Microeconomic Theory (Econ 713)
University of Wisconsin-Madison, Prof. Marzena Rostek
Problem Set 2

Due in (=before) class April 18, 2019

Question 1: Akerlof Model 2.0 **- By Dirk Bergemann**
Consider Akerlof's model as was presented in class. In that model v_S and v_B were the marginal willingness to pay of buyer and seller, which were commonly known. The private information (unobservable by the buyer) to the seller was about the quality of the good (θ). Suppose now that v_S is also private information to the seller and is distributed uniformly on the interval $[\bar{v}_S - \epsilon, \bar{v} + \epsilon]$ for some ϵ

A) Derive necessary and sufficient conditions such that there is positive trade in some equilibrium. *(Hint: It might be helpful to depict the joint distribution of ϵ and θ in a two-dimensional graph.)*

B) How does the introduction of private information about the willingness to pay of the seller affect the conditions. Does the probability of trade decrease or increase with ϵ?

Question 2: Adverse Selection - Exercise MWG $13.B.3$
Consider the competitive labor market with a positive selection version of the adverse selection model in which $r(\cdot)$ is a continuous, strictly decreasing function of θ, where θ captures worker productivity. Let the density of workers of type θ be $f(\theta)$, with $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. In this exercise $r(\theta)$ denotes the reservation utility of a worker of type θ. A worker of type θ will accept employment with wage w, if $w \geq r(\theta)$. Thus, $r(\theta)$ is interpreted as the opportunity cost to a type θ of accepting employment.

1. Show that the more capable workers are the ones choosing to work at any given wage.
2. Show that if $r(\theta) > \theta$ for all θ, then the resulting competitive equilibrium is Pareto efficient.
3. Suppose that there exists a $\hat{\theta}$ such that $r(\theta) < \theta$ for $\theta > \hat{\theta}$and $r(\theta) > \theta$ for $\theta < \hat{\theta}$. Show that any competitive equilibrium with strictly positive employment necessarily involves too much employment relative to the Pareto optimal allocation of workers.

Question 3: Prelim June 2011
Consider the following versions of a signaling model, in which a company wants to hire a worker. Productivity is worker private information and is not observable to the company. The company maximizes expected profits. Worker reservation wage is equal to 0. The labor market is competitive.

A) Suppose that a worker has productivity $\theta \in \{\theta_L, \theta_H\}$, $\theta_H > \theta_L$ and $Pr(\theta = \theta_H) = \mu$. Worker chooses an education level $e \in [0, \infty)$ and is paid $w(e)$. A worker's utility is given by $U(w, e, \theta) = w - c(e, \theta)$, where the cost of education $c(e, \theta) = e\theta$. Verify whether the single-crossing condition holds. Describe all the equilibria of this model.

B) As above, workers have productivity $\theta \in \{\theta_L, \theta_H\}$, $\theta_H > \theta_L$, but there is an equal probability of each type. Assume that the cost of education is the same for both workers, $c(e) = e$. Suppose the utility of worker θ who is paid wage w and undertakes education e is $U(w, e, \theta) = \theta w - e$. Is there an equilibrium where different types of workers choose different education levels? If there is, please describe all such equilibria. If not, please explain why. Show your work.

Question 4: Know your value
Consider a signaling model in which a company wants to hire a worker. The *expected productivity* θ_i is private information of worker i and not observable to the company. The company operates in a competitive labor market, so that *workers must be paid their expected productivity.*

Suppose there is an equal mass of three types of workers $\theta_H > \theta_M > \theta_L$. The costs of education is $c(e, \theta_i) = e/\theta_i$, for $i = H, M, L$.

A) Is there an equilibrium where types θ_L and θ_H choose education level e_L, while θ_M chooses education level $e_M \neq e_L$? If there is, please describe it. If not, explain why. Then, apart from providing an anlytical explanation, use the definition of the single-crossing condition to give a brief intuition for your claim of existence of non-existence.

B) Define and describe the set of equilibria where all three types act identically (i.e. where all three types pool).