Class 17

DATA1220-55, Fall 2024

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Recap: The Central Limit Theorem

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- A distribution of multiple sample means approximates a normal distribution as the sample size for each mean gets larger
- If you take an infinite number of samples of size n from a population, the *sample statistics* (i.e. means $\bar{x}_1, \bar{x}_2, ..., \bar{x}_\infty$) have a probability distribution (i.e. the ***sampling distribution***) that is about normal

Recap: CLT Requirements

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- Requires identically distributed (i.i.d.) observations

Recap: The CLT in Practice

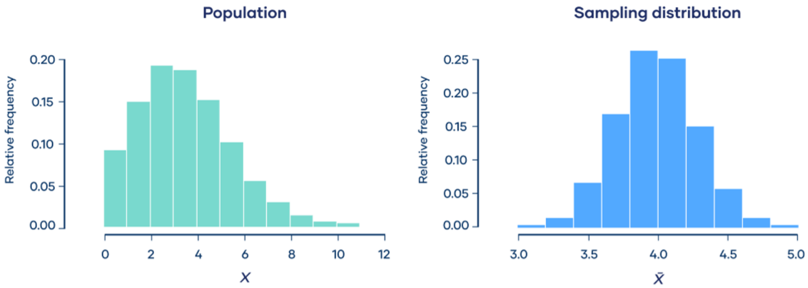

Figure 1: The ***sampling distribution*** of an infinite number of ***sample statistics*** from a population approximates a normal distribution.

Recap: Standard Error of the Sample Statistic

- ***Standard error (SE)*** is the *standard deviation* of the *sample statistic* in a theoretical *sampling distribution*
- If you took an infinite number of samples from a known distribution, the ***standard error*** is the standard deviation of the means of those samples
- Describes the scale (i.e. variability, sampling error) of the sampling distribution

Recap: Calculating the standard error

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- For a distribution of sample proportions, $SE = \sqrt{\frac{p(1-p)}{n}}$

As n increases, the standard error SE decreases.

Recap: Calculating a Z-Score

A ***Z-score*** indicates how many standard deviations σ away from the mean μ a given observation is.

$$Z = \frac{\text{observedvalue} - \text{mean}}{\text{standarddeviation}}$$
$$= \frac{x - \mu}{\sigma}$$

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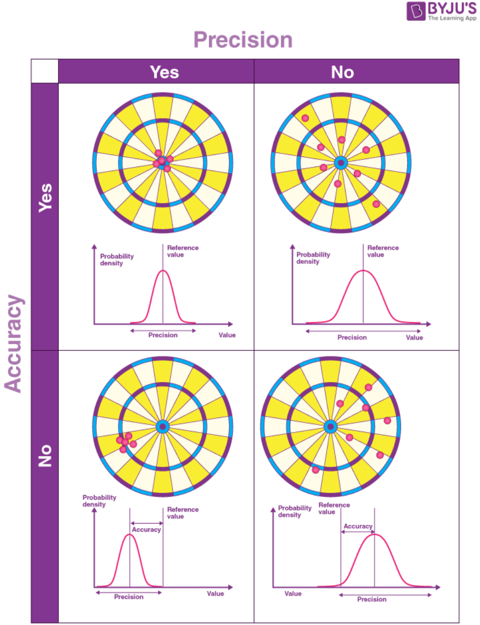
- Accuracy describes how similar an observation or statistic is to the "true" population parameter
- Precision describes how similar the observations or statistics in a distribution are to each other (i.e. the variability of the estimates)

Recap: Accuracy & Precision of Estimates



Recap: Point Estimates & Confidence Intervals

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- A ***point estimate*** describes the ***location*** of an estimate or distribution
- A ***confidence interval*** describes the ***scale*** of an estimate or distribution
- The ***confidence threshold*** or ***confidence level*** describes our uncertainty regarding these values

Recap: Confidence Intervals

A ***confidence interval*** is a numerical range *inside* which a statistic is expected to occur with a given probability $1 - \alpha$ (alpha) in any theoretical sample from a given population

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A ***confidence interval*** is a numerical range *inside* which a statistic is expected to occur with a given probability $1 - \alpha$ (alpha) in any theoretical sample from a given population

- $1 - \alpha$ is the ***confidence level*** and is often expressed as a %
- ***This is only true if your assumptions about the population hold.***

Recap: Confidence Intervals in Practice

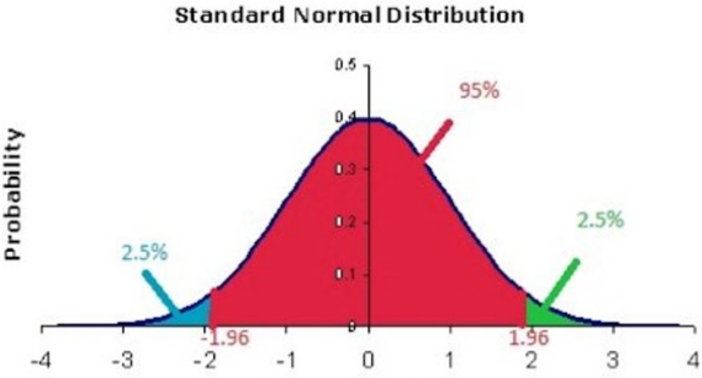

Figure 3: Properties of known distributions, like the 68-95-99.7 Rule, are used to calculate the bounds of a confidence interval.

Recap: Confidence Intervals & Z^*

- A confidence interval is defined as $\text{point estimate} \pm \text{margin of error}$
- $\text{margin of error} = Z^* \times SE$
- $Z^* = \text{Z-Score}_{\alpha/2}$

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3. Your data is ***valid***, so a sampling distribution based on your sample statistics is a ***valid*** estimation of the "true" distribution in the study population.
4. Your data is ***generalizable***, so your estimated sampling distribution for your study population is ***generalizable*** as the "true" sampling distribution for your target population

Statistical Inference and Hypothesis Testing

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- We use sample statistics to describe sample populations and estimate the parameters of the study population's sampling distribution
- We also describe the variability of our measure and quantify our uncertainty regarding our estimate
- We use the overlap between theoretical distributions to decide how meaningful the differences between groups are

Hypothesis Testing Framework

- $\mathbf{H_0}$: The "Null" Hypothesis
  - Represents a position of skepticism, *nothing* is happening here
  - "There is *not* an association between process A and B"

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  - Represents a position of skepticism, *nothing* is happening here
  - "There is *not* an association between process A and B"
- $\mathbf{H_A}$: The "Alternative" Hypothesis
  - The complement of H_0, *something* is happening here
  - "There *is* an association between process A and B"

Conducting a hypothesis test

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Conducting a hypothesis test

- Begin by *assuming* H_0 is the “true” state
- Calculate *the probability that you would see results as extreme or more extreme* than what you saw in your study, assuming the distribution under H_0
- The lower the probability, the less likely it is that we would see these results if H_0 was the “true” state of our population
- If the probability is sufficiently low, we *reject* $\mathbf{H_0}$ and *accept* $\mathbf{H_A}$

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- Predetermined before doing hypothesis test (often $p < 0.05$)
- Also the probability of rejecting the null hypothesis when H_0 is true (i.e. ***Type I Error*** or ***false positive rate***)

Decision Errors

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	*Type 1 Error*
	H_A true	*Type 2 Error*	✓

		Predicted	
		Positive	Negative
Actual	Positive	True Positive (TP)	False Negative (FN)
	Negative	False Positive (FP)	True Negative (TN)

Inference for a Single Proportion

Central Limit Theorem for Proportions: sample proportions $\hat{p}$ will be nearly normally distributed with the mean equal to the population proportion $(\mu = p)$ and the standard deviation equal to the standard error for a proportion $(\sigma = \sqrt{\frac{p(1-p)}{n}})$, such that $\hat{p} \sim N(\mu = p, \sigma = SE_p))$.

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Assumptions: independence, identically distributed, 10+ successes/failures each

Example

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Research Question: Is a hurricane more likely to hit the continental US in 2024?

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What is the study population?

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What is the study population?

All hurricanes which formed in the Atlantic Ocean with the potential to make landfall in the continental US, for which we have records.

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What is the sample population?

298 hurricanes which formed in the Atlantic Ocean between 1980-2023 with the potential to make landfall in the continental United States.

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What is the target population?

Future hurricanes which form in the Atlantic Ocean with the potential to make landfall in the continental US.

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Is it reasonable to assume that the estimated sampling distribution for the study population will be generalizable to the unobserved distribution in the target population?

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- Are the observations *independent*?
- Are the observations *identically distributed*?
- Is the *sample size sufficient*?

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If we assume our data is valid, then we can use our sample statistics to *infer* the sampling distribution for the study population.

If we assume our data is generalizeable, then we can use our sampling distribution to *test the hypothesis* in the target population.

Example

Based on the data from 1980-2023, what is the average probability that a hurricane makes landfall in the continental US?

Step 1: Calculate the sample statistic.

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$$\hat{p} = \frac{72}{298} = 0.242$$

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Step 2: Estimate the sampling distribution.

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$$SE = \sqrt{\frac{0.242(1-0.242)}{298}} = 0.025$$

The sampling distribution for $\hat{p}$ approximates the normal distribution $N(24.2, 2.5)$.

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Step 3: Calculate Z^* for the confidence threshold $\alpha = 0.05$.

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```r
qnorm(0.05 / 2)
```

```
[1] -1.959964
```

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Based on the data from 1980-2023, what is the average probability that a hurricane makes landfall in the continental US?

Step 4: Construct a 95% confidence interval.

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Step 4: Construct a 95% confidence interval.

$\text{point estimate} \pm Z^* \times SE$

```
24.2 - qnorm(0.05 / 2) * 2.5
```

```
[1] 29.09991
```

```
24.2 + qnorm(0.05 / 2) * 2.5
```

```
[1] 19.30009
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With 95% confidence, the probability of a hurricane making landfall in the continental US is 19.3% to 29.1%.

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Step 5: Assume the null hypothesis.

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Based on the data from 1980-2023, what is the average probability that a hurricane makes landfall in the continental US?

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H_0: The probability of a hurricane making landfall in 2024 is 24.2% ($\hat{p} = 24.2\%$).

H_A The probability of a hurricane making landfall in 2024 is *not* 24.2% ($\hat{p} \neq 24.2\%$).

Example

Based on the data from 1980-2023, what is the average probability that a hurricane makes landfall in the continental US?

Step 6: Calculate the sample statistic.

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Based on the data from 1980-2023, what is the average probability that a hurricane makes landfall in the continental US?

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$$\begin{aligned}\hat{p} &= \frac{2}{9} \\ &= 0.222\end{aligned}$$

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Step 7: Calculate the test statistic under H_0.

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$$
\begin{aligned}
Z &= \frac{\hat{p} - p}{SE} \\
&= \frac{22.2 - 24.2}{2.5} \\
&= -0.8
\end{aligned}
$$

Example

Based on the data from 1980-2023, what is the average probability that a hurricane makes landfall in the continental US?

Step 8: Calculate the p-value under H_0.

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```
pnorm(-0.8, mean = 0, sd = 1) * 2
```

```
[1] 0.4237108
```

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Step 9: Reject or fail to reject the null hypothesis.

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The p-value for the observed data under the null hypothesis is $p = 0.423$. As $p > \alpha$ ($\alpha = 0.05$), this is *not* sufficient evidence of a difference.

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Based on the data from 1980-2023, what is the average probability that a hurricane makes landfall in the continental US?

Step 9: Reject or fail to reject the null hypothesis.

The p-value for the observed data under the null hypothesis is $p = 0.423$. As $p > \alpha$ ($\alpha = 0.05$), this is *not* sufficient evidence of a difference.

We fail to reject the null hypothesis that the probability of a hurricane making landfall in 2024 is 24.2%.

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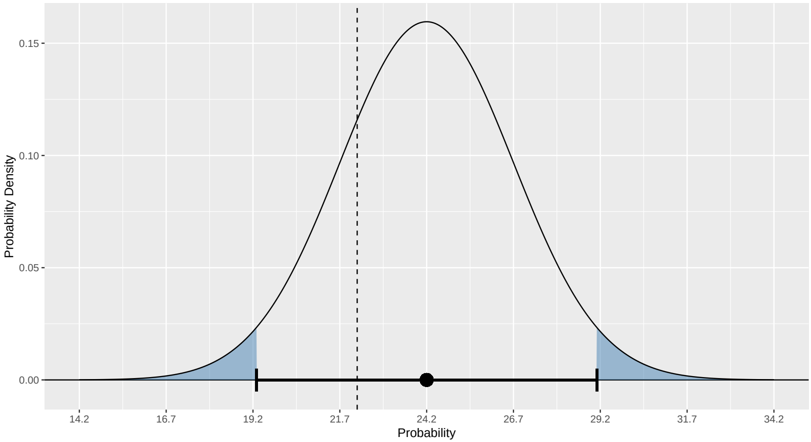

0.15
0.10
0.05
0.00
Probability Density
14.2
16.7
19.2
21.7
24.2
26.7
29.2
31.7
34.2
Probability