$$E\max(x,a) = \underbrace{aF(a)}_{x<a} + \underbrace{(1-F(a))E(x|x\geq a)}_{x\geq a} \quad (1)$$

$$E\max(x,a) = a + \int_a^\infty (1-F(x))\,dx \quad (2)$$

- (2) is the special case where $V(x) = x$, whose general case covered in class.
- (1) is from the law of iterated expectations

$$E(\max(x,a)) = \underbrace{E(\max(x,a)\,|\,x<a)}_{a\,F(a)}\,\underbrace{Pr(x<a)}_{F(a)} + (1-F(a))\,\underbrace{E(\max(x,a)\,|\,x\geq a)}_{E(x|x\geq a)}$$

From (1) = (2),

$$aF(a) + (1-F(a))E(x|x\geq a) = a + \int_a^\infty (1-F(x))\,dx$$

$$\Rightarrow E(x|x\geq a) = a + \frac{\int_a^\infty (1-F(x))\,dx}{1-F(a)}$$