Class 25

DATA1220-55, Fall 2024

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2024-11-01

In-Class Quiz

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- Worth 10% of final grade, will be bonus points available

Statistical Analysis Workflow

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8. Apply results to target population.

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```
p_hat <- 18 / 30

p_hat
```

```
[1] 0.6
```

Inference: Rain Days April 2024

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- $\hat{p} \pm Z^* \times SE_{\hat{p}}$
- $Z^* = Z_{1-\alpha/2}$
- $\text{Confidence} = 1 - \alpha$

Finding Z^*

If our confidence level is 95% (0.95), then our α is 0.05. We need the Z-score that corresponds to the probability $p = 1 - \alpha/2 = 0.975$. Use the `qnorm()` function to find Z^*.

```
z_star <- qnorm(0.975)

z_star
```

```
[1] 1.959964
```

Standard Error for a Single Proportion

```r
se_phat <- sqrt((0.6 * (1 - 0.6)) / 30)

se_phat
```

```
[1] 0.08944272
```

Confidence Interval for $\hat{p}$

Lower bound:

```
p_hat - z_star * se_phat
```

```
[1] 0.4246955
```

Upper bound:

```
p_hat + z_star * se_phat
```

```
[1] 0.7753045
```

Test Statistic

```r
z_test <- (p_hat - 0.5) / 0.09

z_test
```

```
[1] 1.111111
```

P-Value

```
pnorm(z_test, lower.tail = F) * 2
```

```
[1] 0.2665205
```

The Central Limit Theorem

The distribution of the sample statistic $\bar{x}$ or $\hat{p}$ approximates the normal distribution $N\left(\text{populationparameter}, \text{standarderror}\right)$ as $n \to \infty$.

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The ***sampling distribution*** is normal with $\mu = \mathrm{sample statistic}$ and $\sigma = \mathrm{standard error}$.

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- Observations in sample assumed to be ***independent and identically distributed*** (***i.i.d.***)
- Need $n \geq 30$ observations in sample
- Underlying population distribution is normal (less strict as sample n increases)

Sample Means & The Standard Normal (z) Distribution

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- When assumptions met, $\bar{x} \approx \mu$ and $s \approx \sigma$
- $s \approx \sigma$ is a strong assumption!

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- Centered at 0 with the single parameter ***degrees of freedom*** $(df = n - 1)$

The t distribution

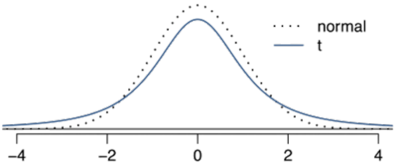


Figure 1: The t distribution versus the standard normal (z) distribution

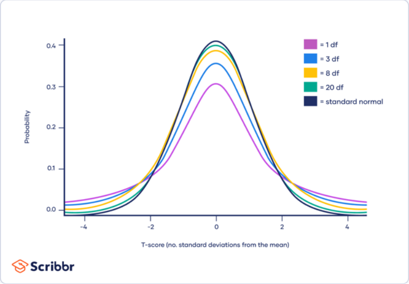


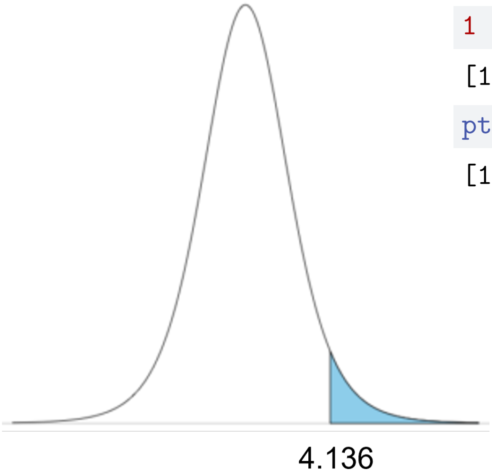
Figure 2: The t distribution is centered at 0 and has the parameter *degrees of freedom* (df)

t Distribution Test Statistic

$$T_{df} = \frac{\text{pointestimate} - \text{nullvalue}}{SE}$$

$$T_{n-1} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

P-Values in R




```r
1 - pt(4.135, 205)
```

```
[1] 2.587688e-05
```

```r
pt(4.135, 205, lower.tail = F)
```

```
[1] 2.587688e-05
```

Confidence Intervals

- When $s \approx \sigma$, the confidence interval is $\text{point estimate} \pm Z^* \times SE$

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- $T^* = T_{1-\alpha/2}$