Econ 710B Final Exam Sarah Bass

1. $Y = X'B + e$

$E[ze] = 0$

$E[e^2 | z] = \exp(\gamma' z)$

I will estimate B using GMM.

For B, $E[g_i(B)] = 0$ where

$g_i(B) = z(Y - X'B)$

The sample moment is

$\bar{g}_n(B) = \frac{1}{n} \sum_{i=1}^{n} z_i(Y_i - X_i'B)$

The criterion is

$J(B) = n\,\bar{g}_n(B)'\,W\,\bar{g}_n(B)$ for some weight matrix $W > 0$.

Then $\hat{B} = \underset{B}{\operatorname{argmin}}\, J(B)$.

Let $W = \Omega^{-1}$ where

$\Omega = E[g_i(B)\,g_i(B)']$ $E[z^2 e^2]$

$= E[zz' \exp(\gamma' z)]$ $E[z^2 E[e^2 | z]]$

$\bar{\Omega} = \frac{1}{n} \sum_{i=1}^{n} z_i z_i' \exp(\gamma' z_i)$. $E[z^2 \exp(\gamma' z)]$

Let $\hat{\gamma} = \underset{\gamma}{\operatorname{argmin}}\, \bar{\Omega}$

2. $Y_{it} = \theta D_{it} + u_i + \delta_t + \varepsilon_{it}$

$$E[\varepsilon_{it} \mid z_{i1}, z_{i2}] = 0$$

$$E[\varepsilon \mid z] = 0$$

$$E[z\varepsilon] = E[z\,E[\varepsilon \mid z]] = 0$$

Then:

$$E[z(Y - \theta D_{it} - u_i - \delta_t)] = 0$$

Converting to a sample moment:

$$\frac{1}{n}\sum zY_i - z\theta D_{it} - zu_i - \delta_t = 0$$

$$\frac{1}{n}\sum z\theta D_{it} = \frac{1}{n}\sum z_i Y_i - zu_i - z\delta_t$$

$$\theta = \left(\frac{1}{n}\sum z_i D_{it}'\right)^{-1}\left(\frac{1}{n} z_i (Y_{it} - u_i - \delta_t)\right)$$

Where $u_i = Y_{i1}$, first time period Y_i

$\delta_t = Y_{12} - Y_{11}$, average ΔY over time for control group $(i=1)$

$$\theta = (Y_{22} - Y_{21}) - (Y_{12} - Y_{11})$$

I have assumed $i=2$ (1 = control group, 2 = treatment).

$$\boxed{\begin{matrix} a & b \\ c & d \end{matrix}}$$

$b - a$

$d - c$

$d - c - b + a$

$a + d - c - (d - c - b + a)$

$= b.$

3. $Y = m(x) + e$

$E[e|x] = 0$

$D(x) = m'(x) = d/dx\, m(x)$

a. Consider a polynomial series regression of order k:

$Y = \hat{\beta}_0 + \hat{\beta}_1 x + \hat{\beta}_2 x^2 + \ldots + \hat{\beta}_k x^k + e$

$\hat{\beta} = \left(\frac{1}{n}\sum_{i=1}^{n} z_i z_i'\right)^{-1} \frac{1}{n}\sum_{i=1}^{n} z_i Y_i$ estimate via OLS

where $z_i = \langle x_i, x_i^2, \ldots, x_i^k \rangle$

Then

$\hat{D}(x) = \hat{\beta}_1 + 2\hat{\beta}_2 x + 3\hat{\beta}_3 x^2 + \ldots + k\hat{\beta}_k x^{k-1}$

$= \sum_{i=0}^{k} i\hat{\beta}_i x^{i-1}$

b. By asymptotic theory, assume $\hat{D}(x)$ is differentiable w.r.t β.

$n(\hat{D}(x) - D(x)) \to N(0, \hat{V}_D)$ where

$\hat{V}_D = d/d\beta\, D(x)'\, V\, d/d\beta\, D(x)$ and

V is the asymptotic covariance matrix.

Then our $SE = \sqrt{\frac{1}{n}\hat{V}_D}$

c. Our 95% CI is:

$\hat{D}(x) \pm 1.96 \cdot SE$

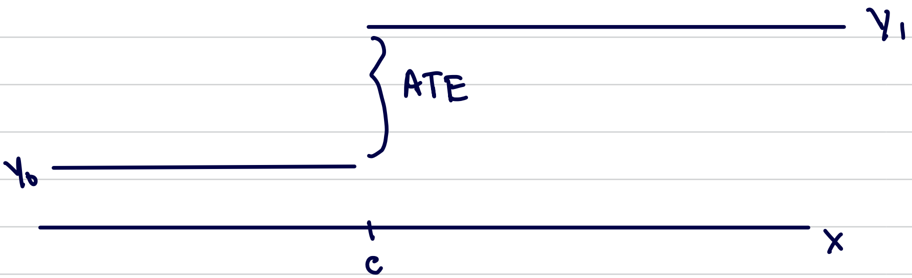
Other CIs can be constructed by using a z-score other than 1.96.

4. $Y = Y_0$ if $D = 0$

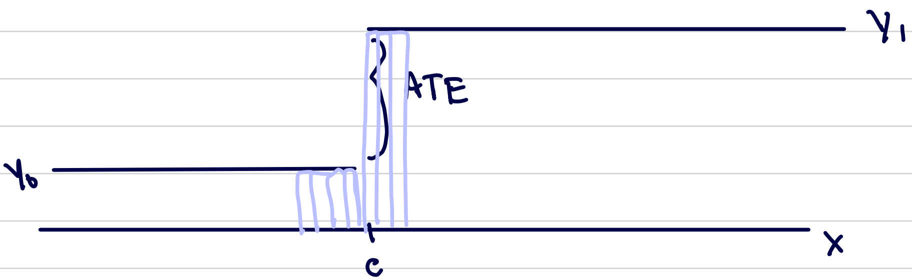
$Y = Y_1$ if $D = 1$

$D = 1\{X \geq c\}$

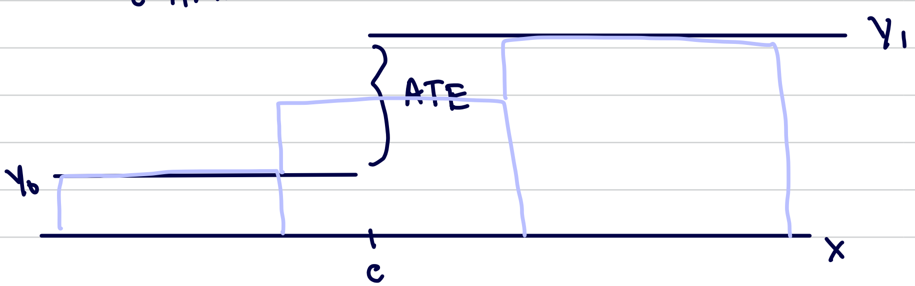
$ATE = E[Y_1 \mid X = c] - E[Y_0 \mid X = c]$



a. As $h \to \infty$, $\hat{\theta} \to \theta$.



b. As $h \to 0$, $\hat{\theta} \to 0$. $E[Y \mid X \approx c] = \frac{1}{2}(Y_1 + Y_0)$. So $E[Y_1 \mid X = c] - E[Y_0 \mid X = c] = 0$.



5. $\log(y^*) = x'\beta + e$ $\qquad y^* = e^{x'\beta + e}$

$e | X \sim N(0, \sigma^2)$ $\qquad = e^{x'\beta} e^{e}$

$$Y = \begin{cases} 1 & \text{if } y^* \in [0, 10k) \\ 2 & \text{if } y^* \in [10k, 20k) \\ 3 & \text{if } y^* \in [20k, 50k) \\ 4 & \text{if } y^* \in [50k, 100k) \\ 5 & \text{if } y^* \in [100k, \infty) \end{cases}$$

a. $$\begin{aligned} \Pr(Y_i = 1 \mid X_i = x) &= \Pr(y^* < 10{,}000 \mid x) \\ &= \Pr(\exp(x'\beta + e) < 10{,}000 \mid x) \\ &= \Pr(\exp(x'\beta) \cdot \exp(e) < 10{,}000 \mid x) \\ &= \Pr(\exp(e) < 10{,}000 / \exp(x'\beta) \mid x) \\ &= \Pr(e < \log(10{,}000) - x'\beta \mid x) \\ &= \Pr\left(\frac{e}{\sigma^2} < \frac{\log(10k) - x'\beta}{\sigma^2} \,\middle|\, x\right) \\ &= \Phi\left(\frac{\log(10{,}000) - x'\beta}{\sigma^2}\right) \end{aligned}$$

So σ^2 is not uniquely identified. From the response probability we only know $\frac{\log(10{,}000) - x'\beta}{\sigma^2}$.

$Y_j = x'\beta_j + e$

b. $$P_j(x) = \frac{x'\beta_j / \sigma^2}{\sum_{\ell=1}^{5} x'\beta_\ell / \sigma^2}$$

$$\ell_n(\beta) = \sum_{i=1}^{n} \sum_{j=1}^{5} 1\{Y_i = j\} \log\left(\frac{x'\beta_j / \sigma^2}{\sum_{\ell=1}^{5} x'\beta_\ell / \sigma^2}\right)$$