Chapter 10

Asymmetric-Key Cryptography

Chapter 10

Objectives

- To distinguish between two cryptosystems: symmetric-key and asymmetric-key
- To introduce trapdoor one-way functions and their use in asymmetric-key cryptosystems
- To introduce the knapsack cryptosystem as one of the first ideas in asymmetric-key cryptography
- To discuss the RSA cryptosystem
- To discuss the ElGamal cryptosystem

10-1 INTRODUCTION

Symmetric and asymmetric-key cryptography will exist in parallel and continue to serve the community. We actually believe that they are complements of each other; the advantages of one can compensate for the disadvantages of the other.

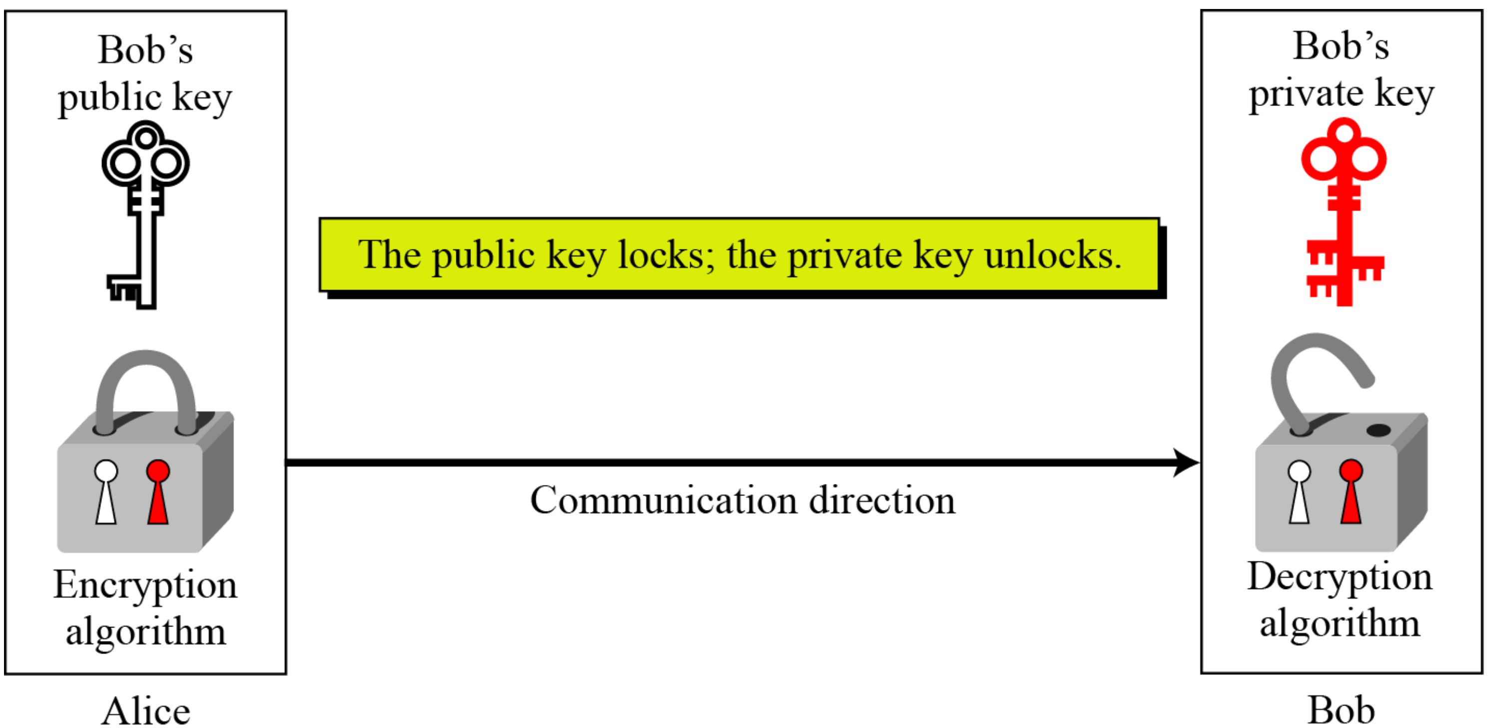
Note

Symmetric-key cryptography is based on sharing secrecy; asymmetric-key cryptography is based on personal secrecy.

10.1.1 Keys

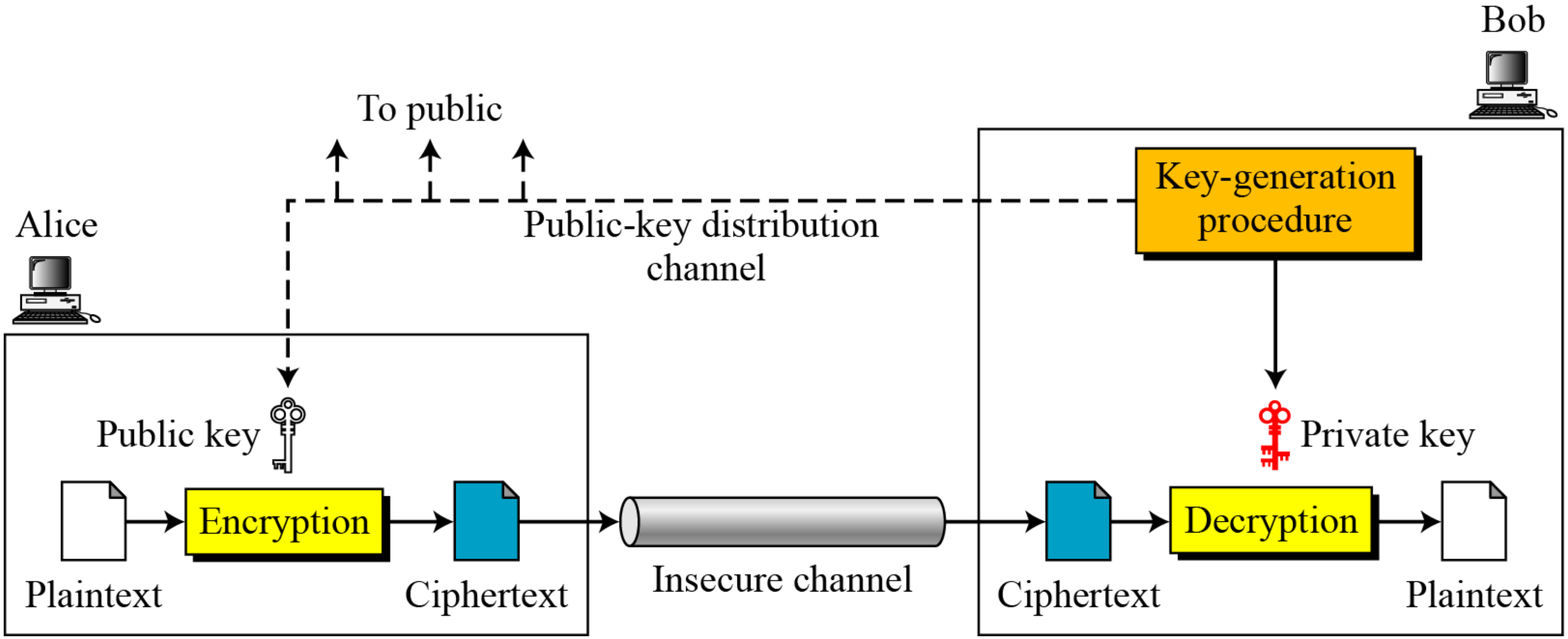
Asymmetric key cryptography uses two separate keys: one private and one public.

Figure 10.1 *Locking and unlocking in asymmetric-key cryptosystem*



10.1.2 General Idea

Figure 10.2 *General idea of asymmetric-key cryptosystem*



10.1.2 Continued

Plaintext/Ciphertext

Unlike in symmetric-key cryptography, plaintext and ciphertext are treated as integers in asymmetric-key cryptography.

Encryption/Decryption

$$\boxed{C = f\,(K_{public}, P) \qquad P = g(K_{private}, C)}$$

10.1.3 Need for Both

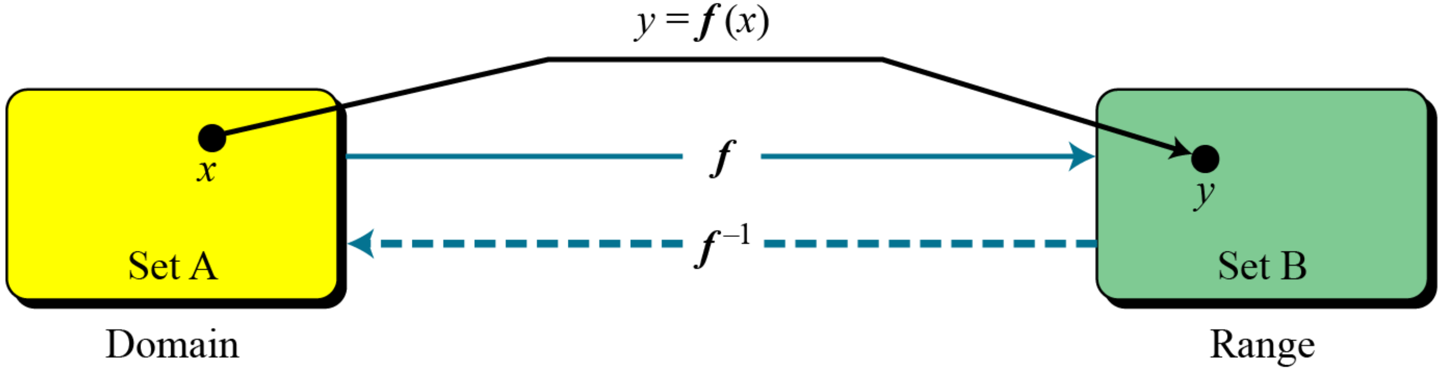
There is a very important fact that is sometimes misunderstood: The advent of asymmetric-key cryptography does not eliminate the need for symmetric-key cryptography.

10.1.4 Trapdoor One-Way Function

The main idea behind asymmetric-key cryptography is the concept of the trapdoor one-way function.

Functions

Figure 10.3 ***A function as rule mapping a domain to a range***



10.1.4 Continued

One-Way Function (OWF)

1. f is easy to compute.
2. f^{-1} is difficult to compute.

Trapdoor One-Way Function (TOWF)

3. Given y and a trapdoor, x can be computed easily.

10.1.4 Continued

Example 10. 1

When n is large, $n = p \times q$ is a one-way function. Given p and q , it is always easy to calculate n ; given n, it is very difficult to compute p and q. This is the factorization problem.

Example 10. 2

When n is large, the function $y = x^k \bmod n$ is a trapdoor one-way function. Given x, k, and n, it is easy to calculate y. Given y, k, and n, it is very difficult to calculate x. This is the discrete logarithm problem. However, if we know the trapdoor, k′ such that $k \times k' = 1 \bmod \phi(n)$, we can use $x = y^{k'} \bmod n$ to find x.

10-2 RSA CRYPTOSYSTEM

The most common public-key algorithm is the RSA cryptosystem, named for its inventors (Rivest, Shamir, and Adleman).

10.2.2 Continued

Algorithm 10.2 *RSA Key Generation*

```
RSA_Key_Generation
{
    Select two large primes p and q such that p ≠ q.
    n ← p × q
    φ(n) ← (p − 1) × (q − 1)
    Select e such that 1 < e < φ(n) and e is coprime to φ(n)
    d ← e^(−1) mod φ(n)                // d is inverse of e modulo φ(n)
    Public_key ← (e, n)                 // To be announced publicly
    Private_key ← d                     // To be kept secret
    return Public_key and Private_key
}
```

10.2.2 Continued

Encryption

Algorithm 10.3 *RSA encryption*

```
RSA_Encryption (P, e, n)                  // P is the plaintext in Zn and P < n
{
    C ← Fast_Exponentiation (P, e, n)     // Calculation of (P^e mod n)
    return C
}
```

In RSA, p and q must be at least 512 bits; n must be at least 1024 bits.

Decryption

Algorithm 10.4 *RSA decryption*

```
RSA_Decryption (C, d, n)                        //C is the ciphertext in Z_n
{
    P ← Fast_Exponentiation (C, d, n)          // Calculation of (C^d mod n)
    return P
}
```

10.2.3 Some Trivial Examples

Example 10. 5

Bob chooses 7 and 11 as *p* and *q* and calculates *n* = 77. The value of $\phi(n) = (7 - 1)(11 - 1)$ or 60. Now he chooses two exponents, *e* and *d*, from $Z_{60}*$. If he chooses *e* to be 13, then d is 37. Note that $e \times d$ mod 60 = 1 (they are inverses of each Now imagine that Alice wants to send the plaintext 5 to Bob. She uses the public exponent 13 to encrypt 5.

Plaintext: 5	$C = 5^{13} = 26 \bmod 77$	Ciphertext: 26

Bob receives the ciphertext 26 and uses the private key 37 to decipher the ciphertext:

Ciphertext: 26	$P = 26^{37} = 5 \bmod 77$	Plaintext: 5

10.2.3 Some Trivial Examples

Example 10. 6

Now assume that another person, John, wants to send a message to Bob. John can use the same public key announced by Bob (probably on his website), 13; John's plaintext is 63. John calculates the following:

Plaintext: 63	$C = 63^{13} = 28 \bmod 77$	Ciphertext: 28

Bob receives the ciphertext 28 and uses his private key 37 to decipher the ciphertext:

Ciphertext: 28	$P = 28^{37} = 63 \bmod 77$	Plaintext: 63

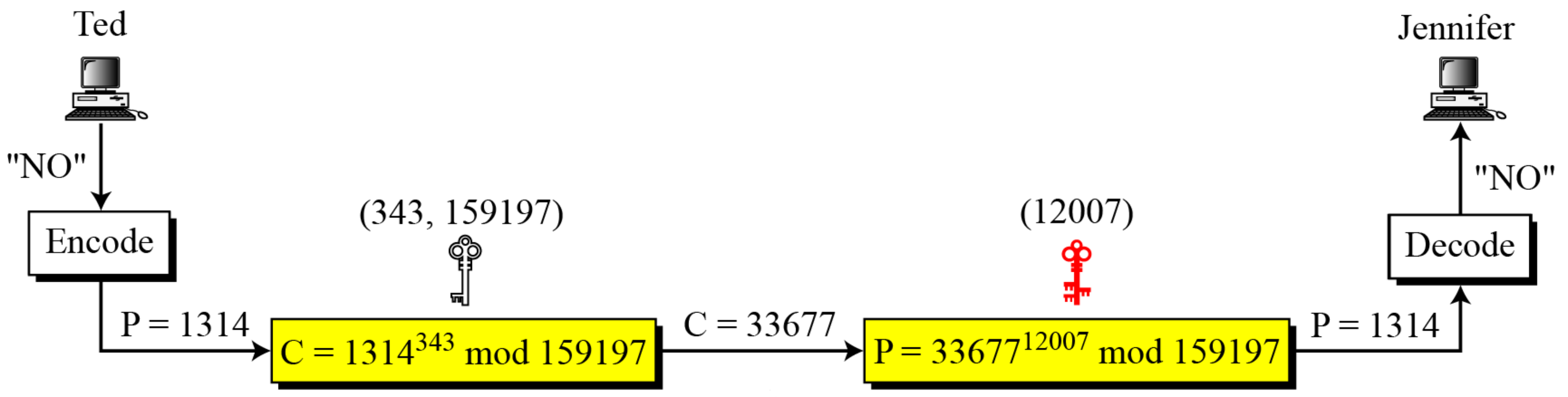
10.2.3 *Some Trivial Examples*

Example 10. 7

Jennifer creates a pair of keys for herself. She chooses p = 397 and q = 401. She calculates n = 159197. She then calculates $\phi(n)$ = 158400. She then chooses e = 343 and d = 12007. Show how Ted can send a message to Jennifer if he knows e and n.

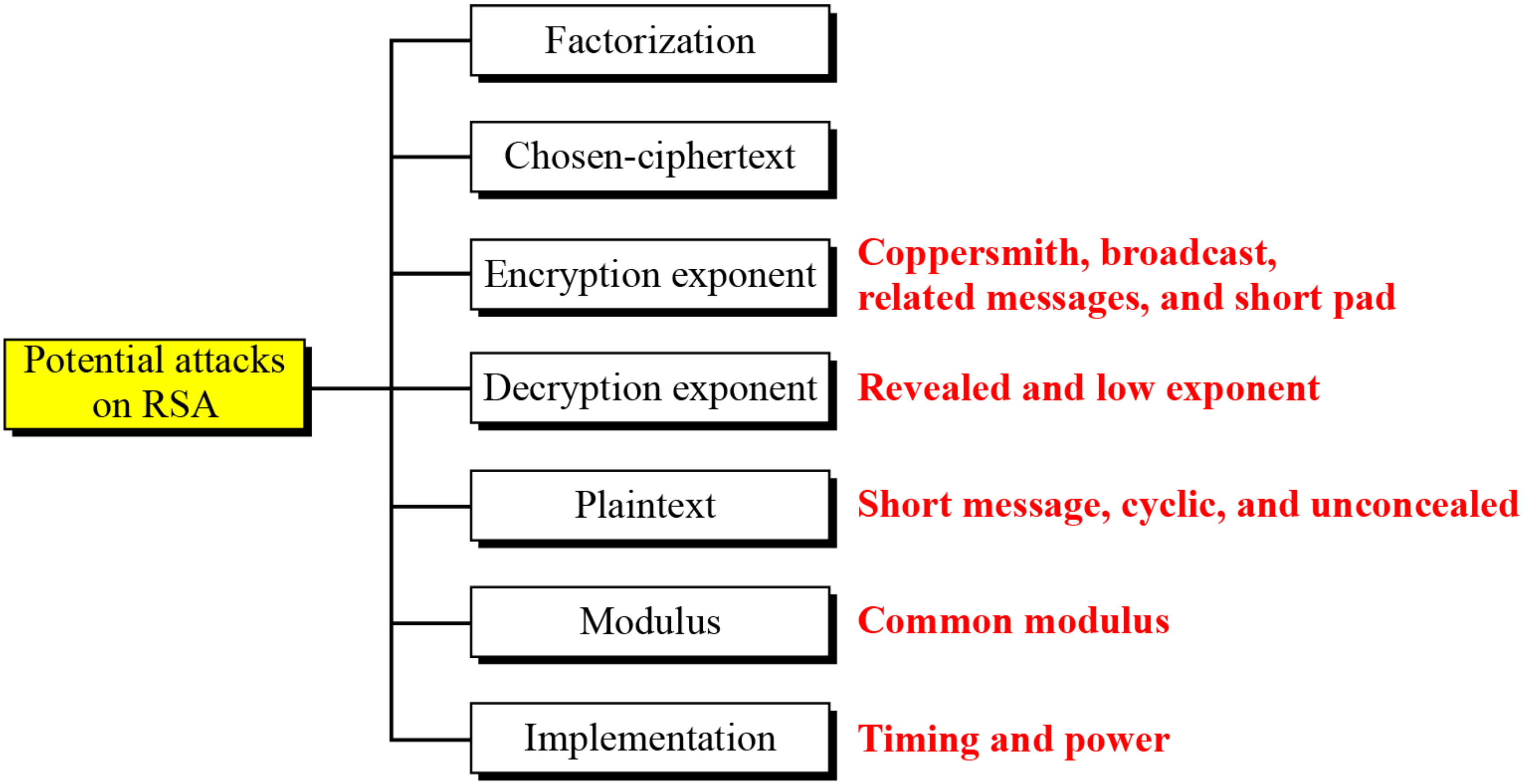
Suppose Ted wants to send the message “NO” to Jennifer. He changes each character to a number (from 00 to 25), with each character coded as two digits. He then concatenates the two coded characters and gets a four-digit number. The plaintext is 1314. Figure 10.7 shows the process.

Figure 10.7 *Encryption and decryption in Example 10.7*



10.2.4 Attacks on RSA

Figure 10.8 ***Taxonomy of potential attacks on RSA***



Numerical

Q. 1 If P=11, Q=3 and e=3,

a. Calculate the private key.

b. What will be the ciphertext for message m=15 securely to B?

- Bob chooses 7 and 11 as p and q. Calculate public and private key. What will be the ciphertext for message M= 5. Verify the plaintext.

Numerical

Q. 2 A chooses public key (e,n) as (7,119). B chooses public key (e,n) as (13,221).

a. Calculate their private keys.

b. What will be the ciphertext sent by A to B if A wishes to send message m=10 securely to B?

c. With what key will A encrypt the message m if A needs to authenticate itself to B?