Microeconomic Theory (Econ 713)
University of Wisconsin-Madison, Prof. Marzena Rostek
Problem Set 2

Due in (=before) class April 18, 2019

Question 1: Akerlof Model 2.0 **- By Dirk Bergemann**

Consider Akerlof's model as was presented in class. In that model v_S and v_B were the marginal willingness to pay of buyer and seller, which were commonly known. The private information (unobservable by the buyer) to the seller was about the quality of the good (θ). Suppose now that v_S is also private information to the seller and is distributed uniformly on the interval $[\bar{v}_S - \epsilon, \bar{v} + \epsilon]$ for some ϵ

A) Derive necessary and sufficient conditions such that there is positive trade in some equilibrium. *(Hint: It might be helpful to depict the joint distribution of ϵ and θ in a two-dimensional graph.)*

B) How does the introduction of private information about the willingness to pay of the seller affect the conditions. Does the probability of trade decrease or increase with ϵ?

A) The seller will sell if $\theta v_s \leq p$. The buyer will buy if $v_b \mathbb{E}[\theta | \theta v_s \leq p] \geq p$. Given the sellers rule, the expectation can be written as,

$$\int_{v_s-\epsilon}^{v_s+\epsilon} \int_0^{\frac{p}{v_s}} \theta \frac{v_s}{p} \frac{1}{2\epsilon} d\theta dv_s$$

Solving the integral gives us,

$$v_b \frac{p}{4\epsilon} \ln\left(\frac{v_s+\epsilon}{v_s-\epsilon}\right) \geq p$$

As in the original setting, the p conveniently cancels and we get

$$\frac{v_b}{4\epsilon} \ln\left(\frac{v_s+\epsilon}{v_s-\epsilon}\right) \geq 1$$

At this stage it is worth confirming that, as $\epsilon \to 0$, we recover the solution we had from the standard model in class.

B) If you plot the LHS of the condition above you will see that it is strictly increasing in $\epsilon > 0$. Thus, we see that the amount of trade is increasing in ϵ. Why is this? One intuition is that as ϵ increases, the fact that the seller is willing to sell gives the buyer less information, therefore the information revelation effect of price is diminished and consequently we get close to the first best outcome.

Question 2:Adverse Selection - Exercise MWG $13.B.3$
Consider the competitive labor market with a positive selection version of the adverse selection model in which $r(\cdot)$ is a continuous, strictly decreasing function of θ. Let the density of workers of type θ be $f(\theta)$, with $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. In this exercise $r(\theta)$ denotes the reservation utility of a worker of type θ. A worker of type θ will accept employment with wage w, if $w \geq r(\theta)$. Thus, $r(\theta)$ is interpreted as the opportunity cost to a type θ of accepting employment.

1. Show that the more capable workers are the ones choosing to work at any given wage.

2. Show that if $r(\theta) > \theta$ for all θ, then the resulting competitive equilibrium is Pareto efficient.

3. Suppose that there exists a $\hat{\theta}$ such that $r(\theta) < \theta$ for $\theta > \hat{\theta}$and $r(\theta) > \theta$ for $\theta < \hat{\theta}$. Show that any competitive equilibrium with strictly positive employment necessarily involves too much employment relative to the Pareto optimal allocation of workers.

1. Suppose firms offer a wage of w. All workers of type θ, with $r(\theta) \leq w$ will accept the offer. Suppose there exists a θ' such that $r(\theta') = w$. Then all workers of type $\theta \geq \theta'$will work due to the decreasing assumption of $r(\theta)$. Thus, the more capable workers will work at any given wage.

2. Firms can offer the wage $w = \bar{\theta}$ and by assumption, $r(\bar{\theta}) > \bar{\theta}$. Thus, no workers will work since $r(\theta) \geq r(\bar{\theta})$ for any $\theta \in [\underline{\theta}, \bar{\theta}]$. Therefore, the competitive equilibrium is Pareto efficient, i.e. nobody will work.

3. If $w = \hat{\theta}$ only workers of type $\theta \geq \hat{\theta}$ will accept the wage w and work. Note that $w = \hat{\theta}$ induces the Pareto efficient allocation. But, $\mathbb{E}[\theta|\theta \geq \hat{\theta}] > \hat{\theta} = w$ implies that it cannot be a competitive equilibrium. If $w < \hat{\theta}$ only works of type $\theta \geq \theta^*$ such that $r(\theta^*) = w$ will accept the offer. But, again, we have $\mathbb{E}[\theta|\theta \geq \theta^*] > \theta^* = w$. Thus, it cannot be a competitive equilibrium. Thus, to obtain market clearing, firms have to offer a wage $w > \hat{\theta}$, which implies that some workers of type $\theta < \hat{\theta}$ will accept the wage. Since $r(\theta) > \theta$ for any $\theta < \hat{\theta}$, there is over employment in the competitive equilibrium.

Question 3: Prelim June 2011
Consider the following versions of a signaling model, in which a company wants to hire a worker. Productivity is worker private information and is not observable to the company. The company maximizes expected profits. Worker reservation wage is equal to 0. The labor market is competitive.

A) Suppose that a worker has productivity $\theta \in \{\theta_L, \theta_H\}$, $\theta_H > \theta_L$ and $Pr(\theta = \theta_H) = \mu$. Worker chooses an education level $e \in [0, \infty)$ and is paid $w(e)$. A worker's utility is given by $U(w, e, \theta) = w - c(e, \theta)$, where the cost of education $c(e, \theta) = e\theta$. Verify whether the single-crossing condition holds. Describe all the equilibria of this model.

B) As above, workers have productivity $\theta \in \{\theta_L, \theta_H\}$, $\theta_H > \theta_L$, but there is an equal probability of each type. Assume that the cost of education is the same for both workers, $c(e) = e$. Suppose the utility of worker θ who is paid wage w and undertakes education e is $U(w, e, \theta) = \theta w - e$. Is there an equilibrium where different types of workers choose different education levels? If there is, please describe all such equlibria. If not, please explain why. Show your work.

A) Let $p(e) \equiv p(\theta = \theta_H|e)$. Note that the single-crossing property does not hold, so there cannot be any separating equilibria. To find the pooling equilibrium e^*, consider the IC conditions, given the competitive wage. For all e, we have:

$$E(\theta) - \theta_H e^* \geq p(e)\theta_H + (1 - p(e))\theta_L - \theta_H e$$

$$E(\theta) - \theta_L e^* \geq p(e)\theta_H + (1 - p(e))\theta_L - \theta_L e$$

Where $E(\theta) = \mu\theta_H + (1-\mu)\theta_L$. the constraints are easiest to satisfy if $p(e) = 0$ for all $e \neq e^*$, which gives effort levels:

$$0 \leq e^* \leq \frac{E(\theta) - \theta_L}{\theta_H}$$

B) Note that all we've done here is essentially re-normalize worker utility. As a result, the single=crossing property still holds, and separating equilibria will exist. Such equilibria must satisfy the following conditions:

$$\theta_H\theta_H - e_H^* \geq p(e)\theta_H\theta_H + (1 - p(e))\theta_L\theta_H - e$$

$$\theta_L\theta_L - e_L^* \geq p(e)\theta_H\theta_L + (1 - p(e))\theta_L\theta_L - e$$

For all e. Setting $p(e) = 0$ if $e \neq e_L^*, e_H^*$, we obtain

$$\theta_H\theta_H - e_H^* \geq \theta_L\theta_H - e,\ e \neq e_L^*, e_H^*$$

$$\theta_L\theta_L - e_L^* \geq \theta_L\theta_L - e, e \neq e_L^*, e_H^*$$

Clearly then $e_L^* = 0$ and $\theta_H(\theta_H - \theta_L) \geq e_H^*$. For the type L to not deviate to e_H^*, we need

$$\theta_L\theta_L \geq \theta_L\theta_H - e_H^*$$

$$e_H^* \geq \theta_L(\theta_H - \theta_L)$$

so $\theta_L(\theta_H - \theta_L) \leq e_H^* \leq \theta_H(\theta_H - \theta_L)$.

Question 4: Know your value

Consider a signaling model in which a company wants to hire a worker. The *expected productivity* θ_i is private information of worker i and not observable to the company. The company operates in a competitive labor market, so that *workers must be paid their expected productivity.*

Suppose there is an equal mass of three types of workers $\theta_H > \theta_M > \theta_L$. The costs of education is $c(e, \theta_i) = e/\theta_i$, for $i = H, M, L$.

A) Is there an equilibrium where types θ_L and θ_H choose education level e_L, while θ_M chooses education level $e_M \neq e_L$? If there is, please describe it. If not, explain why. Then, apart from providing an anlytical explanation, use the definition of the single-crossing condition to give a brief intuition for your claim of existence of non-existence.

B) Define and describe the set of equilibria where all three types act identically (i.e. where all three types pool).

A) The problem set-up is standard and so omitted. We just consider education levels and omit belief determination. We argue that no such equilibrium exists. Suppose otherwise, so that θ_L chooses education level e_L^* and θ_M chooses education level $e_M^* \neq e_L^*$. The incentive constraints for types M, L are

$$w_M - \frac{e_M^*}{\theta_M} \ \geq \ w_L - \frac{e_L^*}{\theta_M}$$

$$w_L - \frac{e_L^*}{\theta_L} \ \geq \ w_M - \frac{e_M^*}{\theta_L}$$

Together, these IC constraints imply

$$\theta_M(w_M - w_L) \geq e_M^* - e_L^* \geq \theta_L(w_M - w_L)$$

Therefore, $w_M \geq w_L$. But then θ_H prefers (e_M, w_M) to (e_L, w_L) since

$$\theta_H(w_M - w_L) \ \geq \ \theta_M(w_M - w_L) \ \geq \ e_M^* - e_L^*$$

implies:

$$w_m - \frac{e_M^*}{\theta_H} \ > \ w_L - \frac{e_L^*}{\theta_H}.$$

This contradcits $e_L = e_H$, which completes the proof.

Finally, the single-crossing condition argues that that if a lower type is indifferent between signal-action pairs, then a higher type strictly prefers to send the higher signal. This guarantees that higher types send weakly higher signals in equilibrium.

B) The common wage is competitively determined, and so $E(\theta) = \frac{1}{3}\theta_H + \frac{1}{3}\theta_M + \frac{1}{3}\theta_L$. Let e^* be the education level chosen by all worker types in a pooling equilibrium. The pooling equilibrium requires th at the low type not deviate:

$$0 \leq e^* \leq [E(\theta) - \theta_L]\theta_L$$

The posterior beliefs equal the prior in equilibrium but must be a point mass on type θ_L for all other education levels $e \neq e^*$.