Econ 761 – Fall 2020
Homework 6

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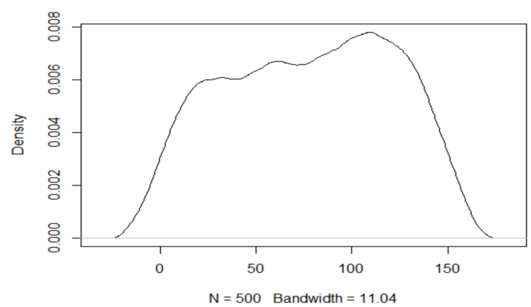
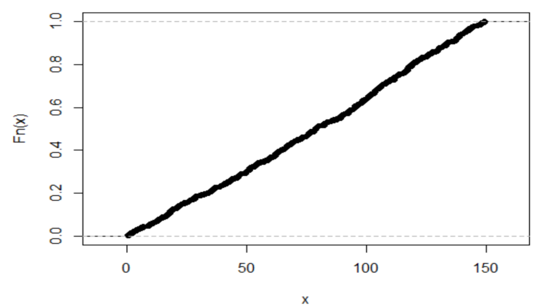
1

I first estimate the underlying distribution of valuations for each bidder from the data. Below are the graphs of the pdf g_{M_1,B_1} (left) and cdf G_{M_1,B_1} (right) for bidder 1.

pdf for bidder 1



cdf for bidder 1



The graphs for the other bidders are similar in terms of maximum and minimum bids, but they differ slightly in exact distribution.

2

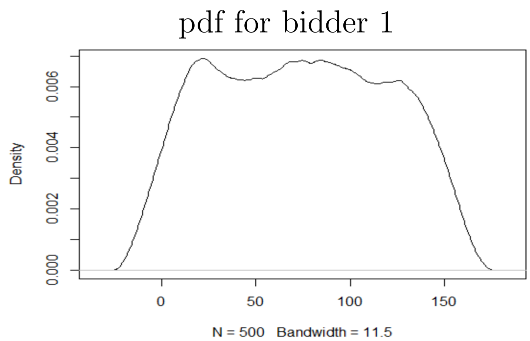
Now we find estimated values of $F_U(u_1, u_2, u_3, u_4)$ at vectors in which each u_i is either the 25th or 75th percentile of the marginal distribution.

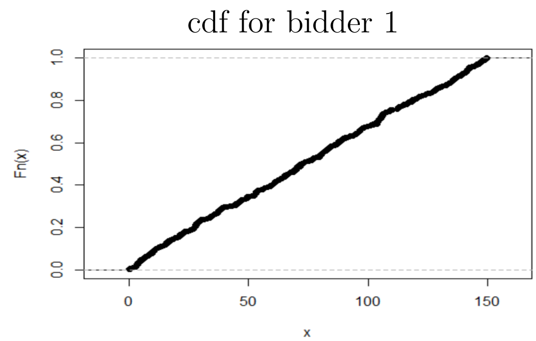
With u_1 at the 25th and 75th percentile, $F_U = 0.279$ and 0.759, respectively
With u_2 at the 25th and 75th percentile, $F_U = 0.228$ and 0.731, respectively
With u_3 at the 25th and 75th percentile, $F_U = 0.253$ and 0.736, respectively
With u_4 at the 25th and 75th percentile, $F_U = 0.242$ and 0.767, respectively

3

Based on the answers from the previous question, it does not seem like the bidders are symmetric. At the 25th and 75th percentiles of various u_i, there is some variation in the joint distribution F_U, especially between u_1 and u_2.

Furthermore, we can compare the graphs for bidders 1 and 2:





We can see from these graphs that bidder 2 bids lower with a much higher frequency while bidder 1 bids higher with a higher frequency. This is also clear from the values of F_U: since they are higher for u_1 at the 25th percentile than for u_2 at the 25th percentile, more of the distribution is below the 25th percentile for bidder 1.

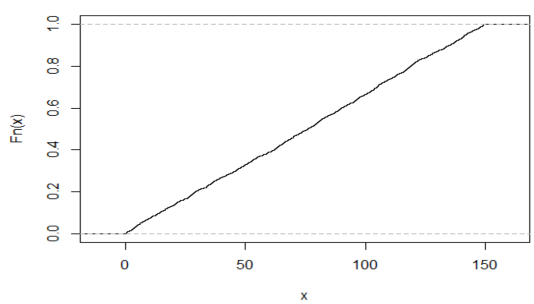
The same is true when comparing values at the 75th percentile. This means bidder 1 tends to bid higher, and lends more evidence that the bidders don't appear to be symmetric.

4

The values do appear to be independent. Without closer inspection, it would be hard to tell if there is collusion in particular auctions among different bidders. In addition, a regression of the bids by bidder 1 on the bids by the other bidders returns coefficient estimates very close to 0, indicating that the values do appear to be independent.

5

Here, we assume symmetry and independence to estimate the joint distribution F_U. Below is the plot of the estimated cdf of the distribution of bidders' values:



6

In this part, rather than assuming symmetry and independence, we impose them in the model. The results are similar to the previous part.

Code

```r
# code prep for hw6

# clear workspace
rm(list=ls())
setwd("C:/z_toshiba/course work/phd/econ 761/hw/hw6/")

library(readr)
bidder_data <- read_table2("fpa.dat", col_names = FALSE)
colnames(bidder_data) <- c("bidder_1", "bidder_2", "bidder_3", "bidder_4")

bidder_data2 <- stack(bidder_data)
colnames(bidder_data2) <- c("bid", "bidder")

# density for each bidder
b1_pdf <- density(bidder_data$bidder_1, kernel = "epanechnikov")
b2_pdf <- density(bidder_data$bidder_2, kernel = "epanechnikov")
b3_pdf <- density(bidder_data$bidder_3, kernel = "epanechnikov")
b4_pdf <- density(bidder_data$bidder_4, kernel = "epanechnikov")
#plot(b1_pdf)
#plot(b2_pdf)
#plot(b3_pdf)
#plot(b4_pdf)

# cdf for each bidder
b1_cdf <- ecdf(bidder_data$bidder_1)
b2_cdf <- ecdf(bidder_data$bidder_2)
b3_cdf <- ecdf(bidder_data$bidder_3)
b4_cdf <- ecdf(bidder_data$bidder_4)
#plot(b1_cdf)
#plot(b2_cdf)
#plot(b3_cdf)
#plot(b4_cdf)

# density for all bidders
all_b_pdf <- density(bidder_data2$bid, kernel = "epanechnikov")
#plot(all_b_pdf)

# cdf for all bidders
all_b_cdf <- ecdf(bidder_data2$bid)
#plot(all_b_cdf)

# estimated values of Fu
all_b_cdf(quantile(bidder_data$bidder_1)[2])
```

```
## [1] 0.2785
```

```r
all_b_cdf(quantile(bidder_data$bidder_2)[2])
```

```
## [1] 0.2275
```

```r
all_b_cdf(quantile(bidder_data$bidder_3)[2])
```

```
## [1] 0.253
```

```r
all_b_cdf(quantile(bidder_data$bidder_4)[2])
```

```
## [1] 0.242
```

```r
all_b_cdf(quantile(bidder_data$bidder_1)[4])
```

```
## [1] 0.759
```

```r
all_b_cdf(quantile(bidder_data$bidder_2)[4])
```

```
## [1] 0.7305
```

```r
all_b_cdf(quantile(bidder_data$bidder_3)[4])
```

```
## [1] 0.736
```

```r
all_b_cdf(quantile(bidder_data$bidder_4)[4])
```

```
## [1] 0.767
```

```r
# correlation between values
val_ind <- lm(bidder_data$bidder_1 ~ bidder_data$bidder_2+
                  bidder_data$bidder_3+bidder_data$bidder_4)
coefs <- summary(val_ind)$coefficients
coefs
```

```
##                         Estimate Std. Error    t value     Pr(>|t|)
## (Intercept)          80.35866286 5.88453069 13.6559170 2.870301e-36
## bidder_data$bidder_2 -0.03152048 0.04304509 -0.7322666 4.643517e-01
## bidder_data$bidder_3 -0.02086055 0.04470880 -0.4665872 6.410001e-01
## bidder_data$bidder_4  0.01355397 0.04347638  0.3117549 7.553578e-01
```