Homework #7

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1. If x thousand dollars is spent on labor and y thousand dollars is spent on equipment, a certain factory produces $Q(x, y) = 50\ x^{\frac{1}{2}} y^{\frac{1}{2}}$ units of output.

   (a) How should \$80,000 be allocated between labor and equipment to yield the largest possible output?

   (b) Use the Envelope Theorem to estimate the change in maximum output if this allocation decreased by \$1000.

   (c) Compute the exact change in (b).

2. Let f, g_1 and g_2 be the following functions from $\mathbb{R}^3 \to \mathbb{R}$: $f(x, y, z) = xyz$, $g_1(x, y, z) = x^2 + y^2 - 1$ and $g_2(x, y, z) = x + z - 1$. Consider the problem of maximizing f on the constraint set given by $g_1 = 0$ and $g_2 = 0$.

   (a) Interpret the constraint set geometrically. Is a maximizer guaranteed to exist?

   (b) Find the set of all points in $\mathbb{R}^3$ on which $Dg(x, y, z)$ does not have full rank, where $g(x, y, z) = (g_1(x, y, z), g_2(x, y, z))$. Do these points belong to the constraint set?

   (c) Use Lagrange's Theorem to find the global maximizer of f on the above constraint set.

3. Sundaram, #4, p. 169.

4. Sundaram, #9, p. 170.