As for everything else, so for a mathematical theory: beauty can be perceived but not explained - Arthur Cayley

1 Review Topics

Vector spaces, linear transformations, isomorphisms

2 Exercises

2.1 Classify each operator as linear or not linear

- $T : \mathcal{C}\,[0,\,1] \to \mathbb{R}$, $Tf\,(x) = \int_0^1 f\,(x)\,dx$.

- Recall that the dot-product between two vectors in $\mathbb{R}^n$ is defined as $\langle x,\, y\rangle := \sum_{i=1}^n x_i y_i$. Define the operator $T_a : \mathbb{R}^n \to \mathbb{R}$, $T_a x = \langle a,\, x\rangle$.

- $T : \mathbb{R} \to \mathbb{R}$, $Tx = mx + b$.

2.2 Characterize the set of solutions to the equations:

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 0 \\ 2x_1 + 2x_3 &= 0 \\ x_1 - 3x_2 + 4x_3 &= 0 \end{aligned}$$

2.3 Show that the set of all polynomials of degree n form a vector space.

2.4 **Show that if $\{u,\, v,\, w\}$ is a set of linearly independent vectors, that $\{u,\, u+v,\, u+v+w\}$ are linearly independent.**

2.5 **Let $\mathcal{P}^n$ be the vector space of polynomials of degree n. Consider the differentiation operator $T : \mathcal{P}^n \to \mathcal{P}^{n-1}$ defined by $Tp\,(x) = \frac{d}{dx} p\,(x)$. Compute $\operatorname{Im} T$, $\ker T$, and $\operatorname{rank} T$.**

2.6 **Prove that a linear map $T : X \to Y$ over two $n-$dimensional vector spaces is 1-to-1 if and only if it is onto.**