GUC German University in Cairo

CSEN 703 - Analysis and Design of Algorithms

Midterm Revision Lecture

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Outline

Bucket Sort

Consider the following bucket sort algorithm.

```
1  BucketSort(A, k)
2  buckets = new array of k empty lists ;
3  max = maximum value in array A ;
4  for i = 0 to length(A) − 1 do
5  |   Insert A[i] in buckets[⌊ A[i]/max ∗ (k − 1)⌋]
6  end
7  for i = 0 to k − 1 do
8  |   Sort buckets[i]
9  end
10 return the concatenation of buckets[1], . . . , buckets[k]
```

Bucket Sort

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Exercise 1

1. Trace the operation of BucketSort on $A = [7, 2, 6, 1, 4, 3]$ and $k = 5$.
2. Prove that BucketSort is correct.
3. What is the best and worst case complexity of BucketSort?

Outline

Asymptotic Notations

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Exercise 2

Prove or disprove the following for any positive $f(n)$ and $g(n)$.

1. If $f(n) = O(g(n))$, then $g(n) = O(f(n))$.

Asymptotic Notations

Exercise 2

Prove or disprove the following for any positive $f(n)$ and $g(n)$.

1. If $f(n) = O(g(n))$, then $g(n) = O(f(n))$.
2. $f(n) + g(n) = \Theta(min(f(n), g(n)))$.

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3. $f(n) = O(f(n)^2)$.

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4. If $f(n) = O(g(n))$ and $g(n) = \Omega(f(n))$, then $f(n) = g(n)$.

Asymptotic Notations

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3. $f(n) = O(f(n)^2)$.
4. If $f(n) = O(g(n))$ and $g(n) = \Omega(f(n))$, then $f(n) = g(n)$.
5. $n^a = \omega(n^b)$ for $a \geq b$.

Asymptotic Notations

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Exercise 3

Prove that $\Sigma_{i=1}^{n} i^p = \Theta(n^{p+1})$ for $p \geq 1$.

Outline

Recursion Tree Method

Exercise 4

Use the recursion tree method to get an upper bound for the following recurrence.

$$T(n) = T(\frac{n}{3}) + T(\frac{2n}{3}) + \Theta(n)$$

Master Method

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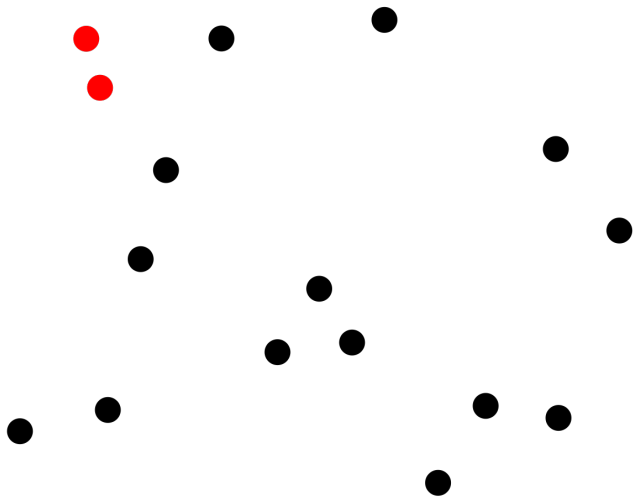
Exercise 5

Consider the following recurrence.

$$\begin{aligned} T(n) &= T(n/2) + 5^{\lfloor \log_5 n \rfloor} \\ T(1) &= \Theta(1) \end{aligned}$$

Can you solve it using the master method? If yes, solve it. If not, explain why.

Divide and Conquer Algorithms Design



Exercise 6

You are given an array of n points in a 2D plane. Design a divide and conquer algorithm to find out the closest pair of points in the array.

Outline

Greedy Algorithms

Exercise 8

Given two arrays representing the arrival and departure times of all trains that reach a railway station, the task is to find the minimum number of platforms required for the railway station so that no train waits. For example:
if A= [9:00, 9:40, 9:50, 11:00, 15:00, 18:00]
and D=[9:10, 12:00, 11:20, 11:30, 19:00, 20:00],
we need a minimum of 3 platforms.

Greedy Algorithms

Exercise 9

Given a value of money V, we want to make a change for V and we have an infinite supply of each of the coin denominations in Euros. That is, we have an infinite supply of $\{1, 2, 5, 10, 20, 50, 100, 200\}$ valued coins. What is the minimum number of coins needed to make the change? For example, if $V = 22$, the minimum number of coins needed is 2 $(20 + 2)$.