February 2, 2024

Audio Filter Design

ELEC-E5620 - Audio Signal Processing, Lecture #3

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Engineering

Sound check

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Course Schedule for 2024 (Periods III-IV)

0.	General issues (Vesa, Gloria & Eloi)	12.1.2024
1.	History and future of audio DSP (Vesa)	19.1.2024
2.	Digital filters in audio (Vesa)	26.1.2024
3.	**Audio filter design (Vesa)**	**2.2.2024**
4.	Analysis of audio signals (Eloi)	9.2.2024
5.	Audio effects processing (Vesa)	16.2.2024
	** No lecture (Evaluation week for Period III)*	**23.2.2024*
6.	Sound synthesis (Dr. Fabian Esqueda, Korg Germany, Berlin)	1.3.2024
7.	Artificial reverberation (Dr. Karolina Prawda, Aalto Acoustics Lab)	8.3.2024
8.	Physics-based sound synthesis (Dr. Max Schäfer, Univ. Erlangen)	15.3.2024
9.	Sampling rate conversion (Vesa)	22.3.2024
	** No lecture (Spring Break/Easter)*	**29.3.2024*
10.	Audio coding (Vesa)	5.4.2024
11.	BONUS: Machine learning in audio (Eloi & Gloria)	12.4.2024

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Demo: Saturating Resonant Filter

Ivan Benc

Xiaojie Pi

Ruijie Wang

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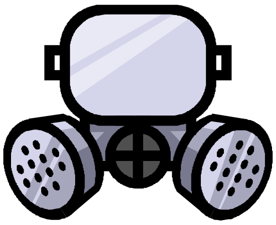
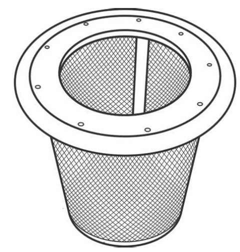
Outline

- Introduction and motivation
- Properties of digital filters
- FIR or IIR?
- Error norms for filter design
- Auditory resolution
- FIR filter design
- IIR filter design
- Graphic equalizers





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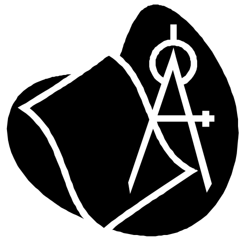
Introduction & Motivation



- Basic audio filters
  - Usually, straightforward low-order design in closed form (see lecture #2 on Digital Filters)
- More advanced filters needed in many real-life applications
  - Equalization of loudspeakers/headphones
  - Room correction (loudspeaker-room EQ)
  - Graphic equalization (for music production, concerts etc.)
  - HRTF (head-related transfer function) filters for 3-D sound
  - Digital musical instruments, e.g., piano soundboard, guitar or violin body
  - Reverberation algorithms (to adjust the T60 curve)
  - Modeling of signal frames, e.g. in audio coding

Example, HRTF Modeling

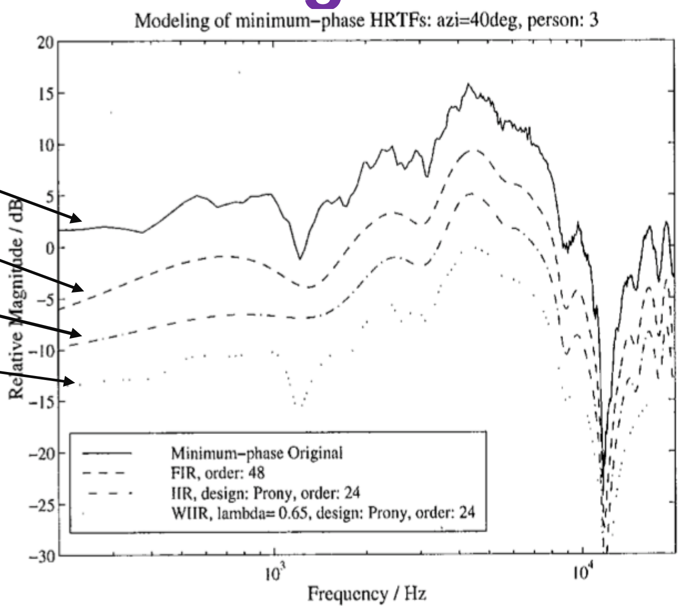
- A simple filter is insufficient

Target response

FIR, order 48

IIR, order 24

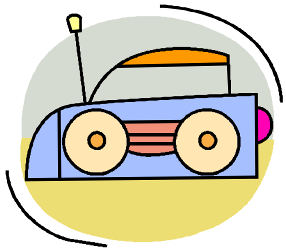
Warped IIR, order 24



• Source: Huopaniemi, J., Zacharov, N., and Karjalainen, M., Objective and Subjective Evaluation of Head-Related-Transfer Function Filter Design, 105th AES Convention

Introduction & Motivation 2

- Which application, what purpose?
  - Requirements help in choosing the design method
- General principle
  - **Quick** methods for real-time applications (aim at designing a low-order filter)
  - **Accurate** methods for off-line processing (high-order filters, when necessary)



- Things to remember in audio filter design
  - Frequency resolution of the hearing system is nonlinear (logarithmic): good at low, poor at high frequencies
  - Conventional ”textbook” filter design techniques (lowpass, highpass, bandpass…) are usually unsuitable

Properties of Filters

- Convolution for linear and time-invariant (LTI) systems:

$$y[n] = h[n] * x[n]$$

Impulse response of the filter

Input signal

- The same in Z-domain

$$Y(z) = H(z)X(z) \Leftrightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{B(z)}{A(z)}$$

FIR part

$$\sum_{k=0}^{M} b_k z^{-k}$$

All-pole part

$$\sum_{k=0}^{N} a_k z^{-k}$$

Properties of Filters: Magnitude

- Frequency response
  - Replace z with $e^{j\omega}$

$$H(e^{j\omega}) = \frac{B(e^{j\omega})}{A(e^{j\omega})}$$

- Magnitude response
  - Take the absolute value

$$|H(e^{j\omega})|$$

- In Matlab
  - `freqz.m`

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Properties of Filters: Phase

- Phase response
  - Radian phase shift

$$\theta(\omega) = \arg\{H(e^{j\omega})\}$$

- Phase delay
  - Time delay of each sinusoidal component (in samples)

$$P(\omega) = -\frac{\theta(\omega)}{\omega}$$

- Group delay
  - Time delay (in samples) of the amplitude envelope of a sinusoid at frequency ω

$$D(\omega) = -\frac{d}{d\omega}\theta(\omega)$$

- In Matlab
  - `phasez.m`
  - `phasedelay.m`
  - `grpdelay.m`

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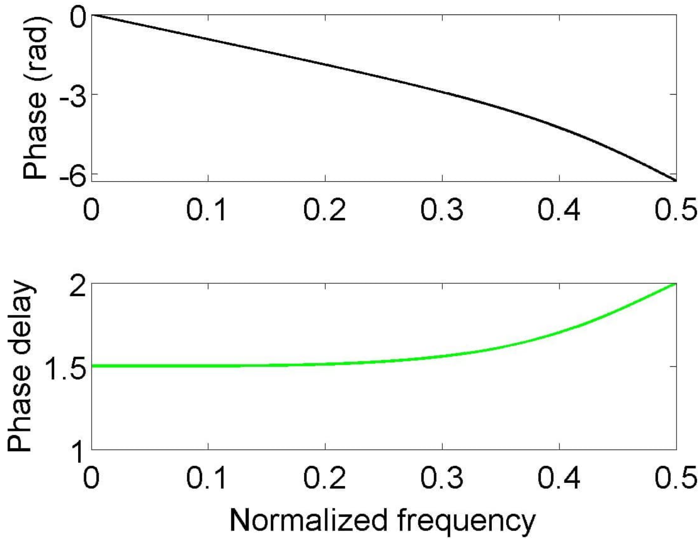
Phase Delay of a Digital Filter

- Phase response and phase delay
  - o 2nd-order allpass filter

$$P(\omega) = -\frac{\theta(\omega)}{\omega}$$



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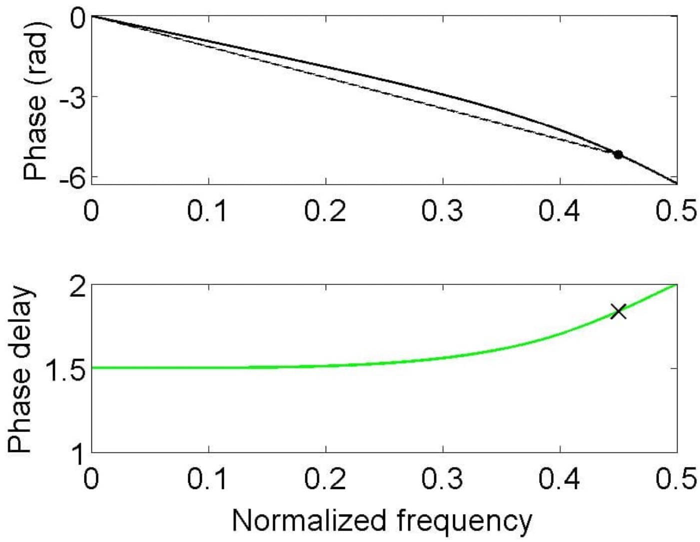
Phase Delay of a Digital Filter (2)

- Phase delay is negative of the slope of a straight line fitted through 0 and $\theta(\omega)$:

$$P(\omega) = -\frac{\theta(\omega)}{\omega}$$

$$= \frac{0-\theta(\omega)}{0-\omega}$$



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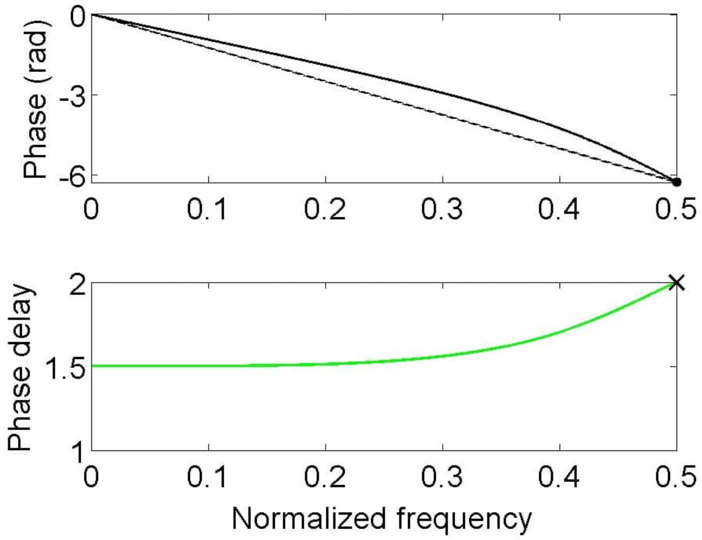
Phase Delay of a Digital Filter (3)

- Phase delay is negative of the slope of a straight line fitted through 0 and $\theta(\omega)$:

$$P(\omega) = -\frac{\theta(\omega)}{\omega}$$
$$= \frac{0 - \theta(\omega)}{0 - \omega}$$



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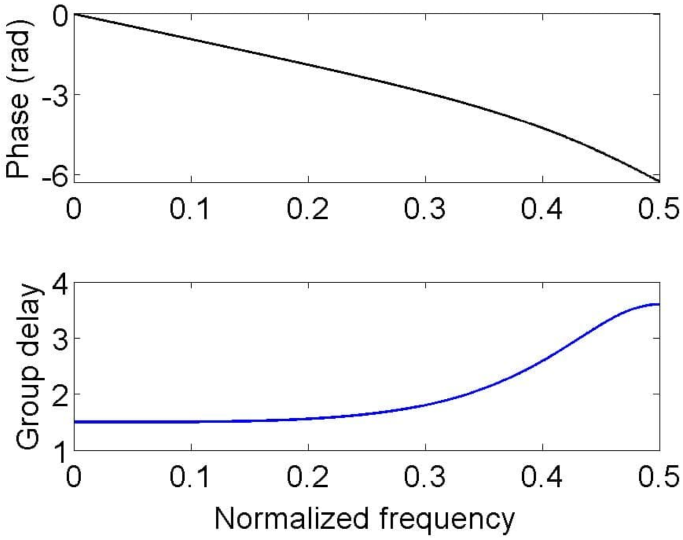
Group delay of a Digital Filter

- Phase response and group delay
  - o The same 2nd-order allpass filter as before

$$D(\omega) = -\frac{d}{d\omega}\theta(\omega)$$



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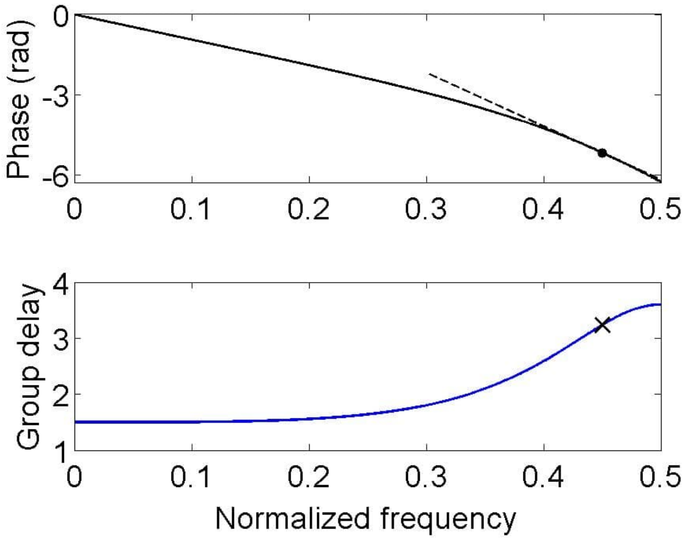
Group delay of a Digital Filter (2)

- Group delay is -1 times the local gradient of the phase function $\theta(\omega)$

$$D(\omega) = -\frac{d}{d\omega}\theta(\omega)$$



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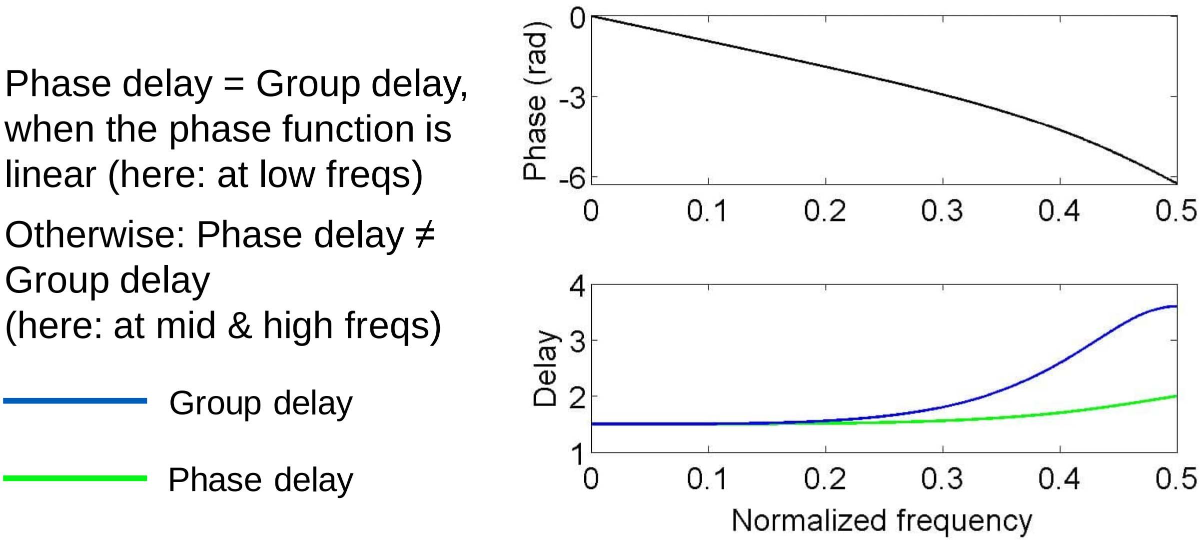
Phase Delay vs. Group delay

- Phase delay = Group delay, when the phase function is linear (here: at low freqs)
- Otherwise: Phase delay ≠ Group delay (here: at mid & high freqs)



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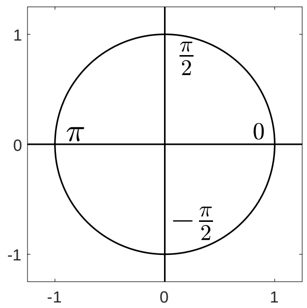
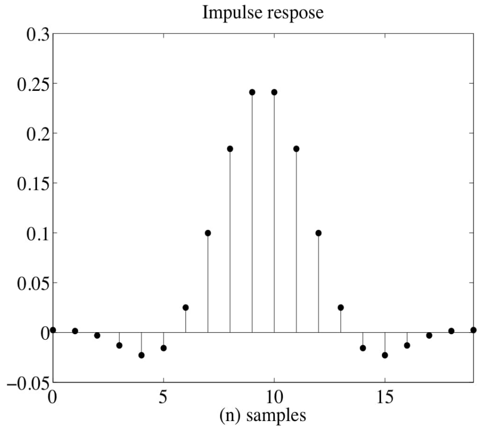
How to Present the Digital Frequency Axis?

- Radians
  - Sampling frequency = 2π → Axis $[0, \pi]$
- Normalized
  - Sampling frequency = 1 → Axis $[0, 0.5]$
  - Easy to convert to Hz by multiplying by the sampling frequency
- Matlab (default, but not a great choice)
  - Sampling frequency = 2 → Axis $[0, 1]$
- Hz
  - E.g., if the sampling frequency is 44.1 kHz → Axis $[0, 22.05]$ kHz



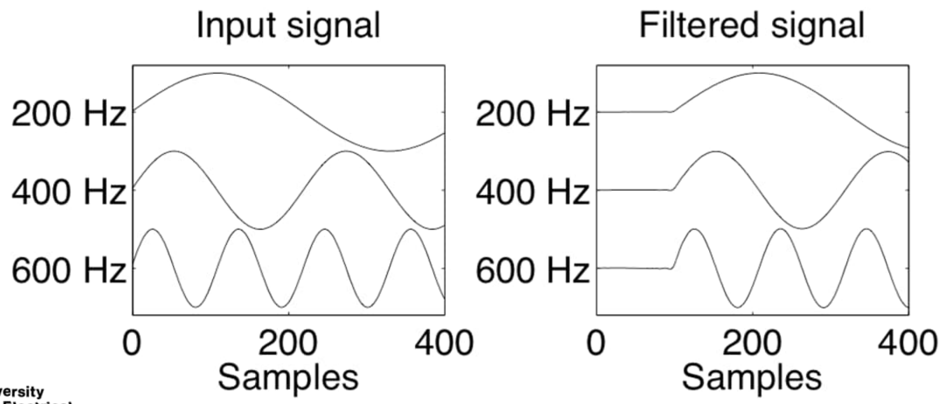
Phase Characteristics (1)

- Linear phase
  - Easy to obtain using FIR filters
  - Symmetric impulse response
  - Phase delay = group delay
  - Sometimes desirable, but why?
- Zero phase
  - Special case of a linear-phase filter
  - Impulse response symmetric w.r.t. 0 → noncausal!
  - Possible offline: `filtfilt.m`



Linear Phase in Practice

- Linear phase → constant phase delay at all frequencies
  - In the example here, the filter causes 100 samples of delay



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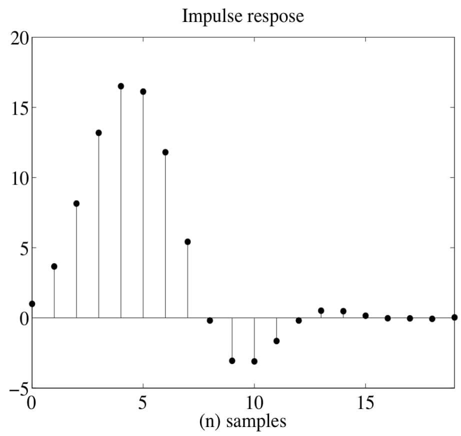
Phase Characteristics (2)



- Minimum phase
  - No zeros outside the unit circle
  - Often desirable, why?
  - Releases energy as fast as possible for the given magnitude response
  - Always stable and causal
  - Also its inverse filter is stable
- Maximum phase
  - All zeros outside the unit circle (or on the unit circle)
  - Time-reversed version of the minimum-phase system

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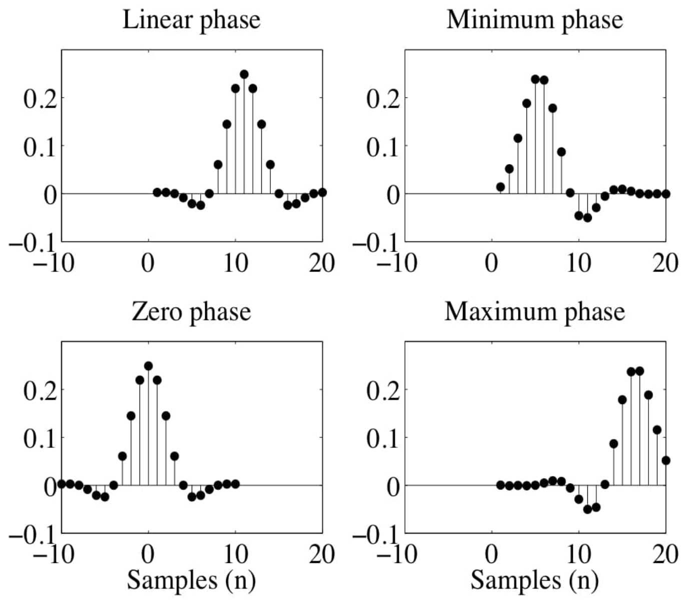
Impulse Response with Different Phase

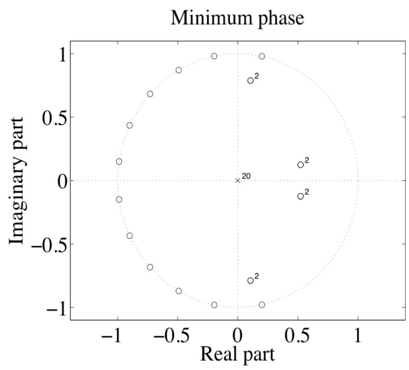


How to Construct a Minimum Phase?

- Option #1: Mirroring the Zeros
  - Zeros outside the unit circle are relocated to inside the unit circle

$$z = re^{j\theta}, r > 1 \ \rightarrow z = \frac{e^{j\theta}}{r}$$

  - Not feasible when the filter order is large, because then it's difficult or impossible to find the zeros (*i.e.*, roots of a polynomial)



How to Construct a Minimum Phase?

- Option #2: The Hilbert Transform Method
  - Logarithm of the magnitude response and the phase function of a minimum-phase system are Hilbert transform pairs
  - To convert a filter to minimum-phase: Replace the filter's phase response with the Hilbert transform of its log magnitude response!
  - The Hilbert transform shifts the phase of the input signal by 90 degrees (π radians)
  - See: https://ccrma.stanford.edu/~jos/sasp/Minimum_Phase_Filter_Design.html

How to Construct a Minimum Phase?

- Option #3: Cepstral Windowing
  - Cepstrum is the inverse Fourier transform of the log of the spectrum
    - F^{-1} is the inverse Discrete Fourier Transform (DFT)
  - Deleting the left side of the cepstrum modifies the phase so that the signal becomes minimum-phase
    - The window $w(n)$ is a unit-step function (selecting the right half)
  - The minimum-phase impulse response is obtained with DFT.
  - Download the codes (`minphase_rceps.m`, `minphase_hilbert.m`) from MyCources and try!
  - See also: https://ccrma.stanford.edu/~jos/fp/Creating_Minimum_Phase_Filters.html

$$c(n) = F^{-1}\{\log|H(z)|\}$$

$$\hat{c}(n) = c(n)w(n)$$

$$h_{\mathrm{mp}}(n) = F^{-1}\{e^{F\{\hat{c}(n)\}}\}$$

Linear-Phase/Minimum-Phase FIR Filter

- Loudspeaker model
  - o Low corner frequency (highpass): ≈500 Hz order 256
  - o High corner frequency (lowpass): ≈20 kHz order 32
- For plotting: `freqz.m, impz.m, zplane.m`
  - o Pay attention to the frequency normalization!
- Download the code (`LS_FIR_demo.m`) from MyCources!

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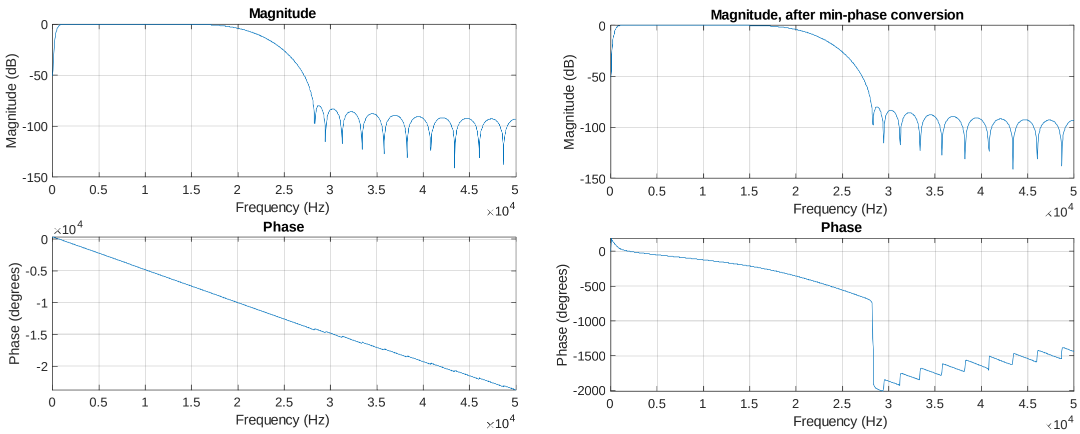
Linear-Phase/Minimum-Phase FIR Filter

Linear phase Minimum phase



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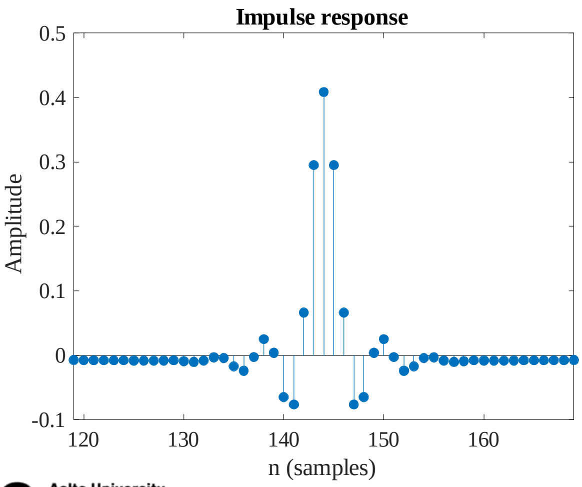
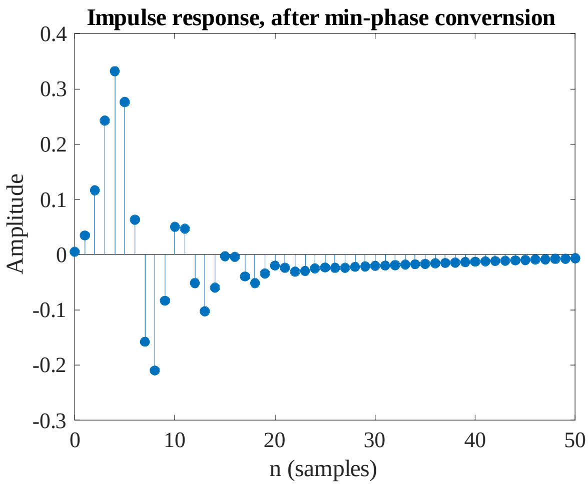
Linear-Phase/Minimum-Phase FIR Filter

Linear phase



Minimum phase



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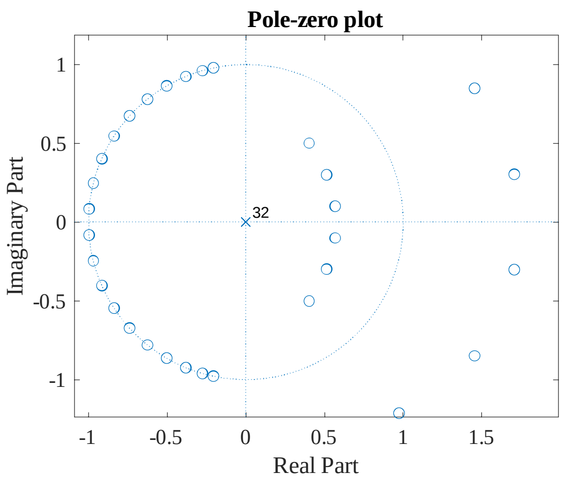
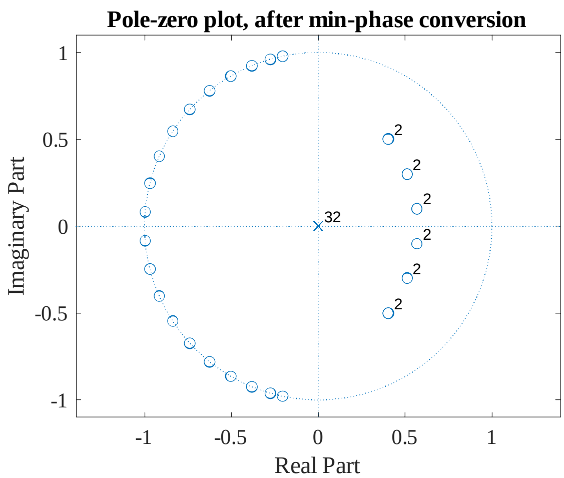
Linear-Phase/Minimum-Phase FIR Filter

Linear phase



Minimum phase



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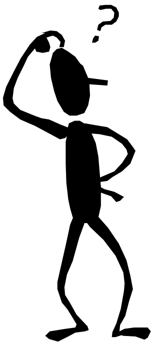
Filter Design Problem

- How to approximate the target response with a digital filter
- Minimize the frequency-response error:

$$E(e^{j\theta}) = H(e^{j\theta}) - \hat{H}(e^{j\theta})$$

Error = target – approximation

- In digital filter design, L_p norms are often used
  - o L_2 leads to the least-squares design
  - o L_∞ leads to the Chebyshev (minimax) design



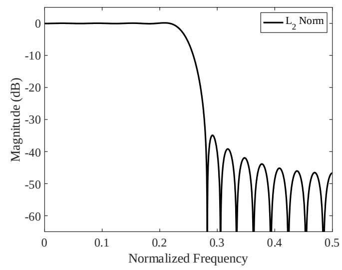
Least-squares (L_2) Norm

$$E_2(e^{j\omega}) = \left\| H(e^{j\omega}) - \hat{H}(e^{j\omega}) \right\|_2^2 \mathrel{\widehat{=}} \sum_{n=0}^{\infty} \left| h(n) - \hat{h}(n) \right|^2$$

- Many popular filter design techniques, such as Prony's method, use this method (`prony.m` in Matlab)
- An easy LS FIR filter design method is to truncate the impulse response of the ideal filter
  - o By including the largest samples, you minimize the squared error!

Solving FIR Filter Coefficients in LS Design

- If $\boldsymbol{h}$ is the desired response vector, $\boldsymbol{\varepsilon}$ is the frequency response error vector, and $\mathbf{F}$ is the discrete Fourier transform (DFT) matrix
- Minimize the squared error to design coefficients $\hat{h}$:



$$\mathbf{F}\hat{\boldsymbol{h}} = \boldsymbol{h} + \boldsymbol{\varepsilon}$$

$$\min\|\boldsymbol{\varepsilon}\|^2 = \min\left\|\mathbf{F}\hat{\boldsymbol{h}} - \boldsymbol{h}\right\|^2$$

$$\frac{\partial}{\partial h}\left(\mathbf{F}\hat{\boldsymbol{h}} - \boldsymbol{h}\right)\left(\mathbf{F}\hat{\boldsymbol{h}} - \boldsymbol{h}\right)^T = 0$$

$$\frac{\partial}{\partial h}\left(\hat{\boldsymbol{h}}^T\mathbf{F}^T\mathbf{F}\hat{\boldsymbol{h}} - 2\boldsymbol{h}^T\mathbf{F}\hat{\boldsymbol{h}} + \boldsymbol{h}^T\boldsymbol{h}\right) = \mathbf{F}^T\mathbf{F}\hat{\boldsymbol{h}} - \mathbf{F}^T\boldsymbol{h} = 0$$

$$\Leftrightarrow \hat{\boldsymbol{h}} = (\mathbf{F}^T\mathbf{F})^{-1}\mathbf{F}^T\boldsymbol{h}^T$$ “pseudoinverse” Matlab: hhat = F\h

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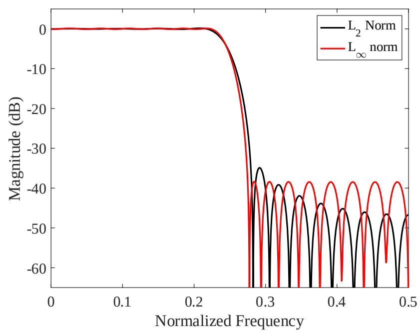
Chebyshev (L_∞) Norm

- Chebyshev (minimax) norm:
  - o Minimize the maximum component of the error norm
  - o Requires an iteration loop and an end rule (for selected accuracy)
  - o The error function will be “equiripple”



$$E_\infty(e^{j\omega}) = \left\|H(e^{j\omega}) - \hat{H}(e^{j\omega})\right\|_\infty \triangleq \max_{-\pi<\omega<\pi}\left|H(e^{j\omega}) - \hat{H}(e^{j\omega})\right|$$

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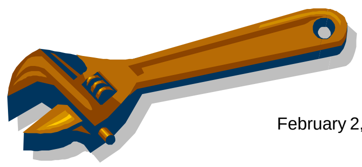
Auditory Resolution

- Account for the properties of the human hearing
  - Linear or non-uniform frequency scale, *e.g.*, critical bands (in Bark)
  - The domain in which the error is matched (*e.g.*, loudness or T60 domain)
- Tools
  - Auditory smoothing: Average of the spectrum with a sliding window (the window size should increase with frequency, as in octaves or Barks
  - Weighting functions: Suppress less important frequencies in the error (e.g., weighting proportional to Bark bandwidths)
  - Frequency warping: Expand the important frequency range; shrink the less important range



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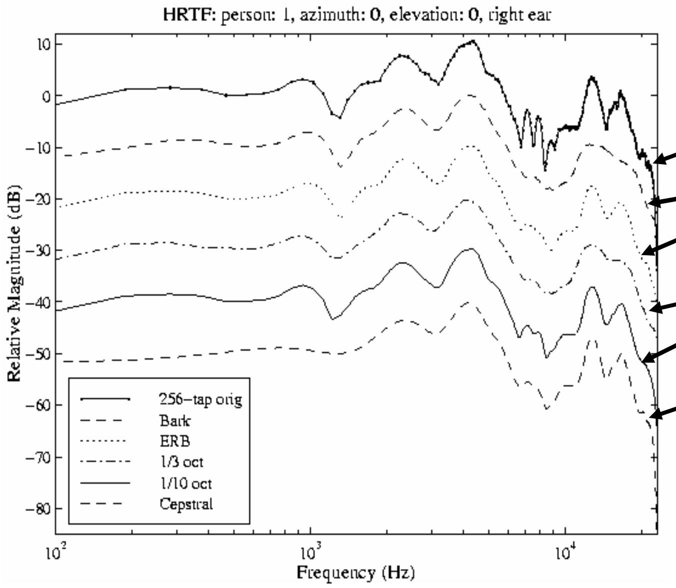
Auditory Smoothing



Example target response

- ERB & Bark smoothing
  (Equivalent Rectangular Bandwidth)
- 1/3 and 1/10-octave smoothing
- Cepstral smoothing

Source: Huopaniemi, J.
Virtual Acoustics and 3-D Sound
in Multimedia Signal Processing.
Doctoral Thesis, TKK, 1999.

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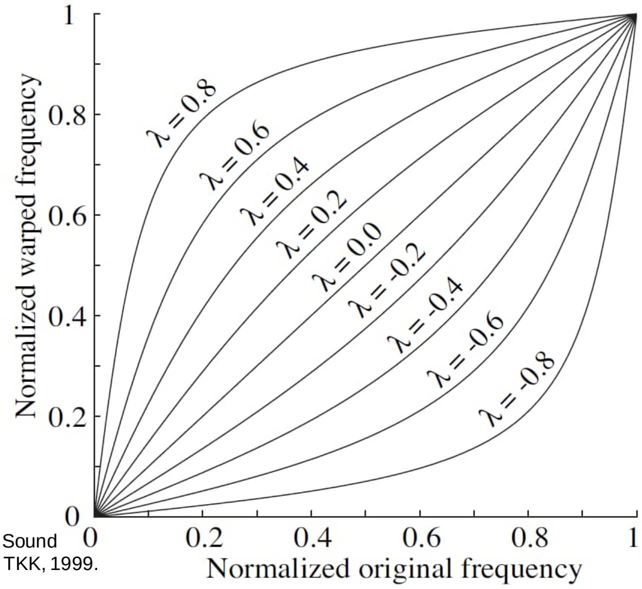
Frequency Warping

- Frequency scale warping before designing the filter
  - o Replace the unit delay with a first-order allpass filter
  - o λ is the warping parameter

$$z^{-1} \rightarrow \frac{z^{-1} - \lambda}{1 - \lambda z^{-1}}$$



Source: Huopaniemi, J. Virtual Acoustics and 3-D Sound in Multimedia Signal Processing. Doctoral Thesis, TKK, 1999.

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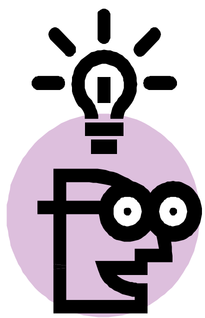
Properties of Warped Filters

- Allpass filter → flat magnitude response with frequency-dependent group delay
- If $\lambda = 0.7661$ (f_s = 48 kHz), the warping closely matches the Bark scale (Smith & Abel 1999)
- Design the filter in warped frequency domain → stretch the important frequencies



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How to Warp?

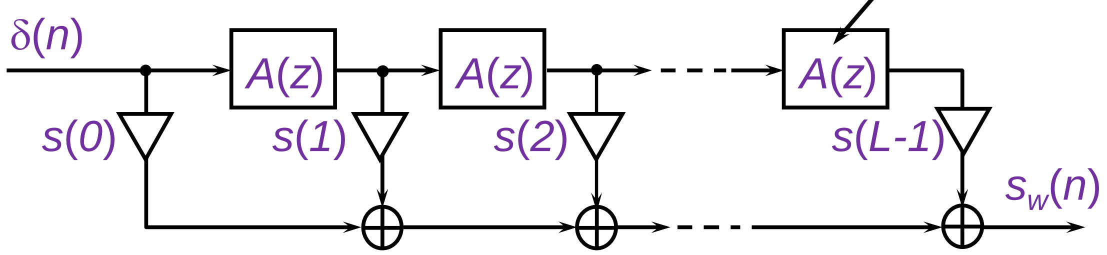
- Implement the filter in the warped frequency domain
  - WFIR or WIIR structures
  - More accuracy, but more computation as well
  - In WIIR structure the delay-free loops must be avoided → remap coefficients
- Unwarp the warped transfer function to a standard filter

Original $\lambda = 0.4$ $\lambda = 0.8$

Warped FIR Filter

- Chain of first-order allpass filters in an FIR structure

$$A(z) = \frac{z^{-1} + \lambda}{1 + \lambda z^{-1}}$$



- $s(n)$ is the signal to be warped, $\delta(n)$ is an impulse, $s_w(n)$ is the warped signal
- The extent of warping is determined by λ
  - With $\lambda = 0$, the warped FIR filter becomes a standard FIR filter

Windowing Methods for FIR Filters

- Truncate the impulse response using a window function
- Many choices for windowing functions!
- Suitable for modeling measured/estimated impulse responses in audio
- In Matlab: fir1.m

Frequency Sampling Method

- Sample the target frequency response uniformly (magnitude & phase)
- Reflect it to the negative frequencies: same magnitude, opposite phase
- Use Inverse DFT (IDFT) to obtain the filter coefficients
- The length of the IDFT specifies the filter length
- Check the frequency response with zero-padded DFT
- Note: there will be “don’t care” regions between the sample points
- In Matlab: fir2.m

Properties of Window Functions

- Rectangular window
  - o Minimizes the truncated time-domain L_2 norm
  - o Gibb's phenomenon (passband ripple)
  - o Low sidelobe rejection
  - o Steep transition band
- Other window functions
  - o Hann
  - o Hamming
  - o Blackman
  - o …

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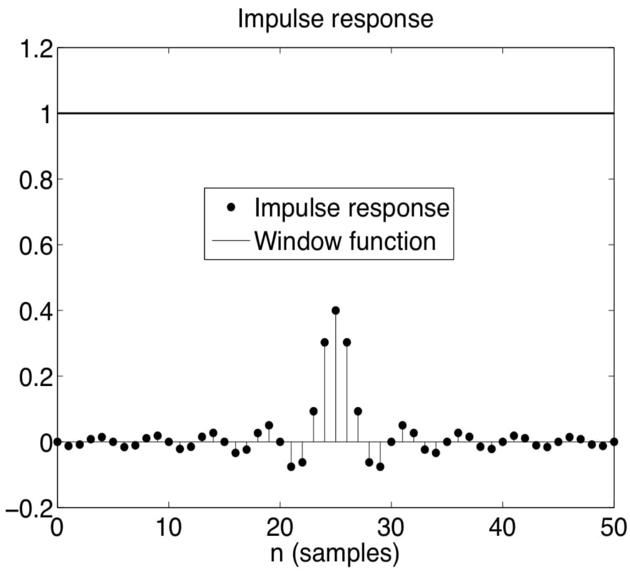
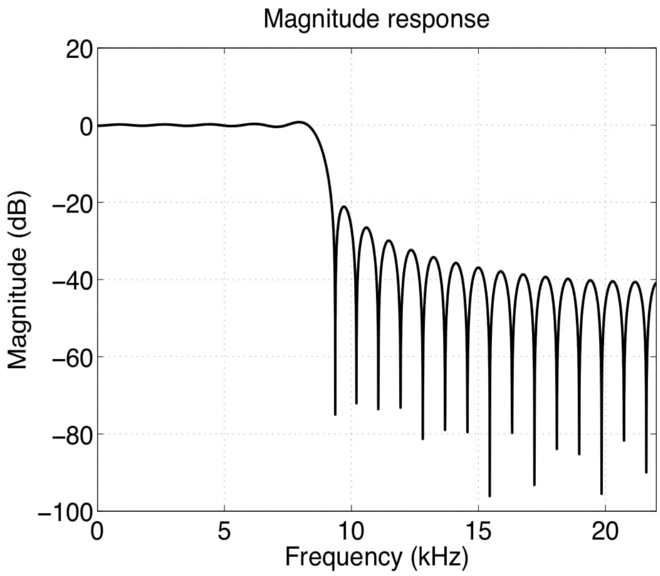
Rectangular window





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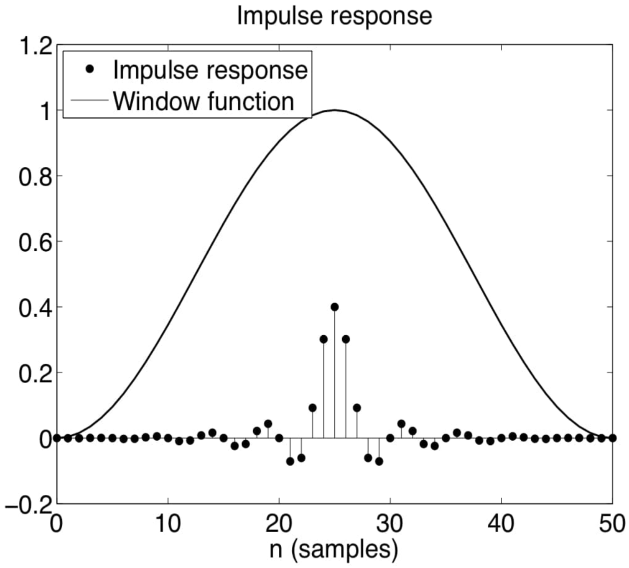
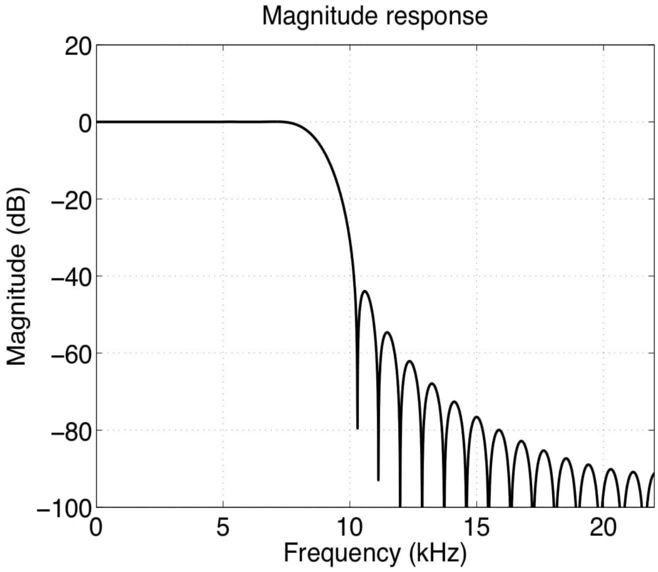
Hann window





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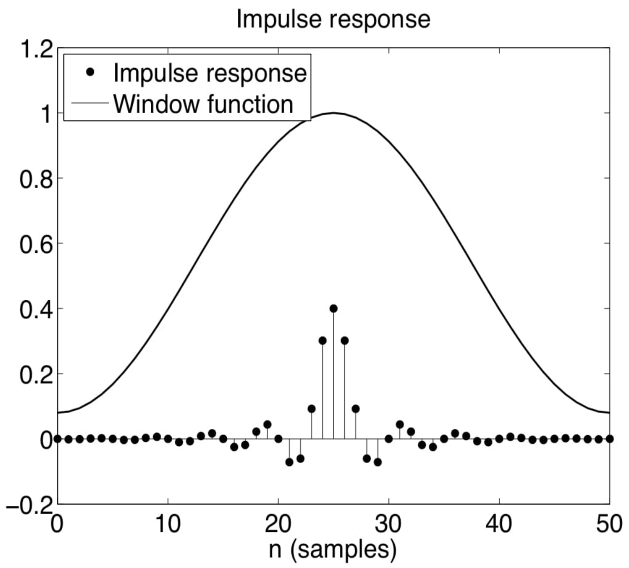
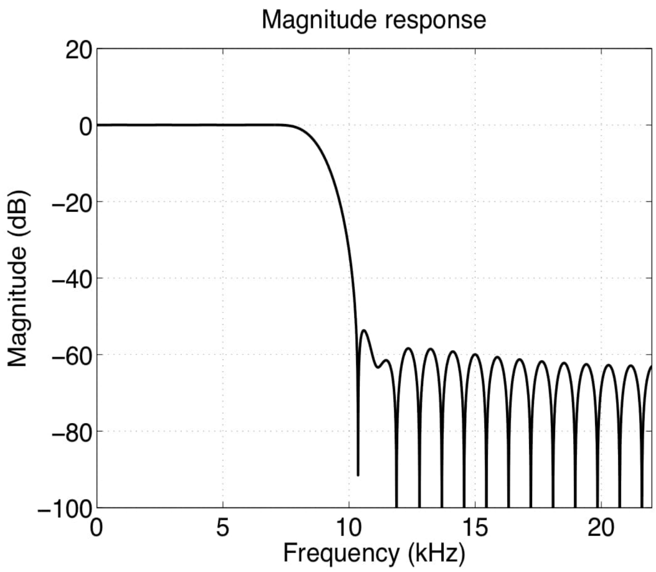
Hamming window





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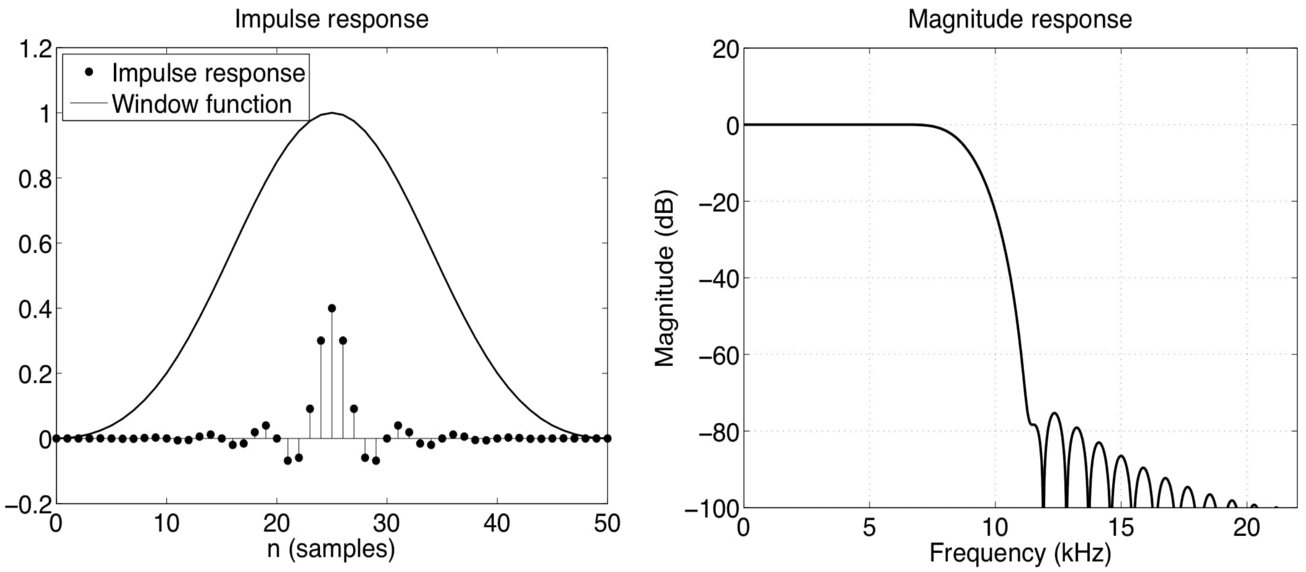
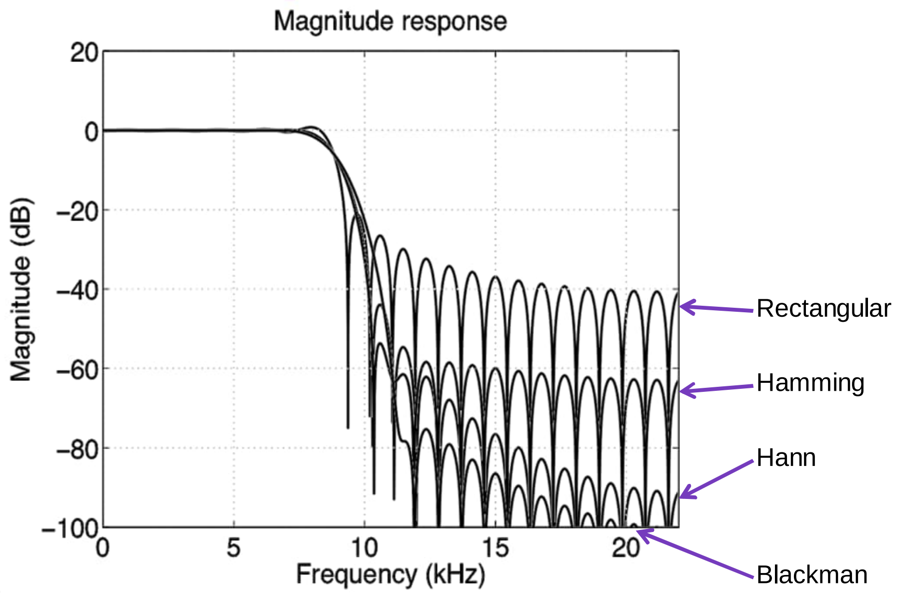
Blackman window



Window Comparison



Weighted LS FIR Filter Design

- Using a non-negative weight function W, minimize

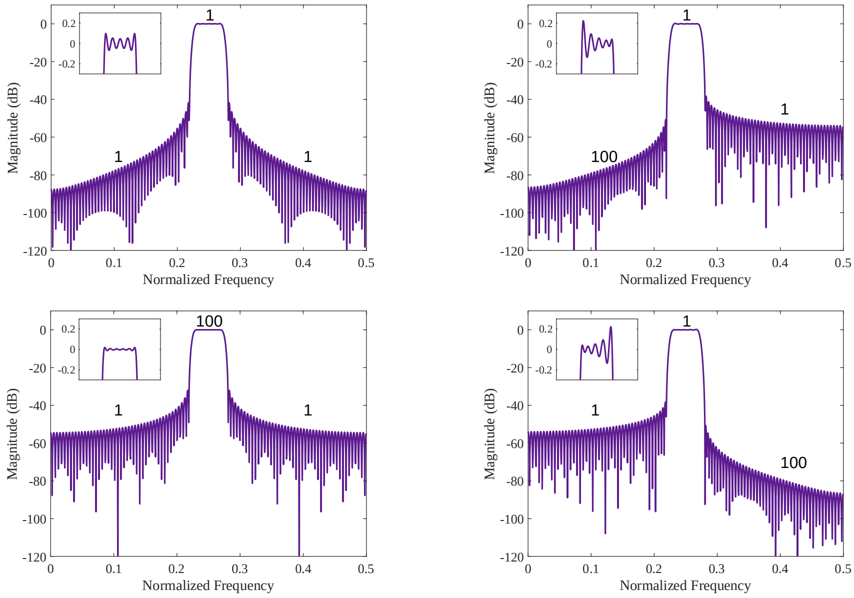
$$E = \sum_{k=0}^{L-1} W(\omega_k)\ \left|H(\omega_k) - \widehat{H}(\omega_k)\right|^2$$

- Again, minimize the squared error:

$$\widehat{\boldsymbol{h}} = (\mathbf{F}^T\mathbf{W}\mathbf{F})^{-1}\mathbf{F}^T\mathbf{W}\boldsymbol{h}^T$$

- Give more weight to important points
  - o Frequency ranges of interest

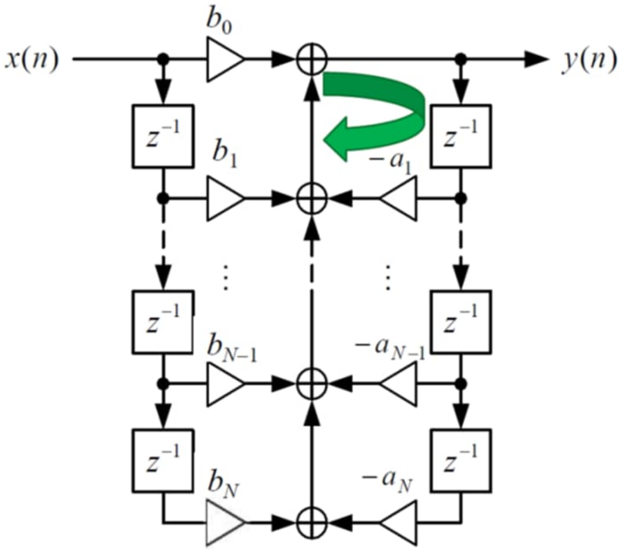
Weighted LS FIR Filter Design



`firls.m`
Least-squares linear-phase FIR filter design

IIR Filter Design Methods

- Linear prediction
- Prony's method
- Fixed-pole IIR filters
  - o Balazs Bank 2007, 2008
- Graphic equalization



Linear Prediction

- Also used in Linear Predictive Coding (LPC)
- Powerful analysis technique, especially in speech processing
- Predict the next sample as a linear combination of previous samples
- Many applications
  - o Spectral envelope estimation
  - o Pitch prediction
  - o Signal modeling for sound synthesis
  - o Detection of clicks in audio restoration

LPC Coefficients

- Compute the next sample as a linear combination of the previous samples

$$\hat{x}(n) = \sum_{k=1}^{P} a_k x(n-k)$$

- Minimize the expected value of the mean-squared prediction error

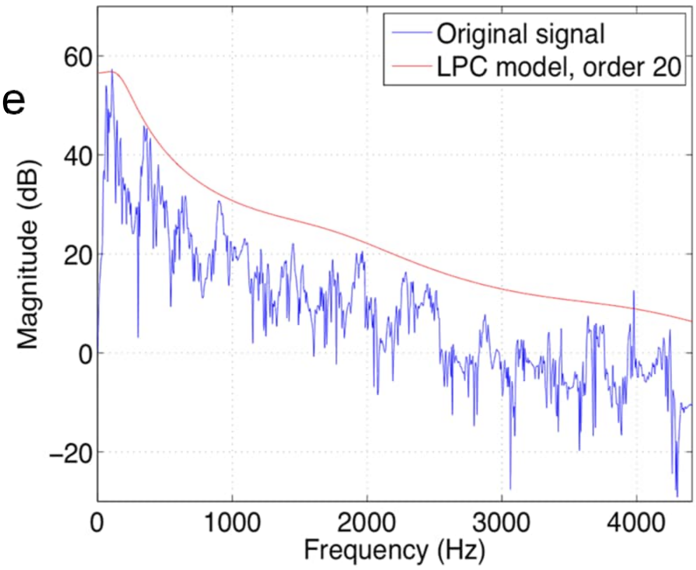
$$E\{e_n^2\} = E\{(x(n) - \hat{x}(n))^2\}$$

LPC Model

- Analysis
  - Use the autocorrelation or covariance method to obtain prediction coefficients
  - FIR filter with prediction coefficients is an inverse filter, which whitens the input signal (decorrelation)
  - Autocorrelation method → minimum phase
- Synthesis
  - Prediction coefficients used in an all-pole IIR filter yield a filter with the spectral characteristics of the input signal
- In Matlab: `lpc.m`

Piano Soundboard Modeling with LPC, $N = 20$

- (Recorded) target response



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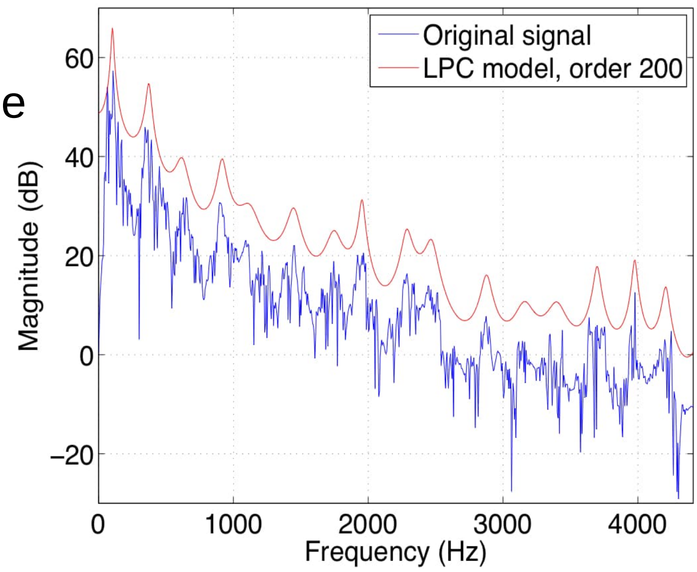
Piano Soundboard Modeling with LPC, $N = 200$

- (Recorded) target response
- Modeled



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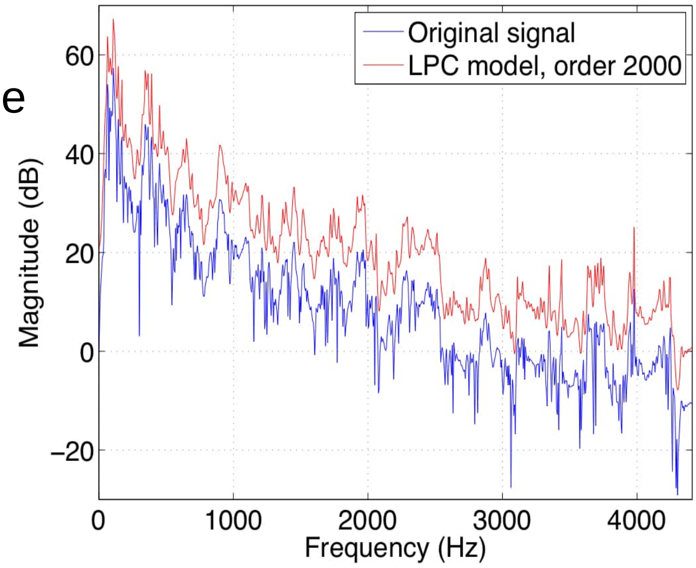
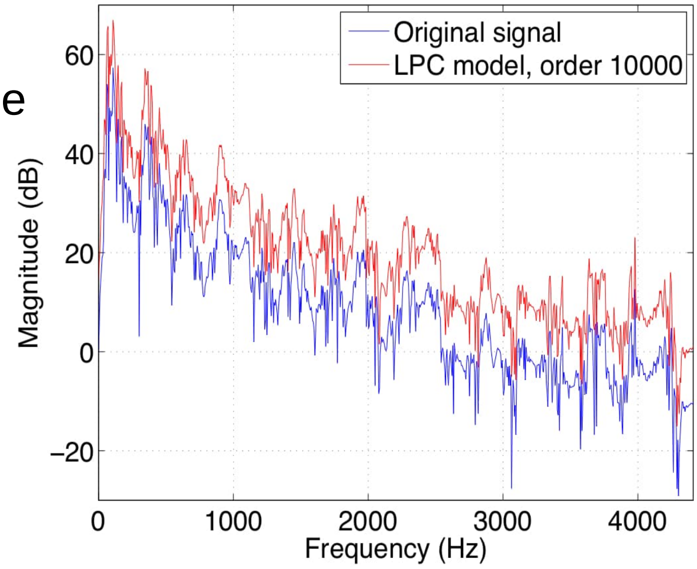
Piano Soundboard Modeling with LPC, $N = 2000$

- (Recorded) target response
- Modeled



Piano Soundboard Modeling with LPC, $N = 10000$

- (Recorded) target response
- Modeled



Prony's Method

$$H(z) = \frac{B(z)}{A(z)}$$

- Design an IIR filter from a given impulse response
- Solve denominator (all-pole IIR) coefficients using linear prediction of order *N*

$$\sum_{k=0}^{N} a_k z^{-k}$$

- Solve numerator (FIR) coefficients by matching the *M* + 1 first samples of the impulse response

$$\sum_{k=0}^{M} b_k z^{-k}$$

- In Matlab: `prony.m`

"Duel of Digital Filters"

- Why is a FIR filter better?
- Why is an IIR filter better?
- …in audio applications
- Brainstorm in small groups:
  - Some groups collect advantages of **FIR** filters
  - Other groups collect advantages of **IIR** filters
  - Please list your ideas, so that we can write them down

"Duel of Digital Filters"

FIR Advantages

- Always stable
- Linear phase

IIR Advantages

- More efficient computationally
- Smaller order
- Parametric control possible
- Analog-like phase response
- Good for real-time applications

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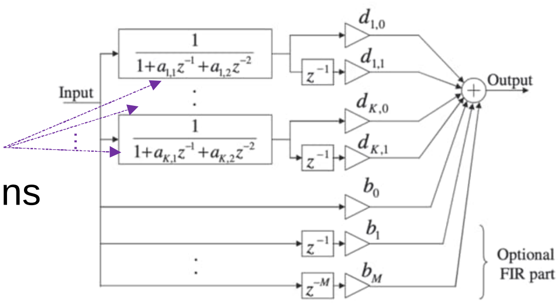
Fixed-Pole IIR Filters

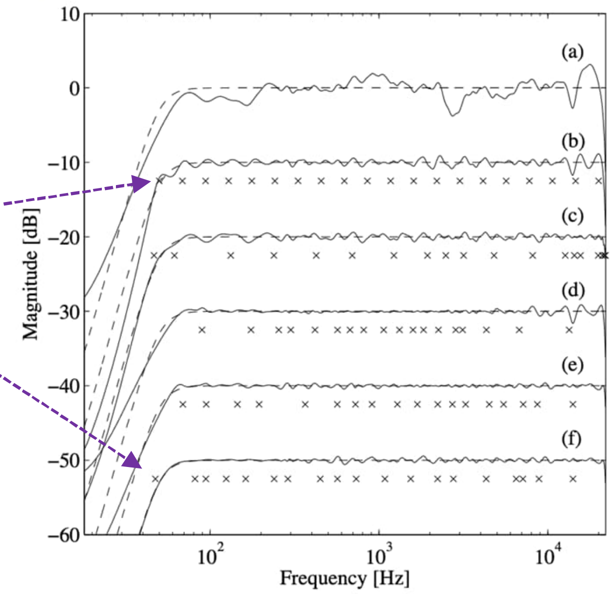
- Perceptually-motivated parallel IIR structure proposed by Bank (2007)
- First, select pole frequencies, e.g. logarithmically: they form basis functions
- Then, solve zeros with LS method, as weights for the basis functions (d_0, d_1)
- That's the final implementation directly; no conversions needed
- Still, the efficiency and resolution can be as good as with warped filters



Source: B. Bank, Audio Equalization with Fixed-Pole Parallel Filters: An Efficient Alternative to Complex Smoothing, *Journal of AES*, 2013

Fixed-Pole IIR Filters

- Frequency resolution can be controlled by the pole positions (x)
  - o Logarithmic pole positions gives a logarithmic frequency resolution
  - o Alternatively, the resolution can be tweaked manually (arbitrarily) to fit the target response



B. Bank, "Loudspeaker and room response equalization using parallel filters: Comparison of pole positioning strategies," in *Proc. AES 51st Int. Conf.*, 2013.

Fixed-Pole IIR Filters

- When the poles are set → linear optimization problem
- Least-squares solution can be used to solve zeros!

$$\mathbf{h}_{\mathrm{t}} = \mathbf{M}\mathbf{p},$$

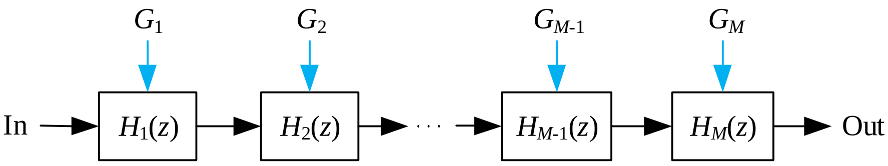
where $\mathbf{h}_{\mathrm{t}}$ is the target impulse response, $\mathbf{M}$ is the modeling signal matrix, and $\mathbf{p}$ contains the zeros (and FIR part coeffs.)

$$\mathbf{p}_{\mathrm{opt}} = (\mathbf{M}^T\mathbf{M})^{-1}\mathbf{M}^T\mathbf{h}_{\mathrm{t}}$$

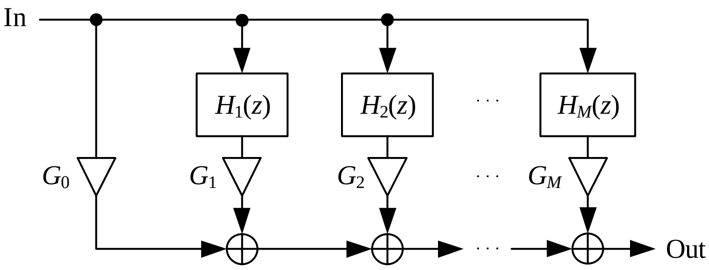
- Design can be done either in the time or frequency domain
  - o Matrix $\mathbf{M}$ contains either impulse responses or the transfer functions of the all-pole second-order sections (basis functions)
  - o In frequency domain: magnitude-only design & frequency-dependent weighting

Graphic Equalizer Design

- Cascade structure: use equalizing filters; well understood today



- Parallel structure: band of resonator (bandpass) filters; difficult to design



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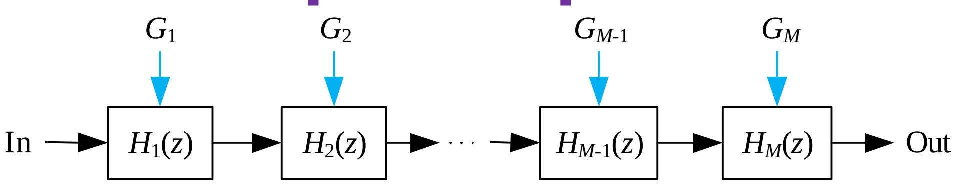
Cascade Graphic Equalizer



- Each subfilter is an equalizing filter with fixed f_c and Q
- Gain G_k at each peak can be adjusted (elsewhere ≈ 0 dB)
- Logarithmic frequency division is used, which is closely related to the auditory frequency resolution
- Problem: **interaction** between bands…

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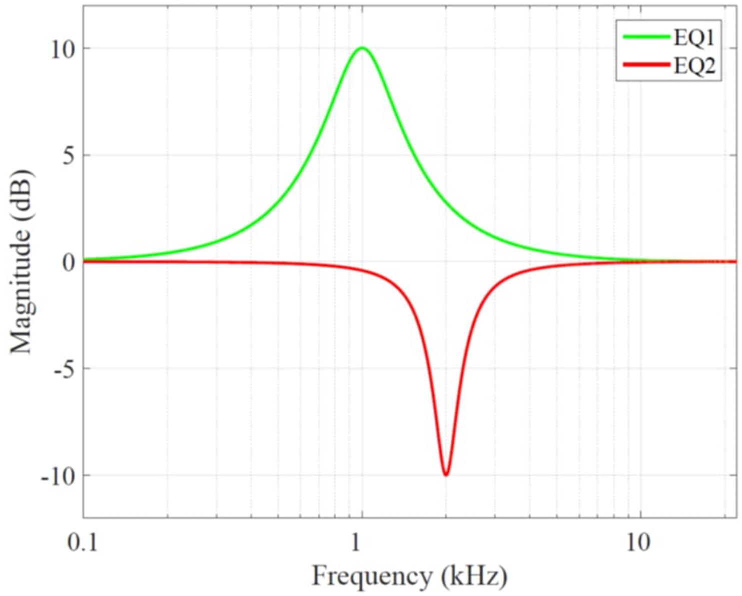
Cascade Graphic Equalizer, Ex. #1

- Two EQ filters
  - EQ1: f_c = 1 kHz & $G = 10$ dB
  - EQ2: f_c = 2 kHz & $G = -10$ dB

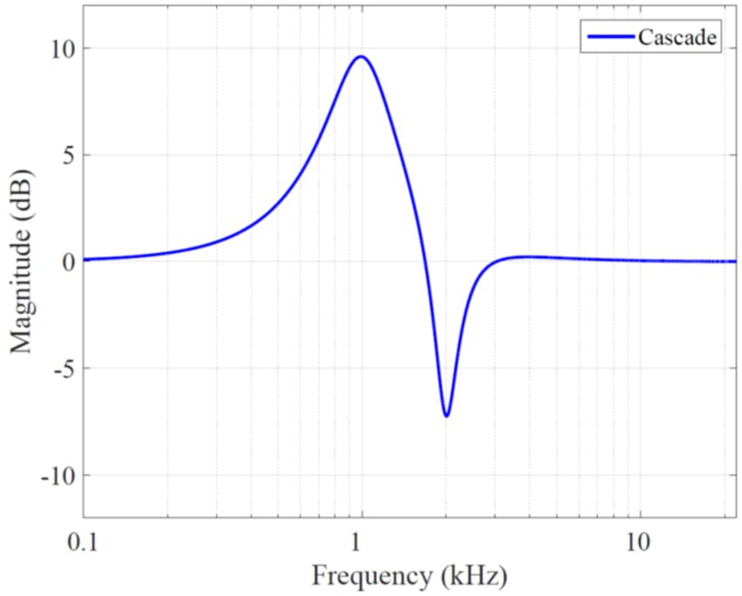


Cascade Graphic Equalizer, Ex. #1

- Responses interact so the overall response is biased:

~~10 dB and –10 dB~~

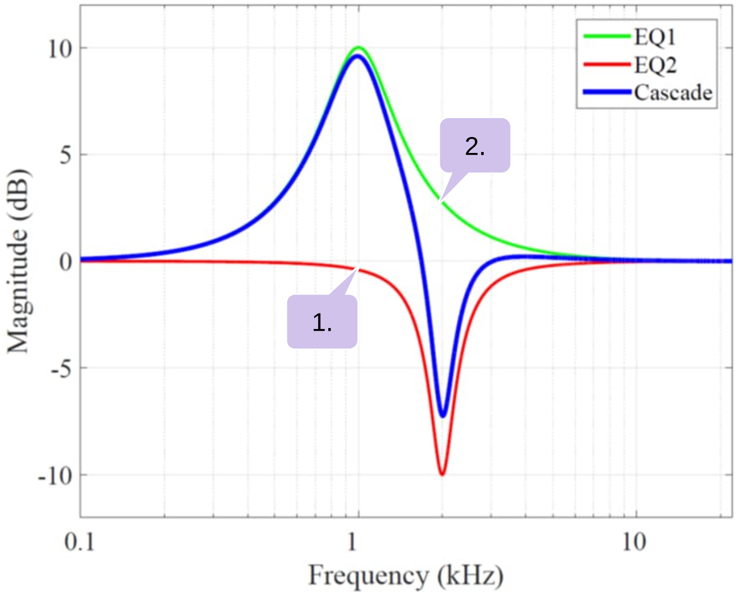
9.6 dB and -7.1 dB



Cascade Graphic Equalizer, Ex. #1

- Interaction problem:
1. Gain of EQ2 at 1 kHz is < 0 dB
2. Gain of EQ1 at 2 kHz is > 0 dB

- Both filters affect the gain at all frequencies!!! (not only around their center frequency)



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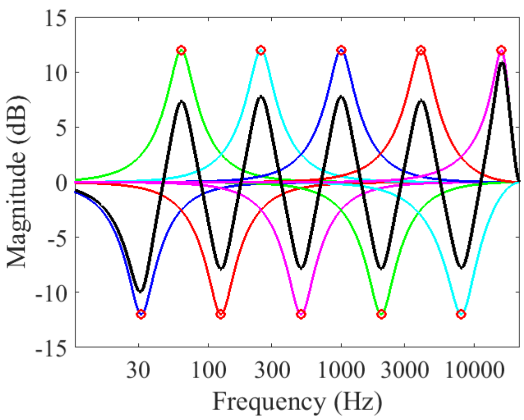
Cascade Graphic EQ for Octave Bands

- Naïve design leads to severe approximation problems
  - o Filter gains = command gains



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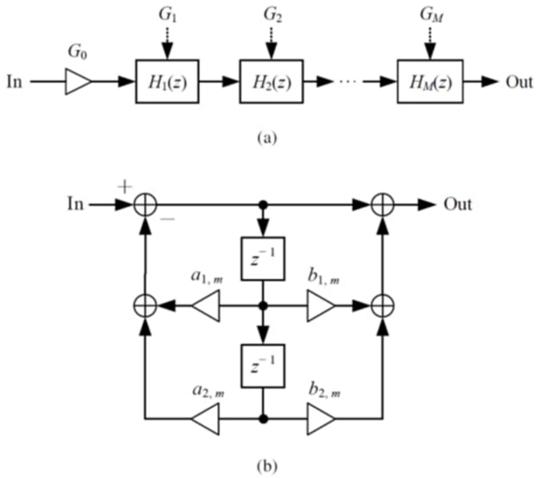
Novel Cascade Graphic EQ Design

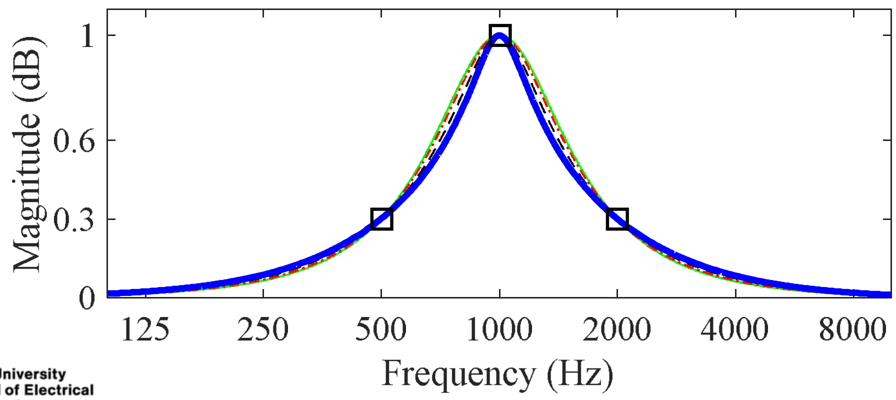
- It is desirable to use 2nd-order band filters
- The most accurate cascade design uses different peak filter shapes than previously (Välimäki & Liski 2017, Liski *et al*. 2019)
- The magnitude response is optimized at center frequencies and at intermediate frequencies



Second-order cell $H_k(z)$

Cascade GEQ with Variable-Q Filters

- Force the magnitude response to be "constant" at its f_c and the 2 neighboring f_c's (Välimäki & Liski 2017)
- Approximately constant basis functions are obtained



Normalized magnitude responses of filters with different gain (-12 dB…12 dB)

Interaction Matrix

- Interaction matrix **B** shows how much each band filter leaks to neighboring bands (Oliver & Jot 2015; Välimäki & Liski 2017)
- Computed with discrete-time Fourier transform, when the band filters are adjusted to a prototype gain
- Matrix **B** is normalized so that it shows how much 1 dB of gain leaks from each band filter to other frequency points

Filter

$$\mathbf{B}_0 = \begin{pmatrix} 1.0 & 0.30 & 0.081 & 0.021 & 0.0052 & 0.0013 & 3.2\times10^{-4} & 7.8\times10^{-5} & 1.7\times10^{-5} & 1.9\times10^{-6} \\ 0.30 & 1.0 & 0.30 & 0.081 & 0.021 & 0.0052 & 0.0013 & 3.1\times10^{-4} & 6.8\times10^{-5} & 7.5\times10^{-6} \\ 0.081 & 0.30 & 1.0 & 0.30 & 0.081 & 0.021 & 0.0052 & 0.0013 & 2.7\times10^{-4} & 3.0\times10^{-5} \\ 0.021 & 0.081 & 0.30 & 1.0 & 0.30 & 0.081 & 0.021 & 0.0050 & 0.0011 & 1.2\times10^{-4} \\ 0.0053 & 0.021 & 0.081 & 0.30 & 1.0 & 0.30 & 0.080 & 0.020 & 0.0043 & 4.8\times10^{-4} \\ 0.0013 & 0.0053 & 0.021 & 0.081 & 0.30 & 1.0 & 0.30 & 0.078 & 0.017 & 0.0019 \\ 3.4\times10^{-4} & 0.0014 & 0.0054 & 0.022 & 0.083 & 0.31 & 1.0 & 0.30 & 0.071 & 0.0081 \\ 8.2\times10^{-5} & 3.3\times10^{-4} & 0.0013 & 0.0052 & 0.021 & 0.08 & 0.30 & 1.0 & 0.27 & 0.033 \\ 2.0\times10^{-5} & 7.9\times10^{-5} & 3.2\times10^{-4} & 0.0013 & 0.0050 & 0.020 & 0.078 & 0.29 & 1.0 & 0.16 \\ 4.6\times10^{-6} & 1.9\times10^{-5} & 7.4\times10^{-5} & 3.0\times10^{-4} & 0.0012 & 0.0047 & 0.019 & 0.072 & 0.26 & 1.0 \end{pmatrix}$$

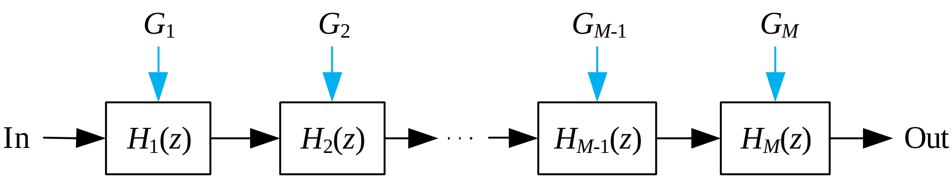
Frequency

Cascade GEQ Based on 2nd-Order Filters

- Use inverse matrix of **B** to solve the optimal dB-gains in the least-squares (LS) sense

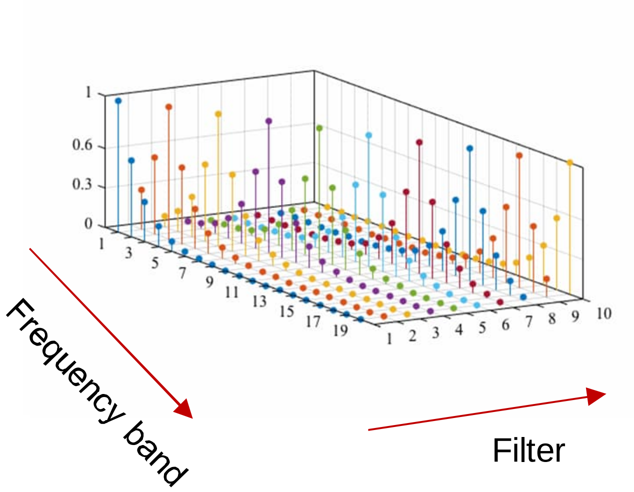
$$\mathbf{g}_{\mathrm{opt}} = \mathbf{B}^{-1}\mathbf{g}$$

- After optimization: filter gains ≠ command gains



Graphic Equalizer Design Using Interaction Matrix

Interaction matrix $\mathbf{B}$



Optimize dB gains in LS sense using pseudoinverse:

$$\mathbf{g} = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{t}_1,$$

where

$\mathbf{t}_1$ = target gains (dB)

$\mathbf{B}$ = interaction matrix

$\mathbf{g}$ = optimized filter gains (dB)

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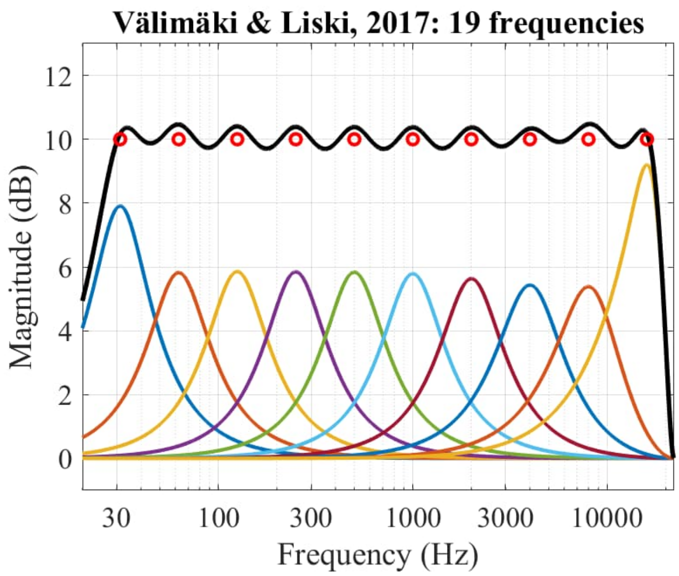
Octave Equalizer: Välimäki & Liski 2017



Peak gains of band filters are now very different from target gains o

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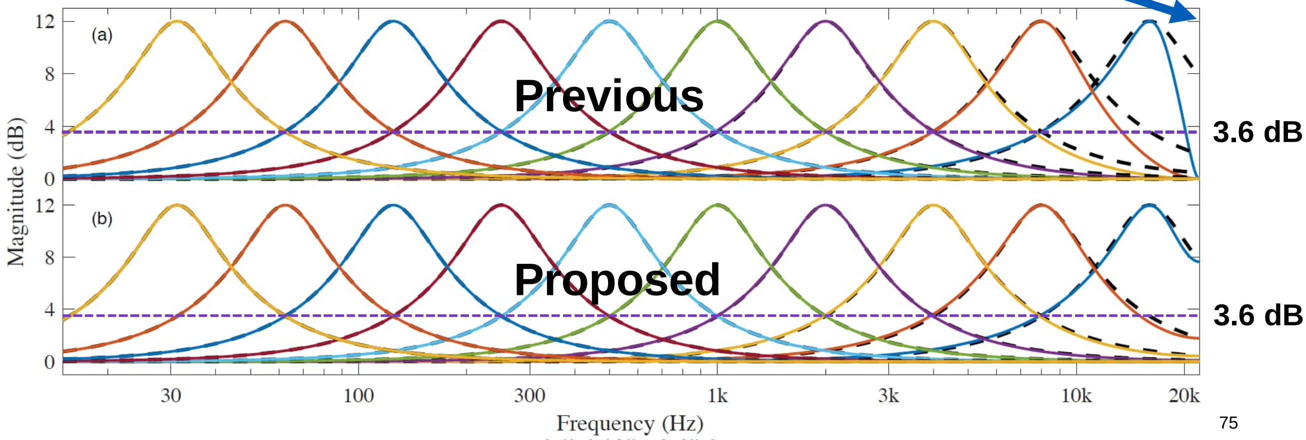
Design with Symmetric Band Filters

- The latest version of the GEQ design (Liski *et al.*, IEEE WASPAA-19) uses symmetric filters having a prescribed Nyquist gain: the ripple is reduced to about 0.8 dB
- The ideal target curve (---) is obtained with oversampling



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Conclusions

- Properties of filters: magnitude and phase
  - Phase delay and group delay are identical when the phase is linear; otherwise, they are different
- Various audio filter design methods are available
  - For FIR filters: Least squares, weighted least squares, frequency sampling, impulse response truncation/windowing
  - The magnitude and phase responses of the FIR filter can be tuned independently
  - For IIR filters: linear prediction, Prony's method, fixed-pole design, graphic EQ
- Auditory resolution can be applied
  - With frequency weighting or warping or with a log frequency scale
- The most suitable method depends on the applications
- Typical design problems are equalizers, HRTF filters, reverberation algorithms, and musical-instrument responses

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Literature

- Easily accessible online book (check this out!)
  - Smith, J. O., "Introduction to Digital Filters with Audio Applications," http://ccrma.stanford.edu/~jos/filters/
- Filter design
  - Parks, T. W. and Burrus, C. S. Digital Filter Design. John Wiley & Sons, 1987.
  - Mitra, S. K. Digital Signal Processing. McGraw-Hill, 2002.
- Frequency warping
  - A. Härmä, M. Karjalainen, L. Savioja, V. Välimäki, U. K. Laine, and J. Huopaniemi, "Frequency-warped signal processing for audio applications," Journal of the Audio Engineering Society, vol. 48, no. 11, pp. 1011–1031, Nov. 2000.
  - Smith, J. O. III and Abel, J. S. "The Bark and ERB bilinear transforms," IEEE Trans. Speech and Audio Process., Vol. 7, No, 6, pp. 697-708, Nov. 1999.
- Graphic equalizers
  - J. Liski, "Equalizer Design for Sound Reproduction," DSc thesis, Aalto University, 2020.

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References

- J. S. Abel and D. P. Berners, "Filter design using second-order peaking and shelving sections," in *Proc. Int. Computer Music Conf.*, Miami, FL, USA, Nov. 2004.
- B. Bank, "Direct design of parallel second-order filters for instrument body modeling," in *Proc. Int. Computer Music Conf.*, Copenhagen, Denmark, Aug. 2007.
- B. Bank, "Perceptually motivated audio equalization using fixed-pole parallel second-order filters," *IEEE Signal Processing Letters*, vol. 15, pp. 477-480, 2008.
- B. Bank, "Audio equalization with fixed-pole parallel filters: An efficient alternative to complex smoothing," *Journal of Audio Engineering Society*, vol. 61, pp. 39-49, 2013.
- B. Bank, "Loudspeaker and room response equalization using parallel filters: Comparison of pole positioning strategies," in *Proc. AES 51st International Conf.*, Helsinki, Finland, Aug. 2013.
- B. Bank, "Warped, Kautz, and Fixed-Pole Parallel Filters: A Review," *J. Audio Eng. Soc.*, vol. 70, no. 6, pp. 414–434, (2022 June). https://doi.org/10.17743/jaes.2022.0016
- Huopaniemi, J., Zacharov, N., and Karjalainen, M., " Objective and subjective evaluation of head-related transfer function filter design," in *Proc. 105th AES Convention*, preprint 4805, San Francisco, CA, USA, September 26-29, 1998. Invited paper.
- J. Huopaniemi, *Virtual Acoustics and 3-D Sound in Multimedia Signal Processing*. Doctoral thesis. Report no. 53 / Helsinki University of Technology, Laboratory of Acoustics and Audio Signal Processing. Espoo, Finland, 1999.
- R. J. Oliver and J.-M. Jot, "Efficient multi-band digital audio graphic equalizer with accurate frequency response control," in *Proc. AES 139th Conv*., New York, NY, USA, Oct. 2015.
- Lehtonen, H.-M., Rauhala, J., and Välimäki, V. "Sparse multi-stage loss filter design for waveguide piano synthesis," In *Proc. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, pp. 331-334, New Paltz, NY, USA, October 16-19, 2005.
- J. Liski, J. Rämö, and V. Välimäki and, "Graphic equalizer design with symmetric biquad filters," in *Proc. IEEE WASPAA*, Oct. 2019.
- V. Välimäki and J. D. Reiss, "All about audio equalization: Solutions and frontiers," *Appl. Sci.*, vol. 6, no. 5, p. 129, May 2016.
- V. Välimäki and J. Liski, "Accurate cascade graphic equalizer," *IEEE Signal Processing Letters*, vol. 24, no. 2, pp. 176–180, Feb. 2017.

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Learning Diary by Next Thursday

- Here are some tips what you could do this time (one of them, not all)
- Convert a linear-phase FIR filter to minimum phase using the Hilbert transform and/or cepstral windowing method and create plots to compare
- Auditory smoothing: compare what a few smoothing techniques can do to a “random” magnitude response, such as 1/3-octave smoothing
- Compare the effect of different window functions on the DFT spectrum (fft.m) of a pure tone or another signal (to truncate it)
- Compare two or more digital filters: the impulse response, the magnitude & phase responses (freqz.m), and/or the group delay (grpdelay.m), and the poles and zeros (zplane.m) could be plotted and studied

Learning Diaries – Submitting Code

- Ensure that all files, including audio and functions, are included in the submitted material
- Verify that the code runs successfully before submission
- When submitting functions, include a file demonstrating their usage
- When renaming files, declare the file extension accurately
- Utilize file compression in .zip format whenever possible
- Avoid using external packages.