Econ 711 Problem Set 4

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October 3, 2020

Question 1

$C(A, \succsim) = \{x \in A : x \succsim y, \forall y \in A\}$ Consider $A, B \subset X$ and $x, y \in A \cap B$. Let $x \in C(A)$ and $y \in C(B)$. Then $x \succsim y$ since $y \in A$ and $y \succsim x$ since $x \in B$. Then by transitivity, $x \in C(B)$ and $y \in C(A)$.

Question 2

$C : P(X) \to P(X)$
Let C satisfy WARP. Then for any $x, y \in A$, either $x \in C(\{x, y\})$ or $y \in C(\{x, y\})$. So either $x \succsim y$ or $y \succsim x$. Thus preferences are complete.

Consider $A, B \subset X$ and $x, y \in A \cap B$ and $z \in B$. Then if $x \in C(A)$ then $x \succsim y$. If $y \in C(B)$, then $y \succsim z$. By WARP, $x \in C(B)$ since $x \in A \cap B$. Then $x \succsim y$ and $y \succsim z$ implies $x \succsim z$. So preferences are transitive.

Let $A \subset X$ and define $C' = \{x \in A : x \succsim y, \forall y \in A\}$. Consider $x \in C'(A)$. Then $x \succsim y \Rightarrow x \in C(A) \Rightarrow C(A) \subset C'(A)$. Now consider $x \in C(A)$. Then $x \succsim y \Rightarrow x \in C'(A) \Rightarrow C'(A) \subset C(A)$. Since $C'(A) \subset C(A)$ and $C(A) \subset C'(A)$, it must be the case that $C'(A) = C(A)$

Question 3

Part A

Let A be a finite set. Consider $A \subset X$ be a singleton set with only one element, a. Then $C(A) = a$ because $a \succsim a$.

Assume $A_n \in X$ with n elements has a choice rule such that $C(A_n) \neq \emptyset$. Now consider $A_{n+1} \in X$ with $n + 1$ elements. Let $a \in A_{n+1}$. Note that $A_{n+1} \setminus \{a\}$ is

*I have discussed this problem set with Emily Case, Michael Nattinger, Alex Von Hafften, and Danny Edgel.

a set with n elements, so $C(A_{n+1} \setminus \{a\}) \neq \emptyset$. Consider $c \in C(A_{n+1} \setminus \{a\})$. Then either $a \succsim c$, or $c \succsim a$. So either $a \in C(A_{n+1})$ or $c \in C(A_{n+1})$. Thus $C(A) \neq \emptyset$ if $A \neq \emptyset$.

Part B

Let A be a finite set. Consider $A \subset X$ be a singleton set with only one element, a. Let $U(a) = 1$. Then $a \succsim a$ implies $U(a) = 1 = U(a)$.

Now assume $A_n \in X$ with n elements has a utility representation with range $\{1, 2, ..., n\}$. Now consider $A_{n+1} \in X$ with $n+1$ elements. Consider $a \in C(A_{n+1})$. Note that $A_{n+1} \setminus \{a\}$ has a utility representation U with range $\{1, 2, ..., n\}$. Let V be the utility representation of A_{n+1} defined as:

V(x) = $\begin{cases} U(x) & \text{if } x \in A_{n+1} \setminus \{a\} \\ n+1 & \text{if } x = a \end{cases}$

If $b, c \in A_{n+1} \setminus \{a\}$, then $V(b) = U(b)$ and $V(c) = U(c)$, and $b \succsim c$ implies $U(b) \geq U(c)$. Since $a \in C(A_{n+1})$, we know that $a \succsim x$ for all $x \in A$, and $V(a) = n+1 \geq V(x)$ for all $x \in A$. Therefore a utility representation of X exists.