Signaling/Screening:

Contracts/adverse selection

A PBE is a set of:

1) beliefs $p(\theta|e)$
2) wages $w(e)$
3) signals e_1, e_2, e_3

Such that:

1) wages are optimal given beliefs
2) signals are optimal given wages
3) beliefs are consistent with given signals

Solve for optimal wage by maximizing firm payoff:

$$\max \sum u_F \, p(\theta|e)$$

Take FOC w.r.t w

Check that SOC is < 0 for a max!

Single crossing property: $\frac{d^2c}{d\theta de} < 0$

- negative cross partial of cost
- A separating equilibrium exists. High types can signal their type θ by sending high signals e.

Intuitive Criterion/Cho-Kreps: Of all the PBE, if high types were to collude and still choose a PBE, what would they choose? If they would stay put, it satisfies the criterion.

For a continuum of e values, see 2018 exam Q1.

To find a range on the e values for a separating equilibrium, use both IC constraints!

Finding separating equilibria: 2020 Q2

① Find w as a function of θ (usually $w = E[\theta|e]$) using firm FOC w.r.t w.

② Set $P(\theta|e) = \begin{cases} 1 & \text{if } \theta=\theta_H \text{ and } e=e_H \ldots \\ 1 & \text{if } \theta=\theta_L \text{ and } e\neq e_H \\ 0 & \text{otherwise} \end{cases}$

③ Then $w_H = E[\theta|e_H] = 1 \cdot \theta_H = \theta_H$

$w_L = E[\theta|e \neq e_H] = 1 \cdot \theta_L = \theta_L$

Note, $e_L = 0$ (no incentive to choose $e \notin \{0, e_H\}$.

④ IR constraint: $u_w \geq 0$

IC constraint: $u_{wi} \geq u_{wj}$

$w_i - c(e_i/\theta_i) \geq w_j - c(e_j/\theta_i)$

⑤ Use IC constraints to solve for $e \neq e_L$

Pooling Equilibria 2018 Q1

① Find w as a function of θ using firm FOC w.r.t w (usually $w = E[\theta|e]$)

② Set $p(\theta|e^*)$ = prior mass of each θ

$\begin{cases} \frac{1}{2} & \text{if } \theta_H \\ \frac{1}{2} & \text{if } \theta_L \end{cases}$

③ Then $w^* = E[\theta|e^*]$. Set $w(e) = 1$ for $e \neq e^*$ and $e = 0$ for $e \neq e^*$.

④ IC constraints: $w^* - c(e^*/\theta_i) \geq 1$

⑤ Solve for e^* using ICs.

Adverse Selection: hidden info (quality, type)

An allocation is ex-post efficient if the person with the highest utility receives the item.

When parameters unknown, sufficient conditions for trade occur using expectations.

$p \leq E[u]$ ← buy

$p \geq E[u]$ ← sell

If one agent knows parameter, other agent will condition expectation. 2020 Q1

Joint conditional expectation:

$$E[x \mid x < a] = \int_0^a x f(x)\,dx \Big/ \int_0^a f(x)\,dx$$

$$E[x \mid x < a, y < b] = \int_0^b \int_0^a x f(x) f(y)\,dx\,dy \Big/ \int_0^b \int_0^a f(x) f(y)\,dx\,dy$$

For $x \sim U[0,1]$, $f(x) = 1$, $F(x) = x$.

Efficiency - surplus is maximized, if there are trades that should have happened actually happened

- optimal if everyone could observe all types continuous effort level

Moral Hazard: hidden action (effort)

If the firm can observe effort, firm absorbs risk since workers are risk averse.

$\max E[\pi_h]$ $\max E[\pi_\ell]$ $\pi = x - w$

st IR_h st IR_ℓ

Take FOC w.r.t w_h, w_ℓ, usually $w_h = w_\ell$, plug into IR, Check to see which π_h, π_ℓ more profitable

If the firm can't observe effort:

If the firm wants to induce high effort,

IC: $E[u_H] \geq E[u_L]$ 2019 Exam Q3

where u_H is a function of w_H,

w_H function of output x_H,

output a stochastic function of effort.

- wage of the non-induced effort = 0 (usually)
- Solve for wage using IC, not FOCs

- same optimization as in first part but w/ IR and IC. IR and IC hold w/ = at max, so we can use these to solve