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Tutorial - 4

(Q1) Solve the following NLPP using Lagrange's multipliers method

Optimize $z = x_1^2 + x_2^2 + x_3^2 - 10x_1 - 6x_2 - 4x_3$

Subject to $x_1 + x_2 + x_3 = 7$ $\quad x_1, x_2, x_3 \geq 0$

Solution :-

$z = x_1^2 + x_2^2 + x_3^2 - 10x_1 - 6x_2 - 4x_3$

$h = x_1 + x_2 + x_3 - \cancel{10}\, 7$

~~$L = z - \lambda h$~~ $\quad L = z - \lambda h$

~~$\frac{\partial L}{\partial x_1} = 2x_1 - 10 = 0$~~ $\quad = (x_1^2 + x_2^2 + x_3^2 - 10x_1 - 6x_2 - 4x_3)$

$\qquad - \lambda(x_1 + x_2 + x_3 - 7)$

$\frac{\partial L}{\partial x_1} = 2x_1 - 10 - \lambda = 0$

$\therefore x_1 = \frac{\lambda + 10}{2}$ ...(I)

$\frac{\partial L}{\partial x_2} = 2x_2 - 6 - \lambda = 0$ $\qquad \frac{\partial L}{\partial x_3} = 2x_3 - 4 - \lambda = 0$

$\therefore x_2 = \frac{\lambda + 6}{2}$ --(II) $\qquad \therefore x_3 = \frac{\lambda + 4}{2}$ --(III)

$\frac{\partial L}{\partial \lambda} = -(x_1 + x_2 + x_3 - 7) = 0$

$\therefore x_1 + x_2 + x_3 = 7$ ...(IV)

~~Putting I, II, III in (IV)~~ Adding I, II and III

$2(x_1 + x_2 + x_3) - 20 - 3\lambda = 0$

From (IV)

$2(7) - 20 - 3\lambda = 0$

$\therefore \lambda = -2$

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Hence $x_1 = 4$, $x_2 = 2$ and $x_3 = 1$

$\therefore$ Xo is $(4, 2, 1)$

$$\Delta_4 = \begin{vmatrix} 0 & \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \frac{\partial h}{\partial x_3} \\ \frac{\partial h}{\partial x_1} & \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \cdot \partial x_2} & \frac{\partial^2 L}{\partial x_1 \cdot \partial x_3} \\ \frac{\partial h}{\partial x_2} & \frac{\partial^2 L}{\partial x_2 \cdot \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} & \frac{\partial^2 L}{\partial x_2 \cdot \partial x_3} \\ \frac{\partial h}{\partial x_3} & \frac{\partial^2 L}{\partial x_3 \cdot \partial x_1} & \frac{\partial^2 L}{\partial x_3 \cdot \partial x_2} & \frac{\partial^2 L}{\partial x_3^2} \end{vmatrix}$$

$$\Delta_4 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{vmatrix}$$

$\therefore \Delta_4 = -12$

$\cancel{\Delta_4 = \Delta_3 =}$

$$\Delta_3 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{vmatrix} = -4$$

Since both Δ_3 and Δ_4 are negative, Xo is minima

$\therefore x_1 = 4 \quad x_2 = 2 \quad x_3 = 1$

$\therefore Z_{min} = 16 + 4 + 1 - 40 - 12 - 4 = -35$

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(Q2) Use method of Lagrange's multiplier to solve NLPP

Optimize $z = -x_1^2 - x_2^2 + 6x_1 + 8x_2$

Subject to $4x_1 + 3x_2 = 16$

$3x_1 + 5x_2 = 15$

$x_1, x_2 \geq 0$

Solution :-

$z = -x_1^2 - x_2^2 + 6x_1 + 8x_2$

$h_1 = 4x_1 + 3x_2 - 16$

$h_2 = 3x_1 + 5x_2 - 15$

$L = z - \lambda_1 h_1 - \lambda_2 h_2$

$L = (-x_1^2 - x_2^2 + 6x_1 + 8x_2) - \lambda_1(4x_1 + 3x_2 - 16) - \lambda_2(3x_1 + 5x_2 - 15)$

$\frac{\partial L}{\partial x_1} = -2x_1 + 6 - 4\lambda_1 - 3\lambda_2 = 0 \quad \cdots (1)$

$\frac{\partial L}{\partial x_2} = -2x_2 + 8 - 3\lambda_1 - 5\lambda_2 = 0 \quad \cdots (2)$

$\frac{\partial L}{\partial \lambda_1} = -(4x_1 + 3x_2 - 16) = 0$ $\qquad \frac{\partial L}{\partial \lambda_2} = -(3x_1 + 5x_2 - 15) = 0$

$\therefore 4x_1 + 3x_2 - 16 = 0 \quad \cdots (3)$ $\qquad \therefore 3x_1 + 5x_2 = 15 \quad \cdots (4)$

On solving (1), (2), (3) and (4) we get

$x_1 = \frac{35}{11} \quad x_2 = \frac{12}{11} \quad \lambda_1 = \frac{-212}{121} \quad \lambda_2 = \frac{268}{121}$

$\therefore X_0(x_1, x_2) = X_0\left(\frac{35}{11}, \frac{12}{11}\right)$

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Here $n=2$ $m=2$ $(m=n)$

Now, $z = -x_1^2 - x_2^2 + 6x_1 + 8x_2$

$$\frac{\partial z}{\partial x_1} = 6 - 2x_1 \qquad \frac{\partial^2 z}{\partial x_1^2} = -2$$

$$\frac{\partial z}{\partial x_2} = 8 - 2x_2 \qquad \frac{\partial^2 z}{\partial x_2^2} = -2$$

$$\frac{\partial^2 z}{\partial x_1 \cdot \partial x_2} = 0 \qquad \frac{\partial^2 z}{\partial x_2 \cdot \partial x_1} = 0$$

$$\begin{vmatrix} \frac{\partial^2 z}{\partial x_1^2} & \frac{\partial^2 z}{\partial x_1 \cdot \partial x_2} \\ \frac{\partial^2 z}{\partial x_2 \cdot \partial x_1} & \frac{\partial^2 z}{\partial x_2^2} \end{vmatrix} = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4 = \Delta_2$$

$\Delta_1 = -2$ $\cancel{\Delta_2 =}$

Since $\cancel{\Delta_1 > 0}$ $\Delta_1 < 0$ $\Delta_2 > 0$

$\therefore X_0 \left(\frac{35}{11}, \frac{12}{11}\right)$ is maxima

$$z_{max} = 6\left(\frac{35}{11}\right) + 8\left(\frac{12}{11}\right) - \left(\frac{35}{11}\right)^2 - \left(\frac{12}{11}\right)^2 = 16.504$$

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(Q3) Solve the following NLPP using Lagrange's multiplier method

~~Optimize $z = 8x_1 + 10x_2 - x_1^2 - x_2^2$~~

~~Subject to $x_1 + x_2 + x_3 = 11$ $x_1, x_2, x_3 \geq 0$~~

Optimize $z = 196 - 24x_1 - 8x_2 - 12x_3 + 2x_1^2 + 2x_2^2 + 2x_3^2$

Subject to $x_1 + x_2 + x_3 = 11$; $x_1, x_2, x_3 \geq 0$

Solution :-

$z = 196 - 24x_1 - 8x_2 - 12x_3 + 2x_1^2 + 2x_2^2 + 2x_3^2$

$h = x_1 + x_2 + x_3 - 11$

$L = z - \lambda h$

$L = (196 - 24x_1 - 8x_2 - 12x_3 + 2x_1^2 + 2x_2^2 + 2x_3^2) - \lambda(x_1 + x_2 + x_3 - 11)$

$\frac{\partial L}{\partial x_1} = -24 + 4x_1 - \lambda = 0$ -- (1)

$\frac{\partial L}{\partial x_2} = -8 + 4x_2 - \lambda = 0$ -- (2)

$\frac{\partial L}{\partial x_3} = -12 + 4x_3 - \lambda = 0$ -- (3)

$\frac{\partial L}{\partial \lambda} = -(x_1 + x_2 + x_3 - 11) = 0$

$\therefore x_1 + x_2 + x_3 = 11$ --- (4)

Adding equation (1), (2) & (3)

$-24 - 8 - 12 + 4(x_1 + x_2 + x_3) - 3\lambda = 0$

From equation (4)

$-24 - 8 - 12 + 4(11) - 3\lambda = 0$

$\lambda = 0$

Putting $\lambda = 0$ in equation (1), (2) and (3), we get

$x_1 = 6$ $x_2 = 2$ $x_3 = 3$

$\therefore X_0(x_1, x_2, x_3) = X_0(6, 2, 3)$

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Here $m=1$ and $n=3$, $m+n=4$

$$\Delta_4 = \begin{vmatrix} 0 & \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \frac{\partial h}{\partial x_3} \\ \frac{\partial h}{\partial x_1} & \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} & \frac{\partial^2 L}{\partial x_1 \cdot \partial x_3} \\ \frac{\partial h}{\partial x_2} & \frac{\partial^2 L}{\partial x_2 \cdot \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} & \frac{\partial^2 L}{\partial x_2 \cdot \partial x_3} \\ \frac{\partial h}{\partial x_3} & \frac{\partial^2 L}{\partial x_3 \cdot \partial x_1} & \frac{\partial^2 L}{\partial x_3 \cdot \partial x_2} & \frac{\partial^2 L}{\partial x_3^2} \end{vmatrix}$$

$$\Delta_4 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 4 & 0 & 0 \\ 1 & 0 & 4 & 0 \\ 1 & 0 & 0 & 4 \end{vmatrix} = -48$$

$$\Delta_3 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 4 & 0 \\ 1 & 0 & 4 \end{vmatrix} = -8$$

Since $\Delta_4 < 0$ and $\Delta_3 < 0$, so X_0 is minima

$\therefore$ X_0 is minima $(6, 2, 3)$

$$Z_{min} = 196 - 24(6) - 8(2) - 12(3) + 2(6)^2 + 2(2)^2 + 2(3)^2$$
$$= 98$$

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(Q4) Solve the following NLPP using Kuhn - Tucker conditions
Maximise $z = 8x_1 + 10x_2 - x_1^2 - x_2^2$
Subject to $3x_1 + 2x_2 \leq 6$; $x_1, x_2 \geq 0$

Solution :-

$$L = 8x_1 + 10x_2 - x_1^2 - x_2^2 - \lambda(3x_1 + 2x_2 - 6)$$

~~Con~~ Kuhn Tucker Conditions :-

$$\frac{\partial L}{\partial x_1} = 8 - 2x_1 - 3\lambda = 0 \quad \text{-- (1)}$$

$$\frac{\partial L}{\partial x_2} = 10 - 2x_2 - 2\lambda = 0 \quad \text{-- (2)}$$

$$\lambda \cdot h(x) = \lambda(3x_1 + 2x_2 - 6) = 0 \quad \text{-- (3)}$$

$$3x_1 + 2x_2 - 6 \leq 0 \quad \text{-- (4)}$$

$$\lambda \geq 0 \quad \text{-- (5)}$$

Case 1 : $\lambda = 0$

From equation (1) & (2)

$x_1 = 4 \quad x_2 = 5$

From (4)

$3(4) + 2(5) - 6$

$= 12 + 10 - 6$

$= 16$

$\therefore 3x_1 + 2x_2 - 6 \nleq 0$

Hence condition not satisfied.

~~satisfied.~~

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Case-2: $\lambda \neq 0$

From equation ① & ②

$x_1 = \frac{8-3\lambda}{2}$ $x_2 = \frac{10-2\lambda}{2}$

From equation ③, since $\lambda \neq 0$

$\therefore 3x_1 + 2x_2 - 6 = 0$

$\therefore 3\left(\frac{8-3\lambda}{2}\right) + 2\left(\frac{10-2\lambda}{2}\right) - 6 = 0$

$\therefore 24 - 9\lambda + 20 - 4\lambda - 12 = 0$

$\therefore -13\lambda + 32 = 0$

$\therefore \lambda = \frac{32}{13}$

$\therefore x_1 = \frac{8 - (32/13)\cdot 3}{2} = \frac{4}{13}$

$\therefore x_2 = \frac{10 - 2\cdot(32/13)}{2} = \frac{33}{13}$

From ④

$3\left(\frac{4}{13}\right) + 2\left(\frac{33}{13}\right) - 6$

$= 0$

Since all conditions were satisfied we accept ~~we can~~ $x_1 = \frac{4}{13}$ and $x_2 = \frac{33}{13}$

$Z_{max} = 8\left(\frac{4}{13}\right) + 10\left(\frac{33}{13}\right) - \left(\frac{4}{13}\right)^2 - \left(\frac{33}{13}\right)^2$

$= 21.307$

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(Q5) Solve the following NLPP using Kuhn Tucker conditions

Maximise $z = 12x_1 \cdot x_2 + 2x_1^2 - 7x_2^2$

Subject to $2x_1 + 5x_2 \leq 98$

$x_1, x_2 \geqslant 0$

Solution :-

~~We rewrite~~ $z = 12x_1x_2 + 2x_1^2 - 7x_2^2 = f(x_1, x_2)$

$h(x_1, x_2) = 2x_1 + 5x_2 - 98$

$L = f(x_1, x_2) - \lambda [h(x_1, x_2)]$

$L = (12x_1x_2 + 2x_1^2 - 7x_2^2) - \lambda(2x_1 + 5x_2 - 98)$

Kuhn-Tucker conditions are

1) $\frac{\partial L}{\partial x_1} = 0$ 2) $\frac{\partial L}{\partial x_2} = 0$

3) $\lambda \cdot h(x_1, x_2) = 0$

4) $h(x_1, x_2) \leq 0$

5) $\lambda \geqslant 0$

We get,

1) $12x_2 + 4x_1 - 2\lambda = 0$

2) $12x_1 - 14x_2 - 5\lambda = 0$

3) $\lambda(2x_1 + 5x_2 - 98) = 0$

4) $2x_1 + 5x_2 - 98 \leq 0$

5) $x_1, x_2, \lambda \geqslant 0$

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Case 1: $\lambda = 0$

From (1) and (2) we get

$4x_1 + 12x_2 = 0$

$12x_1 - 14x_2 = 0$

$\therefore x_1 = 0$, $x_2 = 0$

These values gives $z = 0$

Case 2: $\lambda \neq 0$ so, $2x_1 + 5x_2 - 98 = 0$ ...(6)

Eliminating λ from (1) and (2) we get

$20x_1 + 60x_2 + 28x_2 - 24x_1 = 0$

$\therefore x_1 - 22x_2 = 0$

$\therefore x_1 = 22x_2$ ...(7)

Putting (7) in 6

$x_2 = 2$

$x_1 = 44$

$176 + 24 = 2\lambda$

$\lambda = 100$

$(\lambda > 0)$

These values satisfy all the necessary conditions

$Z_{max} = 2(44)^2 - 7(2)^2 + 12(44)(2) = \underline{\underline{4900}}$