

# Classical Mechanics: Princeton Prelims Edition

## Problem Catalog

### (IN PROGRESS)

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## 1 What Is This?

I am writing this document to solidify my understanding of Classical Mechanics in preparation for the Princeton Prelims for physics Ph.D. study. Many of the conceptual explanations here are paraphrased from my textbooks and Professor Steven Gubser's Fall 2018 *Classical Mechanics: Principles and Problem Solving* class lectures (PHY 503) [1] [2] [3] [4]. When solving some of the problems I referred to the Princeton Prelims Wiki to compare my answers [5]. I also referred to Professor Michael Romalis's teaching website for PHY 203 for extra problems and solutions [6]. I also referred to explanations and solutions given by PHY 503 Teaching Assistant Wentao Fan. I do not guarantee that my solutions are correct, nor do I guarantee that the Wiki solutions are correct. If you are reading this document and have a question or a correction, feel free to email me at [sarafs@princeton.edu](mailto:sarafs@princeton.edu).

## 2 A Few Notes on the Basics

### 2.1 General Advice

**If you get stuck with math:**

- Use a single symbol to represent a more complicated constant to simplify the algebra.
- Taylor expand your expression and approximate to the proper order.

**If your result looks weird:**

- Check units.