Economics 761 **PROBLEM SET 1** September 4, 2020

1. A seller sells an asset to two risk neutral bidders, 1 and 2. Each bidder values the asset at $V > 0, and this fact is common knowledge. Suppose the seller uses a first-price sealed bid auction: the highest bidder wins the asset and pays her bid. There is no reserve price.

a. Formulate the bidding game in normal form (i.e., specify the bidders' strategy spaces and payoffs) and find the equilibrium.

Now suppose the seller uses an all-pay auction: the highest bidder wins the asset, and each bidder pays her bid. There is no reserve price.

b. Determine bidder 1's payoff for any pair of bids (b_1, b_2).
c. Show that a pure strategy Nash equilibrium does not exist.
d. Find the mixed strategy Nash equilibrium.
e. What is the expected revenue to the seller?

2. Consider a homogenous good market with two firms, 1 and 2. The two firms simultaneously choose capacities k_1 and k_2 and then, after observing each other's choice of capacity, compete for customers by setting price. Cost per unit of capacity is c and production costs are zero. The market demand is given by $D(P) = 1$ if $P \leq 1$, and zero otherwise (i.e., it is a step function).

a. Characterize the sets of (k_1, k_2) for which there is a pure strategy Nash equilibrium in prices and determine the equilibrium.
b. Derive closed form solutions for the mixed strategy equilibrium for the remaining set of capacity choices.
c. Derive the profit functions for the first stage under the assumption of Nash equilibrium play in the pricing stage and show that they have the Cournot form. Determine the subgame perfect capacity choices.