Practice Problems 12:

REVIEW

1. Find the lim inf and lim sup for the following:
   (a) $x_n = y_n/n$, where $\{y_n\}$ is a bounded sequence.
   (b) $x_n = \sqrt{1+n^2}/2n$
   (c) $x_n = \cos(1+1/n)$ if n is even, $x_n = 1 - 1/n^2$ otherwise.
2. Show that for any collection of sets (posibly uncountable), $\{E_\alpha\}$, where $\alpha \in A$ is their index: $\left(\bigcap_{\alpha\in A} E_\alpha\right)^c = \bigcup_{\alpha\in A} E_\alpha^c$.
3. Convert the English to math in, "2 is the smallest prime number."
4. Negate the following:
   (a) "Any student will sink unless he or she swims"
   (b) "Most people believe in ghosts after watching a scary movie"
5. Is a fixed point guaranteed for any continuous function, $f : [0,1] \to [0,1]$?
6. Let (X,d) be a metric space, and $E \subseteq X$ non-empty. The distance between a point $x \in X$ and the set E is defined as $\rho(x,E) = \inf\{d(x,y) : y \in E\}$. Is it true that x is a limit point of E if and only if $\rho(x,E) = 0$? In which direction is it true?
7. Prove that $\sqrt{n+1} - \sqrt{n} \to 0$
8. Approximate $\log(2)$ up to two decimal places with a Taylor approximation of 4th degree.
9. Is the set $\{(x,y) \in \mathbb{R}^2 : |xy| \leq 1\}$ compact? If so, provide a proof of it, otherwise find an open cover that lacks a finite subcover.
10. Suppose X and Y are metric spaces and $f : X \to Y$ with X compact and connected. Furthermore, for any $x \in X$ there is an open ball containing x, B_x, such that $f(y) = f(x)$ for all $y \in B_x$. Prove that f is constant on X.

Credit to Alexander Clark