Econ 712 PS 1

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Question 1

Rewriting (1), as $p_{t+1} = f(p_t)$:

$$\begin{aligned} p_t &= \frac{d + p_{t+1}}{1 + r} \\ \Rightarrow p_{t+1} &= (1 + r)p_t - d \\ \Rightarrow p_{t+1} &= ap_t + b \end{aligned}$$

where $a = 1 + r$ and $b = -d$.
Using this autonomous equation, we can find the steady state:

$$\begin{aligned} p^* &= ap^* + b \\ &= \frac{b}{1 - a} \\ &= \frac{-d}{1 - (1 + r)} \\ &= \frac{d}{r} \end{aligned}$$

Question 2

Rewriting (1), as $p_{t+1} = f(p_t)$:

$$\begin{aligned} p_t &= \frac{d + p_{t+1}}{(1 + r)} \\ \implies p_{t+1} &= (1 + r)p_t - d \\ \implies p_{t+1} &= ap_t + b \end{aligned}$$

*I have discussed this problem set with Emily Case, Michael Nattinger, Alex Von Hafften, and Danny Edgel.

where $a = 1 + r$ and $b = -d$.

Use $p_t^c = ca^t$ for the complementary function and use $p_t^p = p^*$ for the particular solution. Thus, the general solution is

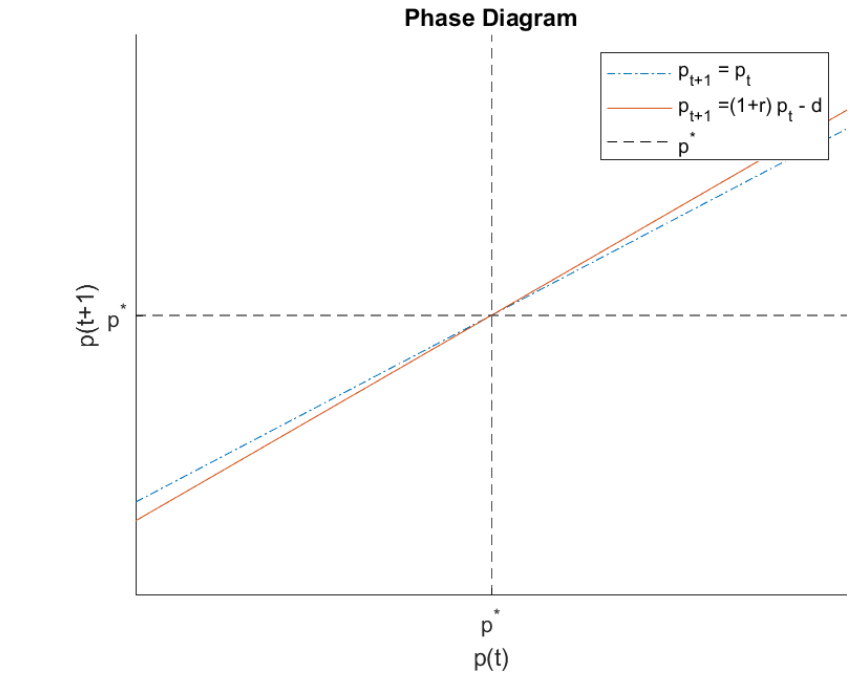
$$p_t^g = p_t^c + p_t^p = ca^t + p^*$$

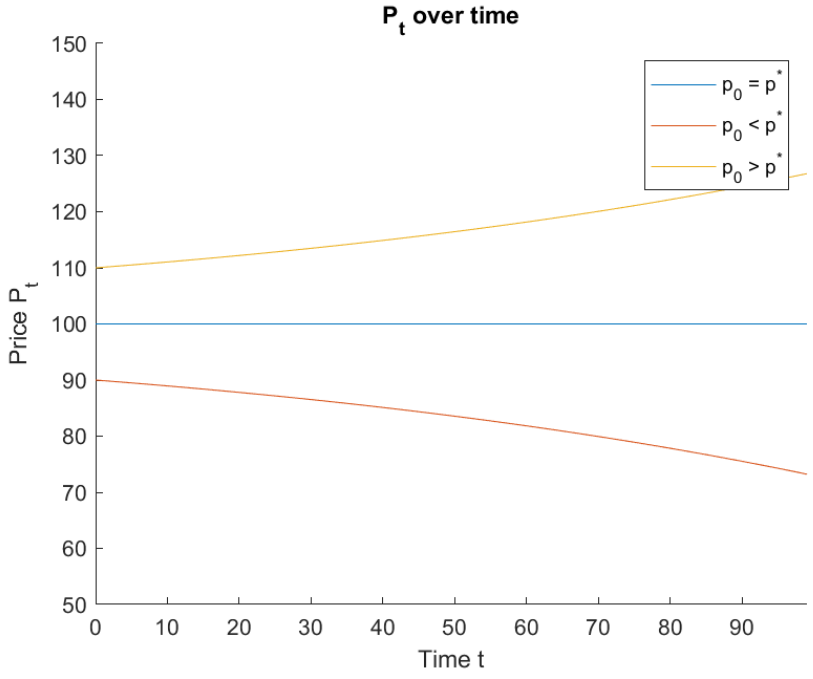
Consider $p_0^g = p_0$. Since $p_0 = ca^0 + p^*$ we know that $c = p_0 - p^*$. Thus,

$$p_t = (p_0 - p^*)a^t + p^*$$

where $a = 1 + r$ and $p^* = \frac{d}{r}$. Because $|a| = |1 + r| > 1$, the price diverges unless $p_0 = p^*$. If $p_0 < p^*$, $p_t \to -\infty$ as $t \to \infty$ and, if $p_0 > p^*$, $p_t \to \infty$ as $t \to \infty$.

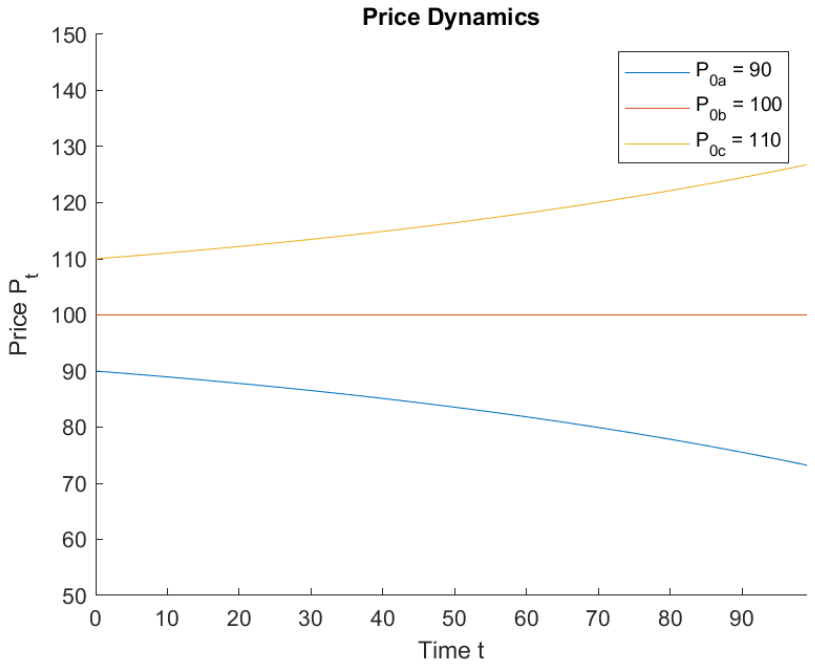
A phase diagram and a graph of p_t against time t are shown below.


Phase Diagram
$p_{t+1} = p_t$
$p_{t+1} = (1+r)\, p_t - d$
p^*
p(t+1)
p^*
p^*
p(t)


P_t over time
$p_0 = p^*$
$p_0 < p^*$
$p_0 > p^*$
Price P_t
150
140
130
120
110
100
90
80
70
60
50
0
10
20
30
40
50
60
70
80
90
Time t

Question 3

The graph below shows the the price dynamics for the first-order difference equation in part with three different initial prices which respectively are below ($p_{0a} = 90$), at ($p_{0b} = 100$), and above ($p_{0c} = 110$) the steady state price level over 100 periods. As we can see, the prices act as predicted in Question 2.



Question 4

In response to an increase in the federal funds rate, the price of the stock will begin and end at a steady state value. However, because the interest rate changes, the stock will begin and end at different steady state values. Before the announcement is made at $t = 20$, the starting steady state will use the initial interest rate $r = 0.01$, so the starting steady state will be $p^* = \frac{1}{0.01} = 100$. After transitioning to the new interest rate in $t = 50$, the ending steady state will use the new interest rate $r = 0.02$, so the ending steady state will be $p^* = \frac{1}{0.02} = 50$.

Because we assume that agents know and believe that prices will be at their fundamental value, when the announcement is made at $t = 20$, agents know that the price of the stock will be at $t = 50$. Knowing the future value, they will adjust the current value of the stock backwards from the value at $t = 50$ during the interest rate transition from $t = 20$ to $t = 50$.

The graph below displays the price dynamics of this change in interest rate.