Econ 703 Homework 6

Fall 2008, University of Wisconsin-Madison

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Due on Oct. 16, Thu. (in the class)

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1. Sundaram, #52, p. 72.

2. Consider two Euclidian spaces $X = \mathbb{R}^n$ and $Y = \mathbb{R}^m$. Let Z be a metric spaces. and let $f : X \times Y = \mathbb{R}^{n+m} \to Z$. We say that f is *continuous in each variable separately* if, for each x_0 in $X = \mathbb{R}^n$, the function $h : \mathbb{R}^m \to Z$ defined by $h(y) = f(x_0, y)$ is continuous and if for each y_0 in $Y = \mathbb{R}^m$ the function $g(x) = f(x, y_0)$ is continuous. Prove that if f is continuous, then f is continuous in each variable separately.

(Remark: whenever considering product spaces of two Euclidian spaces $\mathbb{R}^n$ and $\mathbb{R}^m$ we use the Euclidian metric on $\mathbb{R}^{n+m}$ to define open sets.)

3. Consider two Euclidian spaces $\mathbb{R}^n$ and $\mathbb{R}^m$. Let Y be a compact subset of $\mathbb{R}^m$. Show that $f : \mathbb{R}^n \to Y$ is continuous if and only if the graph of f, $G(f) = \{(x, f(x)) | x \in \mathbb{R}^n\}$, is a closed subset of $\mathbb{R}^n \times Y$.

(HINT: If $G(f)$ is closed, and V is a ball around $f(x_0)$, find a tube about $x_0 \times (Y \setminus V)$ not intersecting $G(f)$).

4. Let $f, g : [0, 1] \to \mathbb{R}$ be continuous functions, and suppose that $f(x) > g(x)$ for all $x \in [0, 1]$. Prove or disprove the following statement: there exists $A > 0$ such that $f(x) \geq g(x) + A$ for all $x \in [0, 1]$.

What if instead f and g were only left continuous?