Econ 703 Homework 7

Fall 2008, University of Wisconsin-Madison

Prof. Raymond Deneckere

Due on Oct. 23, Thu. (in the class)

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1. Prove the following lemma: A metric space (X, d) is connected, iff the only subset of X that are both open and closed are $\emptyset$ and X itself.

2. Let $K_1 \subset \mathbb{R}^n$ and $K_2 \subset \mathbb{R}^m$ be path connected (resp., connected, compact). Show that $K_1 \times K_2$ is path connected (resp., connected, compact).

3. Let $f : (-1, +1) \to \mathbb{R}$ be given by the rule:

$$f(x) = \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

(a) Is f continuous at $x = 0$?
(b) Is f differentiable at $x = 0$?
(c) Investigate the applicability of Taylor's theorem at $x = 0$.

4. Let $I = [0, 1]$, and suppose that $f : I \to I$ is continuous. Prove that there exists $x \in I$ such that $f(x) = x$. (Brouwer fixed point theorem)