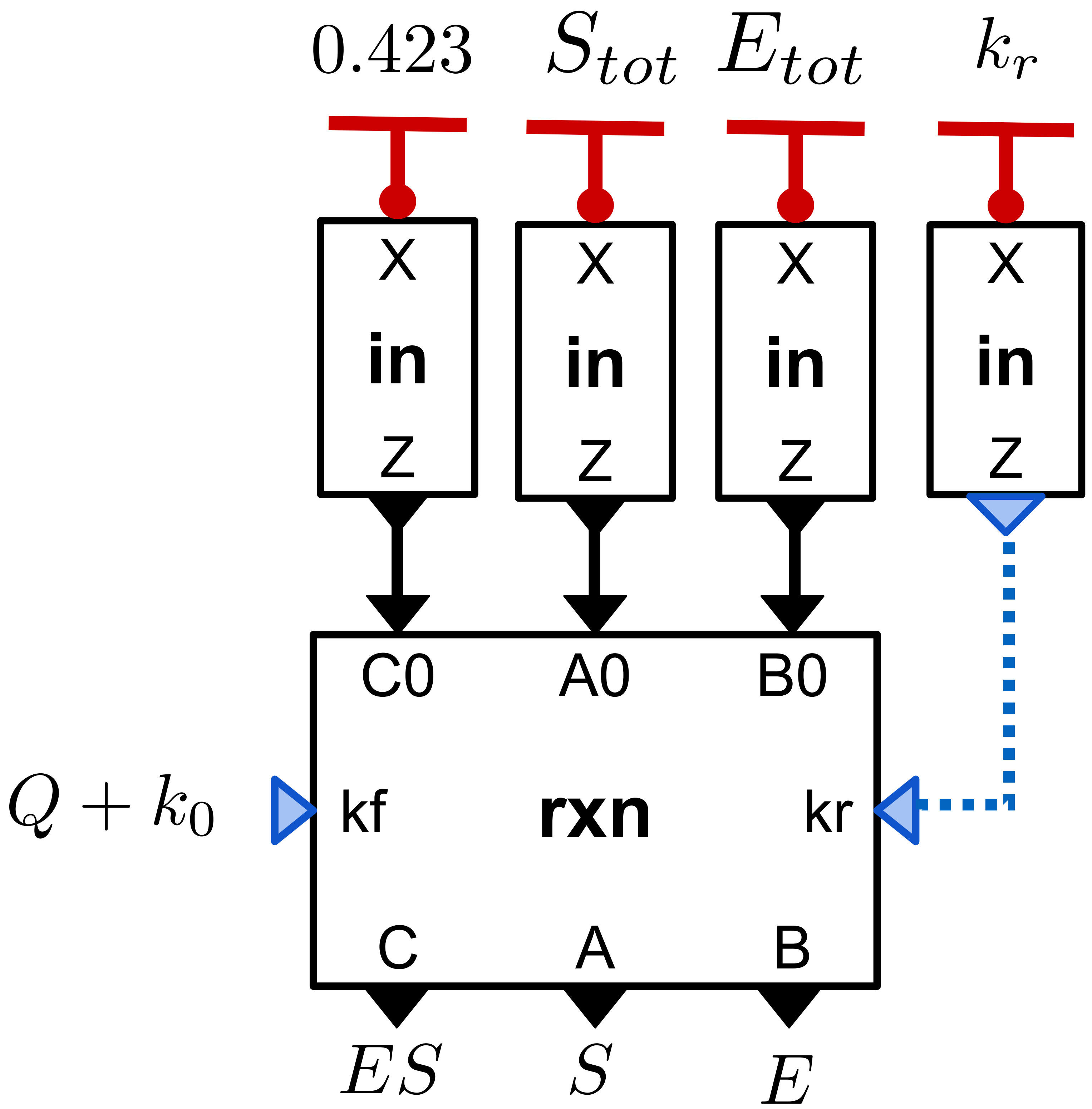
- **<u>Input Components (DACs):</u>** convert sequence of digital values to voltage or current



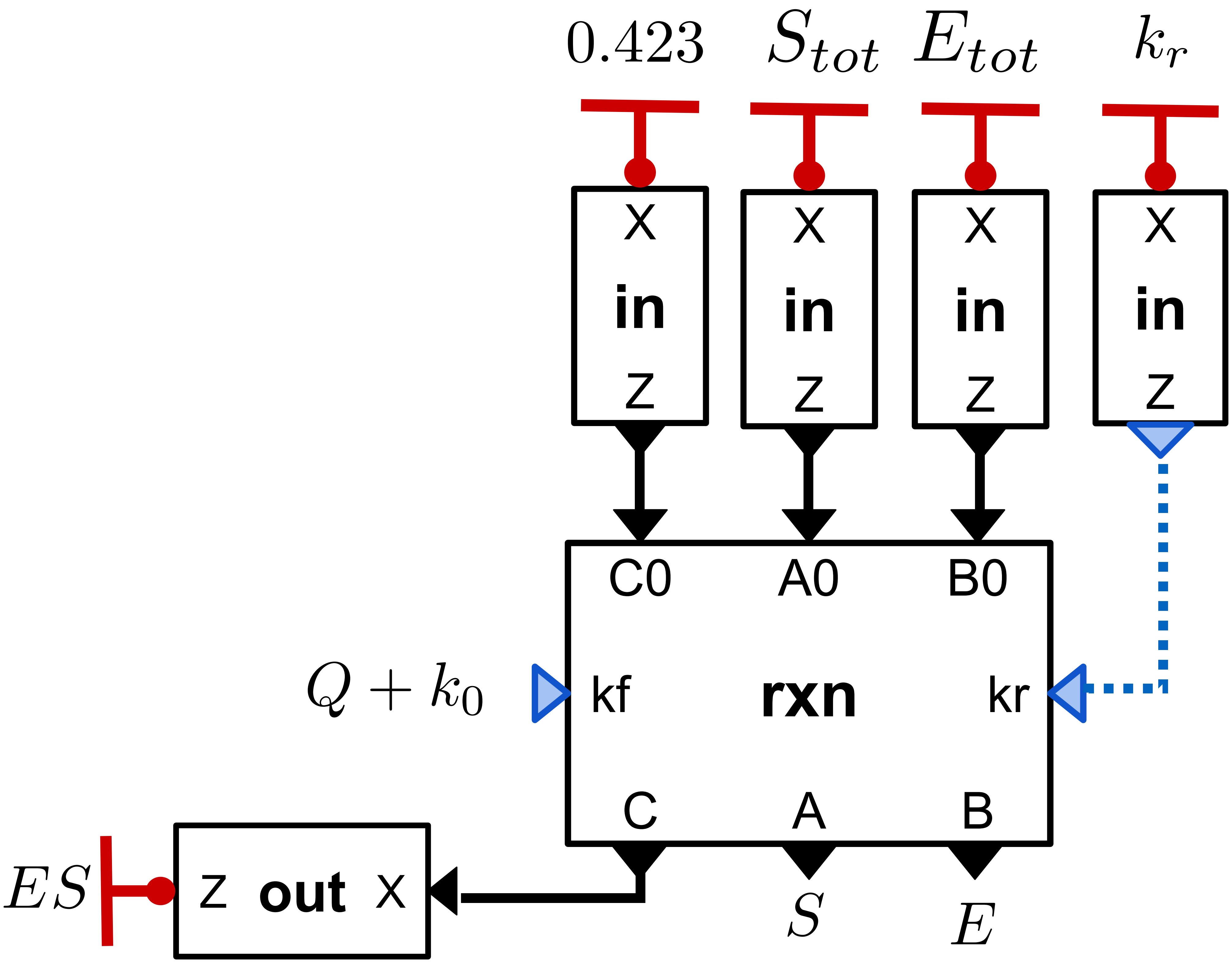
Input Components (DACs): convert sequence of digital values to voltage or current

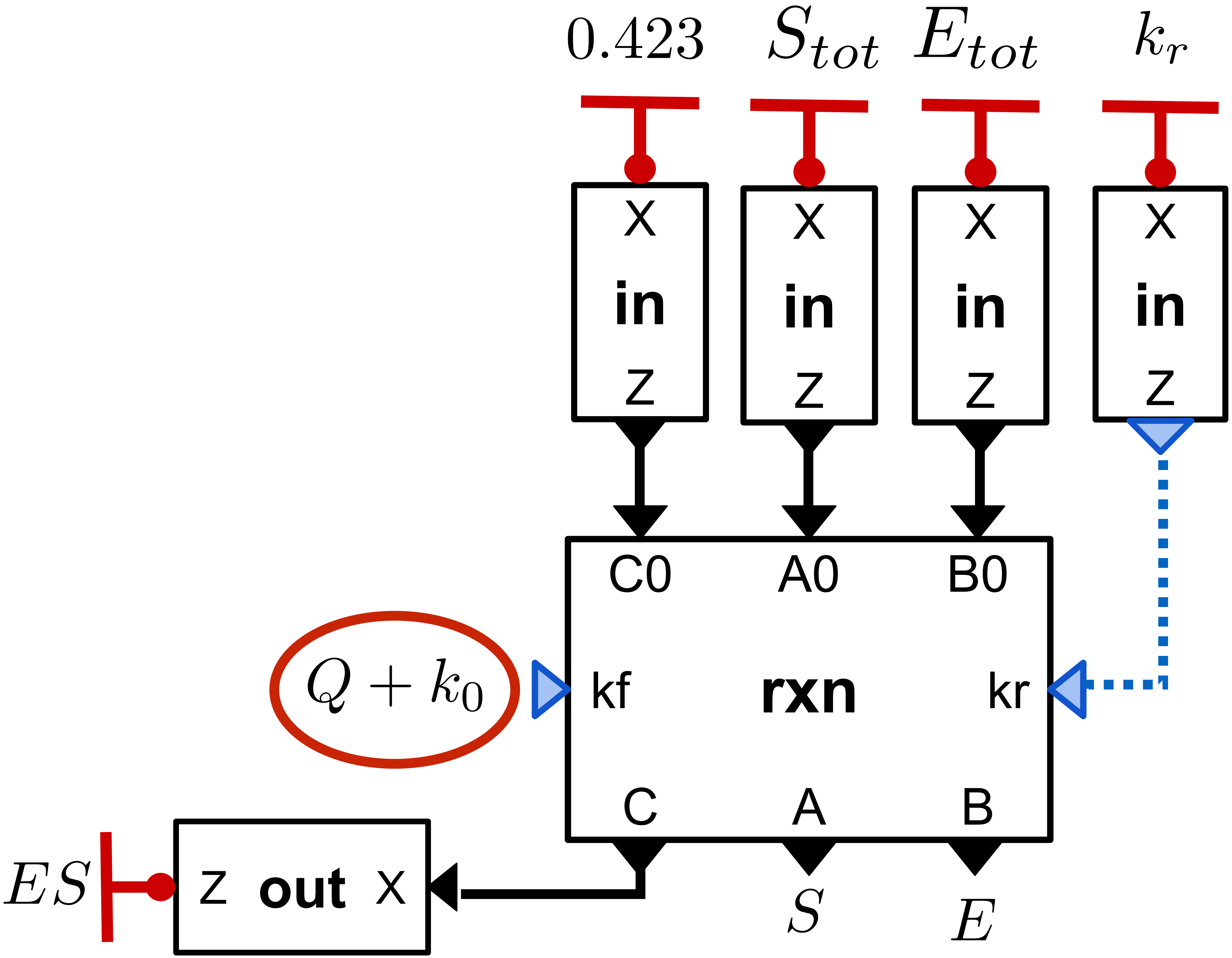


- **Input Components (DACs):** convert sequence of digital values to voltage or current

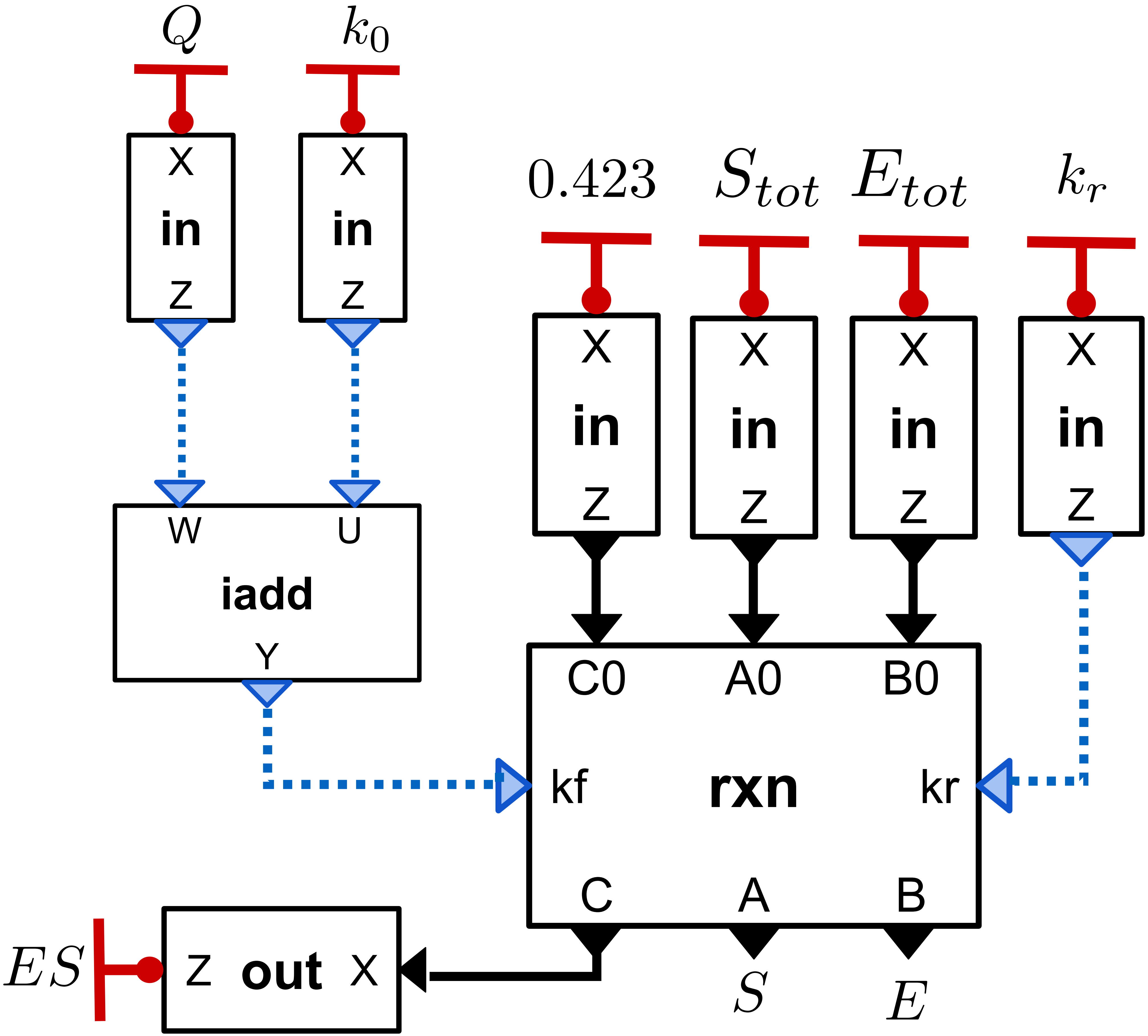


Output Components (ADCs): convert voltage or current to a measurable sequence of digital values





0.423
S_{tot}
E_{tot}
k_r
X
in
Z
X
in
Z
X
in
Z
X
in
Z
C0
A0
B0
$Q + k_0$
kf
rxn
kr
C
A
B
ES
Z
out
X
S
E



Q
k_0
X
in
Z
X
in
Z
W
U
iadd
Y
0.423
S_{tot}
E_{tot}
k_r
X
in
Z
X
in
Z
X
in
Z
X
in
Z
C0
A0
B0
kf
rxn
kr
C
A
B
ES
Z
out
X
S
E

The Search Algorithm

Search over **tableaus**

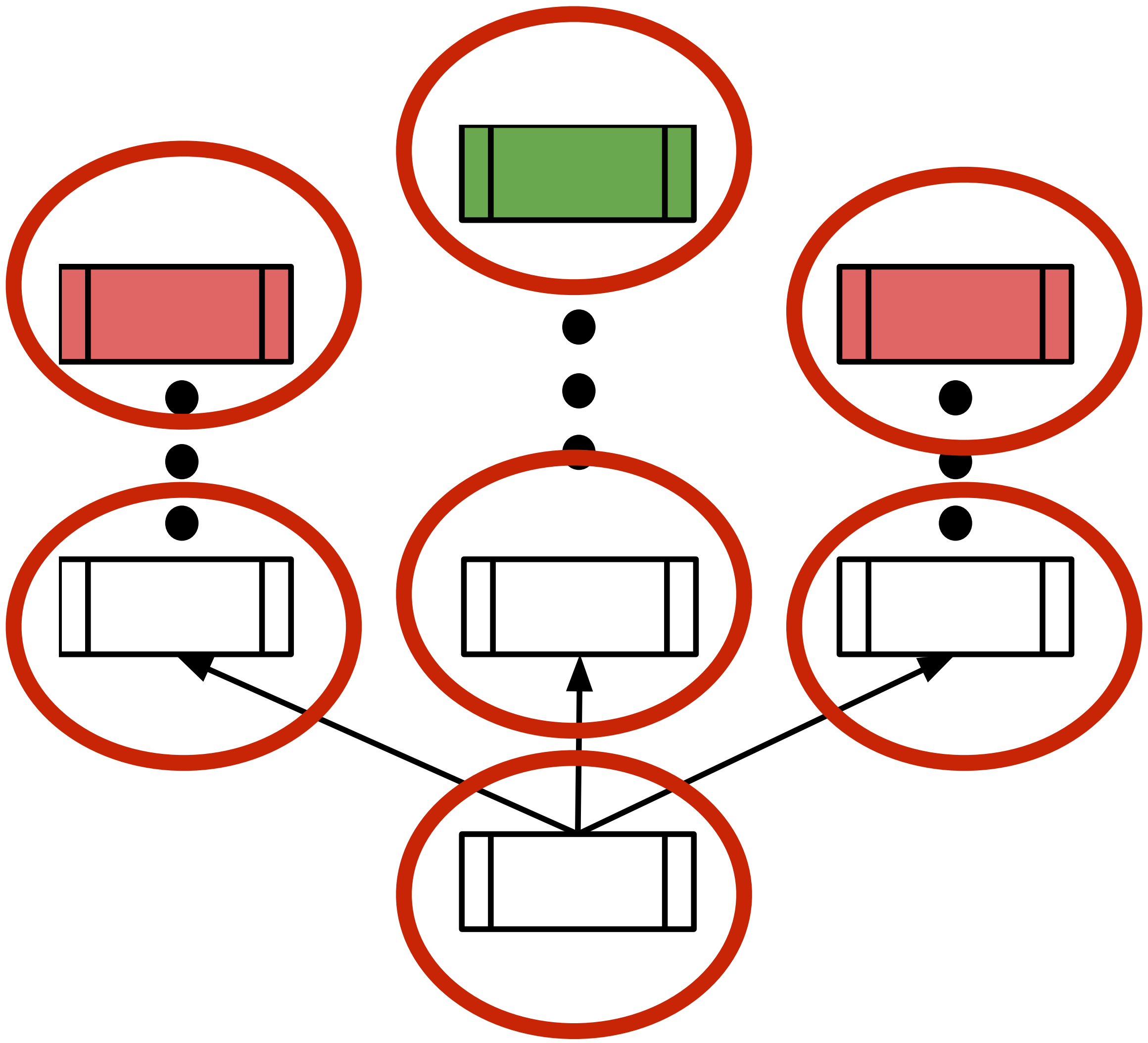
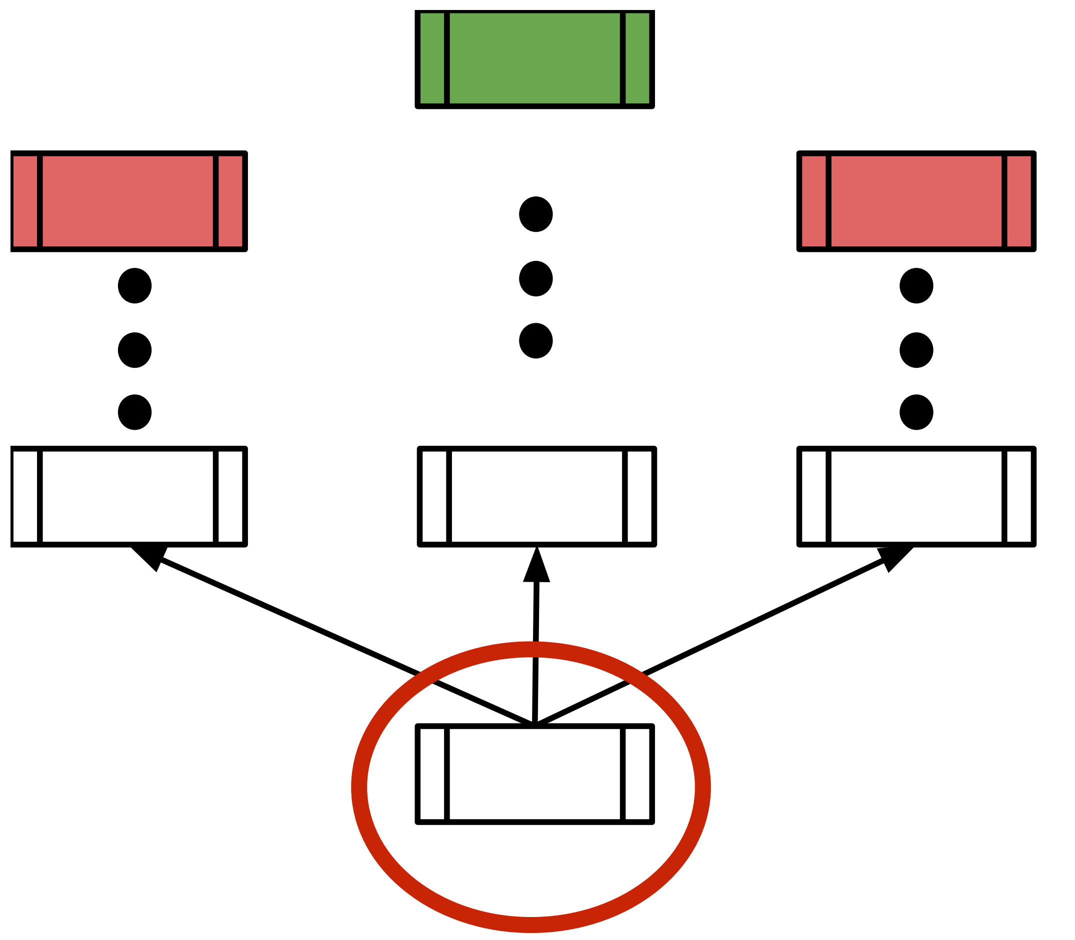


tableau: hardware state, remaining *goals*

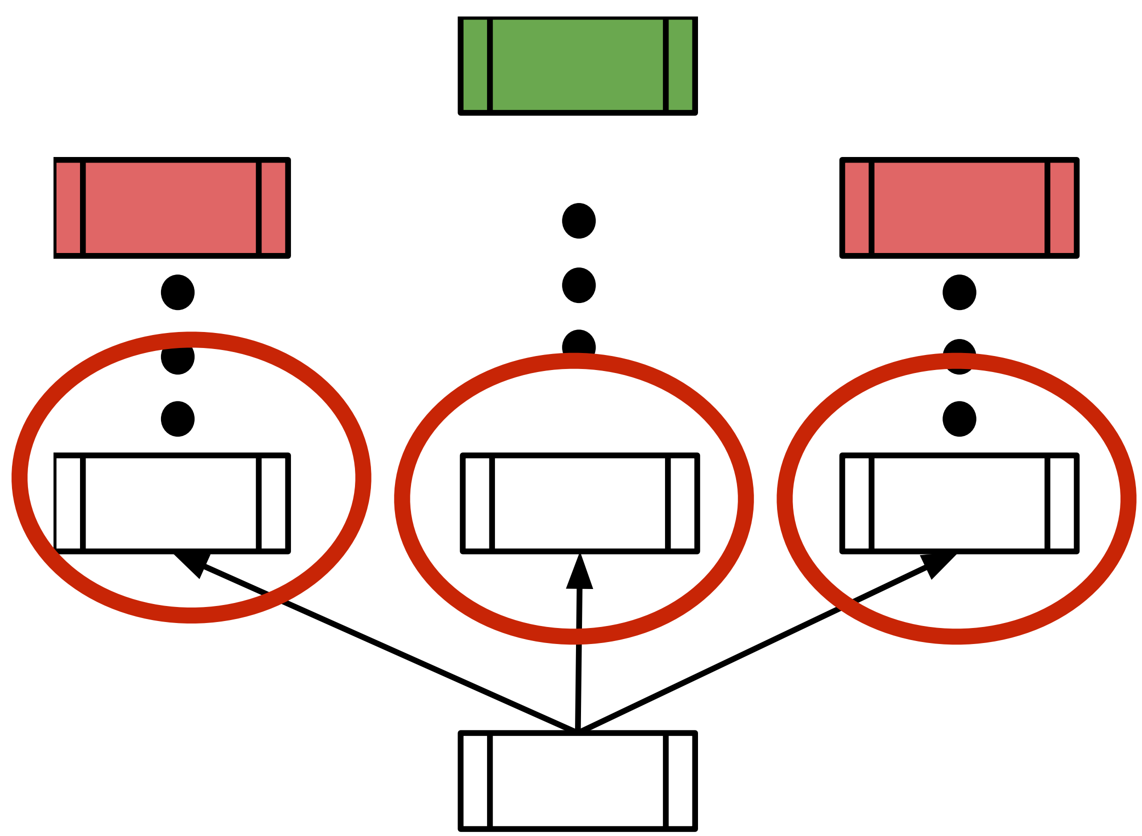
goal: unmapped expression or relation

The Search Algorithm



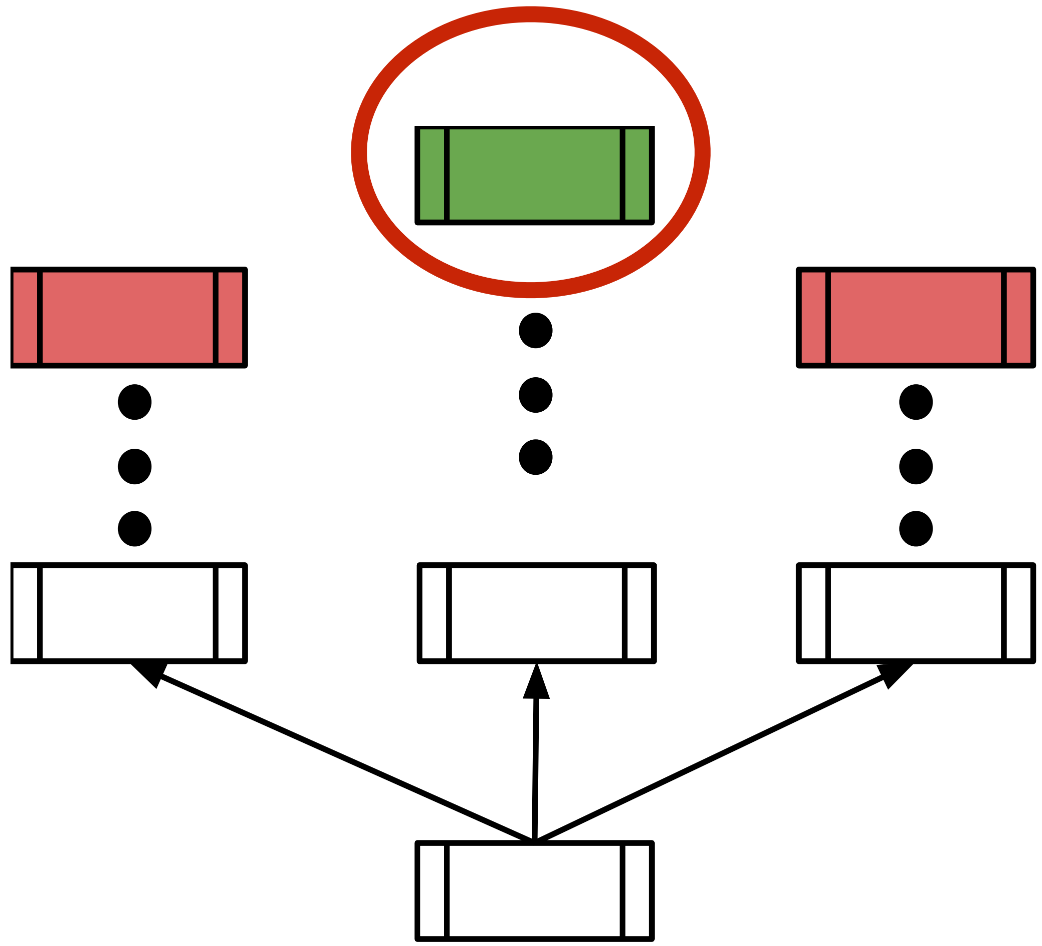
Start with an *initial tableau*

The Search Algorithm



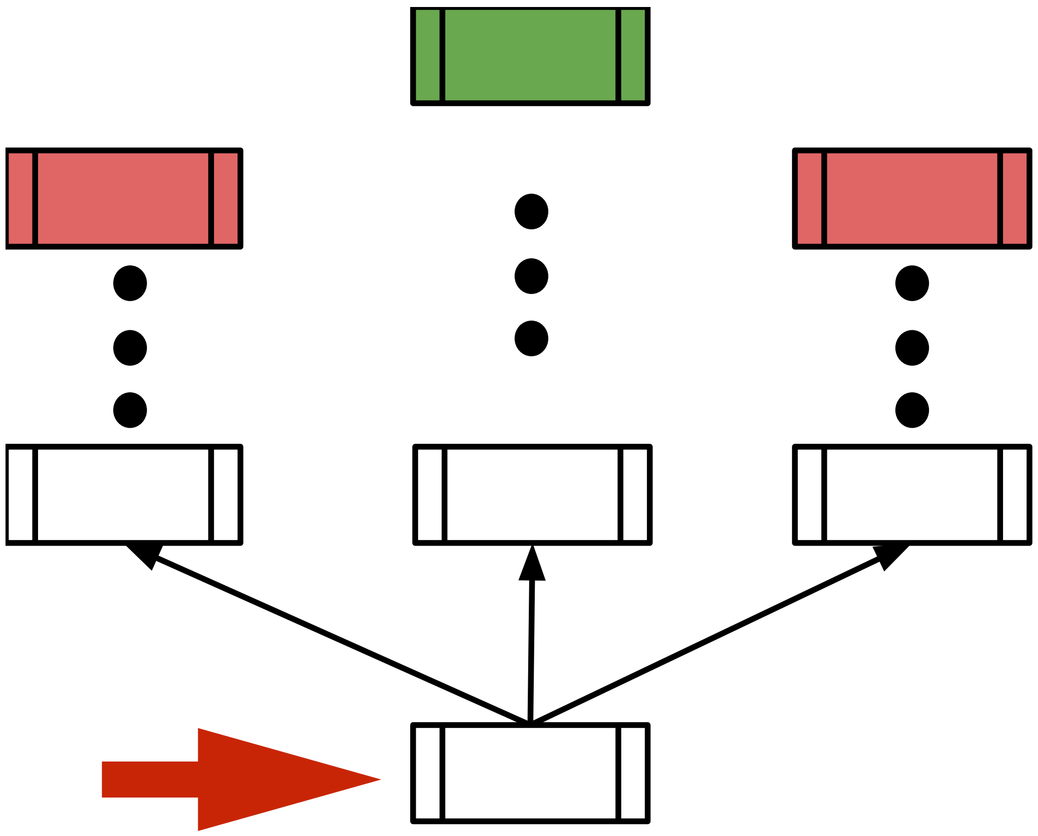
Apply *transitions* over the tableaus

The Search Algorithm



Until a solved tableau is found

The Search Algorithm



1. Initial Tableau
2. Solved Tableau
3. Tableau Transitions
4. Search Algorithm
5. Search Optimizations

1. Initial Tableau

Goals

$$\left\langle \quad \overline{R} \quad \dot{R} \quad W \quad \tilde{R} \quad Z \quad \right\rangle$$

$\overline{R}$	$\dot{R}$	W	$\tilde{R}$	Z
$W_D = U_I$		$\langle B, U \rangle$	$S = 2 \cdot T$	
$Z_I = X_D$		$\langle C, U \rangle$		
$B_I = 2 \cdot A_I$		$\langle Z, A \rangle$		
$C_I = \dfrac{A_I}{2}$				
$F_I = \dfrac{E_I}{4}$				

1. Initial Tableau

Available Hardware   Available Wires

$$\left\langle \quad \overline{R} \qquad \dot{R} \qquad W \qquad \tilde{R} \qquad \tilde{Z} \quad \right\rangle$$

$$W_D = U_I$$
$$Z_I = X_D$$
$$B_I = 2 \cdot A_I$$
$$C_I = \frac{A_I}{2}$$
$$F_I = \frac{E_I}{4}$$

$$\langle B, U \rangle$$
$$\langle C, U \rangle$$
$$\langle Z, A \rangle$$

$$S = 2 \cdot T$$

1. Initial Tableau

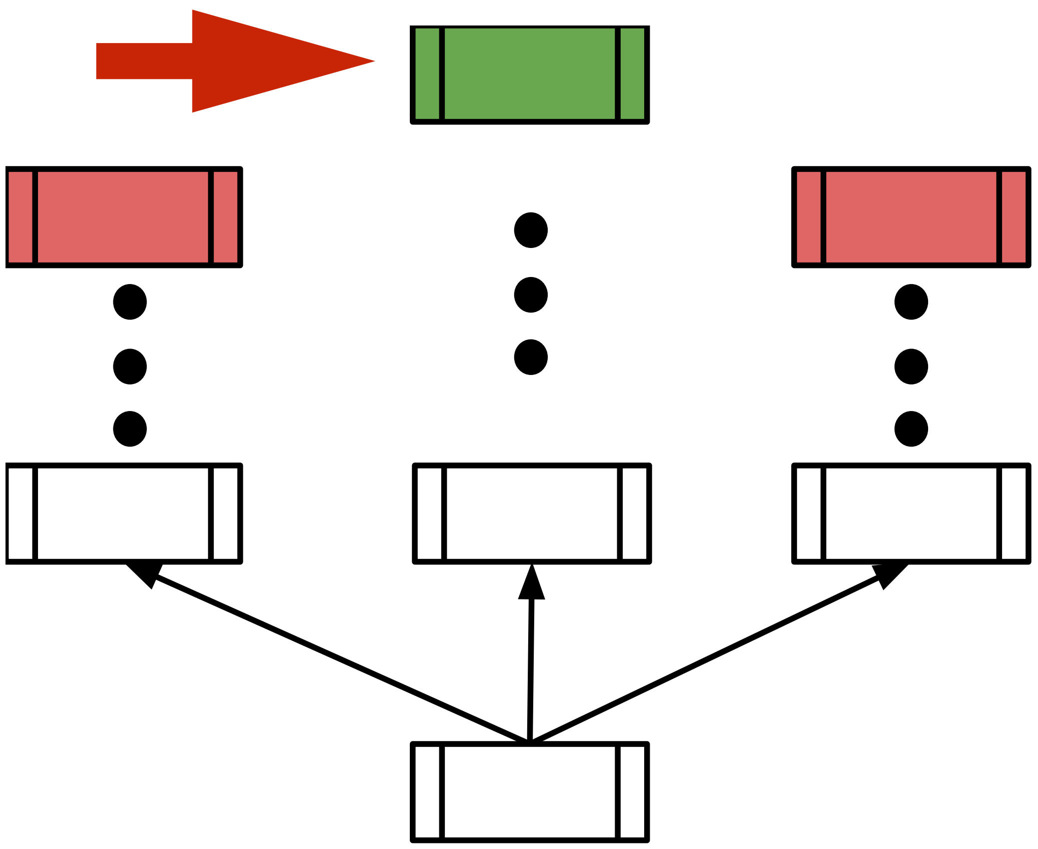
Constrained Hardware

Configuration

$$\left\langle \quad \overline{R} \quad \dot{R} \quad W \quad \tilde{R} \quad Z \quad \right\rangle$$

$\overline{R}$	$\dot{R}$	W	$\tilde{R}$	Z
$W_D = U_I$		$\langle B, U \rangle$	$S = 2 \cdot T$	
$Z_I = X_D$		$\langle C, U \rangle$		
$B_I = 2 \cdot A_I$		$\langle Z, A \rangle$		
$C_I = \frac{A_I}{2}$				
$F_I = \frac{E_I}{4}$				

The Search Algorithm



1. Initial Tableau
2. Solved Tableau
3. Tableau Transitions
4. Search Algorithm
5. Search Optimizations

2. Solved Tableau

No Goals Left

$$\langle \quad \overline{R} \quad \dot{R} \quad W \quad \tilde{R} \quad Z \quad \rangle$$

$$F_I = \frac{E_I}{4}$$

$$C_I = \frac{T}{2}$$

$$Z_I = T$$

$$U_I = S$$

$$B_I = S$$

$$B_I = 2 \cdot T$$

$$\langle C, U \rangle$$

$$T \mapsto X_D$$

$$S \mapsto W_D$$

$$Z \bullet\!\!-\!\!\bullet A$$

$$U \bullet\!\!-\!\!\bullet B$$

2. Solved Tableau

Solution Configuration

$$\langle \quad \overline{R} \qquad \dot{R} \quad W \qquad \tilde{R} \qquad Z \quad \rangle$$

$$F_I = \frac{E_I}{4}$$

$$C_I = \frac{T}{2}$$

$$Z_I = T$$

$$U_I = S$$

$$B_I = S$$

$$B_I = 2 \cdot T$$

$$\langle C, U \rangle$$

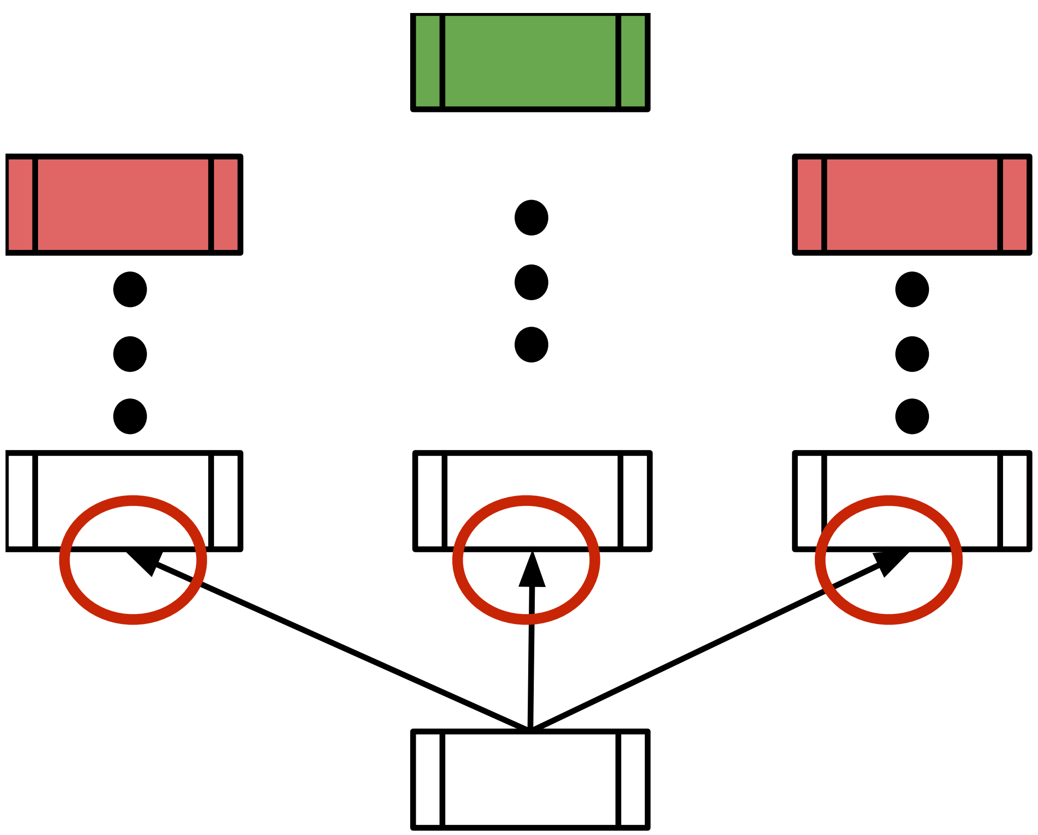
$$T \mapsto X_D$$

$$S \mapsto W_D$$

$$Z \bullet\!\!-\!\!\bullet A$$

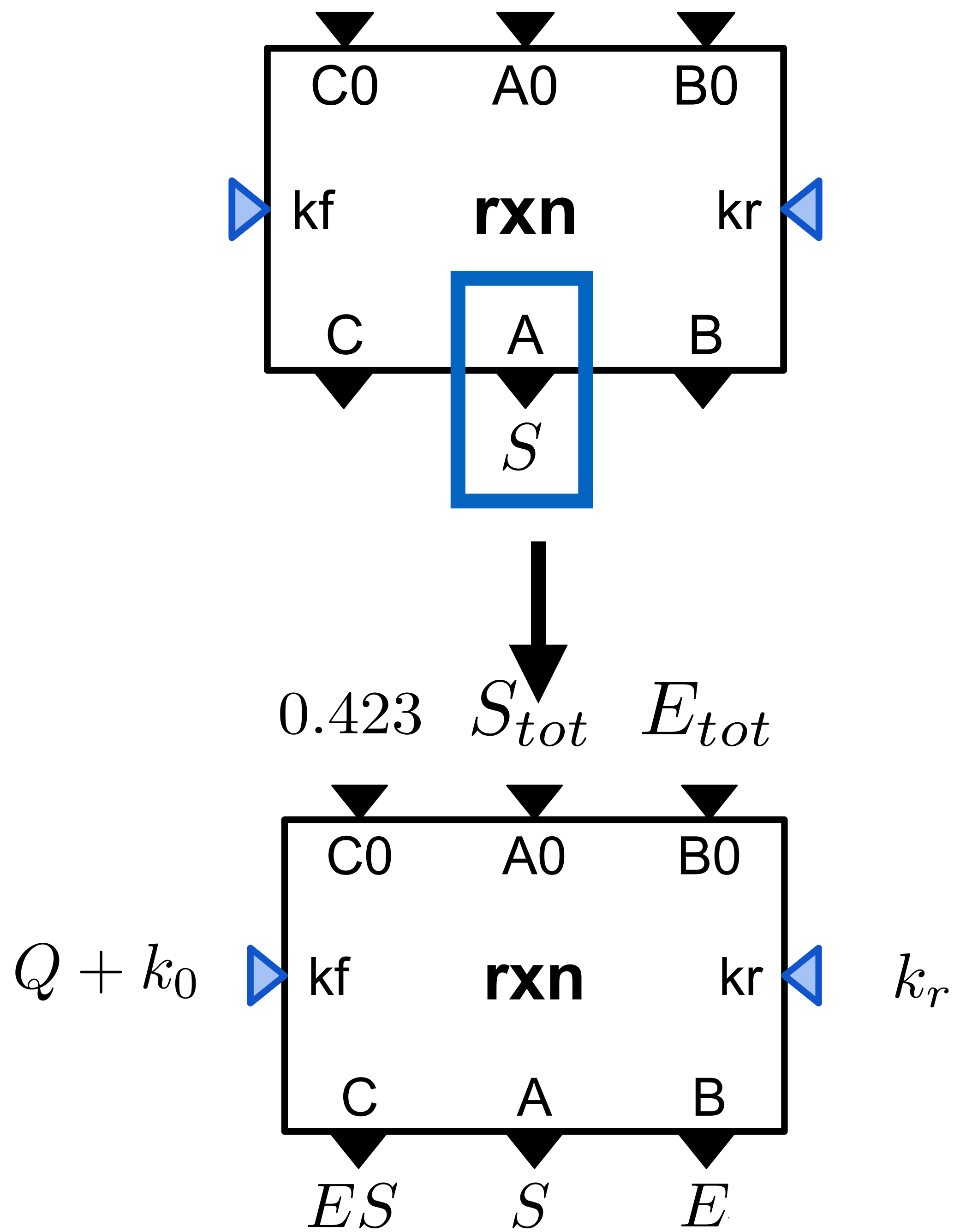
$$U \bullet\!\!-\!\!\bullet B$$

The Search Algorithm



1. Initial Tableau
2. Solved Tableau
3. Tableau Transitions
4. Search Algorithm
5. Search Optimizations

Tableau Transitions



1. Unify
2. Connect
3. Variable Map

1. Transitions Over Tableau

UNIFY

$$\frac{\mathrm{r} \in \overline{\mathrm{R}} \cup \dot{\mathrm{R}} \qquad \widetilde{\mathrm{r}} \in \widetilde{\mathrm{R}} \qquad \text{unify}(\mathrm{r}, \widetilde{\mathrm{r}}, \overline{\mathrm{R}}, \dot{\mathrm{R}}, \widetilde{\mathrm{R}}) = \langle \overline{\mathrm{R}}', \dot{\mathrm{R}}', \widetilde{\mathrm{R}}' \rangle}{\boxed{\langle \overline{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W}, \widetilde{\mathrm{R}}, \mathrm{Z} \rangle} \rightarrow \langle \overline{\mathrm{R}}', \dot{\mathrm{R}}', \mathrm{W}, \widetilde{\mathrm{R}}', \mathrm{Z} \rangle}$$

$$\Big\langle \quad \overline{R} \qquad \dot{R} \qquad W \qquad \tilde{R} \qquad Z \quad \Big\rangle$$

$\overline{R}$	$\dot{R}$	W	$\tilde{R}$	Z
$W_D = U_I$		$\langle B, U \rangle$	$S = 2 \cdot T$	
$Z_I = X_D$		$\langle C, U \rangle$		
$B_I = 2 \cdot A_I$		$\langle Z, A \rangle$		
$C_I = \dfrac{A_I}{2}$				
$F_I = \dfrac{E_I}{4}$				

1. Transitions Over Tableau

$$\frac{\textbf{UNIFY}\quad \mathrm{r} \in \overline{\mathrm{R}} \cup \dot{\mathrm{R}} \qquad \boxed{\widetilde{\mathrm{r}} \in \widetilde{\mathrm{R}}} \qquad \mathsf{unify}(\mathrm{r}, \widetilde{\mathrm{r}}, \overline{\mathrm{R}}, \dot{\mathrm{R}}, \widetilde{\mathrm{R}}) = \langle \overline{\mathrm{R}}', \dot{\mathrm{R}}', \widetilde{\mathrm{R}}' \rangle}{\langle \overline{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W}, \widetilde{\mathrm{R}}, \mathrm{Z} \rangle \rightarrow \langle \overline{\mathrm{R}}', \dot{\mathrm{R}}', \mathrm{W}, \widetilde{\mathrm{R}}', \mathrm{Z} \rangle}$$

$$\left\langle \quad \overline{R} \qquad \dot{R} \qquad W \qquad \tilde{R} \qquad Z \quad \right\rangle$$

$\overline{R}$	$\dot{R}$	W	$\tilde{R}$	Z
$W_D = U_I$		$\langle B, U \rangle$	$S = 2 \cdot T$	
$Z_I = X_D$		$\langle C, U \rangle$		
$B_I = 2 \cdot A_I$		$\langle Z, A \rangle$		
$C_I = \frac{A_I}{2}$				
$F_I = \frac{E_I}{4}$				

1. Transitions Over Tableau

UNIFY

$$\frac{\boxed{r \in \overline{R} \cup \dot{R}} \qquad \widetilde{r} \in \widetilde{R} \qquad \text{unify}(r, \widetilde{r}, \overline{R}, \dot{R}, \widetilde{R}) = \langle \overline{R}', \dot{R}', \widetilde{R}' \rangle}{\langle \overline{R}, \dot{R}, W, \widetilde{R}, Z \rangle \rightarrow \langle \overline{R}', \dot{R}', W, \widetilde{R}', Z \rangle}$$

$$\Big\langle \quad \overline{R} \qquad \dot{R} \qquad W \qquad \tilde{R} \qquad Z \quad \Big\rangle$$

$\overline{R}$	W	$\tilde{R}$
$W_D = U_I$	$\langle B, U \rangle$	$S = 2 \cdot T$
$Z_I = X_D$	$\langle C, U \rangle$	
$B_I = 2 \cdot A_I$	$\langle Z, A \rangle$	
$C_I = \frac{A_I}{2}$		
$F_I = \frac{E_I}{4}$		

1. Transitions Over Tableau

$$\tilde{r}: S = 2 \cdot T \quad \longmapsto \quad r: B_I = 2 \cdot A_I$$

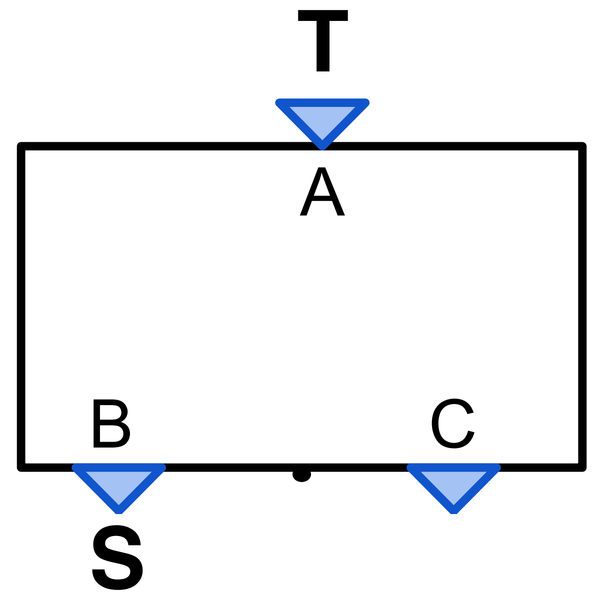
$$\textbf{UNIFY}\ \frac{\mathrm{r} \in \overline{\mathrm{R}} \cup \dot{\mathrm{R}} \qquad \widetilde{\mathrm{r}} \in \widetilde{\mathrm{R}} \qquad \boxed{\mathrm{unify}(\mathrm{r}, \widetilde{\mathrm{r}}, \overline{\mathrm{R}}, \dot{\mathrm{R}}, \widetilde{\mathrm{R}}) = \langle \overline{\mathrm{R}}', \dot{\mathrm{R}}', \widetilde{\mathrm{R}}' \rangle}}{\langle \overline{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W}, \widetilde{\mathrm{R}}, \mathrm{Z} \rangle \to \langle \overline{\mathrm{R}}', \dot{\mathrm{R}}', \mathrm{W}, \widetilde{\mathrm{R}}', \mathrm{Z} \rangle}$$

$$\left\langle \quad \overline{R}' \qquad \dot{R}' \qquad \tilde{R}' \quad \right\rangle$$

$\overline{R}'$	$\dot{R}'$	$\tilde{R}'$
$W_D = U_I$	$B_I = S$	$A_I = T$
$Z_I = X_D$	$B_I = 2 \cdot T$	$S \mapsto B_I$
$F_I = \frac{E_I}{4}$	$C_I = \frac{T}{2}$	

1. Transitions Over Tableau

$$S = 2 \cdot T \longmapsto B_I = 2 \cdot A_I$$



$$\left\langle \quad \overline{R}' \qquad \dot{R}' \qquad \tilde{R}' \quad \right\rangle$$

$$W_D = U_I$$

$$Z_I = X_D$$

$$F_I = \frac{E_I}{4}$$

$$B_I = S$$

$$B_I = 2 \cdot T$$

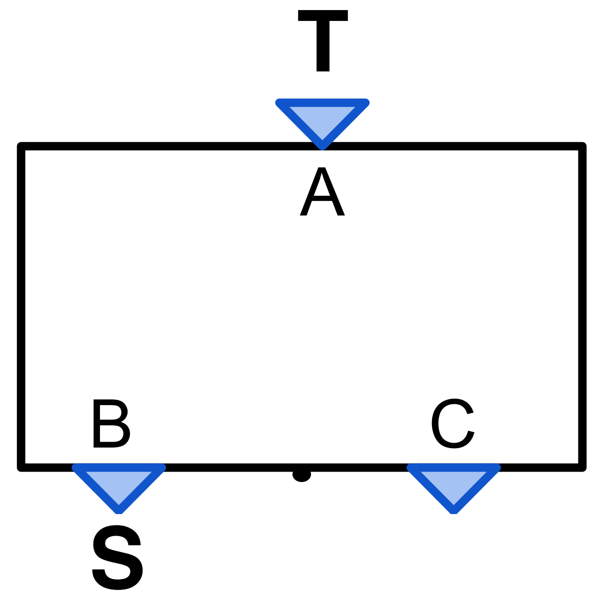
$$C_I = \frac{T}{2}$$

$$A_I = T$$

$$S \mapsto B_I$$

1. Transitions Over Tableau

$$\boxed{S = 2 \cdot T} \longmapsto B_I = 2 \cdot A_I$$

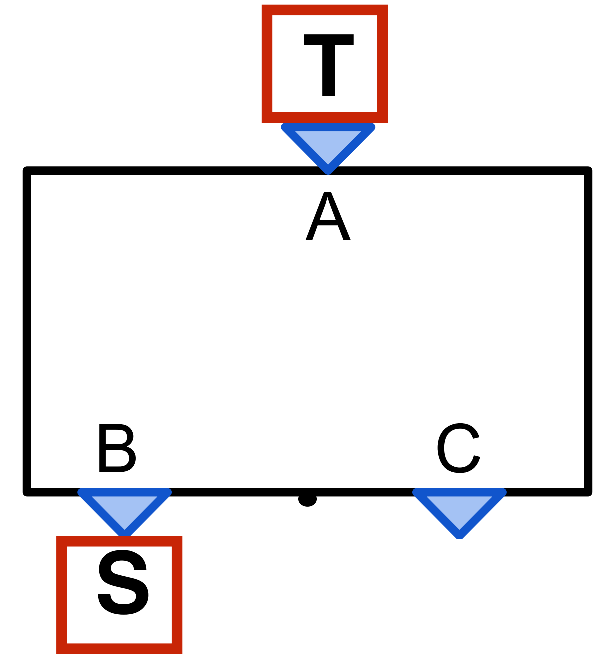


$$\left\langle \quad \overline{R}' \qquad \dot{R}' \qquad \tilde{R}' \quad \right\rangle$$

$\overline{R}'$	$\dot{R}'$	$\tilde{R}'$
$W_D = U_I$	$B_I = S$	$A_I = T$
$Z_I = X_D$	$B_I = 2 \cdot T$	$S \mapsto B_I$
$F_I = \frac{E_I}{4}$	$C_I = \frac{T}{2}$	$S = 2 \cdot T$

1. Transitions Over Tableau

$$S = 2 \cdot T \longmapsto B_I = 2 \cdot A_I$$



$$\left\langle \quad \overline{R}' \qquad \dot{R}' \qquad \tilde{R}' \quad \right\rangle$$

$$W_D = U_I$$

$$Z_I = X_D$$

$$F_I = \frac{E_I}{4}$$

$$B_I = S$$

$$B_I = 2 \cdot T$$

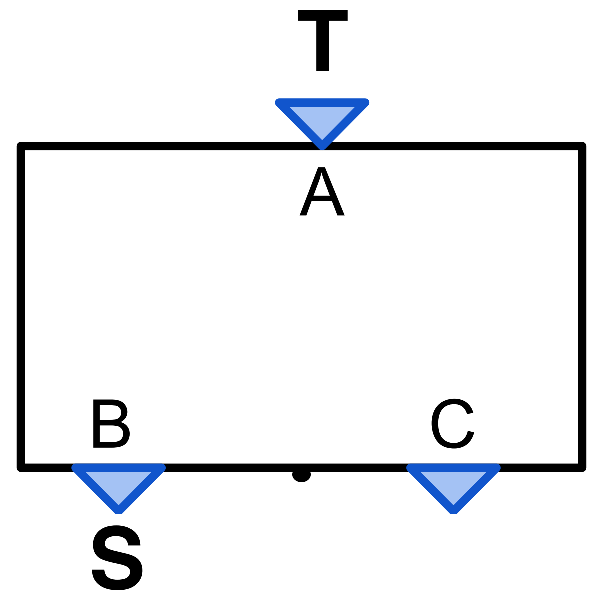
$$C_I = \frac{T}{2}$$

$$A_I = T$$

$$S \mapsto B_I$$

1. Transitions Over Tableau

$$S = 2 \cdot T \longmapsto \boxed{B_I = 2 \cdot A_I}$$



$$\Big\langle \quad \overline{R}' \qquad \dot{R}' \qquad \tilde{R}' \quad \Big\rangle$$

$$W_D = U_I$$

$$Z_I = X_D$$

$$F_I = \frac{E_I}{4}$$

$$B_I = 2 \cdot A_I$$

$$B_I = S$$

$$B_I = 2 \cdot T$$

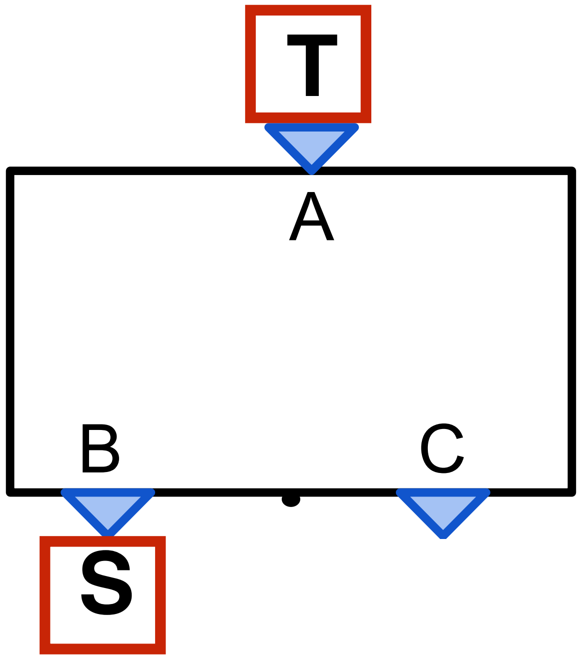
$$C_I = \frac{T}{2}$$

$$A_I = T$$

$$S \mapsto B_I$$

1. Transitions Over Tableau

$$S = 2 \cdot T \longmapsto B_I = 2 \cdot A_I$$



$$\left\langle \quad \overline{R}' \qquad \dot{R}' \qquad \tilde{R}' \quad \right\rangle$$

$$W_D = U_I$$

$$Z_I = X_D$$

$$F_I = \frac{E_I}{4}$$

$$B_I = 2 \cdot A_I$$

$$B_I = S$$

$$B_I = 2 \cdot T$$

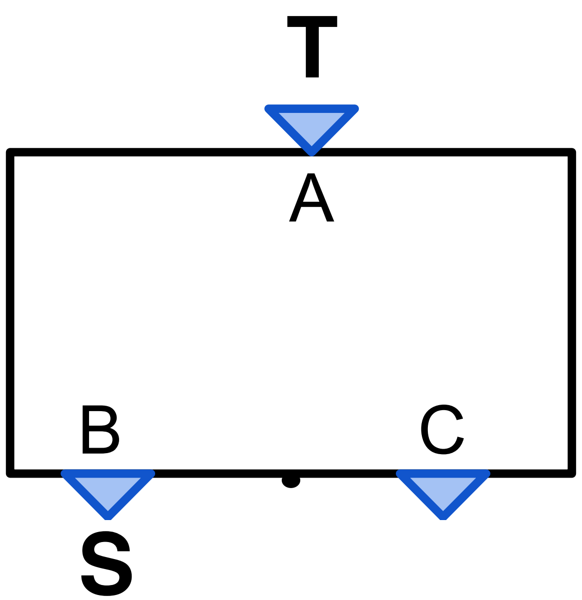
$$C_I = \frac{T}{2}$$

$$A_I = T$$

$$S \mapsto B_I$$

1. Transitions Over Tableau

$$S = 2 \cdot T \longmapsto B_I = 2 \cdot A_I$$



$$B_I = 2 \cdot A_I \rightarrow \begin{array}{l} B_I = S \\ B_I = 2 \cdot T \end{array}$$

$$\left\langle \quad \overline{R}' \qquad \dot{R}' \qquad \tilde{R}' \quad \right\rangle$$

$W_D = U_I$

$Z_I = X_D$

$F_I = \frac{E_I}{4}$

$\cancel{B_I = 2 \cdot A_I}$

$B_I = S$

$B_I = 2 \cdot T$

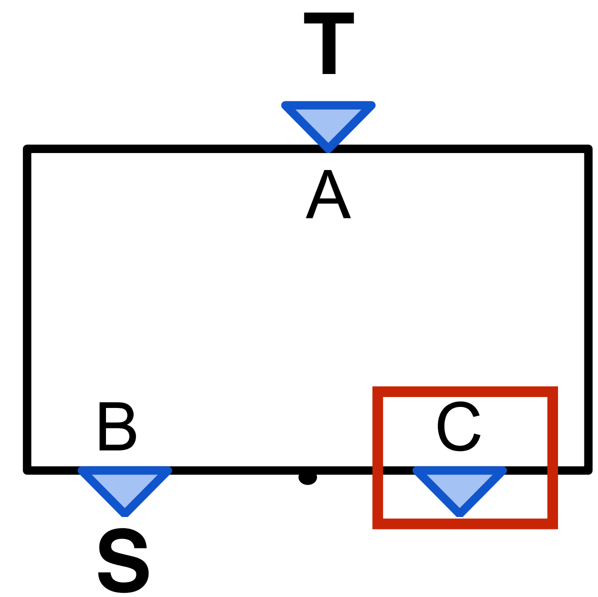
$C_I = \frac{T}{2}$

$A_I = T$

$S \mapsto B_I$

1. Transitions Over Tableau

$$S = 2 \cdot T \longmapsto B_I = 2 \cdot A_I$$



$$\left\langle \overline{R}' \quad \dot{R}' \quad \tilde{R}' \right\rangle$$

$$W_D = U_I$$

$$Z_I = X_D$$

$$F_I = \frac{E_I}{4}$$

$$C_I = \frac{A_I}{2}$$

$$B_I = S$$

$$B_I = 2 \cdot T$$

$$C_I = \frac{T}{2}$$

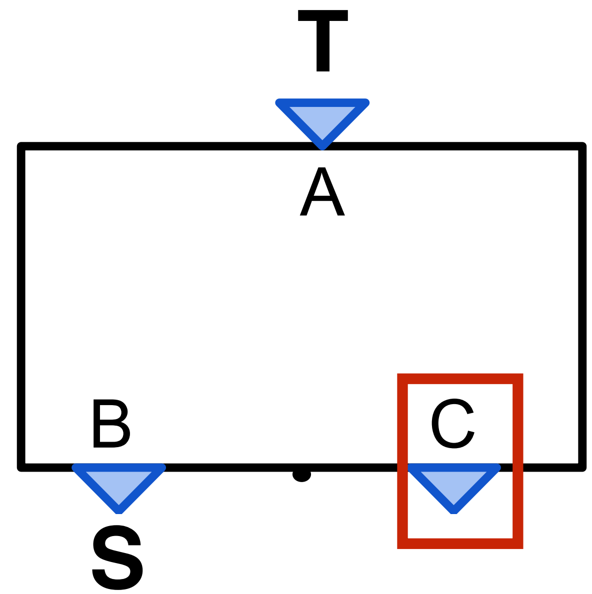
$$A_I = T$$

$$S \mapsto B_I$$

1. Transitions Over Tableau

$$S = 2 \cdot T \longmapsto B_I = 2 \cdot A_I$$

$$C_I = \frac{A_I}{2} \longrightarrow C_I = \frac{T}{2}$$



$$\left\langle \quad \overline{R}' \qquad \dot{R}' \qquad \tilde{R}' \quad \right\rangle$$

$$W_D = U_I$$

$$Z_I = X_D$$

$$F_I = \frac{E_I}{4}$$

$$\cancel{C_I = \frac{A_I}{2}}$$

$$B_I = S$$

$$B_I = 2 \cdot T$$

$$\boxed{C_I = \frac{T}{2}}$$

$$A_I = T$$

$$S \mapsto B_I$$

1. Transitions Over Tableau

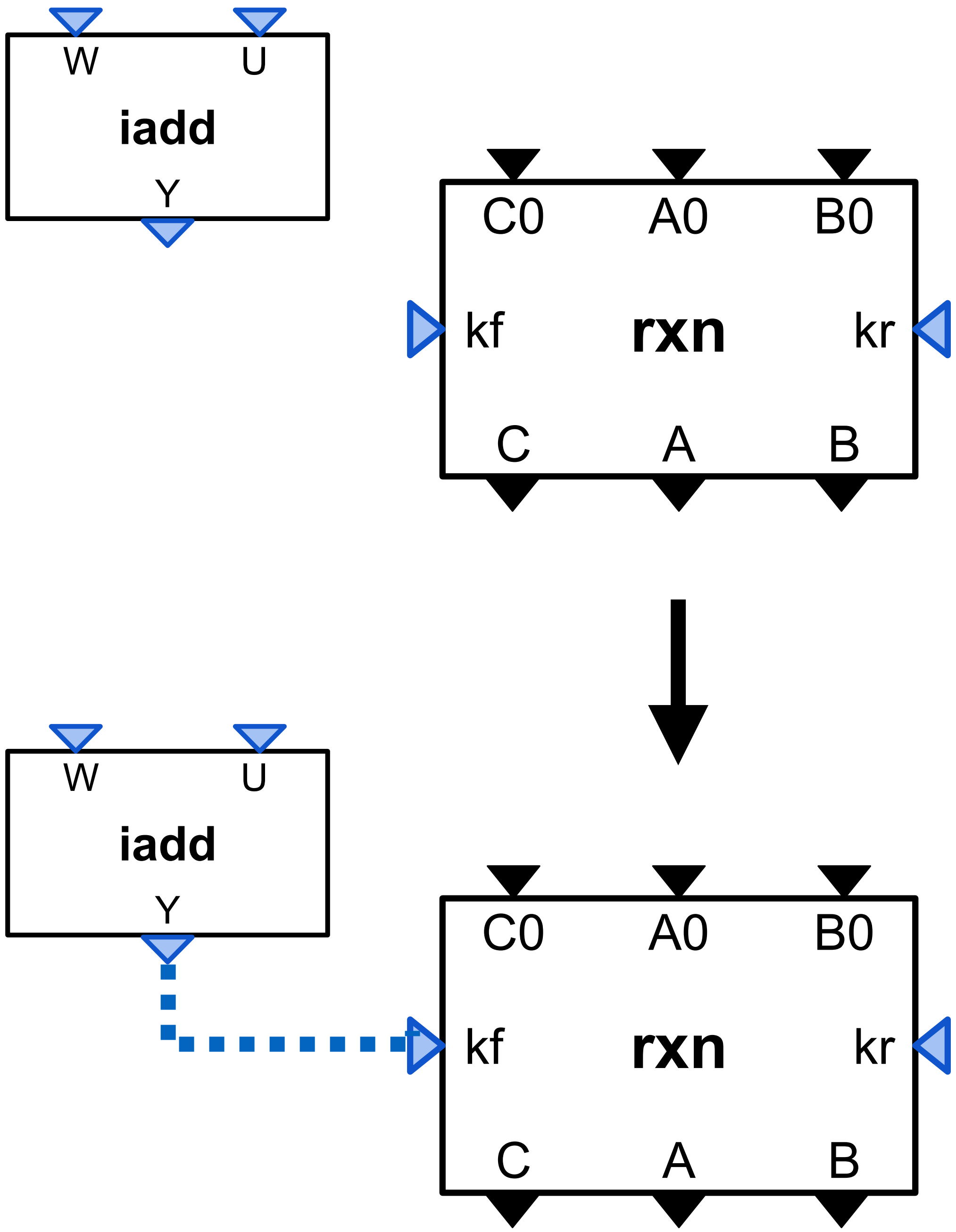
UNIFY

$$\frac{\mathrm{r} \in \overline{\mathrm{R}} \cup \dot{\mathrm{R}} \qquad \widetilde{\mathrm{r}} \in \widetilde{\mathrm{R}} \qquad \text{unify}(\mathrm{r}, \widetilde{\mathrm{r}}, \overline{\mathrm{R}}, \dot{\mathrm{R}}, \widetilde{\mathrm{R}}) = \langle \overline{\mathrm{R}}', \dot{\mathrm{R}}', \widetilde{\mathrm{R}}' \rangle}{\langle \overline{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W}, \widetilde{\mathrm{R}}, \mathrm{Z} \rangle \rightarrow \boxed{\langle \overline{\mathrm{R}}', \dot{\mathrm{R}}', \mathrm{W}, \widetilde{\mathrm{R}}', \mathrm{Z} \rangle}}$$

$$\langle \quad \overline{R}' \qquad \dot{R}' \qquad W \qquad \tilde{R}' \qquad Z \quad \rangle$$

$\overline{R}'$	$\dot{R}'$	W	$\tilde{R}'$	Z
$W_D = U_I$	$B_I = S$	$\langle B, U \rangle$	$A_I = T$	
$Z_I = X_D$	$B_I = 2 \cdot T$	$\langle C, U \rangle$	$S \mapsto B_I$	
$F_I = \frac{E_I}{4}$		$\langle Z, A \rangle$		

Tableau Transitions



1. Unify
2. Connect
3. Variable Map

1. Transitions Over Tableau

$$\textsc{Connect}\ \frac{\boxed{\widetilde{\mathrm{r}} : \overline{\mathtt{i}}_{\mathrm{q}} = \overline{\mathtt{o}}_{\mathrm{q}} \in \widetilde{\mathrm{R}}} \qquad \mathrm{w} : \langle \overline{\mathtt{o}}, \overline{\mathtt{i}} \rangle \in \mathrm{W}}{\langle \overline{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W}, \widetilde{\mathrm{R}}, \mathrm{Z} \rangle \rightarrow \langle \overline{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W} - \{\mathrm{w}\}, \widetilde{\mathrm{R}} - \{\widetilde{\mathrm{r}}\}, \mathrm{Z} \cup \{\overline{\mathtt{o}} \bullet\!\!-\!\!\bullet\ \overline{\mathtt{i}}\} \rangle}$$

$$\langle \quad \overline{R} \quad \dot{R} \quad W \quad \tilde{R} \quad Z \quad \rangle$$

$\overline{R}$	$\dot{R}$	W	$\tilde{R}$
$W_D = U_I$	$C_I = \frac{T}{2}$	$\langle B, U \rangle$	$S \mapsto B_I$
$F_I = \frac{E_I}{4}$	$B_I = S$	$\langle C, U \rangle$	$A_I = Z_I$
	$B_I = 2 \cdot T$	$\langle Z, A \rangle$	$X_D = T$
	$Z_I = T$		

1. Transitions Over Tableau

CONNECT

$$\frac{\widetilde{\mathrm{r}} : \overline{\mathtt{i}}_{\mathrm{q}} = \overline{\mathtt{o}}_{\mathrm{q}} \in \widetilde{\mathrm{R}} \qquad \boxed{\mathrm{w} : \langle \overline{\mathtt{o}}, \overline{\mathtt{i}} \rangle \in \mathrm{W}}}{\langle \overline{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W}, \widetilde{\mathrm{R}}, \mathrm{Z} \rangle \rightarrow \langle \overline{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W} - \{\mathrm{w}\}, \widetilde{\mathrm{R}} - \{\widetilde{\mathrm{r}}\}, \mathrm{Z} \cup \{\overline{\mathtt{o}} \bullet\!\!-\!\!\bullet\ \overline{\mathtt{i}}\} \rangle}$$

$$\langle \quad \overline{R} \quad \dot{R} \quad W \quad \tilde{R} \quad Z \quad \rangle$$

$W_D = U_I$

$F_I = \dfrac{E_I}{4}$

$C_I = \dfrac{T}{2}$

$B_I = S$

$B_I = 2 \cdot T$

$Z_I = T$

$\langle B, U \rangle$

$\langle C, U \rangle$

$\langle Z, A \rangle$

$S \mapsto B_I$

$A_I = Z_I$

$X_D = T$

1. Transitions Over Tableau

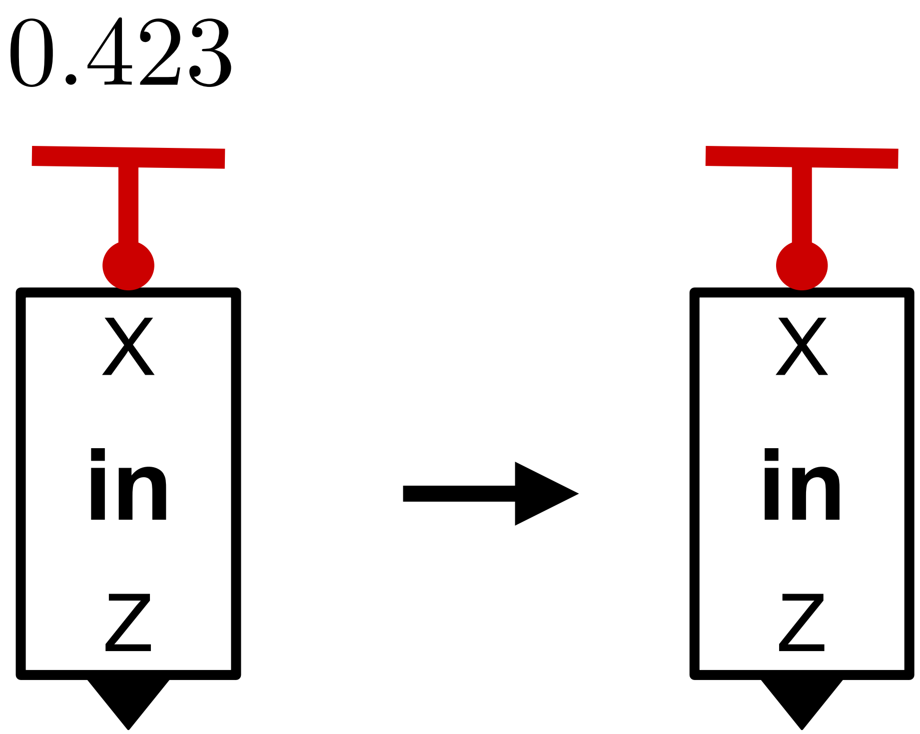
CONNECT

$$\frac{\widetilde{\mathrm{r}} : \overline{\mathtt{i}}_q = \overline{\mathtt{o}}_q \in \widetilde{\mathrm{R}} \qquad \mathrm{w} : \langle \overline{\mathtt{o}}, \overline{\mathtt{i}} \rangle \in \mathrm{W}}{\langle \overline{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W}, \widetilde{\mathrm{R}}, \mathrm{Z} \rangle \rightarrow \boxed{\langle \overline{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W} - \{\mathrm{w}\}, \widetilde{\mathrm{R}} - \{\widetilde{\mathrm{r}}\}, \mathrm{Z} \cup \{\overline{\mathtt{o}} \bullet\!\!-\!\!\bullet \overline{\mathtt{i}}\} \rangle}}$$

$$\langle \quad \overline{R} \qquad \dot{R} \qquad W \qquad \tilde{R} \qquad Z \quad \rangle$$

$W_D = U_I$	$C_I = \frac{T}{2}$	$\langle B, U \rangle$	$S \mapsto B_I$	$\boxed{Z \bullet\!\!-\!\!\bullet A}$
$F_I = \frac{E_I}{4}$	$B_I = S$	$\langle C, U \rangle$	$\cancel{A_I = Z_I}$	
	$B_I = 2 \cdot T$	$\cancel{\langle Z, A \rangle}$	$X_D = T$	
	$Z_I = T$			

Tableau Transitions



1. Unify
2. Connect
3. Variable Map

1. Transitions Over Tableau

$$\dfrac{\textbf{INPUT-VAR-MAP} \quad \widetilde{\mathrm{r}} : \overline{\mathtt{i}}_{\mathrm{d}} = \widehat{i} \in \widetilde{\mathrm{R}} \qquad \widehat{i} \in \widehat{I} \qquad \overline{\mathtt{i}} \mathbin{@} \overline{\mathtt{c}} \qquad \overline{\mathtt{c}} \in \overline{\mathtt{IC}}}{\langle \overline{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W}, \widetilde{\mathrm{R}}, \mathrm{Z} \rangle \rightarrow \langle \overline{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W}, \widetilde{\mathrm{R}} - \{\widetilde{\mathrm{r}}\}, \mathrm{Z} \cup \{\widehat{i} \mapsto \overline{\mathtt{i}}_{\mathrm{d}}\} \rangle}$$

$$\Big\langle \quad \overline{R} \qquad \dot{R} \qquad W \qquad \tilde{R} \qquad Z \quad \Big\rangle$$

$W_D = U_I$	$C_I = \dfrac{T}{2}$	$\langle B, U \rangle$	$S \mapsto B_I$	$Z \bullet\!\!-\!\!\bullet A$
$F_I = \dfrac{E_I}{4}$	$B_I = S$	$\langle C, U \rangle$	$X_D = T$	
	$B_I = 2 \cdot T$			
	$Z_I = T$			

1. Transitions Over Tableau

INPUT-VAR-MAP

$$\frac{\boxed{\tilde{\mathrm{r}} : \bar{\mathtt{i}}_{\mathrm{d}} = \hat{i} \in \tilde{\mathrm{R}}} \qquad \hat{i} \in \hat{I} \qquad \bar{\mathtt{i}} \mathbin{@} \bar{\mathtt{c}} \qquad \bar{\mathtt{c}} \in \overline{\mathtt{IC}}}{\langle \bar{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W}, \tilde{\mathrm{R}}, \mathrm{Z} \rangle \to \langle \bar{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W}, \tilde{\mathrm{R}} - \{\tilde{\mathrm{r}}\}, \mathrm{Z} \cup \{\hat{i} \mapsto \bar{\mathtt{i}}_{\mathrm{d}}\} \rangle}$$

$$\Big\langle \quad \overline{R} \qquad \dot{R} \qquad W \qquad \tilde{R} \qquad \qquad \Big\rangle$$

$$W_D = U_I \qquad F_I = \frac{E_I}{4}$$

$$C_I = \frac{T}{2} \qquad B_I = S \qquad B_I = 2 \cdot T \qquad Z_I = T$$

$$\langle B, U \rangle \qquad \langle C, U \rangle$$

$$S \mapsto B_I \qquad X_D = T$$

$$Z \bullet\!\!-\!\!\bullet A$$

1. Transitions Over Tableau

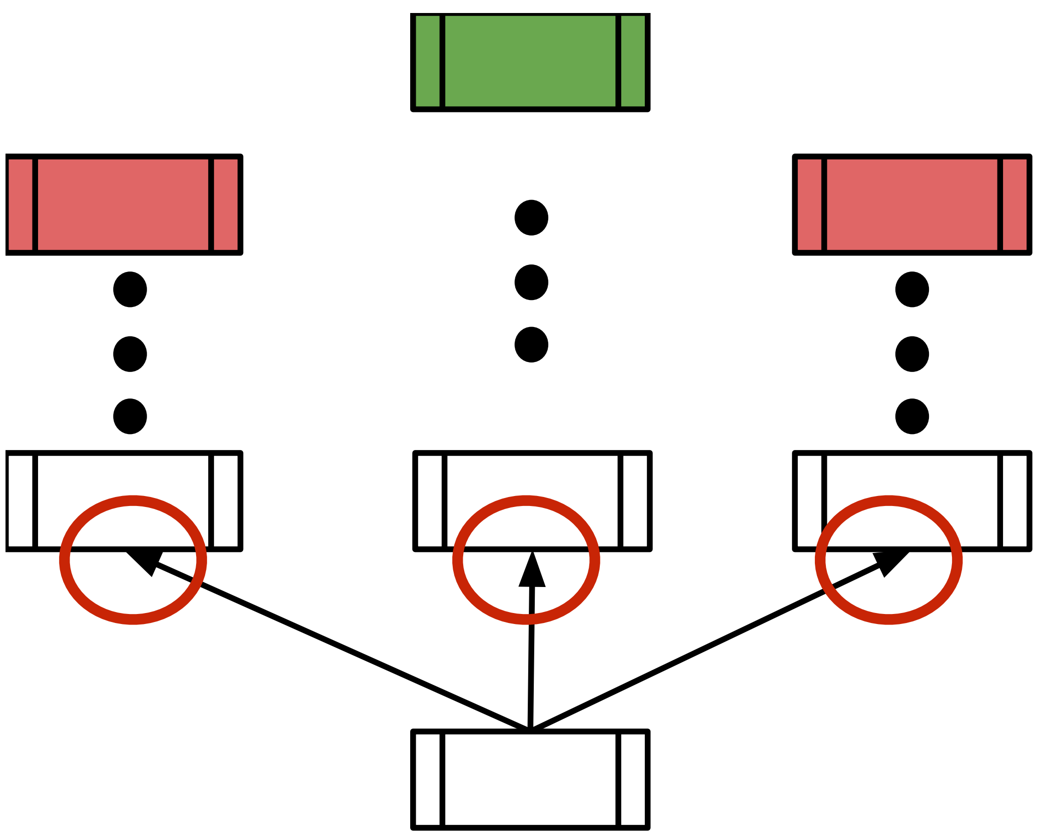
INPUT-VAR-MAP

$$\frac{\widetilde{\mathrm{r}} : \overline{\mathtt{i}}_{\mathrm{d}} = \widehat{i} \in \widetilde{\mathrm{R}} \qquad \widehat{i} \in \widehat{I} \qquad \overline{\mathtt{i}} \mathbin{@} \overline{\mathtt{c}} \qquad \overline{\mathtt{c}} \in \overline{\mathtt{IC}}}{\langle \overline{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W}, \widetilde{\mathrm{R}}, \mathrm{Z} \rangle \to \boxed{\langle \overline{\mathrm{R}}, \dot{\mathrm{R}}, \mathrm{W}, \widetilde{\mathrm{R}} - \{\widetilde{\mathrm{r}}\}, \mathrm{Z} \cup \{\widehat{i} \mapsto \overline{\mathtt{i}}_{\mathrm{d}}\} \rangle}}$$

$$\left\langle \quad \overline{R} \qquad \dot{R} \qquad W \qquad \tilde{R} \qquad Z \quad \right\rangle$$

$\overline{R}$	$\dot{R}$	W	$\tilde{R}$	Z
$W_D = U_I$	$C_I = \frac{T}{2}$	$\langle B, U \rangle$	$S \mapsto B_I$	$Z \bullet\!\!-\!\!\bullet A$
$F_I = \frac{E_I}{4}$	$B_I = S$	$\langle C, U \rangle$	$\cancel{X_D = T}$	$\boxed{T \mapsto X_D}$
	$B_I = 2 \cdot T$			
	$Z_I = T$			

The Search Algorithm



1. Initial Tableau
2. Solved Tableau
3. Tableau Transitions
4. Search Algorithm
5. Search Optimizations

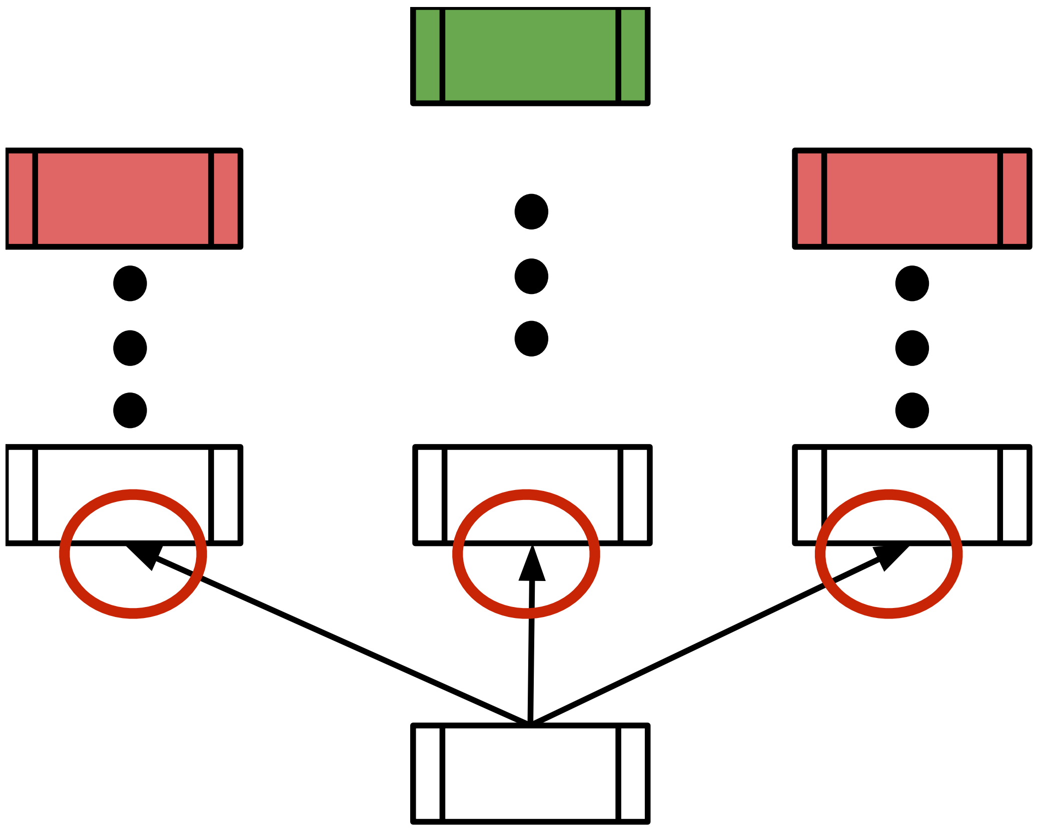
The Search Algorithm

- **<u>Frontier (F):</u>** Tableau configurations to explore
- **choose**: chooses the tableau **t** in **F** to explore.
- **select**: chooses the set of transitions to apply to **t**

<u>Algorithm</u>:

F = initial tableau
while **F**, choose **t** in **F**:
  if **t** is terminal return **Z**
  otherwise:
    select **T:** set of t' where **t**→t'
    remove **t** from **F**, add **T** to **F**

The Search Algorithm



1. Initial Tableau
2. Solved Tableau
3. Tableau Transitions
4. Search Algorithm
5. Search Optimizations

Search Optimizations

- **Search Heuristics**
  - choose lowest complexity tableau configuration.
  - select a simple goal, and prioritize transitions that solve it.

Algorithm:

F = initial tableau

while **F**, *choose* **t** in **F**:

if **t** is terminal return **Z**

otherwise:

select **T:** set of t' where **t**→**t'**

remove **t** from **F**, add **T** to **F**

Search Optimizations

- **Component Aggregation:** aggregate instances of the same component
    - **Pro**: smaller search space
    - **Con**: instance constraints must be handled separately
- **Partial Configuration Caching**
- **Compact Search Tree Data Structure**

Evaluation

Methodology

1. Implemented Arco Compiler
2. Specified Analog Hardware
3. Selected Benchmarks
4. Synthesized Hardware Configurations
5. Analyzed Configurations

Analog Hardware Components

selection of analog components from collaborators, textbooks and publications

Component	Quantity	Description	Relation
iin	25	current input	$Z_I = X_D$
vin	125	voltage input	$Z_V = X_D$
outi	10	current output	$Z_D = X_I$
vout	75	voltage output	$Z_D = X_V$
vgain	40	voltage gain	$O_V = (X_V \cdot Z_V)/(Y_V \cdot 25)$
iadd	30	current adder	$O_I = A_I + B_I + C_I + D_I$
vadd	35	voltage adder	$\partial O2_V/\partial t = 0.1(A_V + B_V - C_V - D_V \cdot O2_V)$
			$O1_V = 0.1(A_V + B_V - C_V - D_V)$
vtoi	30	voltage to current converter	$O_I = X_V/K_V$
itov	30	current to voltage converter	$O_I = K_V \cdot X_I$
ihill	8	hill function for activation/repression	$S_I = M_V(S_I/K_I)^{nv}/((S_I/K_I)^{nv} + 1)$
			$R_I = M_V/((S_I/K_I)^{nv} + 1)$
igenebind	8	gene binding	$O_I = M_I/(1 + K_I \cdot T_I)$
switch	15	genetic switch	$O_I = M_I/(S_I/K_I + 1)^{nv}$
mm	2	Michaelis-Menten dynamics	$X_V = Xt_V - XY_V$
			$Y_V = Yt_V - XY_V$
			$\partial XY_V/\partial t = K_I \cdot X_V \cdot Y_V - R_I \cdot XY_V$

G. Cowan, R. Melville, and Y. Tsividis. A VLSI analog computer/digital computer accelerator. *Solid-State Circuits, IEEE Journal of*, 41(1):42–53, Jan 2006. ISSN 0018-9200. doi: 10.1109/JSSC.

R. Daniel, S. S. Woo, L. Turicchia, and R. Sarpeshkar. Analog transistor models of bacterial genetic circuits. In *Biomedical Circuits and Systems Conference (BioCAS), 2011 IEEE*, IEEE, 2011.

R. Sarpeshkar. Ultra Low Power Bioelectronics: Funda- mentals, Biomedical Applications, and Bio-Inspired Systems. Cambridge University Press, 2010. ISBN 0521857279.

J. J. Y. Teo, S. S. Woo, and R. Sarpeshkar. Synthetic biology: A unifying view and review using analog circuits. *IEEE Trans. Biomed. Circuits and Systems*, 9(4):453–474, 2015.

S. S. Woo, J. Kim, and R. Sarpeshkar. A cytomorphic chip for quantitative modeling of fundamental bio-molecular circuits. *IEEE Trans. Biomed. Circuits and Systems*, 9(4):527–542, 2015.

Dynamical Systems Benchmarks

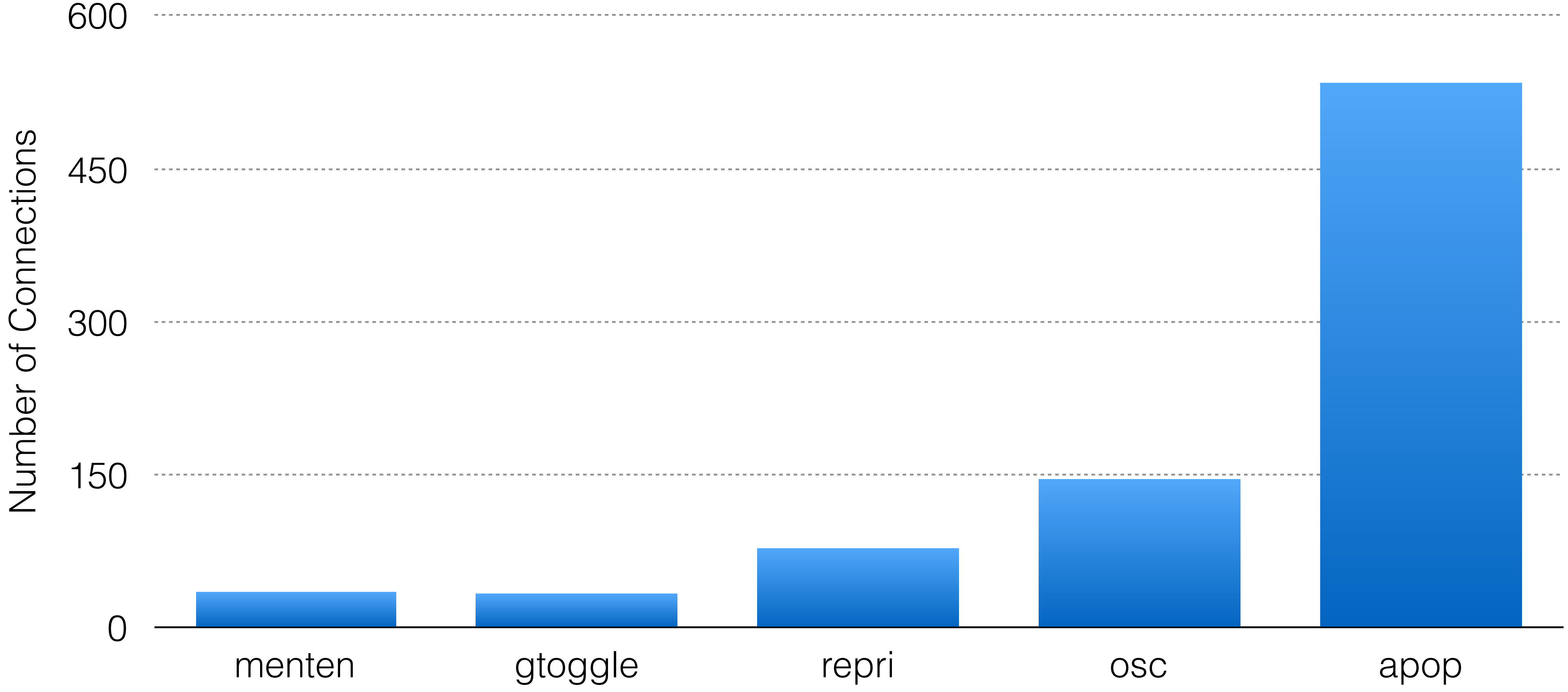
selection of published artifacts from well- cited computational biology papers from `Biomodels` database

Benchmark	Parameters	Functions	Differential Equations
menten	3	0	4
gentoggle	9	3	2
repr	7	3	6
osc	16	16	9
apop	87	48	27

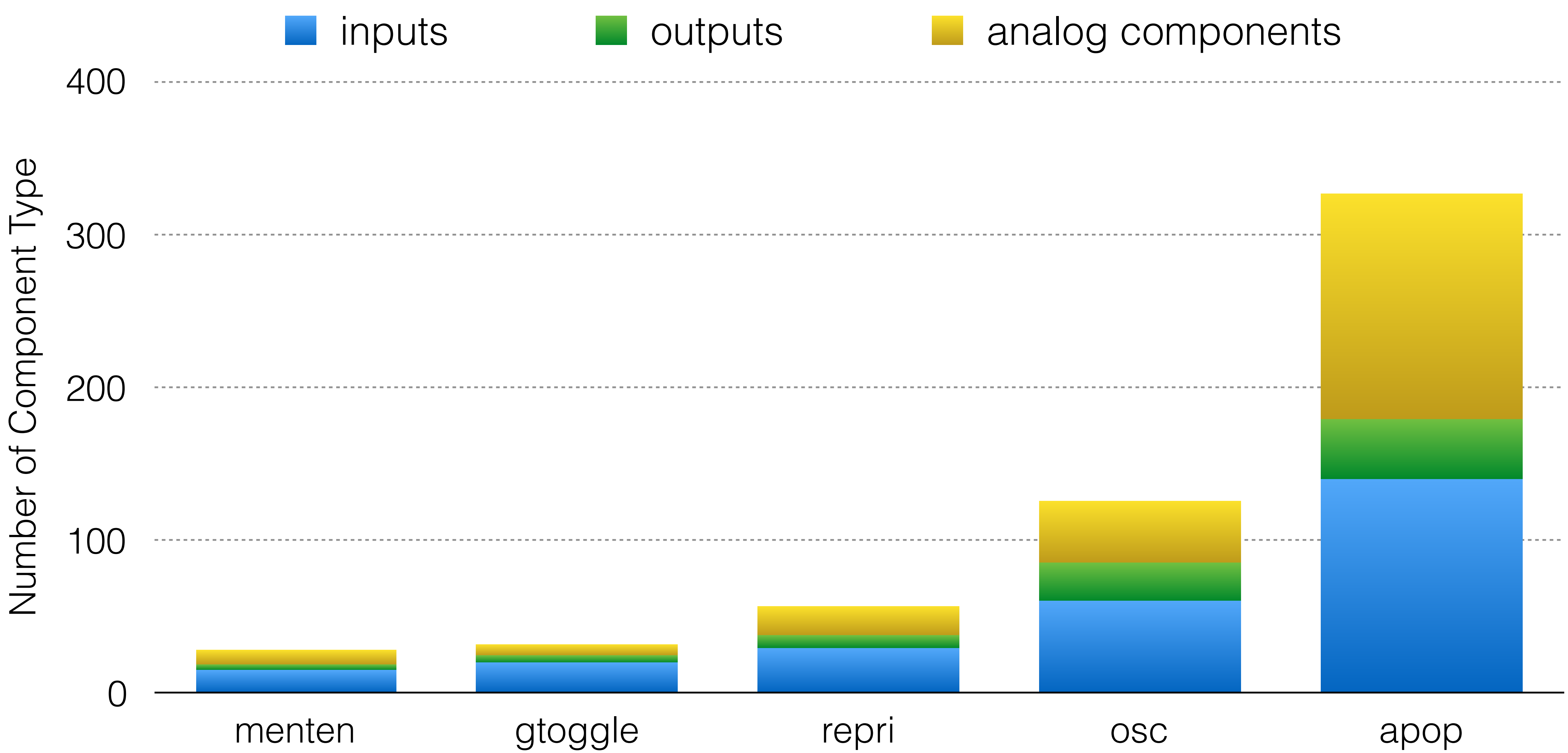
- **menten**: Michaelis-Menten equation reaction. D. R. F. PhD. Biochemistry (Lippincott Illustrated Reviews Series). LWW, 2013. ISBN 1451175620.
- **gentoggle**: genetic toggle switch in E.col. T. S. Gardner, C. R. Cantor, and J. J. Collins. Construction of a genetic toggle switch in escherichia coli. *Nature*, 403(6767): 339–342, 2000.
- **repri**: synthetic oscillatory network of transcriptional regulators. M. B. Elowitz and S. Leibler. A synthetic oscillatory network of transcriptional regulators. *Nature*, 403(6767):335–338, 2000.
- **osc**: circadian oscillation utilizing activator / repressor. J. M. Vilar, H. Y. Kueh, N. Barkai, and S. Leibler. Mechanisms of noise-resistance in genetic oscillators. *Proceedings of the National Academy of Sciences*, 99(9):5988–5992, 2002
- **apop**: protein stress response. K. Erguler, M. Pieri, and C. Deltas. A mathematical model of the unfolded protein stress response reveals the decision mechanism for recovery, adaptation and apoptosis. *BMC systems biology*, 7(1):16, 2013.

https://www.ebi.ac.uk/biomodels-main/

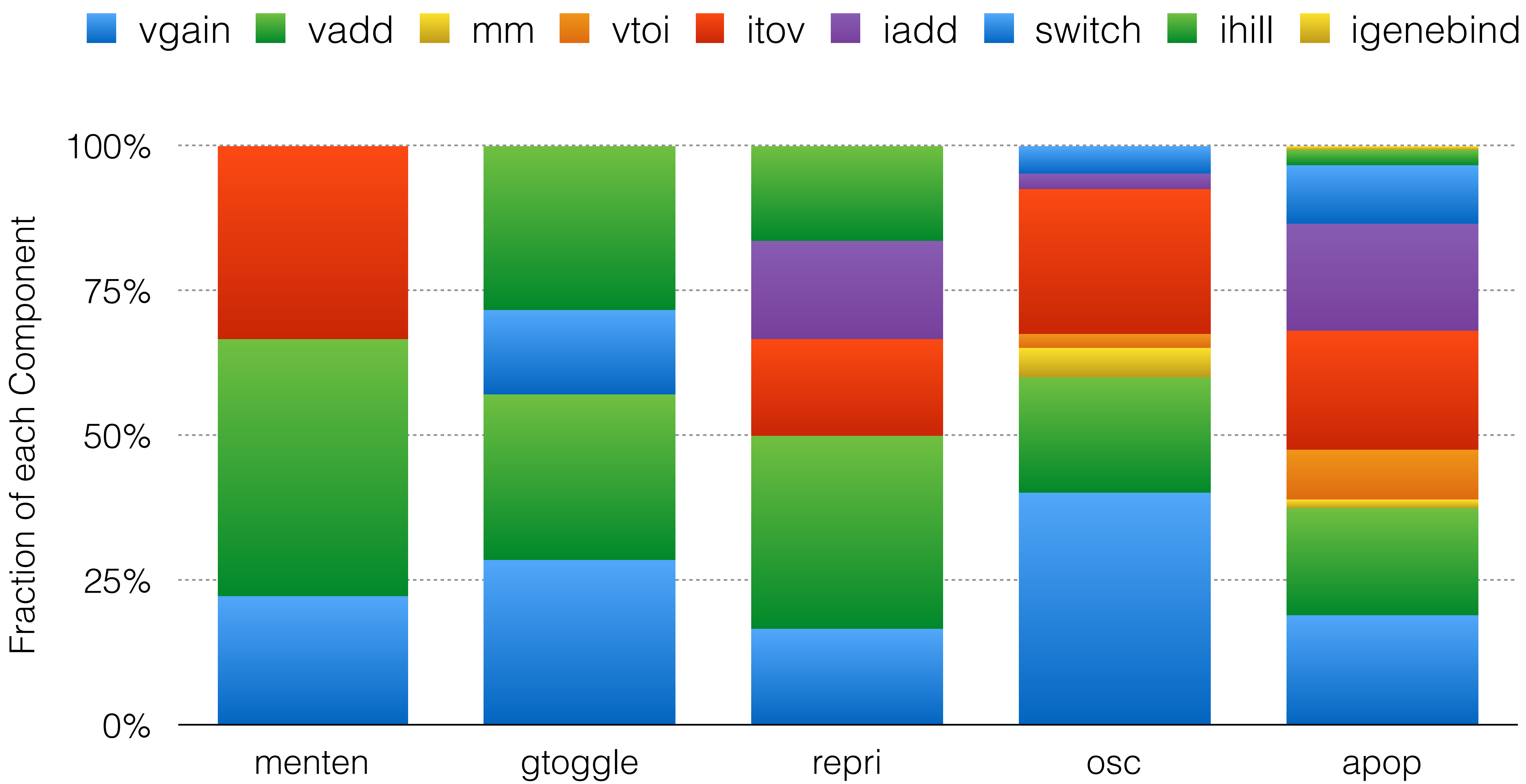
Connections



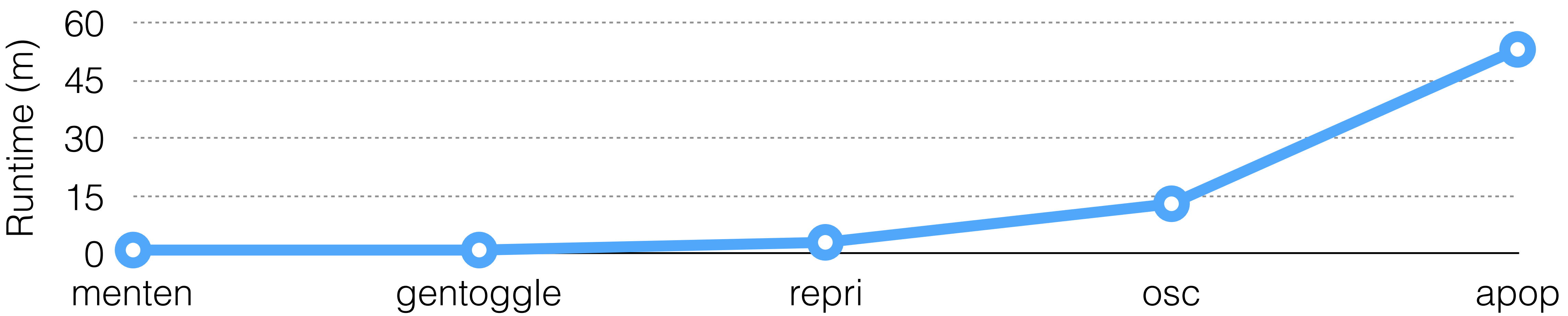
Component Types

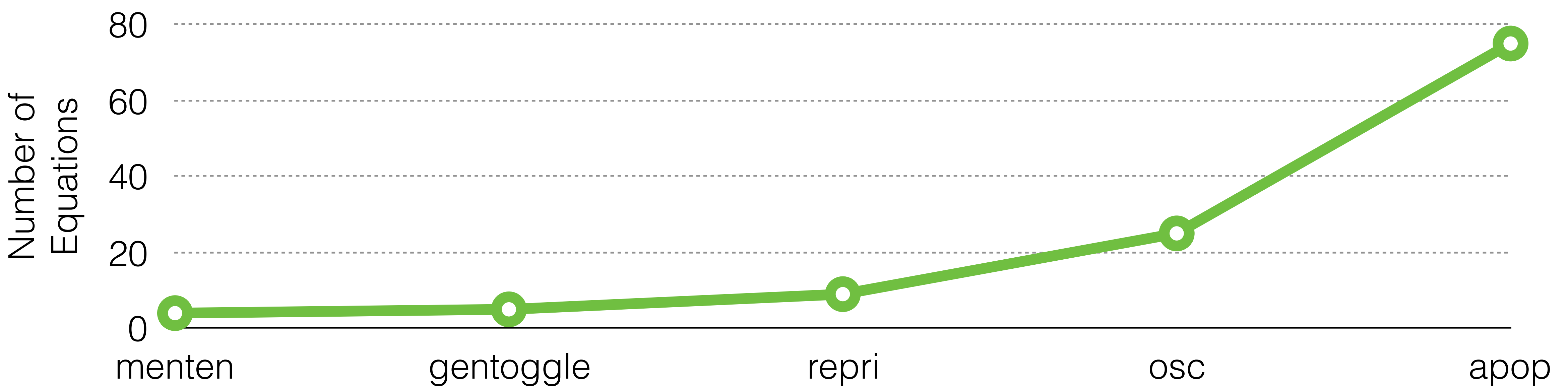


Components


vgain
vadd
mm
vtoi
itov
iadd
switch
ihill
igenebind
100%
75%
50%
25%
0%
Fraction of each Component
menten
gtoggle
repri
osc
apop

Arco Runtime

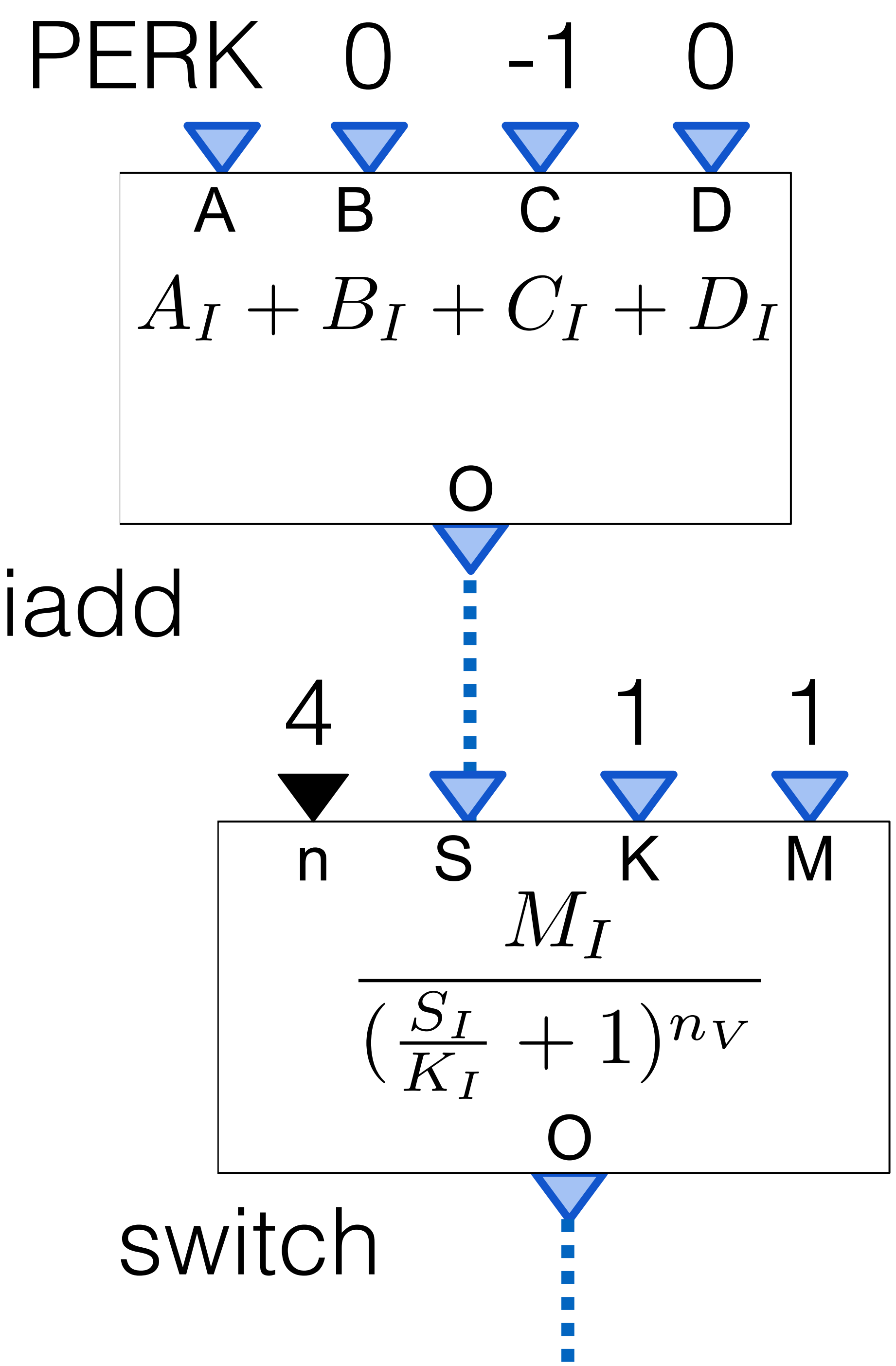




$PERK^{-4}$ term in apop

$$PERK^{-4}$$

$$\frac{1}{(\frac{PERK+0-1-0}{1}+1)^4}$$



Related Work

Analog Neural Network Accelerators for Approximate Computations

1. Esmaeilzadeh, Sampson, Ceze, Burger. Neural acceleration for general-purpose approximate computations. MICRO 2012

2. Amant, Yazdanbakhsh, Park, Thwaites, Esmaeilzadeh, Hassibi, Ceze, Burger. General-purpose code acceleration with limited-precision analog computation. ISCA 2014

	Arco
approximate acceleration of digital subcomputations	**exact mapping of dynamical systems**
training phase	**no training**
analog computation for neural networks	**analog computation for dynamical systems**

Related Work

Discrete Models: Formalisms for modeling digital computing systems, repurposed to model biological systems

1. Fisher, Henzinger. Executable Cell Biology. Nature Biotechnology, 2007
2. Fisher, Harel, Henzinger. Biology as Reactivity. CACM 2011

	Arco
causality of events, no modeling of time	**continuous time analog models based on differential equations**
general-purpose digital hardware	**energy efficient analog hardware**

Conclusion

- same model of computation for the last half century
- **programmable analog devices**: new and powerful model of computation
  - powerful, energy efficient primitives
  - compelling initial application (biology)
- programming language techniques are a key enabling technology
  - we have presented first compiler for this platform
  - deep research area; decades of new and compelling results