Numbers are free creations of the human mind; they serve as a means of apprehending more easily and more sharply the difference of things - Richard Dedekind

1 Review Topics

Suprema and infima, extreme value theorem, intermediate value theorem, monotone functions

2 Exercises

2.1 For each set, compute the supremum or infimum

- $\sup \left\{ x \in \mathbb{R} \mid x^2 < 7 \right\}$ in $\mathbb{R}$.

- $\inf \left\{ x \in \mathbb{Q} \mid x^2 < 7 \right\}$ in $\mathbb{Q}$.

- $\sup \left\{ 2 - \frac{1}{n} \mid n \in \mathbb{N} \right\}$.

2.2 Prove that for a set $A \subset \mathbb{R}$, bounded above, that an upper bound α of A is the supremum of A if and only if for every $\beta < \alpha$, there exists $a \in A$ such that $\beta < a \leq \alpha$.

2.3 Can we apply the Extreme Value Theorem to the function x^2 on $(0, 1)$?

2.4 Prove that the image of an interval $I \subset \mathbb{R}$ under a continuous function $f : \mathbb{R} \to \mathbb{R}$ is also an interval.

2.5 Let f be strictly monotone and continuous on (a, b). Show that f^{-1} exists and is strictly monotone on $f((a, b))$.

2.6 Let $f : [0, 1] \to [0, 1]$ be a continuous function. Then there exists $x \in [0, 1]$ such that $f(x) = x$.

2.7 Let $f : [0, 1] \to \mathbb{R}$ be defined by $f(x) = x \mathbb{1}_{\mathbb{Q}}(x) + (1 - x) \mathbb{1}_{[0, 1] \setminus \mathbb{Q}}(x)$. Show that f is 1-to-1, $f([0, 1]) = [0, 1]$, but f is not monotone on any interval in $[0, 1]$.