Class 30

DATA1220-55, Fall 2024

Sarah E. Grabinski

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- 2-sample t-test for the difference between 2 unpaired means

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- Statistical analysis best practice

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- Rarely calculated by hand. We will perform these exclusively with R.

ANOVA F-Test Hypotheses

- Null hypothesis: the mean outcome μ_i is the same across all k groups, such that each group has the same population mean μ.

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- Alternate hypothesis: at least one mean μ_i is different from the other $k-1$ means, such that there is no single population mean μ.

$$H_A: \text{At least 1 } \mu_i \neq \mu$$

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4. ***Equal variance***: Within-group variance is approximately equal across the k groups. This condition relaxes as sample sizes n_i become more balanced between the k groups ($\frac{n}{k} \to n_i$).

The F-Distribution

- 2 degree of freedom parameters
  - The number of groups k minus 1: $\text{df}_1 = k - 1$
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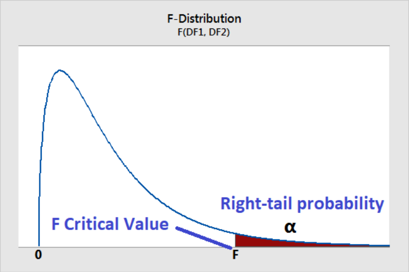
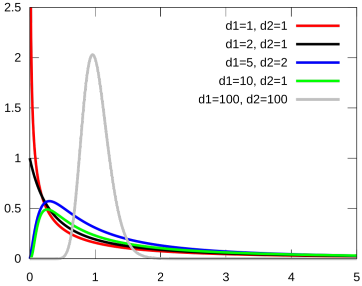
- The larger the test statistic F, the less likely the observed data is under the null hypothesis
- Like the Chi-squared (χ^2) distribution, the upper tail probability is always used for hypothesis testing

The F-Distribution





ANOVA Results Table

Source	df	SS	MS	F	p
Between Groups (Factor)	$k-1$	$\sum_k n_k(\bar{x}_k - \bar{x}.)^2$	$\frac{SS_{Between}}{df_{Between}}$	$\frac{MS_{Between}}{MS_{Within}}$	Area to the right of $F_{k-1,\, n-k}$
Within Groups (Error)	$n-k$	$\sum_k \sum_i (x_{ik} - \bar{x}_k)^2$	$\frac{SS_{Within}}{df_{Within}}$		
Total	$n-1$	$\sum_k \sum_i (x_{ik} - \bar{x}.)^2$			

Legend

k	Number of groups
n	Total sample size (all groups combined)
n_k	Sample size of group k
$\bar{x}_k$	Sample mean of group k
$\bar{x}.$	Grand mean (i.e., mean for all groups combined)
SS	Sum of squares
MS	Mean square
df	Degrees of freedom
F	F-ratio (the test statistic)

Example: Wolf River Sediments

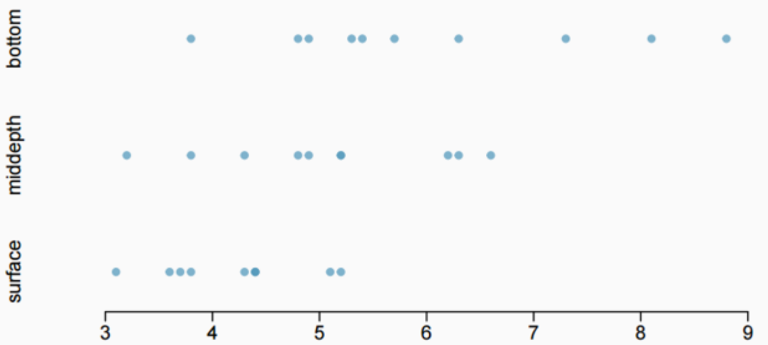
- The Wolf River in Tennessee flows past an abandoned site once used by the pesticide industry for dumping wastes, including chlordane (pesticide), aldrin, and dieldrin (both insecticides)
- These highly toxic organic compounds can cause various cancers and birth defects
- The standard methods to test whether these substances are present in a river is to take samples at six-tenths depth
- Since these compounds are

Figure 1: ***Research Question:*** Does the average aldrin concentration vary between the bottom, mid-depth, and surface?

The Data

	aldrin	depth
1	3.80	bottom
2	4.80	bottom
...		
10	8.80	bottom
11	3.20	middepth
12	3.80	middepth
...		
20	6.60	middepth
21	3.10	surface
22	3.60	surface
...		

Exploratory Analysis & Sample Statistics



	n	mean	sd
bottom	10	6.04	1.58
middepth	10	5.05	1.10
surface	10	4.20	0.66
overall	30	5.1 0	1.37

Equal Variance Assumption

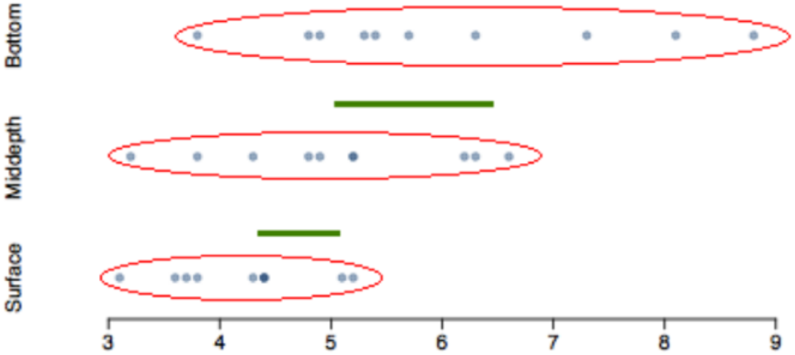


Figure 2: Do these variances look approximately equal?

ANOVA Results

		Df	Sum Sq	Mean Sq	F value	Pr(>F)
(Group)	depth	2	16.96	8.48	*6.14*	0.0063
(Error)	Residuals	27	37.33	1.38		
	Total	29	54.29			

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- But we made some pretty strong assumptions.
- If you want to know which means are different from each other, you will have to do additional pairwise tests between group means.