Homework #3

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1. Is every point of every open set $E \subset \mathbb{R}^2$ a limit point of E? Answer the same question for closed sets in $\mathbb{R}^2$.

2. Let $f, g : [0,1] \to \mathbb{R}$ be continuous functions, and suppose that $f(x) > g(x)$ for all $x \in [0,1]$. Prove or disprove the following statement : There exists $\Delta > 0$ such that $f(x) \geqq g(x) + \Delta$ for all $x \in [0,1]$. What if instead f and g were only left continuous?

3. Suppose that $f'(x)$ exists, $g'(x)$ exists, $g'(x) \neq 0$, and $f(x) = g(x) = 0$. Prove that

$$\lim_{t \to x} \frac{f(t)}{g(t)} = \frac{f'(x)}{g'(x)}$$

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = x^3/(x^2+y^2)$ for $(x,y) \neq (0,0)$, and $f(0,0) = 0$.

   (a) Is f continuous in each variable separately?

   (b) Is f a continuous function?

   (c) Compute the directional derivative of $f(\cdot)$ in the direction of the vector $v = (1,1)$

   (d) Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

   (e) Show that $f(x,y)$ is not differentiable at $(0,0)$

5. Sundaram, p.97, #3