UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MACROECONOMICS THEORY Preliminary Exam

August 1, 2017

9:00 am - 2:00 pm

INSTRUCTIONS

- **Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:**

  (1) your assigned number
  (2) the number of the question you are answering
  (3) the position of the page in the sequence of pages used to answer the questions

Example:	
MACRO THEORY	08/01/17
ASSIGNED # ____________	
Qu # __1__ (Page __2__ of __4__):	

- **Do not answer more than one question on the same page !**
  When you start a new question, start a new page.
- **DO NOT write your name anywhere on your answer sheets!**
  After the examination, the question sheets and answer sheets will be collected.
- **Please DO NOT WRITE on the question sheets.**
- **The number of points for each question is provided on the exam.**
- **Answer all questions.**
- **Do not continue to write answers onto the back of the page – write on one side only.**
- **Answers will be penalized for extraneous material; be concise.**
- **You are not allowed to use notes, books, calculators, or colleagues.**
- **Do NOT use colored pens or pencils.**
- **There are five pages in the exam, including this instruction page—please make sure you have all of them.**

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.

- All scratch paper, unused tablet paper, and exams are to be turned in after the exam. Your proctor will give you directions, listen to your proctor.

- Good luck!

Question 1. This question has two parts:

Consider the following two period overlapping generations model. Agents earn y when young and 0 when old. There is a fixed supply of housing $H^s = 1$. Agents utility function is given by

$$U\left(c_t^t, h_t, c_{t+1}^t\right) = \ln\left(c_t^t\right) + \alpha h_t + \beta c_{t+1}^t$$

where c_t^t is the period t consumption, h_t is the period t housing choice, and c_{t+1}^t is the period $t+1$ consumption of a person born in period $t = 1, 2, 3....$ The initial old hold the stock of housing stock and have preferences βc_1^0. Assume that $1 + \alpha > \beta y$.

A. (10 points) Write down and solve the planner's problem.

B. If p_t is the period t price of a house, solve for a competitive equilibrium with housing in the following parts:

i. (5 points) What is the optimization problem facing a young agent?

ii. (2.5 points) What are the market clearing conditions?

iii. (2.5 points) Define a competitive general equilibrium

iv. (10 points) Solve for an agent's optimal housing and consumption decision rules. How does housing depend on the current and future price of houses? Note: in question **B.vii** you will be asked to verify whether consumption and housing is non-negative, but in this part of the question you can ignore those constraints.

v. (7.5 points) Solve for the law of motion for house prices and graph it in (p_t, p_{t+1}) space. Consider only non-negative prices for which $p_t < y$.

vi. (5 points) Solve for a steady state house price level.

vii. (2.5 points) Check that the non-negativity constraints do not bind in a steady state.

viii. (5 points) Does the competitive equilibrium implement the planner's allocation in a steady state? Why or why not?

Question 2. This question has two parts:

A. (25 points) *Closed Form:* Consider a social planner who faces the problem

$$\max_{\{c_t,k_{t+1},l_t\}_{t=0}^{\infty}} E\sum_{t=0}^{\infty}\beta^t[\ln c_t + b\ln(1-l_t)],\ \ 0<\beta<1$$

subject to

$$c_t + k_{t+1} = A_t k_t^{\alpha} l_t^{1-\alpha},\ \ 0<\alpha<1$$

where A is a stochastic productivity parameter evolving via a first order autoregressive process:

$$\ln A_{t+1} = \rho\ln A_t + \epsilon_{t+1},\ \ 0<\rho<1$$

with $E_t\epsilon_{t+1}=0$ for all t.

i. Formulate the problem using Bellman's functional equation.

ii. Derive the Euler equations and the first order conditions.

iii. Derive a closed form solution for the value and policy function.

iv. Interpret the effects of an increase in ρ on the value and policy functions. Explain.

B. (25 points) *Annuity Markets:* Consider the following problem faced by a retiree. This retiree faces a constant probability of death p in each period. He begins retirement with assets a_0. In every period, he decides how much to consume and save. He receives the constant income l in every period (think of this as social security+pension benefits). This is constant and he receives this until death. Further, assume that annuity markets are incomplete. In other words, insurance against the possibility of living too long is unavailable and all this individual can do is save through one-period risk-free bonds. To make things specific, assume that the momentary utility function is given by $U(c)$ with discount rate β and preferences are time additively separable.

i. Formulate the individual's dynamic programming problem

ii. Write down the first-order and envelope conditions

iii. Rewrite the problem under the assumption that markets are complete - i.e. the individual can borrow and lend freely and that annuity markets are available. Describe the time paths of consumption and asset allocation in both cases - complete and incomplete markets.

iv. In the data, consumption for retirees tends to be flat until age 75 after which it declines. Is there any hope for this model to match this pattern?

Question 3. This question has two parts:

A. (30 total points) Consider the following version of a Lucas asset pricing model, in which an agent wants to minimize fluctuations in consumption around a mean (which we normalize to zero). In other words, a representative agent has preferences:

$$-\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t c_t^2$$

over the single nonstorable consumption good c_t ("fruit"). The endowment process x_t is also mean zero and follows a Gaussian AR process:

$$x_{t+1} = \rho x_t + \varepsilon_{t+1}$$

where $0 < \rho < 1$ and ε_{t+1} is an i.i.d. $N(0, \sigma^2)$ random variable.

i. (10 points) Define a recursive competitive equilibrium with a market in claims to the endowment process ("trees"), with pricing function $p(x)$.

ii. (10 points) Characterize the recursive competitive equilibrium and provide an expression for the pricing function $p(x)$ as explicitly as you can.

iii. (10 points) What is the risk-free one period interest rate in this economy? Interpret your answer.

B. (20 total points) Consider the following variations on the basic (McCall) sequential search model. In each case, suppose that workers are risk-neutral, unemployed workers get constant payments z, jobs last forever (unless specified), and search always yields at least one job offer.

i. (10 points) Suppose that each period an unemployed worker gets two job offers, which are each drawn in an i.i.d. manner from the distribution $F(w)$. Find the worker's Bellman equation and characterize his optimal decision rule. Show that the worker's reservation wage is higher than it would have been if he only received one offer each period.

ii. (10 points) Suppose that we allowed workers to quit, but they would not otherwise separate from firms. That is, at any date an employed worker would have the option to quit his job, become unemployed, and search for a new job. In addition, suppose that wages are no longer known with certainty when a worker accepts a job. That is, unemployed workers receive offers of an expected wage w drawn from $F(w)$. But once employed, the jobs pay $w + \Delta$ with probability $1/2$ and $w - \Delta$ with probability $1/2$ for $\Delta > 0$. The uncertainty is all resolved once the job has begun, that is wages are constant over time but uncertain at the time of acceptance. Find the Bellman equations for employed and unemployed workers and characterize their optimal decision rules. Would employed workers ever quit?

Question 4. This question has four parts:

A. (14 points) In our discussion of Burnside and Eichenbaum (1996), we derived the following equation:

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[\frac{1}{C_{t+1}} \left[(1-\alpha) \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta U_{t+1}^{\phi} \right] \right] \tag{1}$$

Throughout this problem, use X^{ss} to refer to the steady-state version of any endogenous variable, X. And, define $\hat{x}_t = \log\left(\frac{X_t}{X^{ss}}\right)$.

i. Assume that there is no underlying trend in productivity, and that $U^{ss} = 1$. Find an expression for the steady-state capital to output ratio, $\frac{\mathrm{K}^{ss}}{\mathrm{Y}^{ss}}$.

ii. Find the log-linear approximation to Equation 1.

B. (10 points) In the model of Bernanke and Gertler (1989), an optimal contract solves the following constrained maximization problem. In what appears below, π_i is the probability that the output of the project is κ_i units of capital; c_2 is the consumption of the entrepreneur if she reports the good state; c_1 is the consumption of the entrepreneur if she reports the bad state and is not audited; c_a is the consumption of the entrepreneur if she reports the bad state and is audited; and p is the auditing probability:

$$\max_{p, c_1, c_2, c_a} \pi_1 \left(p c_a + (1-p) c_1\right) + \pi_2 c_2$$

subject to

$$r\left(x - S^e\right) \leq \pi_1 \left[\hat{q}\kappa_1 - p\left(c_a + \hat{q}\gamma\right) - (1-p) c_1\right] + \pi_2 \left[\hat{q}\kappa_2 - c_2\right] \tag{2}$$

$$c_2 \geq (1-p)\, \hat{q}\left(\kappa_2 - \kappa_1\right) + c_1 \tag{3}$$

$$c_1 \geq 0;\ c_a \geq 0;\ p \in [0,1]$$

i. Describe, in your own words, what the constraints given by Equations 2 and 3 represent.

ii. Explain why the constraint given by Equation 2 will always bind.

C. (16 points) Consider the problem of an individual firm, as described by the following value function

$$V(A,k) = \max_{k'} A^{1-\alpha}\left(k'\right)^{\alpha} - \left(k' - k\right) - f \cdot \mathbf{1}_{k' \neq k} \cdot k + \mathbf{1}_{k' < k}(1-\lambda)\left(k' - k\right)$$
$$- \frac{\phi k}{2}\left(\frac{k'}{k} - 1\right)^2 + R^{-1} \cdot \int V\left(A', k'\right) dF\left(\frac{A'}{A}\right)$$

Here, A represents the firm's productivity, and k represents the capital stock with which the firm enters the period.. Furthermore, $\alpha \in (0,1)$, $f \geq 0$, $\lambda \in (0,1]$, $\phi \geq 0$, and $R^{-1} \in (0,1)$. In the rest of the problem, use "q" to refer to Tobin's marginal q.

i. Suppose $f = 0$, $\lambda = 1$, $\phi > 0$. Describe the relationship between q and investment rates (measured as $\frac{k'-k}{k}$). Is the relationship increasing, decreasing, or neither?

ii. Suppose $f = 0$, $\lambda < 1$, $\phi = 0$. Describe the relationship between q and investment rates. Is the relationship increasing, decreasing, or neither?

iii. Suppose $f > 0$, $\lambda = 1$, $\phi = 0$. Describe the relationship between q and investment rates. Is the relationship increasing, decreasing, or neither?

iv. Suppose $f = 0$, $\lambda = 1$, $\phi = 0$. Describe the relationship between q and investment rates. Is the relationship increasing, decreasing, or neither?

D. (10 points) In a few sentences, describe Hansen (1985)'s indivisible labor model. Along which dimension does this variant of the RBC model help in "fitting the data"?