Proofs

There are three big kinds of proofs: direct, contradiction, and induction.

1. Contradiction

A proof by contradiction entails supposing the negation of what we want to show. Then if we can find that this leads to a contradiction, the original proposition must be true.

Example: $\sqrt{2}$ is irrational.

Proof: Suppose, by way of contradiction, that $\sqrt{2}$ is rational. Then, by definition, we can find integers p and q such that $\sqrt{2} = \frac{p}{q}$ and that this fraction is fully reduced. Proceeding, $2 = \frac{p^2}{q^2}$, implying p^2 is even. Also, p must be even, so let $p = 2k$ for some integer k. For the fraction to have been fully reduced, this requires q odd. We have

$$p^2 = 4k^2 = 2q^2.$$

Note then

$$q^2 = 2k^2,$$

so q is also even. We have shown that q is both odd an even. This is a contradiction, and so we are done.

2. Induction

From Wikipedia, "The simplest and most common form of mathematical induction infers that a statement involving a natural number n holds for all values of n. The proof consists of two steps:

The basis (base case): prove that the statement holds for the first natural number n. Usually, n = 0 or n = 1.

The inductive step: prove that, if the statement holds for some natural number n, then the statement holds for n + 1."

Example: This is a version of the Well Ordering Principle. For any $n \in \mathbb{N}$, every subset of $\{1, 2, \ldots, n\}$ has a largest ($\geq$) element.
Proof: *to be continued*