Question 1.

Consider the following OG model without population growth. Agents live for two periods, young and old. The time endowment is one for both periods. When an agent of generation t is young, they choose the fraction of time $H_t^t \in [0,1]$ to be invested in human capital accumulation. The cost of acquiring human capital is lost working time. When old, agents work with probability $1-u$ and are unemployed with probability u. Since there is a unit measure of agents and these shocks are identical and independently drawn across agents, this implies the unemployment rate is u for old people of any generation.

Productivity (units of goods produced per unit of time) for young agents is one, whereas productivity for old agents is the amount of human capital accumulated H_t^t, multiplied by a constant A. So output for a young person is $(1-H_t^t)$, for an employed old person is AH_t^t and for an unemployed old person is 0. We will assume that $A(1-u) > 1$ so that the expected net return to acquiring human capital is positive.

Let the (expected) utility of agents of generation t be

$$c_t^t + [(1-u)\log c_{t+1}^{e,t} + u \log c_{t+1}^{u,t}]$$

where c_t^t is generation t consumption when young, $c_{t+1}^{e,t}$ and $c_{t+1}^{u,t}$ is consumption when employed and unemployed when old respectively. Consumption is constrained to be non-negative.

Part 1: Planner's Solution

1. (5 points) State the social planner's problem with equal weights on each generation, taking H_0^0 as given.
2. (17.5 points) Solve the steady-state social planner's problem.
   (a) Which non-negativity constraints on consumption and constraints on human capital can be neglected in solving the planner's problem?
   (b) If you can neglect certain constraints, state the first order necessary conditions for the social planner's solution.
   (c) Use $A(1-u) > 1$ to solve for $c_t^{e,t-1}, c_t^{u,t-1}, c_t^t, H_t^t$.

Part 2: Competitive Equilibrium

Now consider an environment with the same preferences and technologies but we consider a competitive monetary equilibrium with an exogenous money supply $\overline{M}_{t+1} = (1+z)\overline{M}_t > 0$. The injections of money are made proportional to each old agent's holdings of money. Hence if $M_{t+1}^t \geq 0$ denotes the money chosen by an agent of generation t in period t to be used for consumption in period $t+1$, then the amount of money that agent has in old age is $M_{t+1}^t(1+z)$. The price of consumption goods in period t is denoted p_t. Hence any output produced or consumed by an agent is priced at p_t (e.g. a young agent of generation t has value of her production given by $p_t(1-H_t^t)$).

3. (5 points) Write down the optimization problem faced by an agent of generation $t \geq 1$. Hint: Simply state the agent's budget constraints in real rather than nominal terms. That is, let $m_{t+1}^t = M_{t+1}^t/p_{t+1}$ and let $1+\pi_t = p_{t+1}/p_t$.

4. (2.5 points) Define a competitive monetary equilibrium.

5. (15 points) Solve the agent's optimization problem assuming that $\pi_t = z$ (a necessary condition for stationary equilibrium).

   (a) Given the form of the utility function explain which constraints may bind or will definitely not bind in the agent's optimization problem.

   (b) What is the equilibrium allocation of consumption and human capital? How do human capital, consumption, and real money balances vary with inflation? Hint: Use the information from part (5a) to simplify the problem. You might want to start by solving the unconstrained problem and then see which of the constaints actually bind.

Part 3: Comparing Planner Solution and Competitive Equilibrium

6. (5 points) Compare the allocation of consumption and human capital in the planners problem versus the competitive equilibrium. Why might they differ or not differ?

Question 2.

Consider employer provided on-the-job training. Time is discrete. The economy is stationary. Horizons are infinite. Both workers and firms discount according to discount factor $0 < \beta < 1$.

All firms in the economy are identical. A worker is characterized by his/her general human capital level $h \in [0, 1]$. When employed by a firm, a worker produces per period revenue $f(h)$ which on the support of h is assumed bounded, strictly increasing, concave, and at least twice continuously differentiable. In addition, $f(0) = 0$ and $\lim_{h \to 0} f'(h) = \infty$. All workers are born with $h = 0$. A worker survives a given period with probability $1 - \delta \in (0, 1)$. In a period and conditional on survival, with probability $\lambda \in (0, 1)$ a worker switches to another firm effective the beginning of the next period. Workers are always employed with some firm.

During a period, the firm can provide training, τ, to the worker. The law of motion for a worker's human capital is $h_{t+1} = (1 - \kappa)h_t + \tau_t$, where κ is the human capital depreciation rate. The training cost to the firm is $c(\tau)$ where $c(\cdot)$ is strictly increasing, strictly convex, at least twice continuously differentiable, and with $c(0) = c'(0) = 0$. Hence, a firm that is matched with an h type worker has per period profits $f(h) - w(h) - c(\tau)$, where $w(h)$ is the worker's wage. The firm chooses training to maximize the net present value of the forward stream of profits from the match. When a worker dies or the worker quits a match for another firm, the match is destroyed and it has zero value to the firm from then on.

Denote by $W(h)$ the worker's net present value of future wages for a given current match and current human capital h. Denote by $J(h)$ the firm's net present value of future profits from a match with an h type worker given optimally set training choice. Assume regardless of employer, wages are set so that $V(h) = \alpha[W(h) + J(h)] = \alpha S(h)$, where $S(h)$ is match surplus.

1. (10 points) Set up the recursive formulations for $W(h)$, $J(h)$, and $S(h)$.
2. (8 points) Argue that $S(h)$ is a contraction mapping.
3. (5 points) As an aside, can you find sufficient conditions on $f(h)$ and $c(\tau)$ for an upper bound on h instead of the imposed upper bound of 1?
4. (12 points) Prove that $S(h)$ is concave.
5. Consider a social planner whose objective it is to maximize the net present value of a worker's lifetime revenue production, $\sum_{t=0}^{\infty}[\beta(1 - \delta)]^t[f(h_t) - c(\tau_t)]$.
   (a) (12 points) How does the firm's chosen training level compare to that of the planner? Discuss your result.
   (b) (3 points) Are there special parameter cases where training is efficient, in the sense that the planner solution coincides with that of the firms?

Question 3.

1. **(30 points total)** Consider the following economy composed of two agents, A and B, who have deterministically alternating endowments which sum to one. That is, in even periods $t = 0, 2, 4, \dots$ we have $e_t^A = e$ and $e_t^B = 1 - e$ and in odd periods $t = 1, 3, 5, \dots$ we have $e_t^A = 1 - e$ and $e_t^B = e$ where $1/2 < e < 1$. Both agents are risk neutral, but have different discount factors. In particular, the preferences of agents A and B are respectively:

$$U^A = \sum_{t=0}^{\infty} \beta^t c_t^A, \; U^B = \sum_{t=0}^{\infty} \gamma^t c_t^B$$

where $0 < \gamma < \beta < 1$.

   (a) **(10 points)** First analyze Pareto optimal allocations. Consider the case of a planner who solves:

$$v(\theta) = \sup_{\{c_t\}} \left[\theta U^A + (1-\theta) U^B\right]$$

   subject to the feasibility constraint. Here $0 < \theta < 1$. Find $v(\theta)$ and the consumption sequences that attain $v(\theta)$.

   (b) **(10 points)** Now consider a decentralization in which there is a market at date zero for consumption claims, with p_t being the price of consumption at date t. For any θ, construct an equilibrium price sequence so that the equilibrium allocation is the Pareto optimal one. Is such a price sequence unique?

   (c) **(10 points)** Suppose that instead of alternating deterministically, that endowments are stochastic and i.i.d., so with probability p each period agent A gets endowment e but the aggregate endowment is still one. How does this alter your analysis?

2. **(20 points total)** Consider an unemployed worker searching for a job in continuous time. All jobs last forever and have a constant wage w. An unemployed worker consumes the benefit b and chooses search effort a which increases the Poisson arrival rate λ of a job offer linearly: $\lambda(a) = qa$ for some $q > 0$. Search effort incurs a disutility $v(a)$ which is increasing and convex. The agent's preferences are:

$$E_0 \int_0^{\infty} e^{-\rho t} [c_t - v(a_t)] dt$$

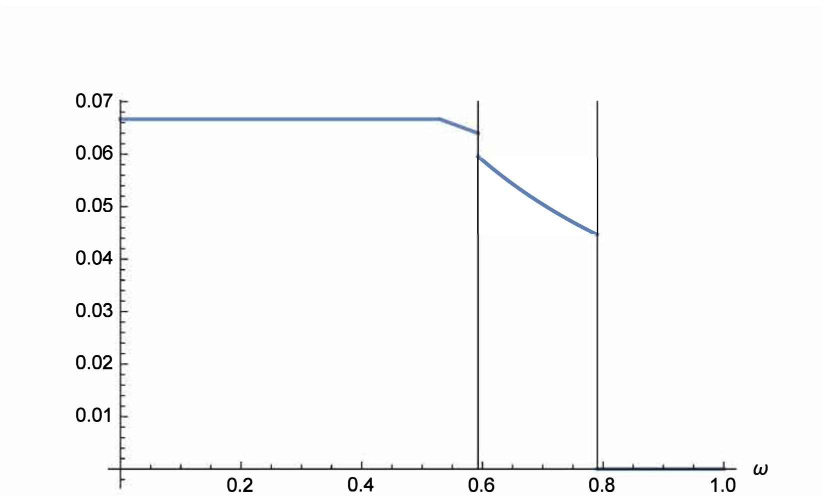
where $\rho > 0$, $c_t = b$ if unemployed and $c_t = w$ if employed, and a_t is zero if employed and is chosen optimally if unemployed.

(a) **(10 points)** Write down the (Hamilton-Jacobi) Bellman equations determining the values of an employed worker and an unemployed worker.

(b) **(10 points)** Find an expression for the optimal choice of search effort. Use this expression to describe the factors that affect the duration of an unemployment spell.

Question 4.

Part A. (30 points; 10 points for each part) The following plot is taken from our discussion of Bernanke and Gertler (1989). The plot was constructed by assuming that each successful investment project produces $\kappa_2 = 0.1$ units of capital; unsuccessful projects produce $\kappa_1 = 0$. The probability of a successful project is $\pi_2 = \frac{2}{3}$ (so $\pi_1 = \frac{1}{3}$); this explains why the capital produced by the most efficient entrepreneurs equals $\frac{2}{3} \cdot 0.1 \approx 0.067$. The horizontal axis gives entrepreneur type, $\omega \in [0, 1]$. The vertical axis plots the expected units of capital which are produced by each entrepreneur of type ω (for a given parameterization, productivity level, etc...).



i. There are two vertical lines (one just below ω =0.6, and a second just below ω=0.8). Call the locations of the vertical lines $\underline{\omega}$, $\bar{\omega}$. Why do the expected units of capital output begin to decline towards the right portion of the $[0, \underline{\omega}]$ interval? In this interval, how does the probability of auditing vary with ω?

ii. Why does the expected units of capital output decrease in ω within the $[\underline{\omega}, \bar{\omega}]$ interval? In this interval, how does the probability of auditing vary with ω?

iii. In your exam booklet, re-draw the picture that I have produced here. Then, draw a new set of lines, now depicting how the relationship between the expected units of capital and ω would look like if π_2 decreased from $\pi_2 = \frac{2}{3}$ to a number slightly smaller than $\frac{2}{3}$? If the thresholds $\underline{\omega}$ and $\bar{\omega}$ change, make sure to incorporate this in your sketch. If the thresholds do not change, state so explicitly. Label your figure clearly.

Part B. (20 points) Consider a simplified version of the model in Benhabib, Rogerson, and Wright (1991) in which there is no capital. The representative consumer has preferences over leisure, market-produced goods, and home (nonmarket) goods. The period utility function is

$$\frac{\sigma}{\sigma-1}\log\left[C_{mt}^{\frac{\sigma-1}{\sigma}}+C_{nt}^{\frac{\sigma-1}{\sigma}}\right]+\log\left(1-H_{mt}-H_{nt}\right).$$

In this equation, C_{mt} and C_{nt} refer to market and home good consumption, while H_{mt} and H_{nt} refer to time spent working in the market and at home. The production functions for market production and home production are:

$$C_{mt}=S_{mt}\cdot H_{mt} \text{ and } C_{nt}=S_{nt}\cdot H_{nt}.$$

In these equations, S_{mt} and S_{nt} refer to exogenous productivity shocks.

Using the terminology of Chari, Kehoe, and McGrattan (2007), compute the efficiency wedge, labor wedge, investment wedge, and government consumption wedge as a function of the two productivity shocks and of the parameters of the model.

Hint: Remember that in Chari, Kehoe, and McGrattan (2007), the wedges are inferred by observing *market* allocations.