University of Wisconsin-Madison
Department of Economics

Econ 703 — Prof. R. Deneckere
Fall 2002

Homework #4
(due on Oct. 1, 2002)

1. A point x is an interior point of set A if there exists a neighbourhood N of x such that $N \subset A$. Let $\mathring{A}$ be the interior of the set A, i.e. the collection of all of its interior points. Prove the following :
   (1) $\mathring{A}$ is an open set;
   (2) A is open iff $A = \mathring{A}$
   (3) If $B \subset A$, and B is open, then $B \subset \mathring{A}$.

2. Let K be the union of the set $\{0\}$ and the set $\{1/n, n \in Z_{++}\}$. Prove that K is compact directly from the definition (i.e., without using the Heine_Borel Theorem).

3. Sundaram, #26, p. 68.

4. Sundaram, #52, p. 72.

5. Consider the set of all rational numbers, Q, and make it into a metric space by defining $d(p,q) = |p-q|$ for all $p,q \in Q$. Let E be the set of all $p \in Q$ such that $2 < p^2 < 3$. Show that E is closed and bounded in Q, but that E is not compact. Conclude that Q is not a compact space. Is E open in Q?
HINT : Be very careful here. The notions closed, open, compact are all with reference to the metric space (Q,d), not the metric space (R,d)!