Econ 703 Homework 5

Fall 2008, University of Wisconsin-Madison

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Due on Oct. 9, Thu. (in the class)

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1. Let $B \subset \mathbb{R}^2$ be as defined as follows: $B = \{(x, y) \in \mathbb{R}^2 : y = \sin \frac{1}{x},\ x > 0\} \cup \{(0, 0)\}$. Is B closed? Open? Bounded? Compact?

2. Let K be the union of the set $\{0\}$ and the set $\{1/n, n \in Z_{++}\}$. Prove that K is compact directly from the definition (i.e., without using the Heine_Borel Theorem).

3. Develop properties of compact sets. For example, is the union of two compact sets compact? The intersection? How about the union (intersection) of a family of compact sets?

4. Let (X, d) be a metric space. For each subset $A \subset X$ and each point $x \in X$, define the distance between A and x as

$$d(A, x) = \inf_{a \in A} d(a, x).$$

(a) Show that $x \in A$ implies $d(A, x) = 0$. (But not conversely.)

(b) Show that $d(A, x)$ is a continuous function of $x \in X$, when A is fixed.

(c) Show that $d(A, x) = 0$ if and only if $x \in A$ or x is a limit point of A.

(d) Show that the closure of A is the union of A and the set M of all points such that $d(A, x) = 0$: $A = A \cup M$.