PROBLEM SET 3

1. Suppose there are three coffee houses along Main Street. The street is one mile. One hundred residents are uniformly distributed along this stretch. The market is covered: each resident buys one cup of coffee per day. A cup of coffee differs only in its location and price and not in any other way. Each customer derives a utility of V = \$3.00 from the cup of coffee. A consumer's (round-trip) cost of travel is quadratic in the distance from home to any of the coffee houses.

Starbucks Coffee House is located at either end of the one mile stretch and Esquire Coffee House is located halfway between the two end points of the street. The prices of coffee at Starbucks' two locations are p_0 and p_1 respectively. Esquire's price of coffee is denoted by q. Marginal costs of a cup of coffee are zero.

(a) Determine the location of the two marginal consumers – the one who is indifferent between purchasing from Esquires and Starbucks located at the left end point and the one who is indifferent between purchasing from Esquires and the Starbucks located the right end point.
(b) Derive the best reply functions to the pricing game in which the coffee houses choose prices simultaneously. Assume that Starbucks can set different prices at its two locations.
(c) Determine the equilibrium prices and market shares.
(d) Suppose Starbucks and Esquires swap houses so that the Starbucks houses are located at one of the endpoints and the halfway point and Esquires is located at the other endpoint. Derive the equilibrium prices and market shares, and explain why it differs (if at all) from (c).

2. Consider a model of vertical differentiation in which a customer of type θ obtains the net benefit $U = s(\theta - p(s))$ from an item of quality s. Here θ is uniformly distributed in the population between zero and one. Marginal costs of providing a good of quality s is $C(s) = cs$ where $c \leq 1/2$.

(a) Find the optimal prices which a monopolist should charge when it offers two goods with qualities s = 1 and s = 2.
(b) Find the optimal price which a monopolist should charge when it offers only one good with quality s = 1.
(c) Determine the conditions on c under which these two solutions are optimal.

Now suppose there are two firms: firm 1 offers low quality product with s = 1 and firm 2 offers high quality product with s = 2. The firms compete in prices.

(d) Determine the relevant range for p_2 and derive the demand for good 1 on that range.
(e) Derive firm 1's best reply as a function of c and plot it in price space. (Hint: it consists of four segments.)
(f) Derive firm 2's best reply and compute the Nash equilibrium prices as a function of c. Can the firms coexist?