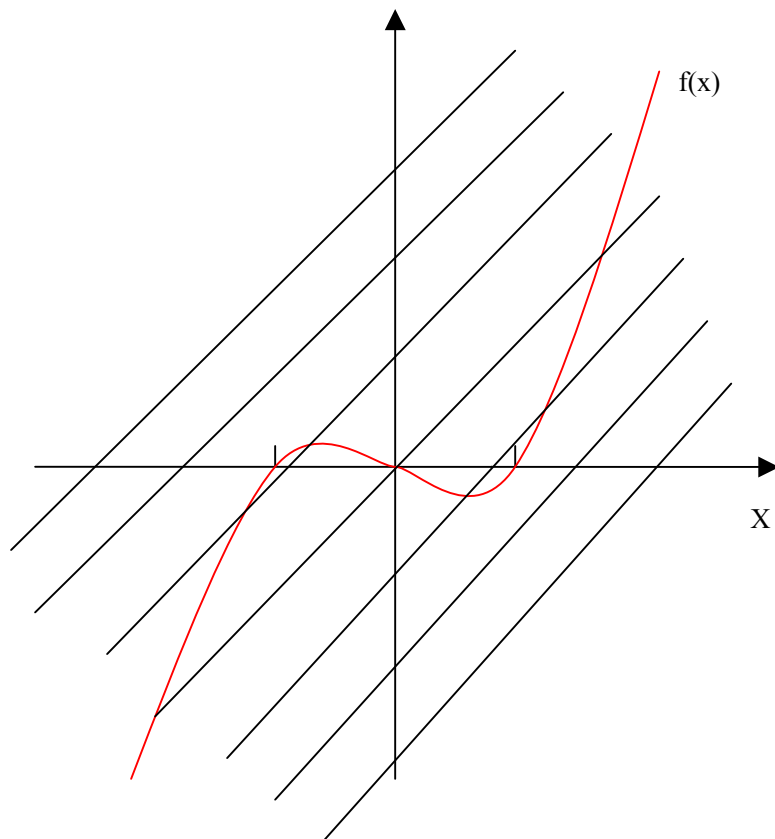


1. The contrapositive statement is: if x is not a square orange, then x doesn't belong to empty set. Because " x doesn't belong to empty set" is always true for any x , it is certainly true when x is not a square orange. Therefore, the contrapositive statement is true. So, the original statement is true.
2. (a) There exists (at least one) $a \in A$, such that $a^2 \notin B$ (or say, such that it is not true that $a^2 \in B$).
 (b) For every $a \in A$, it is true that $a^2 \notin B$ (i.e. it is not true that $a^2 \in B$).
 Another way of negation: There is no $a \in A$ such that $a^2 \in B$.
 (c) There exists (at least one) $a \in A$, such that $a^2 \in B$.
 (d) For every $a \notin A$, it is true that $a^2 \notin B$.
 Another way of negation: There is no $a \notin A$ such that $a^2 \in B$.
3. First, $f(x)$ is as following:



So the function itself is not injective, but it is surjective.

There are many ways to restrict the domain and range to obtain a bijective function g . And for a bijective function, there exists inverse function.

For instance:

$$g: [\sqrt{3}/3, +\infty) \rightarrow [-2\sqrt{3}/9, +\infty), g(x) = x^3 - x.$$

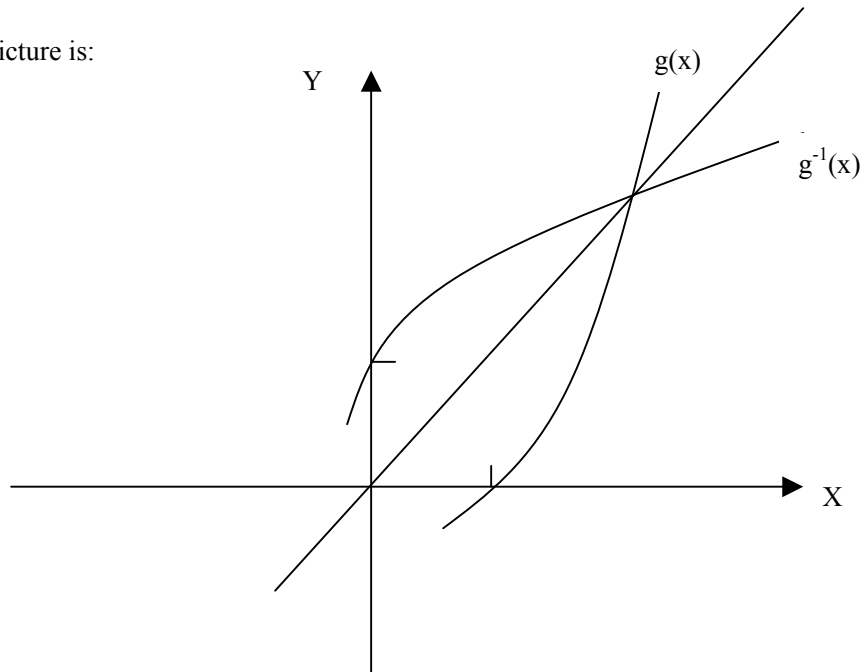
Why is this function injective? Because for any points x and x' in $[\sqrt{3}/3, +\infty)$, and $x \neq x'$, we

must have $g(x) \neq g(x')$.

Why is this function a surjective function? Because, for any $y \in [-2\sqrt{3}/9, +\infty)$, we can find a point x in $[\sqrt{3}/3, +\infty)$ such that y is the image of that point (i.e. $y = g(x)$).

Correspondently, the domain of g^{-1} is $[-2\sqrt{3}/9, +\infty)$, and the range of g^{-1} is $[\sqrt{3}/3, +\infty)$

The picture is:



Some other choices of g

$$g: [1, +\infty) \rightarrow \mathbb{R}_+, g(x) = x^3 - x.$$

$$g: [\sqrt{3}/3, +\infty) \rightarrow [-2\sqrt{3}/9, +\infty), g(x) = x^3 - x.$$

$$g: (-\infty, -1] \rightarrow \mathbb{R}_-, g(x) = x^3 - x.$$

$$g: [-\sqrt{3}/3, -\sqrt{3}/3] \rightarrow [-2\sqrt{3}/9, +2\sqrt{3}/9], g(x) = x^3 - x.$$

4. To prove this relation is an equivalence relation, we need to prove the relation satisfies properties of reflective, symmetry and transitive.

Relation $C = \{((x_0, y_0), (x_1, y_1)) \in \mathbb{R}^2 \times \mathbb{R}^2 \mid y_0 - x_0^2 = y_1 - x_1^2\}$. So if $y_0 - x_0^2 = y_1 - x_1^2$, and $(x_0, y_0), (x_1, y_1) \in \mathbb{R}^2$, then $((x_0, y_0), (x_1, y_1)) \in C$, i.e. $(x_0, y_0) C (x_1, y_1)$.

- 1) Because $y - x^2 = y - x^2$, we have $(x, y) C (x, y)$ for any (x, y) . So relation C has reflective property.

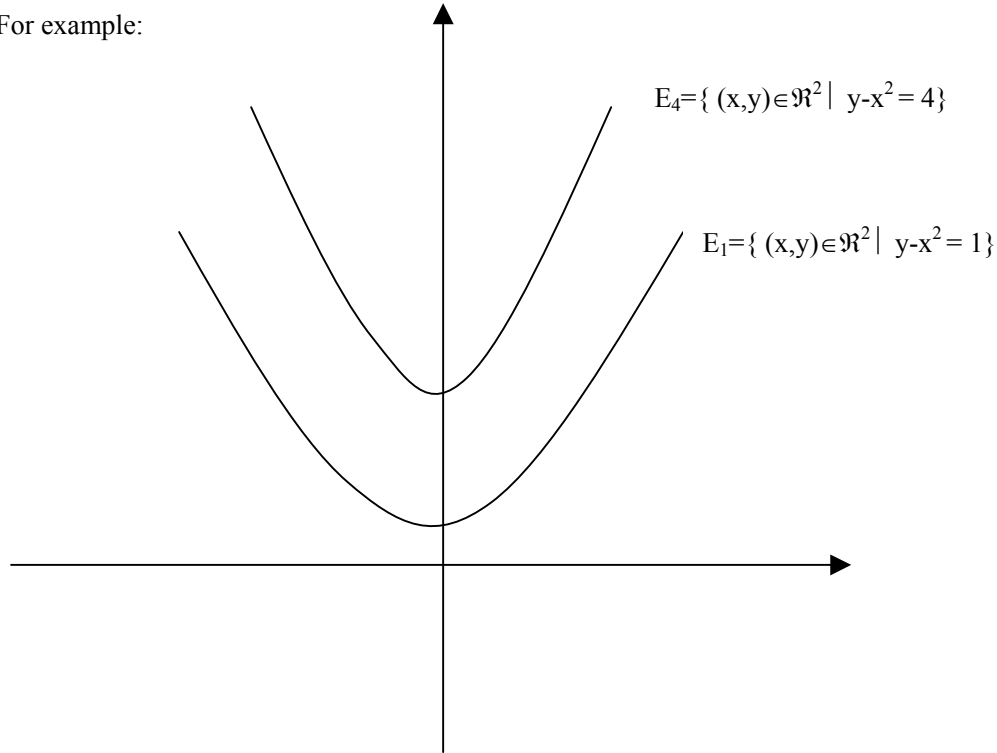
- 2) If $y_0 - x_0^2 = y_1 - x_1^2$, then $y_1 - x_1^2 = y_0 - x_0^2$. That is, if $(x_0, y_0) C (x_1, y_1)$, then $(x_1, y_1) C (x_0, y_0)$. So C has symmetry property

3) If $y_0 - x_0^2 = y_1 - x_1^2$ and $y_1 - x_1^2 = y_2 - x_2^2$, then $y_0 - x_0^2 = y_2 - x_2^2$, that is, if $(x_0, y_0) \in C(x_1, y_1)$, $(x_1, y_1) \in C(x_2, y_2)$, then $(x_0, y_0) \in C(x_2, y_2)$. So C has transitive property

Therefore, this relation is an equivalence relation.

The equivalence classes determined by (x, y) is $E = \{ (x', y') \in \mathbb{R}^2 \mid y' - x'^2 = y - x^2 \}$.

For example:



E_1 is the equivalence classes determined by $(0,1)$, it is also the equivalence determined by $(2,5)$, or $(3,10)$... In fact, it is the equivalence classes determined by any point with $y - x^2 = 1$. We can simply denote it as E_1 . Similarly with E_4 .

5. First, when $n=1$, then unique subset is $\{1\}$, which has the largest element 1.

Second, suppose the statement holds when $n=k$, that is, every nonempty subset of $\{1, 2, 3, \dots, k\}$ $k \in \mathbb{Z}_+$ has a largest element. Now, consider the case of $n=k+1$.

Let S represent the nonempty subsets of $\{1, 2, 3, \dots, k\}$, then $S \cup \{k+1\}$ is a nonempty subset of $\{1, 2, 3, \dots, k, k+1\}$. In fact, the nonempty subset of $\{1, 2, 3, \dots, k, k+1\}$ can be represented by S or $S \cup \{k+1\}$ or $\{k+1\}$. We have known S has a largest element. Suppose the largest element of S is M_s . Then for $S \cup \{k+1\}$, the largest number is $\max \{M_s, k+1\}$, which always exists and equals to $k+1$. For $\{k+1\}$, the largest element is just $k+1$. So the statement is also true for $n=k+1$.

(Another way to state is as following: There are two kinds of nonempty subsets of $\{1, 2, 3, \dots, k, k+1\}$. One is those doesn't include $k+1$, the other is those include $k+1$. Every nonempty subset excluding $k+1$ is also a nonempty subset of $\{1, 2, 3, \dots, k\}$. So it has a largest element. Every nonempty subset including $k+1$ also has a largest number, which is $k+1$. So every nonempty subset of $\{1, 2, 3, \dots, k, k+1\}$ has a largest element.)

Therefore, the original proposition is true for all $n \in \mathbb{Z}_+$.