

R Code

```
# main code file

setwd("C:/z_toshiba/course work/phd/econ 761/hw/hw4/")

rm(list=ls())
library(readxl)
library(tidyverse)
library(MASS)
library(expm)
library(Matrix)
library(dummies)
library(plyr)
library(FixedPoint)

source("blp.R")
source("blpmerger.R")

# import data
x <- read_excel("cereal_ps3.xls")
d <- read_excel("demog_ps3.xls")

x$t <- paste(x$city, x$quarter, sep="_")
d$t <- paste(d$city, d$quarter, sep="_")
x$brand <- paste(x$firm, x$brand, sep="_")

x$brand <- as.factor(x$brand)
x$t <- as.factor(x$t)
x$id <- as.factor(x$id)
x$firm <- as.factor(x$firm)
x <- merge(x, d, by="t")

x$constant <- rep(1, nrow(x))
n <- 20
theta_nlin=rep(0,20)

#####
# OLS without brand FE (1-3)
blp(theta_nlin=theta_nlin, x=x, brandFE=F, iv=F, supply=F, n=n, A=1)
summary(mean_u) # regression results
theta_lin=coefficients(mean_u)[1:4]
c(mean(b), quantile(b, 0.5), sd(b)) # markup statistics
c(mean(mc), quantile(mc, 0.5), sd(mc)) # marginal cost statistics

# Post-Nabisco merger
PNmerger <- x
PNmerger$xi <- residuals(mean_u)
PNmerger$mc <- mc
PNmerger[which(PNmerger$firm==3), "firm"] <- 6
PNmerger$firm <- as.factor(as.character(PNmerger$firm))
mer <- blpmerger(theta_nlin=theta_nlin, theta_lin=theta_lin, x=PNmerger, va=va, n=n, brandFE=F)
c(mean(mer[[1]]), quantile(mer[[1]], 0.5), sd(mer[[1]]))
c(mean(mer[[2]]), quantile(mer[[2]], 0.5), sd(mer[[2]]))

#GM-Quaker merger
```

```

GQmerger <- x
GQmerger$xi <- residuals(mean_u)
GQmerger$mc <- mc
GQmerger[which(GQmerger$firm==2), "firm"] <- 4
GQmerger$firm <- as.factor(as.character(GQmerger$firm))
mer <- blpmerger(theta_nlin=theta_nlin, theta_lin=theta_lin, x=GQmerger, va=va, n=n, brandFE=F)
c(mean(mer[[1]]), quantile(mer[[1]], 0.5), sd(mer[[1]]))
c(mean(mer[[2]]), quantile(mer[[2]], 0.5), sd(mer[[2]]))
#####

#####
# OLS with brand FE (1-3)
Z <- data.matrix(dummy.data.frame(as.data.frame(x$brand)))
blp(theta_nlin=theta_nlin, x=x, brandFE=T, iv=F, supply=F, n=n, A=(t(Z)%*%Z))
coefs # regression results (coefficient estimates)
se # regression results (standard errors)
theta_lin=coefs
c(mean(b), quantile(b, 0.5), sd(b)) # markup statistics
c(mean(mc), quantile(mc, 0.5), sd(mc)) # marginal cost statistics

nam <- names(coefficients(mean_u)[2:length(coefficients(mean_u))])
nam <- gsub("brand", "", nam)
resmd <- data.frame(nam, resmd)
colnames(resmd) <- c("brand", "resmd")

#Post-Nabisco merger
PNmerger <- x
PNmerger$xi <- residuals(mean_u)
PNmerger$mc <- mc
PNmerger[which(PNmerger$firm==3), "firm"] <- 6
PNmerger$firm <- as.factor(as.character(PNmerger$firm))
PNmerger <- join(PNmerger, resmd, by="brand")
mer <- blpmerger(theta_nlin=theta_nlin, theta_lin=theta_lin, x=PNmerger, va=va, n=n, brandFE=T)
c(mean(mer[[1]]), quantile(mer[[1]], 0.5), sd(mer[[1]]))
c(mean(mer[[2]]), quantile(mer[[2]], 0.5), sd(mer[[2]]))

#GM-Quaker merger
GQmerger <- x
GQmerger$xi <- residuals(mean_u)
GQmerger$mc <- mc
GQmerger[which(GQmerger$firm==2), "firm"] <- 4
GQmerger$firm <- as.factor(as.character(GQmerger$firm))
GQmerger <- join(GQmerger, resmd, by="brand")
mer <- blpmerger(theta_nlin=theta_nlin, theta_lin=theta_lin, x=GQmerger, va=va, n=n, brandFE=T)
c(mean(mer[[1]]), quantile(mer[[1]], 0.5), sd(mer[[1]]))
c(mean(mer[[2]]), quantile(mer[[2]], 0.5), sd(mer[[2]]))
#####

#####
# IV without brand FE (1-3)
vars <- seq(1, 20, 1)
vars <- paste("z", vars, sep="")
Z <- data.matrix(x[,vars])
blp(theta_nlin=theta_nlin, x=x, brandFE=F, iv=T, supply=F, n=n, A=(t(Z)%*%Z))
summary(mean_u) # regression results
theta_lin=coefficients(mean_u)

```

```

c(mean(b), quantile(b, 0.5), sd(b)) # markup statistics
c(mean(mc), quantile(mc, 0.5), sd(mc)) # marginal cost statistics

#Post-Nabisco merger
PNmerger <- x
PNmerger$xi <- residuals(mean_u)
PNmerger$mc <- mc
PNmerger[which(PNmerger$firm==3), "firm"] <- 6
PNmerger$firm <- as.factor(as.character(PNmerger$firm))
mer <- blpmerger(theta_nlin=theta_nlin, theta_lin=theta_lin, x=PNmerger, va=va, n=n, brandFE=F)
c(mean(mer[[1]]), quantile(mer[[1]], 0.5), sd(mer[[1]]))
c(mean(mer[[2]]), quantile(mer[[2]], 0.5), sd(mer[[2]]))

#GM-Quaker merger
GQmerger <- x
GQmerger$xi <- residuals(mean_u)
GQmerger$mc <- mc
GQmerger[which(GQmerger$firm==2), "firm"] <- 4
GQmerger$firm <- as.factor(as.character(GQmerger$firm))
mer <- blpmerger(theta_nlin=theta_nlin, theta_lin=theta_lin, x=GQmerger, va=va, n=n, brandFE=F)
c(mean(mer[[1]]), quantile(mer[[1]], 0.5), sd(mer[[1]]))
c(mean(mer[[2]]), quantile(mer[[2]], 0.5), sd(mer[[2]]))
#####

#####
# IV with brand FE (1-3)
vars <- seq(1, 20, 1)
vars <- paste("z", vars, sep="")
Z <- data.matrix(cbind(x[,vars], data.matrix(dummy.data.frame(as.data.frame(x$brand)))))
blp(theta_nlin=theta_nlin, x=x, brandFE=T, iv=T, supply=F, n=n, A=(t(Z)%*%Z))
coefs # regression results (coefficient estimates)
se # regression results (standard errors)
theta_lin=coefs
c(mean(b), quantile(b, 0.5), sd(b)) # markup statistics
c(mean(mc), quantile(mc, 0.5), sd(mc)) # marginal cost statistics

nam <- names(coefficients(mean_u)[2:length(coefficients(mean_u))])
nam <- gsub("brand", "", nam)
resmd <- data.frame(nam, resmd)
colnames(resmd) <- c("brand", "resmd")

#Post-Nabisco merger
PNmerger <- x
PNmerger$xi <- residuals(mean_u)
PNmerger$mc <- mc
PNmerger[which(PNmerger$firm==3), "firm"] <- 6
PNmerger$firm <- as.factor(as.character(PNmerger$firm))
PNmerger <- join(PNmerger, resmd, by="brand")
mer <- blpmerger(theta_nlin=theta_nlin, theta_lin=theta_lin, x=PNmerger, va=va, n=n, brandFE=T)
c(mean(mer[[1]]), quantile(mer[[1]], 0.5), sd(mer[[1]]))
c(mean(mer[[2]]), quantile(mer[[2]], 0.5), sd(mer[[2]]))

#GM-Quaker merger
GQmerger <- x
GQmerger$xi <- residuals(mean_u)
GQmerger$mc <- mc

```

```

GQmerger[which(GQmerger$firm==2), "firm"] <- 4
GQmerger$firm <- as.factor(as.character(GQmerger$firm))
GQmerger <- join(GQmerger, resmd, by="brand")
mer <- blpmerger(theta_nlin=theta_nlin, theta_lin=theta_lin, x=GQmerger, va=va, n=n, brandFE=T)
c(mean(mer[[1]]), quantile(mer[[1]], 0.5), sd(mer[[1]]))
c(mean(mer[[2]]), quantile(mer[[2]], 0.5), sd(mer[[2]]))
#####

#####
# Full model (5-7)
theta_nlin <- c(0, 2, 0, 0, 0.3, 2.2, 0.01, 0.2, 5, 13, -0.2, 1.3, 0, -1, 0, 0, 0.2, 0, 0.3, -0.8)
vars <- seq(1, 20, 1)
vars <- paste("z", vars, sep="")
Z <- data.matrix(cbind(x[,vars], dummy.data.frame(as.data.frame(x$brand))))
params <- optim(par=theta_nlin, fn=blp, x=x, n=n, brand=T, iv=T, supply=F, A=(t(Z)%*%Z),
  method="Nelder-Mead", control=list(reltol=0.1, trace=T))
theta_nlin <- params$par
theta_nlin
blp(theta_nlin=theta_nlin, x=x, n=n, brand=T, iv=T, supply=F, A=(t(Z)%*%Z))
coefs
se
c(mean(b), quantile(b, 0.5), sd(b))
c(mean(mc), quantile(mc, 0.5), sd(mc))

nam <- names(coefficients(mean_u)[2:length(coefficients(mean_u))])
nam <- gsub("brand", "", nam)
resmd <- data.frame(nam, resmd)
colnames(resmd) <- c("brand", "resmd")

#Post-Nabisco merger
PNmerger <- x
PNmerger$xi <- residuals(mean_u)
PNmerger$mc <- mc
PNmerger[which(PNmerger$firm==3), "firm"] <- 6
PNmerger$firm <- as.factor(as.character(PNmerger$firm))
PNmerger <- join(PNmerger, resmd, by="brand")
mer <- blpmerger(theta_nlin=theta_nlin, theta_lin=theta_lin, x=PNmerger, va=va, n=n, brandFE=T)
c(mean(mer[[1]]), quantile(mer[[1]], 0.5), sd(mer[[1]]))
c(mean(mer[[2]], na.rm=T), quantile(mer[[2]], 0.5, na.rm=T), sd(mer[[2]], na.rm=T))

#GM-Quaker merger
GQmerger <- x
GQmerger$xi <- residuals(mean_u)
GQmerger$mc <- mc
GQmerger[which(GQmerger$firm==2), "firm"] <- 4
GQmerger$firm <- as.factor(as.character(GQmerger$firm))
GQmerger <- join(GQmerger, resmd, by="brand")
mer <- blpmerger(theta_nlin=theta_nlin, theta_lin=theta_lin, x=GQmerger, va=va, n=n, brandFE=T)
c(mean(mer[[1]]), quantile(mer[[1]], 0.5), sd(mer[[1]]))
c(mean(mer[[2]], na.rm=T), quantile(mer[[2]], 0.5, na.rm=T), sd(mer[[2]], na.rm=T))

```

```

blp <- function(theta_nlin, x, brandFE=F, iv=T, supply=T, n, A=NULL){

# arrange the non linear coefficients
sig <- diag(theta_nlin[1:4])
pi <- cbind(theta_nlin[5:8], theta_nlin[9:12], theta_nlin[13:16], theta_nlin[17:20])

# place demographic variables in shorter arrays
dj1 <- seq(1, 20, 1)
dj1 <- paste("v", dj1, sep="")
dj2 <- seq(21, 40, 1)
dj2 <- paste("v", dj2, sep="")
dj3 <- seq(41, 60, 1)
dj3 <- paste("v", dj3, sep="")
dj4 <- seq(61, 80, 1)
dj4 <- paste("v", dj4, sep="")

xt <- split(x, as.factor(x$t))

# fixed point algorithm to compute mean utilities
deltas <- NULL
va <- NULL
sij_a <- NULL

for(i in 1:length(xt)){
  xtt <- xt[[i]]
  xi <- data.matrix(xtt[,c("constant", "price", "sugar", "mushy")])
  d <- data.matrix(xtt[,c(dj1, dj2, dj3, dj4)])
  sjo <- c(xtt$share, 1-sum(xtt$share))
  va <- rbind(va,t(rnorm(n)))
  v <- matrix(rep(t(t(rnorm(n)))), ncol(xi)), ncol=n, byrow = TRUE)
  del <- matrix(1, nrow=nrow(xi), ncol=1)
  sij <- matrix(0, ncol=n, nrow=nrow(xi)+1)
  mu <- matrix(0, ncol=n, nrow=nrow(xi))
  for(j in 1:nrow(d)){
    dj <- rbind(d[j,dj1], d[j,dj2], d[j,dj3], d[j,dj4])
    muj <- xi%*%sig%*%v + xi%*%pi%*%dj
    mu <- mu+(muj/(nrow(d)))
    u <- del%*%rep(1, n) + muj
    exp_u <- exp(rbind(u, rep(0, ncol(u))))
    sijt <- sweep(exp_u, 2, colSums(exp_u),`^`)
    sij <- sij + sijt
  }
  sj <- rowMeans(sij)
  delp <- log(sjo) - log(sj) + c(del,1)
  dels <- rbind(c(c(del,1)), c(delp))
  tol <- dist(dels)
  tol <- as.numeric(tol)

repeat{
  del <- delp
  u <- t(t(del[1:(length(del)-1)]))%*%rep(1, n) + mu
  exp_u <- exp(rbind(u, rep(0, ncol(u))))
  sij <- sweep(exp_u, 2, colSums(exp_u),`^`)
  sj <- rowMeans(sij)
  delp <- log(sjo) - log(sj) + del
  dels <- rbind(c(del), c(delp))
}
}

```

```

    tol <- dist(dels)
    tol <- as.numeric(tol)
    if (tol<1e-14) break
  }
  deltas <- rbind(deltas, t(t(delp[1:(length(delp)-1)])))
  u <- t(t(delp[1:(length(delp)-1)]))%*%rep(1, n) + mu
  exp_u <- exp(rbind(u, rep(0, ncol(u))))
  sij_a[[i]] <- sweep(exp_u, 2, colSums(exp_u),`^`)
}
va <- va

# mean utility regression
if(iv==T){
  vars <- seq(1, 20, 1)
  vars <- paste("z", vars, sep="")
  vars <- as.name(paste(vars, collapse="+"))
  form <- paste("price", vars, sep="~")
  p_h <- predict(lm(as.formula(form), data=x))
  if(brandFE==T) { # iv with brand FE
    vars <- c("-1", "p_h", "brand")
  } else { # iv without brand FE
    vars <- c("p_h", "sugar", "mushy")
  }
  vars <- as.name(paste(vars, collapse="+"))
  form <- paste("deltas", vars, sep="~")
} else {
  if(brandFE==T) { # ols with brand FE
    vars <- c("-1", "price", "brand")
  } else { # ols without brand FE
    vars <- c("price", "sugar", "mushy")
  }
  vars <- as.name(paste(vars, collapse="+"))
  form <- paste("deltas", vars, sep="~")
}
mean_u <- lm(as.formula(form), data=x)
nam <- names(coefficients(mean_u))
nam <- which(nam=="price"|nam=="p_h")

#minimum distance estimates for brand dummy FE
if(brandFE==T){
  ymd <- coefficients(mean_u)[2:length(coefficients(mean_u))]
  hvcov <- vcov(mean_u)[2:length(coefficients(mean_u)),2:length(coefficients(mean_u))]
  ymd <- matrix(c(as.numeric(na.omit(ymd))),nrow=nrow(hvcov), ncol=1)
  xmd <- xt[[1]]
  xmd <- data.matrix(xmd[,c("constant", "sugar", "mushy")])
  hdmd <-
solve(t(xmd)%*%solve(hvcov)%*%xmd)%*%t(xmd)%*%solve(hvcov)%*%matrix(c(ymd),nrow=nrow(hvcov), ncol=1)
  resmd <- ymd-xmd%*%hdmd
  semd <- sqrt(diag(solve(t(xmd)%*%solve(hvcov)%*%xmd)))
  coefs <- c(hdmd[1], coefficients(mean_u)[nam], hdmd[2:3])
  se <- c(semd[1], sqrt(vcov(mean_u)[nam,nam]), semd[2:3])
}

# markup and marginal cost estimates
elasticities <- NULL
mc <- NULL

```

```

b <- NULL
for(i in 1:length(xt)){
  xtt <- xt[[i]]
  d <- data.matrix(xtt[,c(dj1, dj2, dj3, dj4)])
  aij <- matrix(0, ncol=n, nrow=nrow(xtt))
  for(j in 1:nrow(d)){
    dj <- rbind(d[j,dj1], d[j,dj2], d[j,dj3], d[j,dj4])
    a <- coefficients(mean_u)[nam] + t(t(xtt$price))%*%sig[2,2]%*%va[i,] + t(t(xtt$price))%*%pi[2,]%*%dj
    aij <- aij+a
  }
  aij <- aij/nrow(d)
  sijt <- sij_a[[i]]
  sijt <- sijt[1:(nrow(sijt)-1),]
  sj <- rowMeans(sijt)
  ep <- rowMeans(aij*sijt*(1-sijt)) #own-price derivatives of demand
  ec <- ((-aij)*sijt)%*%t(sijt)/n #cross-price derivatives of demand
  diag(ec) <- ep
  elasticities[[i]] <- ec*(xtt$price/xtt$share)
  su <- summary(xtt$firm)
  frm <- NULL
  for(k in 1:length(su)) {
    frm[[k]] <- matrix(1, nrow=su[k], ncol=su[k])
  }
  om <- (-1)*data.matrix(bdiag(frm))*ec # omega matrix
  b <- rbind(b, solve(om)%*%sj)
  mc <- rbind(mc, xtt$price-solve(om)%*%sj)
}
b <<- b
mc <<- mc

# estimation of pricing equation
if(supply==T) {
  if(brandFE==T) {
    w <<- lm(mc ~ sugar + mushy + brand + t, data=x)
  } else {
    w <<- lm(mc ~ sugar + mushy + t, data=x)
  }
  g <- matrix(c(residuals(mean_u),residuals(w)), nrow=length(c(residuals(mean_u),residuals(w))), ncol=1)
  ge <<- g
  if(iv==T) {
    vars <- seq(1, 20, 1)
    vars <- paste("z", vars, sep="")
    if(brandFE==T) {
      Z <- data.matrix(rbind(cbind(x[,vars],
dummy.data.frame(as.data.frame(x$brand))),cbind(x[,vars],dummy.data.frame(as.data.frame(x$brand))))))
    } else {
      Z <- data.matrix(rbind(x[,vars],x[,vars]))
    }
  } else {
    Z <- data.matrix(rbind(dummy.data.frame(as.data.frame(x$brand)),dummy.data.frame(as.data.frame(x$brand))))
  }
} else {
  g <- residuals(mean_u)
  ge <<- residuals(mean_u)
  if(iv==T) {
    vars <- seq(1, 20, 1)

```

```

vars <- paste("z", vars, sep="")
if(brandFE==T) {
  Z <- data.matrix(cbind(x[,vars], dummy.data.frame(as.data.frame(x$brand))))
} else {
  Z <- data.matrix(x[,vars])
}
} else {
  if(brandFE==T) {
    Z <- data.matrix(dummy.data.frame(as.data.frame(x$brand)))
  }
}
}

```

```

# criterion function
if(brandFE == T | iv == T) {
  gmm <- ((t(g)%*%Z)%*%solve(A)%*%(t(Z)%*%g))/nrow(xt[[1]])
}
if(brandFE == F & iv == F) {
  gmm <- (t(g)%*%(g))/nrow(xt[[1]])
}
if(length(gmm)==0) {
  gmm <- 1e8
}
return(gmm)
}

```



```

blpmerger <- function(theta_nlin, theta_lin, x, va, n, brandFE=F){

  source("equilibrium.R")
  brandFE <- brandFE

  # arrange the non linear coefficients
  sig <- diag(theta_nlin[1:4])
  hpi <- cbind(theta_nlin[5:8], theta_nlin[9:12], theta_nlin[13:16], theta_nlin[17:20])

  dj1 <- seq(1, 20, 1)
  dj1 <- paste("v", dj1, sep="")
  dj2 <- seq(21, 40, 1)
  dj2 <- paste("v", dj2, sep="")
  dj3 <- seq(41, 60, 1)
  dj3 <- paste("v", dj3, sep="")
  dj4 <- seq(61, 80, 1)
  dj4 <- paste("v", dj4, sep="")

  xt <- split(x, as.factor(x$t))

  # equilibrium after merger
  p_a <- NULL
  sj_a <- NULL

  for(i in 1:length(xt)){
    xtt <- xt[[i]]
    k <- i
    xtt <- xtt[order(xtt$firm, xtt$id),]
    mer <- FixedPoint(Inputs=xtt$price, Function=equilibrium)
    p_a <- rbind(p_a, t(t(mer$FixedPoint)))
    xtt$price <- t(t(mer$FixedPoint))
    xi <- data.matrix(xtt[,c("constant", "price", "sugar", "mushy")])
    d <- data.matrix(xtt[,c(dj1, dj2, dj3, dj4)])

    # predicted market shares
    if(brandFE==F){
      del <- xi%%theta_lin + xtt$xi
    } else {
      hbr <- xi[,c(1,3,4)]%%theta_lin[c(1,3,4)] + xtt$resmd
      del <- data.matrix(cbind(xi[,2], dummy.data.frame(as.data.frame(xtt$brand))))%%c(theta_lin[2], hbr) + xtt$xi
    }
    sij <- matrix(0, ncol=n, nrow=nrow(xi)+1)
    for(j in 1:nrow(d)){
      dj <- rbind(d[j,dj1], d[j,dj2], d[j,dj3], d[j,dj4])
      muj <- xi%%sig%%matrix(rep(va[i,], ncol(xi)), ncol=n, byrow = TRUE) + xi%%hpi%%dj
      u <- del%%rep(1, n) + muj
      exp_u <- exp(rbind(u, rep(0, ncol(u))))
      sijt <- sweep(exp_u, 2, colSums(exp_u), `^`)/nrow(d)
      sij <- sij + sijt
    }
    sj <- rowMeans(sij)
    sj_a <- rbind(sj_a, sj[1:(length(sj)-1)])
  }
  return(list(p_a, sj_a))
}

```

```

equilibrium <- function(p){

  xtt$price <- p
  xi <- data.matrix(xtt[,c("constant", "price", "sugar", "mushy")])
  d <- data.matrix(xtt[,c(dj1, dj2, dj3, dj4)])

  # predicted market shares
  if(brandFE==F){
    del <- xi%%theta_lin + xtt$xi
  } else {
    hbr <- xi[,c(1,3,4)]%%theta_lin[c(1,3,4)] + xtt$resmd
    del <- data.matrix(cbind(xi[,2], dummy.data.frame(as.data.frame(xtt$brand))))%%c(theta_lin[2], hbr) + xtt$xi
  }

  sij <- matrix(0, ncol=n, nrow=nrow(xi)+1)
  for(j in 1:nrow(d)){
    dj <- rbind(d[j,dj1], d[j,dj2], d[j,dj3], d[j,dj4])
    muj <- xi%%sig%%matrix(rep(va[k,], ncol(xi)), ncol=n, byrow = TRUE) + xi%%hpi%%dj
    u <- del%%rep(1, n) + muj
    exp_u <- exp(rbind(u, rep(0, ncol(u))))
    sijt <- sweep(exp_u, 2, colSums(exp_u), `/)/nrow(d)
    sij <- sij + sijt
  }
  sj <- rowMeans(sij)

  # predicted markups
  aij <- matrix(0, ncol=n, nrow=nrow(xtt))
  for(j in 1:nrow(d)) {
    dj <- rbind(d[j,dj1], d[j,dj2], d[j,dj3], d[j,dj4])
    aij <- aij + theta_lin[2] + t(t(xtt$price))%%sig[2,2]%%va[k,] + t(t(xtt$price))%%hpi[2,]%%dj
  }
  aij <- aij/nrow(d)
  sijt <- sij[1:(nrow(sij)-1),]
  ep <- rowMeans(aij*sijt*(1-sijt))
  ec <- (-aij*sijt)%%t(sijt)/n
  diag(ec) <- ep
  su <- summary(xtt$firm)

  frm <- NULL
  for(k in 1:length(su)) {
    frm[[k]] <- matrix(1, nrow=su[k], ncol=su[k])
  }
  om <- (-1)*(data.matrix(bdiag(frm)))*ec
  bt <- solve(om)%%sj[1:(length(sj)-1)]
  hp <- xtt$mc+bt
  return(hp)
}

```