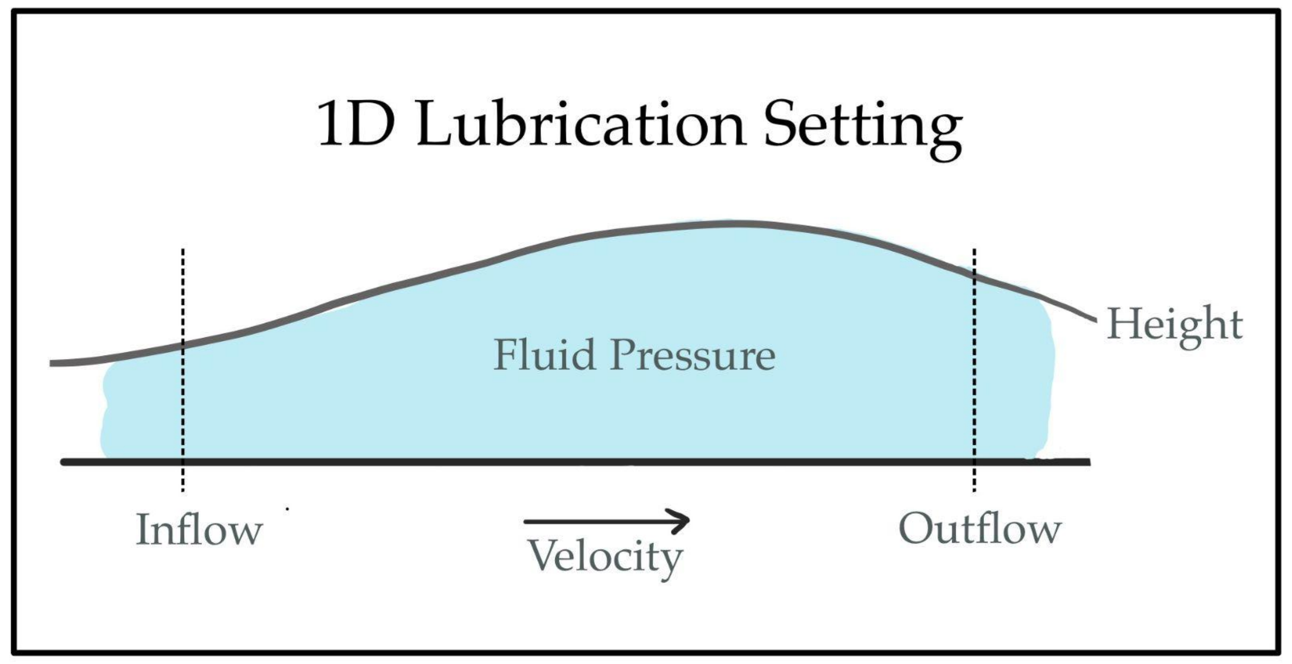
Numerical Methods for Reynolds Equation

Sarah Dennis and Thomas Fai*

Department of Mathematics, Brandeis University, Waltham, MA

Introduction

Lubrication theory is used to model the physical properties of a fluid between narrowly separated surfaces in relative motion.



- Incompressible Newtonian fluid
- Height small enough for laminar flow
- Non-slip boundary at fluid-surface interface

Applications include…

- Bearings and mechanics
- Hemodynamics
- Inkjet Printing

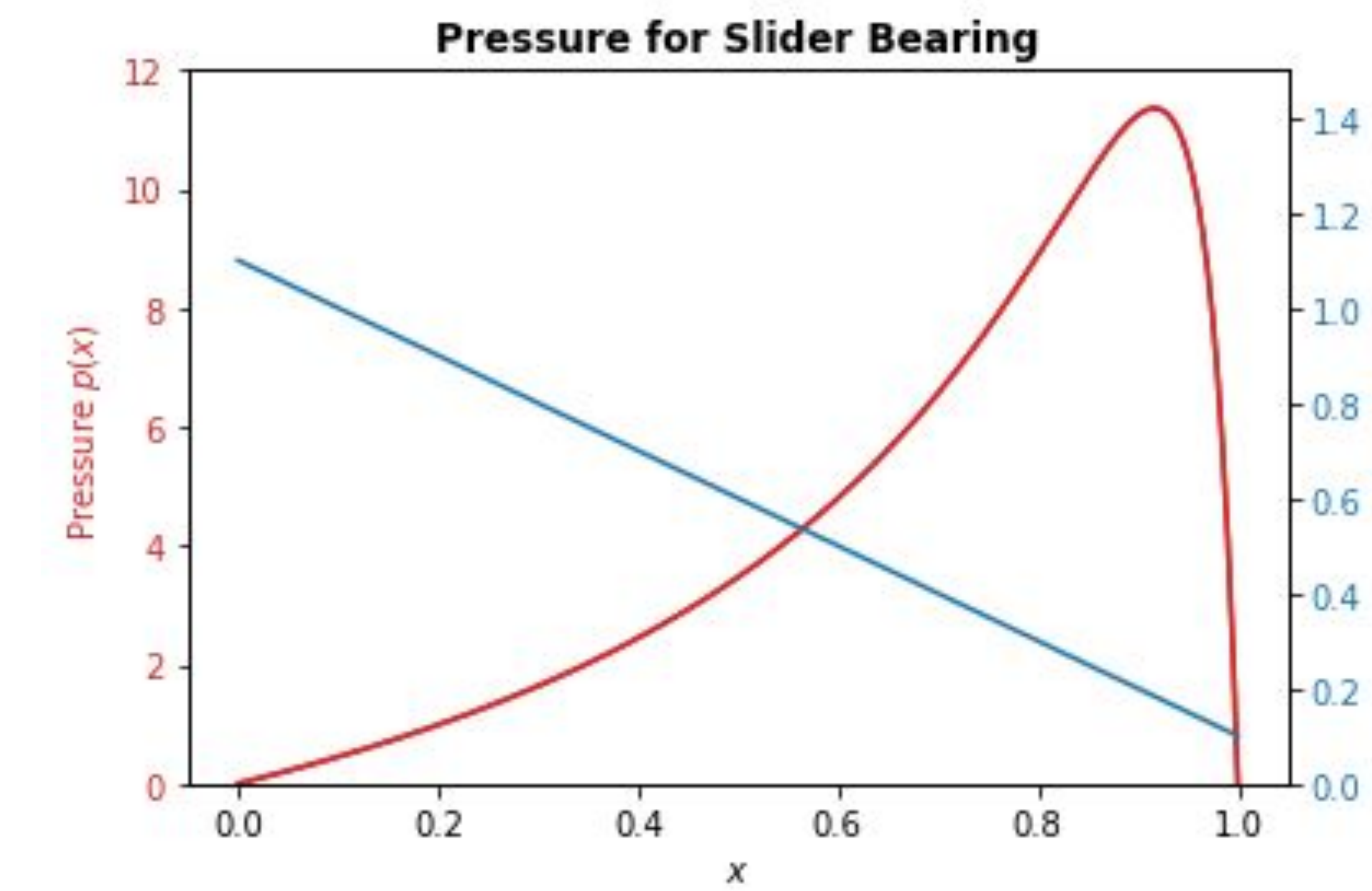
Methods

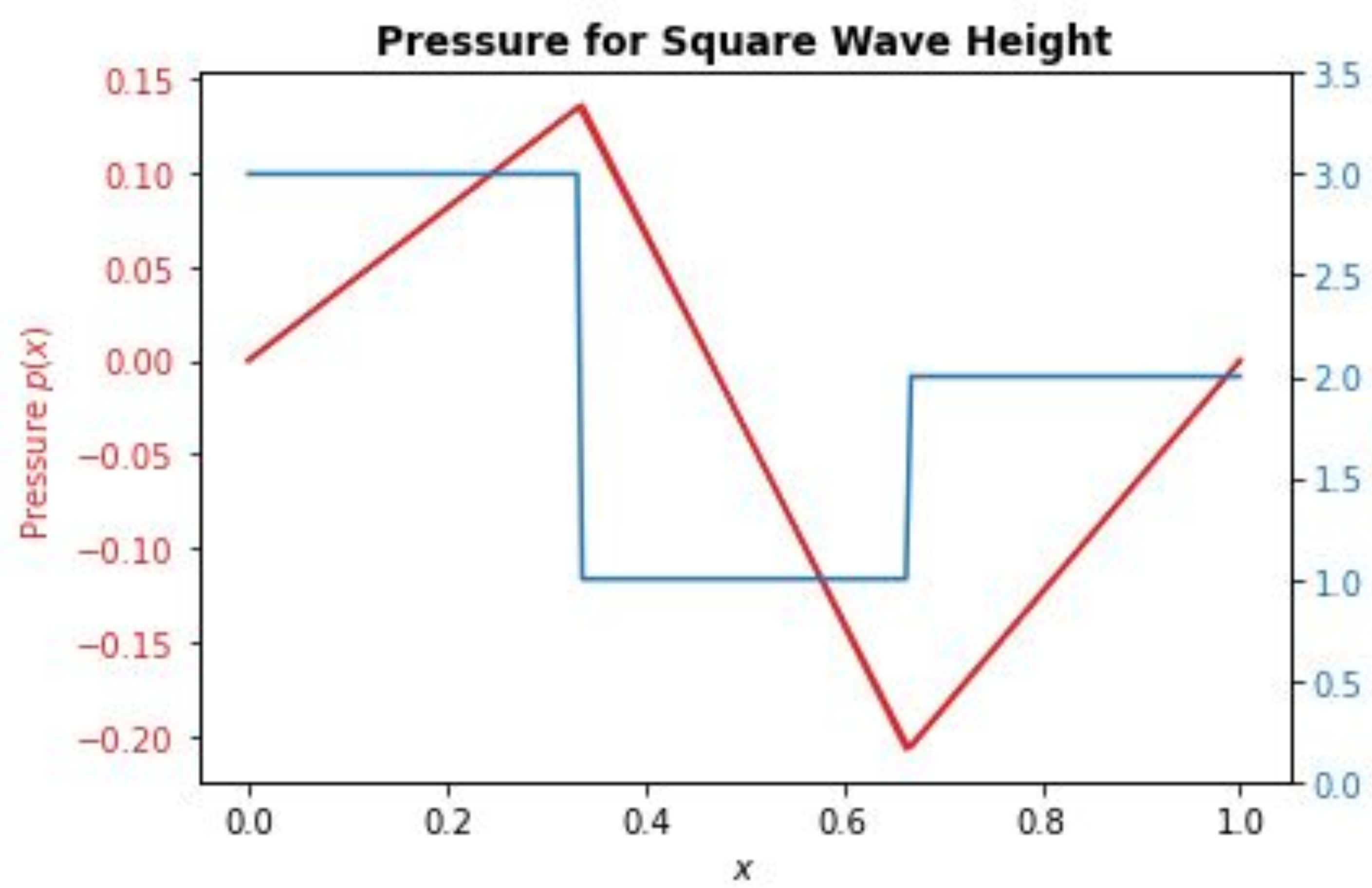
1D Reynolds Equation:

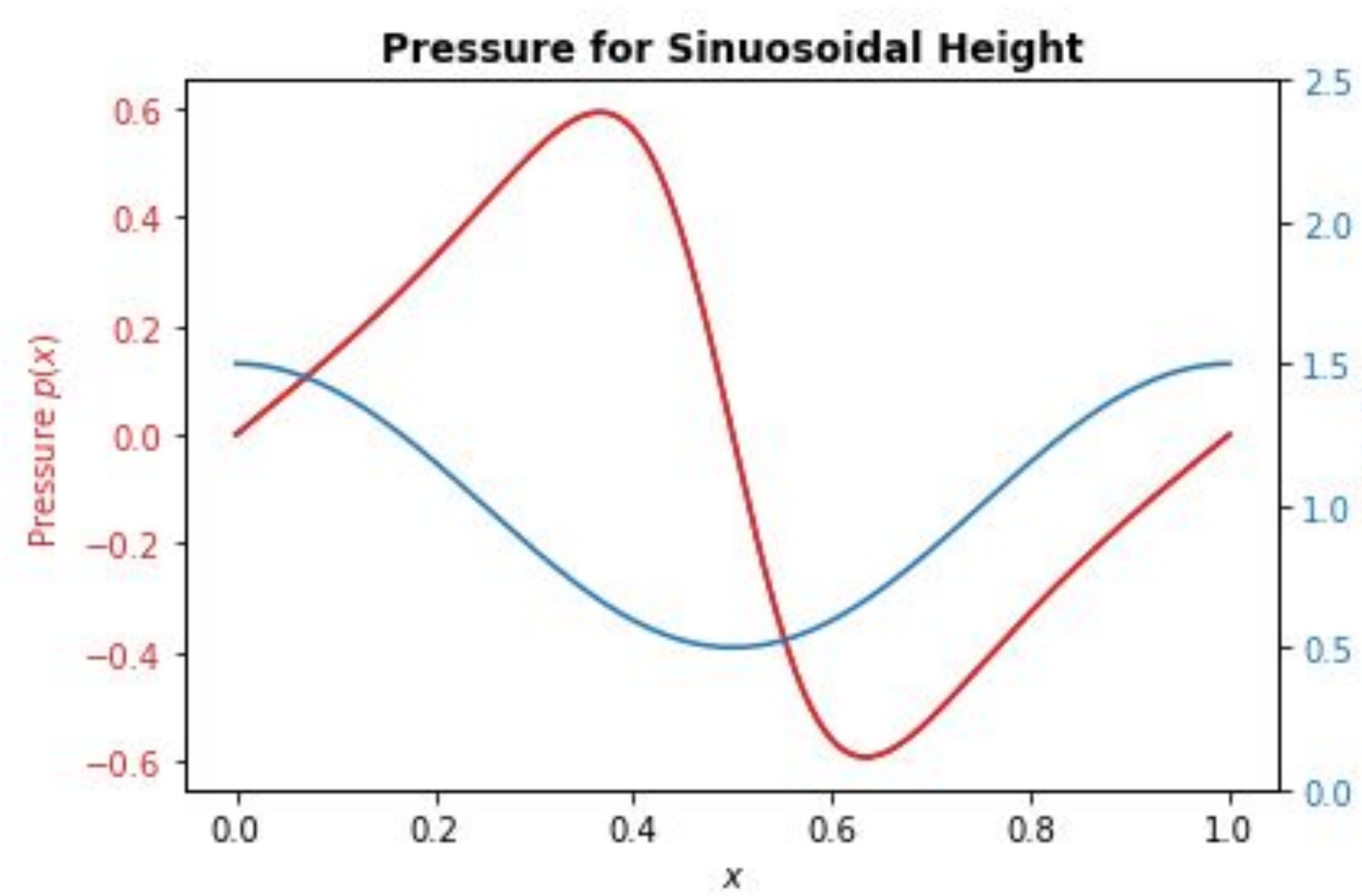
$$\frac{\partial}{\partial x}\left[h^3\frac{\partial p}{\partial x}\right] = 6\eta U\frac{\partial h}{\partial x}$$

- Height: $h(x)$
- Velocity: U
- Pressure: $p(x)$
- Viscosity: η

Results

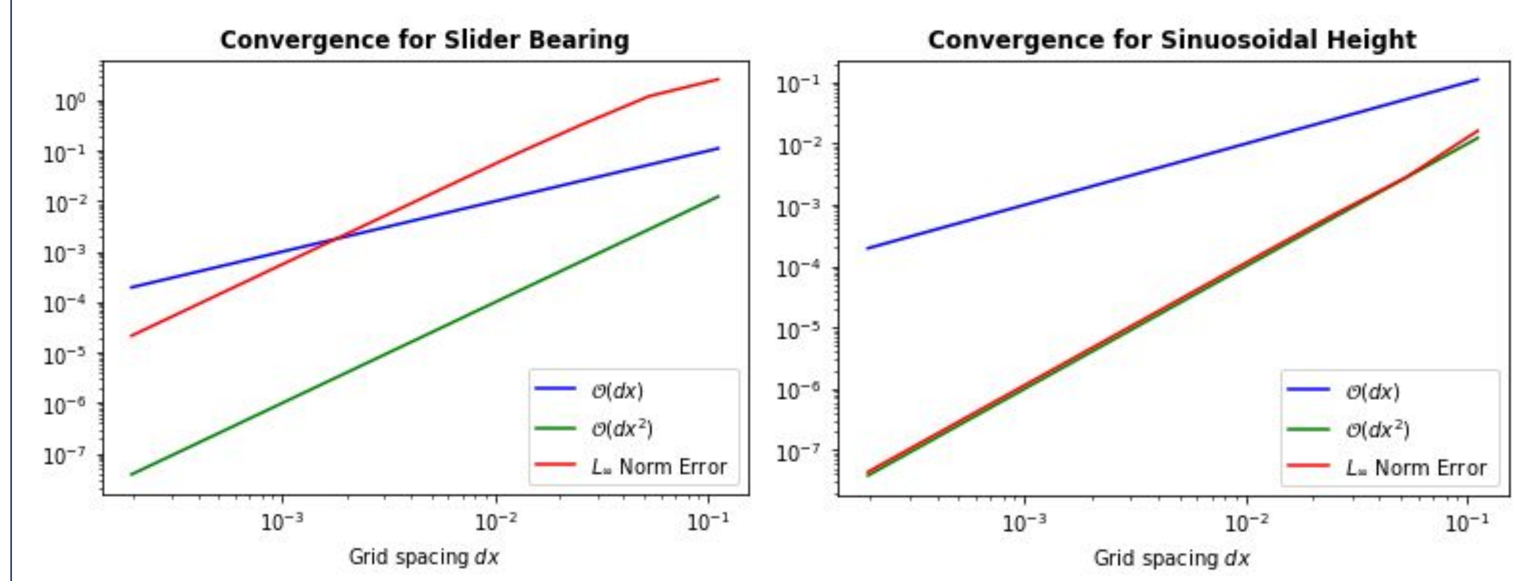




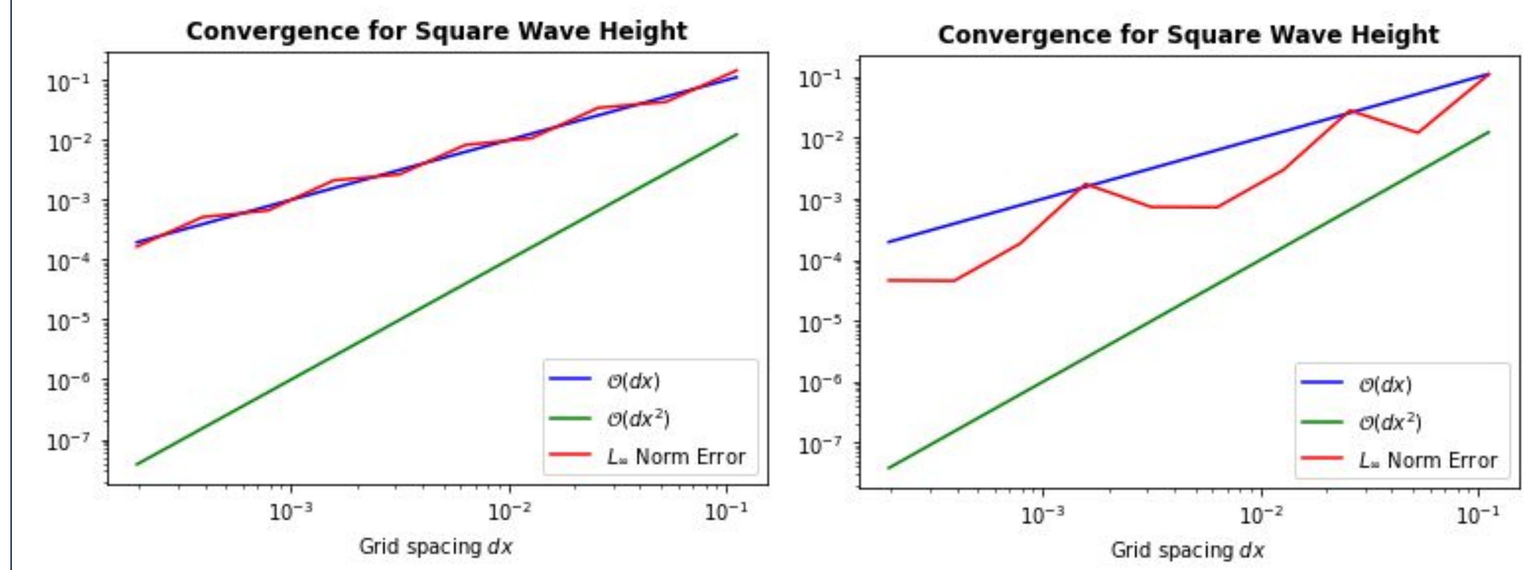


Conclusions

Convergence of Numerical Method to Analytic Solution



Convergence is order two for slider bearings and sinusoidal heights.



Convergence is order one for square wave height functions due to the discontinuous derivative.

Further directions

- Explore limiting behaviour of fluid pressure under a square wave as period decreases
- Modeling of asperity contact using square waves
- Extend to time dependent and 2D surface heights

References

1. Hamrock, B. J. (1991). *Fundamentals of Fluid Film Lubrication*. NASA.
2. San Andrés, L. (2010). One-Dimensional Fluid Film Bearings, Notes 2 Appendix. In *Modern Lubrication Theory*. Texas A&M University Digital Libraries.
3. Szern, A. Z. (1998). *Fluid Film Lubrication: Theory & design*.
4. Takeuchi, S., & Gu, J. (2019). Extended Reynolds lubrication model for incompressible Newtonian fluid. *Physical Review Fluids*, *4*(11), 114101.