Tutorial - 1   18 - Sara Ghorpalkar

1) $A = \begin{bmatrix} 2 & 1 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 3 \end{bmatrix}$

This is a triangular matrix,
Eigen values of A are main diagonal elements.
$\therefore \lambda_1 = 2, \lambda_2 = 1, \lambda_3 = 3$

Using properties of eigen values, we have,

i) A^2
eigen values : $\lambda_1^2, \lambda_2^2, \lambda_3^2$
$= 4, 1, 9$

ii) $-2A$
eigen values : $-2\lambda_1, -2\lambda_2, -2\lambda_3$
$= -2(2), -2(1), -2(3)$
$= -4, -2, -6$

iii) I
eigen values :- 1, 1, 1

Thus, eigen values of $A^2 - 2A + I$
$= 4 - 4 + 1, 1 - 2 + 1, 9 - 6 + 1$
$= 1, 0, 4.$

3) It is given that 2 of the eigen values of 3X3 matrix whose determinant is equal to 6 is 1, 3.

Let c be the third eigen value of matrix.
We know that the product of the eigen values of a square matrix A is determinant A.

So,

$$c \times 1 \times 3 = \det A$$

$$c \times 3 = 6$$

$$c = 2$$

Thus, the third eigen value of matrix = 2.

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4) $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

The ~~a~~ characteristic eqn of $A \Rightarrow |A - \lambda I| = 0$

ie. $\lambda^2 - S_1 \lambda + |A| = 0$ -- ①

where S_1 = Trace of A

$= a_{11} + a_{22}$

$= 3 + 2$

$= 5$

and $|A| = \begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix}$

$= 3 \times 2 - (1 \times -1)$

$= 6 + 1 = 7$

Putting these values in ①, we get

$\lambda^2 - 5\lambda + 7 = 0$ -- ②

This is the characteristic eqn of A.

According to cayley Hamilton theorem, A satisfies ②.

$\therefore A^2 - 5A + 7 = 0$ -- ③

$$
\begin{array}{r|l}
 & 2A^3 + 7A^2 + 21A \cancel{+ 87} + 57 \\
\hline
A^2 - 5A + 7 & 2A^5 - 3A^4 + 0A^3 + A^2 - 4I \\
 & -\underline{2A^5 - 10A^4 + 14A^3} \\
 & 0 \quad 7A^4 - 14A^3 + A^2 - 4I \\
 & -\underline{7A^4 - 35A^3 + 49A^2} \\
 & 0 \quad 21A^3 - 48A^2 + 0A - 4I \\
 & -\underline{21A^3 - 105A^2 + 147A} \\
 & 57A^2 - 147A - 4I \\
 & \cancel{87A^2 - 285A} \\
 & -\underline{57A^2 - 285A + 399I} \\
 & 138A - 403
\end{array}
$$

$2A^5 - 3A^4 + A^2 - 4I$
$= (A^2 - 5A + 7I)(2A^3 + 7A^2 + 21A + 57) + 138A - 403$

$\therefore 2A^5 - 3A^4 + A^2 - 4I$
$= \boxed{138A - 403I}$

$= 138\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 403\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 414 & 138 \\ -138 & 276 \end{bmatrix} - \begin{bmatrix} 403 & 0 \\ 0 & 403 \end{bmatrix}$

$= \begin{bmatrix} 11 & 138 \\ -138 & -127 \end{bmatrix}$

2) $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 2 & -4 \\ -1 & 1 & 4 \end{bmatrix}$

$\lambda^3 - (0+2+4)\lambda^2 + (0+8+0-(-1+2-4))\lambda - 6 = 0$
$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$
$\therefore$ Eigen values $= 1, 3, 2.$

For $\lambda = 1,$
$|A - \lambda I| = 0$

$\begin{bmatrix} -1 & 1 & -2 \\ -1 & 1 & -4 \\ -1 & 1 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\therefore 2x_1 = -3x_3$
$\therefore x_1 = \frac{-3}{2}x_3 \quad / \quad x_3 = \frac{-2}{3}x_1$

$\therefore \frac{9}{2}x_3 + x_2 - 2x_3 = 0$

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$\therefore 5x_3 = -2x_2$

$\therefore x_3 = \frac{-2}{5} x_2$

Let $x_3 = 1$

$\therefore x_1 = -3/2$

$x_2 = -5/2$

$x_2 \begin{pmatrix} -3/2 \\ -5/2 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ -5 \\ 2 \end{pmatrix} t$ AM = GM

For $\lambda = 2$,

$[A - 2I] = 0$

$$\begin{bmatrix} -2 & 1 & -2 \\ -1 & 0 & -4 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 2 & 0 \\ -1 & 0 & -4 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2R_3 + R_2$, $R_3 - R_1$

$$\begin{bmatrix} -3 & 2 & 0 \\ -1 & 0 & -4 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 - R_1$, $R_3 - R_1$, $R_3 + 3R_2$

$$\begin{bmatrix} -1 & 1 & -2 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1 + x_2 - 2x_3 = 0$ -- (1)

$-2x_3 = 0$

$x_3 = 0$

$x_1 = x_2$ -- from (1)

$x_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} t$

AM = GM.

For $\lambda = 3$

$|A - 3I| = 0$

$$\begin{bmatrix} -3 & 1 & -2 \\ -1 & -1 & -4 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 + R_1$, $R_3 - R_1$, $R_3 + 1/2 R_2$

$$\begin{bmatrix} -3 & 1 & -2 \\ -4 & 0 & -6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3x_1 + x_2 - 2x_3 = 0$ -- ①

$-4x_1 - 6x_3 = 0$ -- ②

$-3x_1 + 2x_2 = 0$

$-x_1 - 4x_3 = 0$

$-x_2 = 0$

$x_2 = 0$

$x_3 \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} t$

AM = GM

Since, AM = GM

Hence, diagonisable

$P^{-1} A D = D$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

D = transforming matrix = $\begin{bmatrix} 1 & -3 & 0 \\ 1 & -5 & 0 \\ 0 & 2 & 0 \end{bmatrix}$