Econ 761 HW2

Elasticity of demand is
$$\varepsilon = -\frac{\rho}{Q} \frac{dQ}{dP} = \frac{\rho}{Q} \left(\frac{1}{a_1}\right)$$

 $\Rightarrow \varepsilon = \frac{1}{a_1Q} \left(a_0 - a_1Q + \nu\right) = \frac{a_0}{a_1Q} + \frac{\nu}{a_1Q} - 1$

$$\frac{\partial \mathcal{E}}{\partial Q} = -\frac{Q_0}{\alpha_1 Q^2} - \frac{\gamma}{Q_1 Q^2} = -\frac{Q_0 + \gamma}{Q_1 Q^2}$$

$$\frac{\partial \mathcal{E}}{\partial x} = \frac{1}{a_1 a_2} > 0 \quad \text{for } a_1 > 0 \Rightarrow \frac{\partial \mathcal{E}}{\partial x} \geq 0$$

Taking FOC wrt q;,
$$a_0 - 2a_1q_1 - a_1Q_{-1} + \nu - b_0 - \eta = 0$$

Since firms are symmetric, let $q_1 = q$ $\forall i = 1,..., N \Rightarrow Q = Nq$

$$\Rightarrow a_0 - a_1(N+1)q + y - b_0 - \eta = 0 \Rightarrow q = \frac{a_0 + y - b_0 - \eta}{a_1(N+1)}$$

$$\eta = \frac{1}{a_1} \left[\frac{a_0 + \nu - b_0 - \eta}{N+1} \right]^2 - F$$

c) Firms enter until profits are zero
$$\Rightarrow 0 = \frac{1}{a_1} \left[\frac{a_0 + V - b_0 - \eta}{N+1} \right]^2 - F$$

d) Lerner index:
$$L_{I} = \frac{P - MC}{P} = \frac{a_{0} + v + N(b_{0} + N)}{N + 1} - (b_{0} + N)}{a_{0} + v + N(b_{0} + N)}$$

$$\Rightarrow L_{\mathbf{I}} = \frac{a_0 + \nu - b_0 - \eta}{a_0 + \nu + N(b_0 + \eta)}$$

$$\frac{7}{\sqrt{Fa_1 + b_0 + \eta}} \left(\frac{a_0 + \nu - b_0 - \eta}{a_0 + \nu - b_0 - \eta} \right) = \frac{\sqrt{Fa_1}}{\sqrt{Fa_1} + b_0 + \eta} = L_{I}$$



Herfindahl index:
$$H = \frac{1}{N}$$
 Since firms are symmetric

$$\frac{1}{A + N - b_0 - N - \sqrt{\frac{1}{5}a_1}} = \frac{\sqrt{\frac{1}{5}a_1}}{\sqrt{\frac{1}{5}a_1}} = \frac{\sqrt{\frac{1$$

depend elasticity:
$$\varepsilon = \frac{a_0 + v - a_1Q}{a_1Q}$$

using Q = N (
$$\frac{a_0+\nu-b_0-il}{a_1(N+1)}$$
), $\zeta = \frac{a_0+\nu-il}{N(\frac{a_0+\nu-b_0-il}{N+1})}$

using
$$N=\frac{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}{\sqrt{Fa_1}}$$
 $\frac{N}{\sqrt{N+1}}=\frac{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}{\sqrt{Fa_1}}=\frac{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}{\sqrt{A_0+\nu-b_0-\eta}}$

$$\Rightarrow \xi = \frac{\alpha_0 + \nu - (\alpha_0 + \nu - \delta_0 - \eta - \sqrt{Fa_1})}{\alpha_0 + \nu - \delta_0 - \eta - \sqrt{Fa_1}} = \frac{\delta_0 + \eta + \sqrt{Fa_1}}{\alpha_0 + \nu - \delta_0 - \eta - \sqrt{Fa_1}} = \xi$$

$$\Rightarrow \frac{\partial \mathcal{E}}{\partial F} = \frac{\sqrt{a_1(a_0 + \nu)}}{2\sqrt{F(a_0 + \nu - b_0 - \eta - \sqrt{Fa_1})^2}} \geq 0 \Rightarrow \frac{\partial \mathcal{E}}{\partial F} \geq 0$$

As entry costs rise, potentially fener firms entry the market leading to an inverse in demand elasticity.

$$\frac{\partial \mathcal{E}}{\partial \nu} = -\frac{b_0 + n + \sqrt{Fa_1}}{(n_0 + \nu - b_0 - n - \sqrt{Fa_1})^2} \leq 0 \quad \Rightarrow \quad \frac{\partial \mathcal{E}}{\partial \nu} \leq 0$$

Hence with endogenous number of firms entering, as willingness to pay increases, number of firms entering increases which drives the Lemand elasticity down.

$$\frac{3\varepsilon}{2n} = \frac{(a_0 + v - b_0 - n - |Fa_1|) + (b_0 + n + |Fa_1|)}{(a_0 + v - b_0 - |Fa_1|)^2}$$

$$\frac{\partial \mathcal{E}}{\partial n} = \frac{a_0 + \nu}{(a_0 + \nu - b_0 - \sqrt{Fa_1})^2} \geq 0 \Rightarrow \frac{\partial \mathcal{E}}{\partial n} \geq 0$$

As marginal cost increases, firms produce less and so the demand elasticity increases.

$$\frac{\partial \ln(L_{\rm I})}{\partial F} = \frac{1}{2F} - \frac{\int \alpha_{\rm I}}{(F\alpha_{\rm I} + b_{\rm O} + 1)2\sqrt{F}}$$

$$\frac{\partial \ln(L_{\rm I})}{\partial F} = \frac{1}{2F} + \frac{\int \alpha_{\rm I}}{(\alpha_{\rm O} + 1)^{2}\sqrt{F}}$$

$$\Rightarrow \frac{\partial \ln(L_{\rm I})}{\partial F} \neq \frac{\partial \ln(L_{\rm I})}{\partial F}$$

$$\frac{\partial \ln(L_{\rm L})}{\partial \nu} = -\frac{1}{a_0 + \nu - b_0 - \eta - \sqrt{Fa_1}}$$

$$\frac{\partial \ln(L_{\rm L})}{\partial \nu} \neq \frac{\partial \ln(L_{\rm L})}{\partial \nu} \neq \frac{\partial \ln(H)}{\partial \nu}$$

$$\frac{\partial \ln(Lz)}{\partial n} = -\sqrt{Fa_1} + b_0 + n$$

$$\frac{\partial \ln(H)}{\partial n} = \sqrt{a_0 + \nu - b_0 - n} - \sqrt{Fa_1}$$

$$\Rightarrow \frac{\partial \ln(Lz)}{\partial n} \neq \frac{\partial \ln(Lz)}{\partial n} \neq \frac{\partial \ln(H)}{\partial n}$$

Take due to exogenous changes in F. v. 1

Price is
$$P^* = a_0 - a_1 Q^* + \nu = a_0 - \frac{a_0 + \nu - b_0 - \gamma}{2} + \nu = \frac{1}{2} \left(a_0 + \nu + b_0 + \gamma \right)$$

$$\Rightarrow \text{ profit are } n = P^* \frac{Q^*}{N} - \left(\left(\frac{Q^*}{N} \right) \right)$$

$$= \frac{1}{4} \left(a_0 + \nu + b_0 + \gamma \right) \frac{a_0 + \nu - b_0 - \gamma}{2Na_1} - \left[\left(b_0 + \eta \right) \frac{a_0 + \nu - b_0 - \gamma}{2Na_1} + F \right]$$

$$\Rightarrow n = \frac{1}{4Na_1} \left[a_0 + \nu - b_0 - \gamma \right]^2 - F \text{ for each firm}$$

Firms enfer until zero profits $\Rightarrow \frac{1}{4Na_1} \left[a_0 + \nu - b_0 - \gamma \right]^2 - F = 0$

$$\Rightarrow 4NFa_1 = \left(a_0 + \nu - b_0 - \gamma \right)^2 \Rightarrow N = \frac{1}{4Fa_1} \left(a_0 + \nu - b_0 - \gamma \right)^2$$

Lorser intex: $L_1 = \frac{P - MC}{P} = \frac{\frac{1}{2} \left(a_0 + \nu + b_0 + \gamma \right) - \left(b_0 + \gamma \right)}{\frac{1}{2} \left(a_0 + \nu + b_0 + \gamma \right)}$

$$\Rightarrow L_1 = \frac{a_0 + \nu - b_0 - \gamma}{a_0 + \nu + b_0 + \gamma}$$

Her findall intex: $H = \frac{1}{N} = \frac{4Fa_1}{\left(a_0 + \nu - b_0 - \gamma \right)^2} = H$

denoted elasticity: $-\frac{P}{Q} \frac{dQ}{dP} = \frac{P}{Q} \left(\frac{1}{d_1} \right) = \frac{a_0 + \nu + b_0 + \gamma}{a_0 + \nu - b_0 - \gamma}$

9) (a) $\ln P = c_0 - c_1 \ln Q + c_2 \Rightarrow P = \exp \left(c_0 - c_1 \ln Q + c_2 \right)$
 $-\frac{P}{Q} \ln Q = c_0 - \ln P + c_2 \Rightarrow Q = \exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right] \Rightarrow \frac{1}{2} \left[\exp \left[\frac{1}{4} \left(c_0 - \ln P + c_2 \right) \right$

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(b) problem for firm i is max f(Q) = c(Q)

\Rightarrow \max_{q_i} \exp(c_0 - c_i h(Q_i + Q_{-i}) + g) = [F + (b_0 + \eta) = Q_i]
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$$\frac{7}{7} \exp((c_0-c_1 \ln(Nq)+c_3)) - \frac{qc_1}{Nq} \exp((c_0-c_1 \ln(Nq)+c_3)-b_0-\eta=0)$$

 $\frac{7}{N} \exp((c_0-c_1 \ln(Nq)+c_3)-b_0-\eta=0)$

$$\Rightarrow \exp(c_0 - c_1 \ln(Nq) + c_3) = \frac{N}{N-c_1}(b_0 + \eta)$$

$$\Rightarrow c_0 - c_1 \ln(Nq) + c_3 = \ln\left[\frac{N}{N-c_1}(b_0 + \eta)\right]$$

Price is
$$P^* = \exp((c_0 - c_1 \ln Q^* + \frac{c_3}{3}))$$
 with Q^* defined above
 $\Rightarrow P^* = \exp((c_0 - c_1 \ln [\exp[\frac{c_1}{4}(c_0 + \frac{c_3}{3} - \ln [\frac{N}{N-c_1}(b_0 + \frac{c_3}{3})])] + \frac{c_3}{3})$
 $= \exp((c_0 - [c_0 + \frac{c_3}{3} - \ln [\frac{N}{N-c_1}(b_0 + \frac{c_3}{3})]) + \frac{c_3}{3})$

Profits are
$$\eta = P^* q^* - ((q^*))$$
 for each firm
 $\Rightarrow \eta = \frac{N}{N-c_1}(b_0+\eta) \stackrel{!}{\to} exp\left[\frac{1}{c_1}(c_0+\xi-h(\frac{N}{N-c_1}(b_0+\eta)))\right]$

$$- \left[F + (b_0+\eta) \stackrel{!}{\to} exp\left[\frac{1}{c_1}(c_0+\xi-h(\frac{N}{N-c_1}(b_0+\eta)))\right] \right]$$

Because we will not use the profits anymore, I will not simplify it further.



(d) Lerner index:
$$L_I = \frac{P-MC}{P} = \frac{N}{N-C_1}(b_0+\eta) - (b_0+\eta)$$

$$\Rightarrow \Gamma_{I} = \frac{\frac{N-c'}{N}}{\frac{N}{N-c'}} = \frac{\frac{N-c'}{N-c'}}{\frac{N-c'}{N-c'}} = \frac{\frac{N}{C}}{\frac{C}{L}} = \Gamma_{I}$$

$$\Rightarrow \xi = \frac{c_1 \exp\left[c_1\left(c_0 - \ln\left[\frac{N}{N-c_1}(b_0 + \eta)\right] + c_3\right)\right]}{\exp\left[c_1\left(c_0 + c_3 - \ln\left[\frac{N}{N-c_1}(b_0 + \eta)\right]\right)\right]} = \frac{1}{c_1} \Rightarrow \left[\xi = \frac{1}{c_1}\right]$$

(e) We take N as fixed and exogenous, so the Lorner index LI. Herfindahl index It, and demand clasticity all do not vary with F, v, N.

For E,
$$\frac{\partial i}{\partial F} = \frac{\partial E}{\partial \nu} = \frac{\partial E}{\partial \eta} = 0$$
.

Similarly,
$$\frac{\partial \ln(Lz)}{\partial F} = \frac{\partial \ln(H)}{\partial F} = 0$$
$$\frac{\partial \ln(Lz)}{\partial V} = \frac{\partial \ln(H)}{\partial V} = 0$$
$$\frac{\partial \ln(Lz)}{\partial V} = \frac{\partial \ln(H)}{\partial V} = 0$$

in response to changes in F, V, Z.

2. a) see affached code

b) see attached code

c) The results of the three regressions are presented below.

	Collusion Possible	no Collisian	pooled sample
	N=500	N=500	N=1000
Constant	0.387	-0.163 (0.001)	0.077
In (Herfindahl)	0.619 (0.049)	0.962 (0.01)	0.764
F-Stat for test	61.59	4261.89	47.77
Ho: h(ltofindahl)=]	⇒reject	⇒æject	⇒ reject

When collusion is possible, a one percet invegse in Herindahl index increases the Lerner index by 0.619 percent. This is much higher when there is no collusion (0.962) and in between these two for the pooled sample (0.764).

For this demand function, the elasticity of demand is i = 0.9 = 1.111. Due to the high demand elasticity, monopolies can charge higher prizes leading to a lower correlation between It and Lt in the sample with collusion.

It makes sense that the pooled sample yields a coefficient between either of the individual samples, secause it contains them both. Hence, in all three samples there is a positive correlation between It and LI:

The f-statistic in each test of In(Herfindahl)=1 is large and leads to a rejection of the null. However, the reason it is rejected in the sample with no collusion is because of a very small standard error. By inspection, for the no collusion sample, there is very near a 1-1 correlation between H and Lz.

d) Now we use linear demand, still taking N as fixed.

For nonlinear demand in (c) we had LI = Th, H= IN, E= 4.

In this case we have LE = a + v + N(b + 1), H = 1, E = a + v + N(b + 1)

The results from the structure-conduct-performance paradigm regressions as well as the hypothesis lests are below.

	collusion possible	No collusion	pooled sample
	N=500	N=500	N=1000
constant	- 0.473	-0.658	-0. 5 30
	(0.041)	(0.005)	(0.027)
h (Herfindahl)	0.294 (0.024)	0.480 (0.003)	0.376
F-Stat For test	893.73	28309.22	1481.95
Ho: In (Herindahl)=1	⇒rejed	⇒reject	=> æject

There is still a higher correlation between It and LI for the sample with no collusion. It overer, with linear demand, the magnitude of the relationship in each sample is lower than the respective coefficient with nonlinear demand. In all three samples with linear demand, we reject the null hypothesis that In (Herindahl) = 1, as can be seen in the table above.

In the case of linear demand, demand elasticity depends on quantity whereas in Part (c), elasticity is a constant. With collusion, the equilibrium quantity is lower than when collusion isn't possible, so demand elasticity is higher in the former, leading to pothtially lower markups. These factors explain these results and why they differ from Part (c).

e) From part (d), if we know demand is linear and we suspect collusion in markets 1-250, we can run a structure - conduct -performance paradigm regression and the coefficient on In(Herfindahl) should be quite low, around 0.3.

However, this may not necessarily help us generally.

If we do not know the functional term of demand,

the regression could result in a higher coefficient

(higher than the no collusion sample with linear

demand), which could mislead us in concluding collusion.

Hence, the functional form of demand is important

in the subsequent regression and analysis for us to

be certain whether or not some markets are colluding.

3. a,b) The results of the regressions are below.

	v~U[-1,1]	ν=0
	η =0	η~ U[-1,1]
constant	-0.821	-2.116
	(0.0005)	(0.011)
In (Herfinda AI)	-0.146	-1.359
,	(0,0004)	(0.011)

c) When v=0, maximum willingness to pay is a constant across cities, and the Lerner hall drops by 1.359% for a 1% increase in the Herfindahl index. For 1=0, marginal costs aross cities are the same and the effect of Herfindahl index on Lerner index is much closer to zero.

with more firms, higher competition opposes higher markerps and demand is more elastic when val [-1,1] man 1/2 U[-1,1], leading to coefficient on In(Iterrindahl) that is closer to 0.