Why do we need math as an economic student?

(An answer from Ping Yu.)

- Math provides efficiency and clearance as the language of economics.
  1. Math provides platform of discussion. Math gives any definition and theorem a clear connotation and extension, so we could discuss the same thing by different terms and different things by the same term.
  2. Modern economics is a very complex system, mathematical model helps you to grasp the **essence** about how it works, because it discards many subtle details.

Caveat: Whenever you can't understand something about economics, please rewrite it as a mathematical model, maybe you will have a clearer or deeper understanding.

- Introducing math into economics makes some parts of economics **easier** to understand, and that will be helpful for the further development of economics.
  Modern math is an axiomatic system, using deduction logic, i.e.

$$\text{Assumptions(Axioms)} \overset{\log ic}{\Rightarrow} \text{Conclusions}$$

  If the conclusions are inconsistent with the facts, there must be something wrong with the assumptions, because we must believe logic. Then we must modify the assumptions and reach new conclusions until the conclusions are consistent with the facts.
  In general, where there is logic, there is math. Math is not a cumulus of mathematical symbols, but **logic**.
- For most parts of economics, at least for the first year courses, math is only a tool for economics. But for many parts of economics, math is essential for new theoretical ideas.
  Here I give out some examples.
  i. The existence of competitive equilibrium takes economists 200 years, the kernel is just the fixed point theorem. So only after the Brouwer 's fixed-point theorem was invented in 1911 can we prove the existence theorem of CE, as we know now, they are equivalent for a simple existence theorem.
  ii. Black-Scholes formula of option pricing is just the solution of a partial differential equation.
  iii. The second welfare theorem and the fundamental theorem of asset pricing are just the application of separating hyperplane theorem.

  Note: All these ideas have won the Nobel Prize.

Why is optimization theory most useful in economics?

The following paragraph comes from an introduction of Paul A. Samuelson:

Samuelson's signature method of economic theory, illustrated in his *Foundations of Economic Analysis*(1947), seems to follow two rules which can also been said to

characterize much of Neoclassical economics since: with every economic problem (1) reduce the number of variables and keep only a minumum set of simple economic relations; (2) if possible, rewrite it as a constrained optimization problem.

An alternative explanation comes from the definition of Economics: The traditional view of economics as "the study of human response to scarcity" is, perhaps, too narrow. A more modern definition is that *rationality-based social science studies models of social phenomena in which individual agents are assumed to pursue well specified goals*. Goals are just objective functions. Also, rational individuals must face sorts of constraints: legal, institutional, or, more fundamentally, informational. So, basically, an economic problem is just a constrained optimization problem.

Maybe these will give you an explanation why we will learn optimization theory in this semester.

How should we look on this course from different view of points?

- From the mathematical point, optimization theory includes many aspects: static and dynamic; nonstochastic and stochastic; finite horizon and infinite horizon; discrete time and continuous time etc. This course will study optimization theory that is static, nonstochastic, finite horizon and discrete time.
- From the economic point(maybe only from an first year student's view), economics only studies two problems: optimization and equilibrium, and this view is applicable not only to Micro, but to Macro; not only to price theory, but to game theory. Optimization studies the rational individual's decision theory in certain or uncertain, instant or intertemporal environment. Equilibrium studies the relationship between rational individuals. (The following assertion is arbitrary, but useful for you.)What we will learn this semester is useful for your first part(decision theory) of 711 this semester, but not the second part(equilibrium). Also, Macro will use dynamic programming, which, from the mathematical point, is an optimization technique that is dynamic, nonstochastic(or stochastic), infinite horizon(or finite horizon), discrete time, so what we will learn is not applicable to 712. Although professor Manuelli will teach 712 using Kuhn-Tucker theorem, but it is only pedagogical. The rigorous dynamic programming technique will be taught by professor Seshadri in 714. As to optimal control technique that is used for continuous time case will be taught in professor Brock's 606 and professor Manuelli's Topics in Macroeconomics.

What will we learn from this course?

This course includes two parts:

The first part will include some topological concepts, such as function, order, limit, vector space, open, compact, connected, continuity, and derivative etc., and related theorems. These concepts are basic blocks for mathematical economics, and are the prerequisite of the second part.

In this part, when you study every concept, you must ask yourself **why I should learn this concept**, i.e., why is this concept useful for economic study(Maybe you should build a relationship between economic concepts and mathematical concepts.).

Sometimes I will give you some hints for such questions, but you should learn to think about such questions by yourself. The second caveat for this part is that you must know **which space you are in** whenever you talk about a concept. In Appendix C, the author mainly talked about four kinds of spaces:

Inner Produce Spaces $\subset$ Normed Spaces $\subset$ Metric Spaces $\subset$ Topological Spaces

Actually, there are other spaces between any two neighbouring spaces, e.g., Hausdorff space is a very important space between metric space and topological space, but we will not talk about it. We will mainly concentrate in normed space and metric space. A good way to understand a concept or a theorem is to **find examples and counter-examples** for this concept or theorem. In this course, many good examples are from the following spaces: Euclid space R^n and its subspace, $(l^p, 1 \le p < \infty, d_p)$, (l^∞, d_∞), $(L^p, 1 \le p < \infty, d_p)$, (L^∞, d_∞), $(C[0,1], d_p, 1 \le p < \infty)$, $(C[0,1], d_\infty)$, and these spaces are also very important for the whole mathematics. I will give out the summary for the properties of these spaces.

The second part will be mainly about static optimization theory, and the central theorem will be the well known Kuhn-Tucker Theorem. In general, we will ask the following questions:

- Existence. The main theorem is Weierstrass theorem and Generalized Weierstrass theorem.
- Necessary conditions. Differentiability and Constraint qualifications are important.
- Sufficient conditions. Concavity is important.
- Uniqueness. Concavity is important.
- Globality. Concavity is important.
- Parametric Variation. We have no time to talk about it.

Actually, except for the existence problem, all the other problems are about the characterization of the set of optimal points, about which I will give out a summary. As most of you expected, that will be a useful "cookbook". Try to apply these techniques to the first part of 711.

How to get an A(there is no A+) in this course?

A more general problem is how to study mathematics well. Any effort to answer this question is risky, but some suggestion may be helpful. Except the suggestion above, the following may be useful(maybe useless because of generality).

- Do as many exercises as you can! No one could study a subject, any subject, well if he doesn't do exercises.
- Not only use your hand, but use your **brain**! After studying a course, you should ask yourself what you have learned from this course. If your answer is only one sentence, you must be the best one. But if your answer is a paragraph, maybe you should study harder. For most math learners, what they learned are woods, not a forest. Try for this course.