

# Class 33

## DATA1220-55, Fall 2024

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# Infer Package

- ▶ Functions for “tidy” statistical analysis
- ▶ Specify statistical models, calculate statistics
- ▶ Infer sampling distributions, test hypotheses
- ▶ Uses theoretical or permutation based null distributions



# Tests in the Infer Package

- ▶ 1- or 2-sample proportion- or Z-tests
- ▶ 1- or 2-sample t-tests for means
- ▶ Chi-squared test of independence for categorical variables
- ▶ ANOVA test of independence for numeric variables
- ▶ Correlations and simple linear regression

# Primary Functions

- ▶ `specify()`: set response variable (and explanatory, if needed)
- ▶ `calculate()`: calculate statistics
- ▶ `observe()`: combines `specify()` and `calculate()`
- ▶ `assume()`: sets a null distribution
- ▶ `hypothesize()`: sets a null hypothesis
- ▶ `get_ci()`: calculate a confidence interval from given distribution
- ▶ `visualize()`, `shade_p_value()`: visualize observed statistics vs null hypotheses
- ▶ `get_p_value()`: get p-value for observed statistic under null hypothesis

# Packages for Today

We will be working with the pennies dataset from the moderndive package.

```
library(patchwork) # combining ggplot figures
library(GGally) # for ggpairs() plot matrix
library(Hmisc) # for describe function
library(moderndive) # contains pennies data
library(infer) # statistical functions
library(kableExtra) # for pretty tables
library(tidyverse) # always load last in list

theme_set(theme_bw()) # white background for ggplot2
```

# Research Question

***What is the average year of minting for pennies in circulation in the US in 2019?***

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***What is the average year of minting for pennies in circulation in the US in 2019?***

*Is it reasonable to gather up all the pennies in the US and get the average of the mint years?*

No, we should probably take a sample of pennies and use it to draw inferences and test hypotheses regarding our study and target populations.



# Sampling Methods

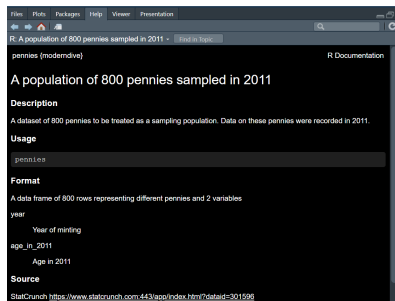


Figure 1: In 2011, Dr. Chester Ismay and Dr. Albert Y. Kim went to a local bank in Northampton, MA and requested all 800 pennies they had available.

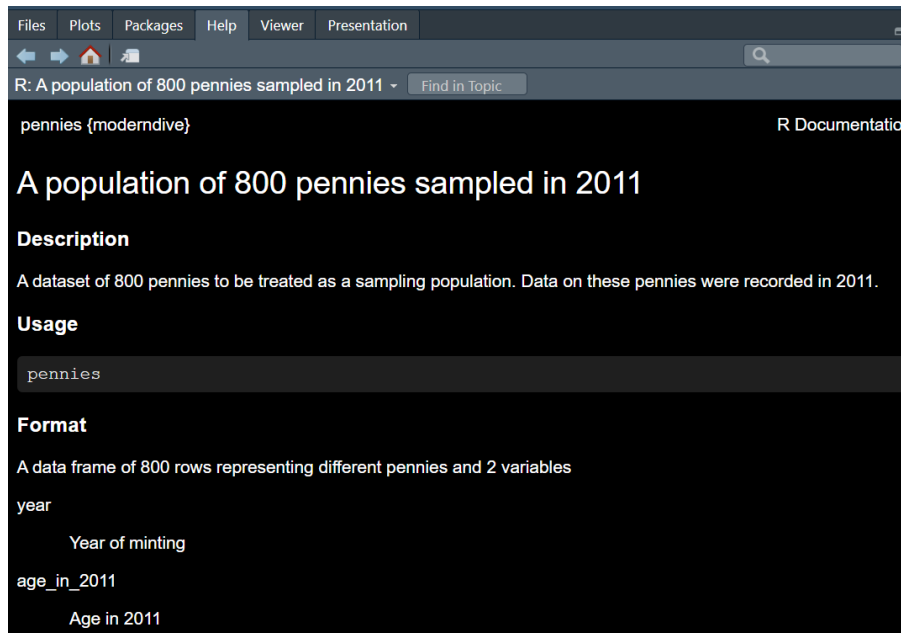
# What's in the data?

Running a question mark before a dataset, function, or package name will do a search in RStudio for help pages on that topic.

```
?moderndive::pennies
```



# What's in the data?



The screenshot shows the RStudio interface with the 'pennies' dataset documentation open. The top menu bar includes 'Files', 'Plots', 'Packages', 'Help', 'Viewer', and 'Presentation'. Below the menu is a toolbar with navigation icons and a search bar. The main window displays the title 'R: A population of 800 pennies sampled in 2011' and a 'Find in Topic' button. The content area shows the dataset name 'pennies {moderndive}' and the R Documentation link. The title 'A population of 800 pennies sampled in 2011' is prominently displayed. Below it, the 'Description' section states: 'A dataset of 800 pennies to be treated as a sampling population. Data on these pennies were recorded in 2011.' The 'Usage' section shows the command 'pennies'. The 'Format' section describes the data frame structure: 'A data frame of 800 rows representing different pennies and 2 variables'. The variables are listed as 'year' (Year of minting) and 'age\_in\_2011' (Age in 2011).

Files Plots Packages Help Viewer Presentation

R: A population of 800 pennies sampled in 2011 Find in Topic

pennies {moderndive} R Documentation

## A population of 800 pennies sampled in 2011

### Description

A dataset of 800 pennies to be treated as a sampling population. Data on these pennies were recorded in 2011.

### Usage

```
pennies
```

### Format

A data frame of 800 rows representing different pennies and 2 variables

year  
Year of minting

age\_in\_2011  
Age in 2011

# A Peek at the Data

```
# from the dplyr package  
glimpse(pennies)
```

Rows: 800

Columns: 2

\$ year <int> 1986, 1996, 1994, 2008, 1999, 2010, 1996

\$ age\_in\_2011 <int> 25, 15, 17, 3, 12, 1, 47, 36, 16, 45, 1

## Another Peek at the Data

```
# from base R  
str(pennies)
```

```
tibble [800 x 2] (S3: tbl_df/tbl/data.frame)  
$ year      : int [1:800] 1986 1996 1994 2008 1999 2010 1  
$ age_in_2011: int [1:800] 25 15 17 3 12 1 47 36 16 45 ...  
- attr(*, "spec")=  
.. cols(  
..   year = col_integer(),  
..   age_in_2011 = col_integer()  
.. )
```

# Codebook

- ▶ `year`: year that the penny was minted
- ▶ `age_in_2011`: the age of the penny in years in 2011

# Exploratory Data Analysis

```
# from the Hmisc package  
describe(pennies$year)
```

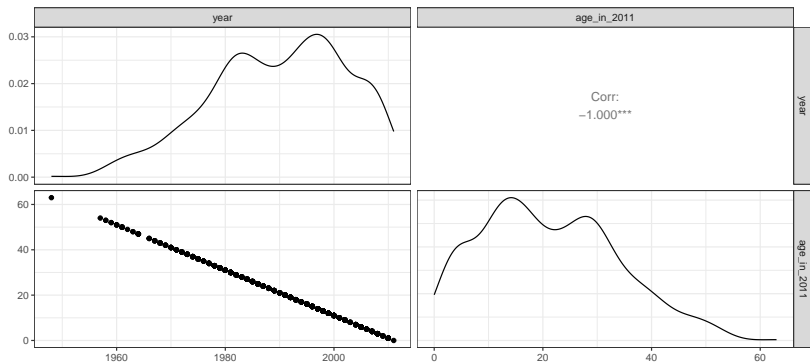
```
pennies$year
```

n	missing	distinct	Info	Mean	Gmd
800	0	55	0.999	1990	14.16
.25	.50	.75	.90	.95	
1981	1991	2000	2006	2008	

```
lowest : 1948 1957 1958 1959 1960, highest: 2007 2008 2009
```

# A Picture's Worth 1000 Words

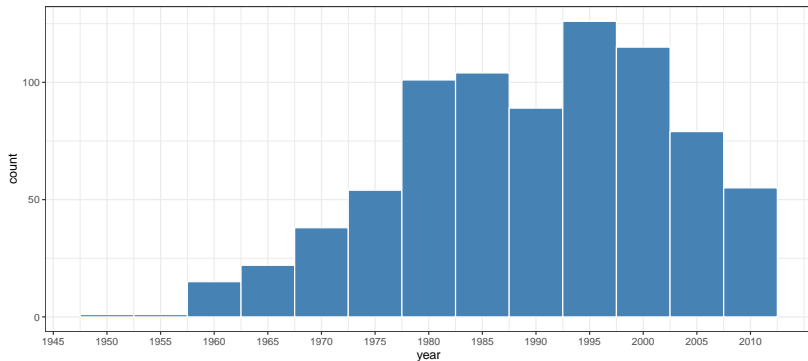
```
# From the GGally package  
ggpairs(pennies)
```





# More Exploratory Data Analysis

What do you see in this population distribution?

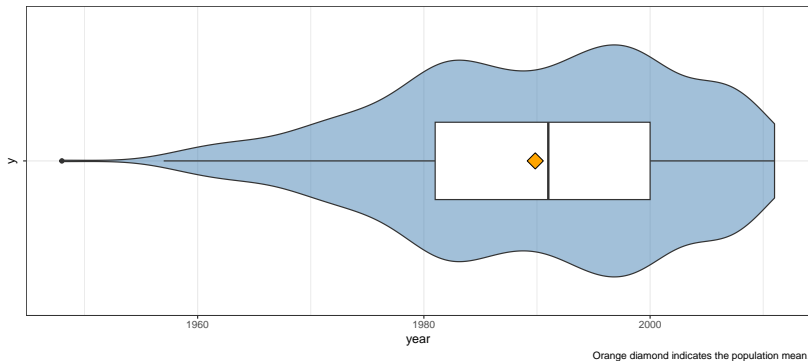


# More Exploratory Data Analysis

```
pennies |>
  ggplot(aes(x = year)) +
  geom_histogram(binwidth = 5,
                 col = 'white',
                 fill = 'steelblue') +
  scale_x_continuous(breaks = seq(1945,
                                  2011,
                                  by = 5))
```

# Keep it Going

Does this population seem normally distributed to you? Why or why not?



# Keep it Going

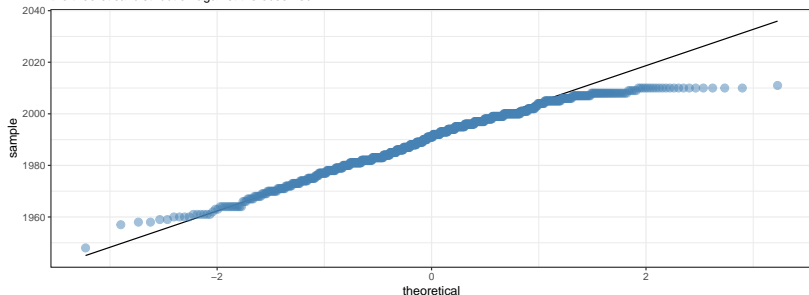
```
pennies |>
  ggplot(aes(x = year, y = '')) +
  geom_violin(fill = 'steelblue',
              alpha = 0.5) +
  geom_boxplot(width = 0.3) +
  stat_summary(fun = 'mean', geom = 'point',
              shape = 23, size = 5, fill = 'orange') +
  labs(caption = 'Orange diamond indicates the population mean')
```

# Last One

If the points in your Q-Q plot don't follow the straight line closely, it may not be reasonable to assume that your population is normally distributed.

Q-Q Plot for a Normal Distribution in Minting Year

Q-Q stands for Quartile-Quantile, because it plots the theoretical distribution against the observed.



The points will all fall on the straight line when the distribution is perfectly normal.

## Last One

```
pennies |>
  ggplot(aes(sample = year)) +
  geom_qq_line() +
  geom_qq(col = 'steelblue', alpha = 0.5, size = 3) +
  labs(title = 'Q-Q Plot for a Normal Distribution in Mint',
        subtitle = paste('Q-Q stands for Quartile-Quartile,
                          'the theoretical distribution against',
                          sep = '\n'),
        caption = 'The points will all fall on the straight line')
```

# Calculating population parameters

You can use functions from the `kableExtra` package to turn your raw R outputs into attractive tables.

n	mean	SD
800	1989.8	12.4

# Calculating population parameters

Find more info here: [kableExtra Vignettes](#)

```
# from the dplyr package
pennies |>
  summarize(
    n = n(),
    mean = mean(year),
    sd = sd(year)) |>
  kable(col.names = c('n', 'mean', 'SD'),
        digits = c(0, 1, 1)) |>
  kable_classic(full_width = F)
```



## Storing population mean as a variable

Use the `pull()` function from `dplyr` to grab just the value(s) from a column, ditching the data frame component.

```
pop_mean <- pennies |>  
  summarize(mean = mean(year)) |>  
  pull(mean)
```

```
pop_mean
```

```
[1] 1989.848
```

## Storing population standard deviation as a variable

Use the `pull()` function from `dplyr` to grab just the value(s) from a column, ditching the data frame component.

```
pop_sd <- pennies |>  
  summarize(sd = sd(year)) |>  
  pull(sd)
```

```
pop_sd
```

```
[1] 12.43956
```

# Can we predict our sampling distribution?

```
# from the dplyr package
pennies |>
  summarize(
    n = 50,
    pop_mean = mean(year),
    sample_se = sd(year) / sqrt(50),
    lower95 = mean(year) - qt(0.975,
                              df = n()) * (sd(year) / sqrt(
upper95 = mean(year) + qt(0.975,
                           df = n()) * (sd(year) / sqrt(
kable(col.names = c('n', 'population mean',
                    'SE for n=50',
                    'Lower 95% CI', 'Upper 95% CI'),
      digits = c(0, 1, 1, 2, 1, 1)) |>
kable_classic(full_width = F)
```

## Can we predict our sampling distribution?

n	population mean	SE for n=50	Lower 95% CI	Upper 95% CI
50	1989.8	1.8	1986.39	1993.3

# Can we predict our sampling distribution?

n	population mean	SE for n=50	Lower 95% CI	Upper 95% CI
50	1989.8	1.8	1986.39	1993.3

The sample means  $\bar{x}$  from a sample of size  $n = 50$  from this population should follow the distribution  $N(1989.8, 1.76)$  if assumptions hold.

## Storing sampling distribution estimates

Use the `pull()` function from `dplyr` to grab just the value(s) from a column, ditching the data frame component.

```
pop_low <- pennies |>
  summarize(lower95 = mean(year) - qt(0.975,
                                     df = n()) * (sd(year)
  pull(lower95)

pop_high <- pennies |>
  summarize(upper95 = mean(year) + qt(0.975,
                                     df = n()) * (sd(year)
  pull(upper95)
```

## Sampling Method



# The Sample





## Taking a sample, pt. 1

When doing random processes in R, you need to use the `set.seed()` function and give it a number. This temporarily “fixes” the randomness so that the function generates the same set of numbers every time.

```
# store number of observations
n <- nrow(pennies)

# randomly sample row numbers/indexes
# set a seed to reproduce exact sample next time
set.seed(123); sample_rows <- sample(seq(1, n),
                                     size = 50)
sort(sample_rows)
```

```
[1] 14 23 26 72 91 118 135 141 143 153 166 179 195 21
[20] 290 294 299 309 348 355 373 374 415 426 463 490 519 52
[39] 590 593 602 603 621 649 665 709 722 766 768 782
```

# Taking a sample, pt. 1

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[20] 290 294 299 309 348 355 373 374 415 426 463 490 519 52
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```

## Taking a sample, pt. 2

Use the `filter()` function from the `dplyr` package to retain observations whose row number matches your generated sample.

```
# filter your table for rows whose  
# number/index is in your sample list  
sample1 <- pennies |>  
  # function from dplyr package  
  filter(row_number() %in% sample_rows)  
  
nrow(sample1)
```

```
[1] 50
```

Or use base R to index the proper rows. The 1st parameter contains the row numbers to keep. The 2nd parameter contains the column numbers to keep. Leave blank if you want them all.

```
sample1 <- pennies[sample_rows, ]
```

## What about another sample?

Set a different seed with the `set.seed()` function to generate a new (but reproducible) random sample.

```
# randomly sample row numbers/indexes
# set a seed to reproduce exact sample next time
set.seed(456); sample_rows2 <- sample(seq(1, n),
                                     size = 50)

# Select sampled rows through indexing
sample2 <- pennies[sample_rows2, ]
```

## Resampling with Infer

You can use the `rep_sample_n()` function from the `infer` package to take reps number of repeated samples of a specified size, with (`replace = T`) or without (`replace = F`) replacement.

```
# don't forget to set a seed!
set.seed(789); samples <- pennies %>%
  rep_sample_n(size = 50,
               replace = TRUE,
               reps = 100)

head(samples)
```

```
# A tibble: 6 x 3
# Groups:   replicate [1]
  replicate year age_in_2011
    <int> <int>      <int>
1         1  2007          4
2         1  1995         16
```

## Summary Statistics & CIs

This function returns a grouped table, so you don't need to use the `.by =` parameter to group by replicate in the `summarize()` function.

replicate	n	point estimate	Lower 95% CI	Upper 95% CI
1	50	1993.9	1990.00	1997.8
2	50	1992.5	1989.33	1995.7
3	50	1988.3	1984.95	1991.7
4	50	1990.8	1987.71	1993.8
5	50	1989.9	1986.13	1993.6
6	50	1989.2	1985.90	1992.4

# Summary Statistics & CIs

Complete the confidence interval calculations within the `summarize()` function.

```
cis <- samples |>
  summarize(
    n = n(),
    mean = mean(year),
    s = sd(year),
    lower95 = mean(year) - qt(0.975,
                              df = n()) * (sd(year) / sqrt(n)),
    upper95 = mean(year) + qt(0.975,
                              df = n()) * (sd(year) / sqrt(n))
```

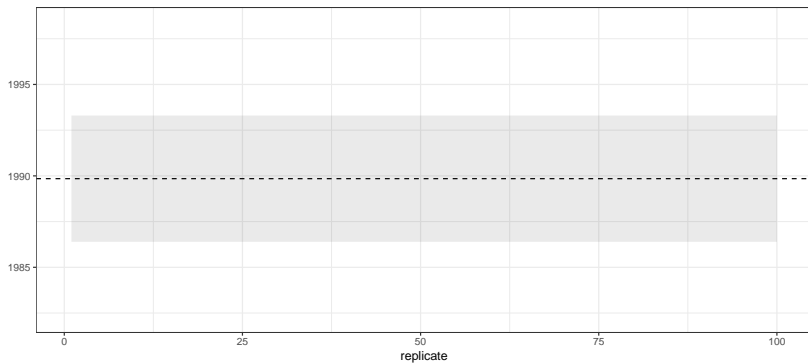
# Summary Statistics & CIs

Remember, the parameters were a mean year of 1989.8 and a standard deviation of 12.4 years in a population of 800. How do these compare?

```
cis |>
  head() |>
  kable(col.names = c('replicate', 'n', 'point estimate',
                     'Lower 95% CI', 'Upper 95% CI'),
        digits = c(0, 1, 1, 2, 1, 1), align = 'c') |>
  kable_classic(full_width = F)
```



# Visualizing the CI's

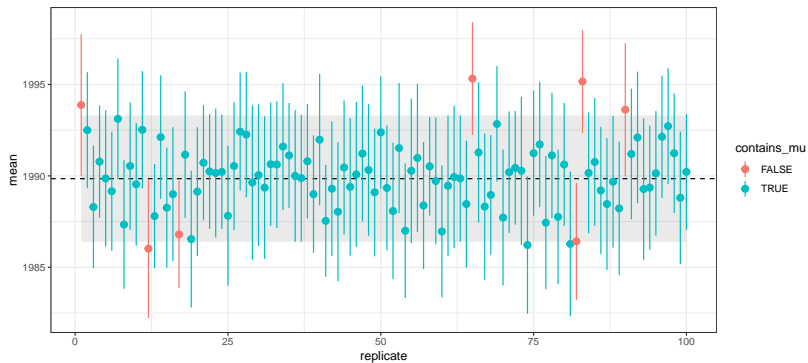


# Visualizing the CI's

```
p1 <- cis |>
  # from the dplyr package, for modifying data
  mutate(contains_mu = ifelse(
    pop_mean >= lower95 & #conditional statement
    pop_mean <= upper95,
    T, F)) |> # value to return if true, if false
  ggplot(aes(x = replicate)) +
  geom_ribbon(aes(ymin = pop_low,
                 ymax = pop_high),
            alpha = 0.1) +
  geom_hline(yintercept = pop_mean,
            linetype = 'dashed') +
  coord_cartesian(ylim = c(min(cis$lower95),
                           max(cis$upper95)))
```

p1

# Visualizing the CI's

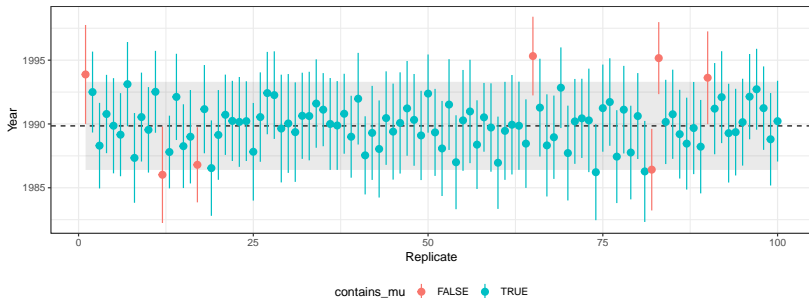


# Visualizing the CI's

```
p2 <- p1 +  
  geom_pointrange(aes(y = mean,  
                      ymin = lower95,  
                      ymax = upper95,  
                      col = contains_mu,  
                      fill = contains_mu))  
  
p2
```

# Visualizing the CI's

95% Confidence Intervals for the Mean Year of Penny Minting  
Inferences from Samples vs Population Parameters



# Visualizing the CI's

```
p3 <- p2 +  
  theme(legend.position = 'bottom') +  
  labs(title = '95% Confidence Intervals for the Mean Year',  
        subtitle = 'Inferences from Samples vs Population Parameters',  
        caption = 'Pop. Mean = 1989.8, Pop. SD = 12.4',  
        x = 'Replicate', y = 'Year')  
  
p3
```

# All Together

95% Confidence Intervals for the Mean Year of Penny Minting  
Inferences from Samples vs Population Parameters



Pop. Mean = 1989.8, Pop. SD = 12.4

# Reordered

95% Confidence Intervals for the Mean Year of Penny Minting  
Inferences from Samples vs Population Parameters



Pop. Mean = 1989.8, Pop. SD = 12.4



## Reordered

```
cis |>
  mutate(contains_mu = ifelse(
    pop_mean >= lower95 &
    pop_mean <= upper95, T, F)) |>
  ggplot(aes(x = reorder(replicate, mean))) +
  geom_ribbon(aes(ymin = pop_low, ymax = pop_high),
    alpha = 0.1) +
  geom_hline(yintercept = pop_mean,
    linetype = 'dashed') +
  coord_cartesian(ylim = c(min(cis$lower95),
    max(cis$upper95))) +
  geom_pointrange(aes(y = mean, ymin = lower95, ymax = upper95,
    col = contains_mu, fill = contains_mu)) +
  theme(legend.position = 'bottom',
    axis.text.x = element_blank()) +
  labs(title = '95% Confidence Intervals for the Mean Year',
    subtitle = 'Inferences from Samples vs Population Parameters',
    caption = 'Pop. Mean = 1989.8, Pop. SD = 12.4',
```

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- ▶ Taking a sample and constructing a confidence interval are considered random processes
- ▶ Confidence is an expression of probability regarding the PROCESS, not the result
- ▶ The confidence level is a pre-analysis statement about the process, not a post-analysis statement about the result
- ▶ 95% confidence means that 95% of the time, confidence intervals constructed using samples of size  $n$  from this population will contain the population parameter...
- ▶ ***...meaning there's a 5% chance ( $\alpha$ ) that your confidence interval does NOT contain the population parameter no matter how good your data is!***

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- ▶ However, if any assumptions are violated, the sample statistics may no longer be accurate estimators of the population parameters.

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- ▶ A “true” population parameter is unknowable. You should not be using this terminology to interpret your results.
- ▶ If you have made reasonable assumptions, theory suggests (and one hopes) the point estimate or sample statistic will be close to that “true” population parameter we can’t know.
- ▶ However, if any assumptions are violated, the sample statistics may no longer be accurate estimators of the population parameters.
- ▶ The confidence level is not the probability that YOUR confidence interval contains the population parameter.

# Activity

Interpreting confidence intervals: an interactive activity:  
<https://rpsychologist.com/d3/ci/>

# Questions

1. What happens to the intervals as you change the confidence level?
2. What happens to the intervals as you change the sample size?
3. What concept from Chapter 3 does the left-middle plot remind you of?
4. Is the confidence interval width normally distributed?
5. How has your understanding of confidence intervals and their interpretation changed?