Homework #7

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1. Sundaram, #8, p.170.

2. Prove the sollowing result (due to J. W. Gibbs, 1876). Consider the problem

$$\max f(x) = \sum_{j=1}^{n} f_j(x_j)$$

$$\text{s.t. } x_j \geq 0, \text{ for all } j = 1, ..., n$$

$$\sum_{j=1}^{n} x_j = 1,$$

where f is a C^1 function.

   (a) Give an economic interpretation of this problem.

   (b) Let x^* be a solution to the above problem. Show that there exists a number μ^* such that $f_j'(x_j^*) = \mu^*$ if $x_j^* > 0$ and $f_j'(x_j^*) \leq \mu^*$ if $x_j^* = 0$.

   (c) Interpret the solution in economics terms.

3. Consider the nonlinear program

$$\min f(x) = \sum_{j=1}^{n} \frac{c_j}{x_j}$$

$$\text{s.t. } \sum_{j=1}^{n} a_j x_j = b \text{ and}$$

$$x_j \geq 0, \text{ for all } j = 1, ..., n,$$

where a_j, b_j and c_j are positive constants for all $j - 1, ..., n$. Show that the optimal value of the objective function is given by

$$f(x^*) = \frac{\left[\sum_{j=1}^{n} \sqrt{a_j c_j})\right]^2}{b}.$$

4. Sundaram, #3, p. 198.

5. Let $C \subset .\mathbb{R}^n$ be a convex set. Show that $X = \{x \in \mathbb{R}^p : x = A\rho,\ \rho \in C\}$, where A is a given $p \times n$ real matrix, is a convex set.