Sem 2 compilation

Taken from so many sources that I'm not citing them properly.

Question 2 (50 points) Consider a static world with a unit continuum of agents. There are two goods: Apples and Bananas. Suppose type 1's have an endowment of 1 apple and 2 bananas and preferences represented by the utility function $u_1(c_a, c_b) = 2\log(c_a) + 2\log(c_b)$. Suppose type 2's have an endowment of 3 apples and 2 bananas and preferences represented by the utility function $u_2(c_a, c_b) = c_a + c_b$. There are exactly a fraction 1/2 of each type.

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1. If endowments are observable, solve for the ex-ante social optimum.
2. Now suppose that type is private. In particular, type 2 agents can claim to be type 1 agents by secretly hiding apples. Is the ex-ante social optimum from the previous question incentive compatible? If not, show why not.
3. Solve for the incentive feasible ex-ante optimum when type 2's can claim to be type 1's. Devise a tax and transfer scheme to implement this.

Question 3

Ramsey Taxation with Heterogeneous Agents

Consider an infinite horizon economy with I types of agents of equal measure. The technology is given by

$$\sum_{i=1}^{I} c_{it} + k_{t+1} - (1-\delta)k_t + g_t = F(k_t, \sum_{i=1}^{I} n_{it})$$

where c_{it} denotes consumption by agent of type i in period t, n_{it} denotes labor input by an agent of type i in period t, k_t denotes the capital stock at the beginning of period t, g_t denotes government consumption in period t, δ the depreciation rate and the production function F is constant returns to scale. Household preferences are given by

$$U^i = \sum_{t=0}^{\infty} \beta^t u^i\left(c_{it}, n_{it}\right)$$

The government has access to proportional taxes on consumption, capital income, labor income and initial wealth. The tax rate is required to be the same on all agents.

1. Define a competitive equilibrium.

2. Suppose the government seeks to maximize a weighted average of utilities of the agents. Define a Ramsey equilibrium and set up the Ramsey problem.

3. Suppose now that household preferences are given by

$$u^i = \frac{c^{1-\sigma}}{1-\sigma} - \psi\left(\frac{n_i}{\theta_i}\right)^{\gamma}$$

where θ_i is a type parameter which is constant over time.

Show that any solution to the Ramsey problem has no intertemporal distortions.

Question 2

Enforcement, Hidden Trading and Externalities

Consider an economy with a measure 1 of agents. Agents can be of two types, A or B, half of the population is of each type. The time horizon is infinite and each period there are two consumption goods, 1 and 2. Preferences are represented by $u(c_1, c_2)$ and satisfy the usual assumptions.

In each period, (including period 0), an exogenous random variable $s \in \{s^1, s^2\}$ is realized. The random varable is *i.i.d.* over time and takes on the value s^1 with probability π. If $s = s^1$, then the endowments of type A agents is (ω_H, ω_H) and that of type B agents is (ω_L, ω_L) where $\omega_H > \omega_L$, and if $s = s^2$, the endowments of type A agents is (ω_L, ω_L) and that of type B agents is (ω_H, ω_H). Markets for trading begin in period 0 before the realization of the period 0 shock.

1. What is an allocation? What is the appropriate notion of resource-feasibility.

2. Define a competitive equilibrium and prove the first and second welfare theorems for this environment.

Now suppose that agents cannot be made worse off from any date onwards than their utility levels associated with consuming their endowments from that date on.

3. Define a competitive equilibrium and prove the first and second welfare theorems for this environment.

Next suppose that agents can trade in a hidden fashion with each other within each period but can be excluded from intertemporal trades.

4. Define a competitive equilibrium for this environment.

5. State conditions on the environment such that the first welfare theorem holds and prove that it does.

6. Provide an example in which the competitive equilibrium is inefficient and explain the source of the inefficiency.

Question 3

Implementing Efficient Allocations

Consider the following two period insurance incentive problem. The economy has a continuum of agents in the unit interval and lasts for two periods, denoted by $t = 0, 1$. All agents are endowed with y_0 units of a single consumption good in period 0. The technology for producing the consumption good in period 1 is

$$y_{1i} = \theta_i l_i$$

where y_i denotes output by agent $i \, \varepsilon \, [0, 1]$, θ_i denotes agent $i's$ productivity, and l_i denotes labor input by agent i. Let θ_i denote the type of agent i. This type can take on two values $\theta_i = 0$ and $\theta_i = 1$. The fraction of the population of type 0 is π. An agent's type is realized at the beginning of period 1 and is private information to each agent. Goods can be stored over time at an exogenous gross interest rate R. Each agent believes the probability of being of type 0 is π. Each agent's preferences over deterministic allocations is given by

$$u(c_{0i}) + \beta\{u(c_{1i}) - v(l_i)\}$$

where u denotes the utility function over consumption, β is the discount factor, and v is the disutility function over labor. Agents maximize expected utility.

1. Define an incentive-feasible allocation.
2. Assume that social welfare is given by the integral over each agent's utility. Set up the planner's problem of solving for an incentive-feasible allocation which maximizes social welfare. What are the first-order conditions that the solution to this problem must satisfy?
3. Consider a government which is restricted to using linear taxes on savings and income, and lump-sum transfers. Can this government implement the solution in part 2? Explain why or why not. If not, what additional instruments would help?

Q1

Consider an economy populated by agents indexed by $\theta \sim f[\underline{\theta}, \overline{\theta}]$. Utility is $c - v(l)$ with v increasing and convex; production is $y = \theta l$. The government wants to implement an allocation of consumption and labour $(l(\theta), c(\theta))$ to maximize a weighted utilitarian criterion:

$$W = \int_{\underline{\theta}}^{\overline{\theta}} (c(\theta) - v(l(\theta))g(\theta)d\theta$$

subject to the resource constraint

$$\int_{\underline{\theta}}^{\overline{\theta}} (c(\theta) - y(\theta))f(\theta)d\theta \leq 0$$

1. Characterize allocations in the full info case, where types are observable
2. Characterize allocations when only production y is observable.
3. Characterize a tax system that implements the solution to (2.). Interpret.

Q2

Consider an economy with a rep agent with preferences

$$\sum_t \beta^t (c_t - u_1 n_t - 0.5 u_2 n_t^2)$$

Production is $y_t = n_t$. The gov levies a flat tax rate τ_t on labour to finance an exogenous stream $\{g_t\}$. The resource constraint is $y_t = c_t + g_t$. Assume a date 0 complete market structure, with date t consumption price q_t. The BC for agents and gov are

$$\sum_t q_t[(1 - \tau_t)n_t - c_t] = 0$$

$$\sum_t q_t(\tau_t n_t - g_t) = 0$$

1. Define a CE with taxes
2. Set up the Ramsey problem and characterize allocs, prices, and taxes under the Ramsey plan
3. Consider two sequences for $\{g_t\}$: $A : \{0, g, 0, g, 0, g, \dots\}$ and $B : \{\beta g, 0, \beta g, 0, \beta g, 0, \dots\}$. Describe how the allocs, prices and taxes differ across the two Ramsey equil associated with the two sequences. Give intuition as to why they differ.

Q3

Log linearize the following equations:

1. $x_t = \frac{w_t}{(1-\alpha)} \left(\frac{\mu_t}{\lambda^{1-\alpha}} + 1 - \mu_t\right)^{-1} \left(1 - \mu_t \frac{\theta}{\gamma} i_t^{\gamma}\right)$
2. $k_{t+1} g_t = y_t - c_t + (1 - \delta - u_t) k_t - F\mu_t$. Linearize wrt μ_t.
3. $\mu_{t+1} = (1 - \mu_t) 2x_t - \mu_t(y_{,t} + \psi) + \mu_t$. Linearize wrt μ_t.
4. $x_t^{\gamma-1} \theta w_t = (1 - y_t) E_t[\beta \frac{c_t}{c_{t+1}} D_{t+1}]$
5. $(\frac{r_t}{\alpha})^{\alpha} (\frac{w_t}{1-\alpha})^{1-\alpha} = \exp(z_t) \exp\left[-\mu_t(1-\alpha) \ln \lambda\right]$. Linearize wrt r_t, z_t, μ_t.
6. $1 = E_t \left[\beta \frac{c_t}{c_{t+1}} g_t [r_{t+1} u_{t+1} - u_{t+1} + 1 - \delta]\right]$. Linearize wrt r_{t+1}.

Q4

Consider an open economy NK model with two countries: Home and Foreign. Under some assumptions and various derivations, we can attain the following objects of interest:

- Home relative to Foreign interest rate (Taylor) rule: $i_t^R = \sigma \pi_t^R + u_t$
- Home relative to Foreign Phillips curve: $\pi_t^R = \delta(q_t - \bar{q}_t) + \beta E_t \pi_{t+1}^R$
- Uncovered interest parity deviations: $i_t^R - E_t s_{t+1} + s_t = \eta_t$

Here q_t is the real exchange rate and s_t is the nominal exchange rate (Home currency per unit of Foreign currency), so that $E_t q_{t+1} - q_t = E_t s_{t+1} - s_t - E_t^R \pi_{t+1}$. $u_t, \bar{q}_t, \eta_t$ are AR(1) shock processes

$$\begin{aligned}
\bar{q}_t &= \mu_q \bar{q}_{t-1} + \epsilon_q \\
u_t &= \mu_u u_{t-1} + \epsilon_u \\
\eta_t &= \mu_e \eta_{t-1} + \epsilon_e
\end{aligned}$$

1. Which vars are state vars, and which are endog vars?
2. Write the model as a linear sytem in terms of $[\pi_t, q_t]$.
3. Solve the model using BK.

Q5

Consider the Eggertson-Krugman model with real debt. 2 types of HHs: Savers with initial assets $-P_1 D_0$, Borrowers with initial assets $P_1 D_0$. Each gets half of the aggregate output Y_t. Preferences are given by $\ln c_1 + \ln c_2$. Borrowing constraint $D_1 \leq P_1 \bar{D}$. Assume that the economy is always in the long run equil in period 2, so $Y_2 = Y$ and $P_2 = 1$. Characterize allocs and prices, and how they change in response to a higher D_0 or lower $\bar{D}$ under 3 scenarios:

1. Flexible prices with $Y_1 = Y$

2. Sticky price P_1 and unconstrained monetary policy, with the planner targeting the natural output $Y_1 = Y$

3. Sticky price P_1 and the ZLB binds

Q6

Some questions from Gali (2015): Chapter 3: 3.3, 3.7; Chapter 5: 5.5