Game Theory Midterm - Sarah Bass

1) $N = \{1, 2\}$

$S_i = \{\text{rock, paper, scissors}\}$

$$u_i(s_i, s_{-i}) = \begin{cases} 2 & \text{if win with S or P} \\ -2 & \text{if lose with S or P} \\ 0 & \text{if tie with S or P} \\ 2+\alpha & \text{if win with R} \\ -2+\alpha & \text{if lose with R} \\ \alpha & \text{if tie with R} \end{cases}$$

	r	p	s
R	(α, α)	$(-2+\alpha, 2)$	$(2+\alpha, -2)$
P	$(2, -2+\alpha)$	$(0, 0)$	$(-2, 2)$
S	$(-2, 2+\alpha)$	$(2, -2)$	$(0, 0)$

Note, this game is symmetric.

If $\alpha > 2$, there is a pure strategy nash equilibrium at (R, r)

$r \to R$

$p \to S \to r \to R \to r$

$s \to R \to r \to R$

If $\alpha = 2$, there are 2 N.Es at (R, r) and (P, r), (P, s), (R, s), (R, p), (S, p), (S, r), (R, r).

$r \to R$

$r \to P \to s \to R$

$p \to S \to r$

$s \to R$

If $\alpha < 2$, there are no N.E.

$r \to P \to S \to R \to p \to S$

2)

a) $N = \{1, 2\}$

$S_i = \{\text{chicken, dare}\}$

$$u_i = \begin{cases} 1 & \text{if } s_i = \text{dare},\ s_j = \text{chicken} \\ -1 & \text{if } s_i = \text{chicken},\ s_j = \text{dare} \\ 0.6 & \text{if } s_i = \text{chicken},\ s_j = \text{chicken} \\ -v_i & \text{if } s_i = \text{dare},\ s_j = \text{dare} \end{cases}$$

	c	d
C	(0.6, 0.6)	(-1, 1)
D	(1, -1)	$(-v_i, -v_j)$

$\underline{v} > 1$

$c \to D \to c$

$d \to C \to d$

The 2 pure strategy N.E are (C, d) and (D, c).

Mixed strategies:

~~Given C, player 2 indifferent b/t c, d if~~

~~$0.6p = (1-p) \Rightarrow p = 0.625$~~

~~Given D, player 2 indif. b/t c, d if~~

~~$-p = -v_j(1-p) \Rightarrow$~~

$0.6\sigma_C \mp (1-\sigma_C) = 1(\sigma_C) + (-v_j)(1-\sigma_C)$ uppercase C

$0.6\sigma_c \mp (1-\sigma_c) = 1(\sigma_c) + (-v_i)(1-\sigma_c)$ lowercase c

$$\sigma_C = \frac{1-v_j}{0.6 \mp v_j} \qquad \sigma_c = \frac{1-v_j}{0.6 \mp v_i}$$

Mixed N.E at $\left(\left(\frac{-v_j+1}{0.6 \mp v_j}\right)(C) + \left(1 - \left(\frac{-v_j+1}{0.6 \mp v_j}\right)\right)(D),\right.$

$\left.\left(\frac{-v_i+1}{0.6 \mp v_i}\right)(c) + \left(1 - \left(\frac{-v_i+1}{0.6 \mp v_i}\right)\right)(d)\right)$

As $v_i \uparrow$, the player 2 less likely to play ~~chicken~~ dare.

2b)

☆

$$U(s_i', s_{-i}, v_i) = \int u_i \, P(v_i > \underline{v})$$

$$s_i(v_j) = \underset{s_i \in S_i}{\operatorname{argmax}} \int (u_i | v_j) \, P(v_j > \underline{v}).$$

$$\boxed{\sigma_i(v_j) = \operatorname{argmax} \int \left[\left(\frac{1-v_j}{0.6-v_j} \right) c + \left(1 - \left(\frac{1-v_j}{0.6-v_j} \right) \right) d \right] F(v_j)}$$

3a) $N = \{Alice, Bob\}$

$s_i = \{R, S\}$

$$u_i = \begin{cases} 4 & \text{if } s_i = S \\ 0 & \text{if } s_i = R,\ s_j = S \\ 6 & \text{if } s_i = R,\ s_j = R \end{cases}$$

	r	s
R	(6,6)	(0, 4)
S	(4,0)	(4,4)

$r \to R \to r$ Pure N.E at (R,r) and (S,s)

$s \to S \to s$

Mixed:

$$6\sigma_R + 0(1-\sigma_R) = 4\sigma_R + 4(1-\sigma_R)$$

$$6\sigma_r + 0(1-\sigma_r) = 4\sigma_r + 4(1-\sigma_r)$$

$$\Rightarrow \sigma_R = 2/3$$

$$\sigma_r = 2/3$$

N.E at $(\frac{2}{3}R + \frac{1}{3}S, \frac{2}{3}r + \frac{1}{3}s)$.

3b)

	r	s	t= r & s
R	(6,6)	(0,4)	(6, 5)
S	(4,0)	(4,4)	(4,3)
T= R&S	(5,6)	(3,4)	(5, 5)

this is not stated in the problem, assumed.

$r \to R \to r$

$s \to S \to s$

$t \to r \to R$

$t \to$

Pure N.E at (R,r), (S,s)

4) Given $v_1 < v_2$,

$$b_1 = \underset{[0,1]}{\operatorname{argmax}} \int (v_1 - b_2)\, P(v_2 < v_1)$$

$$b_2 = \underset{[0,1]}{\operatorname{argmax}} \int (v_2 - b_1)\, P(v_1 < v_2)$$