

ECON 714: Problem Set 4

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Problem 1

This problem asks you to solve a model of *oligopolistic* competition from Atkeson and Burstein (AER'2008), which extends the Dixit-Stiglitz setup and is widely used to analyze heterogeneous markups and incomplete pass-through.

Consider a static model with a continuum of sectors $k \in [0, 1]$ and $i = 1, \dots, N_k$ firms in sector k , each producing a unique variety of the good. Households supply inelastically one unit of labor and have nested-CES preferences:

$$C = \left(\int C_k^{\frac{\rho-1}{\rho}} dk \right)^{\frac{\rho}{\rho-1}}, \quad C_k = \left(\sum_{i=1}^{N_k} C_{ik}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \quad \theta > \rho \geq 1.$$

Production function of firm i in sector k is given by $Y_{ik} = A_{ik}L_{ik}$.

1. Solve household cost minimization problem for the optimal demand C_{ik} , the sectoral price index P_k , and the aggregate price index P as functions of producers' prices.
2. Assume that firms compete à la Bertrand, i.e. choose P_{ik} taking the prices of other firms in a sector P_{jk} , $j \neq i$ as given. Derive demand elasticity for a given firm and the optimal price.
3. Prove that other things equal, firms with higher A_{ik} set higher markups.
4. Assume that $\rho = 1$, $\theta = 5$, $N_k = 20$, and $\log A_{ik} \sim \text{i.i.d. } \mathcal{N}(0, 1)$. Solve the model numerically by approximating the number of sectors with $K = 100,000$. You will need

an efficient algorithm to compute a sectoral equilibrium (search for a fixed point, do not use “solve”) nested in a general equilibrium loop solving for real wages.

5. Compute the aggregate output C of the economy and compare it to the first-best value.
6. *Bonus task*: Does the sectoral equilibrium converge to the one under Bertrand competition with homogeneous goods in the limit $\theta \rightarrow \infty$?