Econ 703 DIS Session 1*

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1 Question 1.

Proof of "if P, then Q" ($P \to Q$).

Proof) Suppose P.

Then, ...

Finally, we have Q.

In conclusion, the hypothesis P induces the claim Q.

e.g. "If , then "

"If-then" propositions $\leftrightarrow$ Sets

$$\text{If } A(x), \quad \text{then} P(x) \quad \Longleftrightarrow$$

where $A = \{x|A(x)\}, P = \{x|P(x)\}$.

2 Question 2.

Making a negation

de Morgan's law (basic)

$$\textbf{not } (P \qquad Q) \quad \Longleftrightarrow \quad (\textbf{not } P) \qquad (\textbf{not } B)$$
$$\textbf{not } (P \qquad Q) \quad \Longleftrightarrow \quad (\textbf{not } P) \qquad (\textbf{not } B)$$

Quantified propositions

$$x \in A \quad P(x) \quad \Longleftrightarrow \quad P(a) \qquad P(b) \qquad \ldots$$
$$x \in A \quad P(x) \quad \Longleftrightarrow \quad P(a) \qquad P(b) \qquad \ldots$$

where $A = \{a, b, \ldots\}$.

de Morgan's law (extended)

$$\textbf{not } (\qquad x \in A \quad P(x)) \quad \Longleftrightarrow \qquad x \in A \quad \textbf{not } P(x)$$
$$\textbf{not } (\qquad x \in A \quad P(x)) \quad \Longleftrightarrow \qquad x \in A \quad \textbf{not } P(x)$$

*Please bring the answer key and this DIS note with you to the TA session.

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e.g.

★ "   of you in this class will be good at math."

⇔"   will be good"   "   will be good"   ...

↓ the negation[1]

"   are bad"   "   are bad"   ...

⇔ "   of you in this class are bad at math."

multiple quantifiers: just repeat de Morgan's law from the beginning.

Make the negation of the statement in the question.[2]

$$
\begin{aligned}
& \textbf{not}[\forall \varepsilon > 0 \ \exists \delta > 0 \ \forall \rho' \in B_R(\rho,\delta) \ \exists p' \in E(\rho') \ \|p - p'\| < \varepsilon.] \\
\Leftrightarrow \quad & \quad \varepsilon > 0 \ \textbf{not}[\exists \delta > 0 \ \forall \rho' \in B_R(\rho,\delta) \ \exists p' \in E(\rho') \ \|p - p'\| < \varepsilon.] \\
\Leftrightarrow \quad & \quad \varepsilon > 0 \qquad \delta > 0 \ \textbf{not}[\forall \rho' \in R(\rho,\delta) \ \exists p' \in E(\rho') \ \|p - p'\| < \varepsilon.] \\
\Leftrightarrow \quad & \quad \varepsilon > 0 \qquad \delta > 0 \qquad \rho' \in B_R(\rho,\delta) \ \textbf{not}[\exists p' \in E(\rho') \ \|p - p'\| < \varepsilon.] \\
\Leftrightarrow \quad & \quad \varepsilon > 0 \qquad \delta > 0 \qquad \rho' \in B_R(\rho,\delta) \qquad p' \in E(\rho') \textbf{not}[\ \|p - p'\| < \varepsilon.] \\
\Leftrightarrow \quad & \quad \varepsilon > 0 \qquad \delta > 0 \qquad \rho' \in B_R(\rho,\delta) \qquad p' \in E(\rho') \ \|p - p'\| \quad \varepsilon.
\end{aligned}
$$

Why do we NOT need to change

Answer 1: Domain of Discourse

$$\forall \quad x \quad \in A \qquad P(x)$$

e.g. The negation of ★:

"Some of you   this class are bad at math."

"Some of you   this class are bad at math."

Answer 2: Conversion from propositions to sets

[1]Here we neglect the tense of the statements.

[2]Here $B_R(\rho,\delta) = \{\rho' | \|\rho - \rho'\| < \delta\}$.

Quantified propositions $\leftrightarrow$ Sets

$$x \in A \quad P(x) \quad \Longleftrightarrow$$
$$x \in A \quad P(x) \quad \Longleftrightarrow$$
$$\text{where } P = \{x|P(x)\}, A = \{x|A(x)\}.$$

Interpreting quantified variables.

For every $\varepsilon > 0$ there exists $\delta > 0$ such that for all ρ' satisfying $||\rho - \rho'|| < \delta$ there exists $p' \in E(\rho')$ such that $||p - p'|| < \varepsilon$.

$$\forall \varepsilon > 0 \quad \exists \delta > 0 \quad \forall \rho' \in B_R(\rho, \delta) \quad \exists p' \in E(\rho') \quad \|p - p'\| < \varepsilon.$$

3 Question 3.

To disprove 'if-then' & 'for all' propositions, You have only to mention *one SPECIFIC counterexample.*

4 Question 4.

One candidate of g is enough. A strictly increasing/decreasing function is a typical injective (one-to-one) function. Check carefully the domain D and the range E of g; is it really a surjective (onto) function, i.e. $g(D) = E$?

5 Question 5.

Consider a sequence $\{x_k\}_{k\in\mathbb{N}}$ in $\mathbb{R}$: i.e. $x_k \in \mathbb{R} \quad \forall k \in \mathbb{N}$.

Subsequences

Def (subsequence): S p.10

A sequence $\{y_k\}_{k\in\mathbb{N}}$ is a *subsequence of* $\{x_k\}_{k\in\mathbb{N}}$, if
its elements y_k are a) choosen from $\{x_n\}$ and b) the order is kept:
i.e. $\exists$ a function $n : \mathbb{N} \to \mathbb{N}$ s.t.

$$\begin{aligned} &y_k = x_{n(k)} \quad \text{for each } k \in \mathbb{N}, \\ &n(k) \text{ is} \qquad\qquad \text{in } k. \end{aligned}$$

e.g. $x_k = k$, i.e. $\{x_k\} = \{1, 2, 3, 4, 5, \ldots\}$.

$$\begin{aligned} &1) \quad \{y_k\} = \{1, -1, 2, 4, 7, \ldots\} \\ &2) \quad \{y_k\} = \{1, 3, 2, 4, 7, \ldots\} \\ &3) \quad \{y_k\} = \{1, 3, 3, 4, 7, \ldots\} \\ &4) \quad \{y_k\} = \{1, 3, 5, 7, 9, \ldots\} \end{aligned}$$

What if $\{z_k\} = \{1, 1, 2, 2, 3, 3, 4, 4, 5, 5, \ldots\}$?

When you confuse multiple subscripts of a subsequence, then give it another name like an "independent" sequence $\{y_k\}$ with an index function $n(\cdot)$; then, use the strict increasingness of $n(\cdot)$.

S Thm. 1.18., p.21

$$\lim_{k\to\infty} x_k = x \qquad \Longleftrightarrow \qquad \lim_{k\to\infty} y_k = \qquad \forall\{y_k\} : \qquad \text{subsequence of } \{x_k\}$$

Cauchy sequences

S Thm. 1.11., p.12

$\{x_k\}$ is a **convergent** sequence $\quad\Longleftrightarrow\quad$ $\{x_k\}$ is a **Cauchy** sequence

- a *convergent* sequence: the limit $x = \lim_{k\to\infty} x_k \in \mathbb{R}$ exists (S p.7).
- a *Cauchy* sequence: for sufficiently large indexes k and l, the distance b/w any two elements $|x_k - x_l|$ is arbitrarily small (S p.11).