Economics 703 : Final Exam

Raymond Deneckere

Fall 2015

Please be very explicit in your answers. Carefully state the appropriate definitions and theorems and argue how they apply. Also, make sure that every step in your argument follows logically and directly from the previous step.

Each question is worth 33 points, with one point given away for free.

1. Let T be some positive integer. Consider the following problem:

$$\text{Maximize } \sum_{t=0}^{T} \beta^t u(c_t)$$

$$\text{subject to } c_1 + x_1 \leq x$$

$$c_t + x_t \leq f(x_{t-1}), \ t = 1, ..., T$$

$$c_t, \ x_t \geq 0, \ t = 0, ..., T,$$

where $x \in \mathbb{R}_{++}$, and $u : \mathbb{R}_+ \to \mathbb{R}$ and $f : \mathbb{R}_+ \to \mathbb{R}_+$ are increasing functions. Derive the Kuhn-Tucker first-order conditions for this problem, and explain under what circumstances these conditions are necessary and sufficient.

2. Let $D = \{x \in \mathbb{R} \mid 0 \leq x \leq 1\}$, and let $f : D \to \mathbb{R}$ be given by the rule $f(x) = -x^2 - x$. Prove or disprove each of the following statements.

   (a) f is a concave function on D

   (b) f is a quasiconvex function on D

   (c) f is a qusiconvexconvex function on D

3. Solve the following problem in $\mathbb{R}^3$. Find the box of given surface area that maximizes volume, by solving the first order conditions associated with the Lagrangean.