Econ 711 Problem Set 3

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Question 1

Part A

Assume prices are strictly positive. Consider the objective function $g(q,\tau) = (1-\tau)pq - c(q)$. Let $t = -\tau$. Assume $t' > t$ and $q' > q$.

$$\begin{aligned} g(q',t') - g(q,t') &= (1+t')pq' - c(q') - ((1+t')pq - c(q)) \\ &= (1+t')p(q'-q) - c(q') + c(q) \\ &> (1+t)p(q'-q) - c(q') + c(q) \\ &= g(q',t) - g(q,t) \end{aligned}$$

So g has strictly increasing differences in q and $-\tau$. For the sake of contradiction, assume $q' < q$ and q and q' are optimal at t and t', respectively. Then $0 \geq g(q,t') - g(q',t') > g(q,t) - g(q',t) \geq 0$, which is a contradiction. Thus a decrease in t (an increase in τ) can never result in an increase in output.

Consequently, even if there are multiple optimal production choices for a given t, all optimal production choices at t' must be greater than all optimal production choices at t. This is a stronger result than Baby Topkis, because in Baby Topkis there may be some production choices that are optimal at t that are larger than some production choices that are optimal at t' (due to overlapping in the SSO).

Part B

Let $t = -\tau$. Consider the objective function $h(q,t) = (1+t)p(q)q - c(q)$. Note that $\frac{\partial h}{\partial t} = p(q)q$. Because we do not know the form of the inverse demand function p(q), we do not know if $\frac{\partial^2 h}{\partial t \partial q}$ is positive or negative. Thus, the firm's objective function does not necessarily have increasing differences in q and t.

*I have discussed this problem set with Emily Case, Michael Nattinger, Alex Von Hafften, and Danny Edgel.

Part C

Consider the objective function $h(q,t) = (1+t)p(q)q - c(q)$ with $c(q)$ strictly increasing. Let $t = -\tau$. Let $0 \leq h(q',t) - h(q,t)$. Assume $t' > t$ and $q' > q$. Then:

$$\begin{aligned}
0 &\leq h(q',t) - h(q,t) \\
&= (1+t)pq' - c(q') - ((1+t)pq - c(q)) \\
&= (1+t)(pq' - pq) - c(q') + c(q) \\
\Rightarrow c(q') - c(q) &\leq (1+t)(pq' - pq) \\
\Rightarrow c(q') - c(q) &< (1+t')(pq' - pq) \\
\Rightarrow 0 &< h(q',t') - h(q,t')
\end{aligned}$$

For the sake of contradiction, let $t' > t$ and assume $q' < q$ where q' and q are optimal at t' and t, respectively. Then, because of optimality at q and q':

$$h(q',t) - h(q,t) \geq 0 \geq h(q,t') - h(q',t')$$

However, $0 \leq h(q',t) - h(q,t) \rightarrow 0 > h(q,t') - h(q',t')$ since $q' < q$, which violates the single-crossing differences. So an increase in τ cannot lead to an increase in output.

Question 2

$f(l,m,r,e) = (l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^z$ with $z = 1.1$.
Let $g = pf(l,m,r,e) - w_l l - w_m m - w_r r - w_e e$

Part A

Consider the following partial derivatives:

$$\begin{aligned}
\frac{\partial g}{\partial l} &= 1.1(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{0.1}(0.5l^{-0.5}m^{0.3}) - w_l \\
\frac{\partial g}{\partial m} &= 1.1(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{0.1}(0.3l^{0.5}m^{-0.7}) - w_m \\
\frac{\partial g}{\partial r} &= 1.1(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{0.1}(0.7r^{-0.3}e^{0.1}) - w_r \\
\frac{\partial g}{\partial e} &= 1.1(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{0.1}(0.1r^{0.7}e^{-0.9}) - w_e
\end{aligned}$$

Because each partial is increasing in the other choice variables (l,m,r,e), the objective function is supermodular. Further, each partial is weakly increasing in $-w_e$, so there are increasing differences. So by the Topkis theorem, the firm will demand weakly more of each input as a result of engineer wage subsidies. Because the firm's problem has a unique solution for each input price vector, then the production set is convex, which means the production function is concave. Since the production function is concave, the objective function will have a unique maximum at each price vector, and the firm will demand more of each input as a result of engineer wage subsidies.

Part B

Now consider if $z = 0.9$. Then we can see the following partial derivatives:

$$
\begin{aligned}
\frac{\partial g}{\partial l} &= 0.9(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{-0.1}(0.5l^{-0.5}m^{0.3}) - w_l \\
\frac{\partial g}{\partial m} &= 0.9(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{-0.1}(0.3l^{0.5}m^{-0.7}) - w_m \\
\frac{\partial g}{\partial r} &= 0.9(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{-0.1}(0.7r^{-0.3}e^{0.1}) - w_r \\
\frac{\partial g}{\partial e} &= 0.9(l^{0.5}m^{0.3} + r^{0.7}e^{0.1})^{-0.1}(0.1r^{0.7}e^{-0.9}) - w_e
\end{aligned}
$$

With this value of z, we can see that our objective function is supermodular in $(l, m, -r, -e)$, and these are all weakly increasing in w_e. So by the Topkis theorem, the firm will demand weakly less of l and m, and weakly more of r and e as a result of engineer wage subsidies. Again the objective function will have a unique maximum.

Part C

Since output is supermodular in l and m and the supply of managers is fixed in the short-run, then by LeChatelier's Principle, the subsidy's effect on unskilled labor would be larger in the long-run than in the short-run.