Tutorial 5

Q1 Use simplex method to solve the following LPP

Maximize $Z = 5x + 3y$

subject to $x + y \leq 2$

$5x + 2y \leq 10$

$3x + 8y \leq 12$

$x, y \geq 0$

Solution

$x + y + s_1 = 2$

$5x + 2y + s_2 = 10$

$3x + 8y + s_3 = 12$

$x, y, s_1, s_2, s_3 \geq 0$

Here basic variables are s_1, s_2 and s_3

Here Non basic variable are x and y

Putting nonbasic variables as 0 in the above equations we get

$s_1 = 2 \qquad s_2 = 10 \qquad s_3 = 12$

$Z = 5x + 3y + 0s_1 + 0s_2 + 0s_3$

		c_j	5	3	0	0	0	Ratio
CB	B.V	X_B	x	y	s_1	s_2	s_3	X_B/x
0	s_1	2	1	1	1	0	0	$2/1 = 2$
0	s_2	10	5	2	0	1	0	$10/5 = 2$
0	s_3	12	3	8	0	0	1	$12/3 = 4$
		z_j	0	0	0	0	0	
		$z_j - c_j$	-5	-3	0	0	0	

5	x	2	1	1	1	0	0
0	S_2	0	0	$-3/5$	-1	$1/5$	0
0	S_3	2	0	$5/3$	-1	0	$1/3$
		z_j	5	5	5	0	0
		$z_j - c_j$	0	2	5	0	0

since all $z_j - c_j \geq 0$

we have $x = 2$ and $y = 0$

since $z = 5x + 3y$

we have $z = 10$

Min $z = x_1 - 3x_2 + 3x_3$

subject to $3x_1 - x_2 + 2x_3 \leq 7$

$2x_1 + 4x_2 \geq -12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1, x_2, x_3 \geq 10$

solution :-

$z' = -x + 3x_2 - 3x_3$

subject to $3x_1 - x_2 + 2x_3 + S_1 = 7$

$2x_1 + 4x_2 - S_2 + A_2 = -12$

$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$

Here non basic variables are x_1, x_2 and S_2

Here basic variables are S_1, A_2 and S_3

Putting non basic variables as 0

$S_1 = 7$ and $A_2 = -12$ $S_3 = 10$

$z' = -x_1 + 3x_2 - 3x_3 + 0S_1 + 0S_2 - MA_2$

~~Tecto~~

CB	BV	X_B	x_1	x_2	x_3	S_1	S_2	S_3	A_2	Ratio
		$C_j \to$	-1	3	-3	0	0	0	$-m$	
										-7
0	S_1	7	3	-1	2	1	0	0	0	-3
$-m$	A_2	-12	2	4	0	0	-1	0	1	$10/3$
0	S_3	10	-4	3	8	0	0	1	0	
		z_j	$-2m$	$4m$	0	0	m	0	$-m$	
		$z_j - c_j$	$-2m+1$	$-4m-3$	3	0	m	1	0	
0	S_1	$31/3$	$5/3$	0	$4/3$	1	0	$1/3$	0	$31/5$
$-m$	A_2	$-76/3$	$22/3$	0	$-32/3$	0	-1	$-4/3$	1	$-76/22$
3	x_2	$10/3$	$-4/3$	1	$8/3$	0	0	$1/3$	0	$10/4 = -5/2$
		z_j	$\frac{-20m}{3} - 4$	3	$\frac{32m}{3} + 8$	0	m	$\frac{4m}{3} + 1$	$-m$	
		$z_j - c_j$	$\frac{22m}{3} - 3$	0	$\frac{32m+11}{3}$	0	m	$\frac{4m}{3} + 1$	0	
-1	x_1	$31/5$	1	0	0	$14/5$	$3/5$	0	$1/5$	0
$-m$	A_2	$-354/5$	0	0	0	$-156/5$	$-24/5$	-1	$-14/5$	1
3	x_2	$58/5$	0	0	1	$32/5$	$4/5$	0	$3/5$	0
		z_j	-1	-1	3	$\frac{156m}{5} + 10/5$	$\frac{22m}{5} + \frac{9}{5}$	m	$\frac{4m}{5} + \frac{8}{5}$	$-m$
		$z_j - c_j$	0	0	0	$\frac{156m}{5} + 33/5$	$\frac{22m}{5} + \frac{9}{5}$	m	$\frac{4m}{5} + \frac{8}{5}$	0

Since all $z_j - c_j \leq 0$ we have

$$x_1 = \frac{31}{5} \qquad x_2 = \frac{58}{5} \qquad x_3 = 0$$

$$z' = -\frac{31}{5} + 3\left(\frac{58}{5}\right) - 3(0)$$

$$= \frac{143}{5}$$

$$Z_{min} = -\left(\frac{143}{5}\right) = \frac{-143}{5}$$

Q4 Write the standard form of the following LPP

Maximize $z = 3x_1 + 5x_2$

subject to $2x_1 + 3x_2 \leq 4$

$3x_1 + 2x_2 \geq I$

$x_1, x_2 \geq 0$

solution :-

Maximize $z = 3x_1 + 5x_2 + 0s_1 + 0s_2$

subject to $2x_1 + 3x_2 + s_1 = 4$

$3x_1 + 2x_2 - s_2 = 12$

$x_1, x_2, s_1, s_2 \geq 0$

Q10 Express the following LPP in standard form.

minimize $z = 3x_1 + 4x_2$

subject to $2x_1 - x_2 - 3x_3 = -4$

$3x_1 + 5x_2 + x_4 = 10$

$x_1 - 4x_2 < 12$

$x_1, x_2, x_3 \leq 0$

Solution

Maximize $z' = -3x_1 - 4x_2$

subject to $-2x + x_2 + 3x_3 = 4$

$3x_1 + 5x_2 + x_4 = 10$

$x_1 - 4x_2 = 12$

$x_1, x_2, x_3 \geq 0$

Q3 Max $Z = 4x_1 + 10x_2$

subject to $2x_1 + x_2 \leq 50$

$2x_1 + 5x_2 \leq 100$

$2x_1 + 3x_2 \leq 90$

$x_1, x_2 \geq 0$

Solution

$2x_1 + x_2 + s_1 = 50$ $\quad$ $Z = 4x_1 + 10x_2 + 0s_1 + 0s_2 + 0s_3$

$2x_1 + 5x_2 + s_2 = 100$

$2x_1 + 3x_2 + s_3 = 90$

Here basic variables are s_1, s_2, s_3

Here non-basic variables are x_1 and x_2

Putting nonbasic variables as 0 in the above equations we get

$s_1 = 50$ $\quad$ $s_2 = 100$ $\quad$ $s_3 = 90$

		$c_j \rightarrow$	4	10	0	0	0	
c_B	B.V	X_B	x_1	x_2	s_1	s_2	s_3	Ratio
0	s_1	50	2	1	1	0	0	50
0	s_2	100	2	5	0	1	0	20
0	s_3	90	2	3	0	0	1	30
		z_j	0	0	0	0	0	
		$z_j - c_j$	-4	-10	0	0	0	

0	s_1	30	8/5	0	1	-1/5	0
10	x_2	20	2/5	1	0	1/5	0
0	s_3	30	4/5	0	0	-3/5	1
		z_j	4	10	0	2	0
		$z_j - c_j$	0	0	0	2	0

Since all $z_j - c_j \geq 0$ we have $x_1 = 0$ and $x_2 = 20$

$Z_{max} = 4(0) + 10(20) = 200$

Q5 Solve the foll LPP by Big-M method

Maximize $z = 3x_1 - x_2$

Subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_1, x_2 \geq 0$

Solution

$2x_1 + x_2 - s_1 + A_1 = 2$

$x_1 + 3x_2 + s_2 = 3$

$x_1, x_2, s_1, s_2, A_1 \geq 0$

Here non-basis variables are x_1, x_2 and s_1

Here non-basis variables are A_1, s_2

Putting non basic variable so in above equation

$A_1 = 2$

$s_2 = 3$

$z = 3x_1 - x_2 + 0s_1 + 0s_2 - mA_1$

		$c_j \rightarrow$	3	-1	0	0	-m	Ratio
CB	BV	XB	x_1	x_2	s_1	s_2	A_1	
-M	A_1	2	2	1	-1	0	1	$2/2=1$
0	s_2	3	1	3	0	1	0	$3/1=3$
		z_j	-2m	-m	m	0	-m	
		$z_j - c_j$	-2m-3	-m+1	m	0	0	
3	x_1	1	1	1/2	-1/2	0		-2
0	s_2	2	0	5/2	1/2	1		4
		z_j	3	3/2	-3/2	0		
		$z_j - c_j$	0	5/2	-3/2	0		

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3	x_1	3	1	3	0	1
0	s_1	4	0	5	1	1
		z_j	3	9	0	3
		$z_j - c_j$	0	10	1	3

Since all $z_j - c_j \geq 0$ the solution is optimal

$x_1 = 3$

$x_2 = 0$

Since $z = 3x_1 - x_2$

$z = 9$

Q6 Solve the following LPP by BigM method

Maximize $Z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

Solution

$2x_1 + x_2 + S_1 = 2$

$3x_1 + 4x_2 - S_2 + A_2 = 12$

Here non basic variables are x_1, x_2 and S_2

Here basic variables are A_2 and S_1

Putting non basic variable as zero

$S_1 = 2$ $A_2 = 12$

$Z = 3x_1 + 2x_2 + 0S_1 + 0S_2 - MA_2$

		C_j	3	2	0	0	$-m$	
CB	BV	XB	x_1	x_2	S_1	S_2	A_2	Ratio
0	S1	2	2	1	1	0	0	2
$-M$	A_2	12	3	4	0	-1	1	3
		z_j	$-3m$	$-4m$	0	m	$-m$	
		$z_j - c_j$	$-3m-3$	$-4m-2$	0	m	0	
2	x_2	2	2	1	1	0	0	
$-M$	A_2	4	-5	0	-4	-1	1	
		z_j	$4+5m$	2	$2+4m$	m	$-m$	
		$z_j - c_j$	$1+5m$	0	$2+4m$	m	0	

Since the artificial variable A_2 appears not at zero level so feasible solution does not exist.

Q7 Write the dual of following LPP

Maximise $Z = x_1 + 3x_2 - 2x_3 + 5x_4$

Subject $3x_1 - x_2 + x_3 - 4x_4 = 2$

$x_1, x_2, x_3 \geq 0$ x_4 unrestricted in sign

Solution Here we have 4 variable and 2 constant

maximize $Z = x_1 + 3x_2 - 2x_3 + 5x_4$

$3x_1 - x_2 + x_3 - 4x_4 \leq 2$ ... (1)

$-3x_1 + x_2 - x_3 + 4x_4 \leq -2$ ... (2)

$5x_1 + 3x_2 - x_3 - 2x_4 \leq 3$ ... (3)

$-5x_1 - 3x_2 + x_3 + 2x_4 \leq -3$ ... (4)

Let $x_4 = x_4' - x_4''$

$x_4', x_4'' \geq 0$

Maximise $Z = x_1 + 3x_2 - 2x_3 + 5(x_4' - x_4'')$

$= x_1 + 3x_2 - 2x_3 + 5x_4' - 5x_4''$

$3x_1 - x_2 + x_3 - 4x_4' + 4x_4'' \leq 2$

$-3x_1 + x_2 - x_3 + 4x_4' - 4x'' \leq 2$

$5x_1 + 3x_2 - x_3 - 2x_4' + 2x_4'' \leq 3$

$-5x_1 - 3x_2 + x_3 + 2x_4' - 2x_4'' \leq -3$

$x_1, x_2, x_3, x_4', x_4'' \geq 0$

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$$
\begin{array}{c|c}
 & \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4' & x_4'' \end{array} \\
\begin{array}{c} y_1' \\ y_1'' \\ y_2' \\ y_2'' \\ \\ \end{array} &
\left[\begin{array}{ccccc}
3 & -1 & 1 & -4 & 4 \\
-3 & 1 & -1 & 4 & -4 \\
5 & 3 & -1 & -2 & 2 \\
-5 & -3 & 1 & 2 & -2 \\
\geq 1 & \geq 3 & \geq -2 & \geq 5 & \geq -5
\end{array}\right.
\left.\begin{array}{c} \leq 2 \\ \leq -2 \\ \leq 3 \\ \leq -3 \\ \\ \end{array}\right\}
\end{array}
$$

Subject to

$3y_1' - 3y_1'' + 5y_2' - 5y_2'' \geq 1$

$-y_1' + y_1'' + 3y_2' - 3y_2'' \geq 3$

$y_1' - y_1'' - y_2' + y_2'' \geq -2$

$-4y_1' + 4y_1'' - 2y_2' + 2y_2'' \geq 5$

$4y_1' - 4y_1'' + 2y_2' - 2y_2'' \geq -5$

Let $y_1 = y_1' - y_1''$ $\quad y_2 = y_2' - y_2''$

$3y_1 + 5y_2 \geq 1$ — 5

$-y_1 + 3y_2 \geq 3$ — 6

$y_1 - y_2 \geq -2$ — 7

$-4y_1 - 2y_2 \geq 5$ — 8

$4y_1 + y_2 \geq -5$ .. 9

Combining 8 & 9

$4y_1 + 2y_2 = -5$ 10

$y_1, y_2 \geq 0$

Here we get 2 variable and 4 constraints

Construct the dual of the following LPP

maximize $z = 4x_1 + 9x_2 + 2x_3$

Subject to $2x_1 + 3x_2 + 3x_3 \le 7$

$3x_1 - 2x_2 + 4x_3 = 5$

$x_1, x_2, x_3 \ge 0$

Solution

Here we have 3 variables and two constraints

Maximize $z = 4x_1 + 9x_2 + 2x_3$

$2x_1 + 3x_2 + 2x_3 \le 7$

$3x_1 - 2x_2 + 4x_3 \le 5$

$-3x_1 + 2x_2 - 4x_3 \le -5$

$x_1, x_2, x_3 \ge 0$

$$\begin{array}{c|ccc|c}
 & x_1 & x_2 & x_3 & \\
y_1 & 2 & 3 & 2 & \le 7 \\
y_2' & 3 & -2 & 4 & \le 5 \\
y_2'' & -3 & 2 & -4 & \le -5 \\
 & \ge 4 & \ge 9 & &
\end{array}$$

Dual

minize $z = 7y_1 + 5y_2' - 5y_2''$

Subject to

$2y_1 + 3y_2' - 3y_2'' \ge 4$

$3y_1 - 2y_2' + 2y_2'' \ge 9$

$2y_1 + 4y_2' - 4y_2 \ge 2$

Let $y_2 = y_2' - y_2''$ so finally

minize $z = 7y_1 + 5y_2$

Subject to $2y_1 + 3y_2 \ge 4$

$3y_1 - 2y_2 \ge 9$

$2y_1 + 4y_2 \ge 2$

Here we get 2 variables and 3 constraints

Q9 Use Dual simplex Method to solve the foll LPP

Minimize $z = 2x_1 + 2x_2 + 4x_3$

subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

Solution

we first express the given problem using $\leq$ in first constraint.

So we have $-2x_1, -3x_2 - 5x_2 \leq -2$

Now

$-2x_1 - 3x_2 - 5x_3 + S_1 = -2$

$3x + x_2 + 7x_3 + S_2 = 3$

$x_1 + 4x_2 + 6x_3 + S_3 = 5$

$z = 2x_1 + 2x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3$

		C_j	2	2	4	0	0	0
C_B	BV	X_B	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	-2	-2	-3	-5	1	0	0
0	S_2	3	3	1	7	0	1	0
0	S_3	5	1	4	6	0	0	1
		z_j	0	0	0	0	0	0
		$z_j - c_j$	-2	-2	-4	0	0	0
		Ratio	1	2/3	4/5			
2	x_2	2/3	2/3	1	5/3	-1/3	0	0
0	S_2	7/3	7/3	0	16/3	1/3	1	0
0	S_3	7/3	-5/3	0	-2/3	4/3	0	1

$x_1 = 0$, $x_2 = \frac{2}{3}$, $x_3 = 0$, $z\ min\ 4/3$