<u>Economics 761 2021</u>

PROBLEM SET 1

1. Consider a market in which the goods are homogenous.

(a) Show that, for all Q, P'(Q) + QP"(Q) is positive for an inverse demand function with constant elasticity, so that assumption (A1) of the Cournot model is violated.
(b) Show that, in the Cournot model in which N firms have identical cost functions, assumptions (A1) and (A2) imply that equilibrium price and per firm quantity are decreasing in N.

2. A seller sells an asset to two risk neutral bidders, 1 and 2. Each bidder values the asset at $V > 0, and this fact is common knowledge. Suppose the seller uses a first-price sealed bid auction: the highest bidder wins the asset and pays her bid. There is no reserve price.

(a) Formulate the bidding game in normal form (i.e., specify the bidders' strategy spaces and payoffs) and find the equilibrium.

Now suppose the seller uses an all-pay auction: the highest bidder wins the asset, and each bidder pays her bid. There is no reserve price.

(b) Determine bidder 1's payoff for any pair of bids (b_1, b_2).
(c) Show that a pure strategy Nash equilibrium does not exist.
(d) Find the mixed strategy Nash equilibrium to the seller and determine the expected revenue to the seller.
(e) Compute the mixed strategy Nash equilibrium under the assumption that the seller sets a positive reserve price R < V. What do you think will happen to the seller's revenues from the auction? No calculation, please, just intuition.

3. Suppose there are three coffee houses along Main Street. The street is one mile. One hundred residents are uniformly distributed along this stretch. Each resident is willing to buy one cup of coffee per day. A cup of coffee differs only in its location and price and not in any other way. Each customer derives a utility of v = $3.00 from the cup of coffee. A consumer's (round-trip) cost of travel is quadratic in the (one-way) distance from home to any of the coffee houses.

Starbucks Coffee House is located at either end of the one mile stretch and Esquire Coffee House is located halfway between the two end points of the street. The prices of coffee at Starbucks' two locations are p_0 and p_1 respectively. Esquire's price of coffee is denoted by q. Marginal costs of a cup of coffee are zero.

(a) Determine the location of the two marginal consumers – the one who is indifferent between purchasing from Esquires and Starbucks located at the left end point and the one who is indifferent between purchasing from Esquires and the Starbucks located the right end point.

(b) Derive the best reply functions to the pricing game in which the coffee houses choose prices simultaneously. Assume that Starbucks can set different prices at its two locations.

(c) Determine the equilibrium prices and market shares.

(d) Suppose Starbucks and Esquires swap houses so that the Starbucks houses are located at one of the endpoints and the halfway point and Esquires is located at the other endpoint. Derive the equilibrium prices and market shares and explain why it differs (if at all) from (c).

(e) Suppose instead of swapping houses, Starbucks sells one of its coffee houses to Seattle Best Coffee. Derive the equilibrium prices and market shares. Explain why it differs (if at all) from (c).

4. A county consists, for all intents and purposes, of two towns, Right and Left. They are connected by a straight road of length one mile. The population is uniformly distributed on this one-mile stretch. Everyone is an imbiber. Jim Beam and Jack Daniels own the only two liquor licenses. Each drinker will go regularly to whichever bar is closest to him. Unfortunately for Jim and Jack, Mr. Nag has convinced the powers that be that if these two gentlemen were unregulated, they would conspire to raise prices. Hence the price of a drink in each bar is equal and fixed at $p by the county Pickled Brain Commission. Thus, the only variable that Jack and Jim have control over is their location. Each drinker's cost of travel is quadratic in the distance from home to the bar. Assume that the utility obtained from a drink is sufficiently high that market is covered for every pair of locations.

(a) Formulate this location game by defining strategies and payoffs. Let a denote the location of Jim Beam and 1-b denote the location of Jack Daniels. Be sure to derive demands for locations (a, 1-b).

(b) Determine the Nash equilibrium locations of the two establishments and explain the intuition behind your answer.

(c) Are the Nash equilibrium locations socially optimal (i.e., do they minimize total travel costs)? Why or why not? If not, set up the social planner's problem and derive the optimal locations.

5. Consider a model of product differentiation along a line segment of length 1. Consumers are uniformly distributed along the line. Consumers have unit demand with valuation 1 – x for a good sold at a distance x from their location. Production costs are zero.

Assume that firm 1 has two products: one (L) located at the left endpoint of the line segment and another (R) at the right endpoint. Firm 1 may charge distinct prices p_{1R} and p_{1L} for its two products. Firm 2 has only a single product, which is also located at the right endpoint, which firm 2 sells at price p_2.

Suppose that the firms compete by simultaneously choosing prices in a one-shot game. Assume that if any consumers are indifferent between a product offered by firm 1 and the product offered by firm 2, they will buy from firm 2.

(a) Find the equilibrium prices and profits of the two firms in equilibrium.
(b) Is firm 1 better or worse off with product R? That is, should firm 1keep or drop product R (assuming no exit costs)? Explain your reasoning.

6. Consider a model of vertical differentiation in which a customer of type θ obtains the net benefit $U = s(\theta - p(s))$ from an item of quality s. Here θ is uniformly distributed in the population between zero and one. Marginal costs of providing a good of quality s is $C(s) = cs$ where $c \leq 1/2$.

(a) Find the optimal prices which a monopolist should charge when it offers two goods with qualities $s = 1$ and $s = 2$.
(b) Find the optimal price which a monopolist should charge when it offers only one good with quality $s = 1$.
(c) Determine the conditions on c under which these two solutions are optimal.

Now suppose there are two firms: firm 1 offers low quality product with $s = 1$ and firm 2 offers high quality product with $s = 2$. The firms compete in prices. Note that relevant range for p_2 is [2c, 1]. At prices below $2c$, firm 2 loses money and at prices above 1, it cannot sell any units, regardless of what p_1 is.

(d) Derive the demand for good 1 for $p_2 \in [2c, 1]$.
(e) Derive firm 1's best reply as a function of c and plot it in price space. (Hint: it consists of four segments.)
(f) Derive firm 2's best reply and compute the Nash equilibrium prices as a function of c. Can the firms coexist?