STAT/MATH 495: Problem Set 03

Sara Culhane

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Contents

Question

For both `data1` and `data2` tibbles (a tibble is a data frame with some metadata attached):

- Find the splines model with the best out-of-sample predictive ability.
- Create a visualizaztion arguing why you chose this particular model.
- Create a visualizaztion of this model plotted over the given (x_i, y_i) points for $i = 1, \ldots, n = 3000$.
- Give your estimate $\widehat{\sigma}$ of σ where the noise component ϵ_i is distributed with mean 0 and standard deviation σ.

Cross Validation function for k=5

```r
set.seed(40) # Set random seed for reproducibility

# Initialize the RMSE vector
r <- rep(0,5)
cv5 <- function(data,df) { # data as full dataset and df as degrees of freedom for spline
  for (i in 1:5) {
    # These indices indicate the interval of the test set
    index<- (((i-1) * round((1/5)*nrow(data))) + 1):((i*round((1/5) * nrow(data))))


    train <- data[-index,] #Remove train


    test <- data[index,] # Keep test

    # A model is learned using each training set
    mod <- smooth.spline(x=train$x, y=train$y, df=df)
    output <- predict(mod, test$x) %>%
      as_data_frame()
    r[i]<- sqrt(mean((test$y-output$y )^2))
  }
return(sum(r)/length(r))
}
```

Calculation of MSE for any df

```
error <- c()
# This is a function create a vector of RMSE for each df

m1 <- function(data,n) {
  for (i in 1:n) {
   error[i] <- cv5(data,i)
  }
  return(error)
}
```

Finding optimal df for both Data1 and Data2

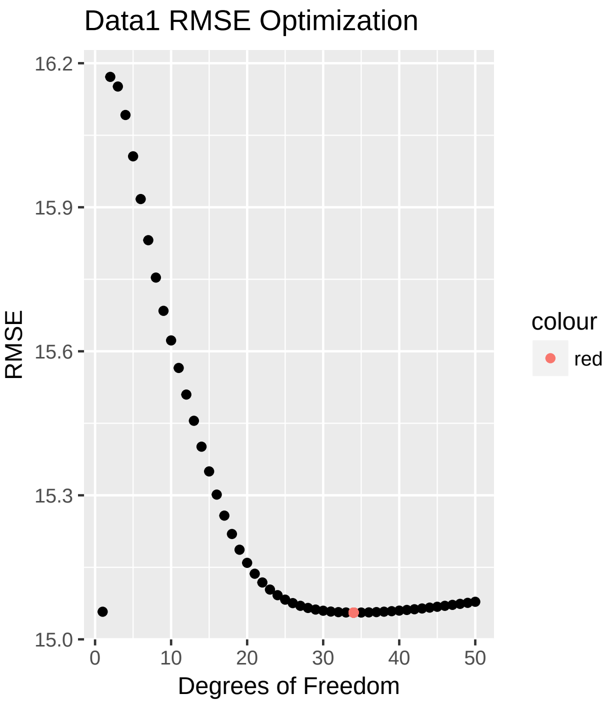
```
# Calculate RMSE for df 1 to 50

d1 <- m1(data1,50)
d2 <- m1(data2,50)

dp1 <- data.frame(x =1:50, y=d1)
dp2 <- data.frame(x =1:50, y=d2)

# Use which.min to find the df with lowest RMSE

low1 <- which.min(m1(data1,50))
low2 <- which.min(m1(data2,50))
```

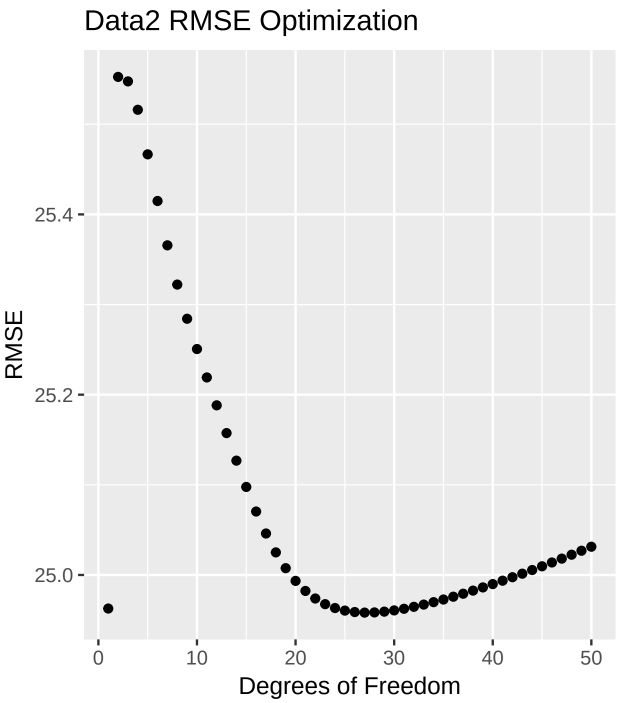
Plots of RMSE for all df examined

```
p1 <- ggplot(dp1, aes(x=x)) +
  geom_point(aes(y=y)) + xlab("Degrees of Freedom") + ylab("RMSE") +
  geom_point(aes(x=low1,y=15.05571, color="red")) + labs(title="Data1 RMSE Optimization")
p2 <-  ggplot(dp2, aes(x=x)) +
  geom_point(aes(y=y))+ xlab("Degrees of Freedom") + ylab("RMSE") + labs(title="Data2 RMSE Optimization
  geom_point(aes(x=low2,y=24.95825, color="red"))
```

```
## mapping: x = low2, y = 24.95825, colour = red
## geom_point: na.rm = FALSE
## stat_identity: na.rm = FALSE
## position_identity
```

```
grid.arrange(p1, p2,ncol=2)
```





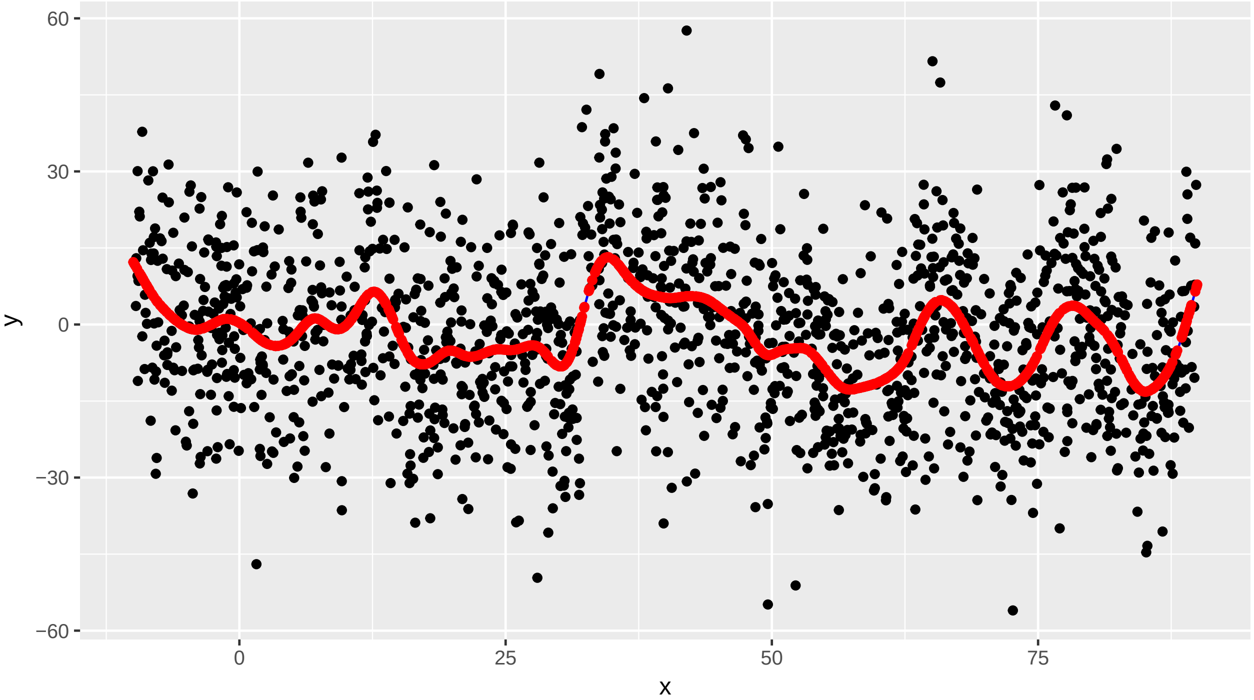
Based on analyis of degrees of freedom from 1 to 50 using our CV 5-fold calculations, we will use 34 and 27 df for data1 and data2 respectively for our df in our spline fit.

Application to full dataset

With this plot, we can make estimates for $f(x)$, which is shown by our fitted spline.

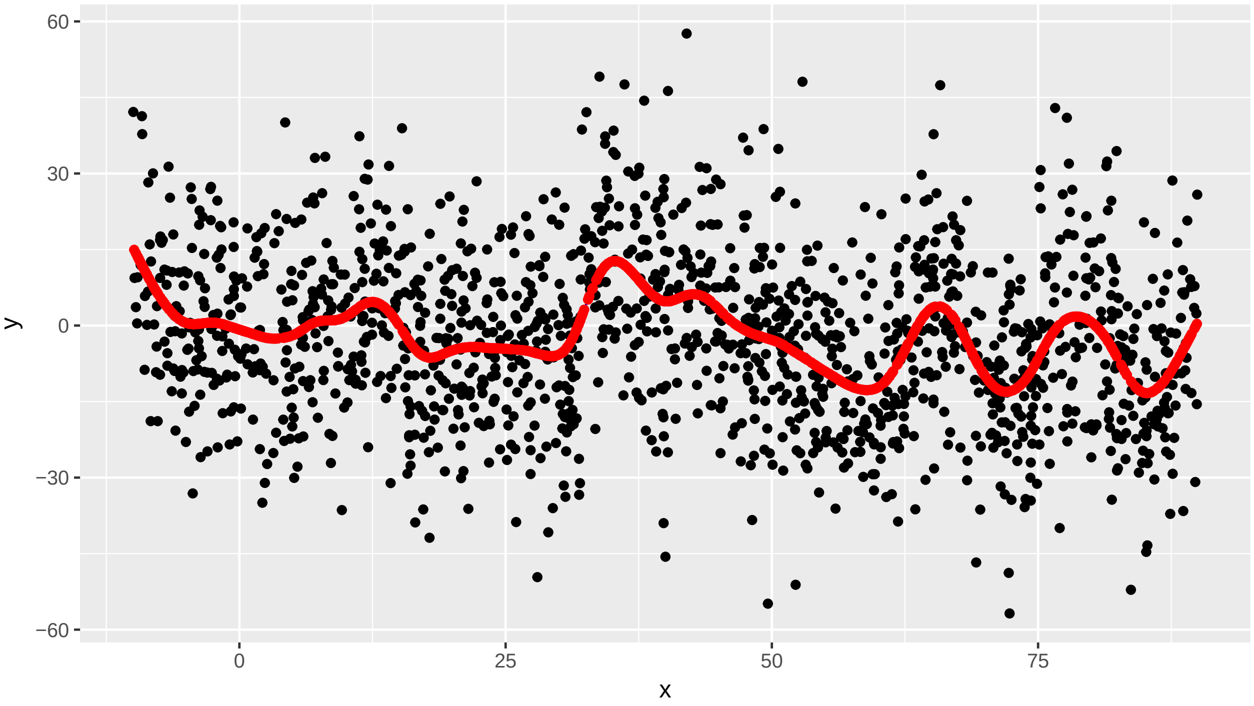
Data1 based on df=34

```
train1 <- data1 %>% sample_n(nrow(data1)/2)
test1 <- anti_join(data1,train1,by="ID")
mod1 <- smooth.spline(x=train1$x, y=train1$y, df=low1)
output <- predict(mod1, test1$x) %>%
  as_data_frame()

mod_t1 <- mod1 %>%
  broom::augment()
p3 <- ggplot(mod_t1, aes(x=x)) +
  geom_point(aes(y=y)) +
  geom_line(aes(y=.fitted), col="blue")
p3 +
  geom_point(data=output, aes(x=x, y=y), col="red")
```



Data2 based on df=27

```
train2 <- data1 %>% sample_n(nrow(data2)/2)
test2 <- anti_join(data2,train2,by="ID")
mod2 <- smooth.spline(x=train2$x, y=train2$y, df=low2)
output <- predict(mod2, test2$x) %>%
  as_data_frame()

mod_t2 <- mod2 %>%
  broom::augment()
p4 <- ggplot(mod_t2, aes(x=x)) +
  geom_point(aes(y=y)) +
  geom_line(aes(y=.fitted), col="blue")
p4 +
  geom_point(data=output, aes(x=x, y=y), col="red")
```



Function for all CV folds

```
set.seed(40) # Set random seed for reproducibility

# Initialize the RMSE vector

cv <- function(data,df,fold){ # data as full dataset,df as degrees of freedom for spline, fold are numb
  r <- rep(0,fold)
  for (i in 1:fold) {
    # These indices indicate the interval of the test set
    index<- (((i-1) * round((1/fold)*nrow(data))) + 1):((i*round((1/fold) * nrow(data))))


    train <- data[-index,] #Remove train


    test <- data[index,] # Keep test

    # A model is learned using each training set
    mod <- smooth.spline(x=train$x, y=train$y, df=df)
    output <- predict(mod, test$x) %>%
      as_data_frame()
    r[i]<- sqrt(mean((test$y-output$y )^2))

  }
return(sum(r)/length(r))
}

error <- c()
# This is a function create a vector of RMSE for each df
```

```r
mx <- function(data,n,folds) {
  for (i in 1:n) {
   error[i] <- cv(data,i,folds)
  }
  return(error)
}
```