

University of Wisconsin-Madison
Department of Economics

Econ 703
Fall 2002

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Final Exam

m

1. Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be concave. Let A be an $n \times m$ matrix, and let $b \in \mathbb{R}^n$. Consider the function $h : \mathbb{R}^m \rightarrow \mathbb{R}$ defined by :

$$h(x) = f(Ax + b), x \in \mathbb{R}^m.$$

Is the function h concave? Why or why not?

2. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ and $g : \mathbb{R}^2 \rightarrow \mathbb{R}$ be given by $f(x,y) = x^2 + y^2$ and $g(x,y) = xy$. Find the minimum of $f(x,y)$ subject to $g(x,y) \geq 16$.

3. The following nonlinear program depends on the three parameters p, q and $r \in \mathbb{R}_+$:

$$\max_{\{x_1, x_2\}} px_1^2 + qx_1x_2, \text{ s.t. } x_1^2 + rx_2^2 \leq 1, x_1 \geq 0, \text{ and } x_2 \geq 0.$$

- a) Does a solution exist? Why or why not?
b) Solve the problem.
c) Determine the sensitivity of the optimum value of the problem to the parameters.
4. Consider the following nonlinear program :

$$\max_{x \in \mathbb{R}} x, \text{ subject to } x^2 \leq 0.$$

Does the corresponding Lagrangean have a saddlepoint? Why or why not?