University of Wisconsin

Microeconomics Prelim Exam

Friday, June 2, 2017: 9AM - 2PM

- There are four parts to the exam. All four parts have equal weight.
- Answer all questions. No questions are optional.
- Hand in 12 pages, written on only one side.
- Write your answers for different parts on different pages. So do not write your answers for questions in different parts on the same page.
- Please place a completed label on the top right corner of each page you hand in. On it, write your assigned number, and the part of the exam you are answering (I,II,III,IV). Do not write your name anywhere on your answer sheets!
- Show your work, briefly justifying your claims. Some solutions might be faster done by drawing a suitable diagram.
- You cannot use notes, books, calculators, electronic devices, or consultation with anyone else except the proctor.
- Please return any unused portions of yellow tablets and question sheets.
- There are six pages on this exam, including this one. Make sure you have all of them.
- Best wishes!

Part I

1. On the TV show "Big Bang Theory", assume that Penny is not indifferent between any pair of her co-stars S, L, or A (Sheldon, Leonard, or Amy). Penny is asked in sequence three questions about her preferences:

   (a) $S \succ L$ or $L \succ S$

   (b) $L \succ A$ or $A \succ L$

   (b) $A \succ S$ or $A \succ S$

   Penny always independently flips a fair *coin* in making each pairwise choice. What is the probability that a decision theorist deems her to be irrational after each stage?

2. You have the following information about Lones' choices from the interval $[5, 25]$:

   $$18 \in C([10, 20]), \quad 11 \in C([5, 15]), \quad 17 \in C([15, 17]), \quad 22 \in C([8, 25]), \quad 6 \in C([5, 18]).$$

   Could Lones have fixed preferences with a single optimizer in every choice interval?

3. Gary has two questions on his five hour prelim. He has enough time to perfectly solve one question, but not enough time to perfectly do both. But perhaps he might wish to partially solve each question. Specifically, question A yields $f(t)$ points if he invests $t \in [0, 4]$ hours in it up to $t = 4$. Question B yields $g(t)$ points if he invests t hours in it, up to $t = 4$. Here, f is increasing and convex and g is increasing and concave; both are differentiable. Gary wishes to maximize the sum of points from A and B. Mathematically and simply characterize his optimal test taking strategy.

   Hint: Consider the four numbers $f'(1), f'(4), g'(1), g'(4)$.

Part II

Consider the following normal form game $\hat{G}$:

		2		
		L	M	H
	L	10,10	3,15	0,7
1	M	15,3	7,7	−4,5
	H	7,0	5,−4	−15,−15

1. Find all rationalizable strategies of $\hat{G}$.
2. Find all Nash equilibria of $\hat{G}$.

Let $\hat{G}^{\infty}(\delta)$ be the infinite repetition of $\hat{G}$ with discount rate $\delta \in (0,1)$. Define play paths:

$$h^0 = \{(L,L),(L,L),(L,L),\ldots\},$$
$$h^1 = \{(M,H),(L,M),(L,M),\ldots\},$$
$$h^2 = \{(H,M),(M,L),(M,L),\ldots\}.$$

Consider the following pure strategy profile $s = (s_1, s_2)$ for $\hat{G}^{\infty}(\delta)$:

(I) Each player begins by playing his action sequence from play path h^0.

(II) If there is a unilateral deviation by player i from h^0, then each player proceeds by playing his action sequence from play path h^i.

(III) If there is a unilateral deviation by player j from play path h^i, then each player proceeds by (re)starting his action sequence from play path h^j. (Both $j \neq i$ and $j = i$ are allowed; in the latter case, the unilateral deviation restarts play path $h^i = h^j$.)

3. For what values of δ is s a subgame perfect equilibrium of $\hat{G}^{\infty}(\delta)$?

Now let $G = \{\{1,2\},\{A_1,A_2\},\{u_1,u_2\}\}$ be a two-player finite action normal form game, and let $G^{\infty}(\delta)$ be its infinite repetition with discount rate $\delta \in (0,1)$, hereafter fixed. Suppose that $G^{\infty}(\delta)$ has at least one pure subgame perfect equilibrium. Let $\Pi_i(\delta)$ be the set of payoffs obtainable by player i in pure subgame perfect equilibria of $G^{\infty}(\delta)$. It can be shown that each player i has a minimal pure subgame perfect equilibrium payoff $\underline{\pi}_i \equiv \min \Pi_i(\delta)$.

Let $s^i = (s^i_1, s^i_2)$ be a pure subgame perfect equilibrium of $G^{\infty}(\delta)$ yielding payoff $\underline{\pi}_i$ to player i. Let $h^i = \{(a^i_1(t), a^i_2(t))\}_{t=0}^{\infty}$ be the play path of s^i.

Let $h^0 = \{(a^0_1(t), a^0_2(t))\}_{t=0}^{\infty}$ be an arbitrary play path of $G^{\infty}(\delta)$. Given h^0, h^1, and h^2, define pure strategy profile $s = (s_1, s_2)$ using (I)–(III) above.

4. Prove that if h^0 is the play path of a subgame perfect equilibrium of $G^\infty(\delta)$, then s is a subgame perfect equilibrium of $G^\infty(\delta)$.

   Hint: You will need to use the facts that h^0, h^1, and h^2 are play paths of pure subgame perfect equilibria of $G^\infty(\delta)$, and that h^1 and h^2 are the worst such play paths for players 1 and 2 respectively.

   Comment: It is possible to answer this question using almost no notation. To answer using notation, let $\pi_i(h) = (1 - \delta) \sum_{t=0}^{\infty} \delta^t u_i(a_1(t), a_2(t))$ denote player i's payoff from following any play path $h = \{(a_1(t), a_2(t))\}_{t=0}^{\infty}$.

Aside: Given this result, to determine whether a play path can arise in a pure subgame perfect equilibrium (at discount rate δ), it suffices to construct the worst equilibrium play paths h^1 and h^2, and then check whether the above strategy profile s is a subgame perfect equilibrium.

Part III

1. Lyft and Uber compete in the Madison ride market in a simultaneous price-setting game. Their ride prices are P_L and P_U. Lyft has the demand $Q_L = 160 - 4P_L + 2P_U$ and Uber has the demand curve $Q_U = 150 - 3P_U + P_L$, where we measure prices in pennies and quantities in miles. Each firm has constant 50 cents per mile cost.

   (a) Find the equilibrium prices and quantities.

   (b) Is Uber's price lower or higher in the two-stage Stackelberg equilibrium in which Uber moves first and Lyft follows. Give an economic intuition.

2. Suppose three men {M1,M2,M3} wish to match with three women {W1,W2,W3}, with the following match payoffs:

	W1	W2	W3
M1	8,5	7,2	6,13
M2	1,3	5,1	4,2
M3	2,5	3,4	1,3

   (a) Is the matching allocation {(M1,W1),(M2,W2),(M3,W3)} stable?

   (b) Find the male-optimal stable allocation.

Part IV

Alvie owns a jar of peanut butter, and Babs owns a jar of jelly. If both of them agree, they can make themselves peanut-butter-and-jelly sandwiches (alternative y). Otherwise, Alvie will make himself a peanut butter sandwich and Babs will make herself a jelly sandwich (alternative n).

It is commonly known that Alvie and Babs each obtain utility $v \in [\frac{1}{2}, 1]$ from eating a peanut-butter-and-jelly sandwich. Alvie's utility θ_A from eating a peanut butter sandwich is his private information, and is drawn from a uniform distribution on $[0, 1]$. Likewise, Babs's utility θ_B from eating a jelly sandwich is her private information, and is also drawn from a uniform distribution on $[0, 1]$.

1. Find the ex post efficient allocation function for this Bayes collective choice problem?
2. For what values of v is there a Bayesian incentive compatible, interim individually rational, and budget balanced mechanism that implements the ex post efficient allocation function?

From now on, suppose that v satisfies the requirement you derived in question 2.

3. Describe the transfer functions of a mechanism that has the desired properties from part (2). How does each agent's payment depend on his own type? Provide intuition for the form of this dependence.
4. Describe the complete set of pairs of interim transfer functions $(\bar{t}_A(\cdot), \bar{t}_B(\cdot))$ that are consistent with achieving the desired properties from part (2).