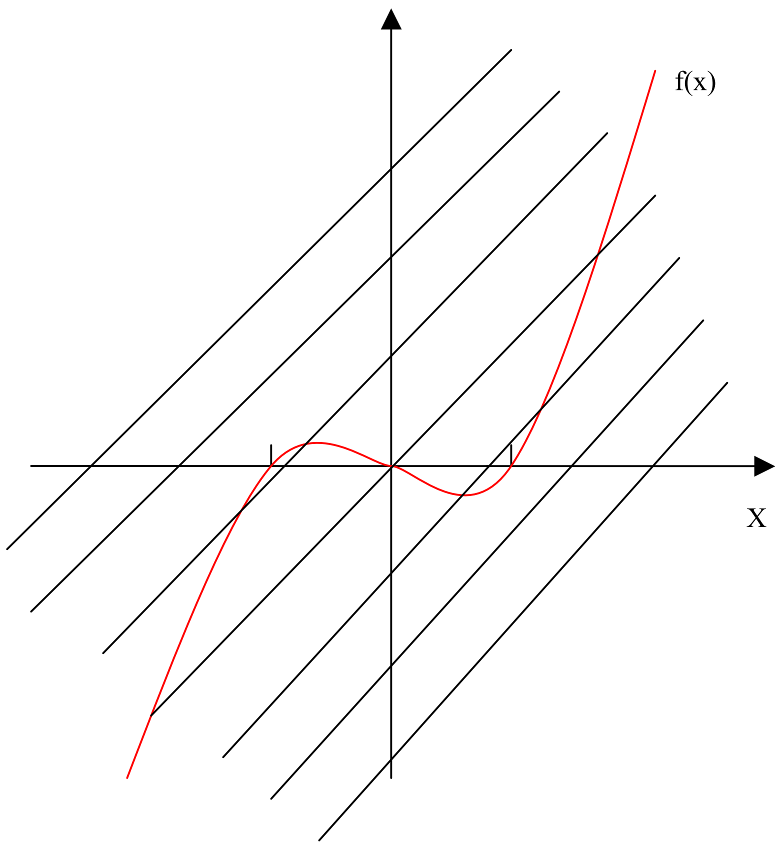
Econ 703----Answer key to homework 1

1. The contrapositive statement is: if x is not a square orange, then x doesn't belong to empty set. Because "x doesn't belong to empty set" is always true for any x, it is certainly true when x is not a square orange. Therefore, the contrapositive statement is true. So, the original statement is true.

2. (a) There exists (at least one) $a \in A$, such that $a^2 \notin B$ (or say, such that it is not true that $a^2 \in B$).
   (b) For every $a \in A$, it is true that $a^2 \notin B$ (i.e. it is not true that $a^2 \in B$).
   Another way of negation: There is no $a \in A$ such that $a^2 \in B$.
   (c) There exists (at least one) $a \in A$, such that $a^2 \in B$.
   (d) For every $a \notin A$, it is true that $a^2 \notin B$.
   Another way of negation: There is no $a \notin A$ such that $a^2 \in B$.

3. First, f (x) is as following:



So the function itself is not injective, but it is surjective.
There are many ways to restrict the domain and range to obtain a bijective function g. And for a bijective function, there exists inverse function.
For instance:

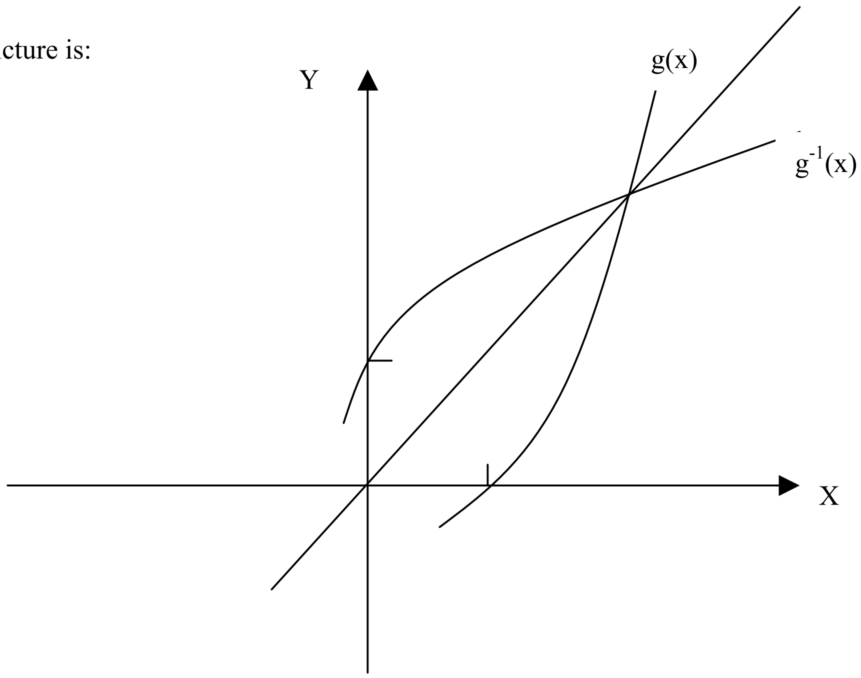
g: $[\sqrt{3}/3, +\infty) \rightarrow [-2\sqrt{3}/9, +\infty)$, g (x)=$x^3$-x.

Why is this function injective? Because for any points x and x' in $[\sqrt{3}/3, +\infty)$, and x≠x', we must have g (x)≠g (x').

Why is this function a surjective function? Because, for any $y \in [-2\sqrt{3}/9,+\infty)$, we can find a point x in $[\sqrt{3}/3,+\infty)$ such that y is the image of that point (i.e. y=g (x)).

Correspondently, the domain of g^{-1} is $[-2\sqrt{3}/9,+\infty)$, and the range of g^{-1} is $[\sqrt{3}/3,+\infty)$

The picture is:



Some other choices of g

g: $[1,+\infty) \rightarrow \Re_{+}$, g (x)=$x^3$-x.

g: $[\sqrt{3}/3,+\infty) \rightarrow [-2\sqrt{3}/9,+\infty)$, g (x)=$x^3$-x.

g: $(-\infty,-1] \rightarrow \Re_{-}$, g (x)=$x^3$-x.

g: $[-\sqrt{3}/3,-\sqrt{3}/3] \rightarrow [-2\sqrt{3}/9,+2\sqrt{3}/9]$, g (x)=$x^3$-x.

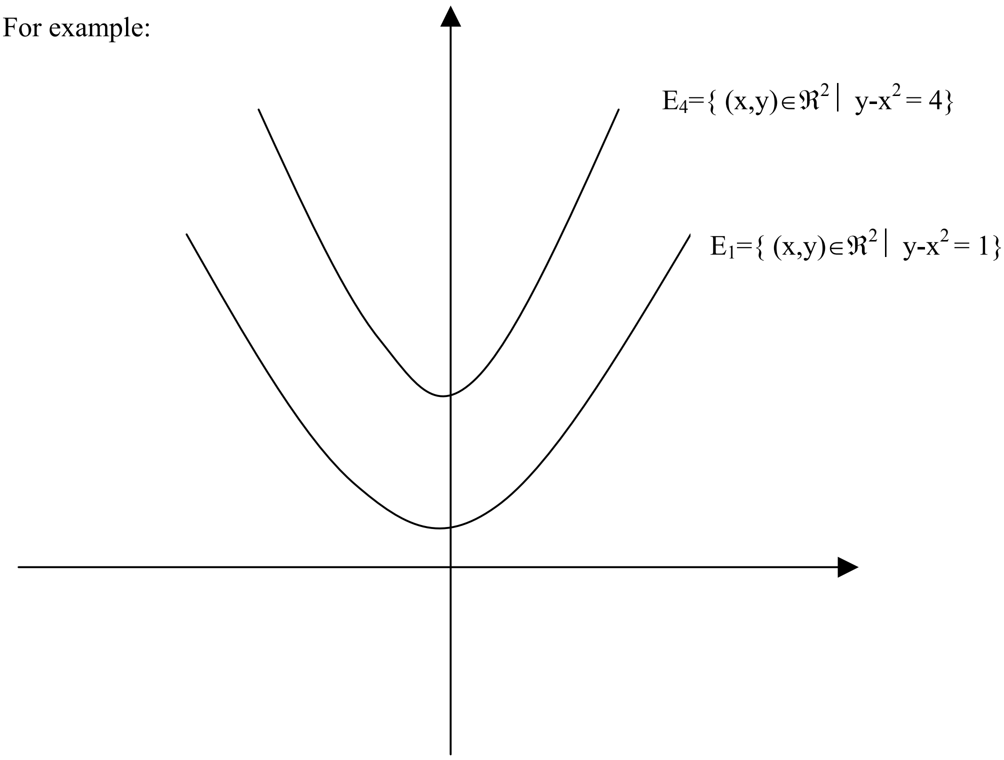
4. To prove this relation is an equivalence relation, we need to prove the relation satisfies properties of reflective, symmetry and transitive.
   Relation C={$((x_0, y_0), (x_1, y_1)) \in \Re 2 \times \Re 2 \mid y_0-x_0^2 = y_1-x_1^2$ }. So if $y_0-x_0^2 = y_1-x_1^2$, and $(x_0, y_0), (x_1, y_1) \in \Re^2$, then $((x_0, y_0), (x_1, y_1)) \in C$, i.e. (x_0, y_0) C (x_1, y_1).
   1) Because $y-x^2=y-x^2$, we have (x,y) C (x,y) for any (x,y). So relation C has reflective property.
   2) If $y_0-x_0^2 = y_1-x_1^2$, then $y_1-x_1^2 = y_0-x_0^2$. That is, if (x_0, y_0) C (x_1, y_1), then (x_1,y_1)C (x_0, y_0). So C has symmetry property

3) If y_0-x_0^2 = y_1-x_1^2 and y_1-x_1^2 = y_2-x_2^2, then y_0-x_0^2 = y_2-x_2^2 , that is, if (x_0, y_0) C (x_1,y_1), (x_1,y_1) C (x_2,y_2), then (x_0, y_0) C (x_2,y_2). So C has transitive property

Therefore, this relation is an equivalence relation.

The equivalence classes determined by (x, y) is $E=\{ (x',y')\in\Re^2 \mid y'-x'^2 = y-x^2\}$.

For example:



E_1 is the equivalence classes determined by (0,1), it is also the equivalence determined by (2,5), or (3,10)... In fact, it is the equivalence classes determined by any point with $y-x^2=1$. We can simply denote it as E_1. Similarly with E_4.

5. First, when n=1, then unique subset is {1}, which has the largest element 1.

   Second, suppose the statement holds when n=k, that is, every nonempty subset of {1,2,3, … , k} $k\in\mathbb{Z}_+$ has a largest element. Now, consider the case of n=k+1.

   Let **S** represent the nonempty subsets of {1,2,3, … , k}, then **S** ∪ {k+1} is a nonempty subset of {1,2,3, … , k, k+1}. In fact, the nonempty subset of {1, 2, 3, … , k, k+1} can be represented by **S** or **S** ∪ {k+1} or {k+1}. We have known **S** has a largest element. Suppose the largest element of **S** is Ms. Then for **S** ∪ {k+1}, the largest number is max {Ms, k+1}, which always exists and equals to k+1. For {k+1}, the largest element is just k+1. So the statement is also true for n=k+1.

   (Another way to state is as following: There are two kinds of nonempty subsets of {1, 2, 3, … , k, k+1}. One is those doesn't include k+1, the other is those include k+1. Every nonempty subset excluding k+1 is also a nonempty subset of {1,2,3, … , k}. So is has a largest element. Every nonempty subset including k+1 also has a largest number, which is k+1. So every nonempty subset of {1, 2, 3, … , k, k+1} has a largest element.)

   Therefore, the original proposition is true for all n $\in\mathbb{Z}_+$.