My special pleasure in mathematics rested particularly on its purely speculative part - Bernard Bolzano

1 Review Topics

Open and closed sets, limits of functions

2 Exercises

2.1 Classify the following sets as open, closed, or neither in $\mathbb{R}$ and provide a brief argument.

- $(-\infty, 0] \cup [1, \infty)$.
- $\bigcup_{n=2}^{\infty} A_n$, where $A_n = \left[\frac{1}{n}, 1 - \frac{1}{n}\right]$
- $\mathbb{Q}$

2.2 Let F be a closed set and E be an open set in a metric space X. Prove that $F \backslash E$ is closed and $E \backslash F$ is open.

2.3 Show that any open set in $\mathbb{R}$ can be re-written as the countable union of disjoint open intervals

2.4 **If** $\lim_{x \to x_0} f(x) = 0$**, and** $g(x)$ **is bounded, show** $\lim_{x \to x_0} f(x) g(x) = 0$**.**

2.5 **Compute** $\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$**.**

2.6 **Prove that if** $f(x) \leq g(x) \leq h(x)$ **for all** x**, and** $\lim_{x \to x_0} f(x) = \lim_{x \to x_0} h(x) = y_0$**, then** $\lim_{x \to x_0} g(x) = y_0$**.**

2.7 **Consider the function** $f(x) = x \mathbb{1}_{\mathbb{Q}}(x)$**, where** $\mathbb{1}_A(x)$ **is equal to** 1 **if** $x \in A$**,** 0 **otherwise. Show that for any** $p \neq 0$**,** $\lim_{x \to p} f(x)$ **does not exist.**