UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MACROECONOMICS THEORY Preliminary Exam

August 1, 2019

9:00 am - 2:00 pm

INSTRUCTIONS

- **Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:**

  (1) your assigned number
  (2) the number of the question you are answering
  (3) the position of the page in the sequence of pages used to answer the questions

Example:	
MACRO THEORY	8/01/19
ASSIGNED # ______________	
Qu # __1__ (Page __2__ of __7__):	

- **Do not answer more than one question on the same page!**
  When you start a new question, start a new page.
- **DO NOT write your name anywhere on your answer sheets!**
  After the examination, the question sheets and answer sheets will be collected.
- **Please DO NOT WRITE on the question sheets.**
- **The number of points for each question is provided on the exam.**
- **Answer all questions.**
- **Do not continue to write answers onto the back of the page – write on one side only.**
- **Answers will be penalized for extraneous material; be concise.**
- **You are not allowed to use notes, books, calculators, or colleagues.**
- **Do NOT use colored pens or pencils.**
- **There are seven pages in the exam, plus this instruction page (8 pages total)—please make sure you have all of them.**

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.
- All scratch paper, unused tablet paper, and exams are to be turned in after the exam. Your proctor will give you directions, listen to your proctor.
- Good luck!

Question 1.

Consider the following simple variant of Diamond's one-sector overlapping generations model with production. Each period there is a new unit measure of 2 period lived agents. There is no population growth.

All agents have identical preferences given by

$$\log(c_t^t) + \beta \log(c_{t+1}^t)$$

where c_j^t is consumption of an agent at time j born in period t. Generation t agents are endowed with one unit of time to work in period t.

There is a constant returns to scale production function $Y_t = K_t^\alpha L_t^{1-\alpha}$ yielding consumption goods where K_t is capital and L_t is labor. Unlike Diamond, capital *does not* depreciate fully every period. In particular, each unit of capital chosen at time t can be used for production at time $t+1$ after which $1-\delta$ remains (this nests Diamond's case where $\delta = 1$). Consumption and capital goods are perfect substitutes (e.g. corn). The initial old are endowed with capital K_0.

Each period, G units of the consumption/capital good is used for "defense" which does not enter the utility function of agents.

Part 1. Social Planner's Solution

Assume the planner weights each generation equally.

Q1 (5 points). State the social planner's problem.

Q2 (12.5 points). Solve the planner's problem. What is the law of motion for capital? What is the steady state level of capital?

Part 2. Competitive Equilibrium

Assume there are competitive markets in consumption, capital, and labor. Firms operate the production technology and rent capital at real interest rate r_t from the old of generation $t-1$ and labor at w_t from the young of generation t. Given constant returns to scale, assume there are no profits to be transferred to households. The government must finance G each period with taxes on households. The proportional tax on capital income paid by the old is denoted τ_t^K and the proportional tax on labor income paid by the young is τ_t^L.

Q3 (2.5 points). State and solve the firm's problem.

Q4 (5 points). State and solve the household's problem. How do taxes affect the household's savings function?

Q5 (2.5 points). State the market clearing conditions and government budget constraint.

Q6 (12.5 points). Solve for the equilibrium law of motion of aggregate capital in a competitive equilibrium. What is the steady state level of capital? How does it differ from Q2?

Q7 (10 points). If there are differences between steady state capital in Q2 and Q6, what would be the optimal way to proportionally tax labor and capital? State the government's problem in a steady state (i.e. as part of a Ramsey equilibrium). If you cannot solve for optimal taxes, provide an intuitive argument of the costs and benefits of raising each tax.

Question 2.

Consider an agent who faces an income process, x_n, where n is duration of the given process. Time is discrete. The process is initialized by $x_0 = 0$ and follows a Markov process where $x_{n+1} = x_n + \tilde{\alpha}_n$ where $\tilde{\alpha}_n$ is a uniformly distributed random variable, $\tilde{\alpha}_n \sim U[-a^n, a^n]$ where $[-a^n, a^n]$ is the support of the distribution and $a \in (0, 1)$. Each $\tilde{\alpha}_n$ is drawn independently. The agent is a net present value income maximizer. In period n the agent receives x_n and at the end of the period/beginning of next period sees the realization of x_{n+1}. Denote by $V_n(x)$ the NPV of the future stream of income given current income x and that the duration of the income process is n. The agent's discount factor is $\beta \in (0, 1)$.

1. **(10 points).** Write the recursive formulation for $V_n(x)$. Find the analytical solution for $V_n(x)$. Does it depend on n?

2. Now, allow the agent the ability to reject the current income process in favor of starting over in a new one (that will have independently drawn income innovations, $\tilde{\alpha}_n$). Specifically upon seeing the realization of next period's income, the agent can choose to accept that realization and stay with the existing income process, or start over in a new process that then resets to duration $n = 0$ and $x_0 = 0$. Denote by r_n the reservation threshold such that the agent is exactly indifferent between continuing with the current income process or choosing to reset, that is $V_n(r_n) = V_0(0)$.

   (a) **(5 points).** Write down the recursive formulation for $V_n(x)$ given this newfound ability to reset the income process in case of unfavorable income realizations.

   (b) **(12 points).** Argue that $V_n(x)$ is the fixed point of a contraction mapping. Be careful to state the mapping. Keep in mind that the income state space is conditional on an initialization of $x_0 = 0$.

   (c) **(18 points).** Characterize $V_n(x)$ along following dimensions: Is the value function increasing/decreasing/constant in x? Is the value function (weakly) convex/concave in x? Is the value function increasing/decreasing/constant in n? Provide proof.

   (d) **(5 points).** How does the reservation threshold evolve in n?

Question 3.

Part 1 (30 points) An economy consists of two types of infinitely lived consumers (each of equal measure) denoted by $i = 1, 2$. There is one nonstorable consumption good. Consumer i consumes c_{it} at time t . Consumer i ranks consumption streams by

$$\sum_{t=0}^{\infty} \beta^t u(c_{it})$$

where $\beta \in (0, 1)$ and $u(c)$ is increasing, strictly concave, and twice continuously differentiable. Consumer 1 is endowed with a stream of the consumption good $y_{it} = 1, 0, 0, 1, 0, 0, 1, ...$ Consumer 2 is endowed with a stream of the consumption good $0, 1, 1, 0, 1, 1, 0, ...$

1. (5 points) Define a competitive equilibrium with time 0 trading. Be careful to include definitions of all the objects of which a competitive equilibrium is composed.
2. (5 points) Compute a competitive equilibrium allocation with time zero trading.
3. (5 points) Prove that the competitive equilibrium is efficient.
4. (5 points) Define a competitive equilibrium with sequential trading of Arrow securities.
5. (5 points) Compute a competitive equilibrium with sequential trading of Arrrow securities.

Part 2 (20 points) Consider the following economy. A unit mass continuum of households lives for two periods. In the first ($t = 0$) each household receives an endowment of one unit of the single consumption good. In the second, one-half will be unable to produce, while the other half can linearly produce the consumption good such that each unit of effort produces one unit of the consumption good. Further, there exists an ability to transfer resources across dates one for one. Those who cant produce have utility over consumption in each date of $u(c_0) + u(c_1)$. Those who can produce have utility over consumption in each date and labor (or output) in the second period of $u(c_0) + u(c_1) - y$. Finally, at the beginning of time (date $t = 0$) every household knows what type it is (whether it can produce at $t = 1$ or not.)

1. (5 points) Characterize, as a system of equations, the solution to the utiliitarian planners problem when household type is observable.
2. (5 points) Suppose a government has the following instruments: A lump sum tax on each type T_i, a linear tax on output τy, and a linear tax on savings for each type, t_i. Can it implement your answer from part 1, and, if so, how?

3. (5 points) Now suppose the type is private to each household. Recharacterize, as a system of equations, the solution to the utilitarian planners problem.

4. (5 points) Given the same instruments as above, can the government implement your answer to part 3, and, if so, how?

Question 4.

This is a question with three parts:

A) (8 points) Consider the AR(2) process given by

$$z_t = \rho z_{t-1} + \beta z_{t-2} + \varepsilon_t \text{ where } \varepsilon_t \text{ is i.i.d. } \mathcal{N}(0,1) \tag{4.1}$$

where z_t is observable but the ε's are unobservable.

Using the state-space formulation given in Equation (4.2) and (4.3), below:

$$y_t = x_1' \alpha_t + x_2' \nu_{1t} \tag{4.2}$$

$$\alpha_t = \mathbb{D}_0 + \mathbb{D}_1 \alpha_{t-1} + \mathbb{D}_2 \nu_{2t} \tag{4.3}$$

carefully define the matrices and vectors y_t, x_1', x_2', α_t Σ_{ν_1}, Σ_{ν_2}, $\mathbb{D}_0$, $\mathbb{D}_1$, and $\mathbb{D}_2$ that describe the AR(2) process given by Equation (4.1).

B) (30 points: 6 points each for parts i-ii; 9 points each for parts iii-iv) In the model of Bernanke and Gertler (1989), an optimal contract solves the following constrained maximization problem. In what appears below, π_i is the probability that the output of the project is κ_i units of capital; c_2 is the consumption of the entrepreneur if she reports the good state; c_1 is the consumption of the entrepreneur if she reports the bad state and is not audited; c_a is the consumption of the entrepreneur if she reports the bad state and is audited; and p is the auditing probability:

$$\max_{p, c_1, c_2, c_a} \pi_1 (p c_a + (1-p) c_1) + \pi_2 c_2$$

subject to

$$r(x(\omega) - S^e) \leq \pi_1 [\hat{q}\kappa_1 - p(c_a + \gamma \hat{q}) - (1-p) c_1] + \pi_2 [\hat{q}\kappa_2 - c_2] \tag{4.4}$$

$$c_2 \geq (1-p)\hat{q}(\kappa_2 - \kappa_1) + c_1 \tag{4.5}$$

$$c_1 \geq 0; \; c_a \geq 0; \; p \in [0,1]$$

For the remainder of the problem, feel free to assume that $x(\omega) = \omega$.

(i) Describe, in your own words, what the constraints given by Equations 4.4 and 4.5 represent.

(ii) Compute the miniumum level of savings $S^*(\omega)$ necessary for full collateralization (so that the entrepreneur of type ω will never be audited).

(iii) Suppose $S^e > S^*(\omega)$. Within the optimal contract, compute $p(\omega, S^e)$, the probability the entrepreneur is audited. This auditing probability is a function of its type ω and savings level S^e.

(iv) Compute the slope(s) of the relationship between the entrepreneur's expected old-age consumption and S^e. `Hint: This slope will differ, depending on whether` $S^e > S^*(\omega)$ `or` $S^e < S^*(\omega)$.

C) (12 points: 6 points for each part)

(i) Regarding the Kiyotaki and Moore (1997) model: Starting from the steady-state allocation, what would happen to the price of land (in terms of fruit) if the indebtedness of farmers unexpectedly increased? Explain why. Your answer should be no more than three sentences long.

(ii) Are these changes in the price of land likely to be larger or smaller when the gatherers' production technology is concave or linear? In at most two sentences explain why.