Sarah Ford

Randomness & Computation Assignment 9

1.

*n* = 2

*n* = 3

*n* = 4

2.

See Code (function *trans(n)*)

3.

The probability of tossing a coin 200 times in succession without seeing a run of length 6 is about 3.47%. Likewise, the probability of tossing a coin 200 times in succession without seeing a run of length 7 is about 20.07%. I computed these probabilities by first creating a transition matrix of size *n* (here, *n* = 6 and *n* = 7) and raising it to the 200th power. The [0][*n*]th entry of the resulting matrix gives the probability that after 200 steps (coin tosses), the coin resulted in a final state *n*. Therefore, I computed 1 – [0][*n*] to get the probability that in 200 sequential tosses of a coin, we did not see a run of length *n*.

See Code (function *prob\_result(trans,state,step)*)

4.

The expected number of tosses until seeing a run of length *n* is equal to the sum of the top row of the resulting matrix of the equation .

*n* = 6

Therefore, the expected number of tosses until we see a run of length 6 is…

*n* = 7

Therefore, the expected number of tosses until we see a run of length 7 is…

5.

See Code (function *avg\_sim(n)*)

6.

The revised transition matrix which reflects an indefinite sequence of tosses, capped at state n changes in its last row so that it does not have an absorbing state, and the probability of moving from state n to state n is ½ (rather than 1 in the previous matrix), and the probability of moving from state n to state 1 is ½ (as opposed to 0 previously).

See Code (function *revised\_trans(n)*)

7.

When we compute the eigenvalues of our Markov Chain for *n* = 6 and *n* = 7, we find…

[

and

such that the stationary distribution for for *n = 6* is 0.03125 and the stationary distribution for for *n* = 7 is 0.015625.

See Code (function *eig\_vals()*)

8.

See Code (function *revised\_sim(tosses,state)*)