Problem Set 2 Sarah Bass

1. This statement is true. If the cost of education were the same for everyone, the payoff for agents $(w - e/a)$ would not differ by ability, and the payoff would be $w - ce$, where c is the universal cost of education. All agents would maximize their payoff w.r.t education, where $w = w(e)$. All workers would choose the same education level, so firms would not be able to differentiate high and low ability workers.

2. a. A buyer's highest willingness to pay is the expected value of all cars (assuming all sellers are willing to sell).

$$E[v] = .50 \cdot 10{,}000 + .50 \cdot 2{,}000 = \$6{,}000.$$

b. If sellers valued high-quality cars at \$8,000, sellers with high-quality cars would not sell (\$8,000 < \$6,000). 50 buyers would buy 50 low-quality cars and would lose $50 \cdot (\$6{,}000 - \$2{,}000) = \$200{,}000$ in surplus. Sellers would gain this \$200,000 surplus.

If sellers valued high-quality cars at \$6,000, all of the cars would sell and markets would clear. On aggregate, buyers and sellers would not gain or lose any surplus.

3.a. $p_h - c_h - q_h c_h \geq p_l - c_h$ IC for high quality

$p_l - c_l \geq p_h - c_l - q_l c_l$ IC for low quality

b. We want the consumer to be willing to buy both types:

$$(1-q_h)s + q_h s - p_h \geq 0$$
$$\rightarrow s \geq p_h$$

$$(1-q_l) - p_l \geq 0$$
$$\rightarrow s \geq \frac{p_l}{1-q_l}$$

$$\rightarrow s = \max\left\{ p_h, \frac{p_l}{1-q_l} \right\}$$

From the incentive constraints in 3a, we have:

$$p_h - c_h - q_h c_h \geq p_l - c_h$$
$$\rightarrow p_h - q_h c_h \geq p_l$$

$$p_l - c_l \geq p_h - c_l - q_l c_l$$
$$\rightarrow p_l \geq p_h - q_l c_l$$

Combining these:

$$p_h - q_h c_h \geq p_h - q_l c_l$$
$$\rightarrow q_h c_h \leq q_l c_l$$

Any equilibrium that satisfies the highlighted equations is a pooling equilibrium.

4.a. If the seller can observe θ, they will set the price $t = \theta q$.

Let $\theta = 1$, the seller maximizes:

$$\max q - q^2$$

FOC w.r.t. q:

$$1 - 2q = 0$$

$$q_1 = 1/2, \quad t_1 = 1/2$$

Let $\theta = 2$, the seller maximizes:

$$\max 2q - q^2$$

FOC w.r.t q:

$$2 - 2q = 0$$

$$q_2 = 1, \quad t_2 = 2$$

which is $2 \cdot q_1$ calculated above.

b. For $\theta = 1$, the IC is $q_1 - t_1 \geq q_2 - t_2$

$$1 \cdot 1/2 - 1/2 = 0$$

$$1 \cdot 1 - 2 = -1$$

$0 \geq -1$, so the IC holds.

For $\theta = 2$, the IC is $2q_2 - t_2 \geq 2q_1 - t_1$

$$2 \cdot 1 - 2 = 0$$

$$2 \cdot 1/2 - 1/2 = 1/2$$

$0 \not\geq 1/2$, so the IC doesn't hold.

c. $q_1 = 1/4$

First note that $\theta = 1$ has no incentive to lie about their type, so

$$q_1 - t_1 = 0 \quad \rightarrow \quad t_1 = 1/4.$$

Next we will bind the IC for $\theta = 2$.

$$2q_2 - t_2 = 2q_1 - t_1$$

$$\rightarrow 2q_2 - t_2 = 1/4$$

$$\rightarrow t_2 = 2q_2 - 1/4$$

So sellers will maximize:

$$\max \quad p\,(t_1 - q_1^2) + (1-p)(t_2 - q_2^2)$$

$$\rightarrow \max \quad p\,(1/4 - 1/16) + (1-p)(2q_2 - 1/4 - q_2^2)$$

where p is the probability $\theta = 1$.

FOC w.r.t q_2:

$$2(1-p) - 2(1-p)\,q_2 = 0$$

$$q_2 = 1$$

$$\rightarrow t_2 = 2 - 1/4 = 7/4$$