Econ 714 Problem Set 1

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Question 1

Part 1

The social planner's problem is to maximize utility subject to the resource constraint:

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$
$$\text{s.t. } C_t = F(K_t) + (1-\delta)K_t - K_{t+1} - D_t$$

The Lagrangian is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta_t \left[U(C_t) + \lambda_t(F(K_t) + (1-\delta)K_t - K_{t+1} - D_t - C_t)\right]$$

Since $U(\cdot)$ and $F(\cdot)$ are strictly increasing and strictly concave, these functions are differentiable. Taking the first order conditions with respect to C_t and K_{t+1}, we see:

$$\lambda_t = U'(C_t)$$
$$\lambda_t = \beta\lambda_{t+1}(F'(K_{t+1}) + (1-\delta))$$

Substituting λ_t from the first equation into the second equation, we have the following Euler equation:

$$U'(C_t) = \beta U'(C_{t+1})(F'(K_{t+1}) + (1-\delta))$$

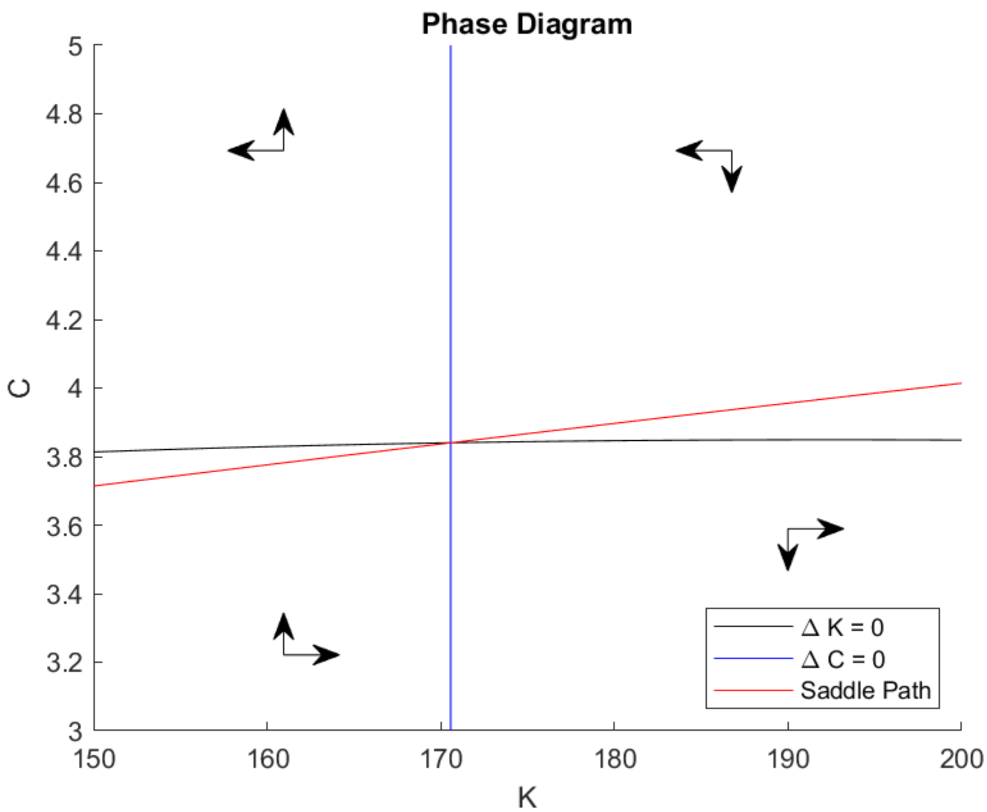
Part 2

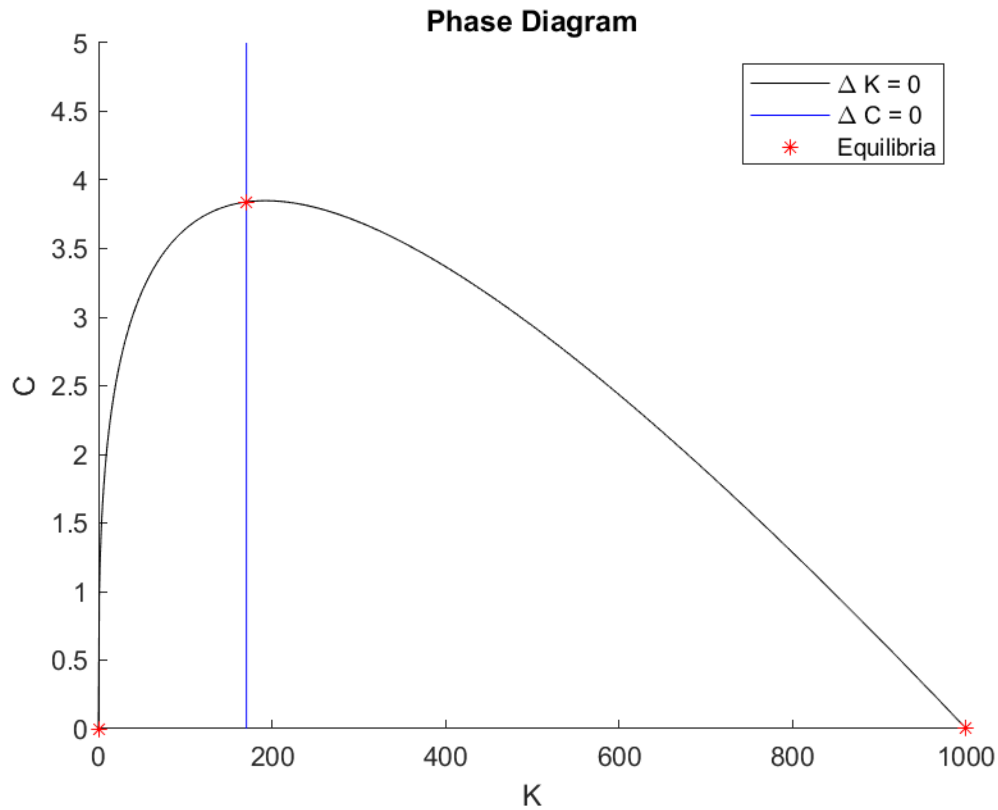
The following system of equations determines the values of capital and consumption in the steady state:

$$C_t = F(K_t) + (1-\delta)K_t - K_{t+1} - D_t$$
$$U'(C_t) = \beta U'(C_{t+1})(F'(K_{t+1}) + (1-\delta))$$
$$\lim_{t\to\infty} \beta^t U'(C_t)K_{t+1} = 0$$

*I have discussed this problem set with Emily Case, Michael Nattinger, Alex Von Hafften, Katherine Kwok, and Danny Edgel.

The phase diagram is shown below. The first graph shows a zoomed-in version of the phase diagram with the saddle path, and the second graph shows a zoomed-out phase diagram with the three equilibrium points indicated.



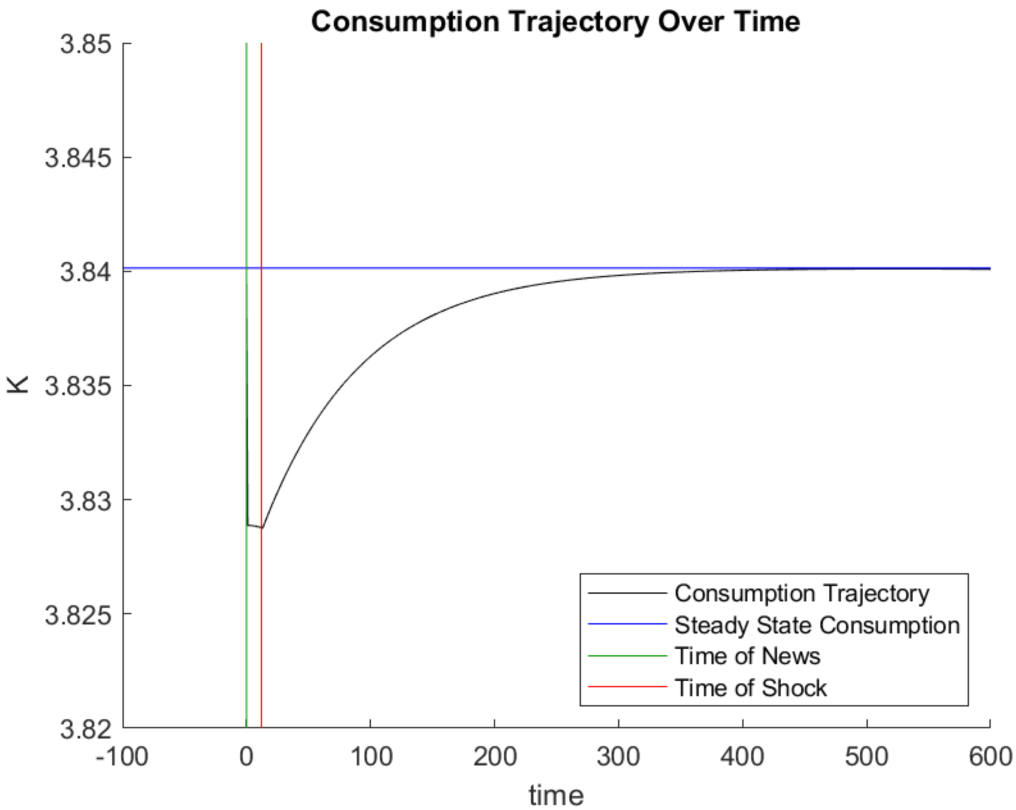


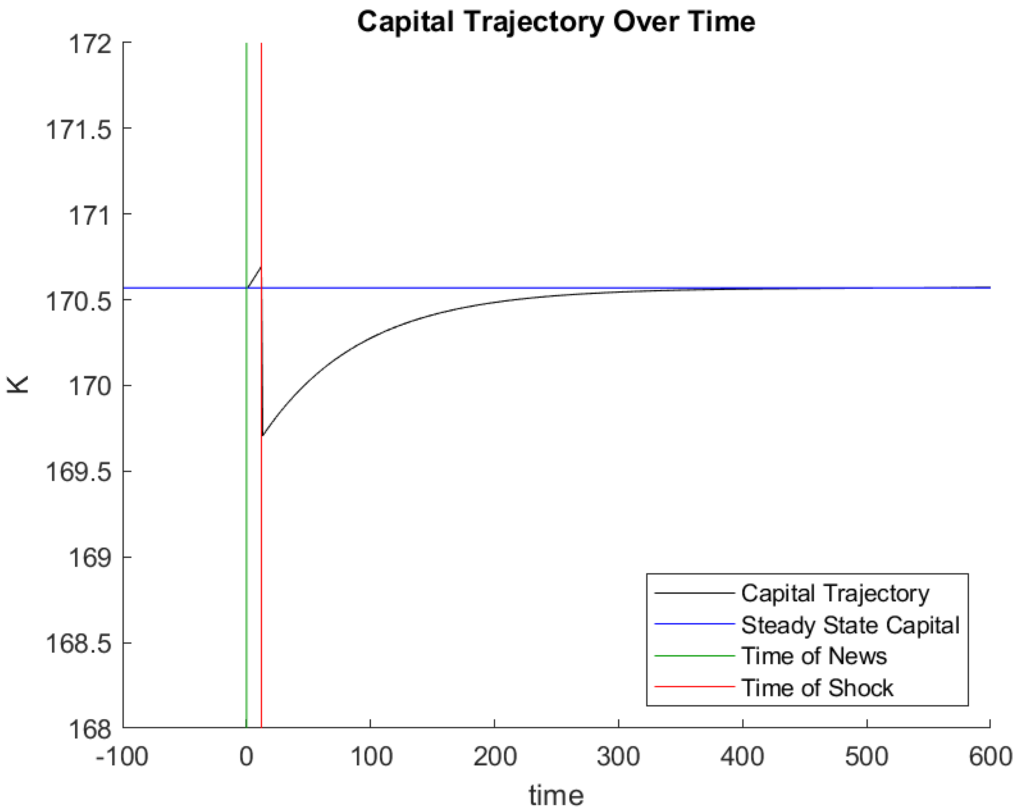
Part 3 & Part 4

The graphs below show the evolution of capital and consumption over time. Since I made these graphs in Matlab, the graphs for part 3 and part 4 of the question are identical, so I only included them once to avoid repetition.

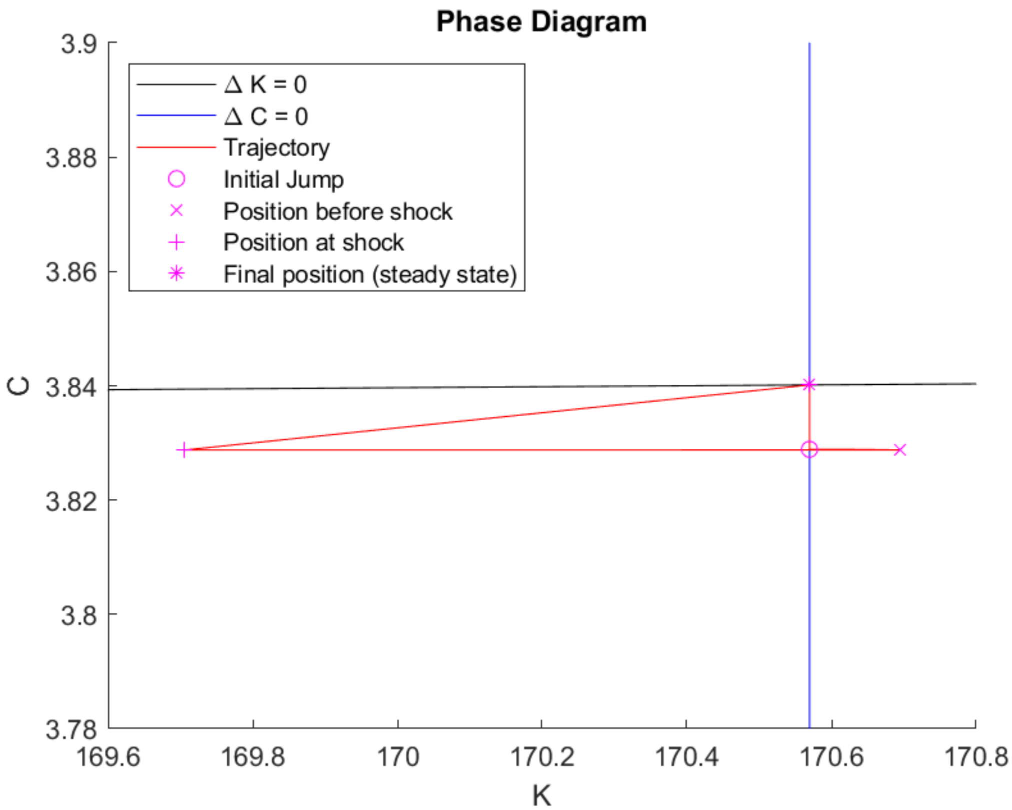
Because agents know that an earthquake is coming in the future, agents will consume less starting at the time of the news and begin stocking up on capital and further reducing consumption (moving in the direction indicated by the arrows in the southeast corner of the phase diagram). When the earthquake hits, the disaster will destroy a fixed amount of the accumulated capital. As a result, agents will shift to the left on the phase diagram, back to the original saddle path. Agents will continue on the saddle path, increasing consumption and accumulating capital, until they are back to the original steady state.

Using the values provided in part 4 of the question and solving in Matlab with the shooting algorithm, the graphs below show the dynamics of consumption and capital over time.


Consumption Trajectory Over Time
3.85
3.845
3.84
3.835
3.83
3.825
3.82
K
-100
0
100
200
300
400
500
600
time
Consumption Trajectory
Steady State Consumption
Time of News
Time of Shock



Capital Trajectory Over Time
172
171.5
171
170.5
170
169.5
169
168.5
168
K
-100
0
100
200
300
400
500
600
time
Capital Trajectory
Steady State Capital
Time of News
Time of Shock


Phase Diagram
Δ K = 0
Δ C = 0
Trajectory
Initial Jump
Position before shock
Position at shock
Final position (steady state)
C
3.9
3.88
3.86
3.84
3.82
3.8
3.78
169.6
169.8
170
170.2
170.4
170.6
170.8
K