

HANK Model

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Eggertsson and Krugman (QJE'2012) is perhaps the first highly cited NK model with heterogeneous agents. The main contribution of the paper is to introduce a model, in which financial frictions play a crucial role on the household side of the economy. This contrasts with the existed models by Kiyotaki and Moore (JPE'1997) and Bernanke, Gertler and Gilchrist (1999) that focused on the financial frictions in production and financial sectors.

Consider a deterministic two-period setting with two types of households – borrowers with initial debt D_0 and savers with savings D_0 – each getting half of the aggregate output Y_t and maximizing log utility $U_i = \log C_1^i + \log C_2^i$ subject to the borrowing constraint $D_1 \leq D$. We are interested in the effect of the deleveraging shock, i.e. a fall in D , which captures the tightening of the borrowing constraint in 2008. Assuming the borrowing constraint is binding for borrowers, their budget constraint implies

If they consumed less,
then would borrow less
→ budget constraint
is binding

$$P_1 C_1^b = \frac{1}{2} P_1 Y_1 + \frac{D}{1+i} - D_0,$$

$$P_2 C_2^b = \frac{1}{2} P_2 Y_2 - D.$$

$$\begin{aligned} \max \log C_1^b + \log C_2^b \\ \text{s.t. } P_1 C_1^b &= \frac{1}{2} P_1 Y_1 + \frac{D}{1+i} - D_0 \\ P_2 C_2^b &= \frac{1}{2} P_2 Y_2 - D \\ D_1 &\leq D \end{aligned} \quad (2)$$

The savers, on the other hand, are always on the Euler equation

can intertemporally change
consumption but stays
on the Euler.

$$\frac{P_1 C_1^s}{P_2 C_2^s} (1+i) = 1.$$

$$\begin{aligned} \max \log C_1^s + \log C_2^s \\ \text{s.t. } P_1 C_1^s &= \frac{1}{2} P_1 Y_1 - \frac{D}{1+i} + D_0 \\ P_2 C_2^s &= \frac{1}{2} P_2 Y_2 + D \\ -D_1 &\leq D \end{aligned} \quad (4)$$

Finally, the market clearing condition implies

Firms:

Given p_1, y_1 ,

$$C_1^s + C_1^b = Y_1, \quad C_2^s + C_2^b = Y_2.$$

$y_2 = y$, $p_2 = 1 \rightarrow$ long run at full potential

Substituting consumption of borrowers (1) and (2) into the market clearing conditions we can back out (nominal) consumption of savers in each period:

$$P_1 C_1^s = \frac{1}{2} P_1 Y_1 - \frac{D}{1+i} + D_0,$$

$$P_2 C_2^s = \frac{1}{2} P_2 Y_2 + D.$$

$$P_1 C_1^s + \left(\frac{1}{2} P_1 Y_1 + \frac{D}{1+i} - D_0 \right) = P_1 Y_1$$

$$P_1 C_1^s = \frac{1}{2} P_1 Y_1 - \frac{D}{1+i} + D_0$$

$$P_2 C_2^s + \left(\frac{1}{2} P_2 Y_2 - D \right) = P_2 Y_2$$

$$P_2 C_2^s = \frac{1}{2} P_2 Y_2 + D$$

$$\frac{\frac{1}{2} P_1 Y_1 - \frac{D}{1+i} + D_0}{\frac{1}{2} P_2 Y_2 + D} (1+i) = 1$$

Plug these expressions into the Euler equation (3) to obtain

$$\frac{(1+i)(\frac{1}{2} P_1 Y_1 + D_0) - D}{\frac{1}{2} P_2 Y_2 + D} = 1.$$

$$\begin{aligned} (1+i)(\frac{1}{2} P_1 Y_1 + D_0) - D &= \frac{1}{2} P_2 Y_2 + D \\ (1+i)(\frac{1}{2} P_1 Y_1 + D_0) &= \frac{1}{2} Y + 2D \\ \frac{1}{2} P_1 Y_1 + D_0 &= \frac{1}{2} Y + 2D / (1+i) \end{aligned}$$

We are going to assume that in the second period the economy is always in the long-run equilibrium with exogenous potential $Y_2 = Y$ and $P_2 = 1$. Substitute this in the previous condition and express the output in the first period:

$$P_1 Y_1 = \frac{Y + 4D}{1+i} - 2D_0. \quad (5)$$

Consider next different scenarios:

1. If prices are flexible and output is exogenous $Y_1 = Y$, equation (5) determines the price level or equivalently, inflation for a given monetary policy i :

$$P_1 = \frac{1 + 4D/Y}{1+i} - 2D_0/Y.$$

deflation
 $i \uparrow \rightarrow P_1 \downarrow$
 $D \downarrow \rightarrow P_1 \downarrow \rightarrow \pi_1 < 0, \pi_2 > 0$

Higher initial debt ($D_0 \uparrow$) and tighter borrowing constraint ($D \downarrow$) lead to deflation.

2. If prices are sticky P_1 , but the monetary policy is unconstrained, the planner can still achieve potential level of output by lowering sufficiently the interest rates and hence, compensating a fall in demand of borrowers due to the deleveraging shock with the a higher demand of savers:

$$1+i = \frac{Y + 4D}{Y + 2D_0}.$$

$D \downarrow \rightarrow Y \downarrow$
 $D \downarrow \rightarrow i \downarrow$

Higher initial debt and a tighter borrowing constraint require a larger decrease in interest rates. In particular, if the deleveraging shock is large $D < \frac{1}{2} D_0$, the interest rate becomes negative $i < 0$.

3. If prices are sticky P_1 and the ZLB binds $i = 0$, the monetary policy can no longer implement the first-best output and there will be a recession:

$$Y_1 = Y + 4D - 2D_0 < Y.$$

ZLB is binding

If you're at ZLB
and prices don't adjust,
you aren't at potential

Werning (2015) "as if" result provides an important benchmark to think about the transmission of monetary shocks in HANK models. To see the point, consider a TANK model with

1. a fraction γ of agents are hand-to-mouth, have no access to asset markets, and consume whatever they earn, $C_{Ht} = Y_t$
2. a fraction $1 - \gamma$ of households are unconstrained, can freely trade the government bonds

and are therefore, on the Euler equation

$$\beta(1 + i_t)\mathbb{E}_t \left(\frac{C_{Ut+1}}{C_{Ut}} \right)^{-\sigma} \frac{P_t}{P_{t+1}} = 1,$$

3. technology $Y_t = L_t$ and each agent is supplying an equal amount of labor, so that the output is equally shared between agents:

$$Y_t = \gamma C_{Ht} + (1 - \gamma)C_{Ut}.$$

Substitute $C_{Ht} = Y_t$ into the market clearing to get

$$Y_t = \gamma Y_t + (1 - \gamma)C_{Ut} \quad \Rightarrow \quad Y_t = C_{Ut},$$

which can be then combined with the Euler equation for unconstrained agents to obtain

$$\beta(1 + i_t)\mathbb{E}_t \left(\frac{Y_{t+1}}{Y_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} = 1.$$