University of Wisconsin

Microeconomics Prelim Exam

Friday, June 1, 2018: 9AM - 2PM

- There are four parts to the exam. All four parts have equal weight.
- Answer all questions. No questions are optional.
- Hand in 12 pages, written on only one side.
- Write your answers for different parts on different pages. So do not write your answers for questions in different parts on the same page.
- Please place a completed label on the top right corner of each page you hand in. On it, write your assigned number, and the part of the exam you are answering (I,II,III,IV). Do not write your name anywhere on your answer sheets!
- Show your work, briefly justifying your claims. Some solutions might be faster done by drawing a suitable diagram.
- You cannot use notes, books, calculators, electronic devices, or consultation with anyone else except the proctor.
- Please return any unused portions of yellow tablets and question sheets.
- There are five pages on this exam, including this one. Be sure you have all of them.
- Best wishes!

Part I

In addition to teaching and research, I run a small distillery called Drips 'n' Drams, located on the Wisconsin River, where I produce artisinal vodka and cold press coffee. The river water is free, and I find the work to be a pleasant break from economics, so we can ignore the costs of water and my labor; my relevant inputs are therefore potatoes (for the vodka), coffee beans, and electricity (for heating and cooling during production).

Each liter of vodka requires 10 pounds of potatoes; each gallon of coffee requires 2 pounds of beans; and given these other inputs, producing v liters of vodka and c gallons of coffee requires $h(v, c)$ kilowatt-hours of electricity.

Assume I'm a price-taker in both input and output markets, prices are non-negative, production satisfies free disposal, and h is strictly increasing in both its arguments.

1. Ordering goods as $y = (v, c, p, b, e) =$ (vodka, coffee, potatoes, beans, electricity), give an expression for my production set $Y \subset \mathbb{R}^5$.
2. Under what conditions on h could you infer h from "data," i.e., from observing my profit level or production choice at all positive price vectors?
3. Given a price vector $(p_v, p_c, p_p, p_b, p_e) \gg 0$, write my profit maximization problem as an unconstrained choice of output levels $(v, c) \geq 0$.

Now suppose that Y is strictly convex, so I have a unique profit-maximizing production plan. Suppose also that h is twice differentiable, and let $h_{vc} = \partial^2 h / \partial v \partial c$.

4. Pick a sign, either $h_{vc} < 0$ or $h_{vc} > 0$. What does it mean (in words) for h_{vc} to have that sign? Give an example of technological details about coffee brewing and vodka-making that would make h_{vc} have that sign.
5. Suppose $h_{vc} < 0$. If the price of potatoes goes up, what effect will this have on my production of each output good? Why?
6. In Fall of 2018, I sign contracts to deliver a fixed amount of vodka (all that I plan to produce) in 2019 to local bars, so for 2019, my production of vodka is fixed, but my production of coffee is still variable. In December of 2018, the price of coffee beans goes up, and is expected to remain that high permanently. If $h_{vc} < 0$, will my optimal level of coffee production be higher in 2019 or 2020? Explain why. Would the answer be the same or different if $h_{vc} > 0$?

Part II

For the normal form game G below, assume throughout the question that $a > 2$, $b > 0$, and $c \in [0, 1)$.

		2		
		A	B	C
1	A	$2, 2$	$0, a$	$c, 0$
	B	$a, 0$	$1, 1$	$0, 0$
	C	$0, c$	$0, 0$	$-b, -b$

Let $G^\infty(\delta)$ be the infinite repetition of G with discount factor $\delta \in (0, 1)$. Let $s_i(L)$ be the stick-and-carrot strategy for player i in $G^\infty(\delta)$ under which (A, A) is chosen on the path of play, pure mutual minmaxing is used as the punishment action profile, and the punishment length is $L \geq 1$. Let $s(L) = (s_1(L), s_2(L))$ be the corresponding strategy profile.

1. Carefully state the result that allows us to evaluate whether a strategy profile is a subgame perfect equilibrium of $G^\infty(\delta)$?

2. What requirements must a, b, c, δ, and L jointly satisfy for $s(L)$ to be a subgame perfect equilibrium of $G^\infty(\delta)$?

3. Fix $\delta \in (0, 1)$ and $L \geq 1$. For each of the parameters a, b, and c, and the inequalities derived for part (2), explain whether increasing the parameter makes the inequality more demanding, has no effect on it, or makes the inequality less demanding. Whenever there is some effect, provide an intuitive explanation for why the effect is in the direction you claim.

4. Assume $b = 1$ and $c = 0$. For which $a > 2$ is the strategy profile $s(1)$ a subgame perfect equilibrium of $G^\infty(\delta)$ for some $\delta \in (0, 1)$?

5. Assume $b = 1$ and $c = 0$. For each $L \geq 1$, for which $a > 2$ is the strategy profile $s(L)$ a subgame perfect equilibrium of $G^\infty(\delta)$ for some $\delta \in (0, 1)$.

6. Suppose that a, $b = b_0$, and c satisfy the conditions stated at the start of the question.. Suppose also that δ and L ensure that $s(L)$ is a subgame perfect equilibrium of $G^\infty(\delta)$. Now change b from b_0 to $100b_0$, leaving a, c, δ, and L fixed. After this change, must $s(L)$ still be a subgame perfect equilibrium of $G^\infty(\delta)$? Must there be some $\delta \in (0, 1)$ under which $s(L)$ is a subgame perfect equilibrium (when $b = 100b_0$ and a, c, and L are fixed)? Explain carefully in words the intertemporal incentives that account for your answers.

Part III

1. At rush hour in a large city, traversing the city streets always takes $m = 120$ minutes, using Waze. The highway is generally faster but subject to variability, and the duration in minutes m is random with mean 100.

   Pat and Chris are in a hurry to cross the city. If Pat takes m minutes, Pat's monetary payoff is $\pi_P(m) = 100 + B_P e^{-\alpha m}$, where $B_P > 0$. If Chris takes m minutes, Chris' monetary payoff is $\pi_C(m) = 100 + B_C e^{-2\alpha m}$, where $B_C > 0$.

   Suppose Chris takes the highway. What can you infer about Pat's driving choice?

2. Millions of people can play the Powerball lottery twice a week. Let us simply assume that *anyone* buys one ticket if the expected prize π plus his/her enjoyment level λ exceeds the ticket price, say \$1 (it rose to \$2 in 2012, but we'll ignore that). People differ in their enjoyment levels. The number of people with enjoyment level at least $1 - \lambda$ is $n = F(\lambda)$, an increasing step function of λ.

   If no one wins the prize, then everyone knows that the (not yet won) jackpot $J > 0$ is added to the next lotto prize (it "rolls over"). When the lotto win chance $p > 0$ is tiny (about one in 11 million), and the rollover from last time is J, assume that the expected prize is
   $$\pi(J, n) = [J/n + 1][1 - e^{-pn}]$$

   (a) Important: What is the shape of the plot of the expected prize $\pi(J, n)$ as a function of the number n of tickets bought? Be as precise as possible to make economic predictions below. Think of behavior for small and large n.
   Hint: Draw a suitable diagram. You may assume that $n \mapsto pn(J + n) - J(e^{pn} - 1)$ single crosses zero from positive to negative as n rises from 1 to ∞.

   (b) Describe a competitive equilibrium in terms of the F and π functions.

   (c) Rigorously identify all Marshallian stable equilibria (i.e. quantity stability).
   Hint: Consider two cases: first, when people are sufficiently heterogeneous in their lotto enjoyment, and second, when they are not too heterogeneous.

   (d) When the jackpot is not won, and thus rolls over, what happens to ticket sales and the expected prize in a stable equilibrium of the next lotto?
   Hint: Illustrate your prediction graphically with jackpots $0 < J_1 < J_2 < J_3 < J_4$.

   (e) What can you identify about the $F(\lambda)$ function by observing the jackpot rollovers and ticket sales each for each lotto? Think graphically.

Part IV

The Math Department is looking to appoint a new chair and decided to hire an outsider with CEO experience. The department will make an offer to Jeff B. If Jeff works $x \geq 0$ hours, this produces the return $2\sqrt{x}$ for the Math Department. Jeff's effort cost of working for x hours is θx. While Jeff has been a star CEO in the industry, the Math Department is uncertain how Jeff's abilities will fit the academic environment. The Department believes that Jeff's cost parameter θ can be high or low (θ_h, θ_l) and that θ_h and θ_l are equally likely; $\theta_h > \theta_l > 0$. Jeff knows his cost type.

The Department will offer a pair of contracts to Jeff. Each contract will specify the number of hours of work and the wage: (x_h, w_h), (x_l, w_l). The Department's objective is to maximize its expected profit (with profit defined as the return to Jeff's work net of his wage). Assume that Jeff's reservation utility is 0.

1. Assume first that the Department knows Jeff's cost parameter. What can the contract condition on? Solve for the optimal contract.
2. From now on, assume that the Department does not know Jeff's type. What can the contract can condition on? Write down the Department's optimization problem, accounting for the participation and incentive compatibility constraints.
3. Can the contract in part (1) be implemented?
4. Does the single-crossing condition holds. Why is it relevant?
5. Do the participation and incentive compatibility constraints bind? Why?

   Hint: Start with participation constraints and check whether x_h or x_l is weakly larger.
6. Solve for the optimal contract. Which type gets the informational rent and why?
7. Is the contract in (4) efficient (i.e., the same as in part (1))? Explain.