Problem Set 9 Sarah Bass

20.1. $Y = -1 + 2X + 5(X-1)\mathbb{1}\{X \geq 1\} - 3(X-2)\mathbb{1}\{X \geq 2\} + e.$

For $X=3$,

$$Y = -1 + 2(3) + 5(2) - 3(1)$$
$$= -1 + 6 + 10 - 3$$
$$= 12$$

20.3. $$m_K(x) = \beta_0 + \beta_1 x + \beta_2 (x - \tau_1)\mathbb{1}\{x \geq \tau_1\} + \beta_3 (x - \tau_2)\mathbb{1}\{x \geq \tau_2\} + \beta_4 (x - \tau_3)\mathbb{1}\{x \geq \tau_3\}$$

$\beta_1 > 0$

$\beta_2 \in (-\beta_1, 0)$

$\beta_3 \in (\beta_2, 0)$

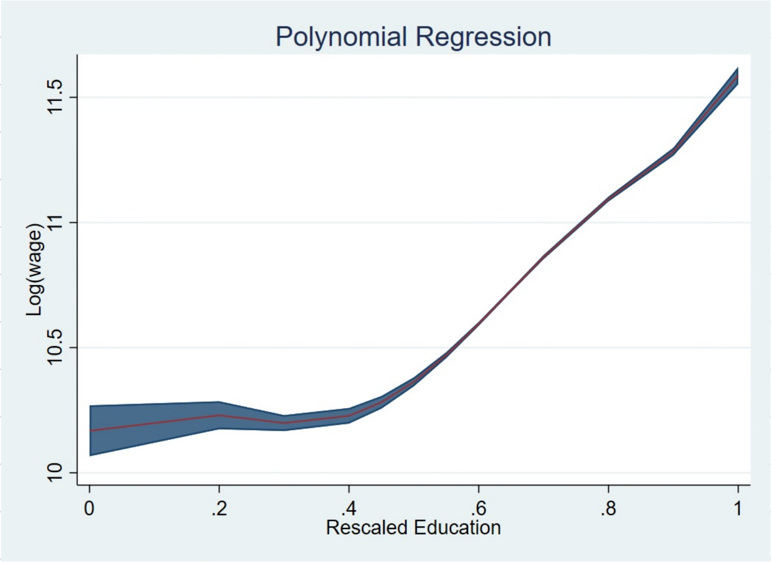
20.11.a.

```
. reg logwage education educ_2 educ_3 educ_4 educ_5 educ_6, r

Linear regression                               Number of obs     =     50,742
                                                F(6, 50735)       =    2032.64
                                                Prob > F          =     0.0000
                                                R-squared         =     0.2011
                                                Root MSE          =     .60538
```

logwage	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
education	1.125813	1.625832	0.69	0.489	-2.060835	4.312461
educ_2	-2.773794	13.81887	-0.20	0.841	-29.85892	24.31134
educ_3	-20.67584	46.28692	-0.45	0.655	-111.3987	70.04703
educ_4	91.36617	74.59538	1.22	0.221	-54.84157	237.5739
educ_5	-111.5427	58.07495	-1.92	0.055	-225.3703	2.2848
educ_6	43.91998	17.54971	2.50	0.012	9.522364	78.3176
_cons	10.16774	.0518213	196.21	0.000	10.06617	10.26931

20.11.6.

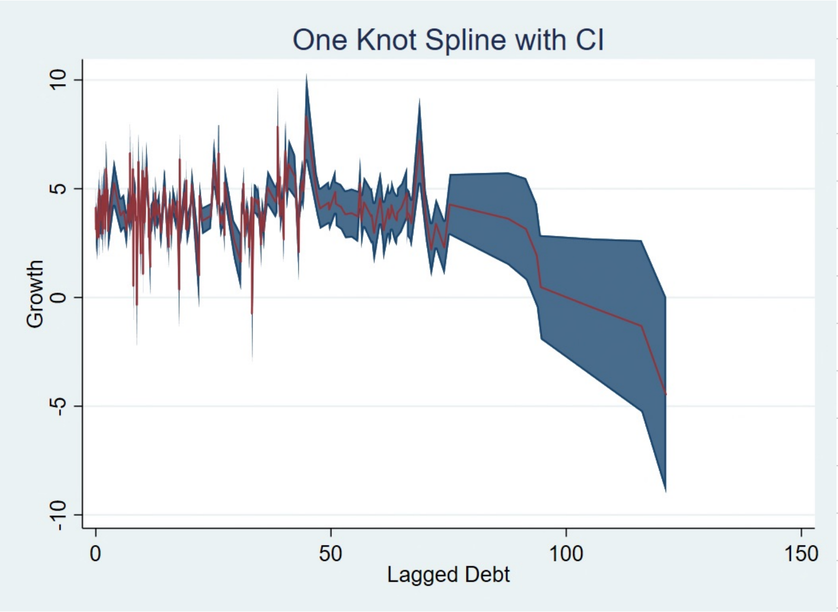


Polynomial Regression
Log(wage)
11.5
11
10.5
10
0
.2
.4
.6
.8
1
Rescaled Education

20.15.a.

Regression Estimates
Growth
10
5
0
-5
0
50
100
150
Lagged Debt
y0
y1
y2

20.15.b



20.15.c.

	AIC
0 knot	1249.774
1 knot	1248.711
2 knot	1248.688

Since the 2 knot spline has the smallest AIC, this is the preferred specification.

20.15.d. All of the models show a drop off in growth with high (90+) values of lagged debt, however this drop off is more pronounced in the models with splines.

21.1 The conditional ATE $\bar{\theta}'$ for $D = 1\{X \le c\}$ would be the negative of $\bar{\theta}$ for $D = 1\{X \ge c\}$.

$$\bar{\theta} = m(c+) - m(c-)$$
$$\bar{\theta}' = m(c-) - m(c+)$$

21.2 The treatment effects that can be identified are at c_1, c_2.

$$\bar{\theta}_1 = m(c_1+) - m(c_1-)$$
$$\bar{\theta}_2 = m(c_2-) - m(c_2+)$$

21.3 $Y = Y_0 1\{X < c\} + Y_1 1\{X \ge c\}$

$$E[Y|X=x] = E[Y_0 1\{X<c\} | X=x] + E[Y_1 1\{X \ge c\} | X=x]$$
$$m(x) = m_0(x) 1\{X<c\} + m_1(x) 1\{X \ge c\}$$

21.4 Consider a rectangular kernel with bandwidth $2h$ and $K(x-c/h) = 1\{|X-c| \le h\}$. Then the LL objective function is:

$$J = \sum_{i=1}^{n} (y_i - B_0 - B_1 x_i - B_2 (x_i - c) D_i - \theta D_i)^2 K\left(\frac{x-c}{h}\right)$$
$$= \sum_{i=1}^{n} (y_i - B_0 - B_1 x_i - B_2 (x_i - c) D_i - \theta D_i)^2 \, 1\{|X-c| \le h\}$$
$$= \sum_{|X-c| \le h} (y_i - B_0 - B_1 x_i - B_2 (x_i - c) D_i - \theta D_i)^2$$

Which is identical to the OLS estimation on the subsample.