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HUMAN FAILURE EVENT DEPENDENCE: WHAT ARE THE LIMITS?

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HUMAN FAILURE EVENT DEPENDENCE: WHAT ARE THE LIMITS?

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The differences between classical human reliability analysis (HRA) dependence and the full spectrum of probabilistic dependence will be examined. Positive dependence suggests as errors increase, the likelihood of subsequent errors also increases, or success increases the likelihood of subsequent success. Conversely, negative dependence suggests begets subsequent error or error begets success. Currently the typical method for dependence in HRA implements the Technique for Human Error Rate Prediction (THERP) via the use of negative dependence equations. THERP defines that the dependence between two human failure events varies at discrete levels between zero and complete dependence (CD).

Dependence in THERP does not consistently span dependence values. In contrast, probabilistic dependence addresses a continuous range. Using the laws of probability, THERP CD and maximum positive dependence (MPD) do not always agree. MPD occurs when two events overlap completely. Maximum negative dependence (MND) is the smallest amount that two events can overlap. When the minimum probability of two events overlapping is less than independence, negative dependence occurs. For example, negative dependence is when an operator fails to actuate Pump A, thereby increasing his or her subsequent chance of successfully actuating Pump B. The initial error may increase the chance of subsequent success.

Comparing THERP and dependence probability theory yields differing results; with the latter addressing negative dependence. Given that most human failure events are rare, the minimum overlap is typically 0. And when the second event is smaller than the first event, the maximum dependence is less than 1, as defined by the axioms of probability. As such, alternative dependence equations are provided along with a look-up table defining the MPD and MND given the probability of two events.

I. INTRODUCTION

Human Reliability Analysis (HRA) is the empirical understanding that humans and systems have unexpected errors. Improved reliability of our systems and human components requires an objective understanding of probability. One of the first steps toward this is providing consistent and concise notation for Human Error Probabilities (HEP). For a value to be considered a probability, it must follow the basic probability laws or axioms. If a value does not follow these laws, then the values are not considered a probability. Probability theory ranges from classical to subjective; however, all identify similar axioms, that when applied arrive at the same empirical response (Ref 1 & 3).

II. DEFINITIONS, AXIOMS, & EXAMPLES

"P(A)" is a common notation in probabilistic risk assessment and HRA. However, the usage does not always appear to be consistent. Thus, for clarity P(A) is the unconditional probability of Event A. Unconditional probabilities are defined as the independent chance a given event results out of several possible outcomes. As such, the unconditional probability of Event A is P(A), and $P(\hat{A})$ is the unconditional probability of Event not A. Likewise denote the unconditional probability of Event B as P(B), and Event not B is $P(\hat{B})$. The relationship between a binary outcome of a single event as P(A) and $P(\hat{A})$ is as follows:

$$P(A) = 1 - P(\hat{A}) \qquad (1)$$

Equation (1) is relevant because it establishes that, in a binary outcome, either Event A, such as a human action, occurs or does not occur. This assumes a certain granularity, at which human tasks are defined and analyzed. All examples illustrating probability concepts for human actions are taken from a nuclear power plant procedural manual (Ref 2). Examples from Ref 2 are steps for operator actions and the completion of a step in the manual is considered an "event."

An example, of equation (1) is the initiating event that triggers the use of the manual; it either does happen or does not. The initiating event is “Any Main Feed water or Condensate System malfunction causing a flow transient…” Thus, the sum of P(Â) and P(A) is 1 and encompasses the entire sample space, as shown in Figure 1. The sample space is the entire space that probability events can inhabit, with the whole sample space probability equal to 1. It is the symbol, of a system during a specific period of time. The system typically contains hardware components and people who control or maintain the hardware.

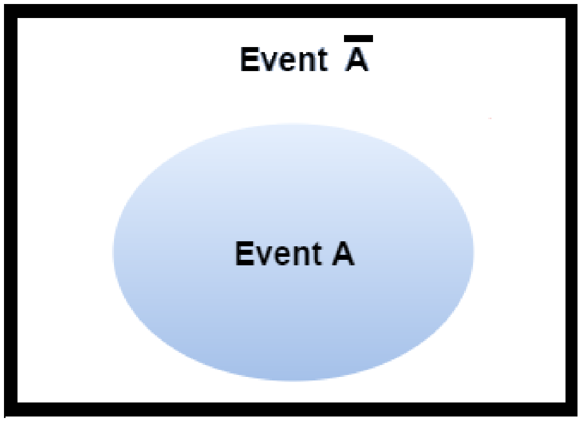

Figure 1. Venn diagram to illustrate the entire sample space is enclosed in the black square. The sum of the probabilities of Event A, and Event not A, is 1. The ellipse size is not drawn to scale and is used for representational purposes.

Two Event scenarios are selected from Ref 2 to illustrate a human action sequence with an associated estimated probability. For illustrative purposes, Events A and B will be considered sequential events, following the alphabetical order with Event A occurring first and Event B second. Example events, or tasks, may occur in other manuals, and thus the examples are limited to Ref 2. Next the axioms and laws of probabilities are defined to aid in the comparison between THERP dependence and probability theory:

- intersection,
- union,
- independence,
- disjoint and,
- dependence.

II.A. Intersection

The intersection (a.k.a., overlap) between two Events, A and B, is the incidence that both Event A and Event B occur. An example of an intersection between Event A and B is:

- Event A “the operator checks whether the main feed water pump tripped” and
- Event B the operator confirms “the initial reactor power is less than 90%.”

In Ref 2 these events only occur once sequentially. The definition of the intersection between Event A and B is denoted A∩B, and shown in Figure 2.

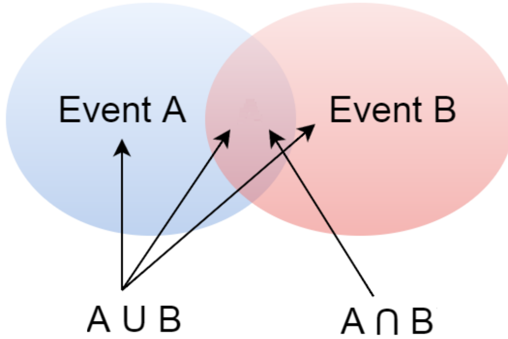

Figure 2. Venn diagrams of event intersection, A∩B and union, AUB.
The ellipse sizes are not drawn to scale and are used for representational purposes.

II.B. Union

The union between two Events, A and B, is when Event A, Event B, or both can occur within the same sample space. The union of events A and B can occur when operator checks if "the main feed water pump trips" or operator checks if "the initial reactor power is less than 90%" or both. More specifically as seen in Figure 2, there are locations in the sample space where Event A, operator checks if "main feed water pump tripped" occurs with Event $\hat{B}$, the operator checks if the"initial reactor power is greater than 90% or is not checked." And, of course, the inverse Event $\hat{A}$, operator checks if "the main feed water pump is not tripped" and Event B, operator checks if "the initial reactor is checked and it's less than 90%" are considered part of the union. The union between events A and B is denoted AUB and labeled in the Venn diagram in Figure 2.

II.C. Independence

Independence between two events is when the probability of one event occurring does not affect the probability of the other event occurring. This is depicted in Figure 2 and in the top right of Figure 3. Independent events do overlap. When the unconditional probabilities of two independent events are greater than 0, a non-zero probability for both events occurring is described as per equation (2).

$$P(A)*P(B)= P(A\cap B) \qquad (2)$$

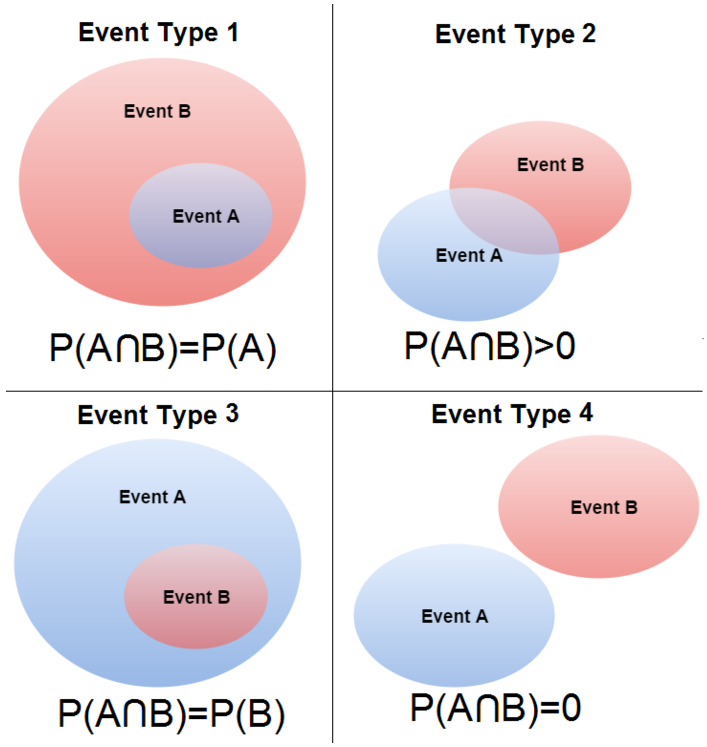

Figure 3. Venn diagrams of four configurations for two events. The Event space is each individual quadrant of the figure, and the probability of quadrant sums to 1.

II.D. Disjoint

Events are disjoint, or mutually exclusive, when two events cannot occur at the same time. For example, Events A and Â cannot occur at the same exact time. When two events are disjoint, such as in Figure 3 (bottom right), the notation for the probability of Event A and Event B is equation (3).

$$P(A \cap B) = 0 \qquad (3)$$

When two events are disjoint they are at the maximum negative dependence (MND). There are cases when events are not disjoint but a MND is still defined. This situation occurs when two event probabilities sum to greater than 1 (i.e., P(A)+P(B)>1). In this specific scenario the events will be forced to have some overlap and are therefore not disjoint. MND is when the intersection, A∩B, is at the lowest posible probability (Figure 3 bottom right). And maximum postive dependence (MPD) is when the intersection, A∩B, is at the largest possible probablity (Figure 3 top and bottom left).

II.E. Dependence

Conditional probability, or dependence, is defined as the success or failure on one task, which is influenced by the success or failure in another task. The notation 'P(A|B)' is the probability of Event A given the probability of Event B, with the virtical bar '|' defined as given. For the purposes of consistent notation, the conditional probability between two Events, A and B, will be symbolized as P(A|B), P(B|A), P(Â|B), P(B|Â), P(A|B̂), P(B̂|A), P(Â|B̂), and P(B̂|Â). Utilizing the definition of intersection, dependence can be defined as in equations (4) and (5).

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \qquad (4)$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \qquad (5)$$

Moreover, all dependence between two probabilities must follow Bayes law as displayed in equation (6).

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)} \qquad (6)$$

And utilizing equations (2), (4) and (5) the following equations (7) and (8) are derived as per the definition of independent events.

$$P(B|A)=P(B|\hat{A})=P(B) \qquad (7)$$

$$P(A|B) = P(A|\hat{B}) = P(A) \qquad (8)$$

II.F. Examples

Examples to better explain the axioms of probability are taken from the nuclear power plant operations manual (Ref 2). Two Events, A and B, have four different configurations considered, as shown in the Venn diagrams of Figure 3. Given the spectrum of possible event arrangements in Figure 3, the event space is each quadrant of the figure, and the probability of each quadrant sums to 1. In Ref 2 there are several examples to expand upon the four configurations of Events A and B in Figure 3:

Event Type 1: Event A can be inside Event B. This is the maximum that Event A can be dependent on B.
- Event A, The operator verifies that the “feedback loops are not in service”
- Event B, The operator “Check(s) the Turbine transient Terminated.” Event B occurs twice and is not always after Event A.
- Thus Event B may occur more frequently than Event A, and can be illustrated as an Event Type 1.

Event Type 2: Event A and B can have some overlap, but not total overlap
- Same example as for the definition of the intersection and union.

Event Type 3: Event B can be inside Event A. This is the maximum that Event B can be dependent on A and in essence this is a reversal of event type 1.
- When restricted to only the event sequences within an operator manual, most events can only be achieved by completing the prior task, as such they are an Event Type 3.
- Event A is The operator checks the “initial reactor power less than 80%” if not true then
- Event B is The operator verifys that the “Isolate steam generator blowdown”
- The only time, in this manual, that “Isolate steam generator blowdown” is completed is after “initial reactor power (is not) less than 80%.”
- Thus, Event B only can happen after Event A.

Event Type 4: A and B can never overlap; this describes a disjoint or negative maximum dependence.
- For example, the initiation is “check DEH controlling Turbine Valves properly”
- If initiation is not obtained, Event A is “go to step 6”
- If initiation is obtained, Event B is “go to step 7”
- Events A and B do not occur within the same space; either step 6 or 7 is completed—but not both—right after step 5.

III. THERP

THERP dependence is used to predict the HEP of an event given the unconditional probabilities of the event and of the steps preceding it, along with the dependence levels. THERP only defined positive dependence at five levels. Once again for notation purposes the first Event is A, and the second Event is B. The five positive dependence levels are as follows: zero (ZD), low (LD), moderate (MD), high (HD) and complete dependence (CD). The THERP dependence equations are (9)-(13) below and based upon Ref 4, which are originally based upon Ref 5 pg 10-27.

$$P(B \mid A \mid ZD) = P(A) * P(B) \qquad (9)$$

$$P(B|A|LD) = P(A) * \frac{1+(19*P(B))}{20} \tag{10}$$

$$P(B|A|MD) = P(A) * \frac{1+(6*P(B))}{7} \tag{11}$$

$$P(B|A|HD) = P(A) * \frac{1+P(B)}{2} \tag{12}$$

$$P(B|A|CD) = P(A) \tag{13}$$

The P(B|A|HD), as per equation (12), is the probability of B given A at HD. Another way to consider THERP is that ZD is the lowest level of dependence (independence), and CD is the largest level of dependence. As such, the range of THERP conditional probabilities, as defined by equations (9)-(13), is between the unconditional probability of Event B and Event A to the unconditional probability of Event A. However, this does not always hold true.

III.A. Example

For example, consider Event A that “the plant is running,” P(A) = 0.7, and Event B “the safety system is triggered while the plant is engaged” as P(B) = 0.04. Then, as per equations (9) and (13), the THERP positive dependence range is 0.028 to 0.7. And, if expert opinion deduced that the level of dependence is low between these events, THERP’s low dependence equation is implemented as per equation (14).

$$P(B|A|LD) = P(A) * \frac{1+(19*P(B))}{20} = 0.7 * \frac{1+(19*0.04)}{20} \tag{14}$$

Next consider the probability theory with the same values of P(A) = 0.7 and P(B) = 0.04. The MPD is when Event B only ever occurs after Event A, as in Figure 4 (left). So, the maximum probability of intersection in this case is P(A∩B)=P(B)=0.04. Thus, when evaluating with equation (4), P(B|A|MPD) = P(A∩B)/P(A) = 0.04/0.7 = 0.057. Then considering MND, Event A and B are disjoint, because P(A)+P(B)=0.74, which is less than 1. So P(A∩B)=0, Figure 4 (right), and using equation (4), P(B|A)=0.

Figure 4. (left) A probability Venn diagram illustrating the MPD. (right) A probability Venn diagram illustrating the MND between Events A and B. These images are drawn to scale to represent P(A)=0.7, and P(B)=0.04.

Thus, the range produced by probability theory (0 to 0.057) is in conflict with the range of THERP (0.028 to 0.7). And the amount of overlap between the two ranges is very small. In addition to the aforementioned difference in dependence, THERP never empirically defines when the conditional probability (dependence) is less than it would be if the events were independent. This situation is negative dependence. Negative dependence can be configured similar to Figure 3 (bottom right) and Figure 4 (right). Intuitively MND is when the events are disjoint, but this can only occur when the two event probabilities summed together are less than 1. When the probability of Events A and B is summed to be greater than 1, the MND of the two events, P(B|A|MND), must be greater than 0.

Another important note is that MPD occurs when one event exists within another, similar to Figure 3 (left top and bottom). However, the MPD changes with the event probabilities, and it cannot always be equal to 1 (see Appendix A). This is different from the definition of CD in THERP. These dependence values, in Appendix A, are based on Bayes Law in equation (6). The next section provides a mathematical proof of the empirical difference between THERP and Bayes Law for background purposes.

IV. COOPERATION BETWEEN BAYES & THERP

Having previously presented the basic framework of probability and corresponding THERP rules an empirical approach, combining the THERP dependence equations and Bayes law will be considered. Proposed is that the THERP equations and Bayes Law do not work together within the same system of equations. First the general form of the THERP equations is defined as:

$$P(B \mid A) = \frac{1+(\Psi * P(B))}{\Psi + 1} * P(A) \tag{15}$$

$$P(A \mid B) = \frac{1+(\Psi * P(A))}{\Psi + 1} * P(B) \tag{16}$$

where Ψ is the 'dependence level' and is discreetly defined from 0 to 19. When a dependence level of 0 is defined, this indicates THERP CD. And when dependence level is 19, this behaves as THERP low dependence. If the variable 'dependence level' in equations (15) and (16) was allowed to increase to infinity, this would be the THERP zero dependence case (Ref 4).

Next Bayes Law is defined in (6). To start the PROOF substitute the THERP general form from equation (15) and (16) into Bayes law in equation (6) and the following is achieved:

$$\frac{1+(\Psi * P(B))}{\Psi + 1} * P(A) = \frac{\frac{1+(\Psi * P(A))}{\Psi + 1} * P(B) * P(B)}{P(A)} \tag{17}$$

Re-arranging equation (17) creates the following forms:

$$\frac{\frac{1+(\Psi * P(B))}{\Psi + 1} * P(A) * P(A)}{\frac{1+(\Psi * P(A))}{\Psi + 1} * P(B) * P(B)} = 0 \tag{18}$$

$$\frac{1+(\Psi * P(B))}{\Psi + 1} * P(A) * P(A) = \frac{1+(\Psi * P(A))}{\Psi + 1} * P(B) * P(B) \tag{19}$$

Then cancel out the "(Ψ+1)" terms from (19):

$$1+(\Psi * P(B)) * P(A) * P(A) = 1+(\Psi * P(A)) * P(B) * P(B) \tag{20}$$

However, equation (20) is clearly false, unless the P(A)= P(B):

$$1+(\Psi * P(B)) * P(B) * P(B) = 1+(\Psi * P(B)) * P(B) * P(B) \quad (21)$$

Cancel out "(1+(Ψ+P(B)))*P(B)" in the numerator and denominator from (22) to achieve the following:

$$P(B) = P(B) \quad (22)$$

Finally, as shown above, this mathematical proof has shown that THERP and Bayes Law do not agree. THERP has existed for several decades and its use has provided insightful answers in HRA. However, THERP equations hold true under certain circumstances. This will become very problematic if a large amount of data is applied to HRA event trees or a dynamic HRA simulation is desired. Moving forward, dependence equations that follow probability laws and act similar to THERP equations are proposed in the next section.

V. SOLUTIONS

Rather than navigate the dependence spectrum unassisted, a reference table has been provided for P(B|A) in Appendix A. The first row of the table is the unconditional probability of Event A, and the first column is the unconditional probability of Event B. The interior cells of the table are the P(B|A) with the form of (MPD, MND). Using the prior example, the unconditional probability of accomplishing Event A is P(A)=0.7, and Event B is P(B)=0.04, the following is recovered from the look-up table: (0, 0.057). Independence (INDEP) is defined as P(B|A|INDEP)=P(B)=0.04. The maximum positive dependence probability is termed MPD, with a notation of P(B|A|MPD) = 0.057. And the maximum negative dependence probability is MND, defined as P(B|A|MND)=0. The spectrum of dependence is displayed Figure 5, and is based on Figure 10-3 in THERP (Ref 5).

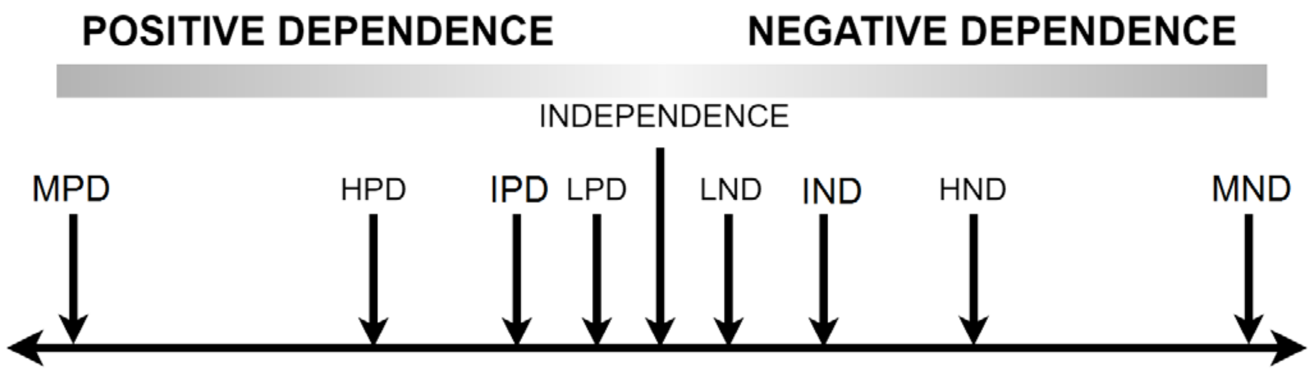

Figure 5. Spectrum of dependence between events.

The following is a proposed scheme for quantifying P(B|A) when the level of dependence (D) is specified as high (H), intermediate (I), or low(L) and as positive (P) or negative (N) (so, for example, "LPD" means "low positive dependence" and "LNP" means "low negative dependence"). For high positive dependence (HPD) when this is identified as the level of dependence between two events, then the mean between MPD and INDEP is calculated:

$$P(\mathrm{B} \mid \mathrm{A} \mid \mathrm{HPD}) = \frac{\mathrm{MPD} + \mathrm{INDEP}}{2} \quad (23)$$

The intermediate positive dependence (IPD) occurring between MPD and INDEP is calculated:

$$P(\mathrm{B} | \mathrm{A} | \mathrm{IPD}) = \frac{\mathrm{HPD} + \mathrm{INDEP}}{2} \quad (24)$$

The low positive dependence (LPD) occurring between MPD and INDEP is calculated:

$$P(\mathrm{B}|\mathrm{A}|\mathrm{LPD}) = \frac{\mathrm{IPD}+\mathrm{INDEP}}{2} \tag{25}$$

And the independence (INDEP), based upon equation (7) and (8) is defined as:

$$P(\mathrm{B}|\mathrm{A}|\mathrm{INDEP}) = P(\mathrm{B}) \tag{26}$$

For negative dependence, simply replace the MPD with MND.

In Figure 6, the axioms of probability system of equations and the THERP system of equations are graphed when P(A)=0.7. As can be seen upon visual inspection there are differences in the quantification methods.

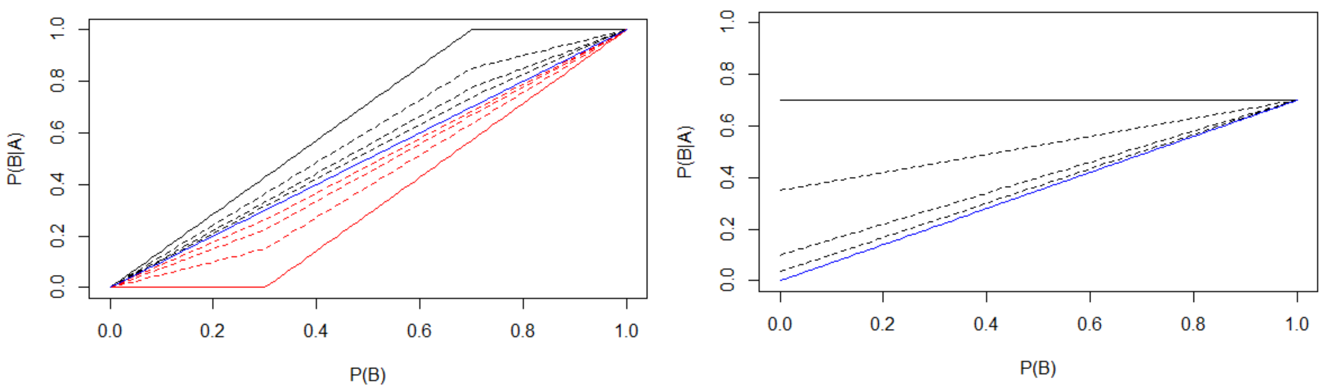

Figure 6. (Left) Implementation of equation (24)-(27) when P(A)=0.7 at 8 different dependence levels and independence. (Right) THERP Equations (9)-(13) are implemented when P(A)=0.7. In both images, independence is indicated by a light blue line.

VI. CONCLUSION

THERP dependence has been used ubiquitously for decades, and has provided approximations of the dependencies between two events. Since its inception, computational abilities have increased exponentially, and alternative approaches that follow the laws of probability dependence need to be implemented. These new approaches need to consider negative dependence and identify when THERP output is not appropriate.

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APPENDIX A

Look up tables defining the maximum positive dependence (MPD) and maximum negative dependence (MND) in the form of (MND,MPD) for the probability of Event B given Event A[P(B|A)]. The top most row that contains numbers, on both tables, houses the unconditional probability of event B, assuming that Event B is the second event or task. The first left column that contains numbers, on both tables, houses the unconditional probability of event A, assuming that Event A is the first event or task. Table 2 is an extension of Table 1, with P(B) values going from 0.05 up to 1.0. In the part shaded black, the (MND,MPD) pairs are all (0,1). In the gray-shaded area, the MND is 0. In the blue area, the MPD is 1. In the white region, 0<MND<MPD<1.

For example, if the following unconditional probabilities were assumed: P(A)=0.5 and P(B)=0.6 then the following cell would be recovered: (0.17,0.83). This indicates that the MPD is 0.83 and the MND is 0.17. This is because P(A)+P(B) = 0.5+0.6=1.1. Thus, the events cannot be disjoint. So the when the equations are applied the following is true P(B|A|HPD)=0.715, P(B|A|IPD)=0.658, P(B|A|LPD)=0.628 and P(B|A|INDP)=0.6. The same holds true for the negative dependence.

Table 1

P(B|A)		P(B)													
		0	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.01	0.02	0.03	0.04
	0	(0,1)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
	0.001	(0,0)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
	0.002	(0,0)	(0,0.5)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
	0.003	(0,0)	(0,0.333)	(0,0.667)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
	0.004	(0,0)	(0,0.25)	(0,0.5)	(0,0.75)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
	0.005	(0,0)	(0,0.2)	(0,0.4)	(0,0.6)	(0,0.8)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
	0.006	(0,0)	(0,0.167)	(0,0.333)	(0,0.5)	(0,0.667)	(0,0.833)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
	0.007	(0,0)	(0,0.143)	(0,0.286)	(0,0.429)	(0,0.571)	(0,0.714)	(0,0.857)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
	0.008	(0,0)	(0,0.125)	(0,0.25)	(0,0.375)	(0,0.5)	(0,0.625)	(0,0.75)	(0,0.875)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
	0.009	(0,0)	(0,0.111)	(0,0.222)	(0,0.333)	(0,0.444)	(0,0.556)	(0,0.667)	(0,0.778)	(0,0.889)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
	0.01	(0,0)	(0,0.1)	(0,0.2)	(0,0.3)	(0,0.4)	(0,0.5)	(0,0.6)	(0,0.7)	(0,0.8)	(0,0.9)	(0,1)	(0,1)	(0,1)	(0,1)
	0.02	(0,0)	(0,0.05)	(0,0.1)	(0,0.15)	(0,0.2)	(0,0.25)	(0,0.3)	(0,0.35)	(0,0.4)	(0,0.45)	(0,0.5)	(0,1)	(0,1)	(0,1)
	0.03	(0,0)	(0,0.033)	(0,0.067)	(0,0.1)	(0,0.133)	(0,0.167)	(0,0.2)	(0,0.233)	(0,0.267)	(0,0.3)	(0,0.333)	(0,0.667)	(0,1)	(0,1)
	0.04	(0,0)	(0,0.025)	(0,0.05)	(0,0.075)	(0,0.1)	(0,0.125)	(0,0.15)	(0,0.175)	(0,0.2)	(0,0.225)	(0,0.25)	(0,0.5)	(0,0.75)	(0,1)
P(A)	0.05	(0,0)	(0,0.02)	(0,0.04)	(0,0.06)	(0,0.08)	(0,0.1)	(0,0.12)	(0,0.14)	(0,0.16)	(0,0.18)	(0,0.2)	(0,0.4)	(0,0.6)	(0,0.8)
	0.06	(0,0)	(0,0.017)	(0,0.033)	(0,0.05)	(0,0.067)	(0,0.083)	(0,0.1)	(0,0.117)	(0,0.133)	(0,0.15)	(0,0.167)	(0,0.333)	(0,0.5)	(0,0.667)
	0.07	(0,0)	(0,0.014)	(0,0.029)	(0,0.043)	(0,0.057)	(0,0.071)	(0,0.086)	(0,0.1)	(0,0.114)	(0,0.129)	(0,0.143)	(0,0.286)	(0,0.429)	(0,0.571)
	0.08	(0,0)	(0,0.012)	(0,0.025)	(0,0.038)	(0,0.05)	(0,0.062)	(0,0.075)	(0,0.088)	(0,0.1)	(0,0.112)	(0,0.125)	(0,0.25)	(0,0.375)	(0,0.5)
	0.09	(0,0)	(0,0.011)	(0,0.022)	(0,0.033)	(0,0.044)	(0,0.056)	(0,0.067)	(0,0.078)	(0,0.089)	(0,0.1)	(0,0.111)	(0,0.222)	(0,0.333)	(0,0.444)
	0.1	(0,0)	(0,0.01)	(0,0.02)	(0,0.03)	(0,0.04)	(0,0.05)	(0,0.06)	(0,0.07)	(0,0.08)	(0,0.09)	(0,0.1)	(0,0.2)	(0,0.3)	(0,0.4)
	0.2	(0,0)	(0,0.005)	(0,0.01)	(0,0.015)	(0,0.02)	(0,0.025)	(0,0.03)	(0,0.035)	(0,0.04)	(0,0.045)	(0,0.05)	(0,0.1)	(0,0.15)	(0,0.2)
	0.3	(0,0)	(0,0.003)	(0,0.007)	(0,0.01)	(0,0.013)	(0,0.017)	(0,0.02)	(0,0.023)	(0,0.027)	(0,0.03)	(0,0.033)	(0,0.067)	(0,0.1)	(0,0.133)
	0.4	(0,0)	(0,0.002)	(0,0.005)	(0,0.008)	(0,0.01)	(0,0.012)	(0,0.015)	(0,0.017)	(0,0.02)	(0,0.022)	(0,0.025)	(0,0.05)	(0,0.075)	(0,0.1)
	0.5	(0,0)	(0,0.002)	(0,0.004)	(0,0.006)	(0,0.008)	(0,0.01)	(0,0.012)	(0,0.014)	(0,0.016)	(0,0.018)	(0,0.02)	(0,0.04)	(0,0.06)	(0,0.08)
	0.6	(0,0)	(0,0.002)	(0,0.003)	(0,0.005)	(0,0.007)	(0,0.008)	(0,0.01)	(0,0.012)	(0,0.013)	(0,0.015)	(0,0.017)	(0,0.033)	(0,0.05)	(0,0.067)
	0.7	(0,0)	(0,0.001)	(0,0.003)	(0,0.004)	(0,0.006)	(0,0.007)	(0,0.009)	(0,0.01)	(0,0.011)	(0,0.013)	(0,0.014)	(0,0.029)	(0,0.043)	(0,0.057)
	0.8	(0,0)	(0,0.001)	(0,0.002)	(0,0.004)	(0,0.005)	(0,0.006)	(0,0.008)	(0,0.009)	(0,0.01)	(0,0.011)	(0,0.012)	(0,0.025)	(0,0.038)	(0,0.05)
	0.9	(0,0)	(0,0.001)	(0,0.002)	(0,0.003)	(0,0.004)	(0,0.006)	(0,0.007)	(0,0.008)	(0,0.009)	(0,0.01)	(0,0.011)	(0,0.022)	(0,0.033)	(0,0.044)
	1	(0,0)	(0.001,0.001)	(0.002,0.002)	(0.003,0.003)	(0.004,0.004)	(0.005,0.005)	(0.006,0.006)	(0.007,0.007)	(0.008,0.008)	(0.009,0.009)	(0.01,0.01)	(0.02,0.02)	(0.03,0.03)	(0.04,0.04)

Table 2

P(B|A)		P(B)														
		0.05	0.06	0.07	0.08	0.09	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
P(A)	0	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
	0	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.01	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.01	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.01	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.01	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.01	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.01	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.02	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.03	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.04	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.05	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.06	(0,0.833)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.07	(0,0.714)	(0,0.857)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.08	(0,0.625)	(0,0.75)	(0,0.875)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.09	(0,0.556)	(0,0.667)	(0,0.778)	(0,0.889)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.1	(0,0.5)	(0,0.6)	(0,0.7)	(0,0.8)	(0,0.9)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
	0.2	(0,0.25)	(0,0.3)	(0,0.35)	(0,0.4)	(0,0.45)	(0,0.5)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0.5,1)	(1,1)
	0.3	(0,0.167)	(0,0.2)	(0,0.233)	(0,0.267)	(0,0.3)	(0,0.333)	(0,0.667)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0.333,1)	(0.667,1)	(1,1)
	0.4	(0,0.125)	(0,0.15)	(0,0.175)	(0,0.2)	(0,0.225)	(0,0.25)	(0,0.5)	(0,0.75)	(0,1)	(0,1)	(0,1)	(0.25,1)	(0.5,1)	(0.75,1)	(1,1)
	0.5	(0,0.1)	(0,0.12)	(0,0.14)	(0,0.16)	(0,0.18)	(0,0.2)	(0,0.4)	(0,0.6)	(0,0.8)	(0,1)	(0.2,1)	(0.4,1)	(0.6,1)	(0.8,1)	(1,1)
	0.6	(0,0.083)	(0,0.1)	(0,0.117)	(0,0.133)	(0,0.15)	(0,0.167)	(0,0.333)	(0,0.5)	(0,0.667)	(0.167,0.833)	(0.333,1)	(0.5,1)	(0.667,1)	(0.833,1)	(1,1)
	0.7	(0,0.071)	(0,0.086)	(0,0.1)	(0,0.114)	(0,0.129)	(0,0.143)	(0,0.286)	(0,0.429)	(0.143,0.571)	(0.286,0.714)	(0.429,0.857)	(0.571,1)	(0.714,1)	(0.857,1)	(1,1)
	0.8	(0,0.062)	(0,0.075)	(0,0.088)	(0,0.1)	(0,0.112)	(0,0.125)	(0,0.25)	(0.125,0.375)	(0.25,0.5)	(0.375,0.625)	(0.5,0.75)	(0.625,0.875)	(0.75,1)	(0.875,1)	(1,1)
	0.9	(0,0.056)	(0,0.067)	(0,0.078)	(0,0.089)	(0,0.1)	(0,0.111)	(0.111,0.222)	(0.222,0.333)	(0.333,0.444)	(0.444,0.556)	(0.556,0.667)	(0.667,0.778)	(0.778,0.889)	(0.889,1)	(1,1)
	1	(0.05,0.05)	(0.06,0.06)	(0.07,0.07)	(0.08,0.08)	(0.09,0.09)	(0.1,0.1)	(0.2,0.2)	(0.3,0.3)	(0.4,0.4)	(0.5,0.5)	(0.6,0.6)	(0.7,0.7)	(0.8,0.8)	(0.9,0.9)	(1,1)