Econ 703 Practice Problem 10

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1 Linear Programming

1.1 Primal Problem

- Consider the optimization problem:

$$V(A, b, c) = \max_{x} c'x \tag{1}$$

$$\text{s.t. } Ax \leq b,\ x \geq 0 \tag{2}$$

where $x, c \in \mathbb{R}^n$,$A \in \mathbb{R}^{mxn}$ and $b \in \mathbb{R}^m$.

- Q1. Consider you have a bakery that makes and sell bread and cake. Each bread is sold at 1\$, each cake is sold at 1\$. To make each bread, you need 1 ounce flour and 0.5 egg and to make a cake you need 0.5 ounce flour, 1 egg. Now you have 10 ounce flour, 15 eggs and you want to maximize your revenue, how much bread and cake you should make (fractions allowed)?

1.2 Dual Problem

- The ducal problem is

$$W(A, b, c) = \min_{\lambda} b'\lambda \tag{3}$$

$$\text{s.t. } A'\lambda \geq c,\ \lambda \geq 0 \tag{4}$$

- Q2. What's the dual problem of the example above? What's the meaning of λ?

2 Fixed point theorems[1]

2.1 Brouwer's Fixed point theorem

- **Theorem** Suppose that $A \subset \mathbb{R}^n$ is a nonempty, compact, convex set and that $f : A \to A$ is a continuous function from A into itself. Then $f()$ has a fixed poit; that is, there is an $x \in A$ such that $x = f(x)$

[1] Mascollel pg. 952~954

2.2 Kakutani's Fixed point theorem

- **Definition** A correspondence $\Phi : X \to P(Y)$ (power set of Y) is said to be **upper-semicontinuous** or u.s.c at a point $x \in X$ if for all open sets V such that $\Phi(x) \subset V$, there exists an open set U containing x such that for all $x' \in U \cap X$, $\Phi(x') \subset V$.

- **Definition** A correspondence $\Phi : X \to P(Y)$ (power set of Y) is said to be **lower-semicontinuous** or l.s.c at a point $x \in X$ if for all open sets V such that $\Phi(x) \cap V \neq \phi$, there exists an open set U containing x such that for all $x' \in U \cap X$, $\Phi(x') \cap V \neq \phi$.

- **Definition** A correspondece is continuous if it's both u.s.c and l.s.c

- Q4. Let $X = [0, 2]$. A correspondence $\Phi : X \to P(Y)$ is defined as below is u.s.c? l.s.c?

$$\Phi(x) \begin{cases} \{1\}, & 0 \leq x \leq 1 \\ X, & 1 \leq x \leq 2 \end{cases}$$

- Q5. Let $X = [0, 2]$. A correspondence $\Phi : X \to P(Y)$ is defined as below is u.s.c? l.s.c?

$$\Phi(x) \begin{cases} \{1\}, & 0 \leq x \leq 1 \\ X, & 1 < x \leq 2 \end{cases}$$

- **Theorem** Suppose that $A \subset \mathbb{R}^n$ is a nonempty, compact, convex set and that $\Phi : A \to A$ is an u.s.c correspondence from A into itself with the property that the set $\Phi(x) \subset A$ is nonempty and convex for every $x \in A$. Then $\Phi()$ has a fixed point; that is, there is an $x \in A$ such that $x \in \Phi(x)$