

Game Theory Midterm - Sarah Bass

1)

$$N = \{1, 2, 3\}$$

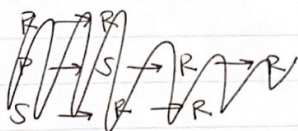
$$S_i = \{\text{rock, paper, scissors}\}$$

$$u_i(s_i, s_{-i}) = \begin{cases} 2 & \text{if win with } S \text{ or } P \\ -2 & \text{if lose with } S \text{ or } P \\ 0 & \text{if tie with } S \text{ or } P \\ 2+\alpha & \text{if win with } R \\ -2+\alpha & \text{if lose with } R \\ \alpha & \text{if tie with } R \end{cases}$$

	R	P	S
R	(α, α)	$(-2+\alpha, 2)$	$(2+\alpha, -2)$
P	$(2, -2+\alpha)$	$(0, 0)$	$(-2, 2)$
S	$(-2, 2+\alpha)$	$(2, -2)$	$(0, 0)$

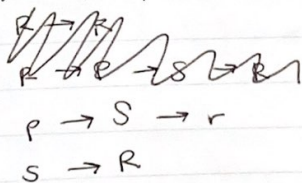
Note, this game is symmetric.

If $\alpha > 2$, there is a pure strategy nash equilibrium at ~~(R, R)~~ (R, r)



$r \rightarrow R$
 $p \rightarrow S \rightarrow r \rightarrow R \rightarrow r$
 $s \rightarrow R \rightarrow r \rightarrow R$

If $\alpha = 2$, there are 2 N.E.s at (R, r) and ~~$(P, r), (P, s), (R, s), (R, p), (S, p), (S, r), (R, r)$~~



$r \rightarrow R$
 $r \rightarrow P \rightarrow s \rightarrow R$

$p \rightarrow S \rightarrow r$
 $s \rightarrow R$

\otimes

If $\alpha < 2$, there are no N.E.

$r \rightarrow P \rightarrow S \rightarrow R \rightarrow p \rightarrow S$

2)
a)

$$N = \{1, 2\}$$

$$S_i = \{\text{chicken}, \text{dare}\}$$

$$u_i = \begin{cases} 1 & \text{if } S_i = \text{dare}, S_j = \text{chicken} \\ -1 & \text{if } S_i = \text{chicken}, S_j = \text{dare} \\ 0.6 & \text{if } S_i = \text{chicken}, S_j = \text{chicken} \\ -v_i & \text{if } S_i = \text{dare}, S_j = \text{dare} \end{cases}$$

	c	d
C	(0.6, 0.6)	(-1, 1)
D	(1, -1)	(-v_i, -v_j)

$v > 1$

$c \rightarrow D \rightarrow c$
 $d \rightarrow c \rightarrow d$

The 2 pure strategy N.E are (C, d) and (D, c).

Mixed strategies:

Given C, Player 2 indifferent b/w c, d if
 $0.6p = (1-p) \Rightarrow p = 0.625$
 Given D, player 2 indif. b/w c, d if
 $-p = -v_j(1-p) \Rightarrow$

$$0.6\sigma_c + (1-\sigma_c) = 1(\sigma_c) + (-v_j)(1-\sigma_c) \quad \text{uppercase c}$$

$$0.6\sigma_c + (1-\sigma_c) = 1(\sigma_c) + (-v_i)(1-\sigma_c) \quad \text{lowercase c}$$

$$\sigma_c = \frac{1-v_j}{0.6-v_j} \quad \sigma_c = \frac{1-v_i}{0.6-v_i}$$

$$\text{Mixed N.E at } \left(\frac{v_j+1}{0.6-v_j} \right)(c) + \left(1 - \frac{v_j+1}{0.6-v_j} \right)(d),$$

$$\left(\frac{-v_i+1}{0.6-v_i} \right)(c) + \left(1 - \frac{-v_i+1}{0.6-v_i} \right)(d)$$

As $v_i \uparrow$, the ~~the~~ player 2 less likely to play ~~chicken~~ dare.

2b)

*

$$U(s_i', s_i, v_i) = \int u_i P(v_i > v)$$

$$s_i(v_j) = \operatorname{argmax}_{s_i \in S_i} \int u_i(v_j) P(v_j > v).$$

$$\sigma_i(v_j) = \operatorname{argmax} \int \left[\left(\frac{1-v_j}{0.6-v_j} \right) c + \left(1 - \left(\frac{1-v_j}{0.6-v_j} \right) \right) d \right] \frac{F(v_j)}{F(v_j)}$$

3a)

$$N = \{ \text{Alice}, \text{Bob} \}$$

$$S_i = \{ R, S \}$$

$$u_i = \begin{cases} 4 & \text{if } s_i = S \\ 0 & \text{if } s_i = R, s_j = S \\ 0 & \text{if } s_i = R, s_j = R \end{cases}$$

	r	s
R	(0, 0)	(0, 4)
S	(4, 0)	(4, 4)

$r \rightarrow R \rightarrow r$ Pure N.E at (R, r) and (S, s)
 $s \rightarrow S \rightarrow s$

Mixed:

$$6\sigma_R + 0(1-\sigma_R) = 4\sigma_R + 4(1-\sigma_R)$$

$$6\sigma_R + 0(1-\sigma_R) = 4\sigma_R + 4(1-\sigma_R)$$

$$\Rightarrow \sigma_R = \frac{2}{3}$$

$$\sigma_R = \frac{2}{3}$$

N.E at $(\frac{2}{3}R + \frac{1}{3}S, \frac{2}{3}r + \frac{1}{3}s)$.

3b)

	r	s	$t=r \& s$	$r \rightarrow R \rightarrow r$
R	(0, 0)	(0, 4)	(0, 5)	$s \rightarrow S \rightarrow s$
S	(4, 0)	(4, 4)	(4, 3)	$t \rightarrow r \rightarrow R$
$t = R \& S$	(5, 0)	(3, 4)	(5, 5)	$t \rightarrow$

this is not stated in the problem, assumed.

pure N.E at $(R, r), (S, s)$

4) Given $v_1 < v_2$,

$$b_1 = \operatorname{argmax}_{b_1 \in [0,1]} \int \cancel{P(v_1 < v_2)} (v_1 - b_2) P(v_2 < v_1)$$

$$b_2 = \operatorname{argmax}_{b_2 \in [0,1]} \int (v_2 - b_1) P(v_1 < v_2)$$