Class 12

DATA1220-55, Fall 2024

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Probability Review

- (General) Addition Rule
- (General) Multiplication Rule
- Dependence vs Independence

The General Addition Rule

The probability of event A ***or*** event B occurring is the sum of the probability that A occurs and the probability that B occurs minus the probability that A *and* B occurs.

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= P(A) + P(B) - P(A \cup B) \\ &= P(A \cap B) \end{aligned}$$

The Addition Rule for Disjoint Events

When events A and B are ***disjoint***, the probability of event A ***or*** event B occurring is just the sum of the probability that A occurs and the probability that B occurs, because the probability that event A *and* event B occurs is 0.

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= P(A) + P(B) \\ &= P(A \cap B) \end{aligned}$$

The General Multiplication Rule

The probability of event A ***and*** event B occurring is the product of the probability that A occurs and the *conditional probability* that B occurs given that A has already occurred.

$$\begin{aligned} P(A \text{ and } B) &= P(A) \times P(B \text{ given } A) \\ &= P(A) \times P(B|A) \\ &= P(A \cup B) \end{aligned}$$

Independent Processes

- If random process B is ***independent*** of random process A, then the probability of random process B does NOT vary based on the outcome of random process A
- *i.e. knowing the outcome of A does NOT provide additional information about the probability of B*
- Example: When listening to a playlist using a "true shuffle", the probability that the next song will be by a particular artist *does not* change based on whether or not the last song played was also by that artist.

Multiplication Rule for Independent Processes

The probability of event A ***and*** event B occurring is the product of the probability that A occurs and the probability that B occurs, because the probability of B does not change based on the outcome of A.

$$\begin{aligned} P(A \text{ and } B) &= P(A) \times P(B \text{ given } A) \\ &= P(A) \times P(B|A) \\ &= P(A) \times P(B) \\ &= P(A \cup B) \end{aligned}$$

How do you know if two random processes are independent?

- Compare the conditional probabilities of B given the different possible outcomes of A. If $P(B|A) \approx P(B)$ for all values of A, then the two random processes are likely independent.
- Calculate the probability that event A and B occur under both an independence model ($P(A \text{ and } B) = P(A) \times P(B)$) and a dependence model ($P(A \text{ and } B) = P(A) \times P(B|A)$.
  - If $P(A) \times P(B) \approx P(A) \times P(B|A)$, then A and B are likely ***independent processes***.
  - If $P(A) \times P(B) \neq P(A) \times P(B|A)$, then A and B are likely ***dependent*** processes.

Practice: Swing Voters

- Pew Research survey asked 2,373 randomly sampled registered voters about their...
  - Political affilitation (Democrat, Republican, Independent)
  - Whether they consider themselves a swing voter (Yes, No)
- 35% responded Independent, 23% identified as swing voters, and 11% identified as both

Pratice: Swing Voters

- Are these events disjoint or non-disjoint?
- What does the sample space look like?
- What do the contingency tables look like?
- What % of voters identify as an Independent *or* a swing voter?
- What % of voters identify as *neither* an Independent *nor* a swing voter?
- Are identifying as an Independent and identifying as a swing voter dependent or independent processes?

Pratice: Poverty and Language

The American Community Survey (ACS) provides public data each year to give communities demographic information to plan investments and services. The 2010 ACS estimates that...

- 14.6% of Americans live below the poverty line
- 20.7% of Americans speak a language other than English at home
- 31.1% of Americans live below the poverty line *or* speak a language other than English at home

Pratice: Poverty and Language

- Are these events disjoint or non-disjoint?
- What does the sample space look like?
- What do the contingency tables look like?
- What % of Americans live below the poverty line *and* speak a language other than English at home?
- What % of Americans live below the poverty line *and* speak only English at home?
- Are living below the poverty line and speaking a language other than English at home dependent or independent?

Chapter 4 - Distributions

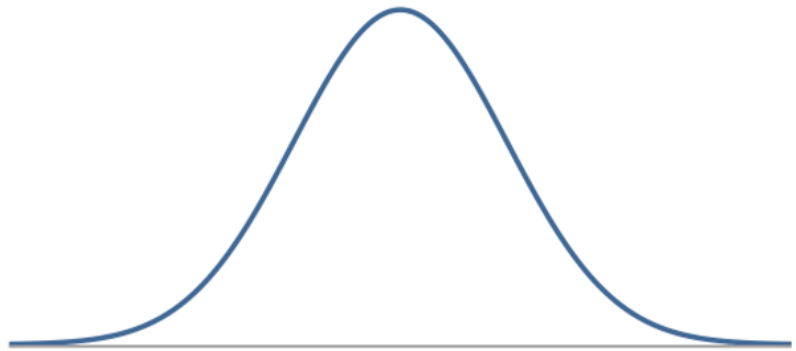
- We will *only* be covering Chapter 4.1 on the normal distribution in your textbook
- If you have an interest in math or statistics, you may want to read the rest of Chapter 4
  - 4.2 - Geometric distribution
  - 4.3 - Binomial distribution
  - 4.4 - Negative binomial distribution
  - 4.5 - Poisson distribution

Chapter 4 Objectives

- Identify and describe the standard normal and normal distributions
- Standardize normal distributions and calculate Z-scores
- Calculate percentiles and exact probabilities
- Apply the 68-95-99.7 Rule
- Read a QQ-Plot (not in book)

The Normal Distribution

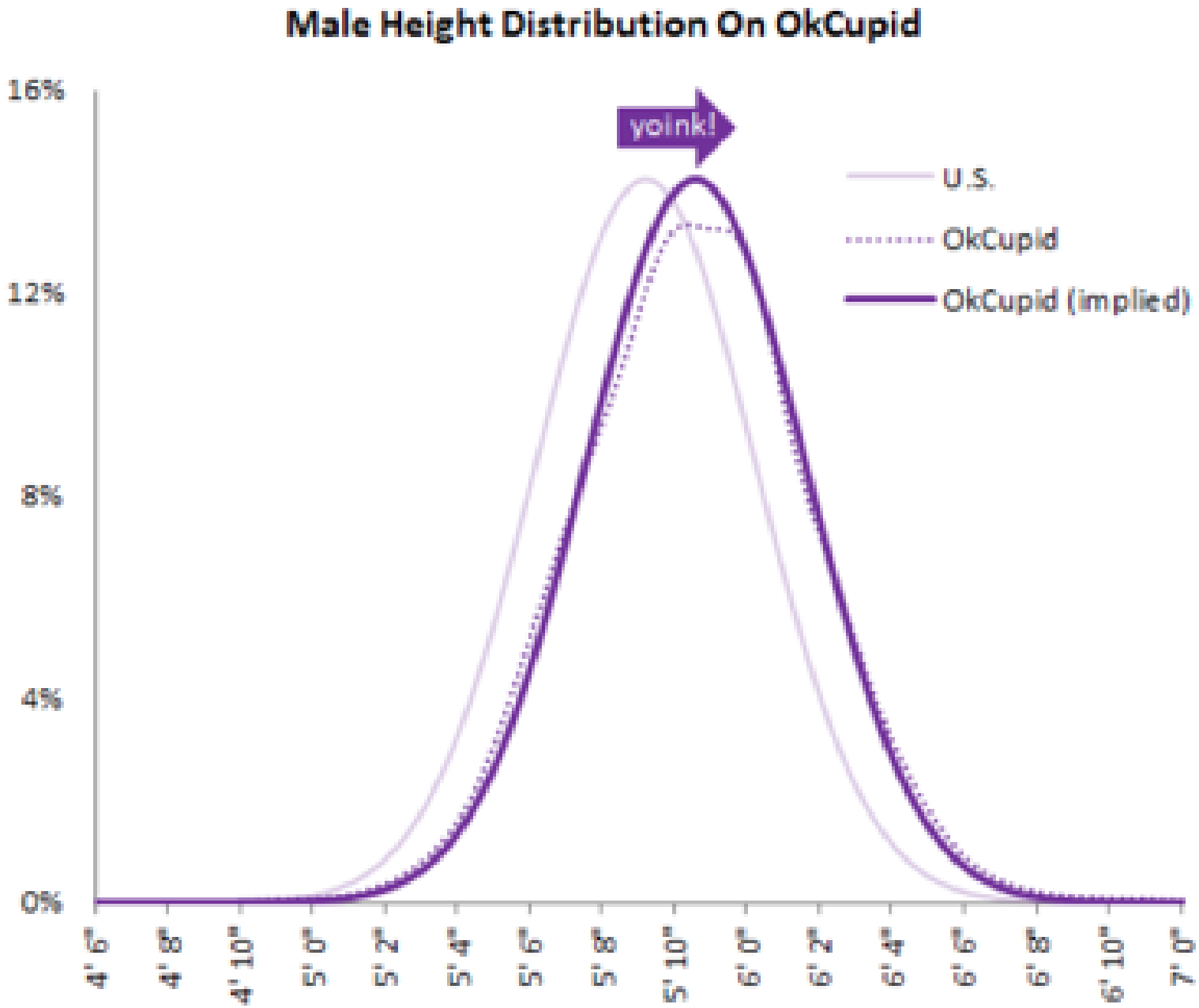
- Symmetric, unimodal, "bell-shaped"
- Not as common as people think in real data
- Strong assumption in small sample sizes ($ 20)
- Powerful statistical tests available when outcome approximates normal distribution



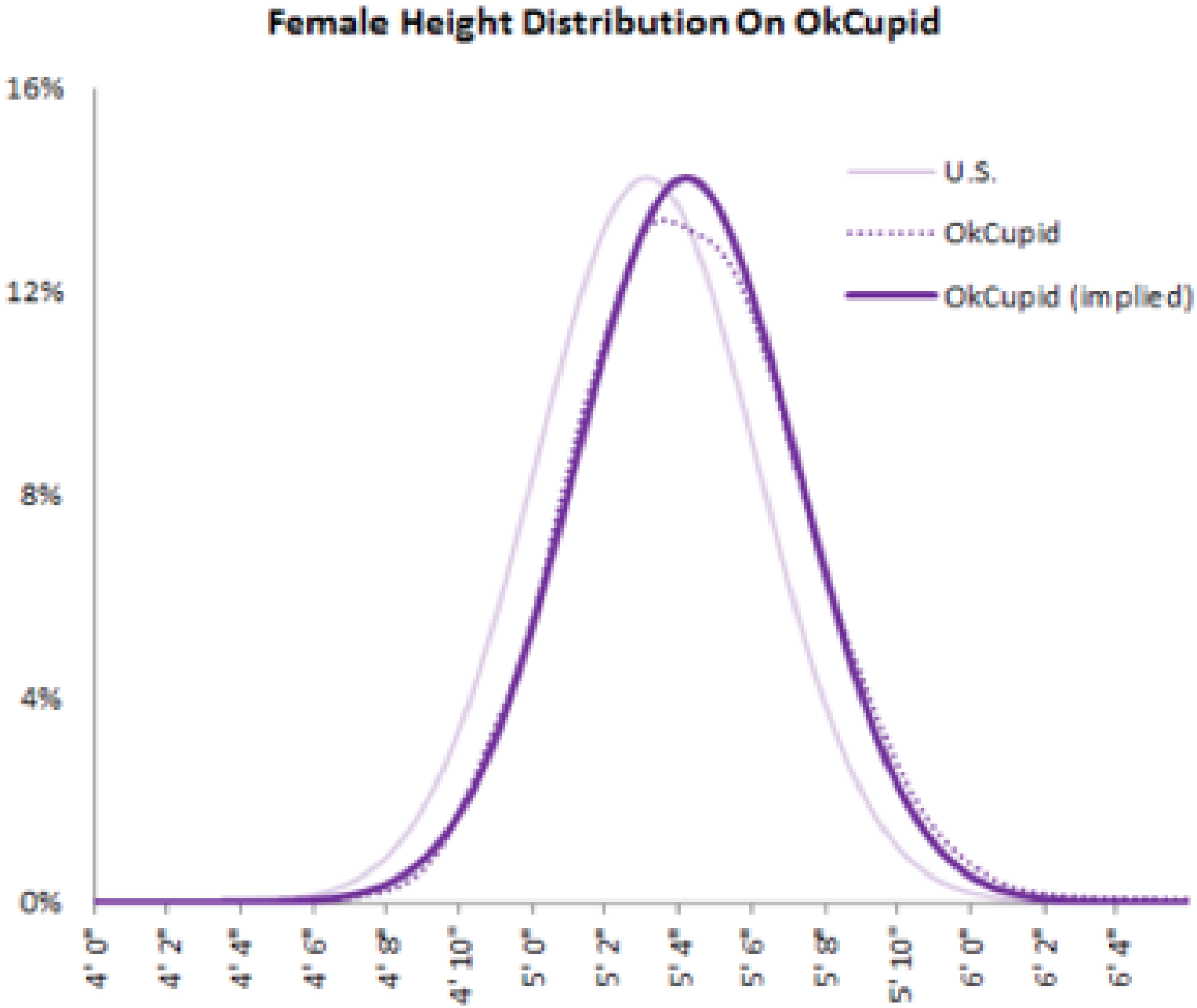
Notation

- μ (Greek letter mu) represents the mean
- σ (Greek letter sigma) represents the standard deviation of the mean
- $N(\mu, \sigma)$ stands for a normal distribution with mean μ and standard deviation σ

Histograms and Density Curves



Example: OkCupid, Heights of Males



The shape of a normal distribution varies by location (mean) and scale (standard deviation)

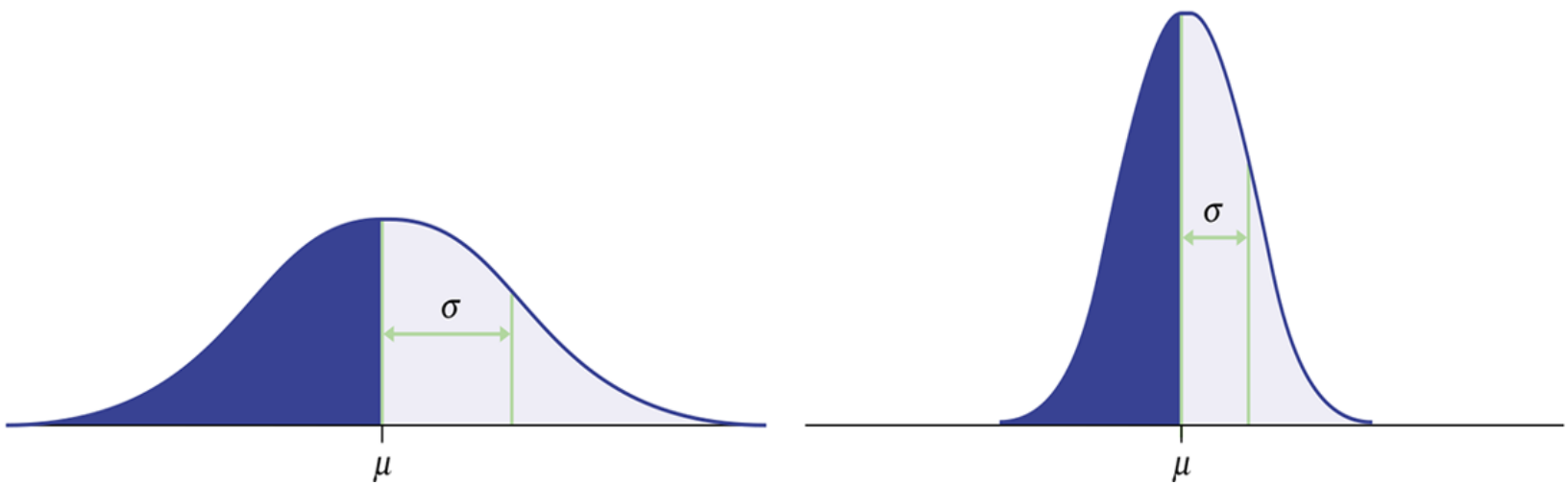


Figure 3: Changing the mean shifts the "center" of the distribution. Changing the standard deviation alters the "width" of the distribution (i.e. variability).

Standardizing Normal Distributions with Z-Scores

- A ***Z-score*** is the number of standard deviations a value falls above (when positive) or below (when negative) the mean of the data
- Z-scores standardize a normal distribution by...
  - Centering the data at 0 by subtracting the mean from each score
  - Scaling the units of the data to 1 by dividing the centered data by the standard deviation

Calculating the Z-Score

$$
\begin{aligned}
Z &= \frac{\mathrm{observedvalue} - \mathrm{mean}}{\mathrm{standarddeviation}} \\
&= \frac{x - \mu}{\sigma}
\end{aligned}
$$

Example: Test Scores

- SAT scores are normally distributed with $\mu = 1500$ and $\sigma = 300$ ($N(\mu = 1500, \sigma = 300)$)
- ACT scores are normally distributed with $\mu = 21$ and $\sigma = 5$ ($N(21, 5)$)

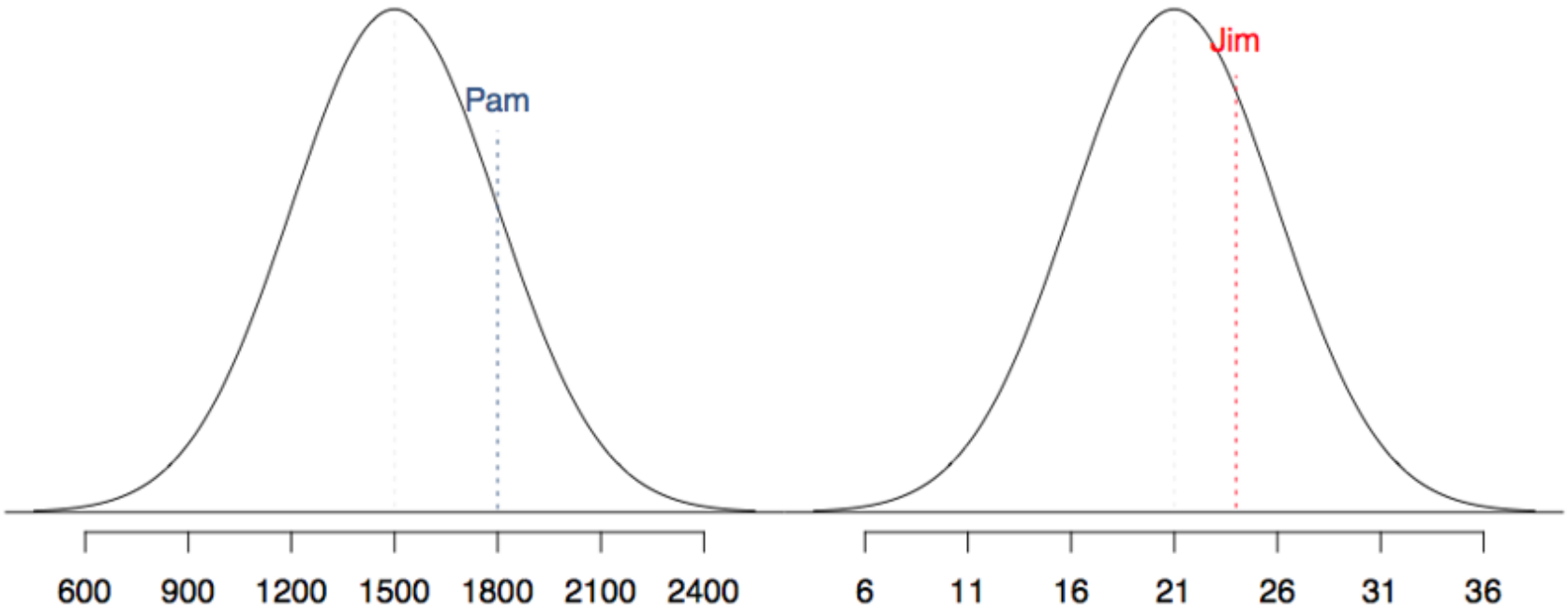


Figure 4: How do we compare normal distributions with different locations and scales?

Visualizing Z-Scores

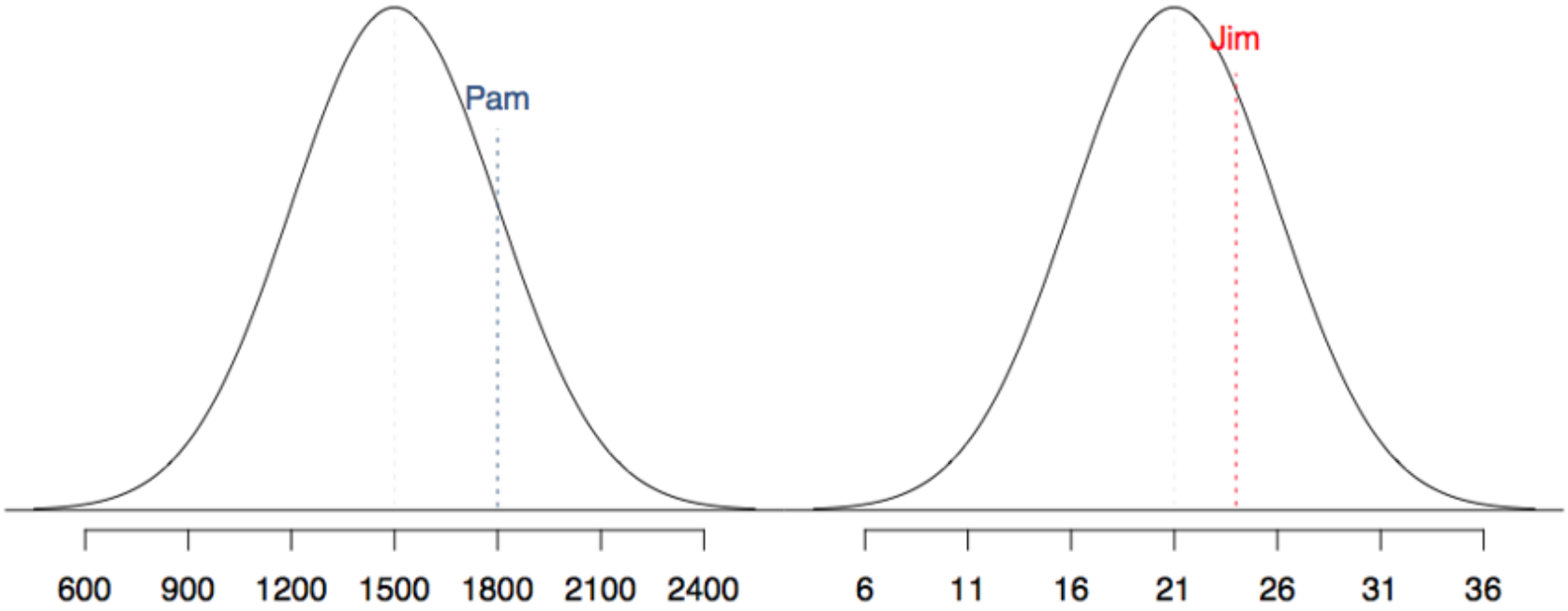


Figure 5: Standardizing the data by converting values to Z-scores puts different distributions on the same scale.

The Standard Normal Distribution

- The ***standard normal distribution*** is a normal distribution with $\mu = 0$ and $\sigma = 1$ (written $N(\mu = 0, \sigma = 1)$)
- Units of the standard normal distribution are standard deviations (Z-scores) (i.e. 1 unit = 1 SD)
- Observations that are 2+ standard deviations from the mean are considered unusual

The 68-95-99.7 Rule

When data is (nearly) normally distributed...

- ~68% of the observations are within 1 standard deviation of the mean ($\mu \pm \sigma$)
- ~95% of the observations are within 2 standard deviations of the mean ($\mu \pm 2\sigma$)
- 99.7% of the observations are within 3 standard deviations of the mean ($\mu \pm 3\sigma$)

The 68-95-99.7 Rule

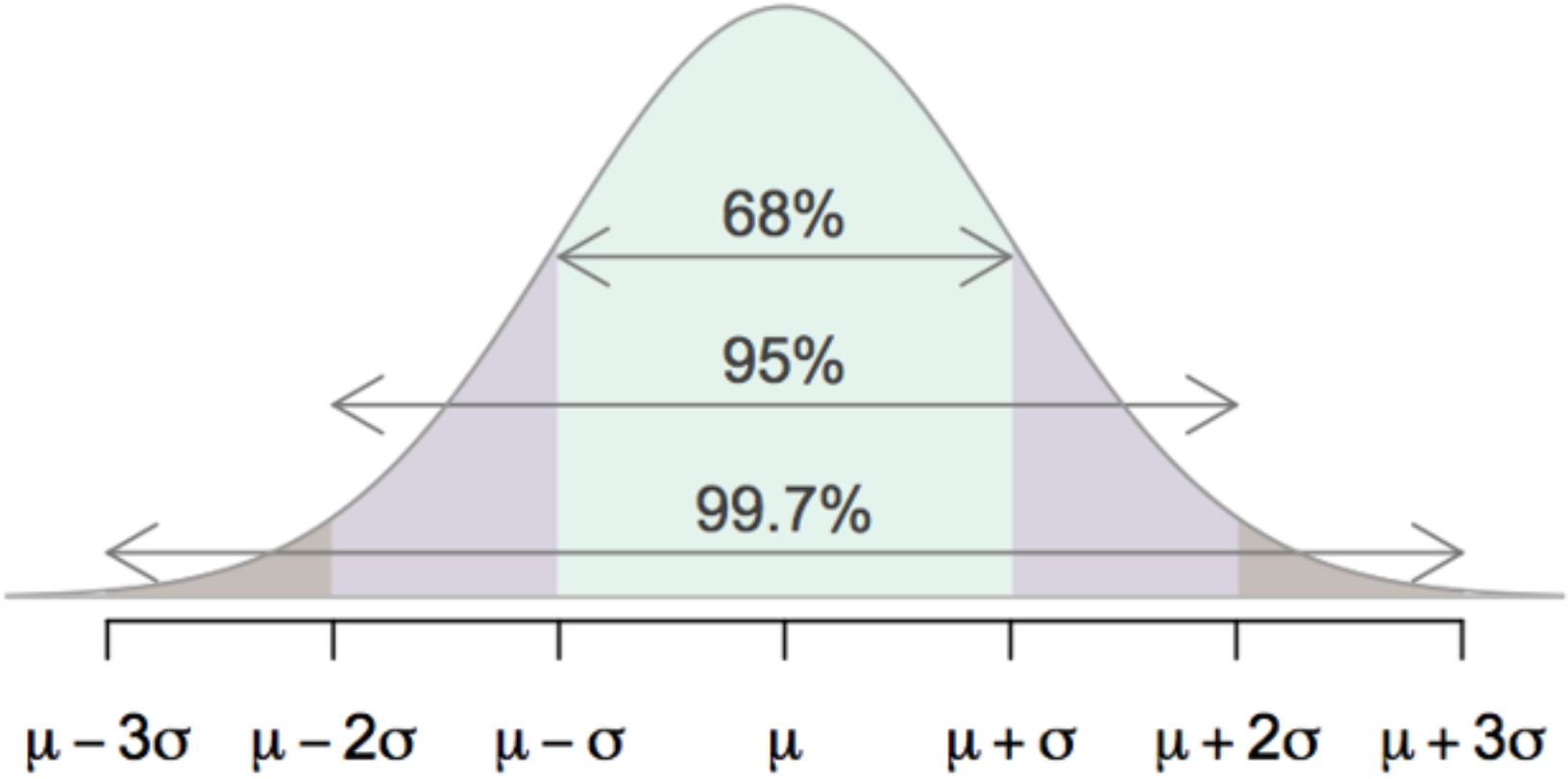


Figure 6: The 68-95-99.7 Rule describes approximately what proportion of the observations should lie within 1, 2, and 3 standard deviations of the mean respectively, if the data is normally distributed

Example: Test Scores

- SAT scores have the distribution $N(1500, 300)$
- ~68% of scores will be 1200-1800
- 95% of scores will be 900-2100
- 99.7% of scores will be 600-2400

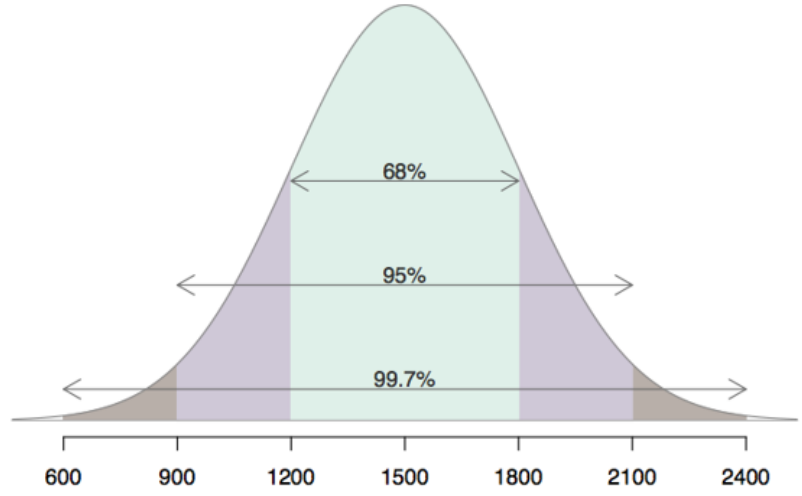


Figure 7: The 68-95-99.7 Rule describes approximately what proportion of the observations should lie within 1, 2, and 3 standard deviations of the mean respectively, if the data is normally distributed

Proportions, Probabilities, and Percentiles

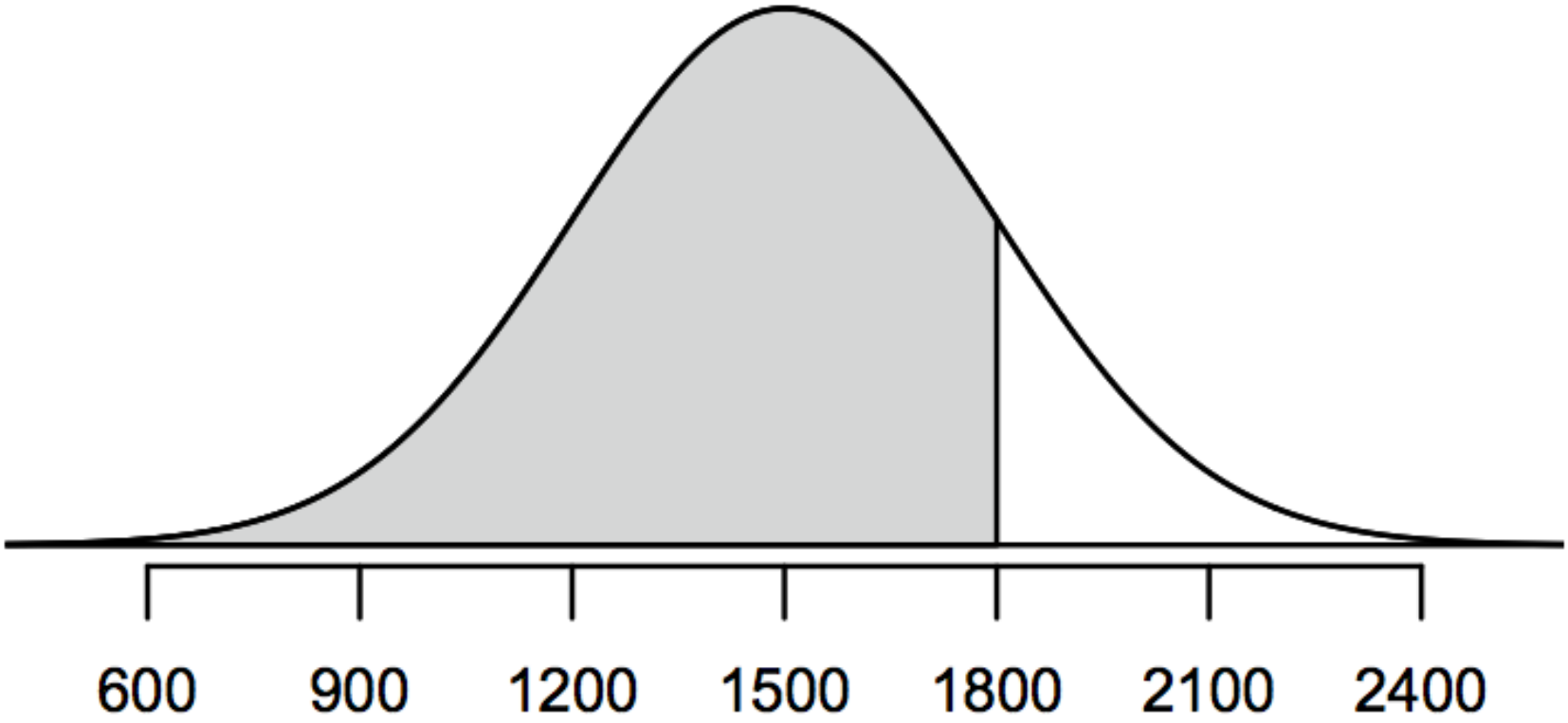


Figure 8: A ***percentile*** is the proportion or percentage of observations that fall *below* a given value in a distribution.

Probability Density and Cumulative Density Functions

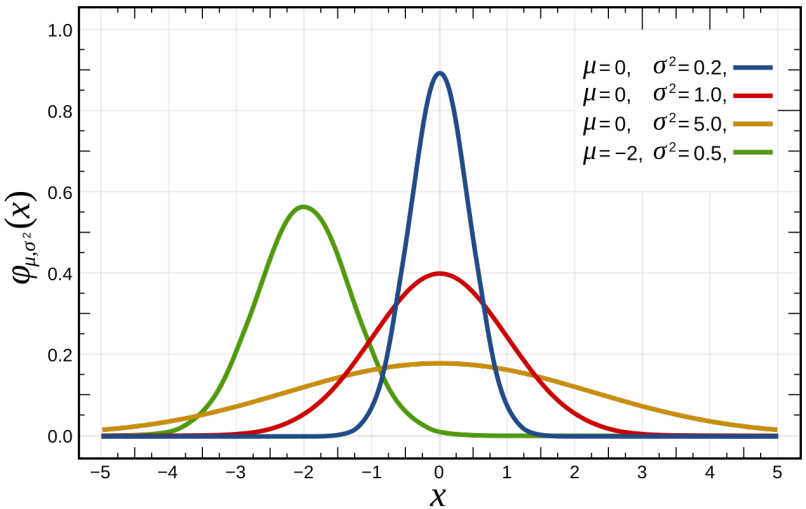


Figure 9: We can calculate the exact probability for a particular value or range of values.

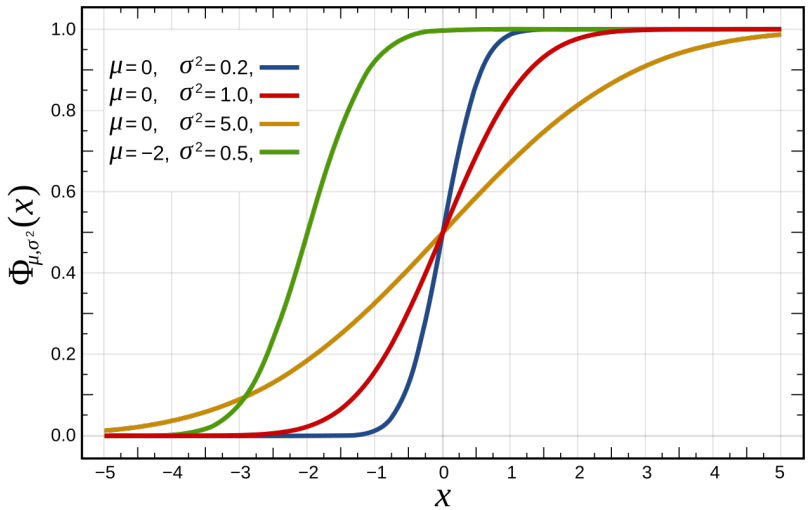


Figure 10: We can calculate the cumulative probability of a variable being less than a given value.

Calculating Percentiles with Z-Score Tables

	Second decimal place of Z									
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	*0.8413*	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015

Figure 11: You can use a Z-Score Table to look up the percentile that corresponds to a particular Z-Score.

Calculating Probabilities with Z-Score Tables

Second decimal place of Z										
0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	Z
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	−2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	−2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	−2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	−2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	−2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	−2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	−2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	−2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	−2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	−2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	−1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	−1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	−1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	−1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	−1.5

Figure 12: You can use a Z-Score Table to look up the probability of a particular Z-Score.

Example: Discrete Numeric Variables

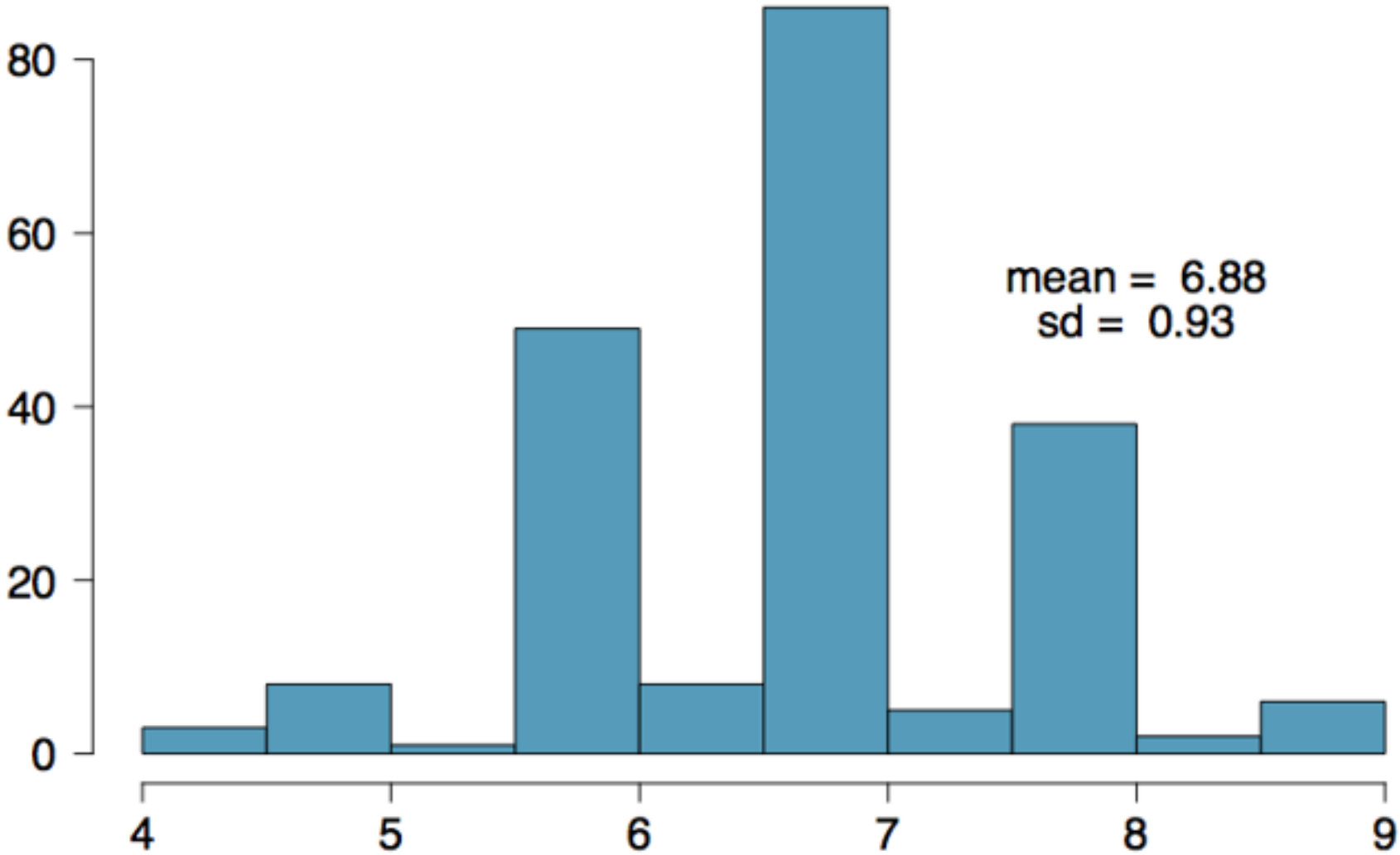


Figure 13: Sometimes the normal distribution is an acceptable approximation of a discrete numeric variable, but other distributions may be more appropriate.

Example: QQ-Plot

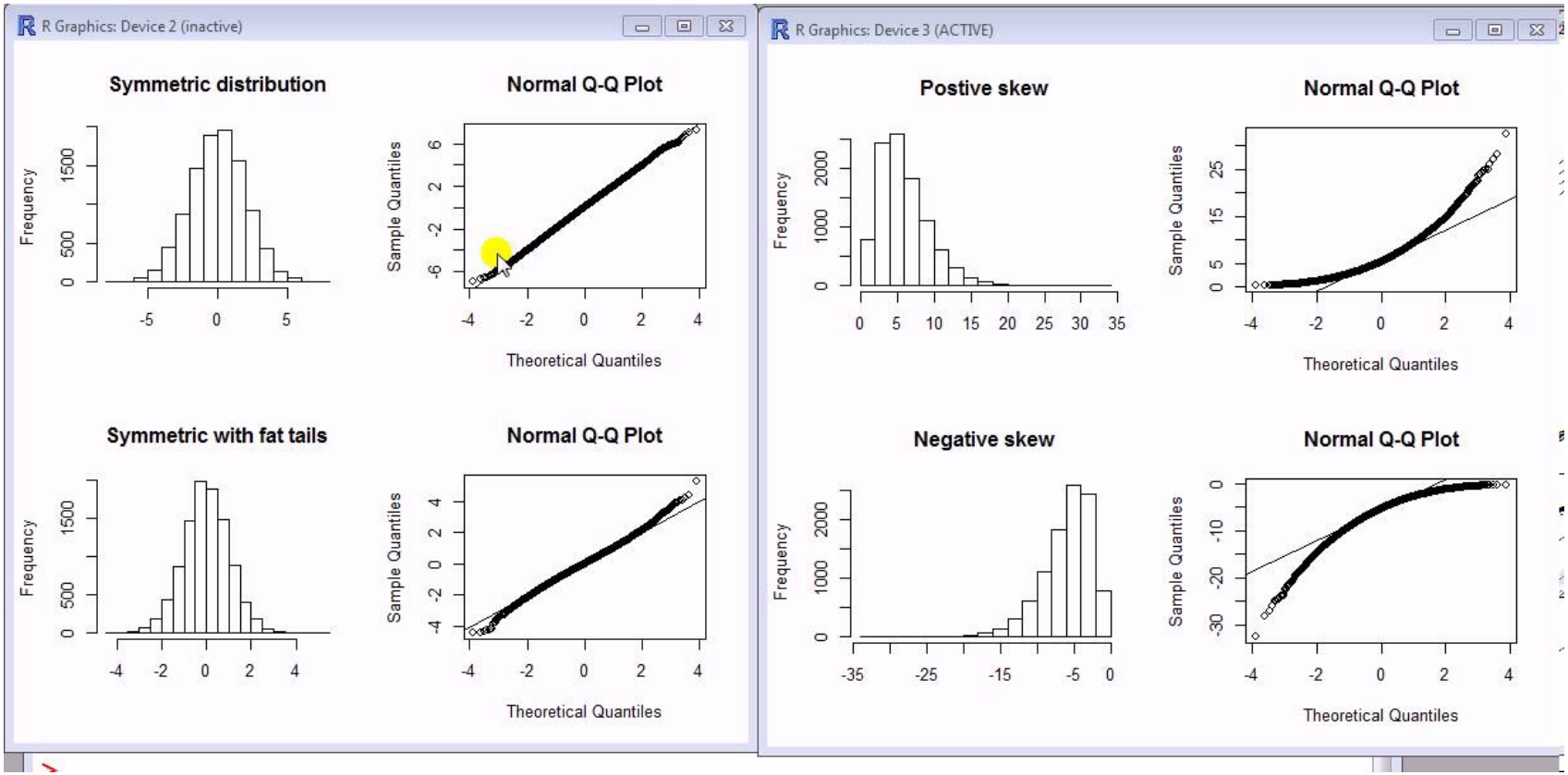


Figure 14: Quantile-Quantile (QQ) Plots can help easily identify when you can and cannot assume normality.