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Tutorial - 5

(Q1) Use Simplex Method to solve the following LPP

Maximize $z = 5x + 3y$

Subject to $x + y \leq 2$

$5x + 2y \leq 10$

$3x + 8y \leq 12$

$x, y \geq 0$

Solution :-

$x + y + s_1 = 2$

$5x + 2y + s_2 = 10$

$3x + 8y + s_3 = 12$

$x, y, s_1, s_2, s_3 \geq 0$

Here basic variables are s_1, s_2 and s_3

Here Non basic variables are x and y

putting non basic variables as 0 in the above equations we get

$s_1 = 2$ $s_2 = 10$ $s_3 = 12$

$z = 5x + 3y + 0s_1 + 0s_2 + 0s_3$

		$C_j \rightarrow$	5	3	0	0	0	Ratio
CB	B.V	x_B	x	y	s_1	s_2	s_3	x_B / x_K
0	s_1	2	1	1	1	0	0	$2/1 = 2$
0	s_2	10	5	2	0	1	0	$10/5 = 2$
0	s_3	12	3	8	0	0	1	$12/3 = 4$
		z_j	0	0	0	0	0	
		$z_j - c_j$	-5	-3	0	0	0	

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S	x	2	1	1	1	0	0
0	S2	0	0	-3/5	-1	1/5	0
0	S3	2	0	5/3	-1	0	1/3
		z_j	5	5	5	0	0
		$z_j - C_j$	0	2	5	0	0

Since all $z_j - C_j \geq 0$

we have $x = 2$ and $y = 0$

Since $z = 5x + 3y$

We have $z = 10$

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(Q2) Min $z = x_1 - 3x_2 + 3x_3$

Subject to $3x_1 - x_2 + 2x_3 \leq 7$

$2x_1 + 4x_2 \geq -12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$

Solution :-

$z' = -x_1 + 3x_2 - 3x_3$

Subject to $3x_1 - x_2 + 2x_3 + S_1 = 7$

$2x_1 + 4x_2 - S_2 + A_2 = -12$

$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$

Here non basic variables are x_1, x_2 and S_2

Here basic variables are S_1 ~~and~~, A_2 and S_3

Putting non basic variables as 0

$S_1 = 7$ and ~~8~~ $A_2 = -12$ $S_3 = 10$

$z' = -x_1 + 3x_2 - 3x_3 + 0S_1 + 0S_2 - mA_2$

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CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	A_2	Ratio
		$C_j \rightarrow$	-1	3	-3	0	0	0	$-m$	
0	S_1	7	3	-1	2	1	0	0	0	-7
$-m$	A_2	-12	2	4	0	0	-1	0	1	-3
0	S_3	10	-4	3	8	0	0	1	0	$10/3$
		Z_j	$-2m$	$-4m$	0	0	m	0	$-m$	
		$Z_j - C_j$	$-2m+1$	$-4m-3$	3	0	m	0	0	
0	S_1	$31/3$	$5/3$	0	$4/3$	1	0	$1/3$	0	$31/5$
$-m$	A_2	$-76/3$	$22/3$	0	$-32/3$	0	-1	$-4/3$	1	$-76/22$
3	x_2	$10/3$	$-4/3$	1	$8/3$	0	0	$1/3$	0	$10/-4 = -5/2$
		Z_j	$\frac{-22m}{3} - 4$	3	$\frac{32m}{3} + 8$	0	m	$\frac{4m}{3} + 1$	$-m$	
		$Z_j - C_j$	$\frac{-22m}{3} - 3$	0	$\frac{32m}{3} + 11$	0	m	$\frac{4m}{3} + 1$	0	
-1	x_1	$31/5$	1	0	$14/5$	$3/5$	0	$1/5$	0	
$-m$	A_2	$-354/5$	0	0	$\frac{-156}{5}$	$-22/5$	-1	$-14/5$	1	
3	x_2	$58/5$	0	1	$32/5$	$4/5$	0	$3/5$	0	
		Z_j	-1	3	$\frac{156m}{5} + \frac{18}{5}$	$\frac{22m}{3} + \frac{9}{5}$	m	$\frac{14m}{5} + \frac{8}{5}$	$-m$	
		$Z_j - C_j$	0	0	$\frac{156m}{5} + \frac{33}{5}$	$\frac{22m}{5} + \frac{9}{5}$	m	$\frac{14m}{5} + \frac{8}{5}$	0	

Since all $Z_j - C_j \geq 0$, we have

$$x_1 = \frac{31}{5} \qquad x_2 = \frac{58}{5} \qquad x_3 = 0$$

$$\therefore \cancel{Z_{min}} \; Z' = -\frac{31}{5} + 3\left(\frac{58}{5}\right) \cancel{-3(0)} - 3(0)$$

$$= \frac{143}{5}$$

$$\therefore Z_{min} = -\left(\frac{143}{5}\right) = -\frac{143}{5}$$

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(Q4) Write standard form of the following LPP

Maximize $z = 3x_1 + 5x_2$

Subject to $2x_1 + 3x_2 \leq 4$

$3x_1 + 2x_2 \geq 7$

~~and~~ $x_1, x_2 \geq 0$

Solution:-

Maximise $z = 3x_1 + 5x_2 + 0s_1 + 0s_2$

Subject to ~~3~~ $2x_1 + 3x_2 + s_1 = 4$

$3x_1 + 2x_2 - s_2 = 12$

$x_1, x_2, s_1, s_2 \geq 0$

(Q10) Express the following LPP in standard form

Minimize $z = 3x_1 + 4x_2$

Subject to $2x_1 - x_2 - 3x_3 = -4$

$3x_1 + 5x_2 + x_4 = 10$

$x_1 - 4x_2 = 12$

$x_1, x_2, x_3 \geq 0$

Solution:-

Maximize $z' = -3x_1$ ~~+~~ $-4x_2$

Subject to $-2x_1 + x_2 + 3x_3 = 4$

$3x_1 + 5x_2 + x_4 = 10$

x_1 ~~+~~ $- 4x_2 = 12$

$x_1, x_2, x_3 \geq 0$

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(Q3) Max $z = 4x_1 + 10x_2$

Subject to
$2x_1 + x_2 \le 50$
$2x_1 + 5x_2 \le 100$
$2x_1 + 3x_2 \le 90$
$x_1, x_2 \ge 0$

Solution :-

$2x_1 + x_2 + S_1 = 50$ $\quad Z = 4x_1 + 10x_2 + 0S_1 + 0S_2 + 0S_3$
$2x_1 + 5x_2 + S_2 = 100$
$2x_1 + 3x_2 + S_3 = 90$

Here basic variables are S_1, S_2, S_3
Here non basic variables are x_1 and x_2

Putting non basic variables as 0 in the above equations we get
$S_1 = 50 \quad S_2 = 100 \quad S_3 = 90$

		$C_j \rightarrow$	4	10	0	0	0	
CB	B.V	X_B	x_1	x_2	S_1	S_2	S_3	Ratio
0	S_1	50	2	1	1	0	0	50
0	S_2	100	2	5	0	1	0	20
0	S_3	90	2	3	0	0	1	30
		Z_j	0	0	0	0	0	
		$Z_j - C_j$	-4	-10	0	0	0	
0	S_1	30	8/5	0	1	-1/5	0	
10	x_2	20	2/5	1	0	1/5	0	
0	S_3	30	4/5	0	0	-3/5	1	
		Z_j	4	10	0	2	0	
		$Z_j - C_j$	0	0	0	2	0	

Since all $z_j - c_j \ge 0$, we have $x_1 = 0$ and $x_2 = 20$
$\therefore Z_{max} = 4(0) + 10(20) = \underline{200}$

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(Q5) Solve the following LPP by Big-M method

Maximise $z = 3x_1 - x_2$

Subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_1, x_2 \geq 0$

Solution:-

$2x_1 + x_2 - S_1 + A_1 = 2$

$x_1 + 3x_2 + S_2 = 3$

$x_1, x_2, S_1, S_2, A_1 \geq 0$

~~Non BV~~ Here non basic variables are x_1, x_2 and S_1

Here basic variables are A_1, S_2.

Putting non basic ~~varib~~ variable as 0 in ~~to~~ above equation

$A_1 = 2$

$S_2 = 3$

$z = 3x_1 - x_2 + 0S_1 + 0S_2 - MA_1$

		$C_j \rightarrow$	3	-1	0	0	-M	Ratio
CB	BV	X_B	x_1	x_2	S_1	S_2	A_1	
-M	A_1	2	2	1	-1	0	1	$2/2 = 1$
0	S_2	3	1	3	0	1	0	$3/1 = 3$
		Z_j	-2M	-M	M	0	-M	
		$Z_j - C_j$	-2M-3	-M+1	M	0	0	
3	x_1	1	1	1/2	-1/2	0		-2
0	S_2	2	0	5/2	1/2	1		4
		Z_j	3	3/2	-3/2	0		
		$Z_j - C_j$	0	5/2	-3/2	0		

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3	x_1	3	1	3	0	1
0	s_1	4	0	5	1	1
		z_j	3	9	0	3
		$z_j - c_j$	0	10	0	3

Since all $z_j - c_j \geq 0$ the solution is optimal

$x_1 = 3$

$x_2 = 0$

Since $z = 3x_1 - x_2$

$z = 9$

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(Q6) Solve the following LPP by big M method

Maximize $Z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \le 2$

$3x_1 + 4x_2 \ge 12$

$x_1, x_2 \ge 0$

Solution :-

$2x_1 + x_2 + S_1 = 2$

$3x_1 + 4x_2 - S_2 + A_2 = 12$

Here non basic variables are x_1, x_2 and S_2

Here basic variables are A_2 and S_1

Putting non basic variable as zero

$S_1 = 2$ $A_2 = 12$

$Z = 3x_1 + 2x_2 + 0S_1 + 0S_2 - MA_2$

		$C_j \rightarrow$	3	2	0	0	$-m$	
CB	BV	XB	x_1	x_2	S_1	S_2	A_2	Ratio
0	S_1	2	2	1	1	0	0	2
$-m$	A_2	12	3	4	0	-1	1	3
		Z_j	$-3m$	$-4m$	0	m	$-m$	
		$Z_j - C_j$	$-3m-3$	$-4m-2$	0	m	0	
2	x_2	2	2	1	1	0	0	
$-m$	A_2	4	-5	0	-4	-1	1	
		Z_j	$4+5m$	2	$2+4m$	m	$-m$	
		$Z_j - C_j$	$1+5m$	0	$2+4m$	m	0	

Since the artificial variable A_2 appears not at zero level ~~and~~, ~~fe~~ so feasible solution does not exist.

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(Q7) Write the dual of the following LPP

Maximise $z = x_1 + 3x_2 - 2x_3 + 5x_4$

Subject to $3x_1 - x_2 + x_3 - 4x_4 = 2$

$5x_1 + 3x_2 - x_3 - 2x_4 = 3$

$x_1, x_2, x_3 \geq 0$, x_4 unrestricted in sign

Solution :- Here we have 4 variable and 2 constraint

Maximise $z = x_1 + 3x_2 - 2x_3 + 5x_4$

$3x_1 - x_2 + x_3 - 4x_4 \leq 2$ --- (1)

$-3x_1 + x_2 - x_3 + 4x_4 \leq -2$ --- (2)

$5x_1 + 3x_2 - x_3 - 2x_4 \leq 3$ --- (3)

$-5x_1 - 3x_2 + x_3 + 2x_4 \leq -3$ --- (4)

Let $x_4 = x_4' - x_4''$

$x_4', x_4'' \geq 0$

Maximise $z = x_1 + 3x_2 - 2x_3 + 5(x_4' - x_4'')$

$= x_1 + 3x_2 - 2x_3 + 5x_4' - 5x_4''$

$3x_1 - x_2 + x_3 - 4x_4' + 4x_4'' \leq 2$

$-3x_1 + x_2 - x_3 + 4x_4' - 4x_4'' \leq -2$

$5x_1 + 3x_2 - x_3 - 2x_4' + 2x_4'' \leq 3$

$-5x_1 - 3x_2 + x_3 + 2x_4' - 2x_4'' \leq -3$

$x_1, x_2, x_3, x_4', x_4'' \geq 0$

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$$
\begin{array}{c} \\ y_1' \\ y_1'' \\ y_2' \\ y_2'' \\ \\ \end{array}
\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4' & x_4'' \\ \end{array}
$$

$$
\begin{array}{c|ccccc|c}
 & x_1 & x_2 & x_3 & x_4' & x_4'' & \\
\hline
y_1' & 3 & -1 & 1 & -4 & 4 & \leq 2 \\
y_1'' & -3 & 1 & -1 & 4 & -4 & \leq -2 \\
y_2' & 5 & 3 & -1 & -2 & 2 & \leq 3 \\
y_2'' & -5 & -3 & 1 & 2 & -2 & \leq -3 \\
\hline
 & \geq 1 & \geq 3 & \geq -2 & \geq 5 & \geq -5 &
\end{array}
$$

Dual

$$\min z = 2y_1' - 2y_1'' + 3y_2' - 3y_2''$$
$$= 2(y_1' - y_1'') + 3(y_2' - y_2'')$$

Subject to

$$3y_1' - 3y_1'' + 5y_2' - 5y_2'' \geq 1$$
$$-y_1' + y_1'' + 3y_2' - 3y_2'' \geq 3$$
$$y_1' - y_1'' - y_2' + y_2'' \geq -2$$
$$-4y_1' + 4y_1'' - 2y_2' + 2y_2'' \geq 5$$
$$4y_1' - 4y_1'' + 2y_2' - 2y_2'' \geq -5$$

Let $y_1 = y_1' - y_1''$ $\quad y_2 = y_2' - y_2''$

$$3y_1 + 5y_2 \geq 1 \quad \cdots (5)$$
$$-y_1 + 3y_2 \geq 3 \quad \cdots (6)$$
$$y_1 - y_2 \geq -2 \quad \cdots (7)$$
$$-4y_1 - 2y_2 \geq 5 \quad \cdots (8)$$
$$4y_1 + 2y_2 \geq -5 \quad \cdots (9)$$

Combining 8 & 9

$$4y_1 + 2y_2 = -5 \quad \cdots (10)$$

$y_1, y_2 \geq 0$

Here we get 2 variable and 4 constraints.

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(Q8) Construct the dual of the following LPP

maximize $z = 4x_1 + 9x_2 + 2x_3$

Subject to $2x_1 + 3x_2 + 2x_3 \le 7$

$3x_1 - 2x_2 + 4x_3 = 5$

$x_1, x_2, x_3 \geq 0$

Solution :-

Here we have 3 variables and two constraints

maximize $z = 4x_1 + 9x_2 + 2x_3$

Subject to $2x_1 + 3x_2 + 2x_3 \le 7$

$3x_1 - 2x_2 + 4x_3 \le 5$

$-3x_1 + 2x_2 - 4x_3 \le -5$

$x_1, x_2, x_3 \geq 0$

$$
\begin{array}{c|ccc|c}
 & x_1 & x_2 & x_3 & \\
\hline
y_1 & 2 & 3 & 2 & \le 7 \\
y_2' & 3 & -2 & 4 & \le 5 \\
y_2'' & -3 & 2 & -4 & \le -5 \\
\hline
 & \geq 4 & \geq 9 & \geq 2 &
\end{array}
$$

Dual:-

minimize $z = 7y_1 + 5y_2' - 5y_2''$

Subject to

$2y_1 + 3y_2' - 3y_2'' \geq 4$

$3y_1 - 2y_2' + 2y_2'' \geq 9$

$2y_1 + 4y_2' - 4y_2'' \geq 2$

Let $y_2 = y_2' - y_2''$

So finally,

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Minimize $z = 7y_1 + 5y_2$

subject to

$\cancel{3y_1 +}\ 2y_1 + 3y_2 \geq 4$

$3y_1 - 2y_2 \geq 9$

$2y_1 + 4y_2 \geq 2$

Here we get 2 variables and 3 constraints.

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(Q9) Use Duel Simplex Method to solve the following LPP

Minimize $z = 2x_1 + 2x_2 + 4x_3$

Subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

Solution :-

We first express the given problem using $\leq$ in first constraint

So we have, $-2x_1 - 3x_2 - 5x_3 \leq -2$

Now,

$-2x_1 - 3x_2 - 5x_3 + S_1 = -2$

$3x_1 + x_2 + 7x_3 + S_2 = 3$

$x_1 + 4x_2 + 6x_3 + S_3 = 5$

$Z = 2x_1 + 2x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3$

		$C_j \rightarrow$	2	2	4	0	0	0	
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	~~Ratio~~
0	S_1	-2	-2	-3	-5	1	0	0	
0	S_2	3	3	1	7	0	1	0	
0	S_3	5	1	4	6	0	0	1	
	~~Zj~~	Z_j	0	0	0	0	0	0	
	~~Zj-Cj~~	$Z_j - C_j$	-2	-2	-4	0	0	0	
		Ratio	1	2/3	4/5				

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2	x_2	2/3	2/3	1	5/3	-1/3	0	0
0	S_2	7/3	7/3	0	16/3	1/3	1	0
0	S_3	7/3	-5/3	0	-2/3	4/3	0	1

$\therefore \quad x_1 = 0 \quad x_2 = \frac{2}{3} \quad x_3 = 0 \quad Z_{min} = \frac{4}{3}$

(Q10) (On Pgno 5)