University of Wisconsin-Madison
Department of Economics

Econ 703
Fall 2000

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Final Exam

1. A consumer with a fixed income $I > 0$ consumes two commodities. If he purchases q_i units of commodity i ($i=1,2$), the per unit price he pays is $p_i(q_i)$, where $p_i(.)$ is a strictly increasing C_1 function. The consumer's utility function is given by $u(q_1,q_2) = \ln q_1 + \ln q_2$.

   (a) Describe the consumer's utility maximization problem. Does the Weierstrass theorem apply to yield existence of a maximum? Prove that a maximizer exists.

   (b) Write down the Kuhn-Tucker conditions for a maximum.

   (c) Under what conditions on $p_1(.)$ and $p_2(.)$ are these conditions also sufficient? (Specify the most general conditions possible).

   (d) Suppose $p_1(q_1) = \sqrt{q_1}$ and $p_2(q_2) = \sqrt{q_2}$. Are the sufficient conditions you gave met by this specification? Calculate the optimal consumption bundle in this case.

   (e) Interpret the Lagrange multiplier on the budget constraint.

2. Consider the Cobb-Douglas production function $f(x,y) = x_1^{1/4}x_2^{3/4}$. Compute the second order Taylor expansion of $f(x,y)$ around the point $(x,y) = (1,1)$. Can you provide a Taylor expansion around the point $(x,y) = (0,0)$?

3. Let $f: \mathbf{R}^2 \to \mathbf{R}$ and $g: \mathbf{R}^2 \to \mathbf{R}$ be given by $f(x,y) = x + y$ and $g(x,y) = xy$. Find the maximum and minimum of $f(x,y)$ subject to $g(x,y) = 16$.

4. Define $f : \mathbf{R}^3 \to \mathbf{R}$ by $f(x,y,z) = x^2y + e^x + z$, and let K belong to the range of $f(.)$.

   (a) For which points $(x,y,z) \in \mathbf{R}^3$ can the equation $f(x,y,z) = K$ be solved for x in terms of y and z?

   (b) Letting $(x,y,z)=(0,1,-1)$, compute the partial derivatives of x with respect to y and z at the point $(y,z)=(1,-1)$.

$\frac{\partial x}{\partial(y,z)} = -\frac{1}{F_x} F_{(y,z)} = -\frac{1}{2xy+e^x}\begin{pmatrix} x^2 \\ 1 \end{pmatrix}$

$\frac{\partial x}{\partial(y,z)}\Big|_{(1,-1)} = -(0,1) = (0,-1)$