Econ 761 Problem Set 2

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Question 1

Part (a)

First note, $Q = \frac{a_0 - P - \nu}{a_1}$. Solving for price elasticity of demand, we have:

$$\begin{aligned}
\epsilon &= -\frac{P\partial Q}{Q\partial P} \\
&= \frac{P}{Qa_1} \\
&= \frac{a_0 - a_1 Q + \nu}{Qa_1} \\
&= \frac{a_0 + \nu}{Qa_1} - 1
\end{aligned}$$

Using this equation for elasticity, we can see how elasticity of demand changes with Q and ν.

$$\begin{aligned}
\Rightarrow \frac{\partial \epsilon}{\partial Q} &= -\frac{\nu + a_0}{Q^2 a_1} \leq 0 \\
\Rightarrow \frac{\partial \epsilon}{\partial \nu} &= \frac{1}{a_1 Q} \geq 0
\end{aligned}$$

Thus, as Q increases, elasticity of demand decreases; and as ν increases, the elasticity of demand increases.

Part (b)

Total costs are given by $c = F + (b_0 + \nu)Q$. The firm takes the quantities of the other firms as given and maximizes profit:

$$\begin{aligned}
&\max_q [a_0 - a_1(q + Q_{-i})]q - [F + (b_0 + \eta)q] \\
&\Rightarrow a_0 - 2a_1 q - a_1 Q_{-i} - b_0 - \eta = 0
\end{aligned}$$

*I have discussed this problem set with Emily Case, Michael Nattinger, and Andrew Smith.

Applying symmetry,

$$\begin{aligned}
Q &= Nq \\
\Rightarrow q &= \frac{a_0 + \nu - b_0 - \nu}{a_1(N+1)} \\
\Rightarrow Q &= N\frac{a_0 + \nu - b_0 - \nu}{a_1(N+1)} \\
\Rightarrow P &= \frac{a_0 + \nu + N(b_0 + \eta)}{N+1}
\end{aligned}$$

Part (c)

Individual firm profits are:

$$\begin{aligned}
Pq - C &= \frac{a_0 + \nu + N(b_0 + \eta)}{N+1}\frac{a_0 + \nu - b_0 - \nu}{a_1(N+1)} - F - \frac{(b_0 + \eta)(a_0 + \nu - b_0 - \eta)}{a_1(N+1)} \\
&= \frac{1}{a_1}\left[\frac{a_0 + \nu - b_0 - \eta}{N+1}\right]^2 - F
\end{aligned}$$

Firms enter until profits are zero:

$$\begin{aligned}
0 &= \frac{1}{a_1}\left[\frac{a_0 + \nu - b_0 - \eta}{N+1}\right]^2 - F \\
\Rightarrow N &= \frac{1}{\sqrt{Fa_1}}(a_0 + \nu - b_0 - \eta) - 1
\end{aligned}$$

Part (d)

$$\begin{aligned}
L_I = \frac{P - mc}{P} &= \frac{\frac{a_0 + \nu + N(b_0 + \eta)}{N+1} - (b_0 + \eta)}{\frac{a_0 + \nu + N(b_0 + \eta)}{N+1}} \\
&= \frac{a_0 + \nu - b_0 - \eta}{a_0 + \nu + N(b_0 + \eta)} \\
&= \frac{a_0 + \nu - b_0 - \eta}{a_0 + \nu + \left(\frac{1}{\sqrt{Fa_1}}(a_0 + \nu - b_0 - \eta) - 1\right)(b_0 + \eta)} \\
&= \frac{\sqrt{Fa_1}}{\sqrt{Fa_1} + b_0 + \eta}.
\end{aligned}$$

Firms are identical so the Herfindahl index is $H = \frac{1}{N} = \frac{\sqrt{Fa_1}}{a_0 + \nu - b_0 - \eta - \sqrt{Fa_1}}$.

$$\begin{aligned}
\epsilon &= \frac{a_0 + \nu - N\left(\frac{a_0 + \nu - b_0 - \eta}{N+1}\right)}{N\left(\frac{a_0 + \nu - b_0 - \eta}{N+1}\right)} \\
&= \frac{b_0 + \eta + \sqrt{Fa_1}}{a_0 + \nu - b_0 - \eta - \sqrt{Fa_1}}.
\end{aligned}$$

Part (e)

$$\begin{aligned}
\frac{\partial \epsilon}{\partial F} &= \frac{\sqrt{a_1}(a_0+\nu)}{2\sqrt{F}(a_0+\nu-b_0-\eta-\sqrt{Fa_1})^2} \geq 0\\
\frac{\partial \epsilon}{\partial \nu} &= -\frac{b_0+\eta+\sqrt{Fa_1}}{(a_0+\nu-b_0-\eta-\sqrt{Fa_1})^2} \leq 0\\
\frac{\partial \epsilon}{\partial \eta} &= \frac{a_0+\nu}{(a_0+\nu-b_0-\eta-\sqrt{Fa_1})^2} \geq 0
\end{aligned}$$

$$\begin{aligned}
\log(L_I) &= (1/2)\log(Fa_1) - \log(\sqrt{Fa_1}+b_0+\eta),\\
\log(H) &= (1/2)\log(Fa_1) - \log(a_0+\nu-b_0-\eta-\sqrt{Fa_1})
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \log(L_I)}{\partial F} &= \frac{1}{2F} - \frac{\sqrt{a_1}}{(\sqrt{Fa_1}+b_0+\eta)2\sqrt{F}}\\
\frac{\partial \log(H)}{\partial F} &= \frac{1}{2F} + \frac{\sqrt{a_1}}{(-\sqrt{Fa_1}+a_0+\nu-b_0-\eta)2\sqrt{F}}\\
&\neq \frac{\partial \log(L_I)}{\partial F}\\
\frac{\partial \log(L_I)}{\partial \nu} &= 0\\
\frac{\partial \log(H)}{\partial \nu} &= -\frac{1}{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}\\
&\neq \frac{\partial \log(L_I)}{\partial \nu}\\
\frac{\partial \log(L_I)}{\partial \eta} &= -\frac{1}{\sqrt{Fa_1}+b_0+\eta}\\
\frac{\partial \log(H)}{\partial \eta} &= \frac{1}{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}\\
&\neq \frac{\partial \log(L_I)}{\partial \eta}
\end{aligned}$$

As we can see above, $\log(L_I), \log(H)$ do not change at the same rate in responses to F, ν, η.

Part (f)

Colluding firms solve:

$$\max_Q (a_0 - a_1 Q + \nu)Q - (b_0+\eta)Q - F$$

Taking first order conditions, we have:

$$\begin{aligned}
a_0 - 2a_1 Q + \nu - b_0 - \eta &= 0 \\
\Rightarrow Q &= \frac{a_0 + \nu - b_0 - \eta}{2a_1} \\
\Rightarrow P &= (1/2)(a_0 + \nu + b_0 + \eta) \\
\Rightarrow \pi &= \frac{(a_0 + \nu - b_0 - \eta)^2}{4Na_1} - F
\end{aligned}$$

Firms enter until profits are zero, which implies:

$$\begin{aligned}
N &= \frac{(a_0 + \nu - b_0 - \eta)^2}{4Fa_1} \\
L_I &= \frac{P - mc}{P} \\
&= \frac{a_0 + \nu - b_0 - \eta}{a_0 + \nu + b_0 + \eta} \\
H &= \frac{1}{N} \\
&= \frac{4Fa_1}{(a_0 + \nu - b_0 - \eta)^2} \\
\epsilon &= -\frac{P\partial Q}{Q\partial P} \\
&= \frac{a_0 + \nu + b_0 + \eta}{a_0 + \nu - b_0 - \eta}
\end{aligned}$$

Part (g)

We now have

$$\begin{aligned}
Q &= \exp((1/c_1)(c_0 - \log(P) + \xi)) \\
\epsilon &= (P/Q)(\exp((1/c_1)(c_0 - \log(P) + \xi))) = (1/Q)\exp((1/c_1)(c_0 - \log(P) + \xi))
\end{aligned}$$

Taking partial derivatives, we have:

$$\begin{aligned}
\frac{\partial \epsilon}{\partial Q} &= -\frac{1}{Q^2}\exp((1/c_1)(c_0 - \log(P) + \xi)) \leq 0 \\
\frac{\partial \epsilon}{\partial \nu} &= (1/Q)\exp((1/c_1)(c_0 - \log(P) + \chi)) \geq 0
\end{aligned}$$

Under the new inverse supply curve, the firm maximizes profit:

$$\begin{aligned}
&\max_q \exp(c_0 - c_1 \log(q + Q_{-i}) + \xi)q - (b_0 + \eta)q - F \\
\Rightarrow \exp(c_0 - c_1 \log(q + Q_{-i}) + \xi) - &\exp(c_0 - c_1 \log(q + Q_{-i}) + \xi)\frac{qc_1}{Q} - b_0 - \eta = 0
\end{aligned}$$

Applying symmetry across firms:

$$
\begin{aligned}
\exp(c_0 - c_1 \log(Nq) + \xi) &= \frac{N}{N - c_1}(b_0 + \eta) \\
\Rightarrow q &= \frac{1}{N} \exp((1/c_1)(c_0 + \xi - \log((N/(N - c_1)(b_0 + \eta))))) \\
\Rightarrow Q &= \exp((1/c_1)(c_0 + \xi - \log((N/(N - c_1)(b_0 + \eta))))) \\
\Rightarrow P &= \exp(c_0 - c_1 \log(\exp((1/c_1)(c_0 + \xi - \log((N/(N - c_1)(b_0 + \eta)))))) + \xi) \\
&= \frac{N}{N - c_1}(b_0 + \eta).
\end{aligned}
$$

Note, this implies:

$$
\begin{aligned}
L_I &= \frac{\frac{N}{N-c_1}(b_0 + \eta) - (b_0 + \eta)}{\frac{N}{N-c_1}(b_0 + \eta)} = \frac{c_1}{N} \\
H &= \frac{1}{N} \\
\epsilon &= (1/Q) \exp((1/c_1)(c_0 - \log(P) + \xi)) = \frac{1}{c_1}
\end{aligned}
$$

Since N is exogenously determined, all of the partial derivatives are zero:

$$
\frac{\partial \epsilon}{\partial F} = \frac{\partial \epsilon}{\partial \nu} = \frac{\partial \epsilon}{\partial \eta} = \frac{\partial \log(L_I)}{\partial F} = \frac{\partial \log(L_I)}{\partial \nu} = \frac{\partial \log(L_I)}{\partial \eta} = \frac{\partial \log(H)}{\partial F} = \frac{\partial \log(H)}{\partial \nu} = \frac{\partial \log(H)}{\partial \eta} = 0
$$

Therefore, $\log(L_I), \log(H)$ change at the same rate in response to changes in $F, \nu, \eta : 0$.

Question 2

I solved question 2 by adapting the Stata code provided. My results are as follows:

Figure 1: Regression tables and F test results for price given by equations 3 and 1.

Sample	β	SE	p	N
Equation (3)				
No Collusion	1.002	0.002	0.346	583
Collusion	-0.005	0.002	0.000	417
Pooled	0.840	0.035	0.000	1000
Equation (1)				
No Collusion	0.542	0.004	0.000	583
Collusion	-0.005	0.002	0.000	417
Pooled	0.436	0.017	0.000	1000

In our regression using equation 3, we can see that the results vary across groups. For cities with no collusion, the β is very close to 1, and we can't reject the F test with the null hypothesis that the coefficient is exactly 1. With collusion, the β is not statistically different from 0, and we can reject the null hypothesis that the coefficient is 1. In the pooled case, the β is in the middle, and we can still reject the null hypothesis that the coefficient is 1.

In the regression using equation 1, we can see that the collusion case is still close to 0 and the pooled case is in between the collusion and no collusion cases. For both the collusion and pooled cases we still reject the null hypothesis that the coefficient is 1. However, using this equation drastically alters the no collusion case. Now, the coefficient in the no collusion case is well below 1, and we can reject the null hypothesis that the coefficient is 1.

Under both equations, the OLS coefficients are higher without collusion than with. If we want to show that a subset of markets features collusion then we can run these regressions on those markets and the magnitude of the coefficient can be indicative of likely presence of collusion. I would hesitate to extrapolate this fact to more general model.

Question 3

The results from the simulated regressions are as follows:

Figure 2: Regression results with varied ν and η

	$\nu \sim U[-1,1]$	$\eta \sim U[-1,1]$
cons	-0.652***	-2.23***
	(1.5e-08)	(0.0121)
log(H)	-8.57e-17	-1.51***
	(1.37e-08)	(0.011)

The results vary greatly and in line with theory. Our theory predicts that $H = \frac{\sqrt{Fa_1}}{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}$ and $L_I = \frac{\sqrt{Fa_1}}{\sqrt{Fa_1}+b_0+\eta}$. The only differences across markets are in ν in the first case, and in η in the second case.

In the first case, as ν varies, H is affected but L_I is unaffected as it is independent of ν. Therefore, we expect no statistically significant relationship between H and L_I.

In the second case, H is increasing in η and L_I is decreasing in η, so we expect a negative sign on the regression coefficient.

The elasticities are increasing in η and decreasing in ν, so we expect a positive correlation between ϵ and $\log(H)$ in both experiments, and negative correlation with L_I only in the experiment where η varies and ν is fixed.