

## Context Free Grammar

09 January 2022 10:43

- Create a grammar from language
- CFL (Context free language) CFG / Relationship b/w grammars

⇒ Grammar & its classifications  
Ram reads

$\langle \text{noun} \rangle \langle \text{verb} \rangle$

Sentences  $\rightarrow \langle \text{noun} \rangle \langle \text{verb} \rangle$

$\langle \text{noun} \rangle \rightarrow \text{Ram} / \text{Sita} / \text{Sara} / \text{Rohit}$

$\langle \text{verb} \rangle \rightarrow \text{read} / \text{talking} / \text{sitting} / \text{walking}$

Based on this Grammar

$$G = (V, \Sigma, P, S)$$

$$= (V, T, P, S)$$

$V \rightarrow$  set of variables  $\{S, A, B, C\} \Rightarrow$  (Capital letters)

$\Sigma / T \rightarrow$  set of terminals  $\{a, b, c\} \Rightarrow$  (Small letters)

$S \rightarrow$  starting symbol /  $S \in V$

$\times \rightarrow \text{zero}$   
 $+$   $\rightarrow \text{one}$

$P \rightarrow$  set of production rule

$$A \rightarrow \alpha \quad \text{where } A \in (V \cup T)^+$$

$$\alpha \Rightarrow \alpha \in (V \cup T)^*$$

$$\checkmark \quad \underline{S \rightarrow aA}$$

## Structure of Grammar

$L$  is language over alphabet  $A$ , then  $G$  for  
Consist of rules

$$x \rightarrow y$$

$x/y \rightarrow$  denote string of symbols  
 $S \rightarrow y$

Ex  $A = \{a, b, c\}$  then  $G$  for language  $A^*$

can be described by following production rule

$$\begin{aligned} S &\rightarrow \epsilon & (i) \\ S &\rightarrow aS & (ii) \\ S &\rightarrow bS & (iii) \\ S &\rightarrow cS & (iv) \end{aligned}$$

aacb

$$\begin{aligned} S &\Rightarrow aS & (ii) \\ &\Rightarrow aaS & (ii) \\ &\Rightarrow aaaS & (iv) \\ &\Rightarrow aacbS & (iii) \\ &\Rightarrow aacb & (i) \end{aligned}$$

① bbca

② ababccab

③ cabcabac

$$G = (V, T, P, S)$$

$$V = \{S\} \quad T = \{0, 1\}$$

P is defined as

$$(i) S \rightarrow \epsilon$$

$$(ii) S \rightarrow 0S1 \quad \text{Derive } 000111$$

$$S \Rightarrow 0S1 \quad (ii)$$

$$\Rightarrow 00S11 \quad (ii)$$

$$\Rightarrow 000S111 \quad (ii)$$

$$\Rightarrow 000111 \quad (i)$$

$$L = \{a^n b^n \mid n \geq 0\}$$

ab

aabb

aaabbb...

$$L = \{(ab)^n \mid n \geq 0\}$$

Leftmost & Rightmost Derivation

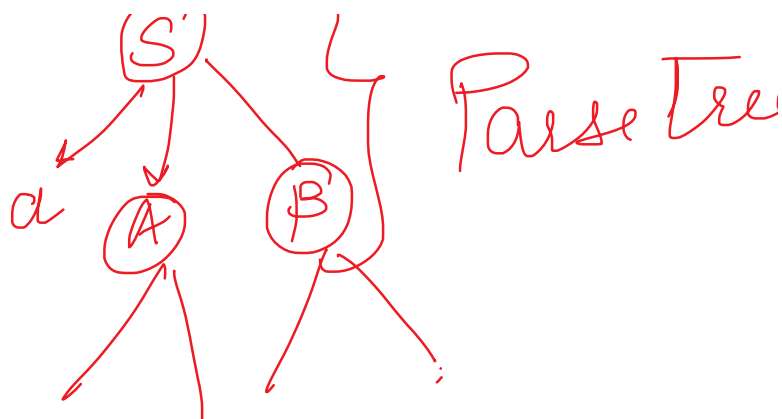
FasbsAB

$$\begin{aligned} &\Rightarrow aab\overline{S}AB \\ &\Rightarrow aab\overline{b}AB \\ &\Rightarrow aab\overline{ba}AB \\ &\Rightarrow aab\overline{bab}AB \end{aligned}$$

$$\longrightarrow A \quad B \quad C$$

$$\longleftarrow$$

$\Rightarrow aabbaab$



## Notations

- ① Non-terminal  $\Rightarrow$  Capital letter
  - ② Terminals/Auxiliary letter  $\Rightarrow$  smallcase
  - ③ String of terminals  $uvwxyz$
  - ④ Sentential form (collection)
- $S \Rightarrow aSBb$

- ⑤  $\rightarrow$  production
- ⑥  $\Rightarrow$  process of derivation
- $\xRightarrow{*}$  Zero or many
- $\xRightarrow{+}$  one or more

## Leftmost / Rightmost Derivation

### ⑥ Leftmost Derivation

$\rightarrow$  If at each step in derivation, a production is applied to the leftmost variable then it is leftmost derivation

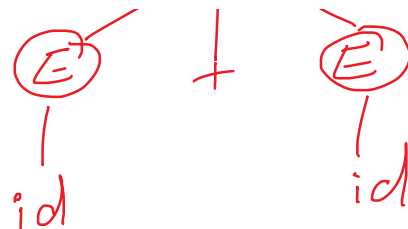
$(\{E\}, \{+, *, id\}, P, E)$

- ①  $E \rightarrow E + E$
- ②  $E \rightarrow E * E$
- ③  $E \rightarrow id$



id+id

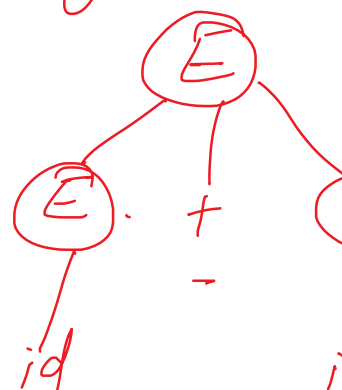
$$\begin{aligned}
 E &\Rightarrow E + E \quad (1) \\
 &\Rightarrow id + E \quad (3) \\
 &\Rightarrow id + id \quad (3)
 \end{aligned}$$



### Rightmost Derivation :-

If at each step in the Derivation, a production rule is applied to rightmost variable, then it is rightmost.

$$\begin{aligned}
 E &\Rightarrow E + E \quad (1) \\
 &\Rightarrow E + id \quad (3) \\
 &\Rightarrow id + id \quad (3)
 \end{aligned}$$



$$G = (\{S, A\}, \{a, b\}, P, S)$$

Production rule is

$$S \rightarrow aAS/a$$

$$A \rightarrow SbA/SS/ba$$

"aabbba"

Soln  $\Rightarrow$  S is start symbol & Production rule are

$$(1) S \rightarrow aAS$$

$$(2) S \rightarrow a$$

$$(3) A \rightarrow SbA$$

$$(4) A \rightarrow SS$$

⑤  $A \rightarrow ba$

Leftmost

$S \Rightarrow aAS \text{ (1)}$   
 $\Rightarrow ceSbAS \text{ (3)}$   
 $\Rightarrow aabAS \text{ (2)}$   
 $\Rightarrow aabbaS \text{ (5)}$   
 $\Rightarrow aabbba \text{ (2)}$

$S \rightarrow aB \mid bA$   
 $A \rightarrow a \mid aS \mid bAA$   
 $B \rightarrow b \mid bS \mid aBB$

② CFL

Rightmost

$S \Rightarrow aAS \text{ (1)}$   
 $\Rightarrow aAa \text{ (2)}$   
 $\Rightarrow asbAa \text{ (1)}$   
 $\Rightarrow asbbaa \text{ (1)}$   
 $\Rightarrow aabbba \text{ (1)}$

Leftmost

Rule applied  
 2  
 5  
 4  
 1  
 6  
 3

✓

Rightmost

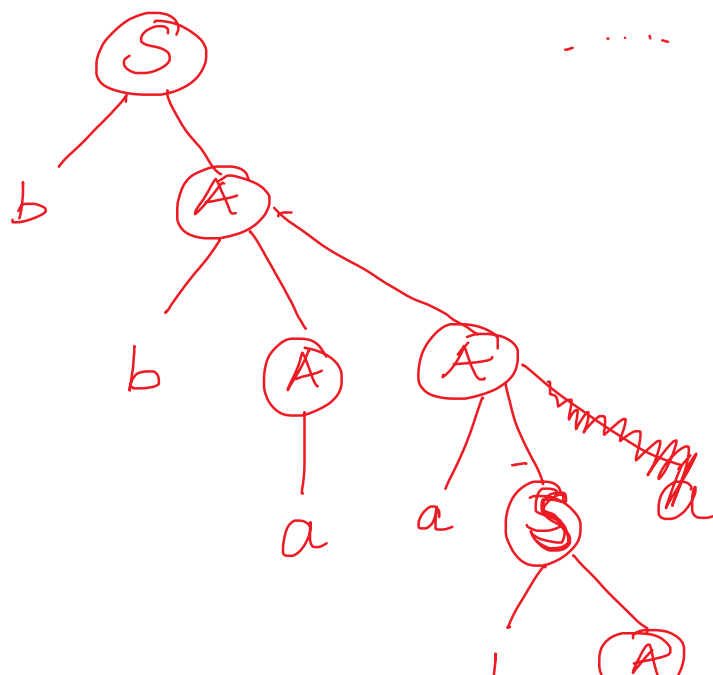
2  
 5  
 4

$S \Rightarrow bA \text{ (2)}$   
 $\Rightarrow bbAA \text{ (5)}$   
 $\Rightarrow bbAA \text{ (3)}$   
 $\Rightarrow bbasa \text{ (4)}$   
 $\Rightarrow bbaaba \text{ (1)}$   
 $\Rightarrow bbaaba \text{ (6)}$

Derivation

Rule

Leftmost  
Parse  
true





aaabbbb

$S \rightarrow aSb$

$S \rightarrow ab$

In this case both Derivations are same

Context Free language

$\Rightarrow$  It is language generated by CFG



$L = \{ w \mid w \in T^* \text{ \& is derivable from start symbol } \}$

CFG  $S \rightarrow aSb \mid ab$

Soln: -

(1)  $S \rightarrow aSb$

(2)  $S \rightarrow ab$

(a)  $S \Rightarrow \underline{ab}$

(b)  $S \Rightarrow aSb \quad (1)$

$\Rightarrow aabb \quad (2)$

(c)  $S \Rightarrow aSb \quad (1)$

$\Rightarrow aasbb \quad (1)$

$\Rightarrow aaabbbb \quad (2)$

(d)  $S \Rightarrow aSb \quad (1)$

$$\Rightarrow aasbb \quad (1)$$

$$\Rightarrow aaasbbb \quad (2)$$

$$\Rightarrow aaaaabbbbb$$

The language generated is

$$L = \{ ab, aabb, aaabbb, \dots \}$$

$$a^n b^n \quad n \geq 1$$

$$L = \{ a^n b^n \mid n \geq 1 \}$$

②

Start deriving string

$$\textcircled{a} S \Rightarrow aB \quad (1)$$

$$\Rightarrow ab \quad (6)$$

$$\textcircled{b} S \Rightarrow bA \quad (2)$$

$$\Rightarrow ba \quad (3)$$

$$L = \{ ab, ba, abab, baab, \dots \}$$

$$\textcircled{c} S \Rightarrow aB \quad (1)$$

$$\Rightarrow \cancel{ab} abS \quad (7)$$

$$\Rightarrow ababB \quad (1)$$

$$\Rightarrow abab$$

$$\textcircled{d} S \Rightarrow bA \quad (2)$$

$$\Rightarrow baS \quad (4)$$

$$\Rightarrow \cancel{baab} A \quad (1)$$

$$\Rightarrow baabB \quad (1)$$

$$\Rightarrow baab$$



$L = \{ ab, ba, abab, abba, baab, bbaa, bbaaba, babbba \}$   
 $L =$  Here we are getting string of different length  
 but with equal no of a's & b's  
 $L = \{ x \mid x \text{ contains equal no of a's \& b's} \}$