CONSTRUCT SEQUENCE

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. How to construct a sequence in A that converges to a limit point x of A?
see $HW3\#3$ part of $(\leftarrow)$
way1: $x_1 \in B(x,r), x_2 \in B(x,r/2), ..., x_k = B(x, 1/2^k)$.
way2: $x_k \in B(x, 1/k)$.

. How to construct a sequence in A that converges converge to SupA=x?
Let $x_n \in (x - 1/k, x]$.

. How to construct a subsequence that converges to the limsup of the sequence?
suppose $a = limsup x_k$, and assume $a < \infty$. We want to find a subsequence $\{x_{n_k}\}$ that converges to a.
Define $a_k = sup\{x_k, x_{k+1}, ...\}$, so $a_{n_k} = sup\{x_{n_k}, x_{(n_k)+1} ...\}$.
Now, pick, $x_{n_1} = x_1$, pick $x_{n_2} s.t. \mid x_{n_2} - a_{n_1} \mid < \frac{1}{2}$, pick $x_{n_3} s.t. \mid x_{n_3} - a_{n_2} \mid < \frac{1}{2^2}$, ... pick $x_{n_{(k+1)}} s.t. \mid x_{n_{(k+1)}} - a_{n_k} \mid < \frac{1}{2^k}$, ...
We know $\{a_n\}$ converge to a, then $\{a_{n_k}\}$ also converge to a. So for any $\epsilon, \exists N_1$, s.t. for any $n_k > N_1$, we have $\mid a_{n_k} - a \mid < \epsilon/2$. we also know, that $\exists N_2$, s.t. for any $n_{k+1} > N_2$, we have $\mid x_{n_{(k+1)}} - a_{n_k} \mid < \epsilon/2$. Choose $N = Max(N_1, N_2)$, we will get $\mid x_{n_{(k+1)}} - a \mid < \mid x_{n_{(k+1)}} - a_{n_k} \mid + \mid a_{n_k} - a \mid < \epsilon$ for all $n_{k+1} > N_2$. Therefore x_{n_k} converges to a.

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