UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MACROECONOMICS THEORY Preliminary Exam

June 11, 2019

9:00 am - 2:00 pm

INSTRUCTIONS

- **Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:**

  (1) your assigned number
  (2) the number of the question you are answering
  (3) the position of the page in the sequence of pages used to answer the questions

Example:	
MACRO THEORY	6/11/19
ASSIGNED # ____________	
Qu # __1__ (Page __2__ of __7__):	

- **Do not answer more than one question on the same page!**
  When you start a new question, start a new page.
- **DO NOT write your name anywhere on your answer sheets!**
  After the examination, the question sheets and answer sheets will be collected.
- **Please DO NOT WRITE on the question sheets.**
- **The number of points for each question is provided on the exam.**
- **Answer all questions.**
- **Do not continue to write answers onto the back of the page – write on one side only.**
- **Answers will be penalized for extraneous material; be concise.**
- **You are not allowed to use notes, books, calculators, or colleagues.**
- **Do NOT use colored pens or pencils.**
- **There are seven pages in the exam, plus this instruction page (8 pages total)—please make sure you have all of them.**

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.

- All scratch paper, unused tablet paper, and exams are to be turned in after the exam. Your proctor will give you directions, listen to your proctor.

- Good luck!

Question 1.

Consider the following simple variant of Diamond's one-sector overlapping generations model with production. Each period there is a new unit measure of 2 period lived agents. There is no population growth. Within each generation, agents are divided into two types $i \in \{\ell, e\}$. A fraction $\gamma \in (\frac{1}{2}, 1]$ are of type ℓ and $1-\gamma$ are of type e. Given that there are measures γ of type ℓ agents and $1-\gamma$ of type e agents, we assume perfect competition in all markets.

All agents have identical preferences. They care linearly only about old age consumption and have no disutility from work.

There are three technologies. One technology (call it the "production technology") is a constant returns to scale production function $Y_t = K_t^{\alpha} L_t^{1-\alpha}$ yielding consumption goods where K_t is capital <u>per firm</u> and L_t is labor <u>per firm</u> as in Diamond.

The second technology (call it the "investment technology") converts a unit of consumption goods at time t into a unit of capital at time $t+1$ as in Diamond. All time t capital depreciates fully so gross investment equals net investment as in Diamond's model .

The third is a storage technology (call it the "pillow technology") where one unit of the consumption good stored in period t yields A units of consumption goods in period $t+1$. This is not in Diamond's model. Assume that $\alpha\left[\gamma(1-\alpha)\right]^{-1} \geq A$.

Agent type implies several things:

1. Type ℓ agents are endowed with 1 unit of labor when young and zero labor when old. Type e agents have zero endowment of labor when young and 1 unit when old.

2. Only type e agents have access to the investment technology and only type ℓ agents have access to the pillow technology.

Notice that type ℓ agents (laborers) are natural lender/savers since they need to save their wages paid when young for future consumption. Type e agents (entrepreneurs) are natural borrowers since they have nothing in youth but can borrow to invest in capital which is productive in old age. Without loss of generality we will assume that type e agents run the production technology at $t+1$, supplying their unit of labor in old age and hiring N_{t+1} young agents of type ℓ.. Assume that each initial old of type e is each endowed with capital K_0.

<u>Part 1. Full Information</u>

First assume there is full information and full commitment. In particular, agent type and all actions are publicly observed (and punishable so that it is impossible to not repay one's borrowings).

Q1 (7.5 points). What is the optimization problem facing a type ℓ agent? Let c_{t+1}^i denote consumption at time $t+1$ of a type i agent born at time t,

w_t denote the wage rate at time t, s_{t+1} denote type ℓ lending chosen at time t returning R_{t+1} units of consumption per unit saved at $t+1$, p_{t+1} denote pillow storage at time t returning A units of consumption per unit under the pillow at $t+1$. Under what conditions on R_{t+1} and A will the type ℓ agent choose to lend to entrepreneurs? What does this imply about saving with full information?

Q2 (10 points). What is the optimization problem facing a type e agent at time t? Let B_{t+1} denote borrowing by an entrepreneur at time t (having to pay back R_{t+1} per unit borrowed at $t+1$), K_{t+1} denote the entrepreneur's capital chosen at time t which is an input into his/her $t+1$ production function, N_{t+1} denote the entrepreneur's demand for workers when he/she is running the production technology at $t+1$. What are the first order conditions of the entrepreneur/firm with respect to labor and capital? Hint: recall the entrepreneur has a unit of labor at time $t+1$ and can hire young workers too.

Q3 (2.5 points). What are the labor and capital market clearing conditions? Hint: Since there are $(1-\gamma)$ entrepreneurs/firms, aggregate variables are given by $\overline{X}_{t+1} = (1-\gamma)X_{t+1}$ where X_{t+1} is the per firm quantity.

Q4 (7.5 points) What is the equilibrium law of motion for capital under the assumption that $R_{t+1} \geq A$?

Q5 (7.5 points). In a steady state equilibrium, under what condition is $R^* \geq A$? What does the capital market clearing condition imply about whether borrowing by an entrepreneur B^* is bigger or smaller than saving s^* by a worker?

Part 2. Private Information

Now assume that there is private information about agent type and some actions. In particular, pillow storage activity is private information (*so that an agent who falsely borrows can eat their borrowings next period without punishment*). Labor market transactons however are publicly observed.

Q6 (7.5 points). If borrowing and lending were at the full information levels in question 5, why might a young type ℓ agent want to lie and claim to be a type e agent? What incentive constraint must be imposed to prevent such behavior by a type ℓ agent? Is there a problem with type e claiming to be type ℓ?

Q7 (7.5 points). Suppose that A is sufficiently high that the incentive compatibility constraint binds. What does that imply for the capital stock of the economy in a steady state?

Question 2.

Consider a firm with production function $f(k) = Ak^\alpha$, where $k \geq 0$ is capital, $A \geq 0$ and $0 < \alpha \leq 1$. Time is discrete. Horizon is infinite. The firm's discount factor is $0 < \beta < 1$. Each period the firm's capital stock depreciates at rate δ. In order to build up capital, the firm must invest in capital. The law of motion for the firm's capital is $k_{t+1} = (1-\delta)k_t + i_t$, where i_t is the firm's investment in period t. The investment cost is convex, $c(i) = i^2$. A firm's period t profits are $\pi_t = f(k_t) - c(i_t)$. The firm makes the period t investment choice so as to maximize the value of the firm. A firm's value is the net present value of the discounted stream of future profits. All firms enter with initial capital stock $k_0 = 0$. The economy that the firm is in is stationary.

1. (5 points) State the recursive formulation of the value of a firm that currently has capital stock k. Denote it $V(k)$, where time subscripts have been eliminated given the stationary environment.

2. (7 points) Argue that the $V(k)$ is a fixed point of a functional mapping T, and prove that T is a contraction mapping.

3. (16 points) Using that T is a contraction mapping, prove that $V(k)$ is monotonically increasing and concave in k.

4. (4 points) Denote by $\hat{k}(k)$ the firm's choice of next period's capital stock conditional on the current stock of k. State the first order condition for the optimal choice of $\hat{k}(k)$.

5. (6 points) Argue that there exists a unique stationary firm size for the firm. Denote it by k^*. Solve for k^*. How does stationary firm size depend on TFP, A?

6. (4 points) Suppose the economy is populated by a mass of firms in which A is varying across firms. Suppose all firms have realized their stationary firm size. How does the marginal value of a unit of capital vary across firms in the population of firms?

7. (8 points) Suppose instead that investment cost is linear, $c(i) = i$ and restrict discussion to $0 < \alpha < 1$. For this case, determine how the marginal value of a unit of capital varies across firms, as in question 6. Explain how and why the results are different.

Question 3.

Part A. (30 points) Consider a pure endowment economy with two types of consumers. Consumers of type 1 have the following preferences over consumption goods

$$\sum_{t=0}^{\infty} \beta^t c_{1,t}$$

and consumers of type 2 have preferences

$$\sum_{t=0}^{\infty} \beta^t \ln (c_{2,t})$$

where $c_{i,t} \geq 0$ is the consumption of a type i consumer and $\beta \in (0,1)$ is the common discount factor. The consumption good is tradable but non-storable. Both types of consumers have equal measure. The consumer of type 1 has endowments

$$y_{1,t} = \mu > 0, \, \forall t \geq 0$$

while consumer 2 has endowments

$$y_{2,t} = \begin{cases} 0 & \text{if } t \geq 0 \text{ is even} \\ \alpha & \text{if } t \geq 0 \text{ is odd} \end{cases}$$

where $\alpha = \mu \left(1 + \beta^{-1}\right)$.

1. (5 points) Define a competitive equilibrium with time 0 trading. Be careful to include definitions of all the objects of which a competitive equilibrium is composed.
2. (5 points) Compute (i.e. solve in closed form) a competitive equilibrium allocation with time zero trading.
3. (5 points) Compute the time 0 wealths of the two types of consumers using the competitive equilibrium prices.
4. (5 points) Prove that the competitive equilibrium is efficient.
5. (5 points) Define a competitive equilibrium with sequential trading of Arrow securities.
6. (5 points) Compute a competitive equilibrium with sequential trading of Arrow securities.

Part B. (20 points) Consider a simplified version of the Mirrlees problem where all agents are ex ante identical, but differ ex post by their labor productivity, θ. There are only two possible values of the type, $\theta_H > \theta_L$. Assume that utility is given by: $u(c, l) = u(c) - v(l)$ where c is consumption, l is hours worked and u and v satisfy all of the usual assumptions. Assume that there are three periods, 0, 1, and 2. In period 1, θ is realized. In period 2, output is produced and consumption occurs. Output of a type θ that works l hours is $y = \theta l$. In the remainder of the problem, you are asked to study three sets of assumption about timing and information revelation.

1. (5 points) First, assume that no contracting or exchange is possible at time 0. Rather, assume that θ is public information and that agents can make transfers among themselves after θ is realized. What will the consumption and work hours of each type be in this case. In particular how do these compare across the two types?

2. (5 points) Next, assume that contracting is done at time zero (when all agents are identical) and that at that time, it is known that the type (θ) of each agent will be publicly known at time 1. What will the consumption and work hours of each type be in this case? Compare the welfare, both ex-ante and ex-post, of each type, to the allocation from part 1.

3. (10 points) Assume that contracting is done at time 0, but that at time 1, θ will be private information.

   (a) What is the contracting problem in this case?

   (b) Which IC is binding? Show this

   (c) How do the MRS's (Marginal rates of substitution) between consumption and leisure compare to that from Part 1 for each type. In other words, what are the implicit marginal tax rates?

Question 4.

A) (16 points: 4 points for each part) In our discussion of RBC models, we considered period utility functions of the form

$$u(c,l) = \log c + v(l)$$

Here, c denotes consumption of the household and l denotes labor supplied by the household. Consider the special case in which $v(l) = \log(1-l)$ so that

$$u(c,l) = \log c + \log(1-l) \tag{1}$$

(i) Consider a household who is trying to maximize the u in Equation (1) subject to the following budget constraint:

$$c = w \cdot l$$

Compute $\frac{\partial(1-l)}{\partial w}$. This derivative is the total effect of wages on leisure.

(ii) Consider a household who is trying to maximize the u in Equation (1) subject to the following budget constraint:

$$c + w(1-l) = I$$

Compute $\frac{\partial(1-l)}{\partial I}$. This derivative is proportional to the income (wealth) effect of wages on leisure.

(iii) Re-do parts (i)-(ii) for the alternate utility function:

$$\tilde{u}(c,l) = \log[c + \log[1-l]] \tag{2}$$

(This is a special case of what are known as Greenwood–Hercowitz–Huffman preferences.)

(iv) Is the utility function given by Equation (1) consistent with a balanced-growth path? Is the utility function given by Equation (2) consistent with a balanced-growth path? Explain in one-two sentences your answer to these yes/no questions.

B) (24 points: 8 points for each part) Consider the problem of an infinitely-lived firm with periodic access to a capital market. The firm discounts future profits with discount factor $R^{-1} \in (0,1)$. It starts each period with productivity a and capital k. It has access to a capital market with probability $\gamma \in [0,1]$, which allows it to buy or sell capital at unit price q; the firm's period profits are $a^{\chi} \cdot f(k') - q \cdot (k' - k)$; here $\chi \in (0,1)$ and $q > 0$. With probability $1-\gamma$, $k' = k$ (the firm is not allowed to change its capital stock) and the firm earns period profits $a^{\chi} \cdot f(k)$. Independent of whether the firm accessed the capital market, $\frac{a'}{a}$ is i.i.d., drawn from some distribution $F\left(\frac{a'}{a}\right)$. (Here, a' denotes the firm's productivity in the subsequent period.)

(i) Write a Bellman equation which describes the firm's present-discounted-value maximization problem as a function of a and k.

(ii) What additional conditions, if any, do you need to impose on f so that your value function from part (i) is homogeneous of degree 1 in a?

(iii) For the conditions that you imposed in part (ii), re-write your maximization problem using a single-state-variable value function.

C) (10 points) Summarize, in at most four sentences, the differences between Kiyotaki and Moore (1997) and Bernanke and Gertler (1989). What are the sources of (reasons behind) the borrowing constraint in the two papers? Describe the channel(s) through which increases in borrowers' net worth relax the borrowing constraint.