Econ 711 Problem Set 2

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Question 1

$$N \geq 2$$
$$s_i \in [0, w], w > 0$$
$$\pi_i = n \min\{s_1, s_2, ..., s_n\} - s_i$$

Player i will choose their contribution based on the other players actions. Player i will not play less than the minimum of the other players because this would reduce the payoff, and player i would not play more than the minimum of the other players because this would reduce the payoff. So player i will play exactly the minimum of the other players. Since all players will act this way, utlimately all players will choose the same $s \in [0, w]$.

Question 2

(a)

$$N = \{\text{Joe, Donald}\}$$
$$a_i = \{(a_1, a_2, a_3) \text{ s.t. } a_1 + a_2 + a_3 = 1\}$$
$$b_i = \{(b_1, b_2, b_3) \text{ s.t. } b_1 + b_2 + b_3 = 1\}$$
$$U_{Donald} = 1\{1\{a_1 > b_1\} + 1\{a_2 > b_2\} + 1\{a_3 > b_3\} > 1\{a_1 < b_1\} + 1\{a_2 < b_2\} + 1\{a_3 < b_3\}\}$$
$$U_{Joe} = 1\{1\{a_1 < b_1\} + 1\{a_2 < b_2\} + 1\{a_3 < b_3\} > 1\{a_1 > b_1\} + 1\{a_2 > b_2\} + 1\{a_3 > b_3\}\}$$

(b)

Assume there is a Nash equilibrium. Then if Donald does not win, Donald would be better off by allocating more funding to two of the voters to secure a win. If Joe does not win, Joe would be better off by allocating more funding to two of the voters to secure a win. Since it will always be the case that either Joe or Donald does not win, at least one player could always improve their outcome by changing their allocation. So a Nash equilibrium does not exist.

*I have discussed this problem set with Emily Case, Michael Nattinger, Alex Von Hafften, and Danny Edgel.

Question 3

(a)

$$N = \{\text{Alice, Bob}\}$$
$$s_i = \{T_1, T_2, T_3\}$$
$$u(s_i, s_{-i}) = i * 1\{s_i = s_{-i}\}$$

(b)

All strategies are best responses if the other player plays the same strategy, so there are three pure strategy Nash equilibria: (1,1), (2,2), (3,3).

Next we'll consider the existence of mixed strategy Nash equilibria. Let p_1, p_2, p_3 represent the probabilities of playing T_1, T_2, T_3, respectively.

In order to play a mixture of T_1 and T_2, it must be the case that $p_1 = 2p_2$ such that $p_1+p_2+p_3 = 1$. So $(\frac{2}{3}(1) + \frac{1}{3}(2), \frac{2}{3}(1) + \frac{1}{3}(2))$ is a Nash equilibrium.

In order to play a mixture of T_1 and T_3, it must be the case that $p_1 = 3p_3$ such that $p_1+p_2+p_3 = 1$. So $(\frac{3}{4}(1) + \frac{1}{4}(3), \frac{3}{4}(1) + \frac{1}{4}(3))$ is a Nash equilibrium.

In order to play a mixture of T_2 and T_3, it must be the case that $2p_2 = 3p_3$ such that $p_1+p_2+p_3 = 1$. So $(\frac{3}{5}(2) + \frac{2}{5}(3), \frac{3}{5}(2) + \frac{2}{5}(3))$ is a Nash equilibrium.

In order to play a mixture of T_1, T_2 and T_3, it must be the case that $p_1 = 2p_2 = 3p_3$ such that $p_1 + p_2 + p_3 = 1$. So $(\frac{6}{11}(1) + \frac{3}{11}(2) + \frac{2}{11}(3), \frac{6}{11}(1) + \frac{3}{11}(2) + \frac{2}{11}(3))$ is a Nash equilibrium.

Question 4

(a)

First note that setting $q_i \geq 2$ will result in negative profit, so neither firm will choose this. As long as $q_{-i} < 2$, firm i can make a profit by choosing $q_i \in (0, 2 - q_{-i})$. Since corner solutions will not be optimal, we can take first order conditions. Firm i faces the following maximization problem:

$$\max\{q_i(2 - q_i - q_{-i}) - q_i\}$$

Taking first order conditions:

$$\frac{\partial \pi}{\partial q_i} = 1 - 2q_i - q_{-i} = 0$$
$$\Rightarrow q_i = \frac{1 - q_{-i}}{2}$$
$$\Rightarrow q_{-i} = \frac{1 - q_i}{2}$$
$$\Rightarrow q_i = \frac{1 - \frac{1-q_i}{2}}{2} = \frac{1}{3}$$
$$\Rightarrow q_{-i} = \frac{1}{3}$$
$$\Rightarrow \pi = \frac{1}{3}(2 - \frac{1}{3} - \frac{1}{3}) - \frac{1}{3} = \frac{1}{9}$$

(b)

Now firm 1 is trying to solve the following maximization problem:

$$\max\{q_1(2 - q_1 - q_2) - \frac{3}{4}q_1\}$$

Taking first order conditions:

$$\frac{\partial}{\partial q_1} = 2 - 2q_1 - q_2 - \frac{3}{4} = 0$$
$$\Rightarrow q_1 = \frac{\frac{5}{4} - q_2}{2}$$
$$= \frac{\frac{5}{4} - \frac{1-q_1}{2}}{2}$$
$$= \frac{1}{2}$$
$$\Rightarrow q_2 = \frac{1 - \frac{1}{2}}{2}$$
$$= \frac{1}{4}$$
$$\Rightarrow \pi_1 = \frac{1}{2}(2 - \frac{1}{2} - \frac{1}{4}) - \frac{1}{2}$$
$$= \frac{1}{8}$$
$$\Rightarrow \pi_1 = \frac{1}{4}(2 - \frac{1}{2} - \frac{1}{4}) - \frac{1}{4}$$
$$= \frac{1}{16}$$

(c)

In part A, both firms act to maximize profits so they produce equal amounts and have equal profits. However, in part B, firm 1 expands their market share. As a result, when firm 1 expands their market share, firm 2 reduces production to stay profit maximizing. Consequently the profits for firm 1 increase when they increase market share, the profits for firm 2 decrease, and the total profit decreases.

Question 5

(a)

We'll first find the time that maximizes $u(t)$. Taking first order conditions:

$$\begin{aligned} 1 - 2t &= 0 \\ \Rightarrow t &= \frac{1}{2} \end{aligned}$$

Clearly, this dominates the corners since $u(0) = 0$ and $u(1) = 0$. Next we'll solve for $Q(t)$ such that participants are indifferent between following $Q(t)$ for all t and joining at $t = \frac{1}{2}$.

$$\begin{aligned} v(Q(t))u(t) &= v(0)u(\frac{1}{2}) \\ \Rightarrow (1 + Q(t) + \frac{1}{2}Q(t)^2)t(1-t) &= \frac{1}{2}(1 - \frac{1}{2}) \\ \Rightarrow (1 + Q(t) + \frac{1}{2}Q(t)^2)t(1-t) &= \frac{1}{4} \\ \Rightarrow 1 - \frac{1}{4}\left(\frac{1}{t(1-t)}\right) + Q(t) + \frac{1}{4}Q(t)^2 &= 0 \\ \Rightarrow Q(t) &= \frac{-1 \pm \sqrt{1 - 4(\frac{1}{4})(1 - \left(\frac{1}{4t(1-t)}\right))}}{2(\frac{1}{4})} \\ \Rightarrow Q(t) &= -2 \pm \sqrt{\frac{1}{t(1-t)}} \end{aligned}$$

Note, $\max Q(t) = 1$, so

$$\begin{aligned}
1 &= -2 \pm \sqrt{\frac{1}{t(1-t)}} \\
3 &= \pm\sqrt{\frac{1}{t(1-t)}} \\
\Rightarrow 9 &= \frac{1}{t(1-t)} \\
\Rightarrow -9t^2 + 9t - 1 = 0 & \\
\Rightarrow t &= \frac{-9 \pm \sqrt{9^2 - 4(-1)(-9)}}{2(-9)} \\
&= \frac{-9 \pm \sqrt{81-36}}{-18} \\
&= \frac{-9 \pm \sqrt{45}}{-18} \\
&= \frac{-9 \pm 3\sqrt{5}}{-18} \\
&= \frac{3+\sqrt{5}}{6}
\end{aligned}$$

Thus there cannot be a terminal rush.

(b)

The support of Q is $[\frac{1}{2}, \frac{3+\sqrt{5}}{6}]$.