Practice Problems 1

Office Hours: Tuesdays, Thursdays from 4:30 to 5:30 at SS 7143.
E-mail: gabriel.martinez@wisc.edu
If you need help, reach out: your classmates, the TA, textbooks, or the Professor.

- Common Symbols

$\forall$: for all	$\in$: element of	$>$: grater than	$\Rightarrow$: implies	$\equiv$: equivalent to
$\wedge$: and	$\vee$: or	$\subset$: subset	$\cup$: union	$\cap$: intersection
$\exists$ exists	$\exists!$ exists a unique	$\emptyset$: empty set	$\neg P$: not P	A^c : complement of A

$A \backslash B = A \cap B^c$: A minus B
$\mathcal{P}(A) \equiv 2^A$: the power set of A
$f(A)^{-1}$: the pre-image of A

NEGATIONS

1. Negate the following:
   (a) * Exists $x \in \mathbb{R}$ such that $\log x = 30$
   (b) $\forall\ a \in \mathbb{Q},\ \sqrt{a} \in \mathbb{Q}$
   (c) * If you're Madisonian, then you were born in Wisconsin.
   (d) A person can be happy while not loving spicy food.
   (e) * $\forall \epsilon \in \mathbb{R}$ such that $\epsilon > 0,\ \exists N \in \mathbb{N}$ such that $\forall\ n \in \mathbb{N}$, satisfying $n \geq N,\ 1/n < \epsilon$.
   (f) Between any rational numbers, there exists another rational.

SETS

2. For any sets A, B, C, prove that:
   (a) * $(A \cap B) \cap C = A \cap (B \cap C)$
   (b) * $A \cup B = A \Leftrightarrow B \subseteq A$
   (c) $(A \cup B)^c = A^c \cap B^c$
   (d) $A \backslash B \subseteq A$
3. * Let Q be the statement $2x > 4$ and $P : 10x + 2 > 15$. Show that $Q \implies P$ using:
   (a) a direct proof
   (b) contrapositive principle
   (c) contradiction

FUNCTIONS

4. Let $f : S \to T$, $U_1, U_2 \subset S$ and $V_1, V_2 \subset T$.

   (a) * Prove that $V_1 \subset V_2 \implies f^{-1}(V_1) \subset f^{-1}(V_2)$.

   (b) Prove that $f(U_1 \cap U_2) \subset f(U_1) \cap f(U_2)$.

   (c) $f^{-1}(V_1 \cup V_2) = f^{-1}(V_1) \cup f^{-1}(V_2)$.

5. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. Give an example of the following or show that it is impossible to do so:

   (a) a function, $f : X \to Y$, that is neither injective nor surjective

   (b) a one-to-one function, $f : X \to Y$, that is not onto

   (c) a bijection, $f : X \to Y$

   (d) a surjection , $f : X \to Y$, that is not one-to-one