University of Wisconsin-Madison Department of Economics

Econ 703 Prof. R. Deneckere Fall 2003

COURSE PLAN Mathematics for Economists

I. Enrollment Requirement

All first graduate students in economics are required to take this course. Waivers will be given only if a student has previously taken a course in analysis and one in optimization, and obtained satisfactory course grades in each.

II. Prerequisites

One year of multivariate calculus and one semester of linear algebra. Students should be intimately familiar with this material, and review it over the summer, prior to entering graduate school. Good textbooks that cover the relevant material, and lend themselves well to self-study are:

Apostol, T., "Calculus," Volumes I and II, Blaisdell Publishing Company, Waltham, Massachusetts, 1969.

Munkres, J., "Elementary Linear Algebra," Addison-Wesley, Reading, Massachusetts, 1964.

III. Office Hours

By appointment, or drop in the day of class. My office phone number is 263-6724. My office location is Social Science 6422. You can reach me via email at rjdeneck@facstaff.wisc.edu.

IV. Reading Materials

The required text for the course is:

Sundaram, R., "A First Course in Optimization Theory," Cambridge University Press, Cambridge, 1996.

The following two texts are not required, but are highly recommended:

Simon, C. and L. Blume, "Mathematics for Economists," W.E. Norton & Co., New York, 1994.

Rudin, W., "Principles of Mathematical Analysis," Mc Graw-Hill, New York, 1976,

The Simon and Blume text provides a more elementary exposition of much of the material in Sundaram. The Rudin text is a classic, treating Analysis at the undergraduate level. Students wishing to see a lengthier, and somewhat more elementary analysis text may also wish to consult:

Marsden, J., "Elementary Classical Analysis," W.H. Freeman and Company, San Francisco, 1974.

The background material on set theory and logic can be found in:

Munkres, J., "*Topology : A First Course*," Prentice Hall, Englewood Cliffs, New Jersey, 1975, Chapter 1.

V. Grading

The course grade will be a weighted average of the grades on the midterm (40%) and the final (60%).

VI. Course Outline

Below, the required readings from Sundaram's text are indicated with a *. The other readings provide supplementary material that is highly recommended, but not required.

Week 1: Elements of Set Theory and Logic

Fundamental Concepts, Functions, Order and Equivalence Relations, The Real Numbers, Finite Sets, Countable and Uncountable Sets, Induction and Recursion

*Sundaram, Appendix A and B, pp. 315-331. Simon and Blume, Appendix A1, pp. 847-855. Rudin, Chapter 1, pp. 1-21. Munkres, Chapter 1, pp. 3-78.

Week 2: Properties of Rⁿ; Metric spaces

Sequences in R, Lim Inf and Lim Sup, Limits Sequences in Rⁿ; Limit Points, Limits Vectorspaces, Norms, Metric Spaces

*Sundaram, Sections 1.1 and 1.2, pp 1-24. *Sundaram, Appendix C, pp. 330-348. Simon and Blume, Chapter 10 and 12, pp. 199-236 and 253-272. Rudin, Chapter 3, pp. 47-78.

Week 3: Topology of Rⁿ

Basic open sets, Open sets, Closed Sets Compact Sets, Connected Sets, Convex Sets

Simon and Blume, Ch. 29, pp. 803-821.

Week 4 : Continuity and Differentiability of Functions

Continuity, Discontinuities, Monotonic Functions, Upper and Lower Semi-continuity, Lipschitz Continuity, Linear Transformations, Differentiation

*Sundaram, Section 1.4, pp. 41-50. Simon and Blume, Chapter 13, pp. 273-299, Simon and Blume, Sections 14.1-14.4, pp 305-312. Rudin, Chapter 4, pp. 83-102.

Week 5: Differential Calculus

Partial and Directional Derivatives, Chain Rule, Higher Order Derivatives

Simon and Blume, Sections 14.5-14.9, pp. 313-333. Rudin, Chapter 5, pp. 103-119.

Week 6: Functions of Several Variables: Some Important Results

Intermediate and Mean Value Theorems, Taylor's Theorem, Contraction Mapping Theorem, Inverse and Implicit Function Theorem

*Sundaram, Sections 1.5 and 1.6, pp. 49-66. Simon and Blume, Chapter 15, pp. 334-374. Rudin, Chapter 9, pp. 204-238.

Week 7: Existence of Solutions to Optimization Problems; Unconstrained Optimization

Midterm

The Weierstrass Theorem, Upper Semicontinuity and the Generalized Weierstrass Theorem First Order Conditions, Second Order Conditions

*Sundaram, Chapters 2-4, pp. 74-111. Simon and Blume, Chapter 17, pp. 396-410.

Week 8: Local Theory of Constrained Optimization I: Equality Constraints

The Theorem of Lagrange, Constraint Qualifications, Second Order Conditions, Sensitivity Analysis

*Sundaram, Ch. 5, pp. 112-144. Simon and Blume, Chs. 18 and 19, pp. 411-482.

Week 9: Local Theory of Constrained Optimization I: Inequality Constraints

The Kuhn-Tucker Theorem, Mixed Constraints

*Sundaram, Ch. 6, pp. 145-171.

Week 10: Global Theory of Optimization: Concavity

Saddle point Theorem, Concave Functions, Conjugate Functions, Duality, Constrained Optimization

*Sundaram, Ch. 7, pp.172-202. Simon and Blume, Ch. 21, pp. 505-543.

Week 11: Generalized Concavity

Quasiconcavity, Pseudoconcavity, Concave Transformable Functions

*Sundaram, Ch. 8, pp. 203-223.

Week 12 : Parametric Continuity

Correspondences and the Maximum Theorem, Fixed Point Theorems, Sensitivity Analysis

*Sundaram, Ch. 9, pp. 224-241

Week 13: Supermodularity and Parametric Monotonicity

Supermodularity, Parametric Monotonicity, Tarski fixed point theorem.

*Sundaram, Chapter 10, pp. 253-267.

Week 14 : Dynamic Programming

The Maximum Principle, Bellman Equation, Contraction Mappings, Existence of an Optimal Strategy, Continuity and Differentiability Properties of an Optimal Strategy, Dependence upon Parameters

*Sundaram, Chapters 11-12, pp. 268-314.