

# Time Dilation and Contraction for Programmable Analog Devices with Jaunt

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$$E = 6800 - ES$$

$$S = 4400 - ES$$

$$\partial ES / \partial t = 0.0001 \cdot E \cdot S - 0.01 \cdot ES$$

$$ES(0) = 0$$

Dynamical  
System

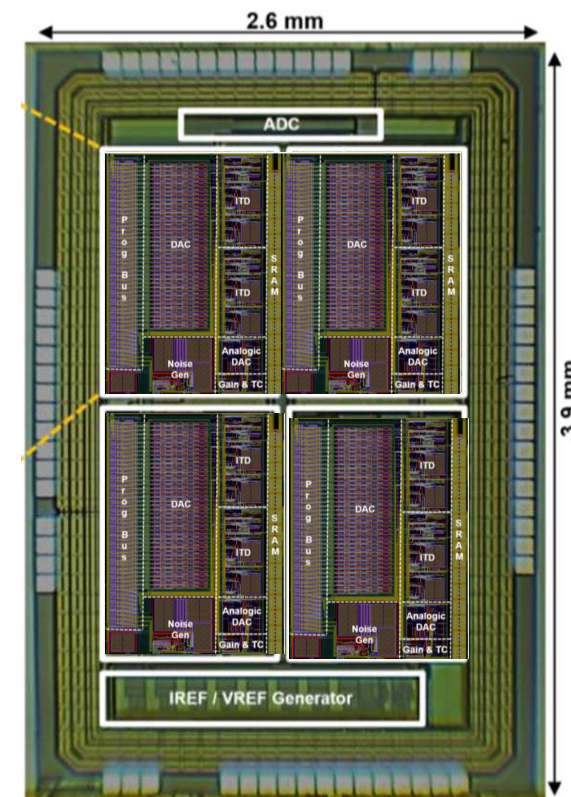
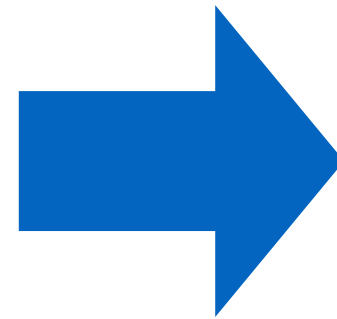
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Dynamical  
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Programmable  
Analog Devices

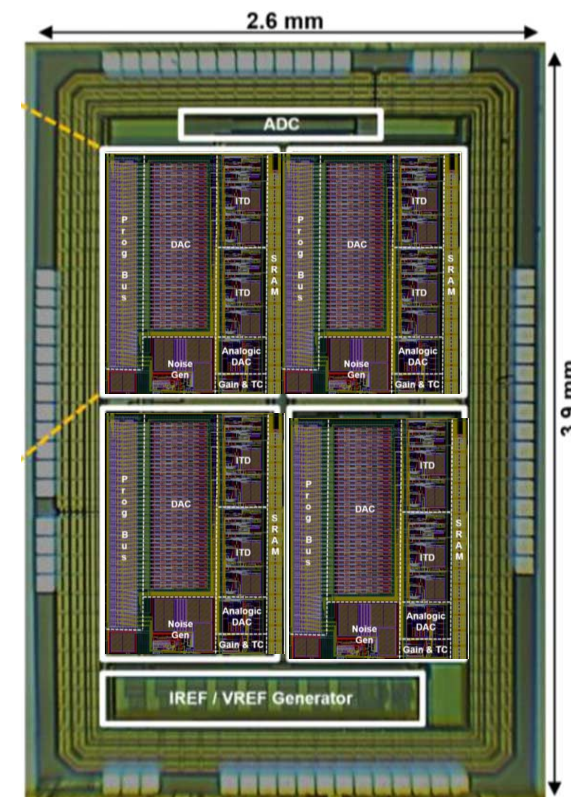
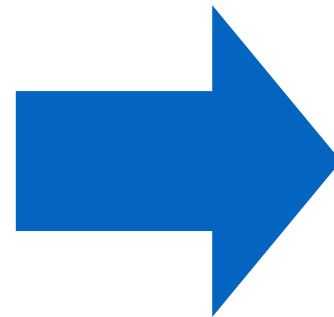
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Dynamical  
System



Programmable  
Analog Devices  
**With Physical Limitations**

# Change Dynamic Range of Variables

## Change Speed of Simulation

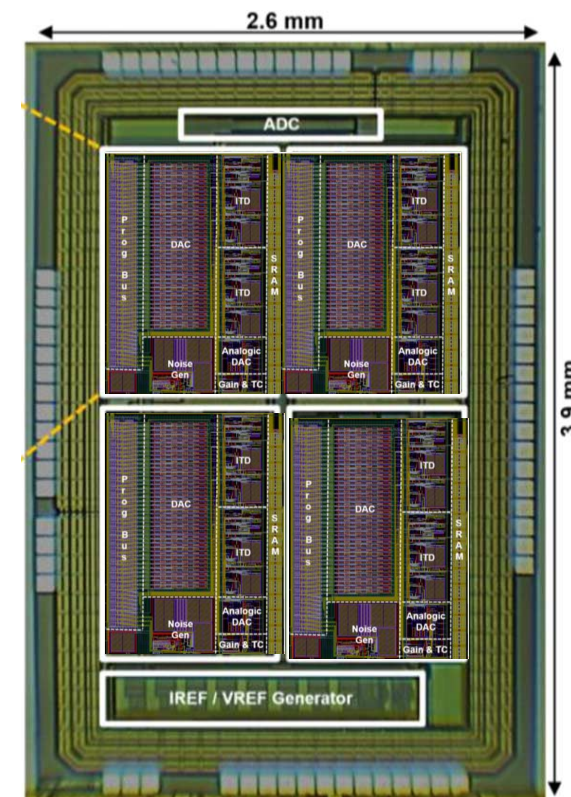
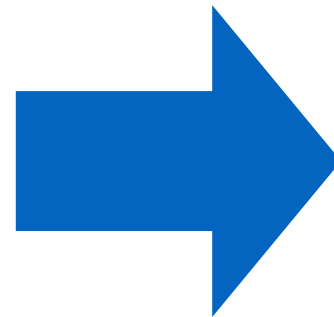
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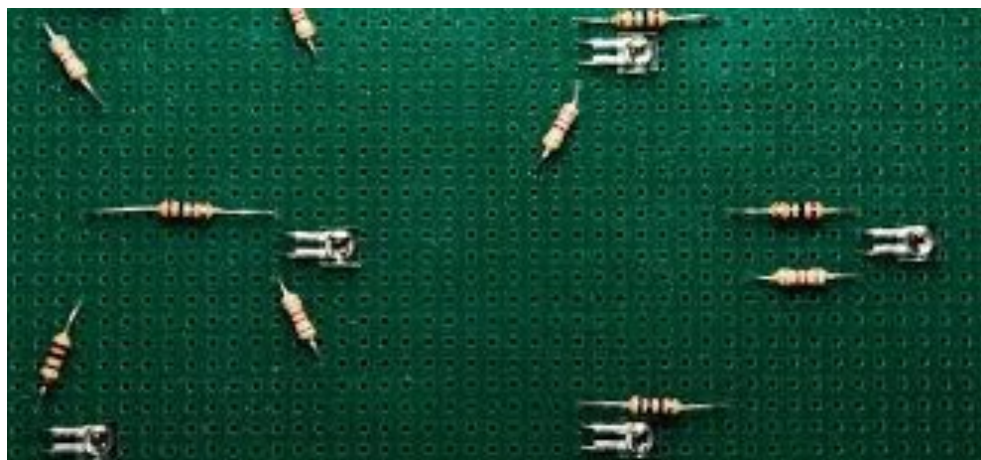
Dynamical  
System



Programmable  
Analog Devices  
**With Physical Limitations**



# Modeling the Physical World



# Dynamical Systems Model Biological Processes

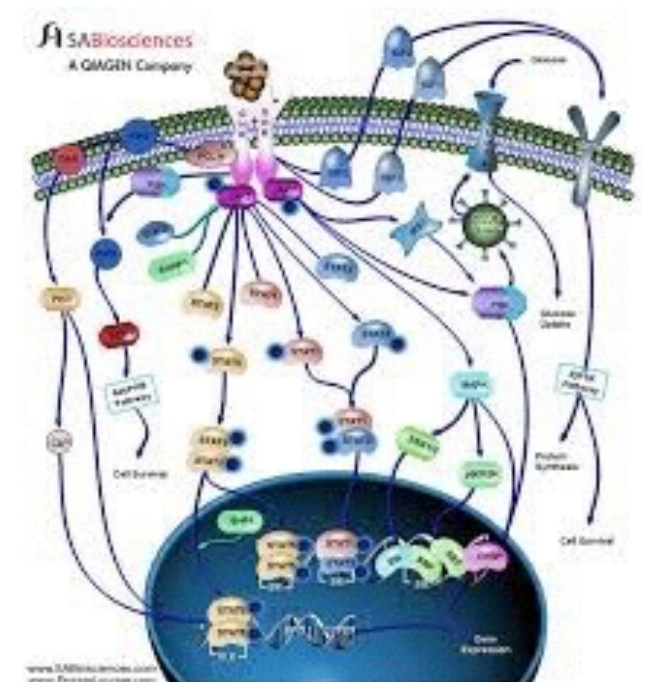
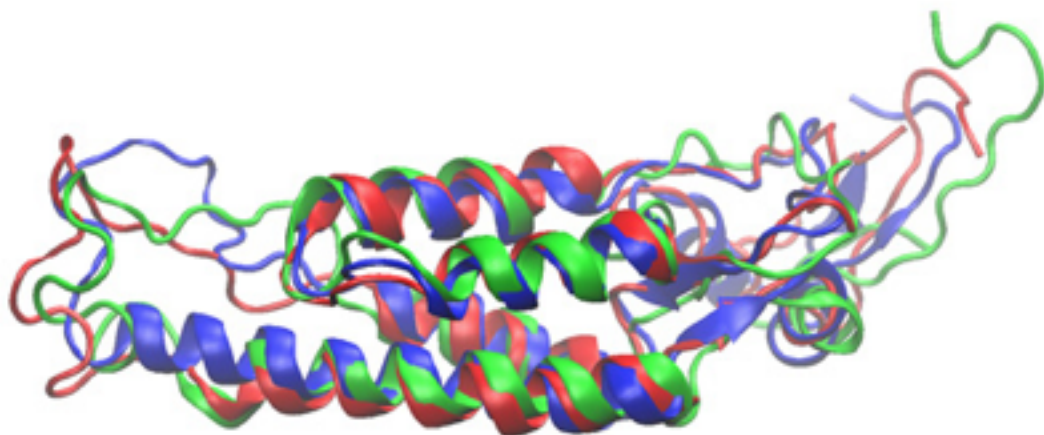
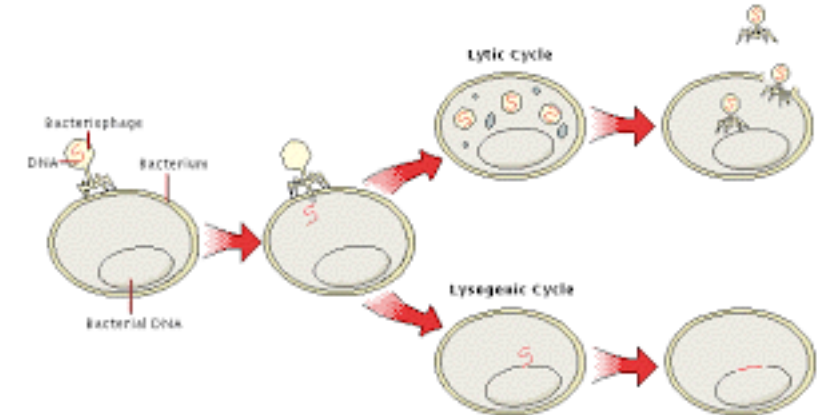


$$E = 6800 - ES$$

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$$\partial ES / \partial t = 0.0001 \cdot E \cdot S - 0.01 \cdot ES$$

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# Dynamical Systems

$$E = 6800 - ES$$

$$S = 4400 - ES$$

$$\partial ES / \partial t = 0.0001 \cdot E \cdot S - 0.01 \cdot ES$$

$$ES(0) = 0$$

- **state variables** that model physical quantities (E, S, ES)
- **differential equations** that specify continuous dynamics of *state variables* over time



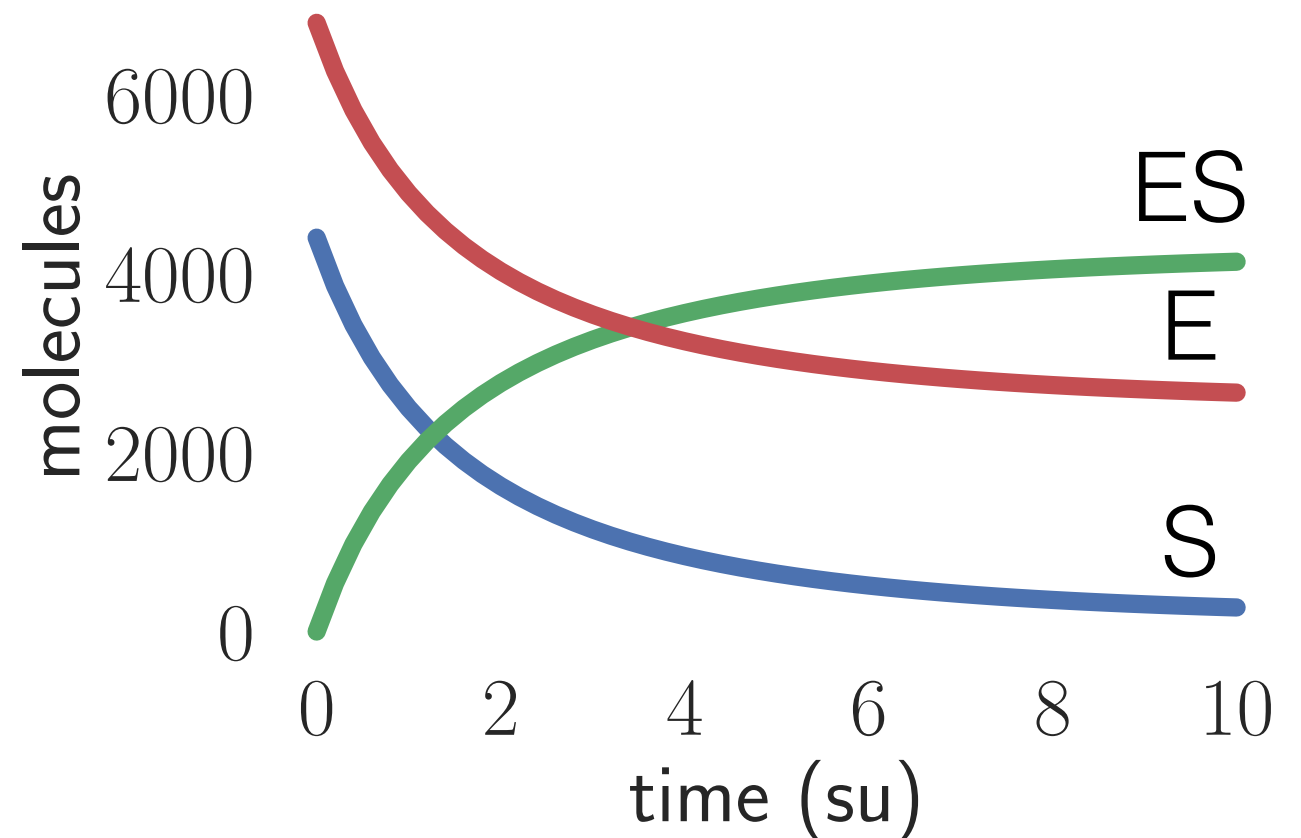
# Goal: Simulate Dynamical System

$$E = 6800 - ES$$

$$S = 4400 - ES$$

$$\partial ES / \partial t = 0.0001 \cdot E \cdot S - 0.01 \cdot ES$$

$$ES(0) = 0$$



- Compute dynamics of state variables over time
- Plot concentrations of compounds over time

# Analog Computing circa 1950

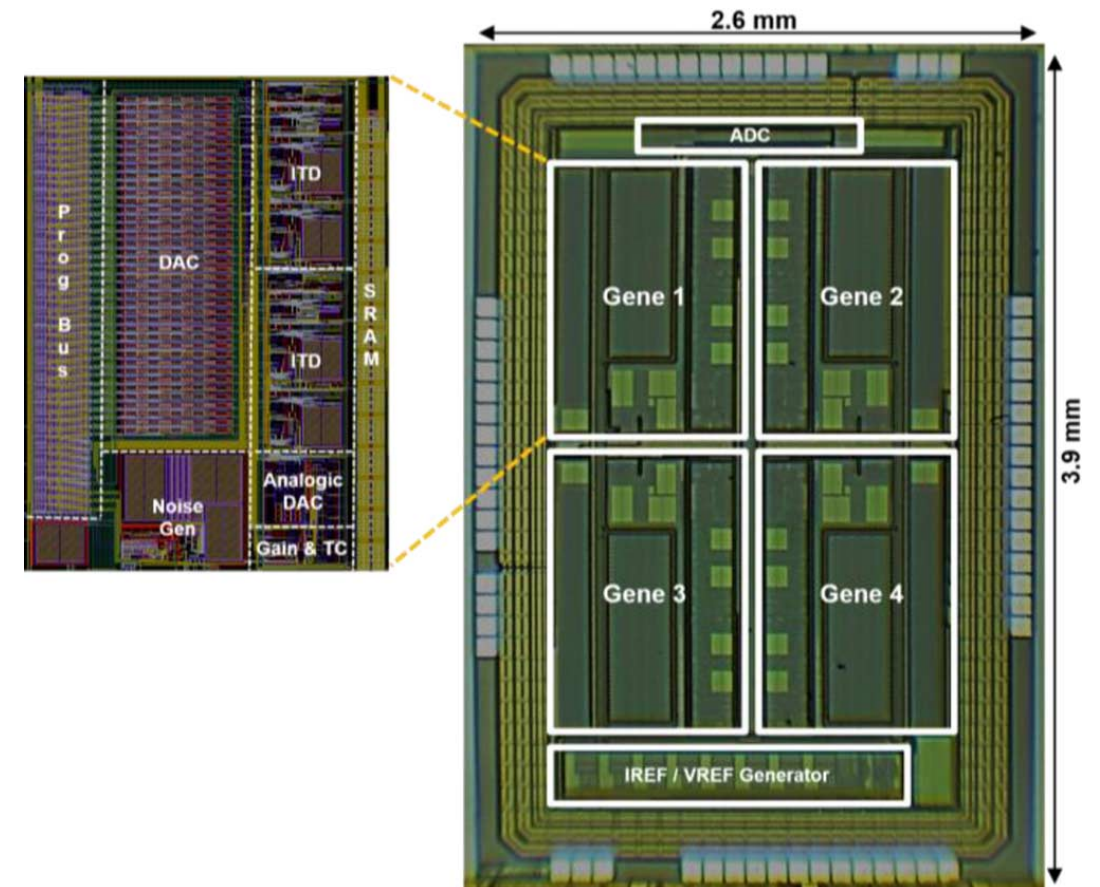
Telefunken RAT 700

- **direct mapping**
  - variables → properties of wires
  - properties: voltage, current
  - dynamics → circuit dynamics
- **straightforward simulation:**
  - power up circuit
  - measure circuit properties over time
- **1970-2010:** *Age of Digital Computers*
  - Analog computers out of fashion



# Programmable Analog Devices

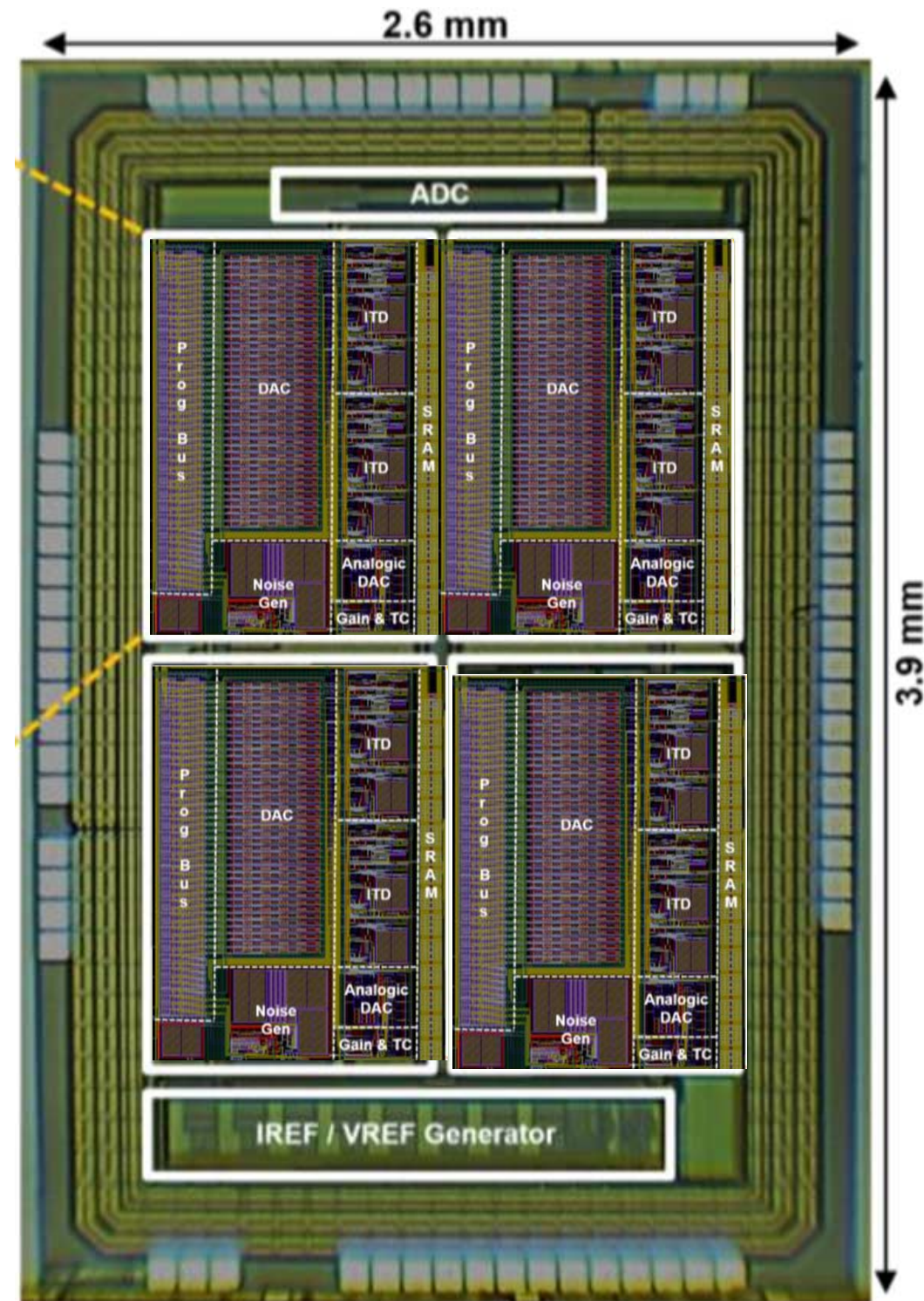
- modernized hardware
  - modern semiconductor technology
- new capabilities
  - powerful, heavily optimized analog components
  - digital reprogrammability
  - exploit analog noise



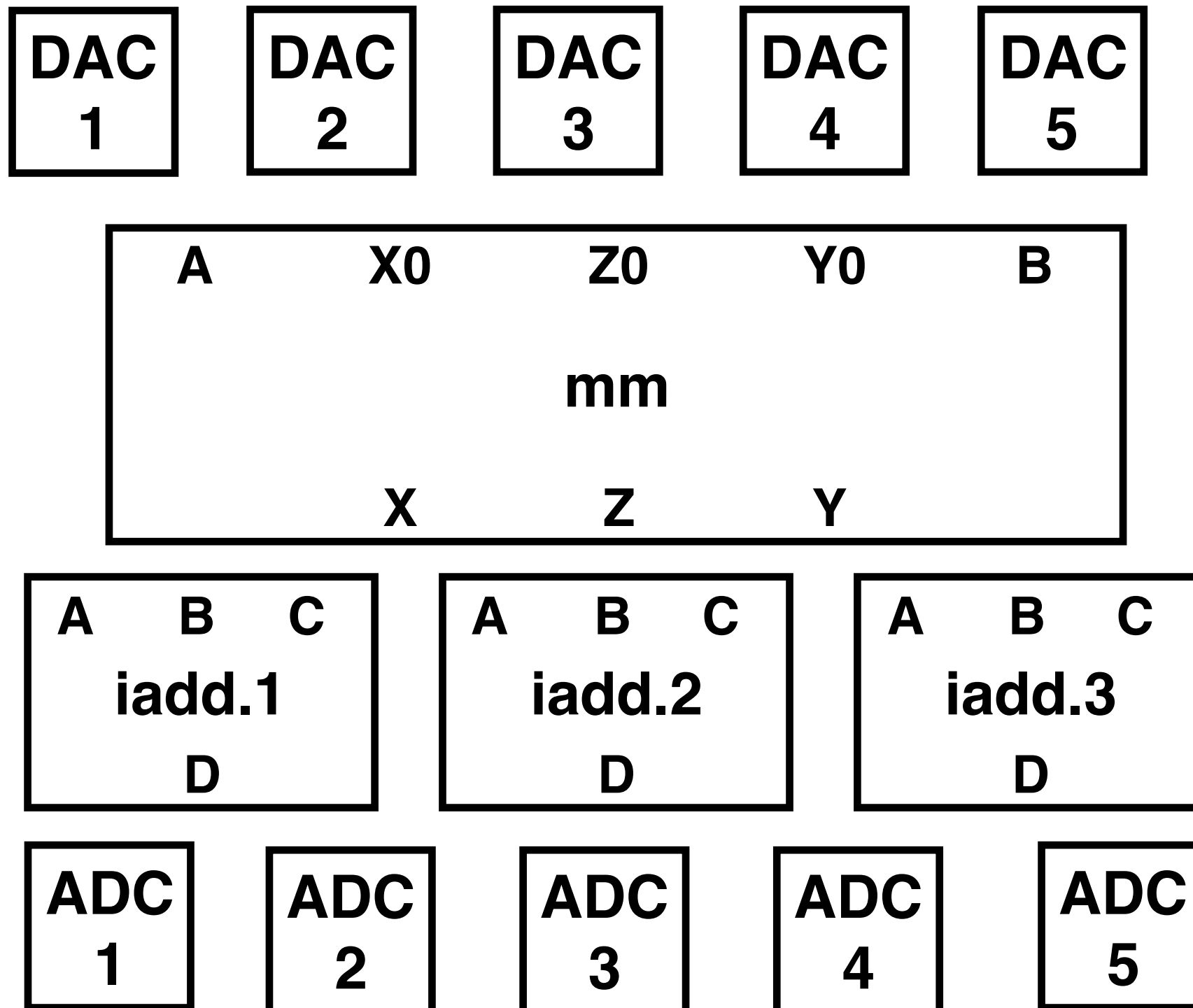
S. S. Woo, J. Kim, and R. Sarpeshkar. A cytomorphic chip for quantitative modeling of fundamental bio-molecular circuits. *IEEE Trans. Biomed. Circuits and Systems*, 9(4):527–542, 2015.



# Analog Device

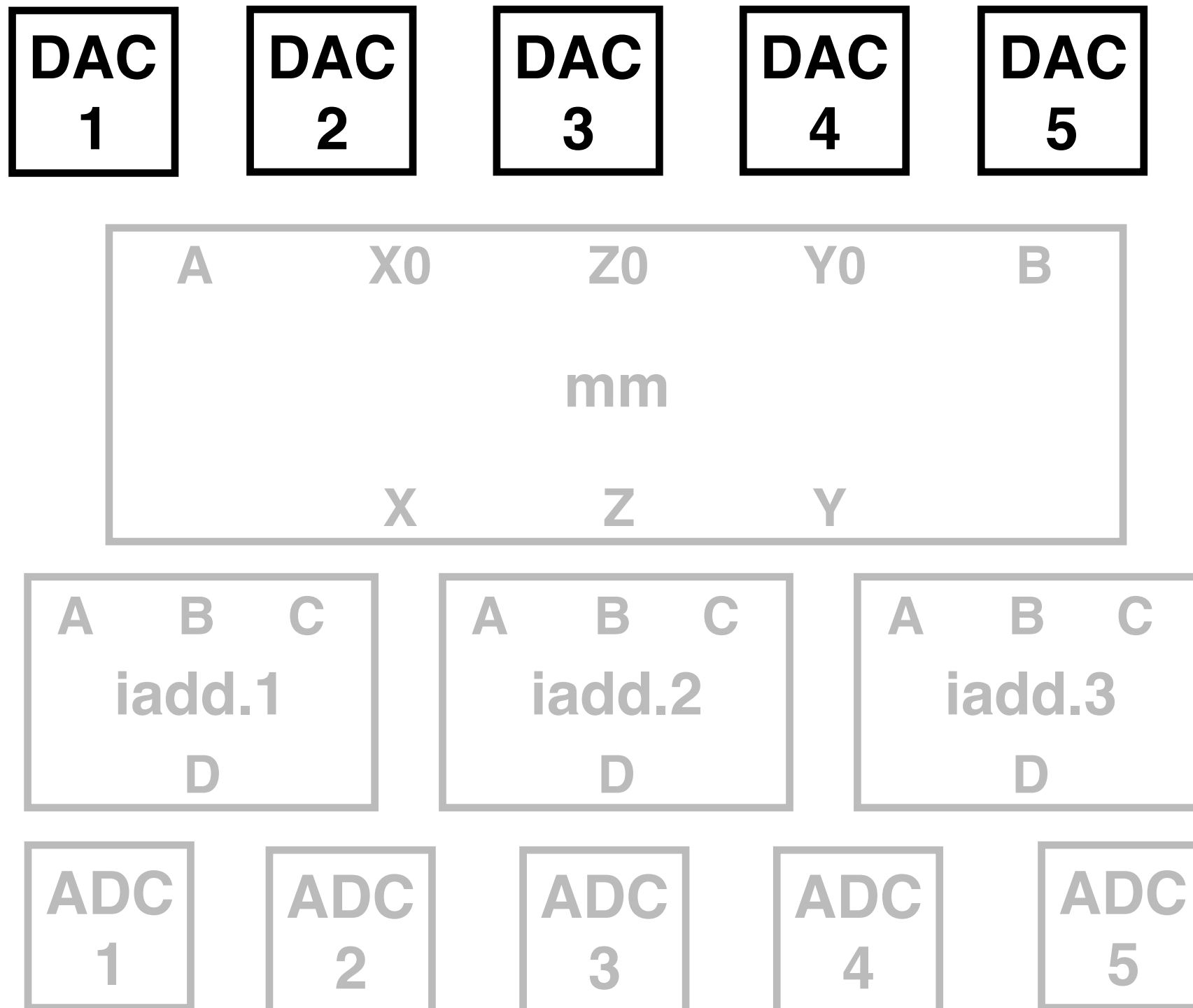


# Analog Device



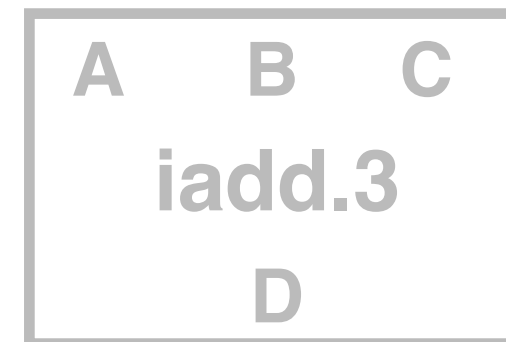
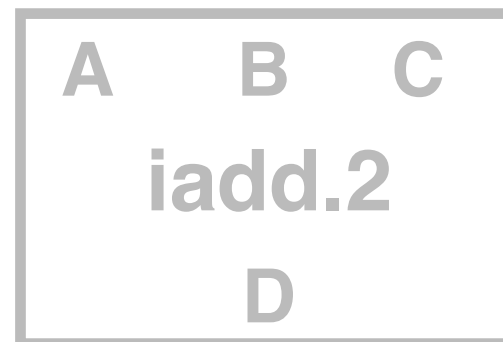
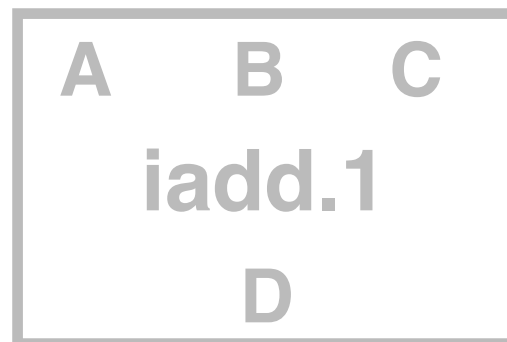
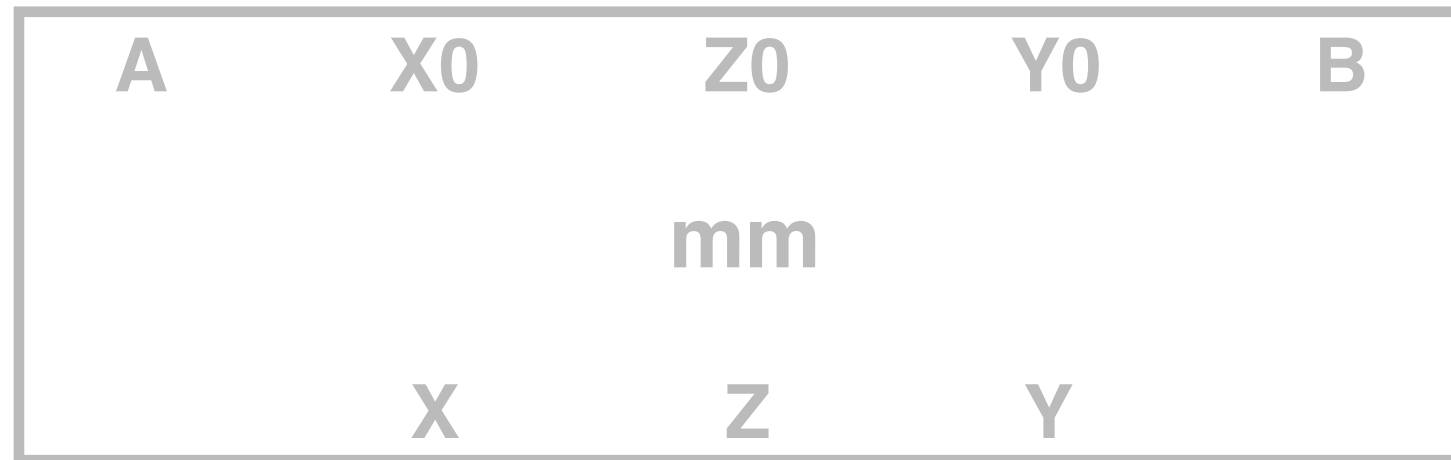


# Analog Device

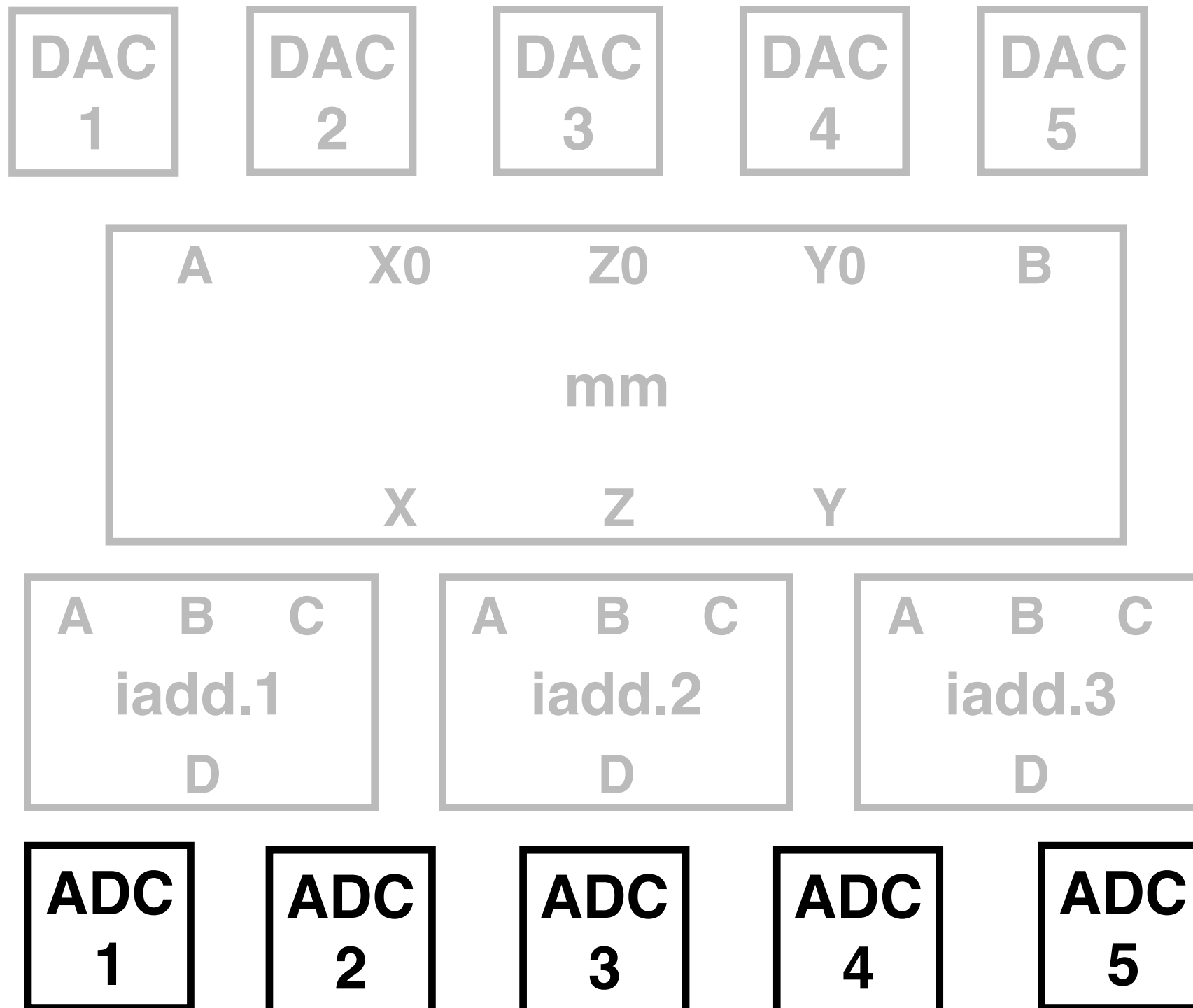


# Analog Device

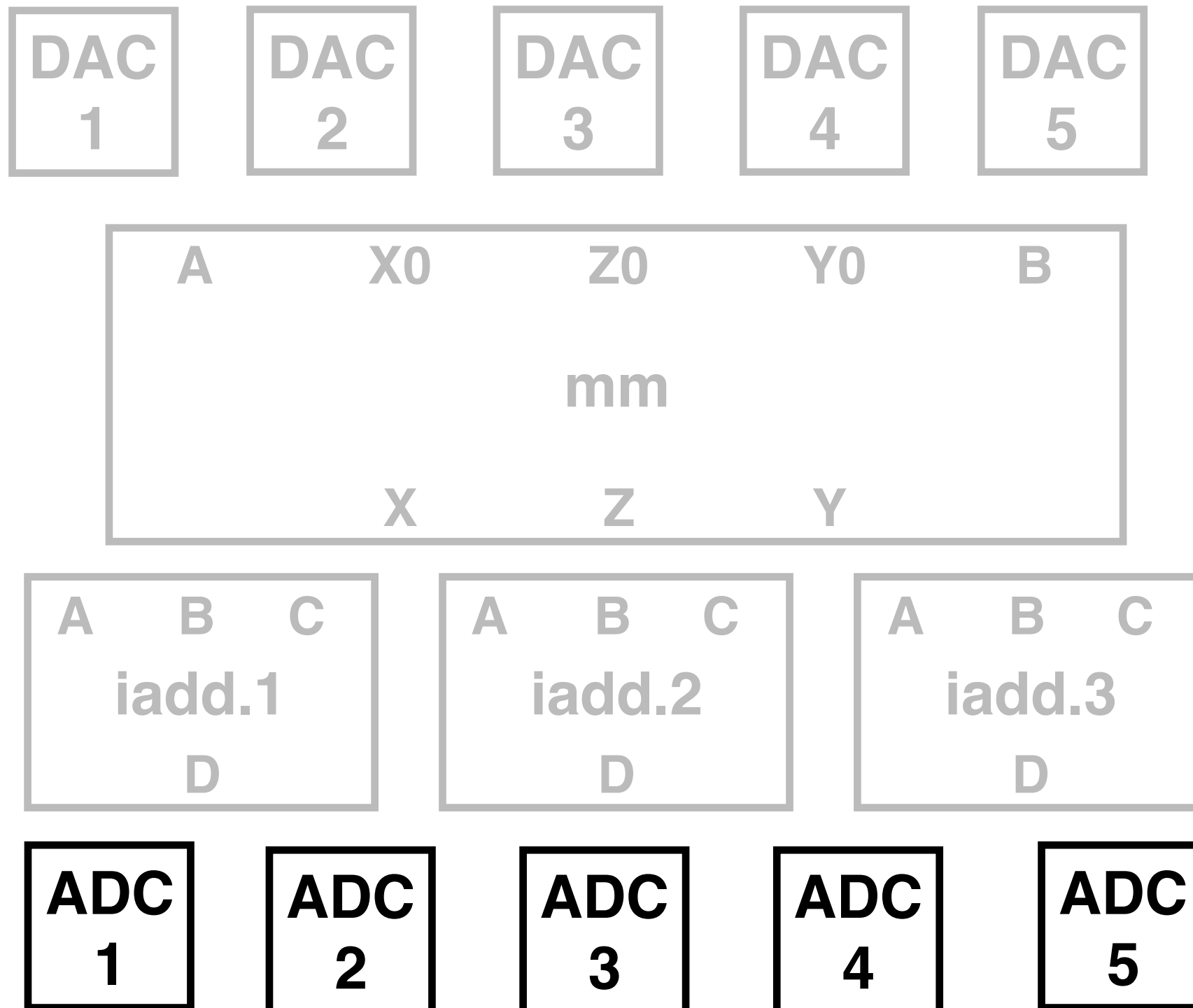
every 0.1 hu



# Analog Device



# Analog Device



every 1 hu

# Analog Device

DAC  
1

DAC  
2

DAC  
3

DAC  
4

DAC  
5

| A  | X0 | Z0           | Y0 | B |
|--|----|--------------|----|---|
| $Z = \int A \cdot X \cdot Y - B \cdot Z \, dt$ |    | $X = X0 - Z$ |    |   |
| $Z = Z0 \text{ @ } t=0$                        |    | $Y = Y0 - Z$ |    |   |
|  | X  | Z            | Y  |   |

| A               | B | C |
|-----------------|---|---|
| $D = A + B - C$ |   |   |
| D               |   |   |

| A               | B | C |
|-----------------|---|---|
| $D = A + B - C$ |   |   |
| D               |   |   |

| A               | B | C |
|-----------------|---|---|
| $D = A + B - C$ |   |   |
| D               |   |   |

ADC  
1

ADC  
2

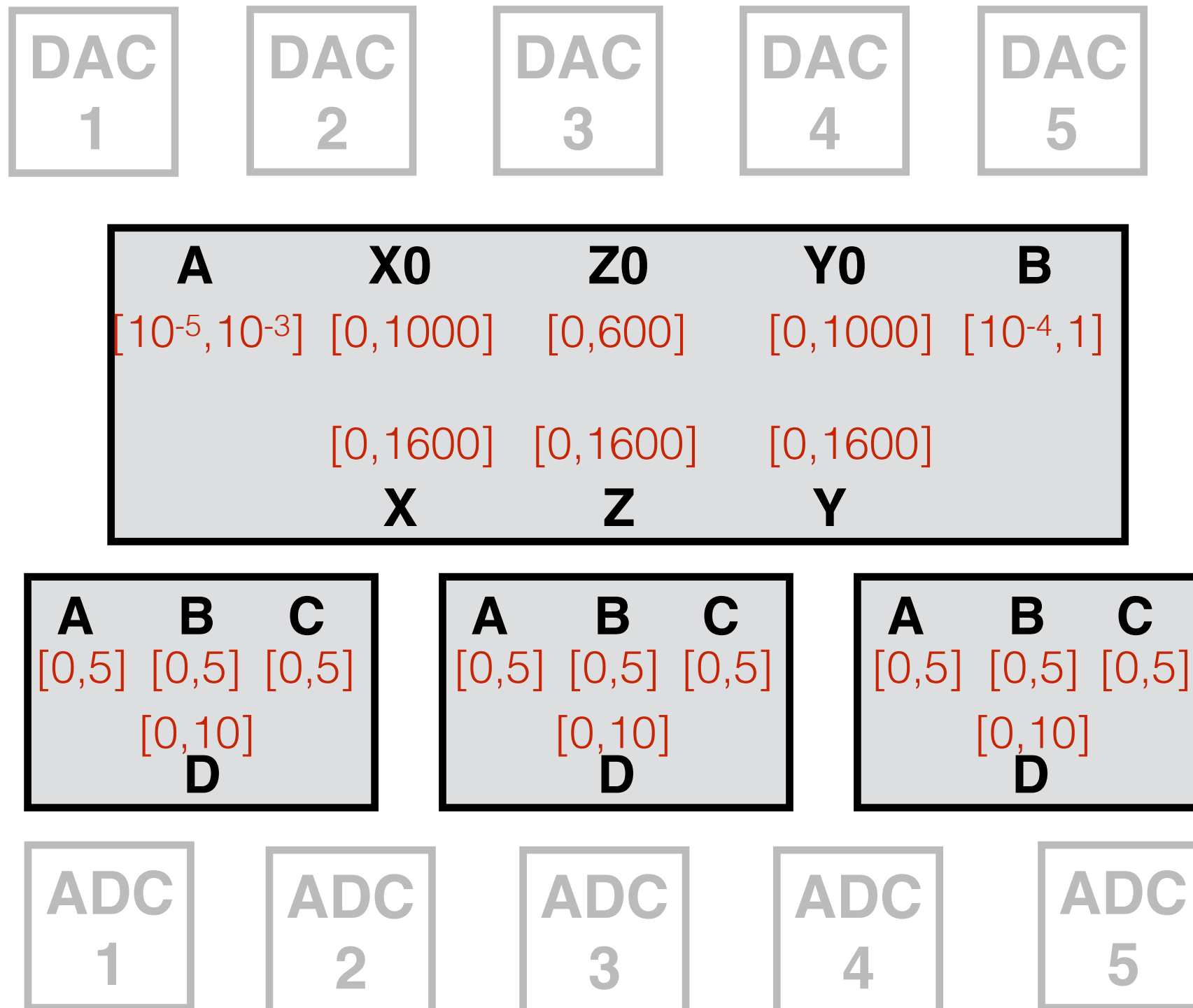
ADC  
3

ADC  
4

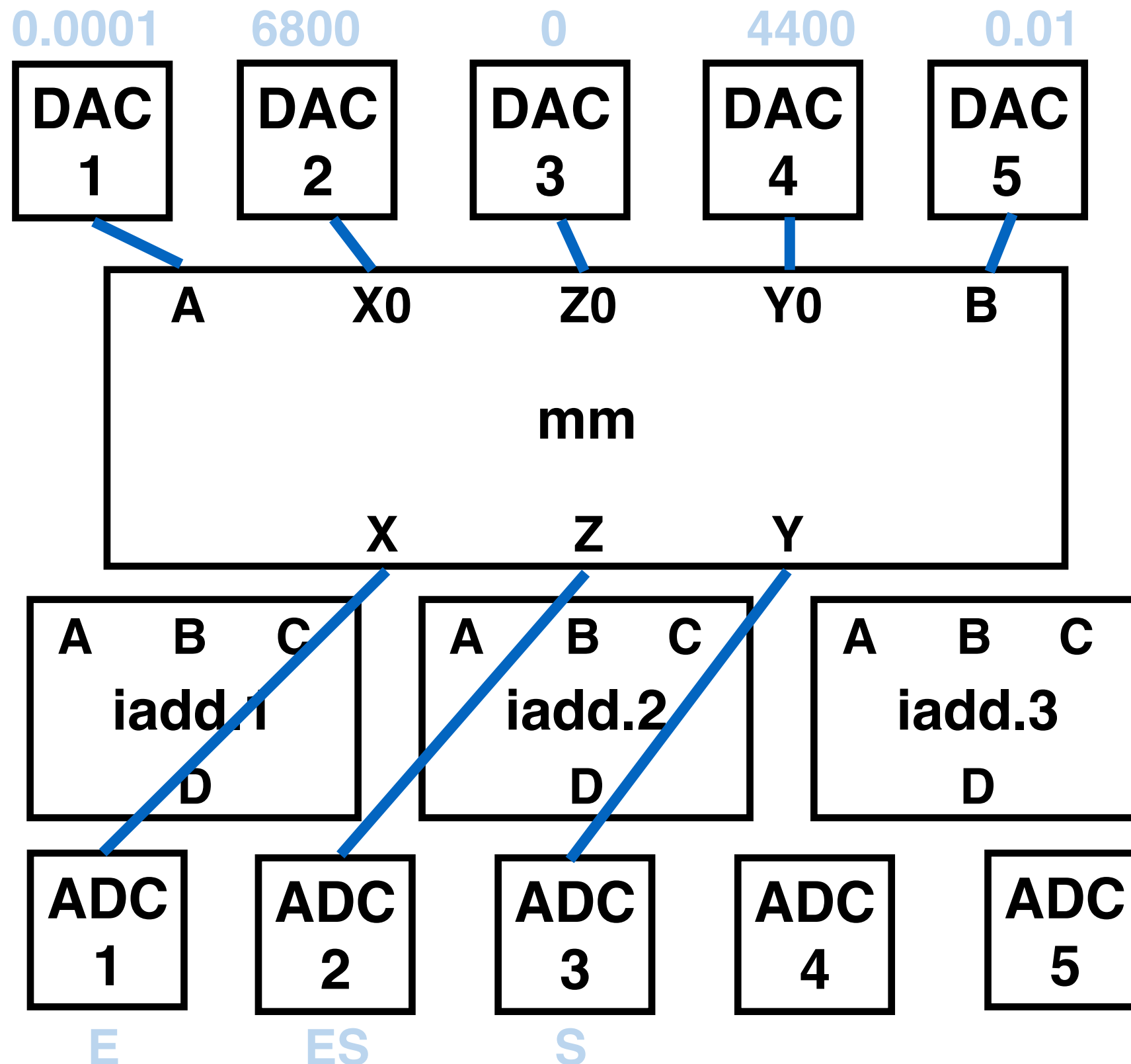
ADC  
5



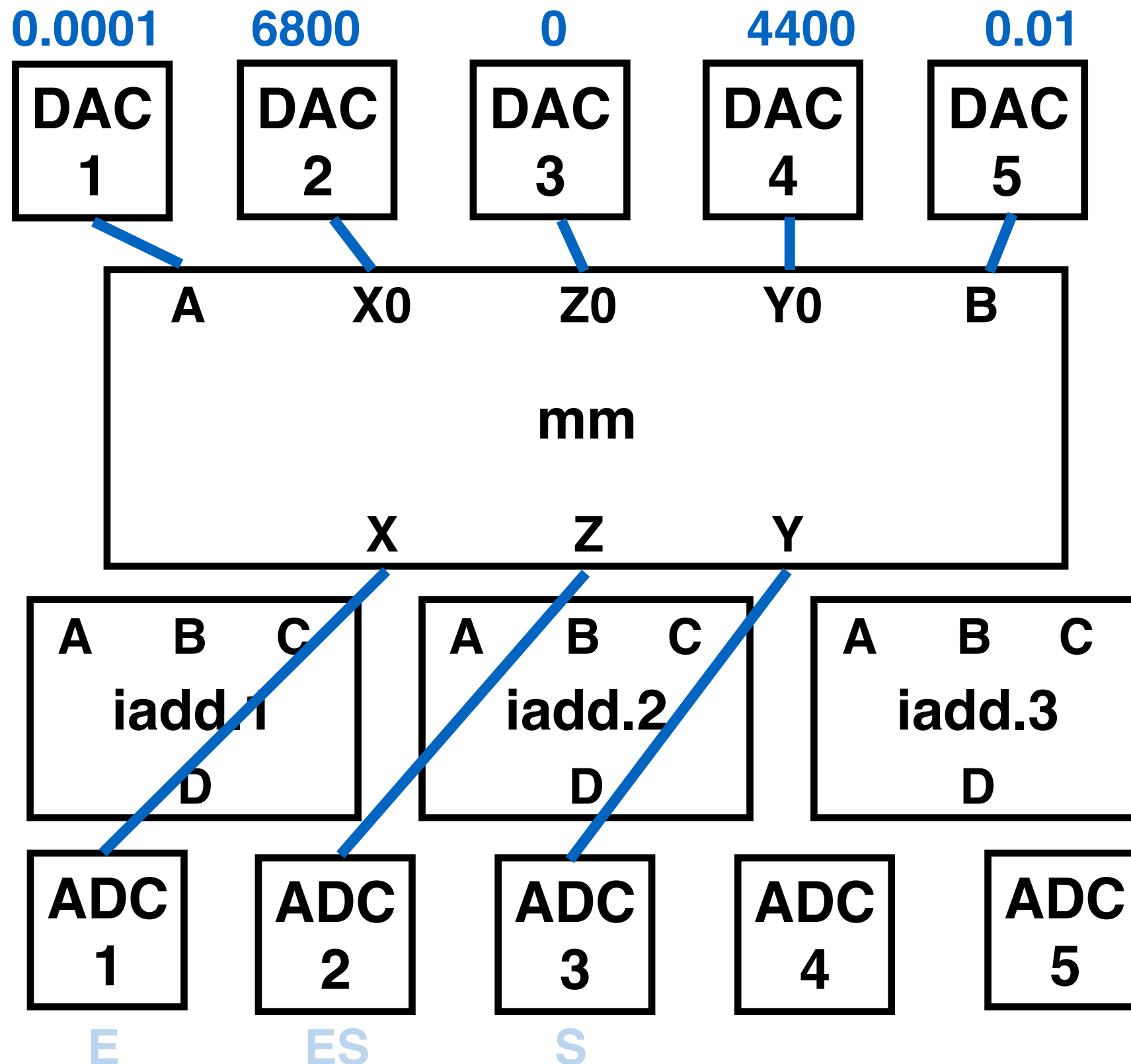
# Analog Device



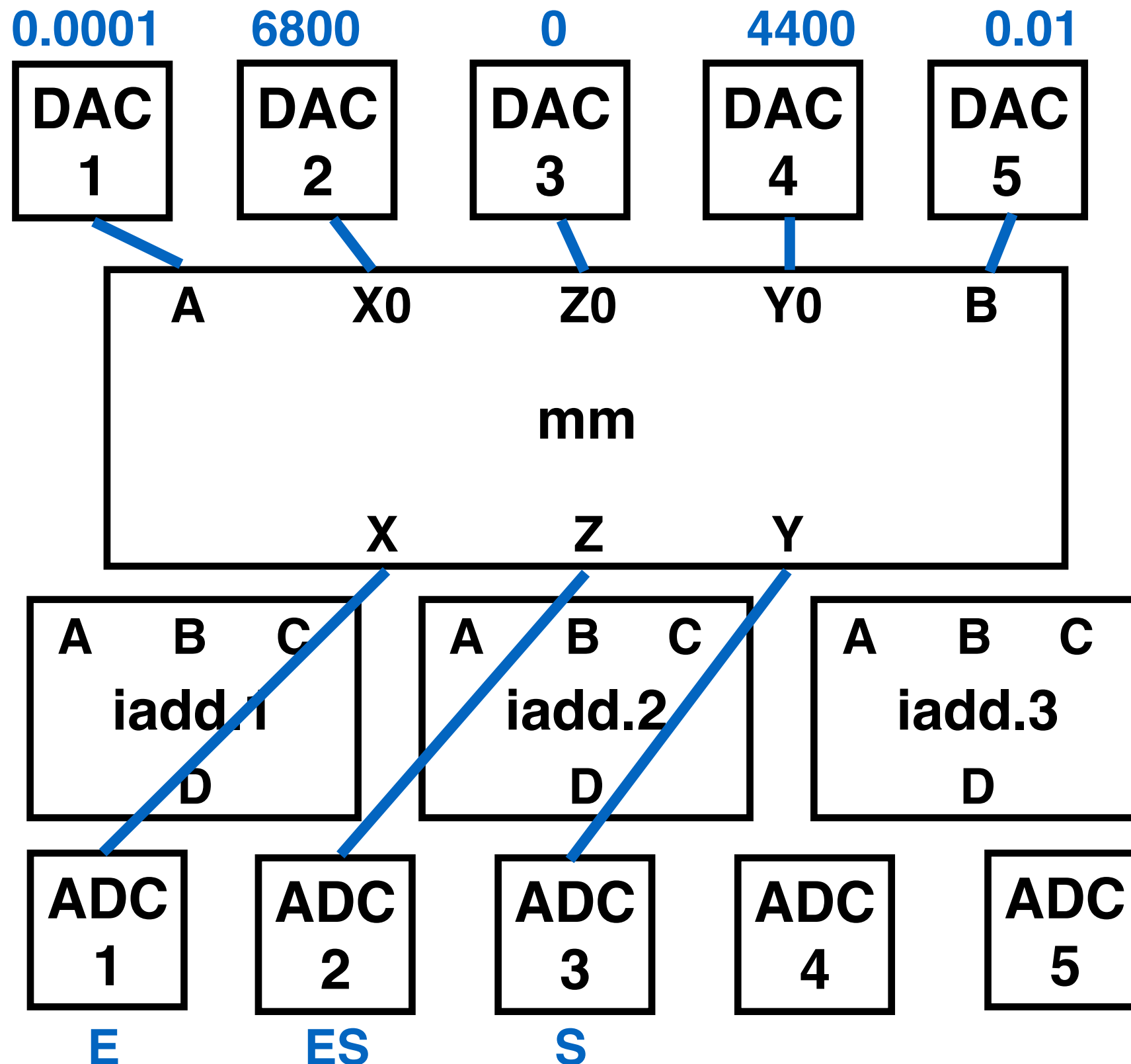
# Configuring the Analog Device



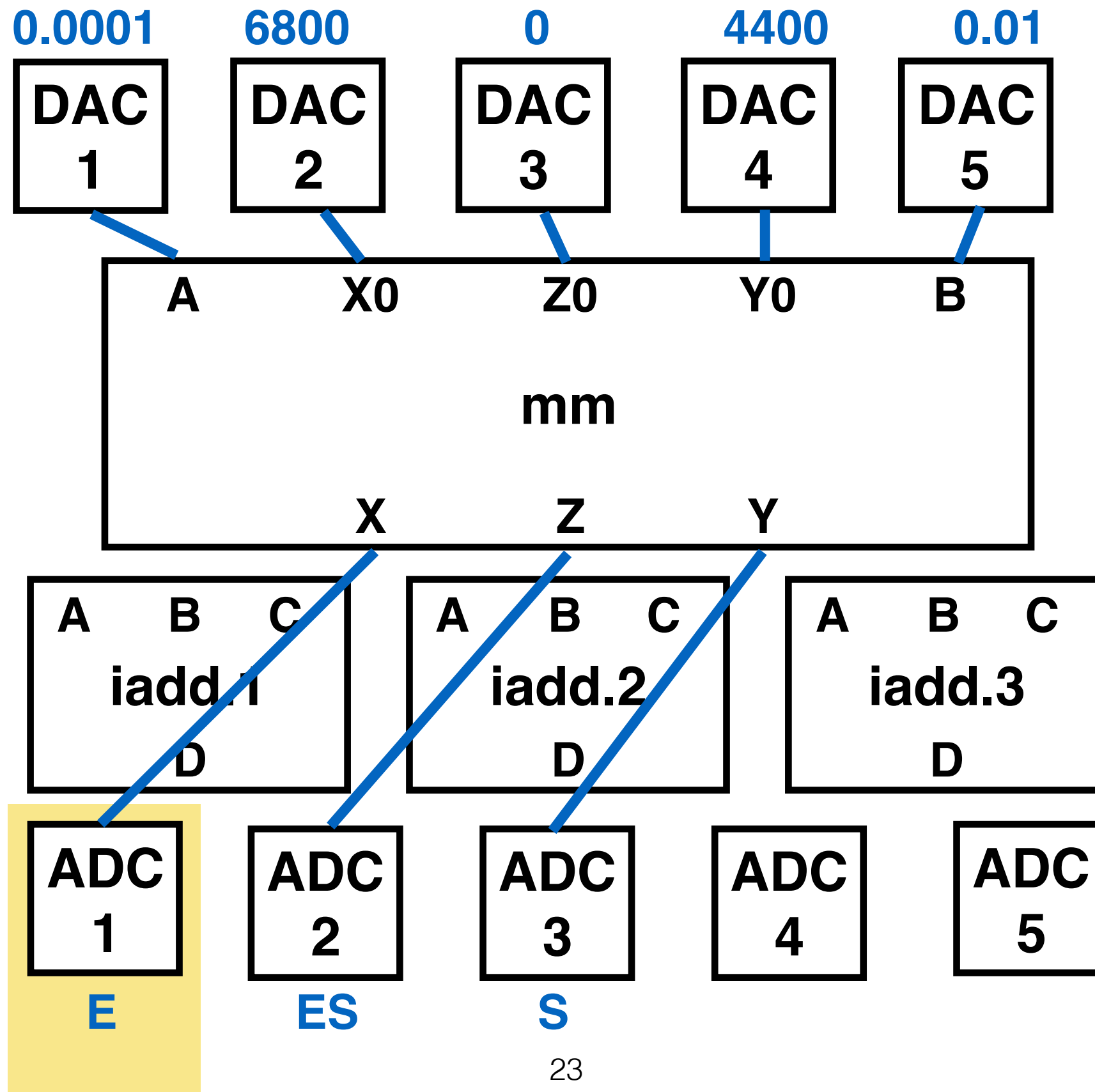
# Configuring the Analog Device



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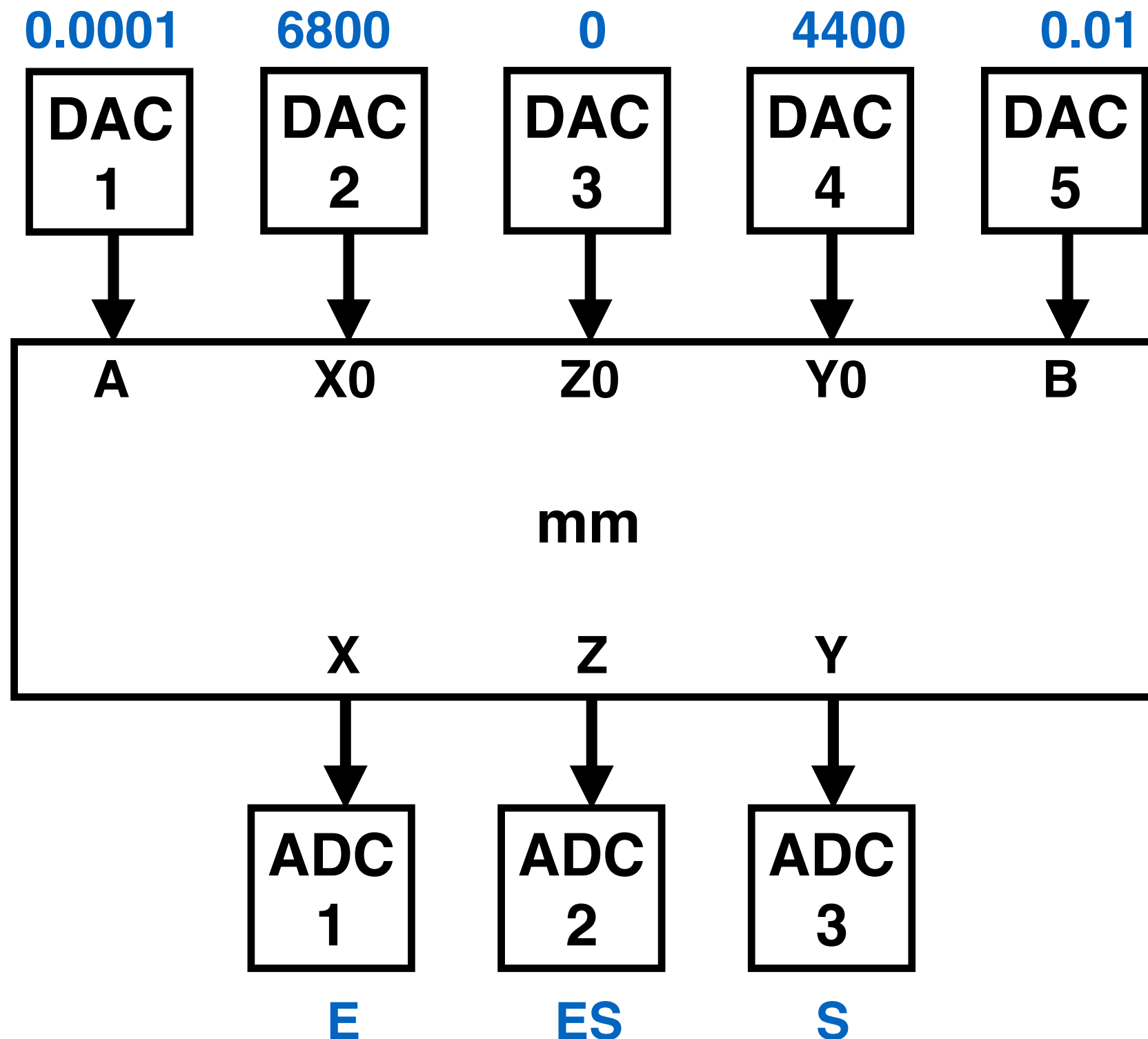


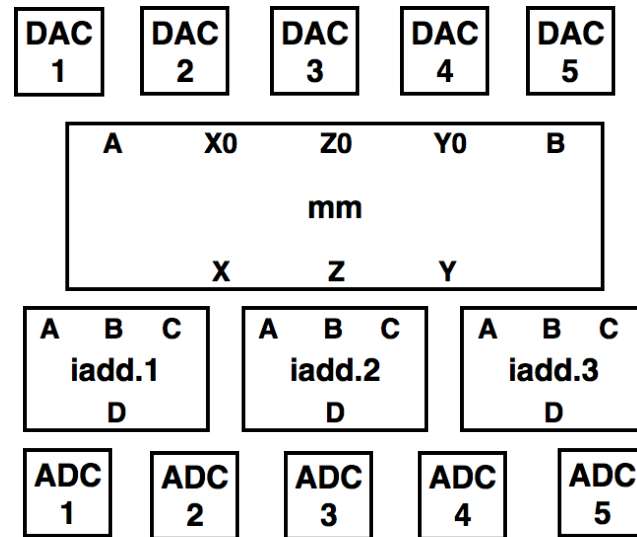
# Configuring the Analog Device





# Analog Device Configuration

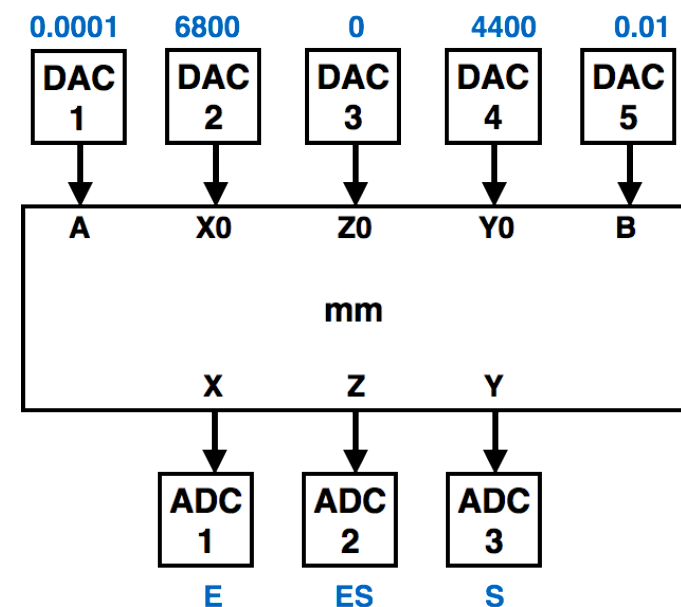




## Analog Device Specification

$$\begin{aligned}
 E &= 6800 - ES \\
 S &= 4400 - ES \\
 \partial ES / \partial t &= 0.0001 \cdot E \cdot \\
 &\quad - 0.01 \cdot ES \\
 ES(0) &= 0
 \end{aligned}$$

Dynamical  
System



Analog Device  
Configuration

[1] Sara Achour, Rahul Sarpeshkar, and Martin C. Rinard. Configuration synthesis for programmable analog devices with Arco. PLDI 2016

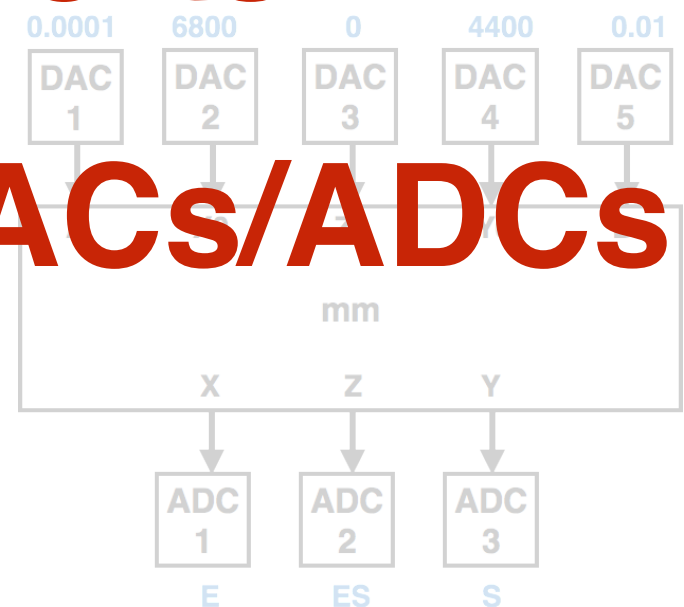
**does not consider**  
**physical limitations of hardware**

**- operating range of ports**

**- sampling rates of DACs/ADCs**

$$\begin{aligned}
 E &= 6800 - ES \\
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 \partial ES / \partial t &= 0.0001 \cdot E \cdot \\
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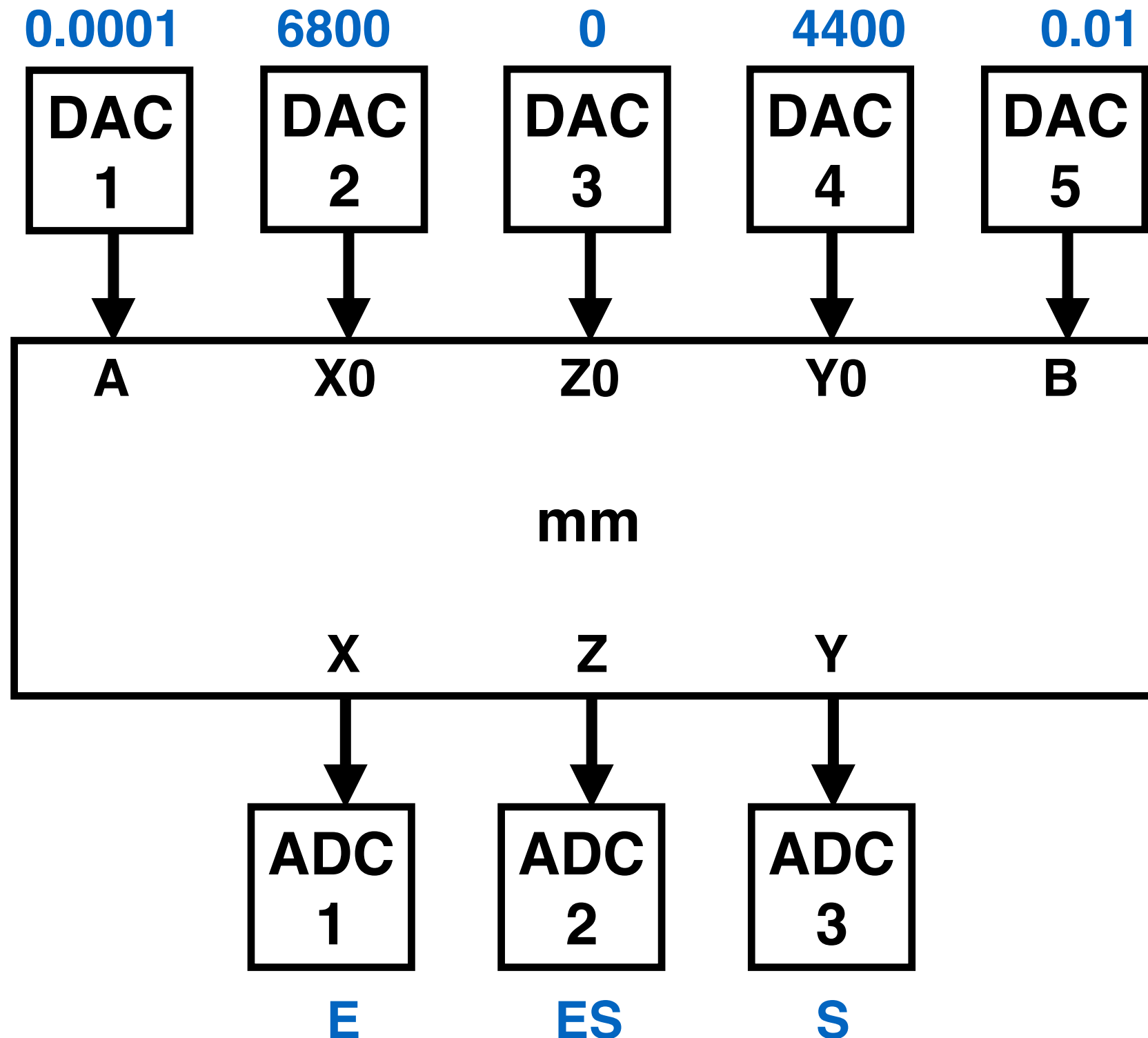
Dynamical  
System



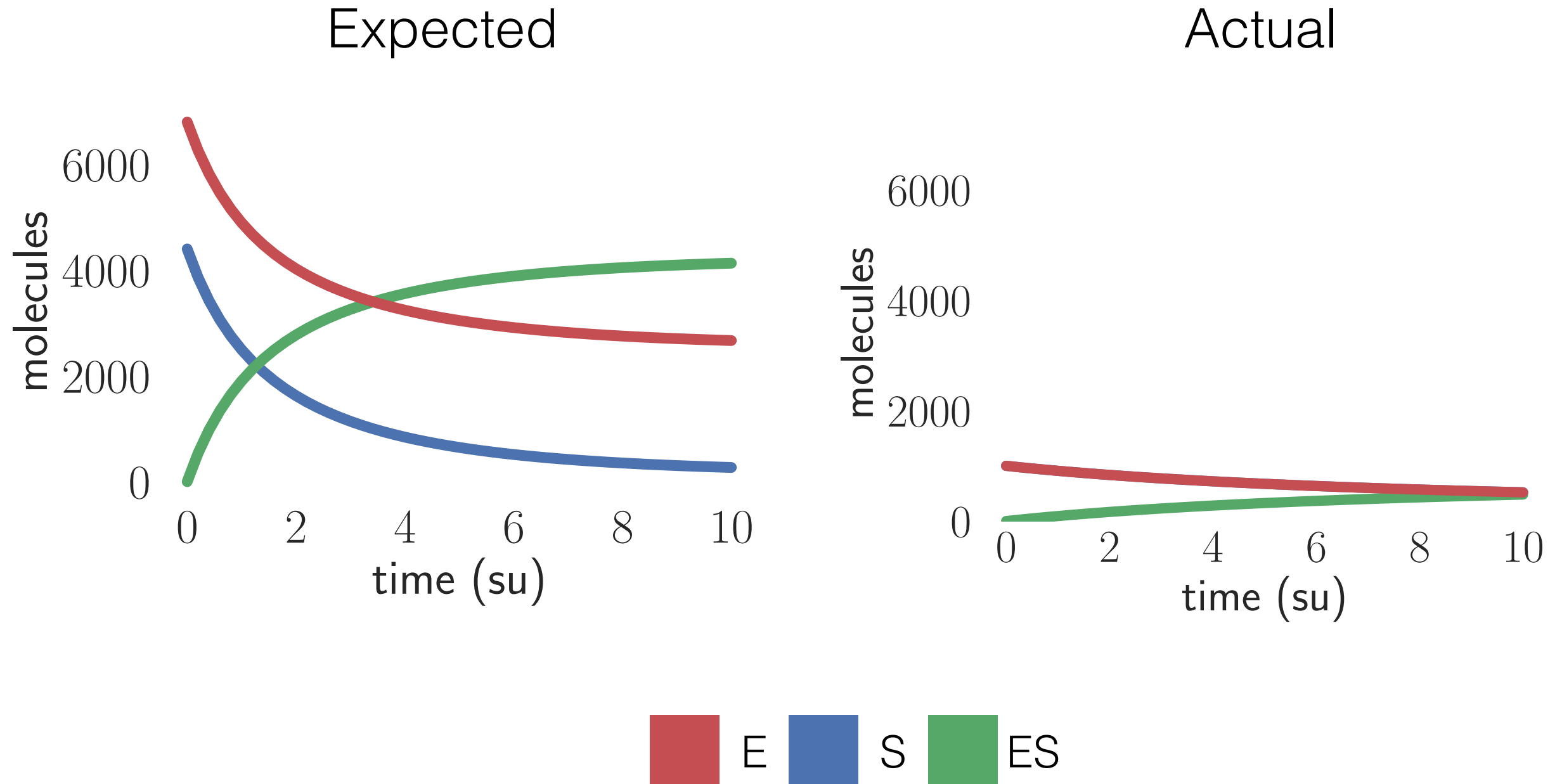
Analog Device  
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# Analog Device Configuration

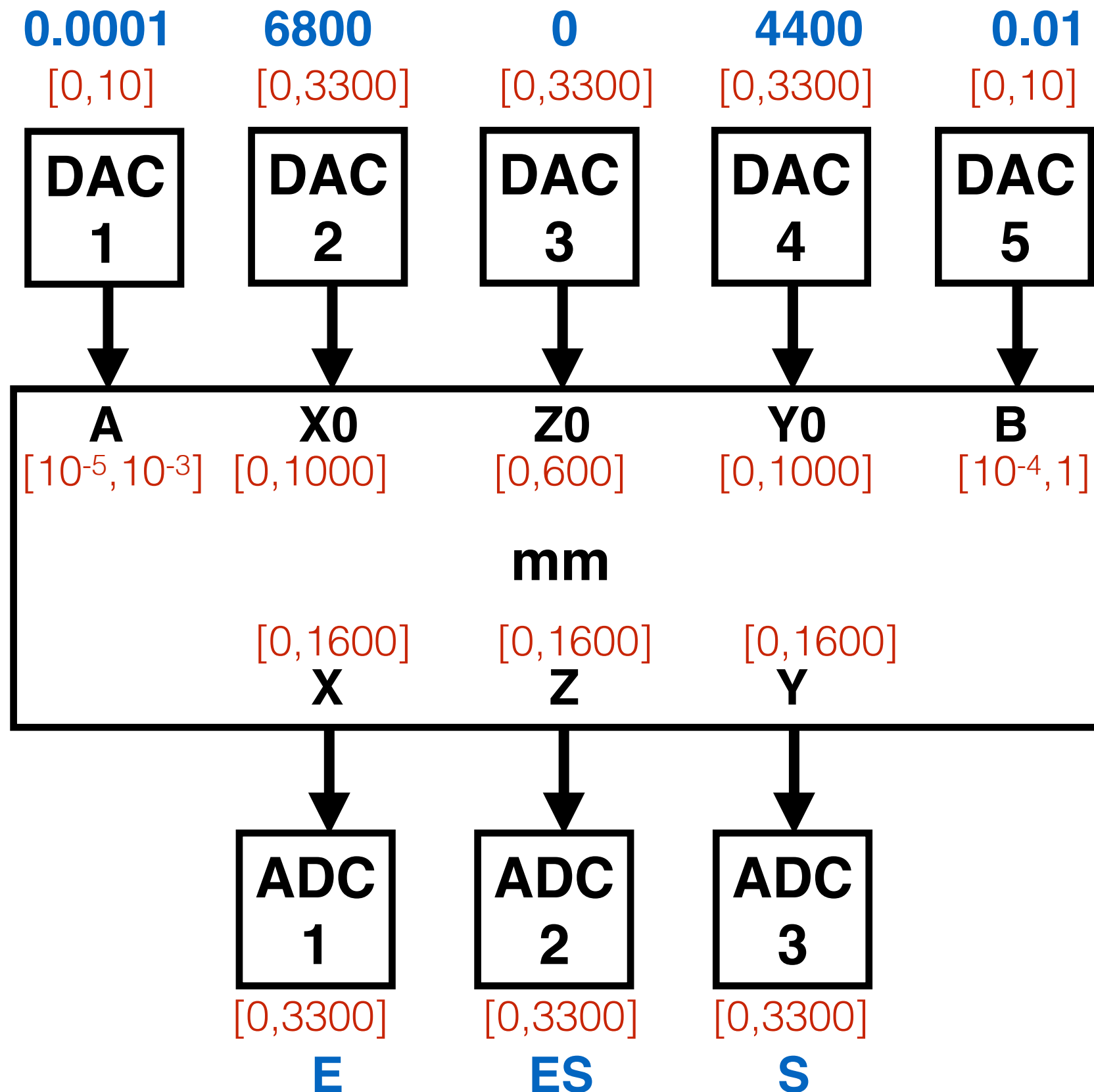


# Execution on Analog Device

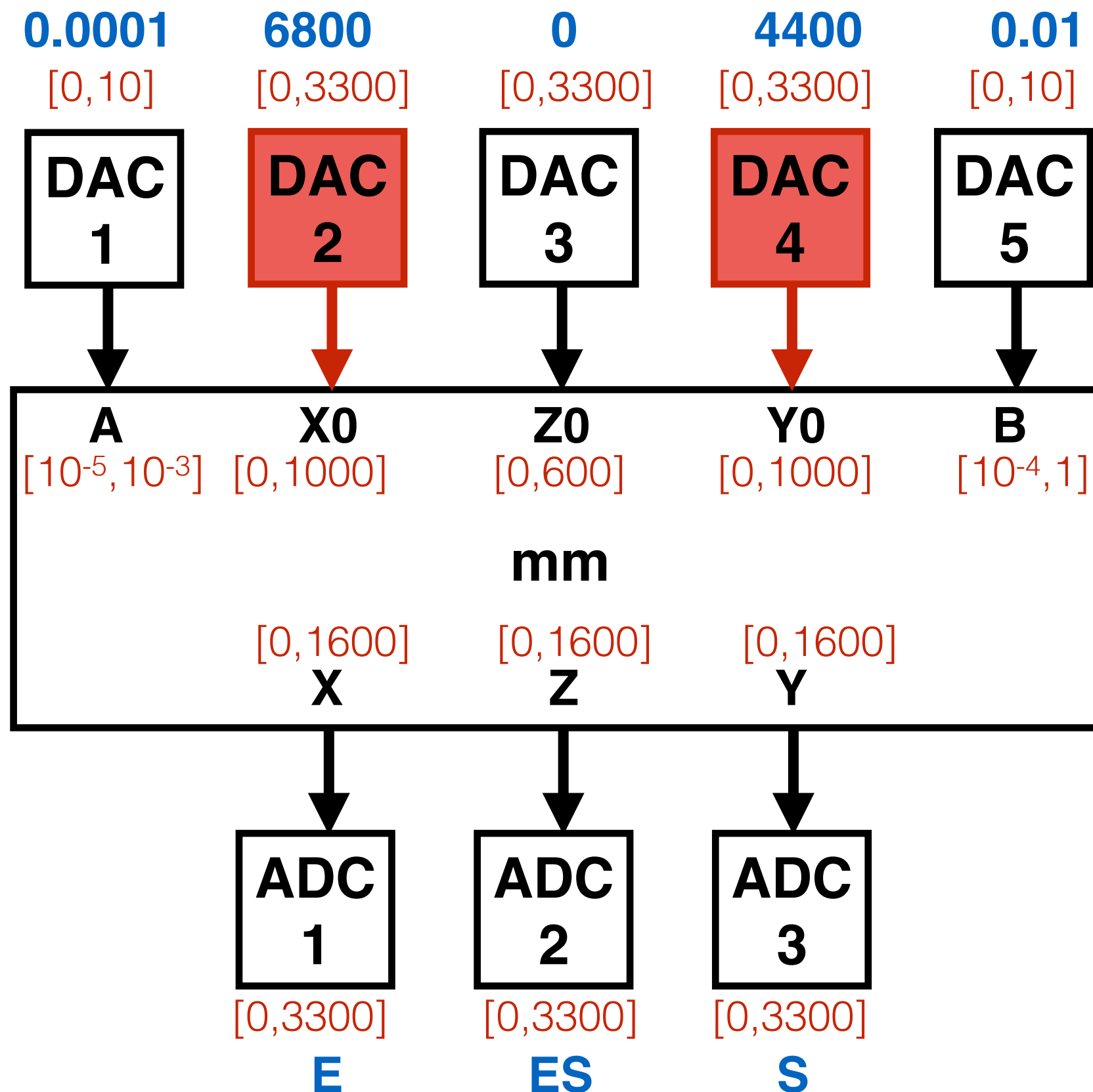




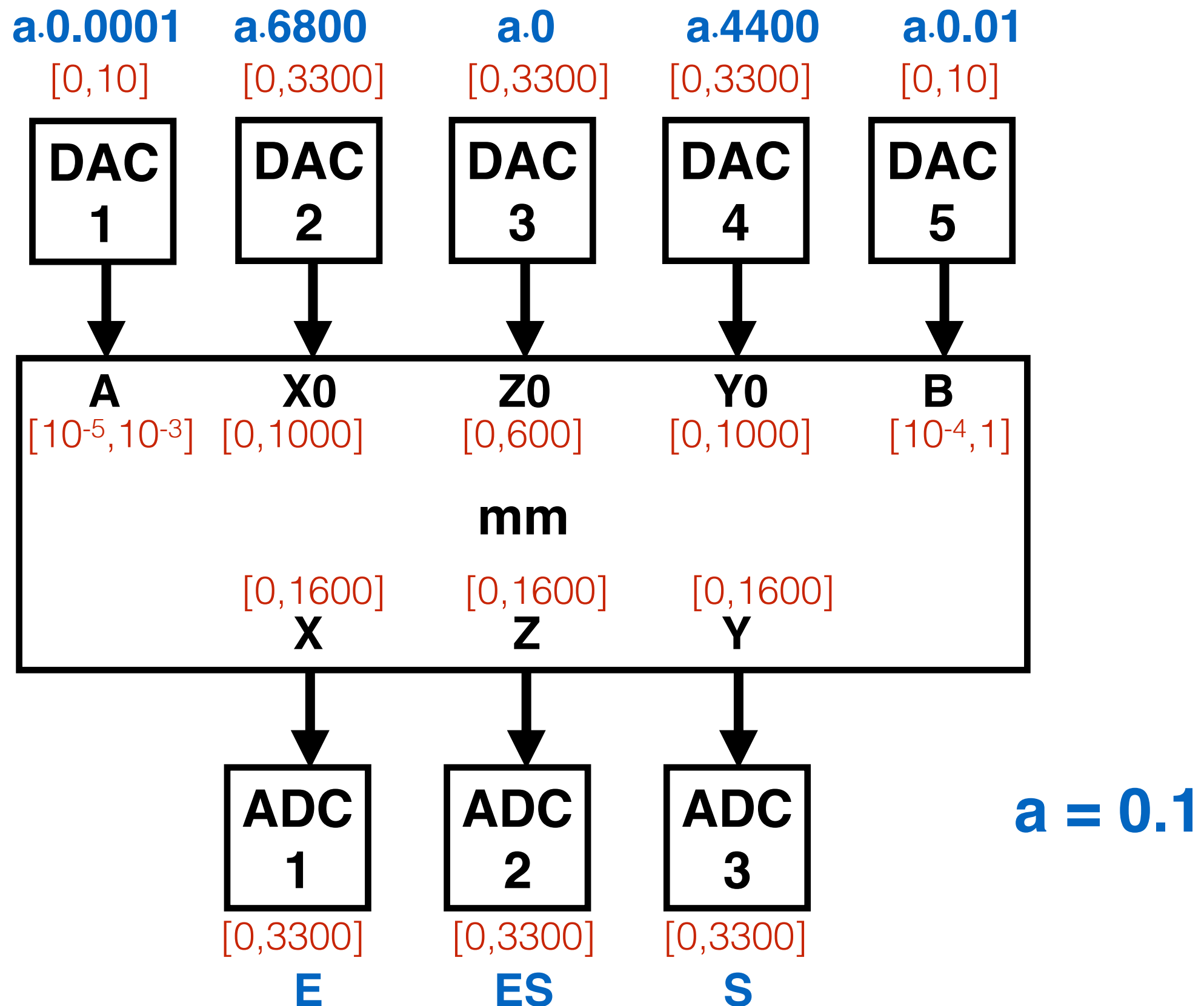
# Analog Device has Physical Limitations



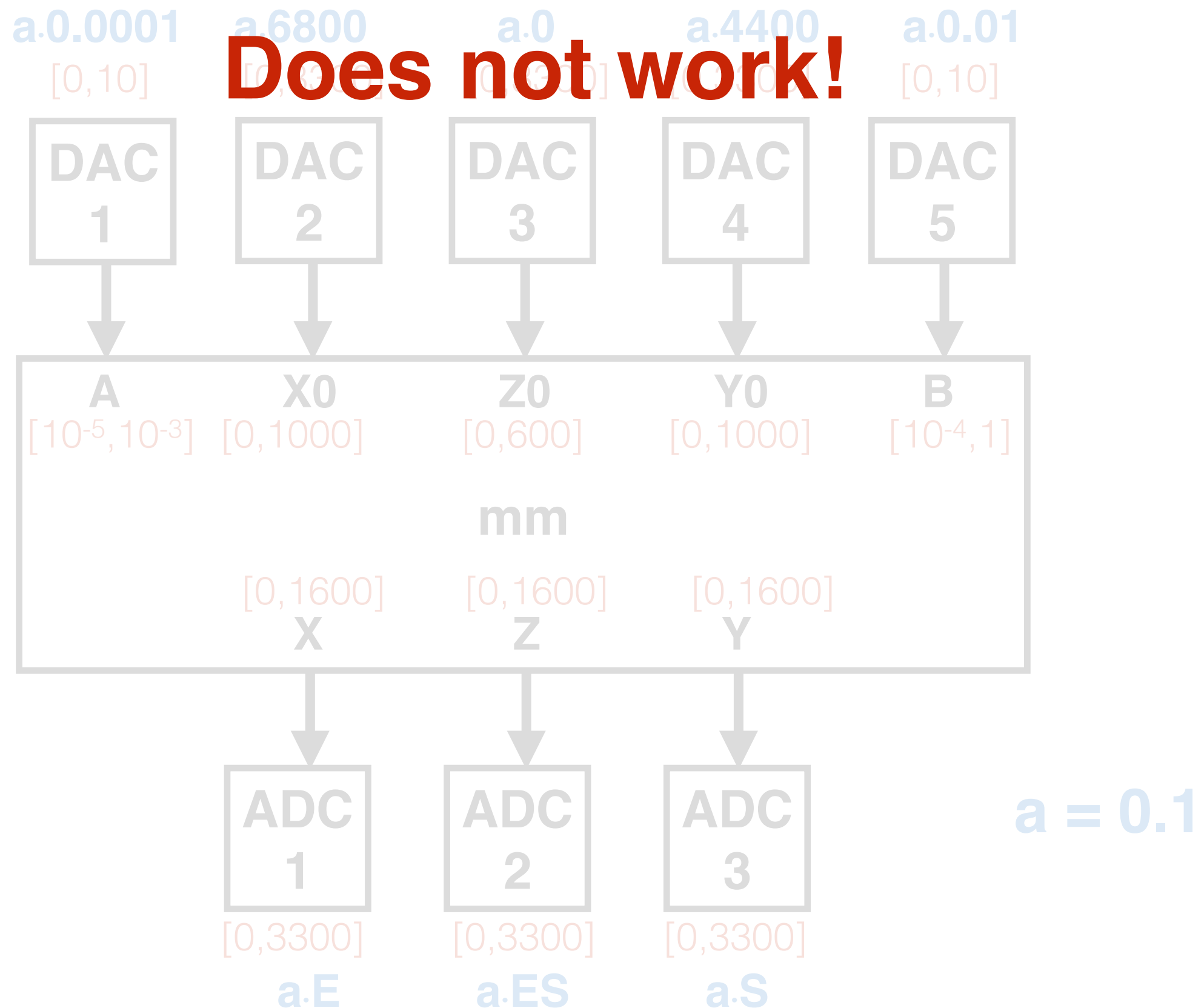
# Analog Device has Physical Limitations



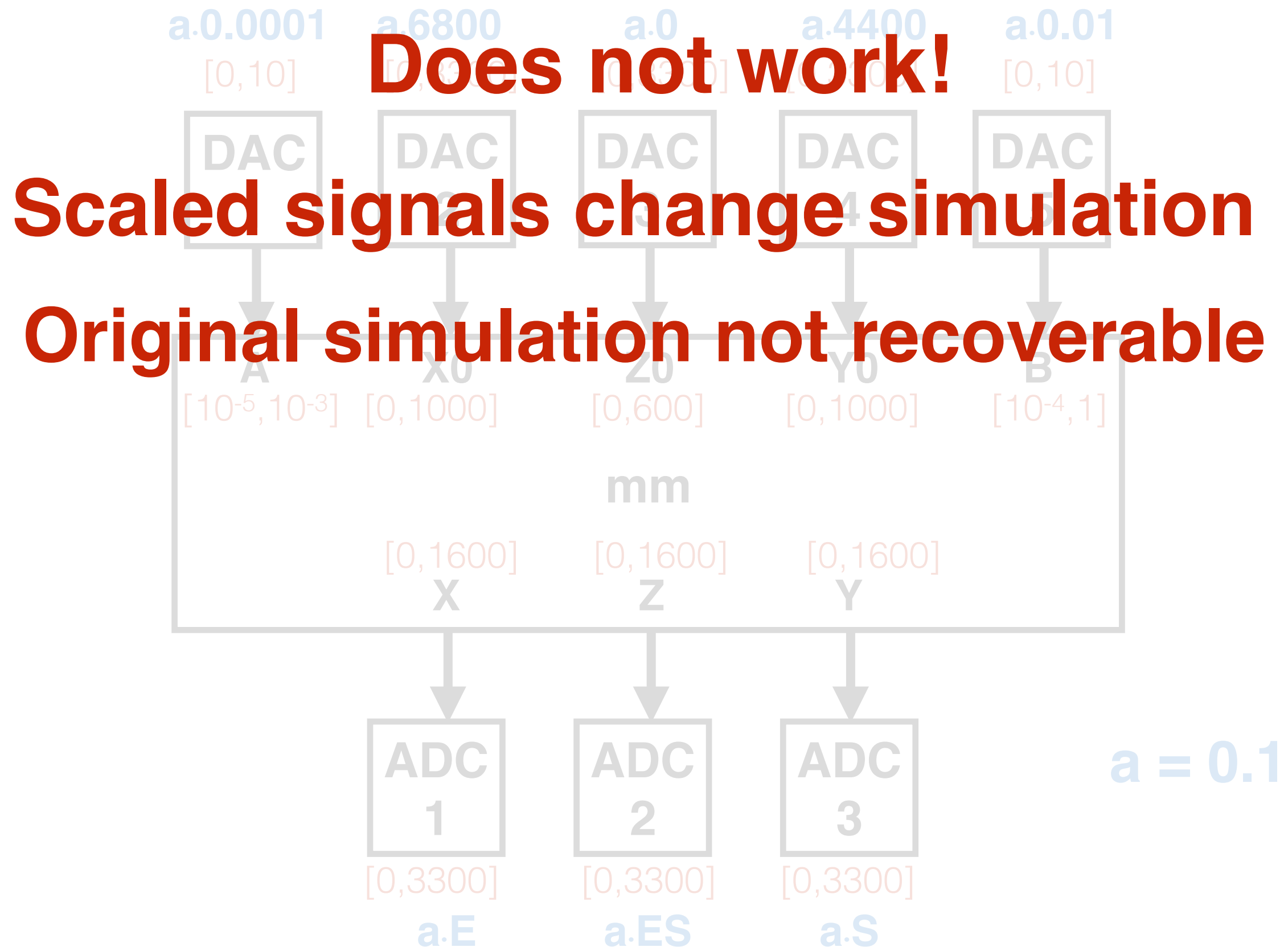
# Uniformly scaling the configuration



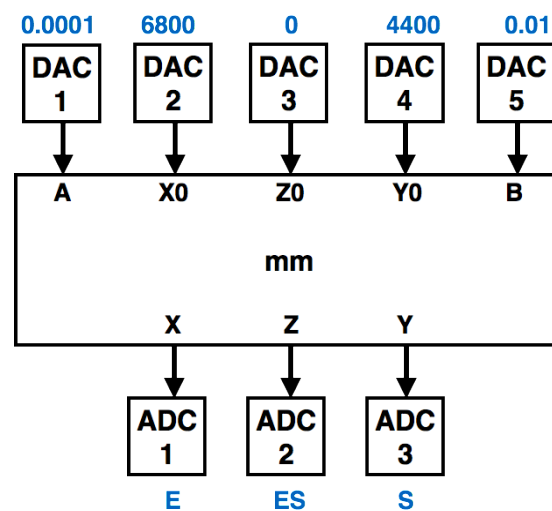
# Uniformly scaling the configuration



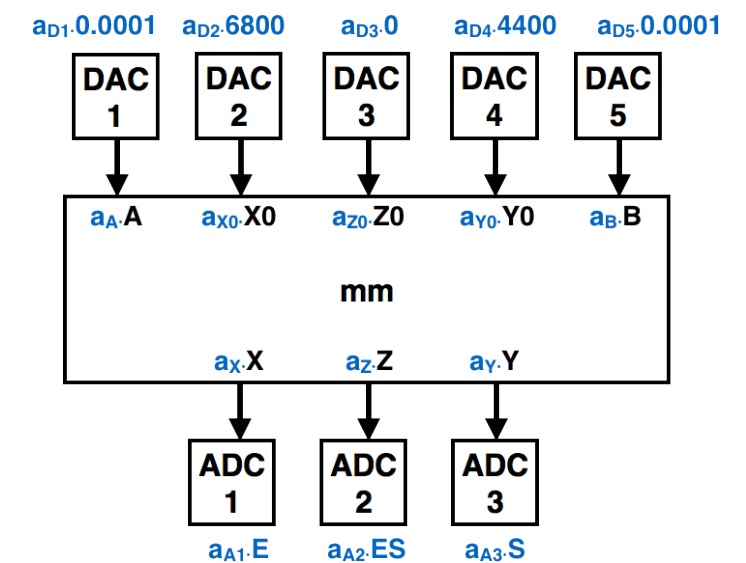
# Uniformly scaling the configuration



## Analog Device Configuration



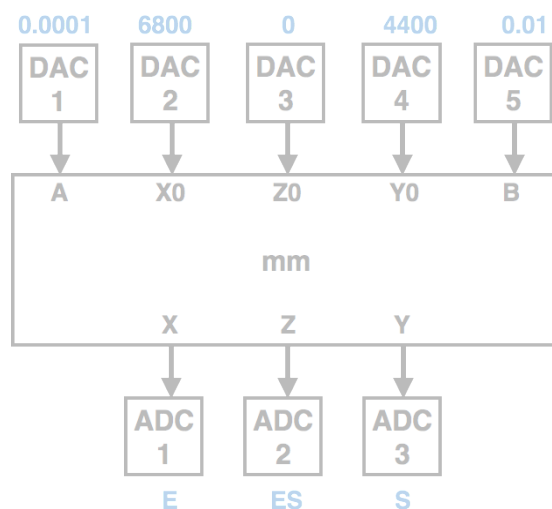
## Scaled Analog Device Configuration



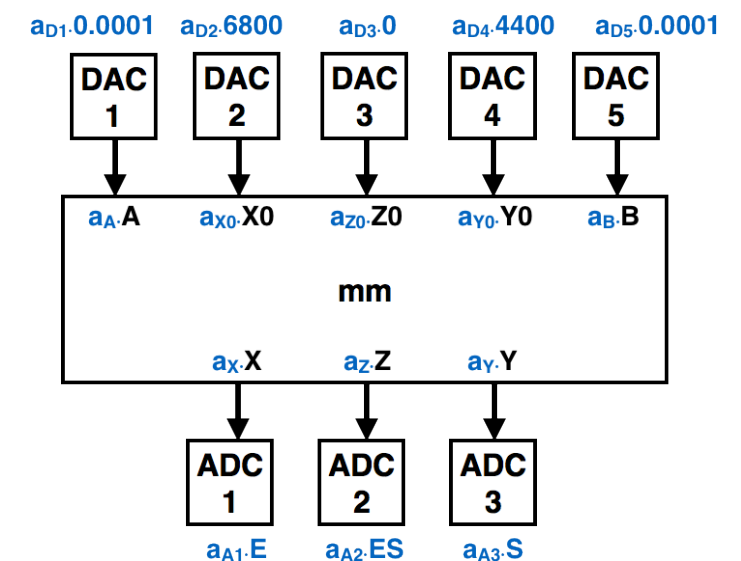
physically realizable: signals are within port operating ranges

recoverable: recover original simulation at digital interface

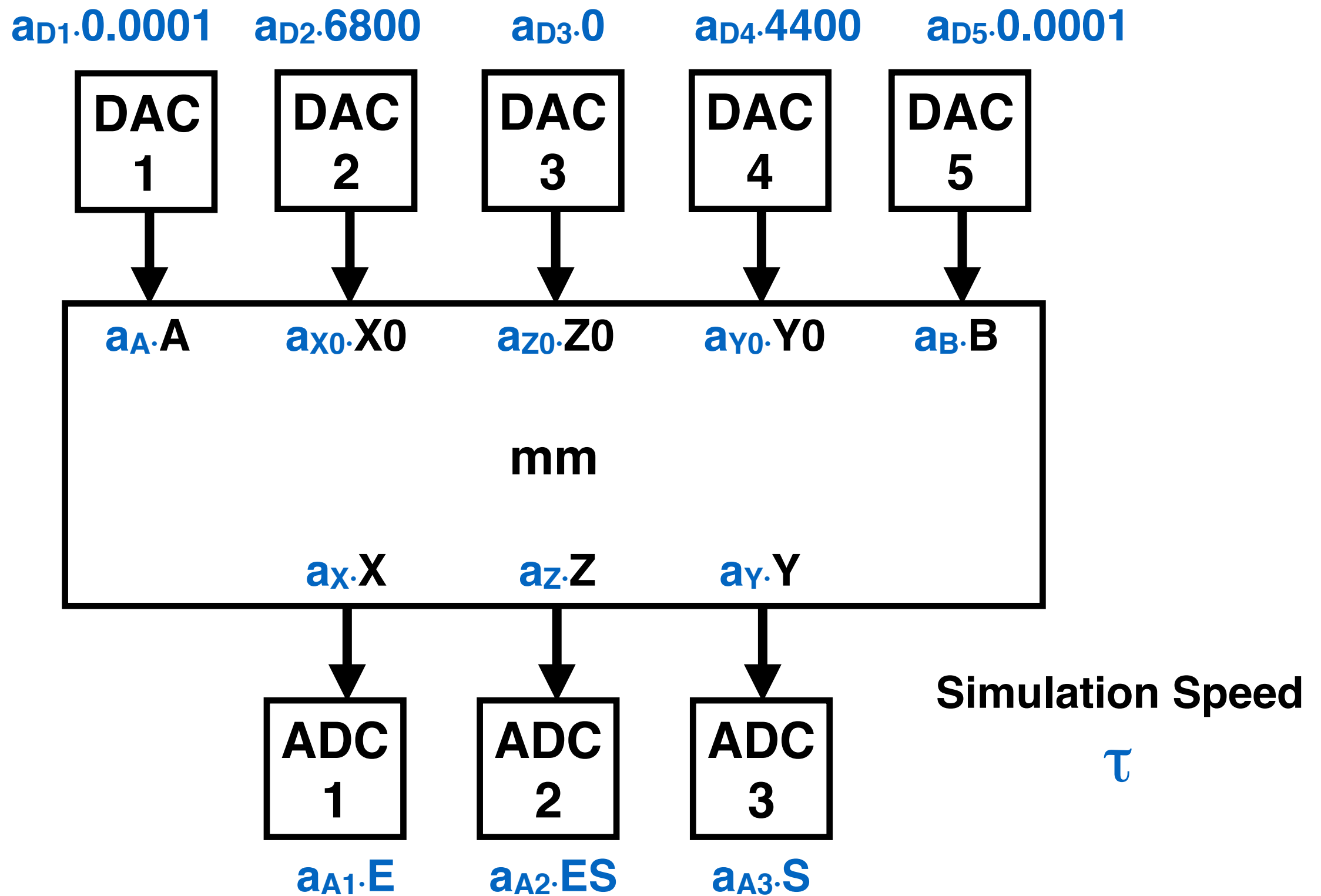
Analog Device Configuration



**Scaled** Analog Device Configuration

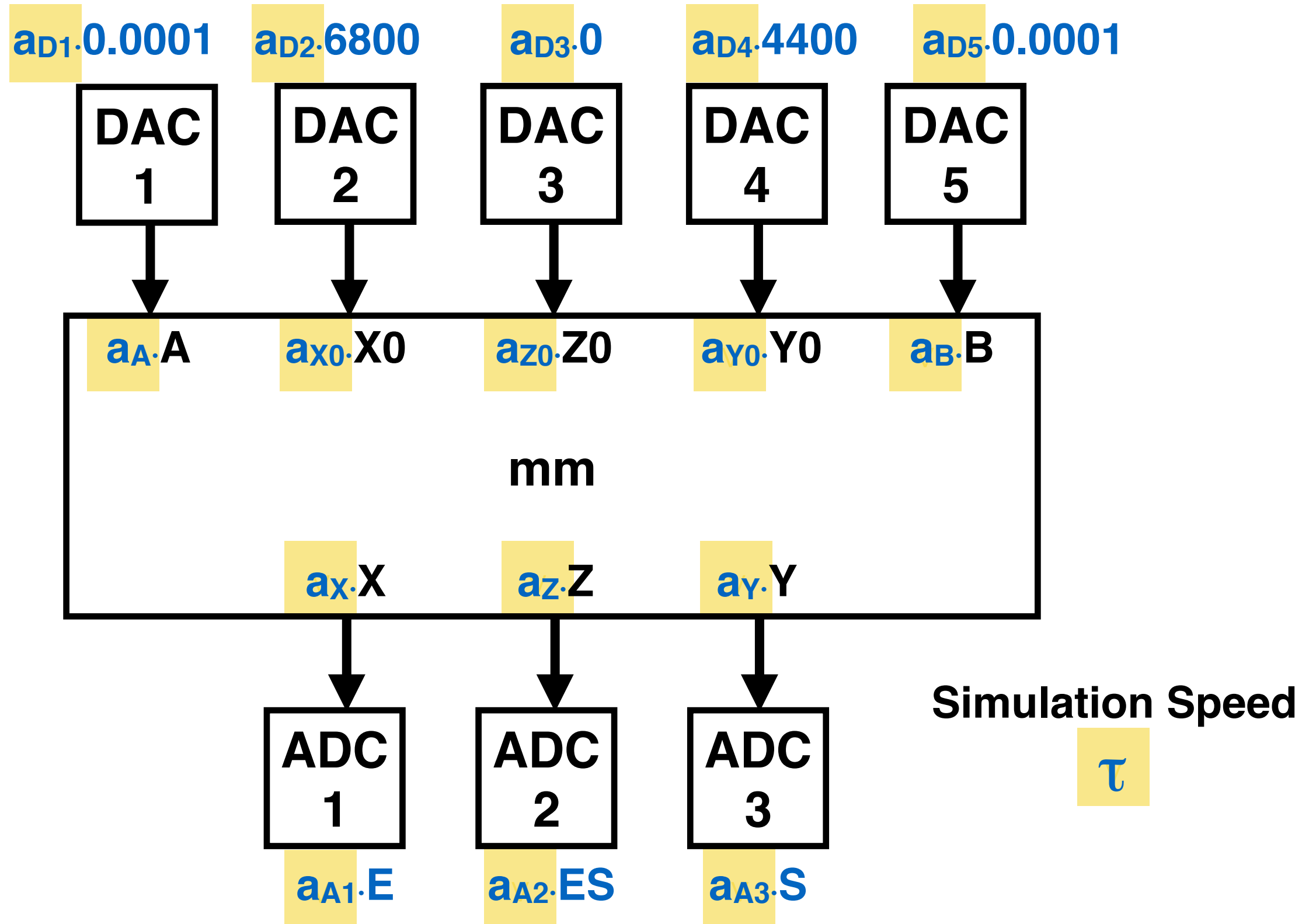


# Scaled Configuration

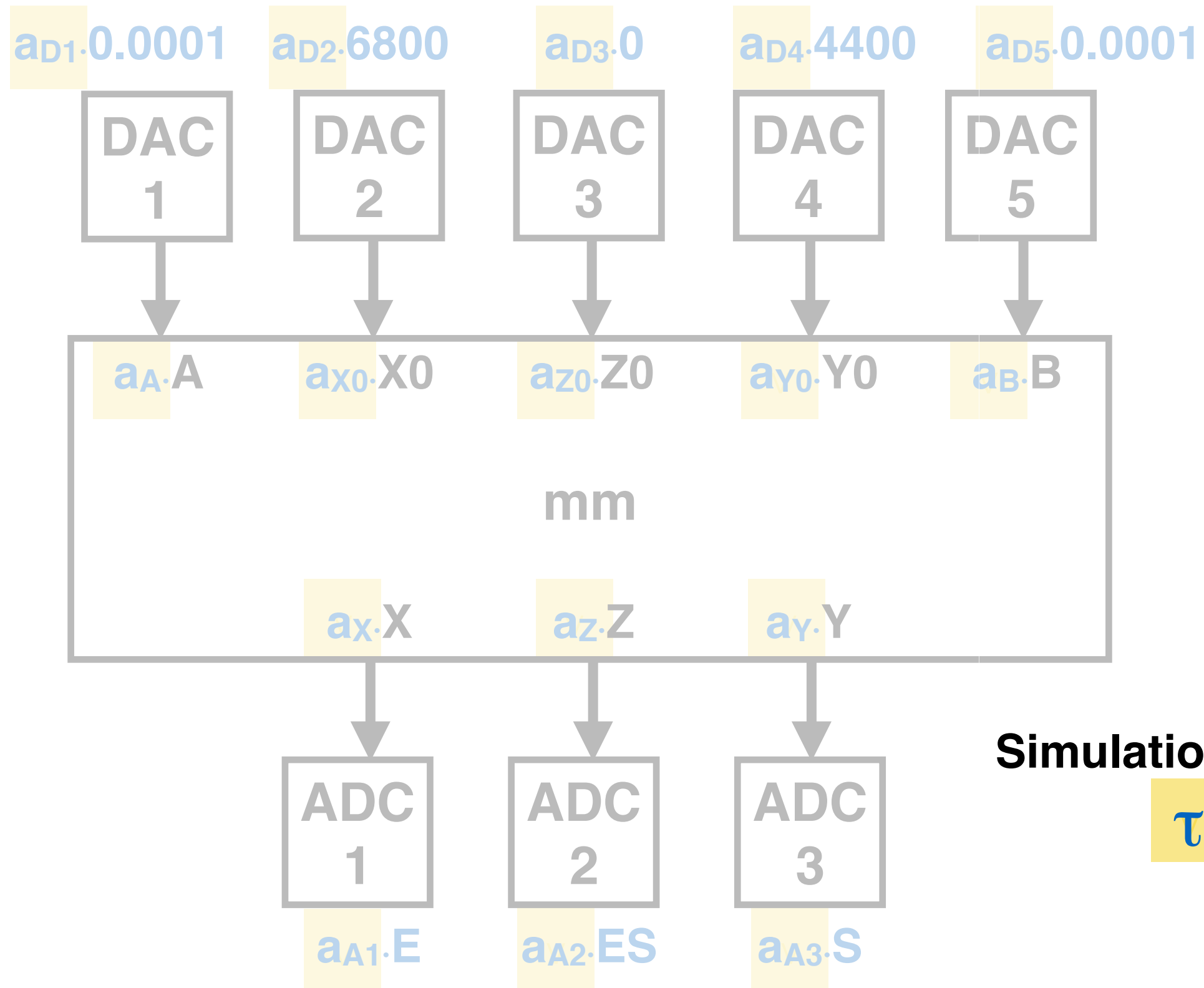




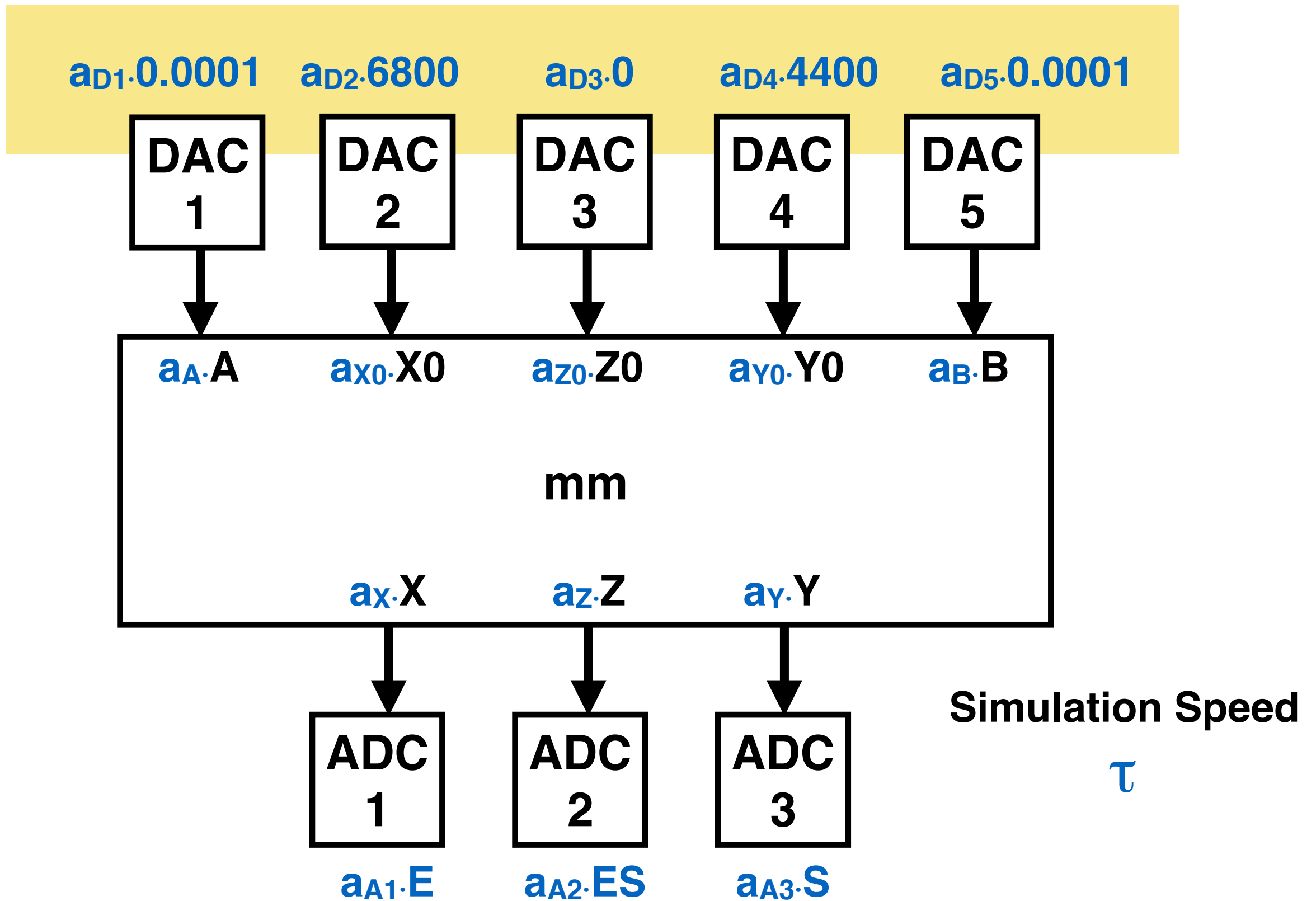
# Scaled Configuration



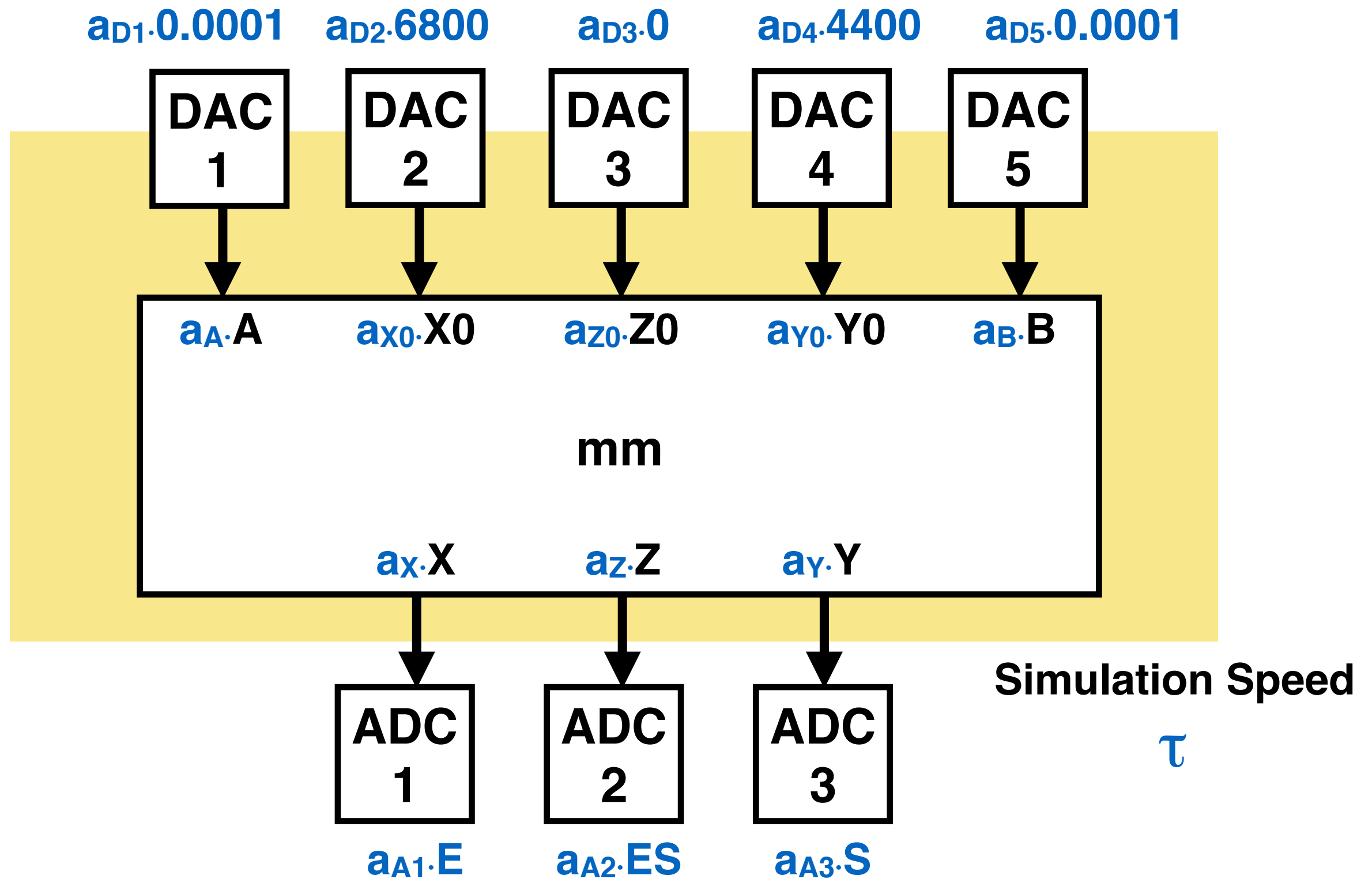
# Time Scaling Factor, $\tau$



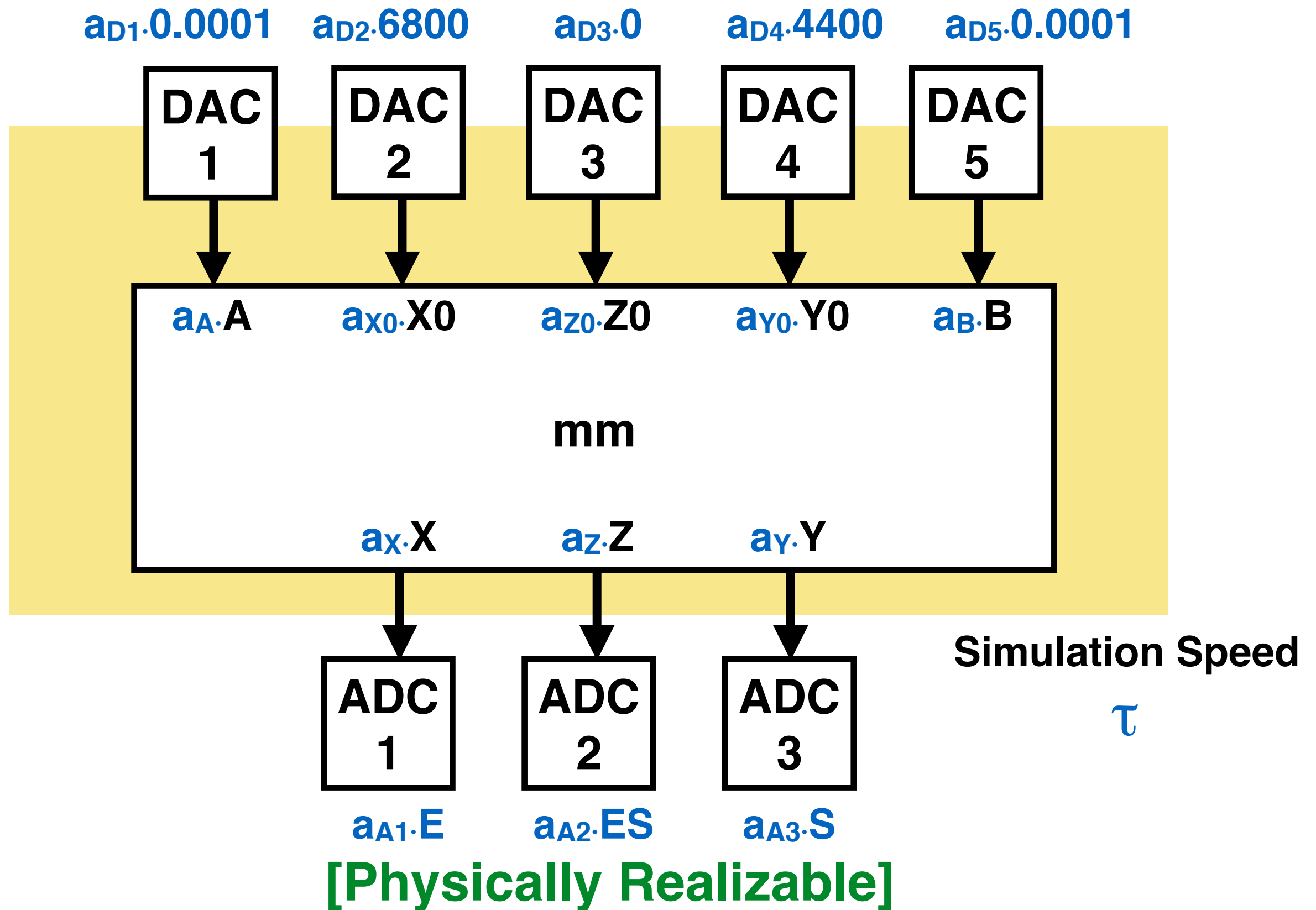
Multiply each DAC value by its scaling factor

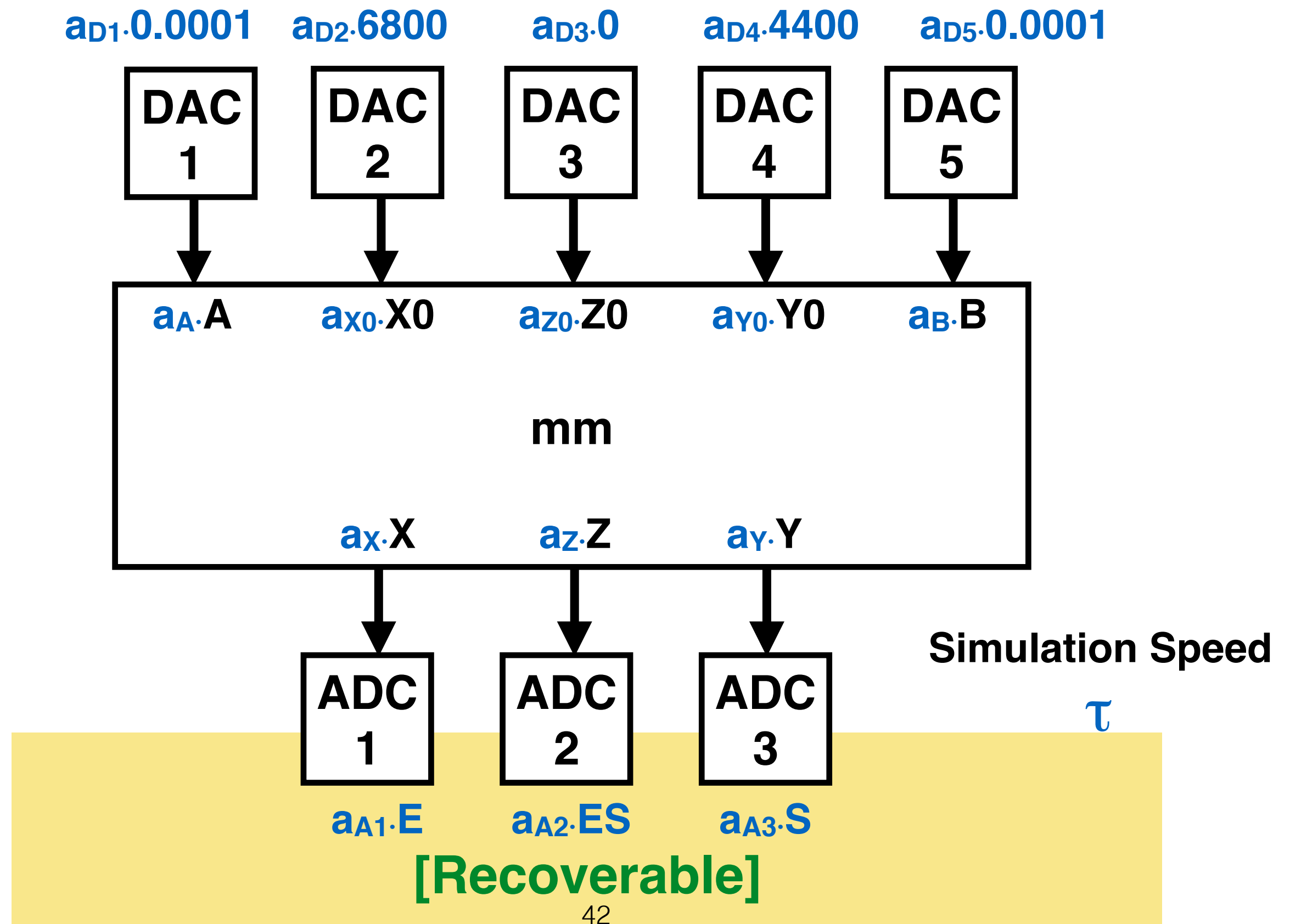


The scaled signals propagate through the analog device

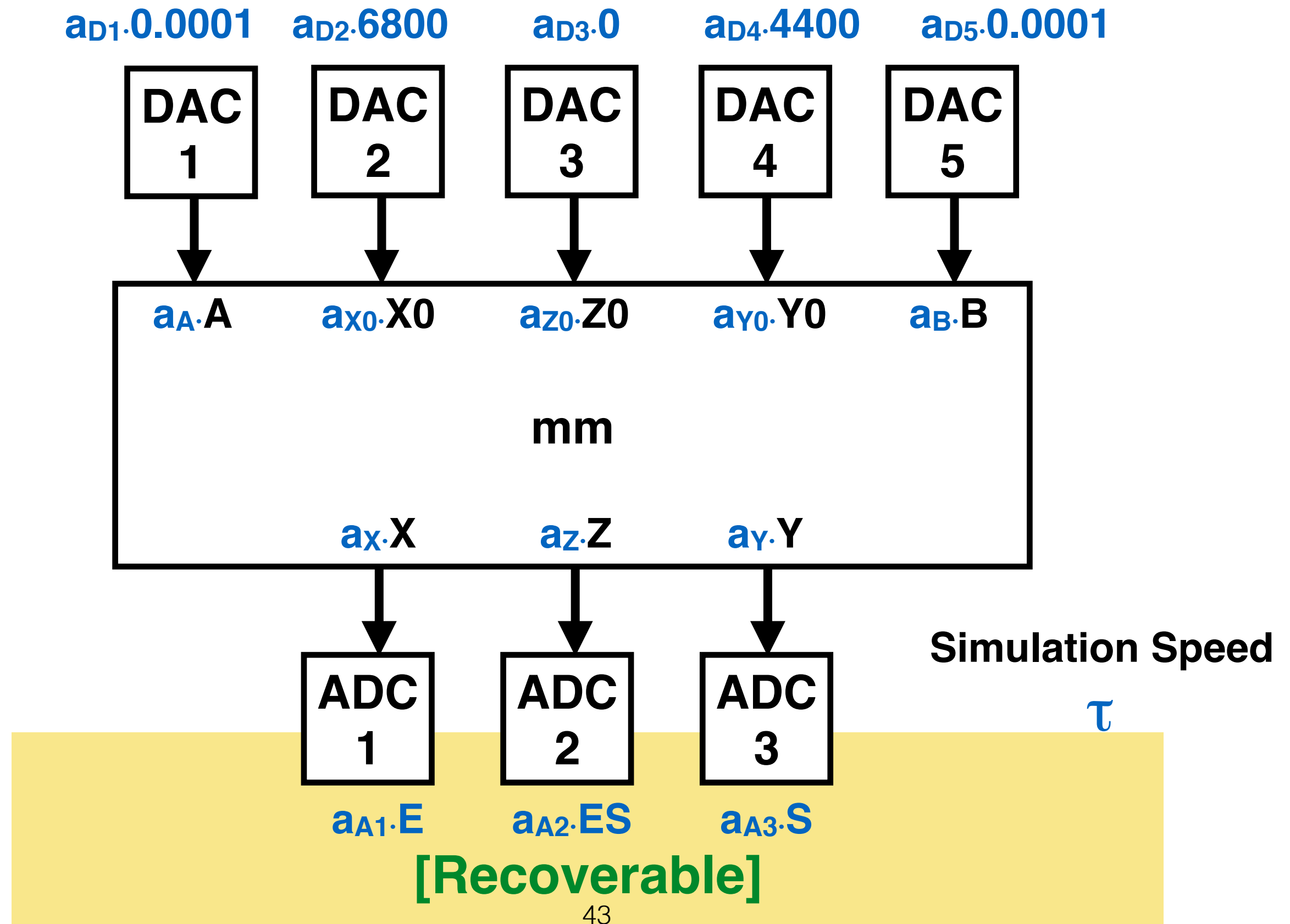


All of the internal signals fall in the port operating ranges

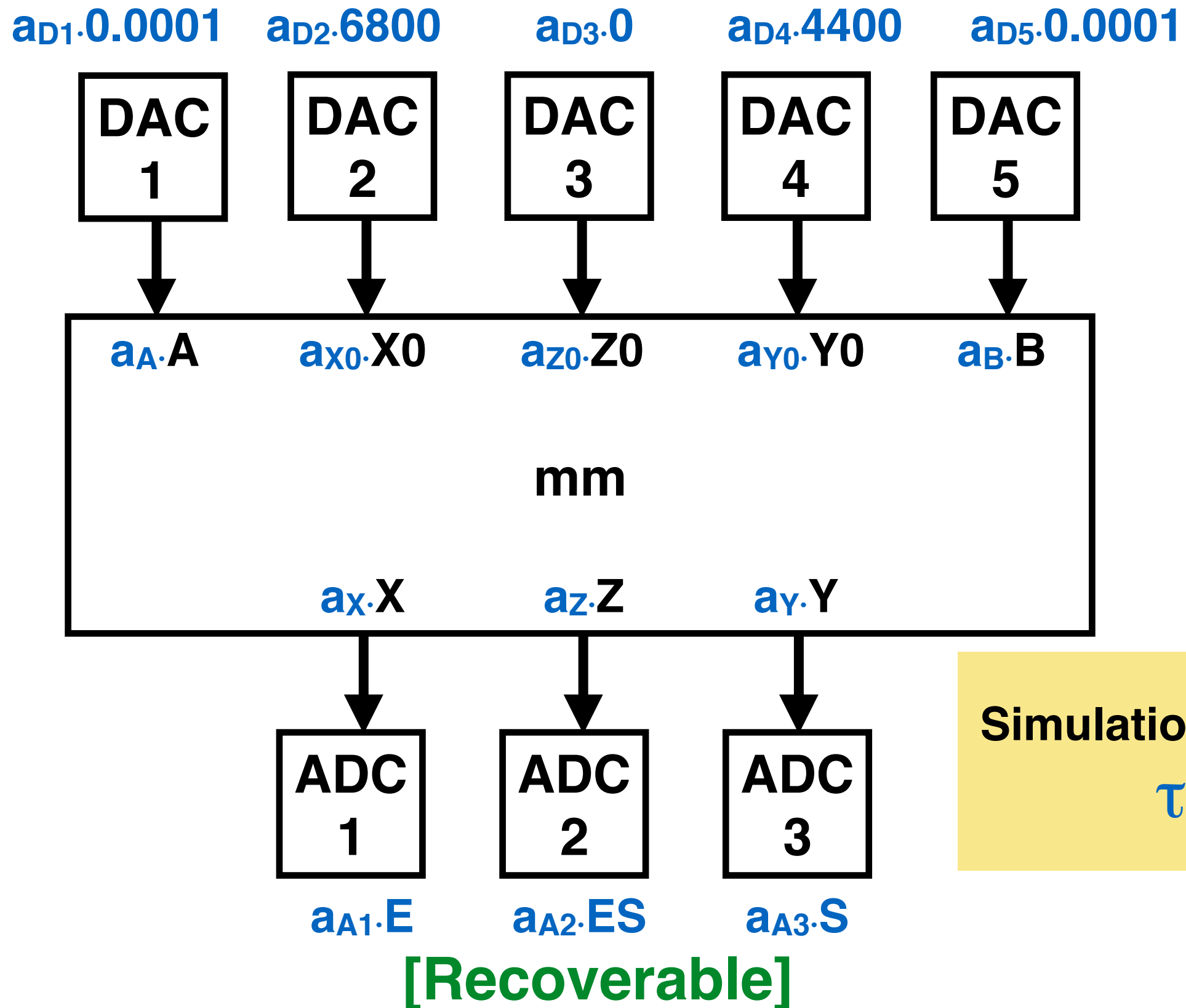




Divide ADC samples by scaling factors



Multiply hardware time by  $\tau$





# Jaunt

- **Objective**: Find numerical assignments for scaling factors that produces fastest simulation
- Scaling Factors:  $\mathbf{T, a_{D1}, \dots, a_{D5}, a_{A1}, \dots, a_{A3}, a_A, \dots, a_z}$
- Values: real numbers  $> 0$

# Jaunt

- **Objective**: Find numerical assignments for scaling factors that produces fastest simulation
  - Scaling Factors:  $\mathbf{T}, \mathbf{a}_{D1}, \dots, \mathbf{a}_{D5}, \mathbf{a}_{A1}, \dots, \mathbf{a}_{A3}, \mathbf{a}_A, \dots, \mathbf{a}_Z$
  - Values: real numbers  $> 0$
- **Geometric Program**: Convex optimization problem
  - Device Configuration  $\rightarrow$  Geometric Program

# Geometric Program Generation

**maximize  $\tau$**

**subject to:**

Operating Range Constraints

Sampling Constraints

Connection Constraints

Factor Constraints

} **physically realizable**

} **recoverable**

# Geometric Program Generation

**maximize  $\tau$**

**subject to:**

**Operating Range Constraints**

Sampling Constraints

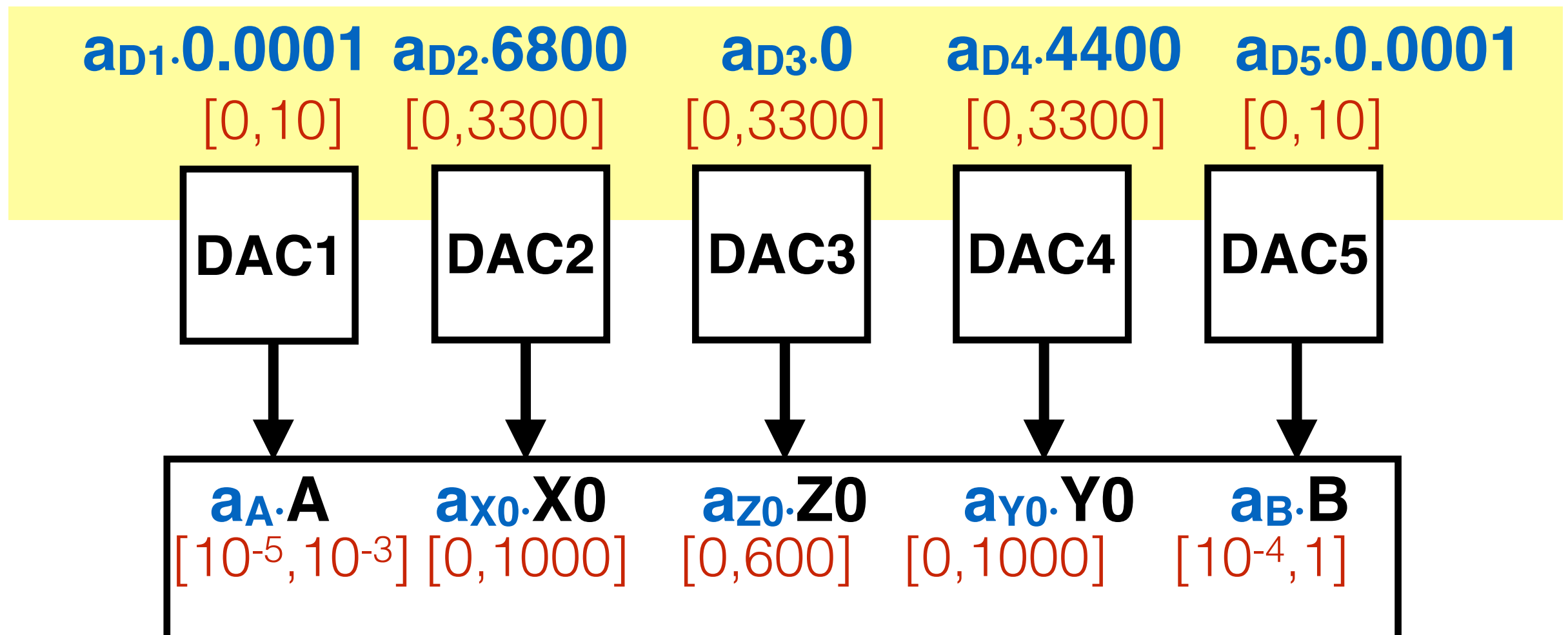
Connection Constraints

Factor Constraints

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# Operating Range Constraints



$$a_{D1} \cdot 0.0001 \in [0, 10]$$

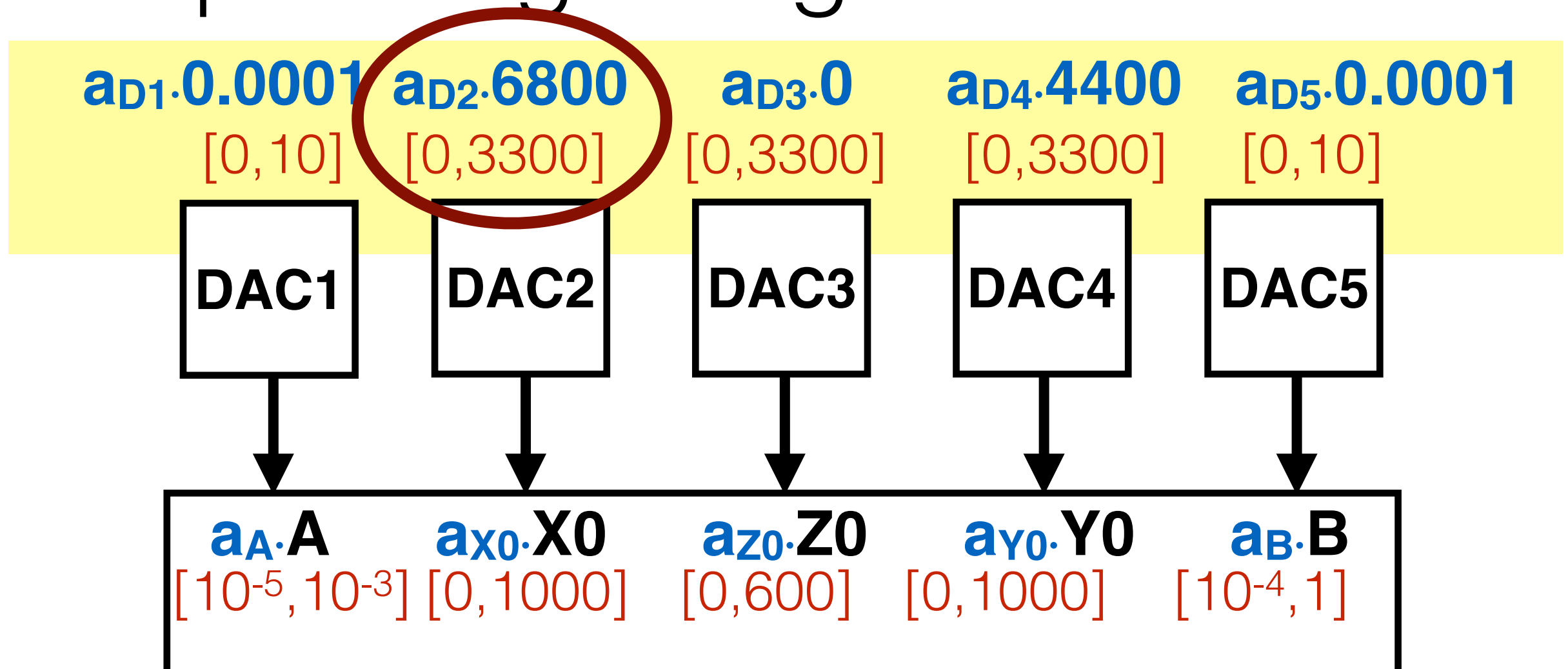
$$a_{D4} \cdot 4400 \in [0, 3300]$$

$$a_{D2} \cdot 6800 \in [0, 3300]$$

$$a_{D5} \cdot 0.0001 \in [0, 10]$$

$$a_{D3} \cdot 0 \in [0, 3300]$$

# Operating Range Constraints



$$a_{D1} \cdot 0.00001 \in [0, 10]$$

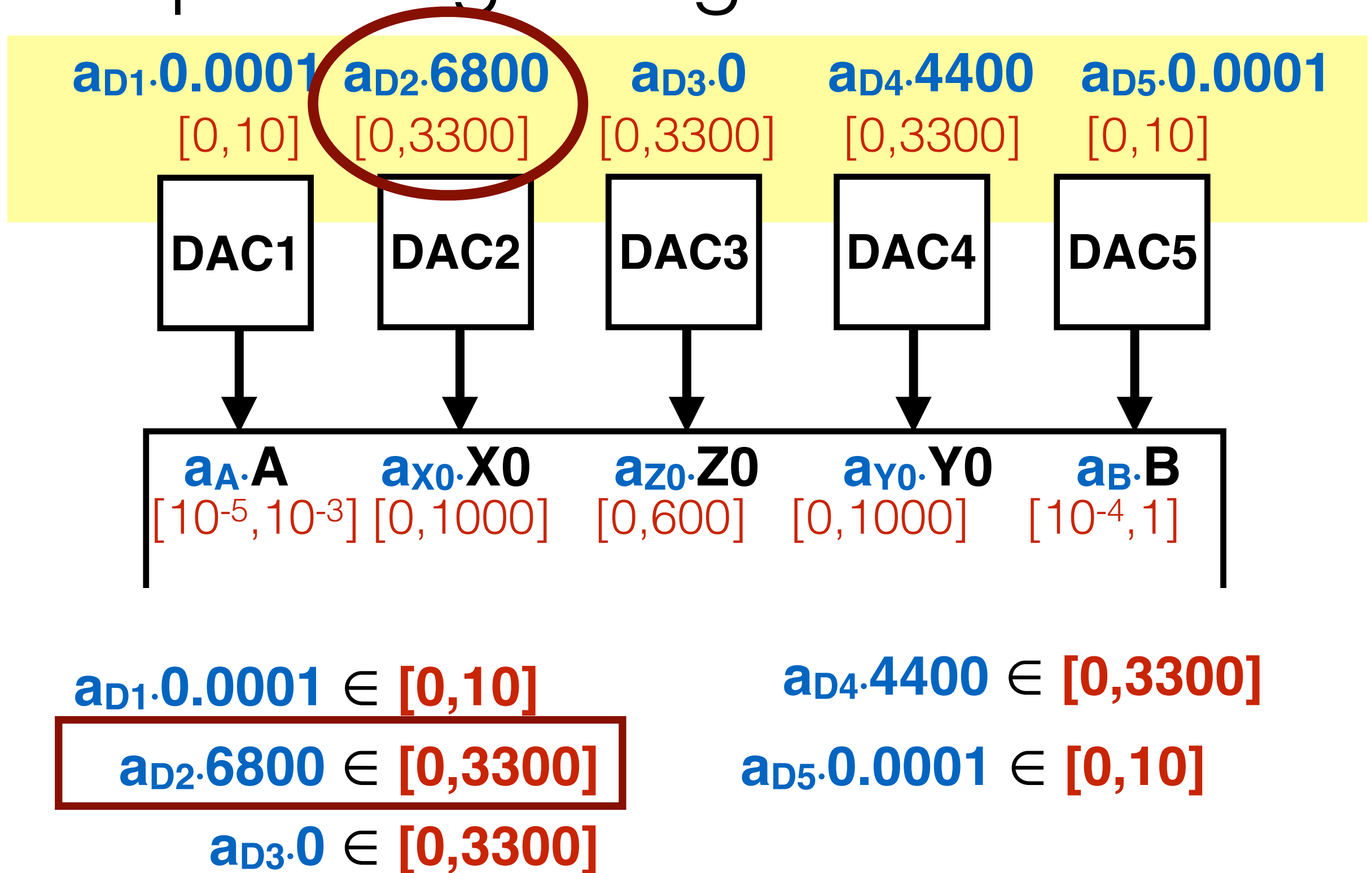
$$a_{D4} \cdot 4400 \in [0, 3300]$$

$$a_{D2} \cdot 6800 \in [0, 3300]$$

$$a_{D5} \cdot 0.00001 \in [0, 10]$$

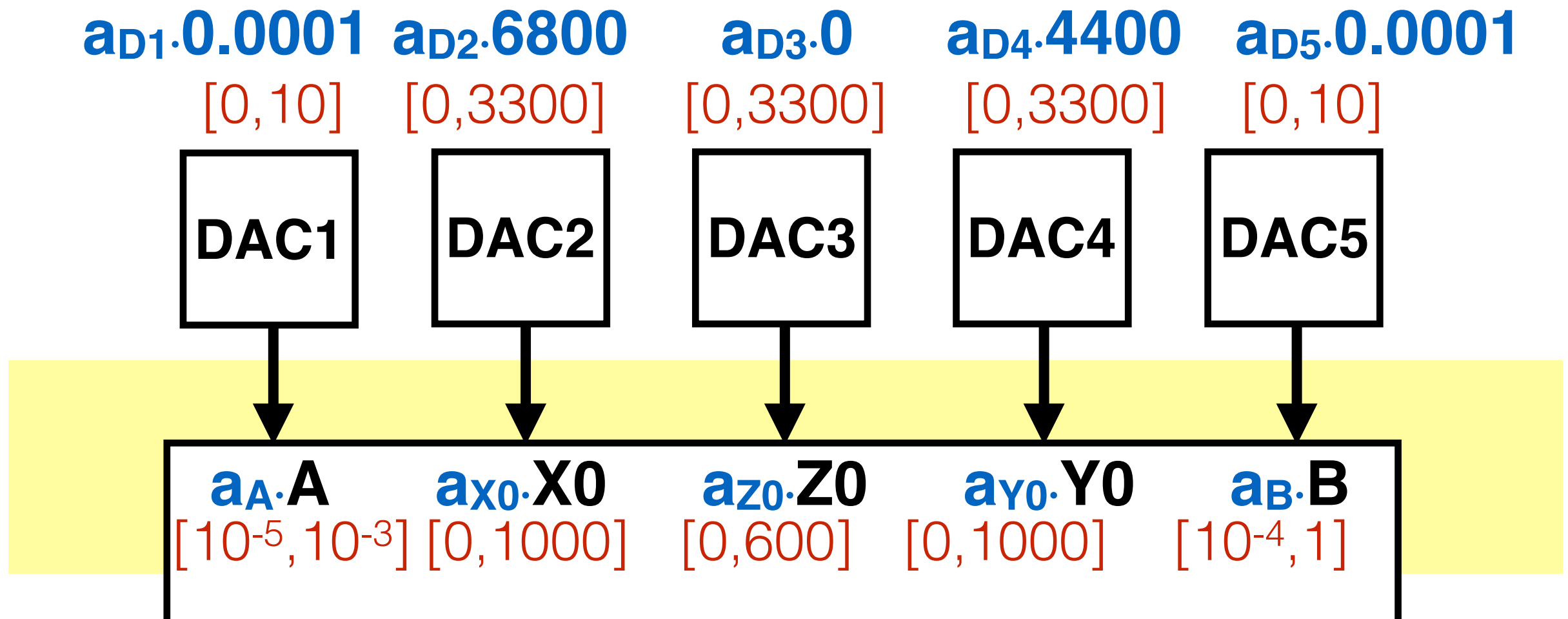
$$a_{D3} \cdot 0 \in [0, 3300]$$

# Operating Range Constraints





# Operating Range Constraints



$$a_A \cdot 0.0001 \in [10^{-5}, 10^{-3}]$$

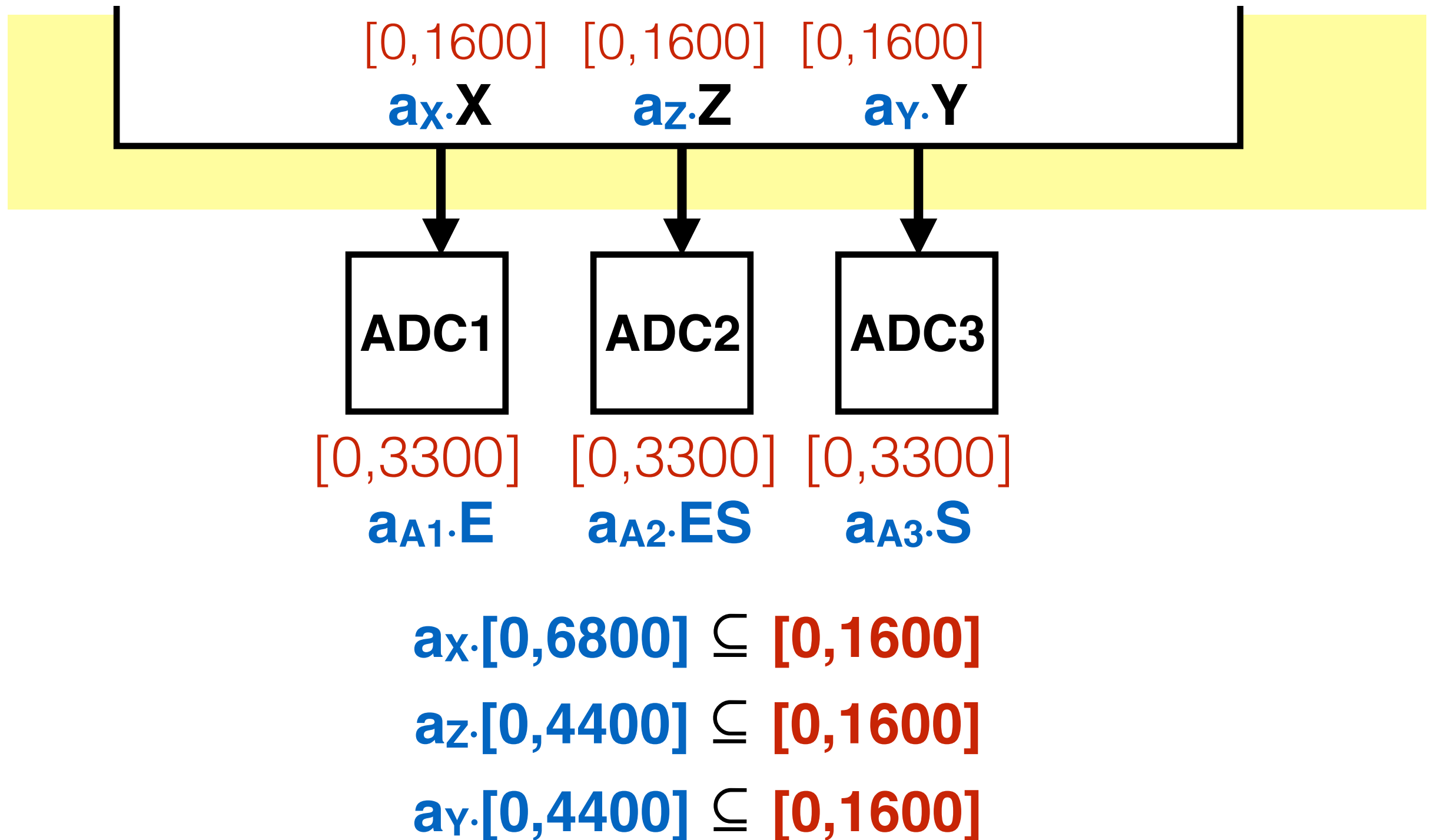
$$a_{Y0} \cdot 4400 \in [0, 1000]$$

$$a_{X0} \cdot 6800 \in [0, 1000]$$

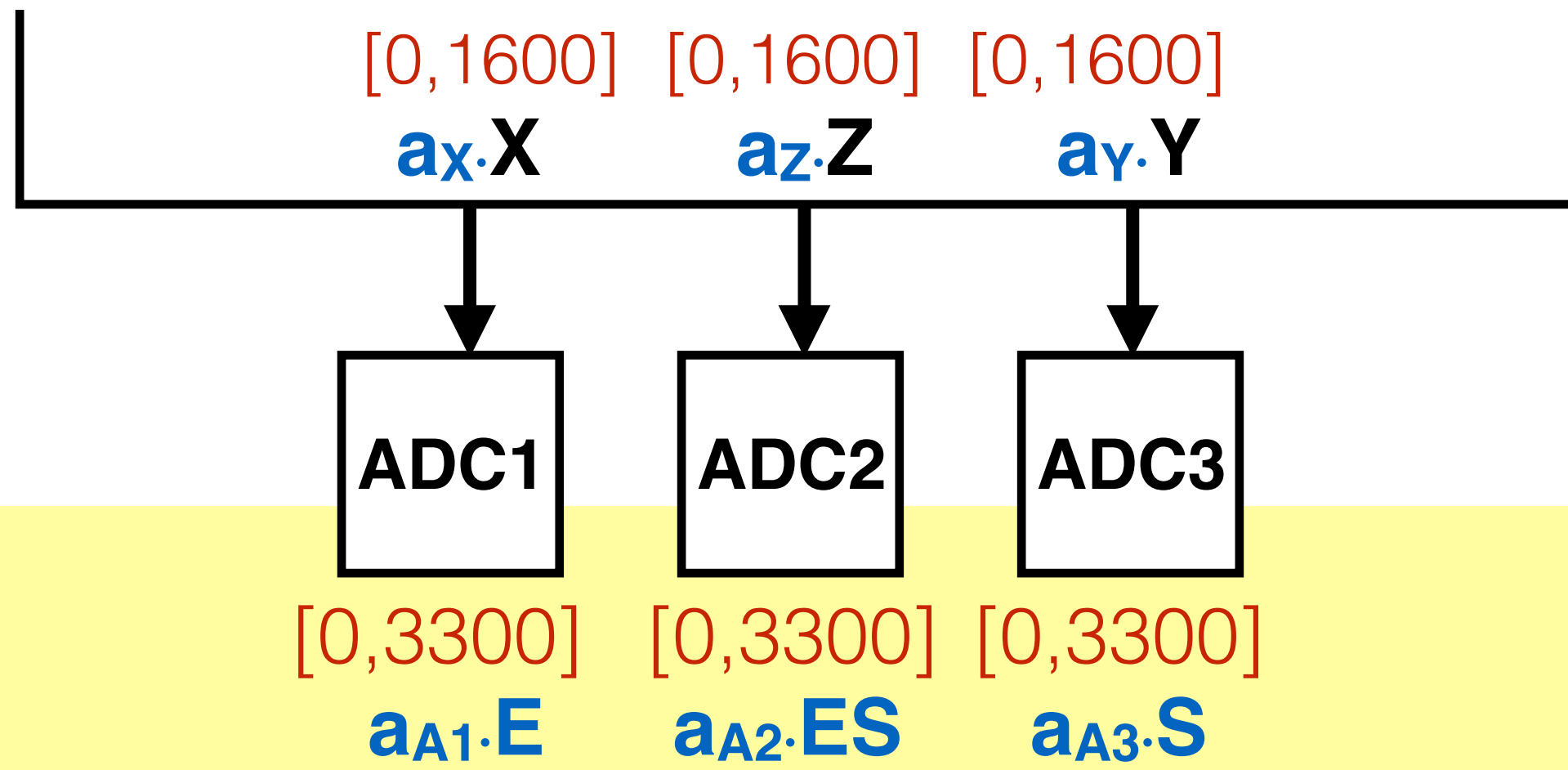
$$a_B \cdot 0.0001 \in [10^{-4}, 1]$$

$$a_{Z0} \cdot 0 \in [0, 600]$$

# Operating Range Constraints



# Operating Range Constraints



$$a_{A1} \cdot [0, 6800] \subseteq [0, 3300]$$

$$a_{A2} \cdot [0, 4400] \subseteq [0, 3300]$$

$$a_{A3} \cdot [0, 4400] \subseteq [0, 3300]$$

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**maximize  $\tau$**

**subject to:**

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Sampling Constraints

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**maximize  $\tau$**

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**Sampling Constraints**

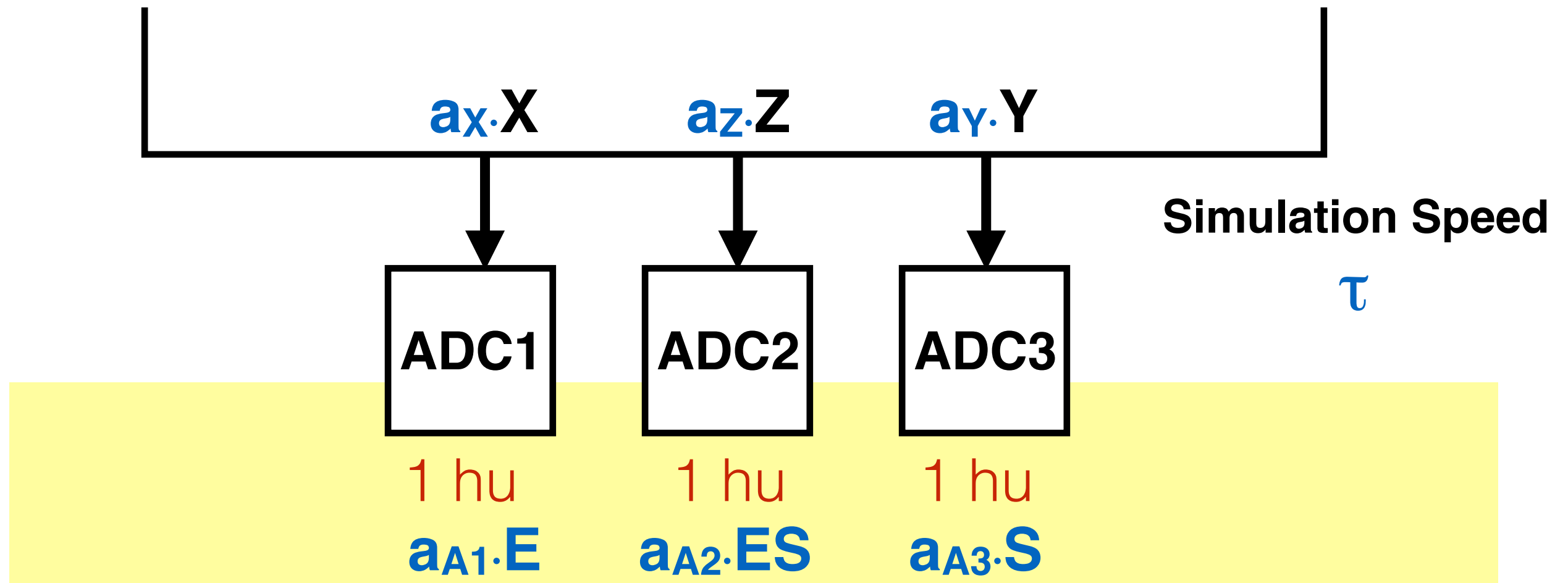
Connection Constraints

Factor Constraints

} **physically realizable**

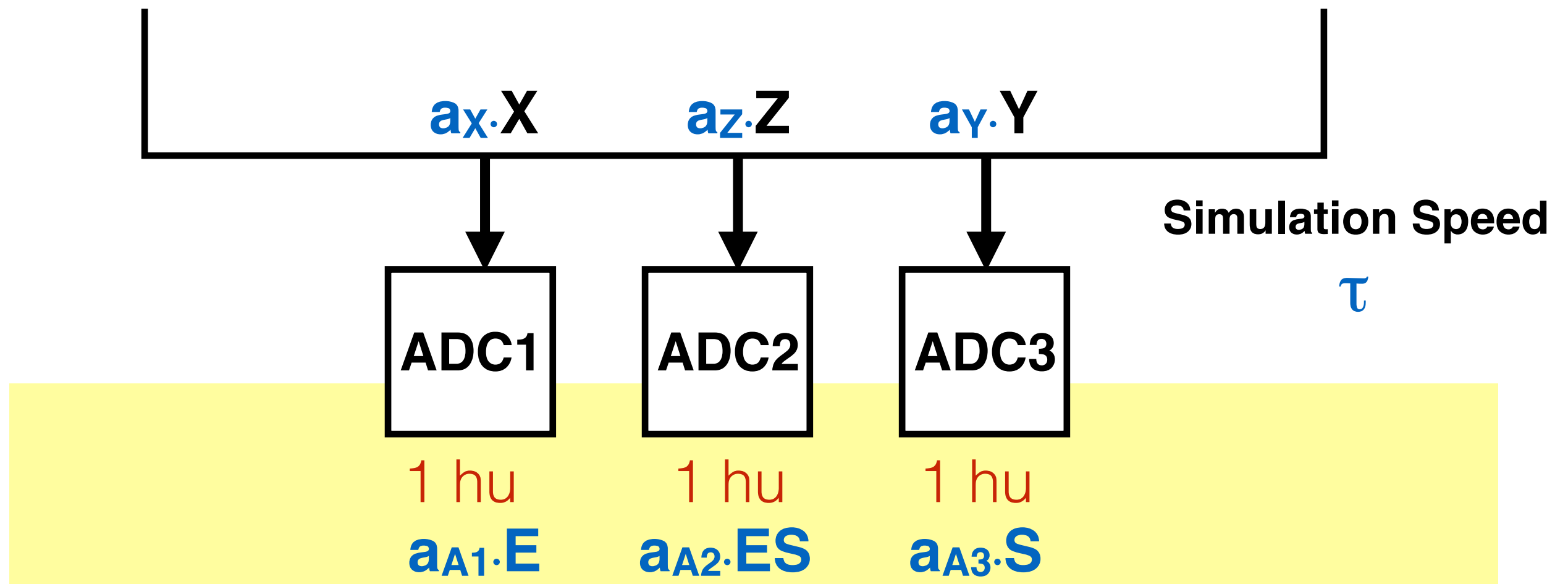
} **recoverable**

# Sampling Constraints



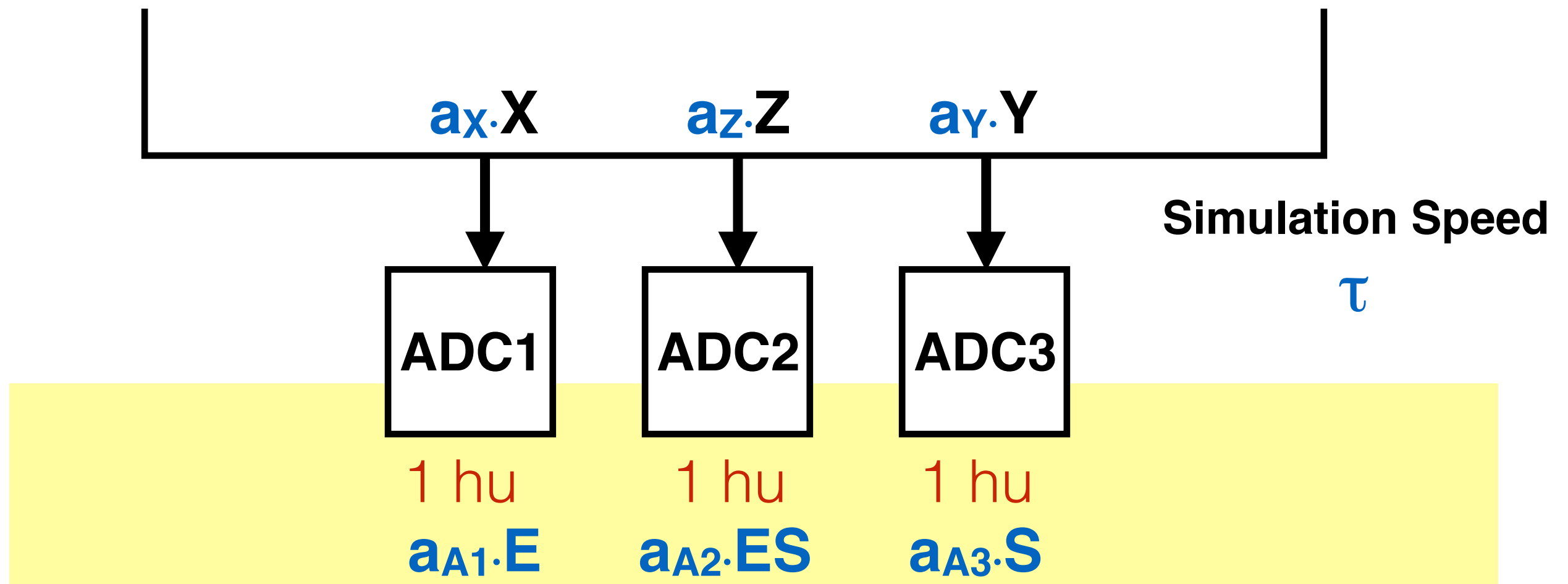
$$1 \text{ hu} \cdot \tau \leq 0.5 \text{ su}$$

# Sampling Constraints



$$1 \text{ hu} \cdot \tau \leq 0.5 \text{ su} \quad \tau: \text{hu} \rightarrow \text{su}$$

# Sampling Constraints

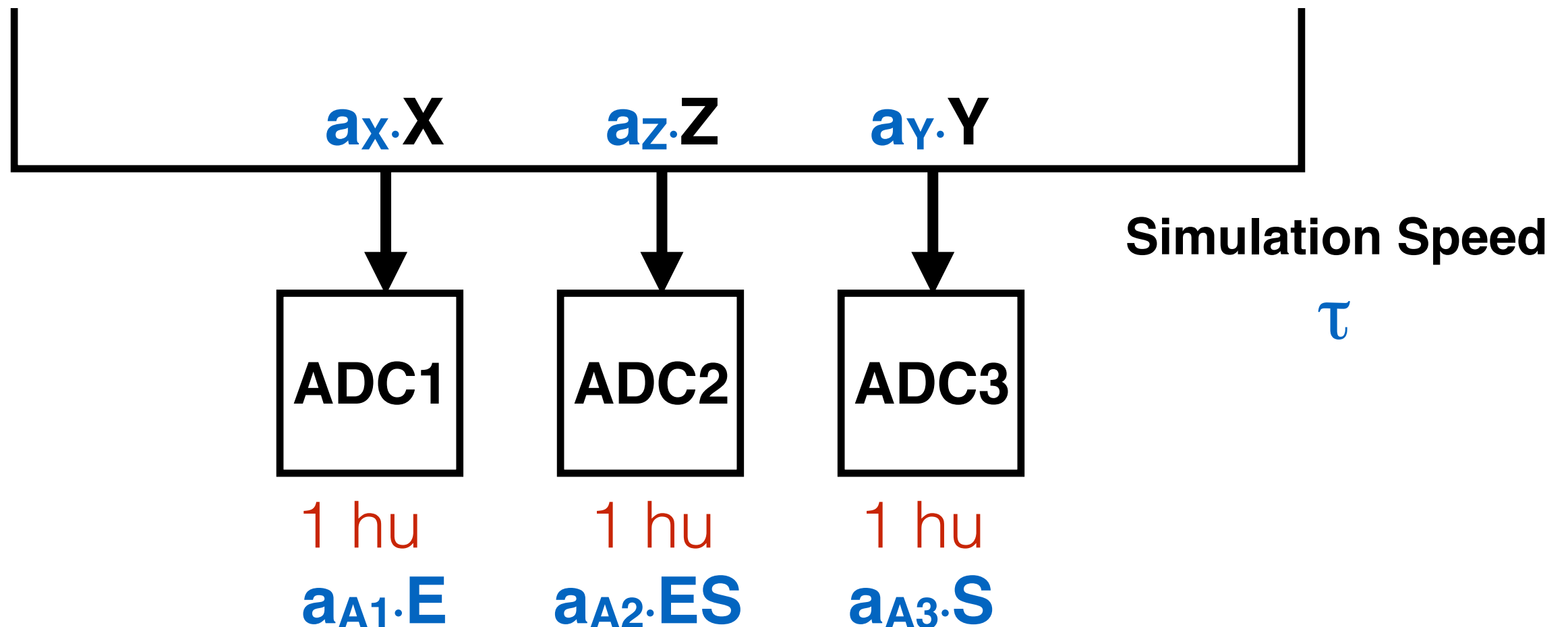


$$1 \text{ hu} \cdot \tau \leq 0.5 \text{ su} \quad \tau: \text{hu} \rightarrow \text{su}$$

time between simulation observations (su)



# Sampling Constraints



$$1 \text{ hu} \cdot \tau \leq 0.5 \text{ su} \quad \tau: \text{hu} \rightarrow \text{su}$$

user defined maximum time between samples

Limits simulation speed

Improves sample fidelity

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Sampling Constraints

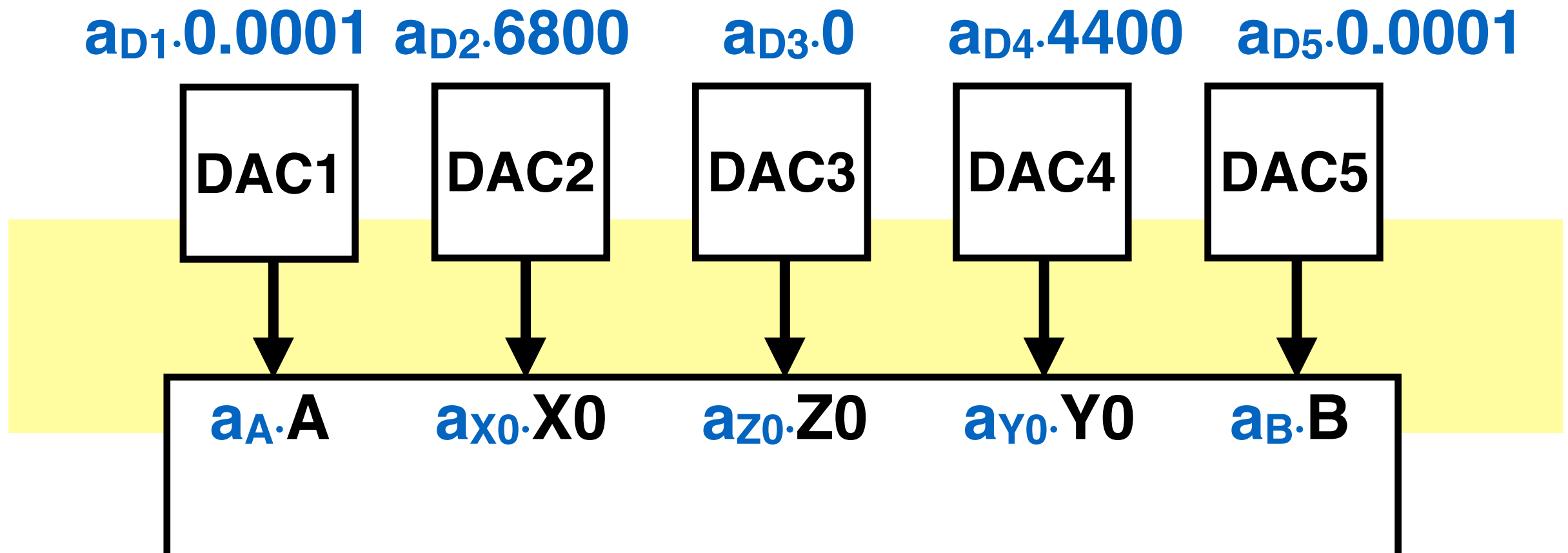
**Connection Constraints**

Factor Constraints

} **physically realizable**

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# Connection Constraints



$$a_{D1} = a_A$$

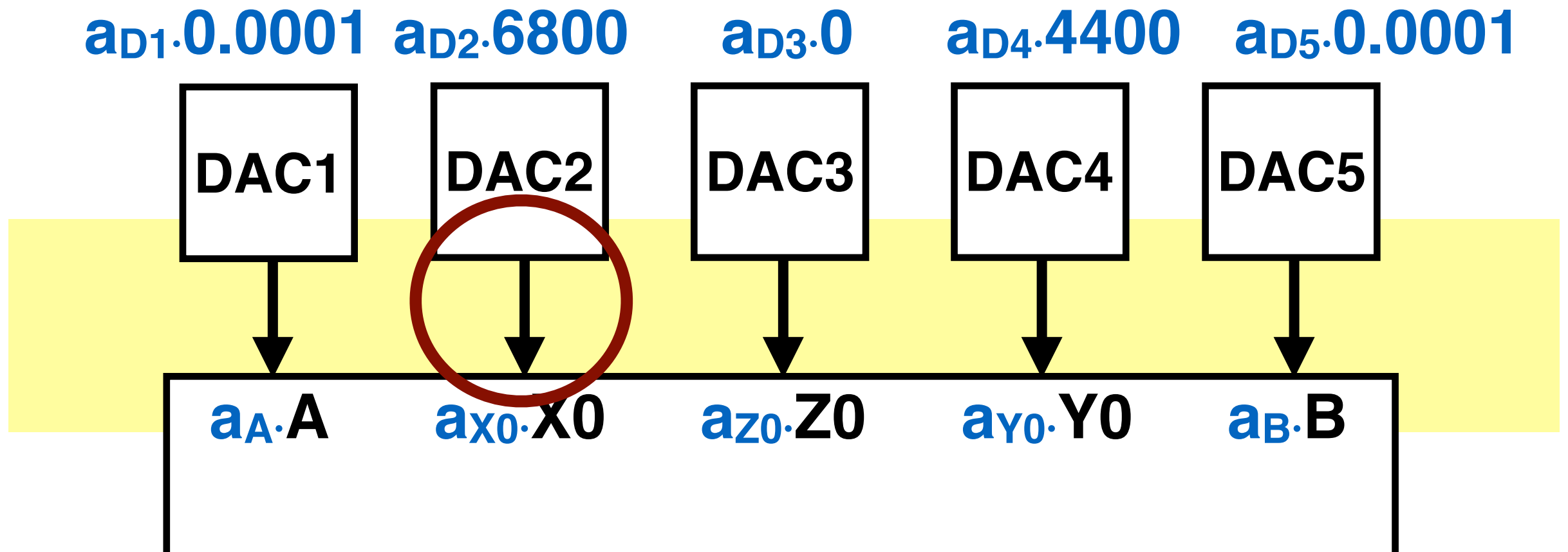
$$a_{D3} = a_{z0}$$

$$a_{D5} = a_B$$

$$a_{D2} = a_{x0}$$

$$a_{D4} = a_{y0}$$

# Connection Constraints



$$a_{D1} = a_A$$

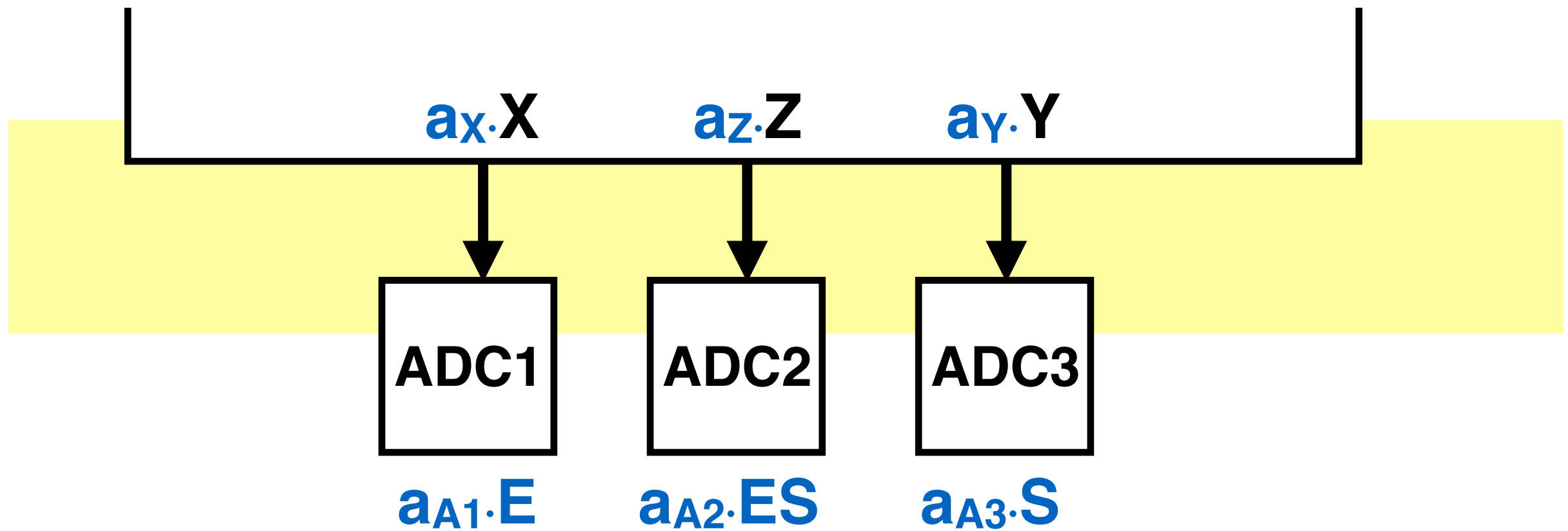
$$a_{D2} = a_{x0}$$

$$a_{D3} = a_{z0}$$

$$a_{D4} = a_{y0}$$

$$a_{D5} = a_B$$

# Connection Constraints



$$a_x = a_{A1} \quad a_z = a_{A2} \quad a_y = a_{A3}$$

# Geometric Program Generation

**maximize  $\tau$**

**subject to:**

Operating Range Constraints

Sampling Constraints

Connection Constraints

Factor Constraints

} **physically realizable**

} **recoverable**

# Geometric Program Generation

**maximize  $\tau$**

**subject to:**

Operating Range Constraints

Sampling Constraints

Connection Constraints

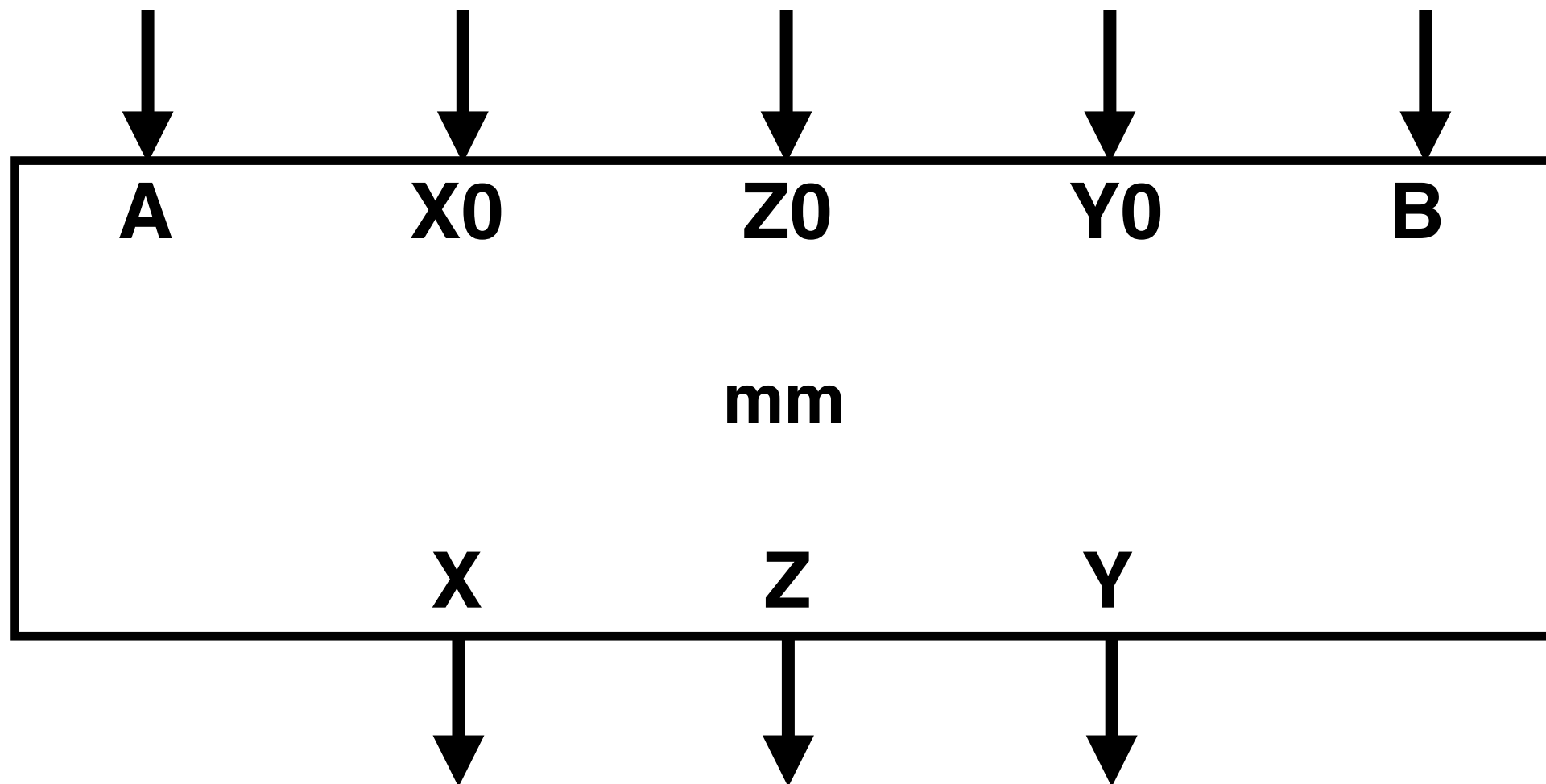
**Factor Constraints**

} **physically realizable**

} **recoverable**



# Factor Constraints



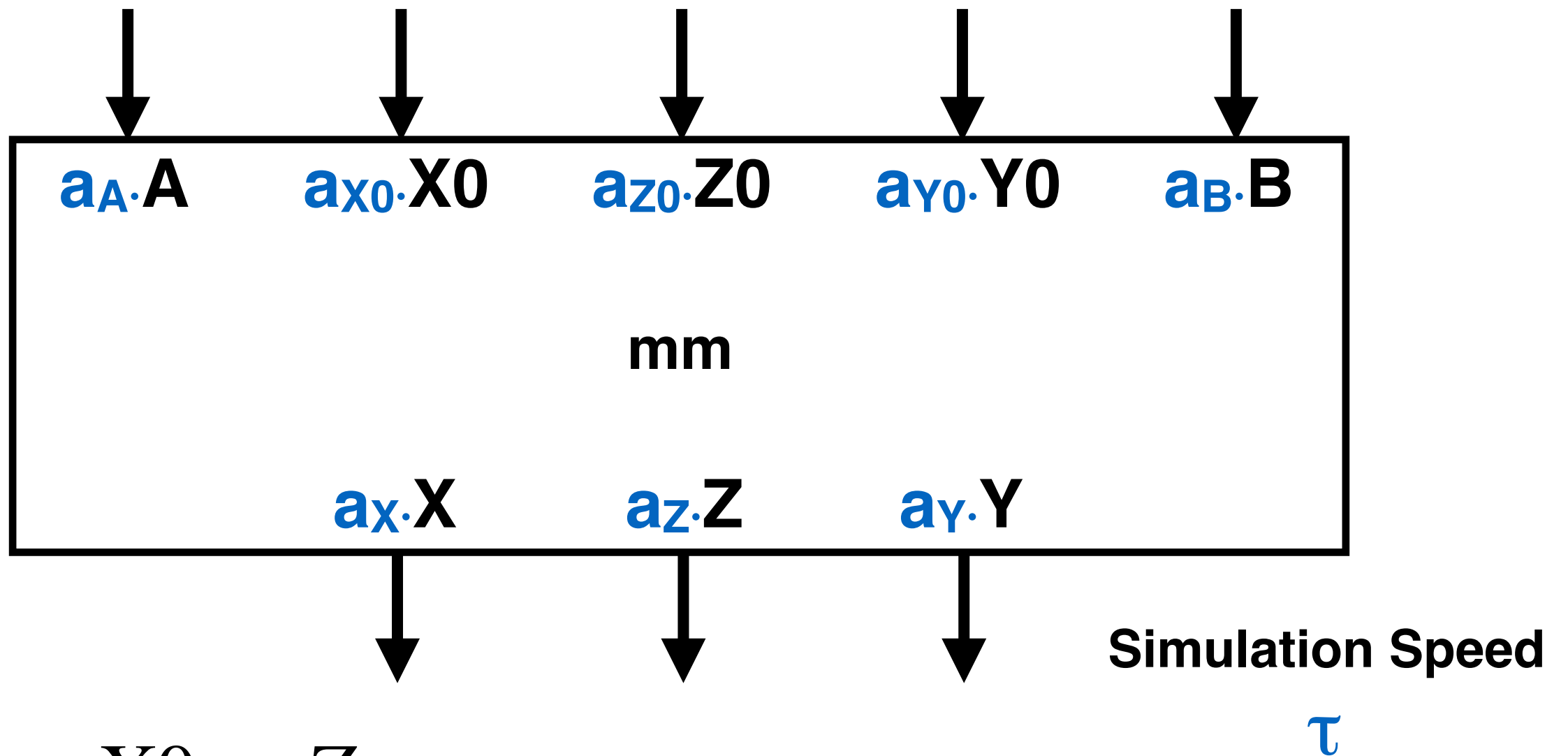
$$X = X0 - Z$$

$$Y = Y0 - Z$$

$$Z = \int A \cdot X \cdot Y - B \cdot Z \, dt$$

$$Z = Z0 \text{ @ } t=0$$

# Factor Constraints



$$a_X X = a_{X0} X0 - a_Z Z$$

$$a_Y Y = a_{Y0} Y0 - a_Z Z$$

$$a_Z Z = \int [a_A A \cdot a_X X \cdot a_Y Y - a_B B \cdot a_Z Z] \tau^{-1} dt$$

$$a_Z Z = a_{Z0} Z0 @ t=0$$

# Factor Constraints

$$a_X X = a_{X0} X_0 - a_Z Z$$

# Factor Constraints

$$a_X X = a_{X0} [X0-Z]$$

$$a_{X0} = a_Z$$

# Factor Constraints

$$a_x[X = X_0 - Z]$$

$$a_{x_0} = a_z$$

$$a_x = a_{x_0}$$

# Factor Constraints

$$a_X[X = X_0 - Z]$$

$$a_{X_0} = a_Z$$

$$a_X = a_{X_0}$$

$$a_Y Y = a_{Y_0} Y_0 - a_Z Z$$

# Factor Constraints

$$a_X[X = X_0 - Z]$$

$$a_{X_0} = a_Z$$

$$a_X = a_{X_0}$$

$$a_Y[Y = Y_0 - Z]$$

$$a_{Y_0} = a_Y$$

$$a_{Y_0} = a_Z$$

# Factor Constraints

$$a_X[X = X_0 - Z]$$

$$a_{X_0} = a_Z$$

$$a_X = a_{X_0}$$

$$a_Y[Y = Y_0 - Z]$$

$$a_{Y_0} = a_Y$$

$$a_{Y_0} = a_Z$$

$$a_Z Z = \int [a_A A \cdot a_X X \cdot a_Y Y - a_B B \cdot a_Z Z] \tau^{-1} dt$$



# Factor Constraints

$$a_X[X = X_0 - Z]$$

$$a_{X_0} = a_Z$$

$$a_X = a_{X_0}$$

$$a_Y[Y = Y_0 - Z]$$

$$a_{Y_0} = a_Y$$

$$a_{Y_0} = a_Z$$

$$a_Z Z = \int a_B \cdot a_Z [A \cdot X \cdot Y - B \cdot Z] \tau^{-1} dt$$

$$a_A \cdot a_X \cdot a_Y = a_B \cdot a_Z$$

# Factor Constraints

$$a_X[X = X_0 - Z]$$

$$a_{X_0} = a_Z$$

$$a_X = a_{X_0}$$

$$a_Y[Y = Y_0 - Z]$$

$$a_{Y_0} = a_Y$$

$$a_{Y_0} = a_Z$$

$$a_Z Z = a_B \cdot a_Z \cdot \tau^{-1} \int [A \cdot X \cdot Y - B \cdot Z] dt$$

$$a_A \cdot a_X \cdot a_Y = a_B \cdot a_Z$$

# Factor Constraints

$$a_X[X = X_0 - Z]$$

$$a_{X_0} = a_Z$$

$$a_X = a_{X_0}$$

$$a_Y[Y = Y_0 - Z]$$

$$a_{Y_0} = a_Y$$

$$a_{Y_0} = a_Z$$

$$a_Z[Z = \int[A \cdot X \cdot Y - B \cdot Z \, dt]]$$

$$a_A \cdot a_X \cdot a_Y = a_B \cdot a_Z \quad a_Z = a_B \cdot a_Z \cdot \tau^{-1}$$

# Factor Constraints

$$a_X[X = X_0 - Z]$$

$$a_{X_0} = a_Z$$

$$a_X = a_{X_0}$$

$$a_Y[Y = Y_0 - Z]$$

$$a_{Y_0} = a_Y$$

$$a_{Y_0} = a_Z$$

$$a_Z[Z = \int [A \cdot X \cdot Y - B \cdot Z \, dt]]$$

$$a_A \cdot a_X \cdot a_Y = a_B \cdot a_Z \quad a_Z = a_B \cdot a_Z \cdot \tau^{-1}$$

$$a_Z Z = a_{Z_0} Z_0 @ t=0$$

$$a_{Z_0} = a_Z$$

## Geometric Program

maximize  $\tau$   
subject to:  
 $a_z = a_B \cdot a_z \cdot \tau^{-1}$   
 $a_A \cdot a_x \cdot a_y = a_B \cdot a_z$   
 $a_{y0} = a_y$   
 $a_{y0} = a_z$   
...

### Geometric Programming Library (GPKit)

Convert to  
Convex  
Program

Convex  
Solver  
(cvxopt)

## Numerical Assignments

$\tau = 0.50$   
 $a_{D1} = 8.28$   
 $a_A = 8.28$   
 $a_{D5} = 0.50$   
 $a_B = 0.50$   
 $a_{D2-D4} = 0.06$   
 $a_{x0} = 0.06$   
 $a_{y0} = 0.06$   
 $a_{z0} = 0.06$   
 $a_x = 0.06$   
 $a_y = 0.06$   
 $a_z = 0.06$   
 $a_{A1-A3} = 0.06$

## Geometric Program

maximize  $\tau$   
subject to:  
 $a_z = a_B \cdot a_z \cdot \tau^{-1}$   
 $a_A \cdot a_x \cdot a_y = a_B \cdot a_z$   
 $a_{y0} = a_y$   
 $a_{y0} = a_z$   
...

### Geometric Programming Library (GPKit)

Convert to  
Convex  
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Convex  
Solver  
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## Numerical Assignments

$\tau = 0.50$   
 $a_{D1} = 8.28$   
 $a_A = 8.28$   
 $a_{D5} = 0.50$   
 $a_B = 0.50$   
 $a_{D2-D4} = 0.06$   
 $a_{x0} = 0.06$   
 $a_{y0} = 0.06$   
 $a_{z0} = 0.06$   
 $a_x = 0.06$   
 $a_y = 0.06$   
 $a_z = 0.06$   
 $a_{A1-A3} = 0.06$

## Geometric Program

maximize  $\tau$   
subject to:  
 $a_z = a_B \cdot a_z \cdot \tau^{-1}$   
 $a_A \cdot a_x \cdot a_y = a_B \cdot a_z$   
 $a_{y0} = a_y$   
 $a_{y0} = a_z$   
...

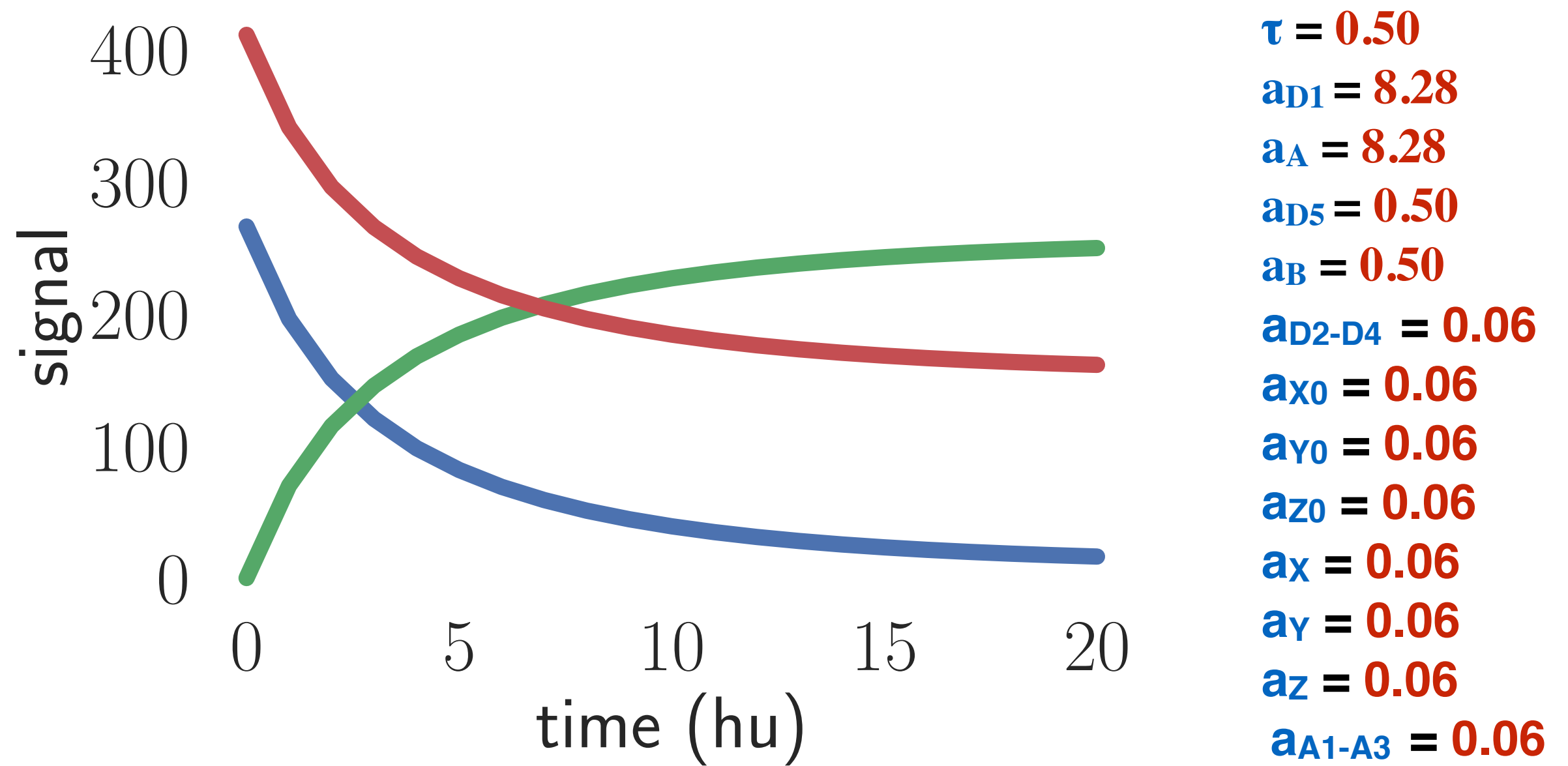
### Geometric Programming Library (GPKit)

Convert to  
Convex  
Program

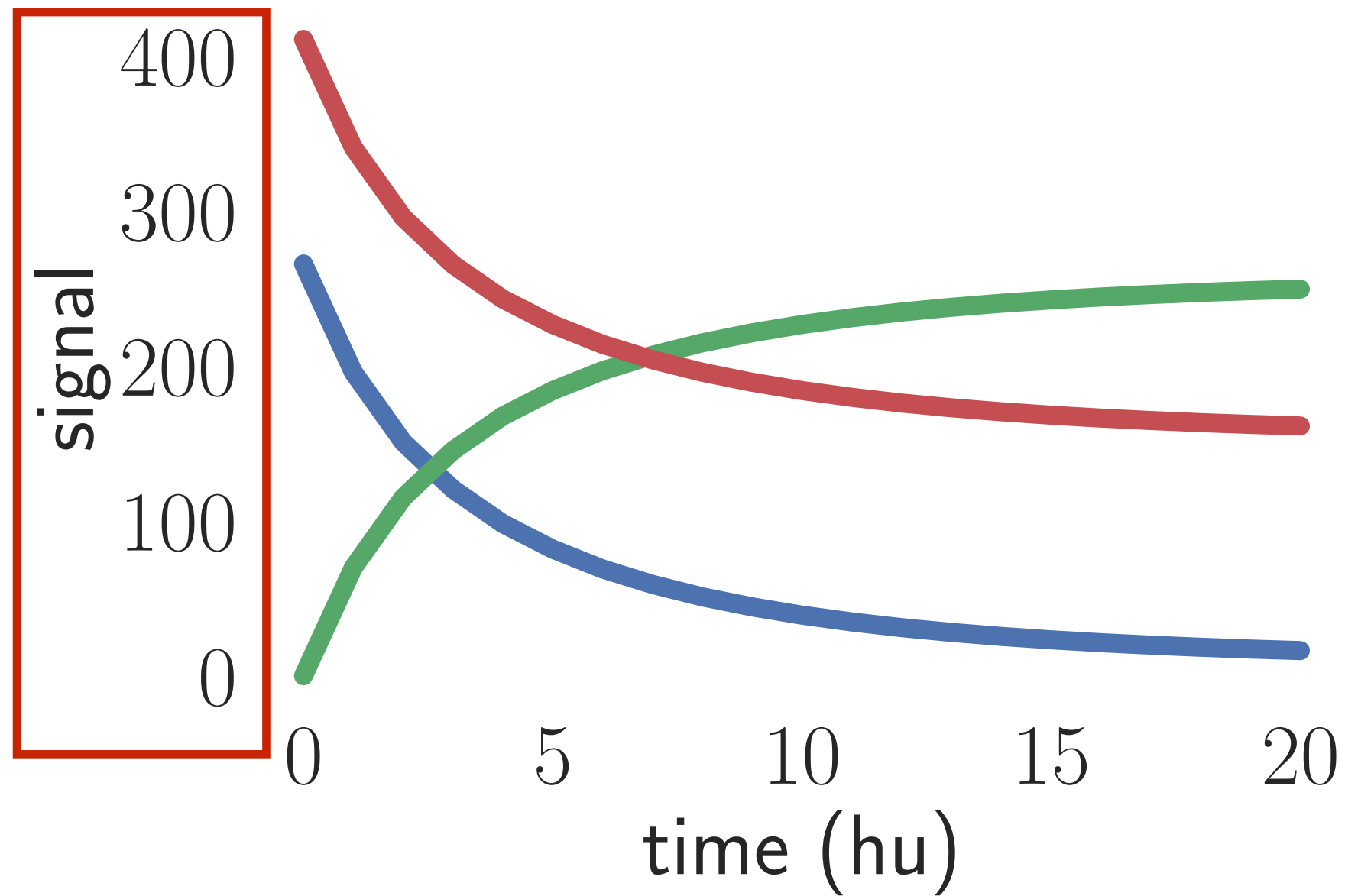
Convex  
Solver  
(cvxopt)

## Numerical Assignments

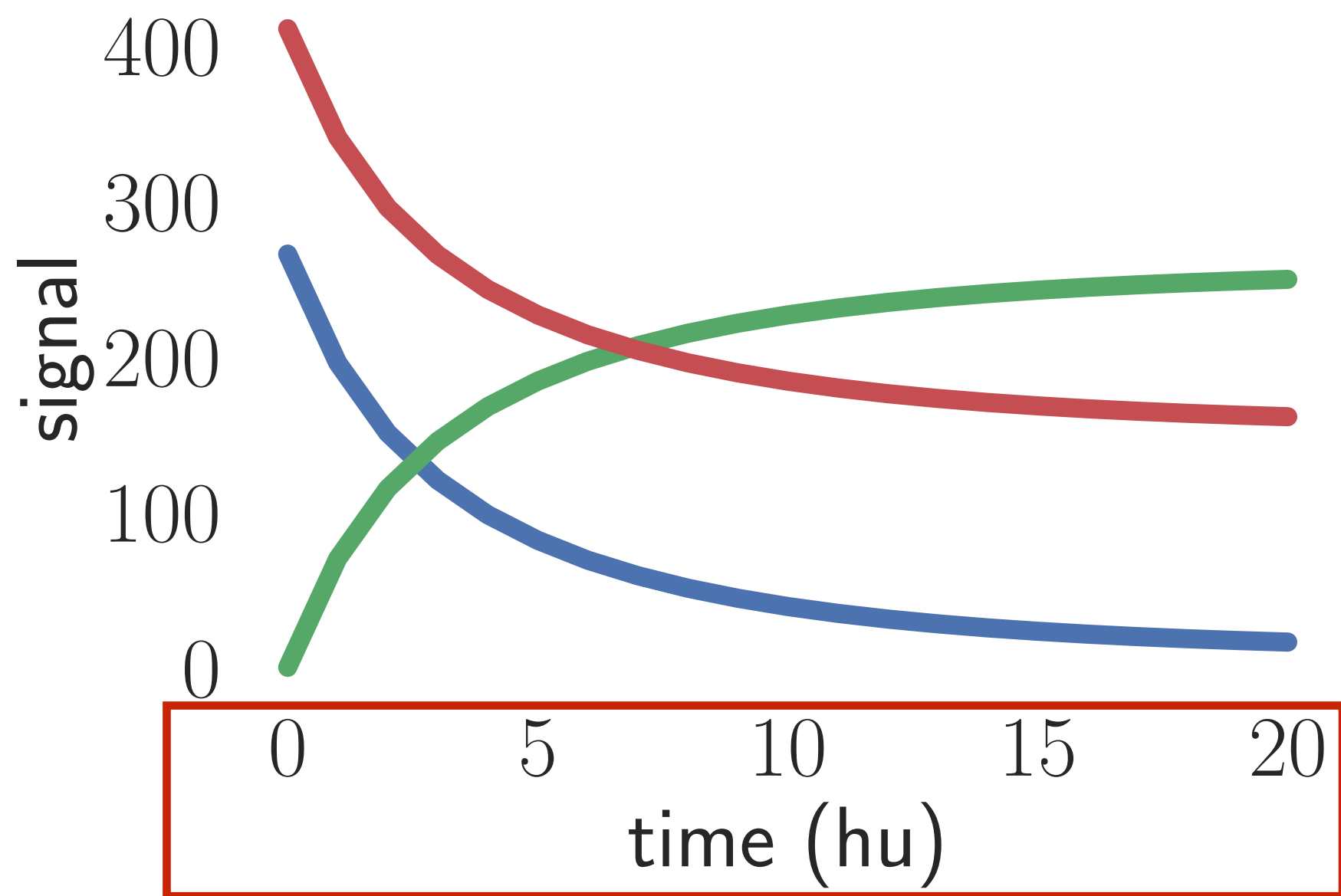
$\tau = 0.50$   
 $a_{D1} = 8.28$   
 $a_A = 8.28$   
 $a_{D5} = 0.50$   
 $a_B = 0.50$   
 $a_{D2-D4} = 0.06$   
 $a_{x0} = 0.06$   
 $a_{y0} = 0.06$   
 $a_{z0} = 0.06$   
 $a_x = 0.06$   
 $a_y = 0.06$   
 $a_z = 0.06$   
 $a_{A1-A3} = 0.06$







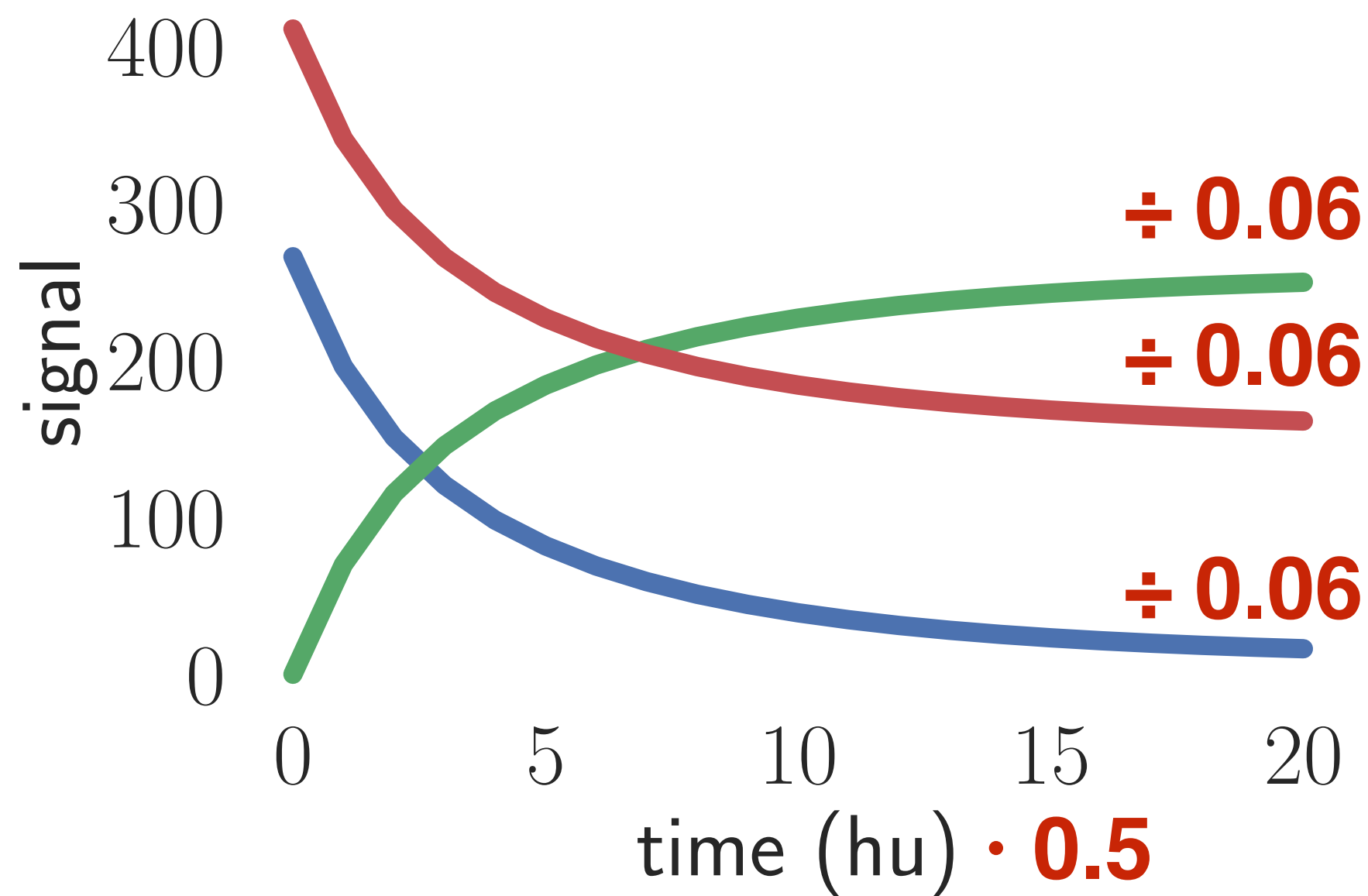
$\tau = 0.50$   
 $a_{D1} = 8.28$   
 $a_A = 8.28$   
 $a_{D5} = 0.50$   
 $a_B = 0.50$   
 $a_{D2-D4} = 0.06$   
 $a_{x0} = 0.06$   
 $a_{y0} = 0.06$   
 $a_{z0} = 0.06$   
 $a_x = 0.06$   
 $a_y = 0.06$   
 $a_z = 0.06$   
 $a_{A1-A3} = 0.06$



**$\tau = 0.50$**   
 **$a_{D1} = 8.28$**   
 **$a_A = 8.28$**   
 **$a_{D5} = 0.50$**   
 **$a_B = 0.50$**   
 **$a_{D2-D4} = 0.06$**   
 **$a_{X0} = 0.06$**   
 **$a_{Y0} = 0.06$**   
 **$a_{Z0} = 0.06$**   
 **$a_X = 0.06$**   
 **$a_Y = 0.06$**   
 **$a_Z = 0.06$**   
 **$a_{A1-A3} = 0.06$**

2x slower than real-time

# Recover Original Simulation



$$\tau = 0.50$$

$$a_{D1} = 8.28$$

$$a_A = 8.28$$

$$a_{D5} = 0.50$$

$$a_B = 0.50$$

$$a_{D2-D4} = 0.06$$

$$a_{x0} = 0.06$$

$$a_{y0} = 0.06$$

$$a_{z0} = 0.06$$

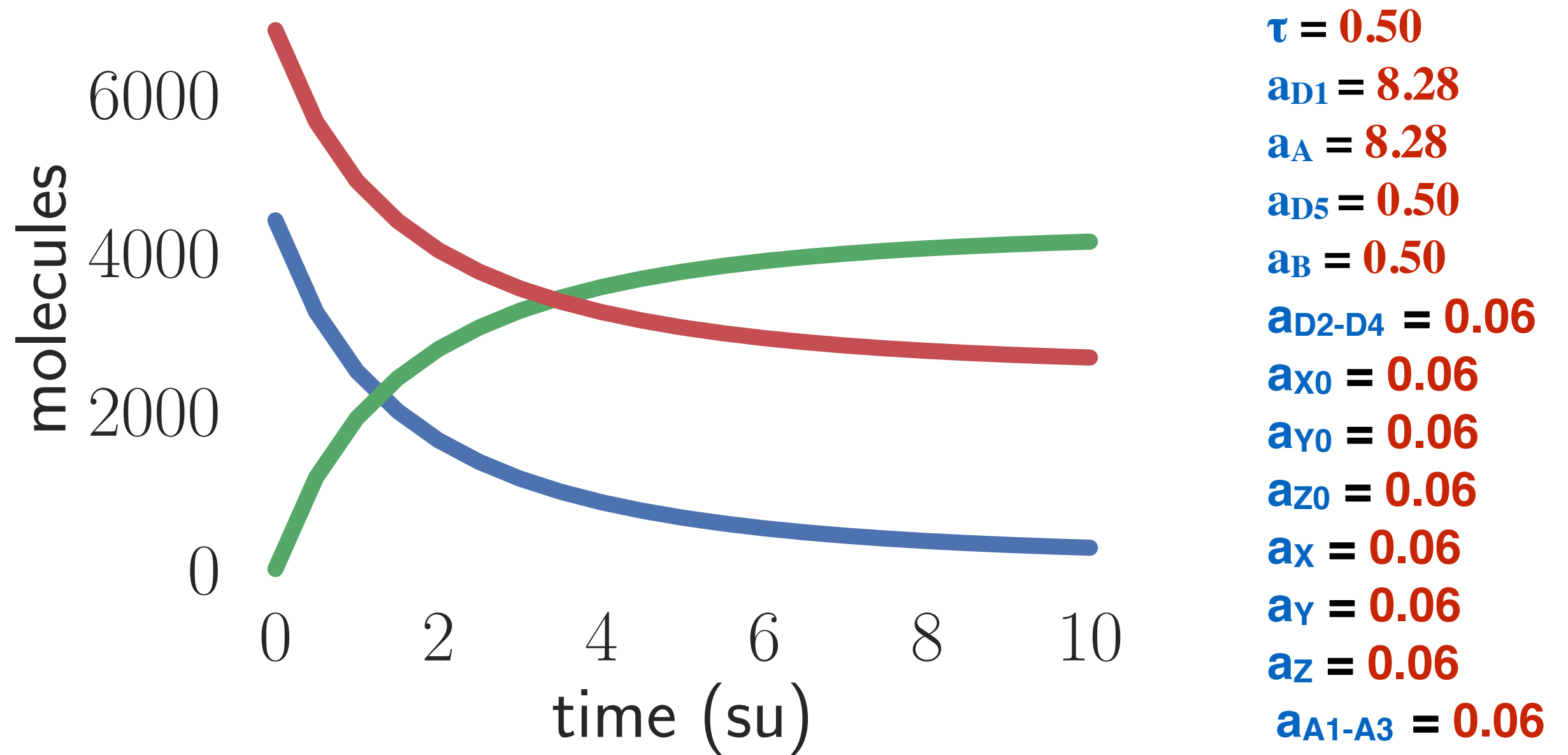
$$a_x = 0.06$$

$$a_y = 0.06$$

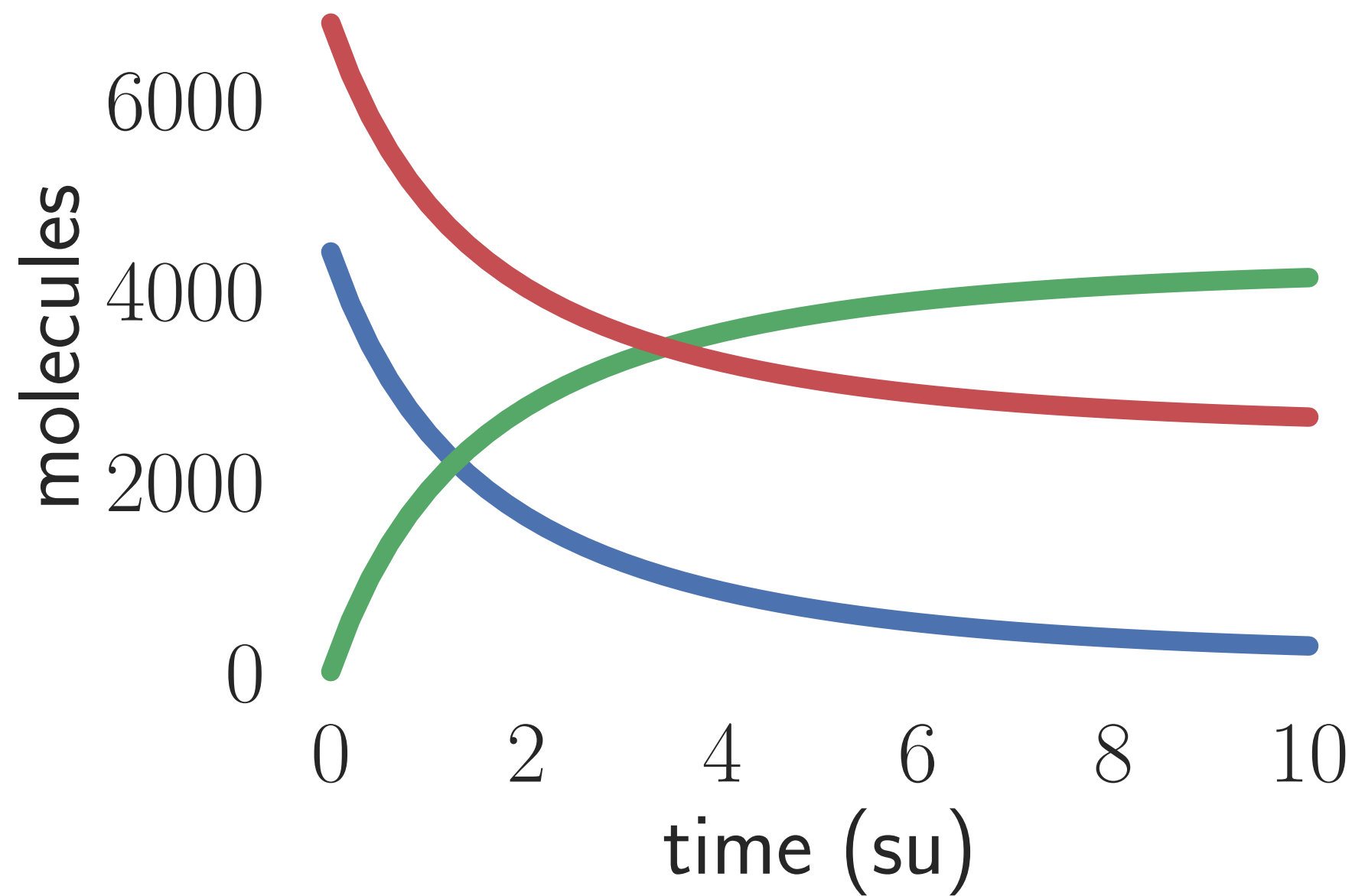
$$a_z = 0.06$$

$$a_{A1-A3} = 0.06$$

# Recovered Simulation



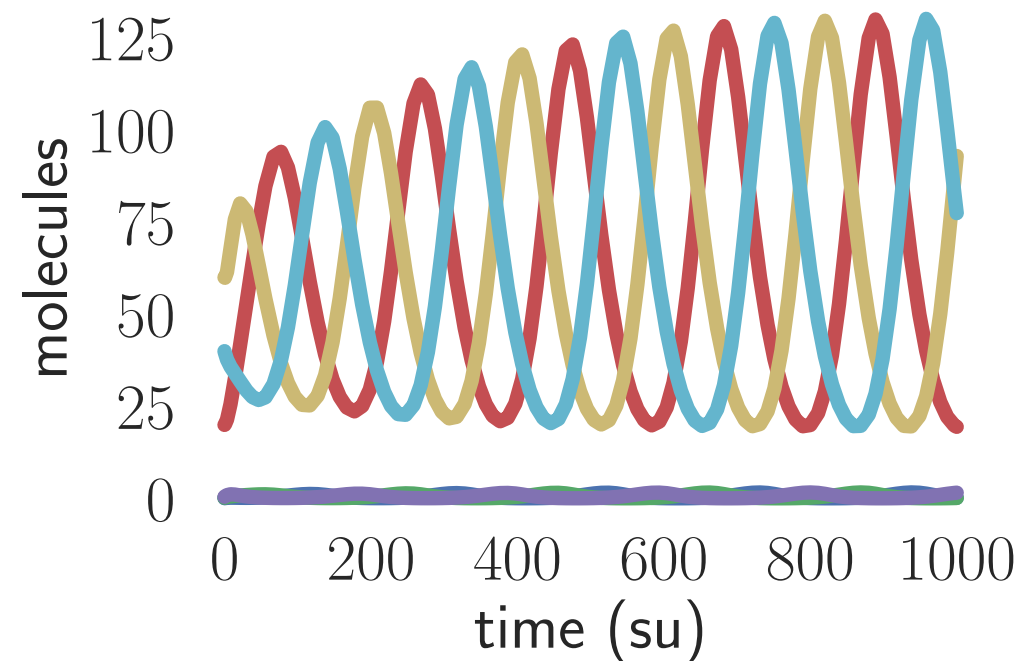
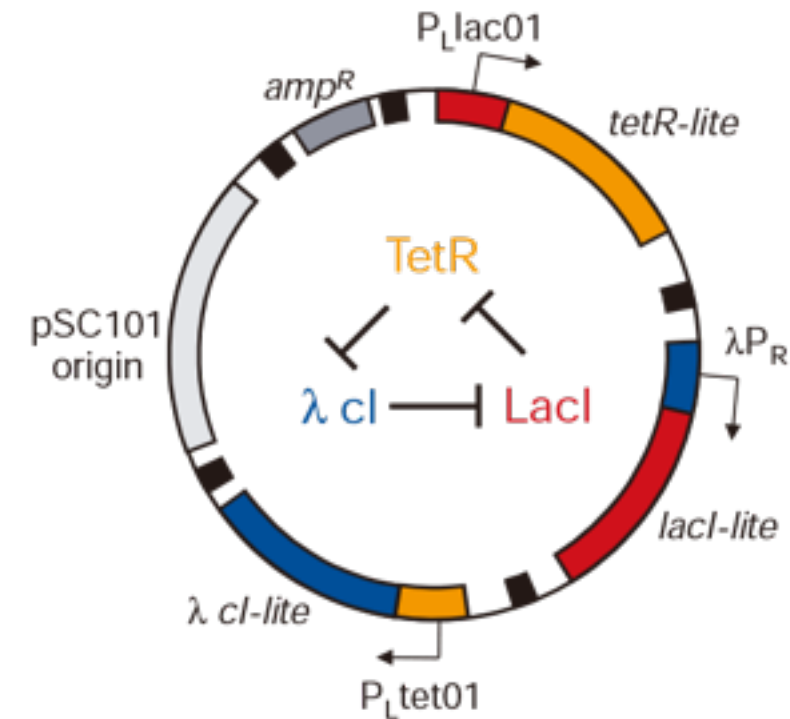
# Expected Simulation



# Results

# reprissilator

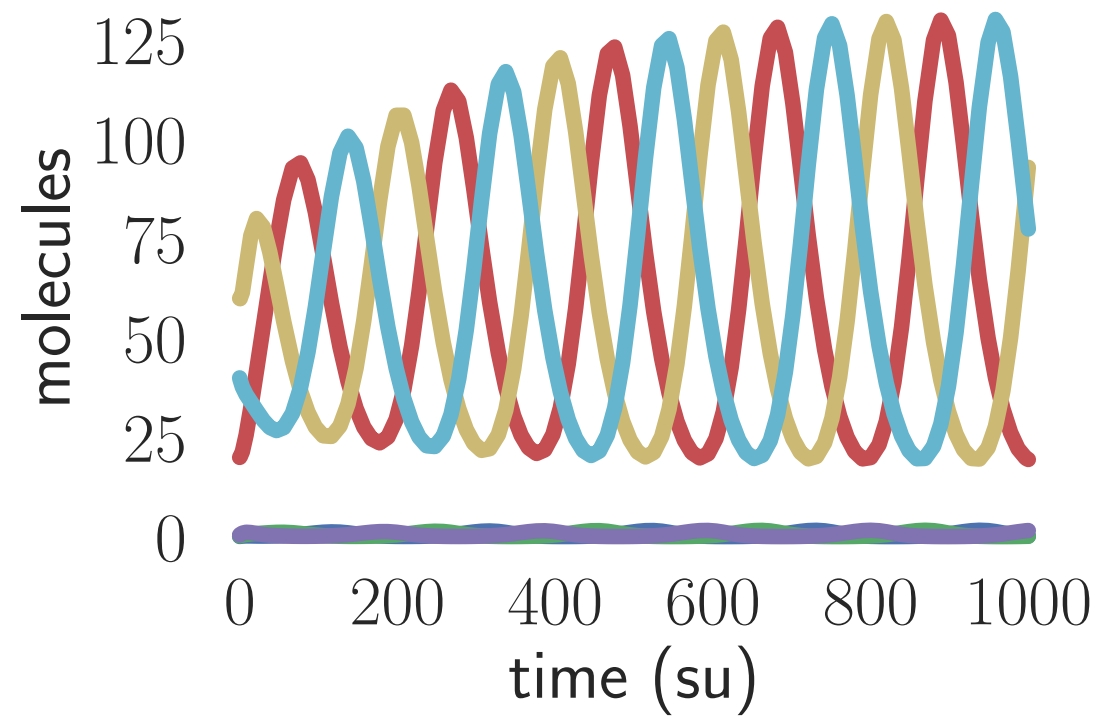
- Gene regulatory network that generates oscillations
  - synthetic genetic clock
- Model:
  - 9 diffeqs
  - 7 parameters
- Simulation
  - time: 1000 su
  - sample: 20 su



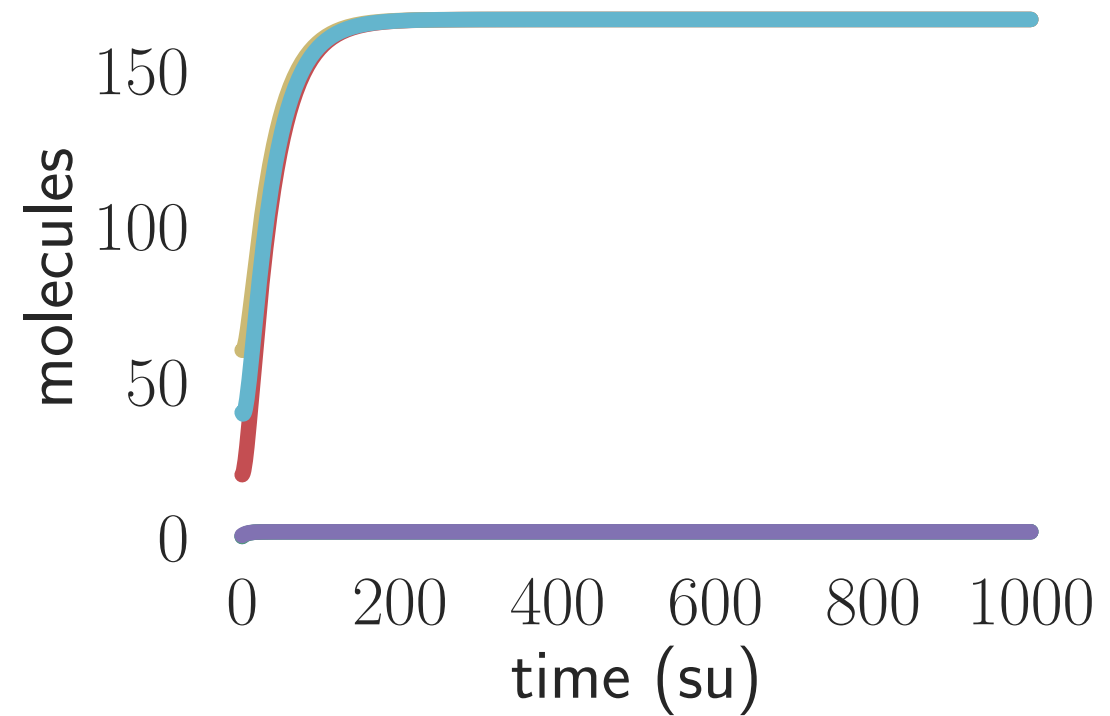
[1] Michael B Elowitz and Stanislas Leibler. A synthetic oscillatory network of transcriptional regulators. *Nature*, 403(6767):335–338, 2000.

# reprissilator

## Reference



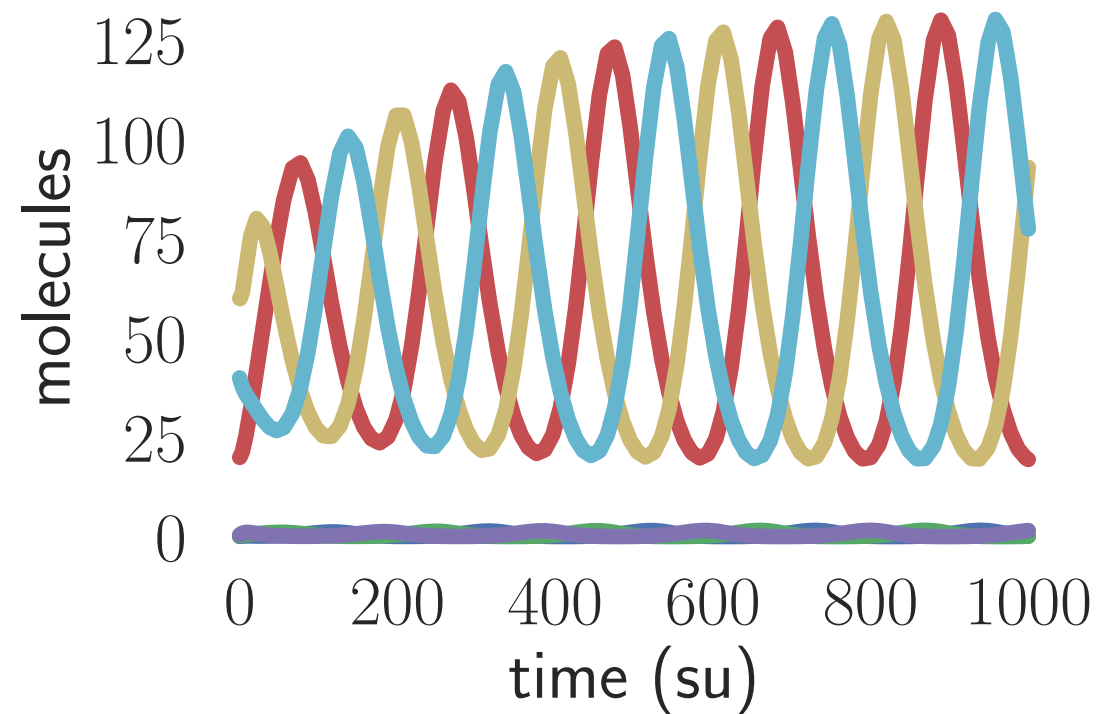
## No Jaunt



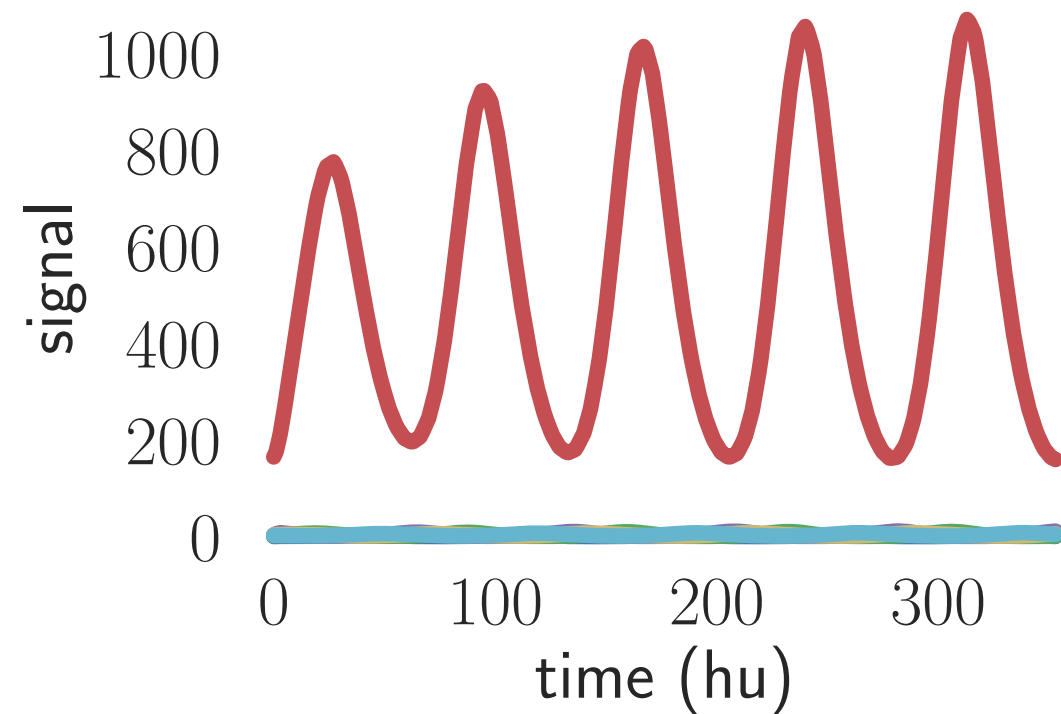


# reprissilator

Reference



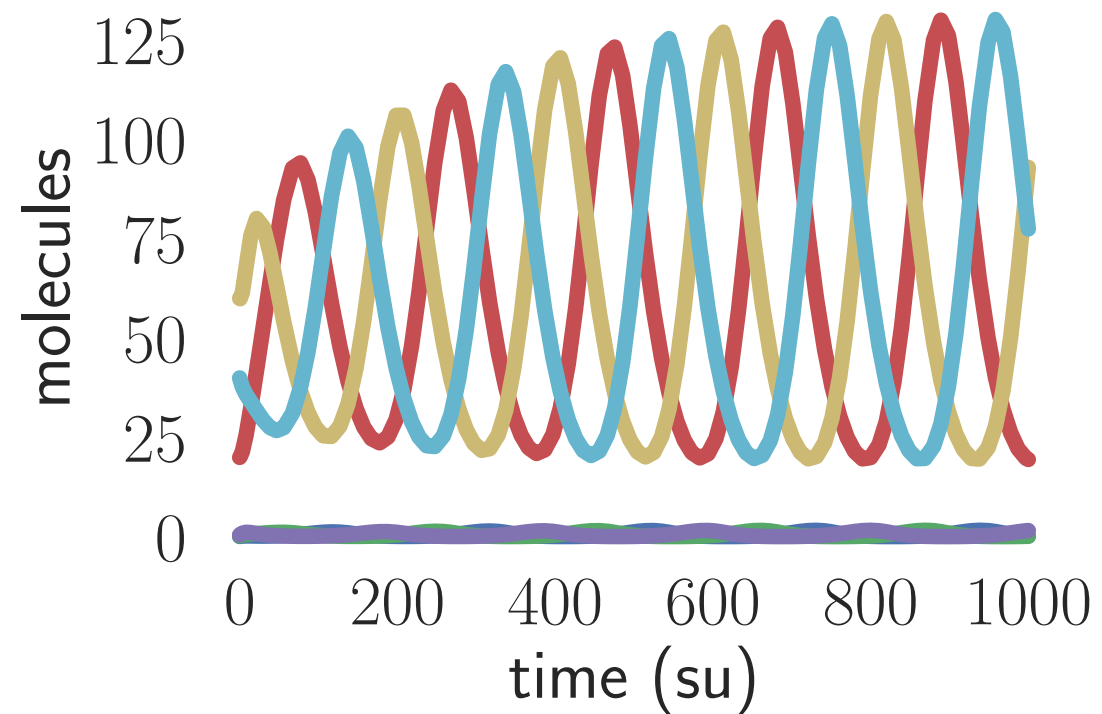
Jaunt + No Recover



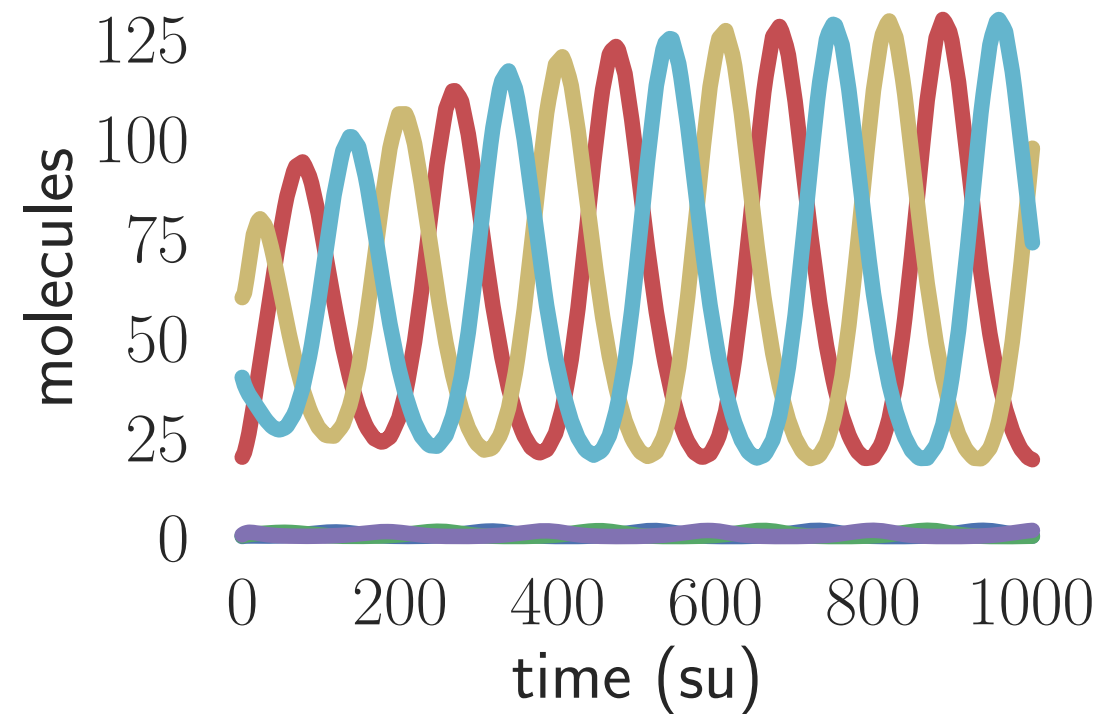
2.839x faster

# reprissilator

## Reference

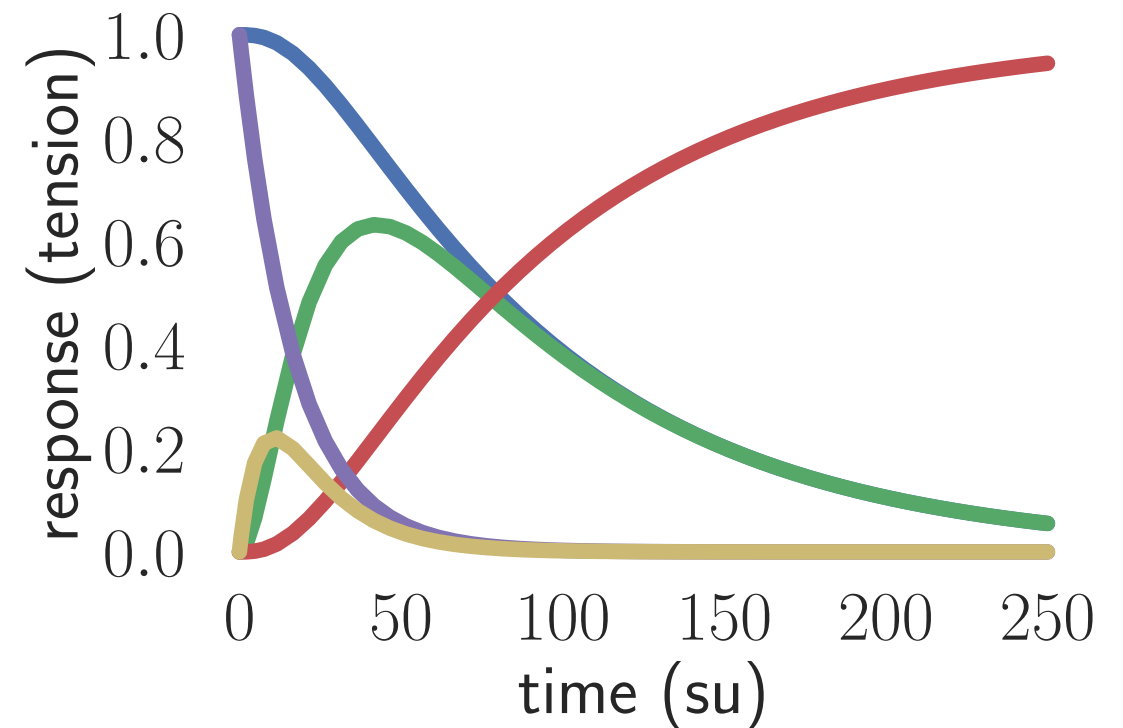


## Jaunt + Recover



# Botulinum Neurotoxin A Response

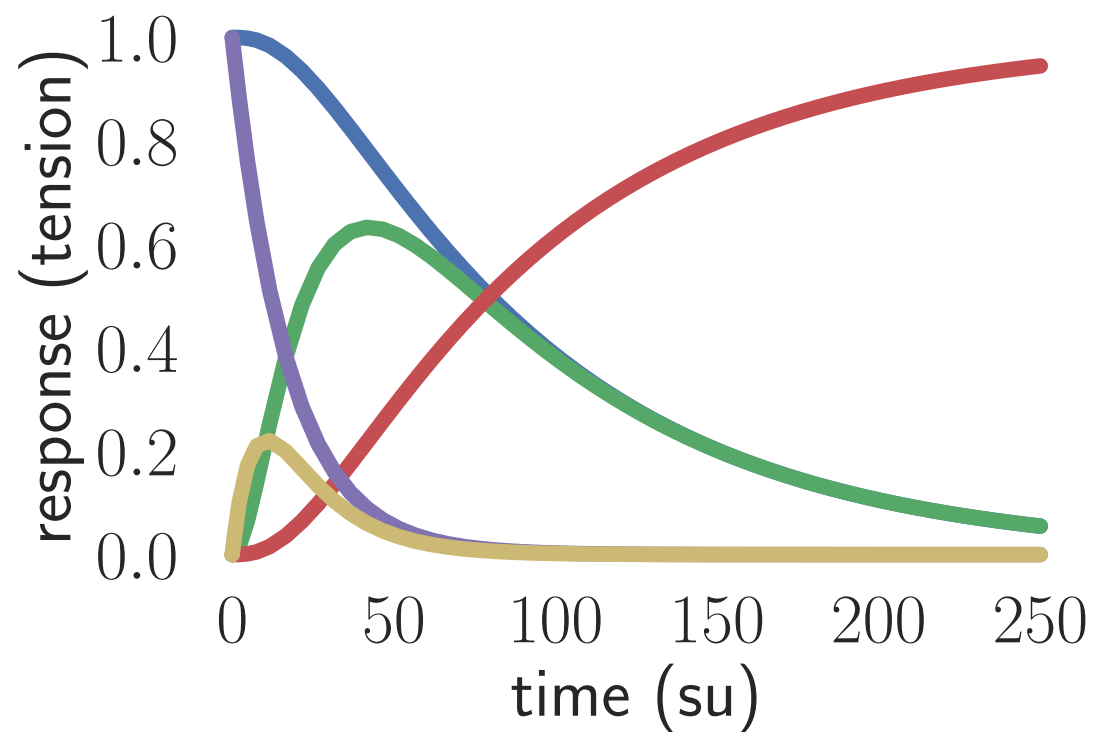
- Models the Paralysis of skeletal muscles after the introduction on Botulinum Neurotoxin A
- Model:
  - 5 diffeqs
  - 1 function
  - 8 parameters
- Simulation
  - time: 250 su
  - sample: 5 su



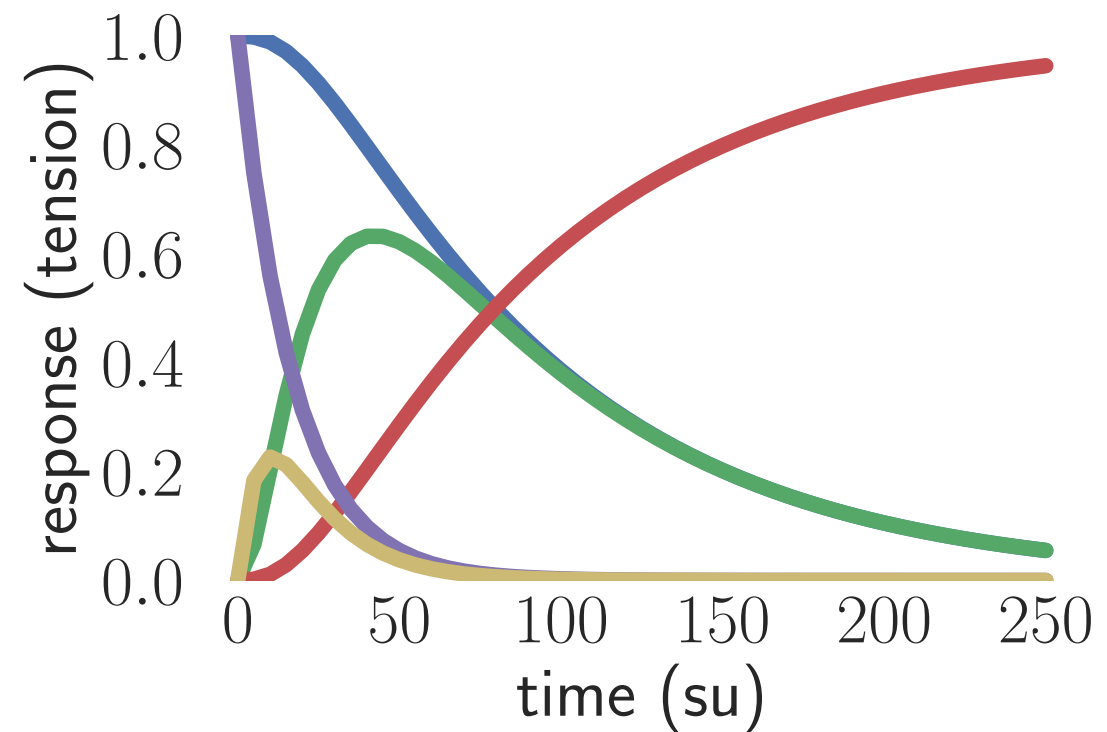
[1] Frank J Lebeda, Michael Adler, Keith Erickson, and Yaroslav Chushak. Onset dynamics of type a botulinum neurotoxin-induced paralysis. *Journal of pharmacokinetics and pharmacodynamics*, 35(3):251, 2008.

# Botulinum Neurotoxin A Response

Reference



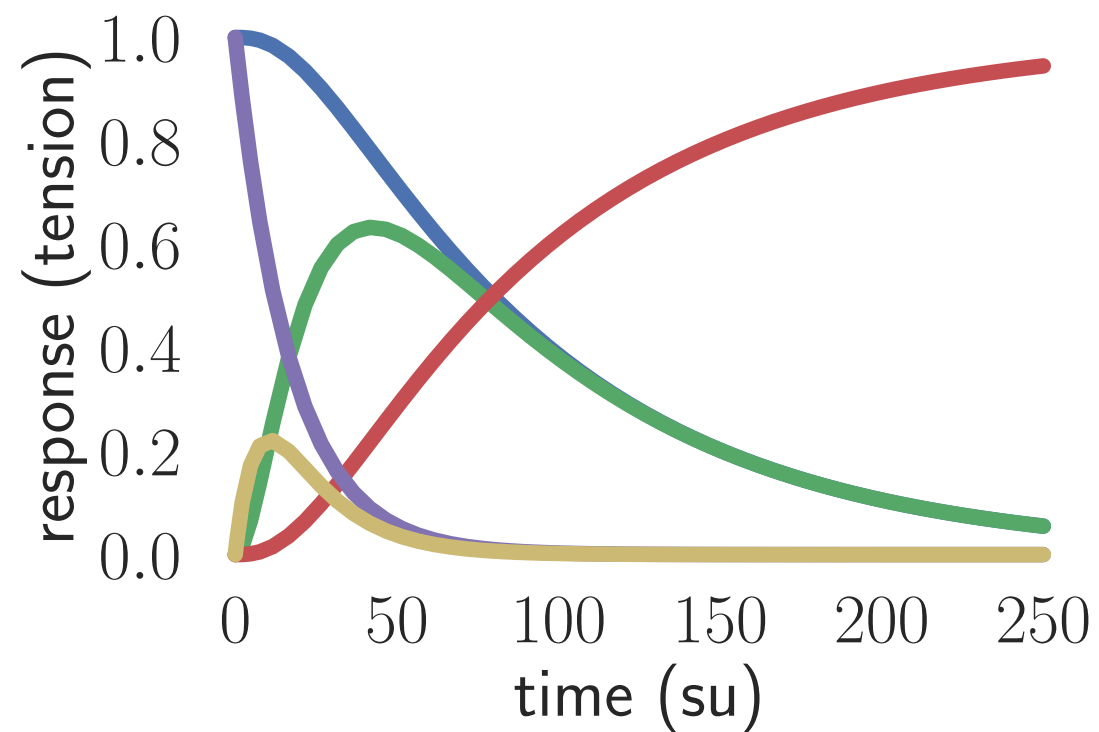
No Jaunt



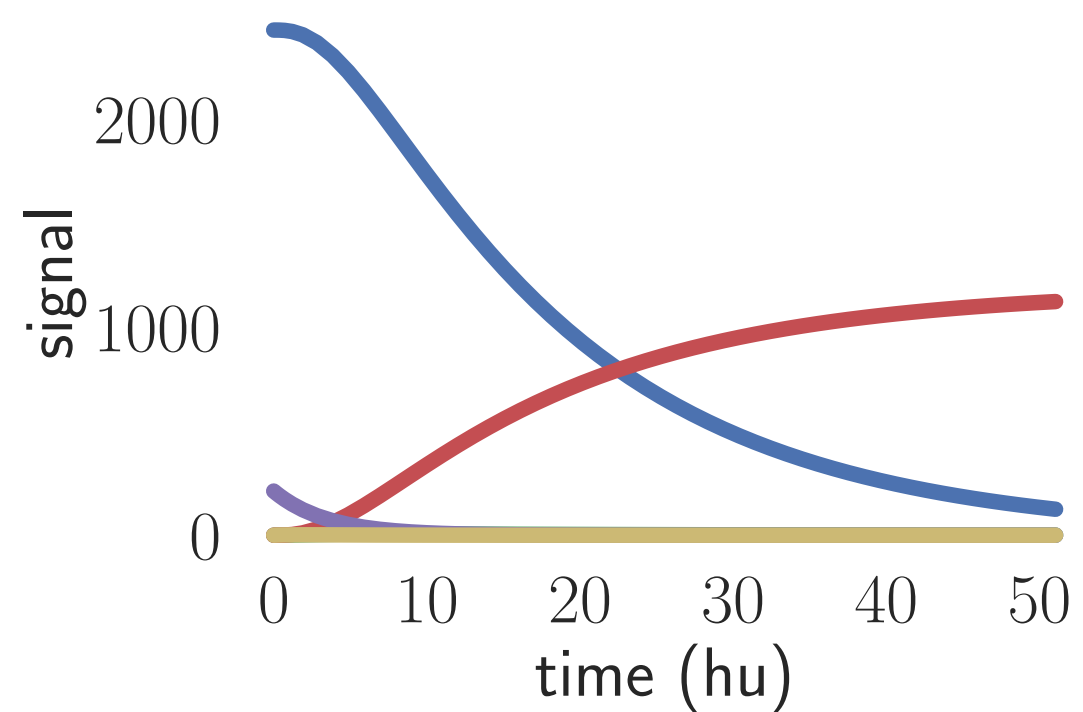
\*Jaunt still used for pruning search space

# Botulinum Neurotoxin A Response

Reference



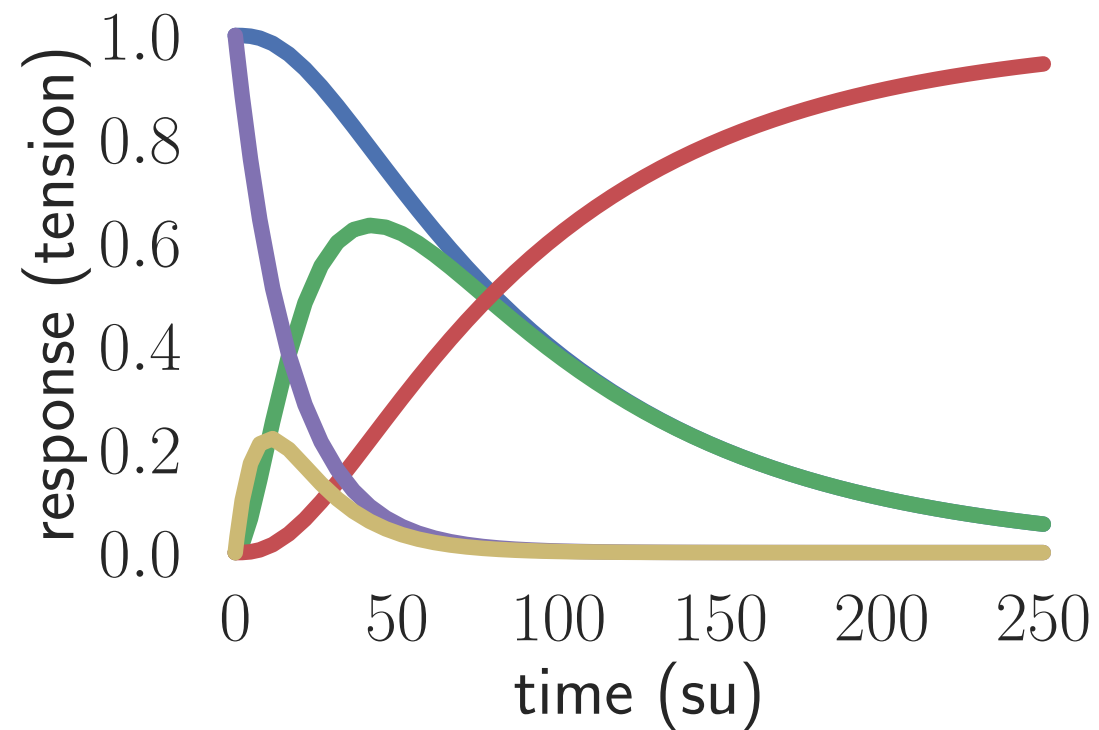
Jaunt + No Recover



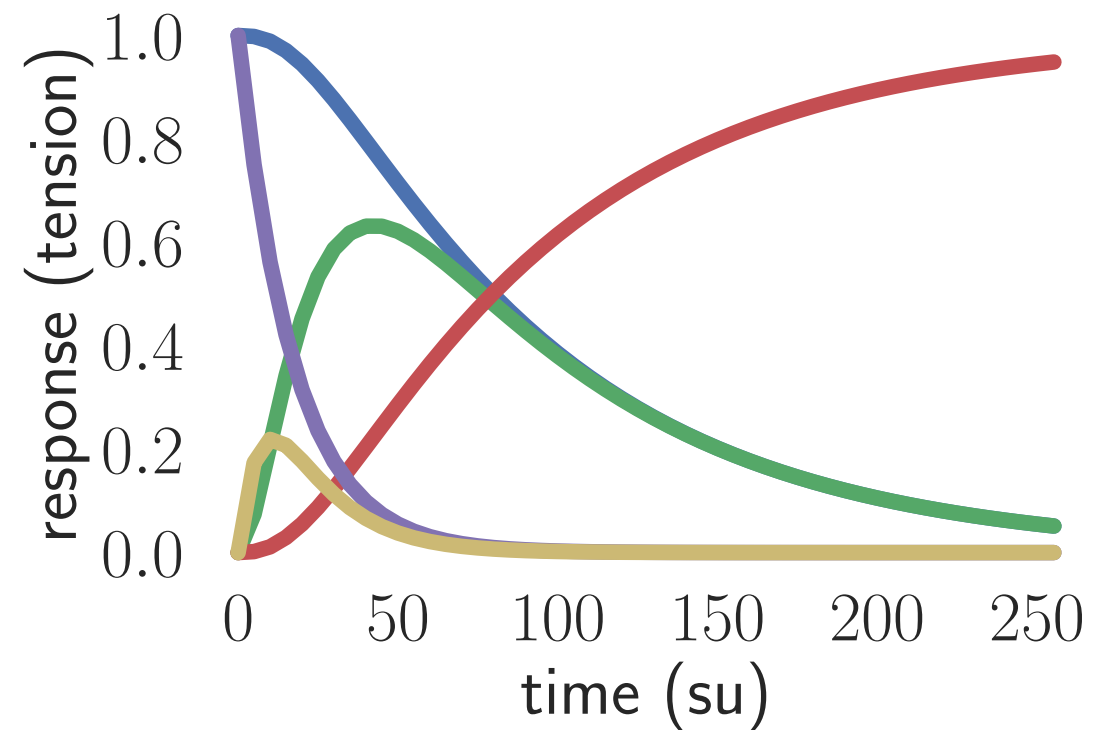
5x faster

# Botulinum Neurotoxin A Response

Reference



Jaunt + Recover



# Correctness+Speedup Results

| benchmark | speedup      | No Jaunt | Jaunt |
|-----------|--------------|----------|-------|
| smol      | <b>0.50x</b> |          | ✓     |
| sconc     | <b>1.00x</b> |          | ✓     |
| mmrxnp    | 77.48x       |          | ✓     |
| repri     | 2.839x       |          | ✓     |
| bont      | <b>5.00x</b> | ✓*       | ✓     |
| epor      | 0.142x       |          | ✓     |
| gtoggle   | <b>0.1x</b>  |          | ✓     |

# Conclusion

- Powerful new computing substrate for biochemical models
- Jaunt transforms mapped simulations to:
  - respect physical limitations of hardware
  - accelerate simulation