Econ 712 Problem Set 6

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Question 1

Part 1

In the Ramsey equilibrium, the government chooses first based on households' optimal predicted response to each potential choice. If the government chooses y_H, households will choose x_H, which yields a utility of 24. If the government chooses y_L, households will choose x_L, which yields a utility of 12. Since (x_H, y_H) yields the higher utility, the government will choose (ξ_H, x_H, y_H).

In the no commitment equilibrium, the households choose first based on the government's predicted response to each potential choice. Households will assume that the government will choose y_L. Then each individual household will select ξ_L, which implies that households will aggregately choose x_L by consistency. So we will end up with (ξ_L, x_L, y_L) and a utility of 12.

Part 2

Since there are finite periods, the government will choose y_L in the last period to maximize utility, but households will predict this and choose x_L. In the second to last period, households will have no incentive to choose y_H, so they will again choose y_L, so households will also choose x_L. This pattern will occur for each preceding period, so the economy cannot support a Ramsey equilibrium in any period.

Part 3

Households will propose to support the Ramsey equilibrium starting in period 1. However, since there are finite periods, the government will choose y_L in the last period to maximize utility, and households will predict this and choose x_L. If the government were to deviate from the Ramsey path, the households would punish the government. As a result, each subsequent period would reach equilibrium at $(\xi_{LL}, x_{LL}, y_{LL})$. Since there are 3 periods in this economy, there 3 possible points at which the government could deviate from the Ramsey equilibrium (period 1, period 2, period 3). Comparing the utilities across these equilibria, we see that:

- Utility from Period 1 Deviation: $25 + 0.9(2) + 0.9^2(2) = 28.42$

*I have discussed this problem set with Emily Case, Michael Nattinger, Alex Von Hafften, and Danny Edgel.

- Utility from Period 2 Deviation: $24 + 0.9(25) + 0.9^2(2) = 48.12$
- Utility from Period 3 Deviation: $24 + 0.9(24) + 0.9^2(12) = 55.32$

Since the government can maximize utility by maintaining a Ramsey equilibrium until the last period, this is what the government will do.

Question 2

Part 1

The Planner's Problem is to solve the following maximization:

$$\begin{aligned} &\max(\ln(l) + \ln(\alpha + c) + \ln(\alpha + g)) \\ &\text{subject to } l + g + c = 1 \end{aligned}$$

The Lagrangian for this problem is:

$$\mathcal{L} = \ln(l) + \ln(\alpha + c) + \ln(\alpha + g) + \lambda(1 - l - c - g)$$

Then the first order conditions:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial l} &= \frac{1}{l} - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial c} &= \frac{1}{\alpha + c} - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial g} &= \frac{1}{\alpha + g} - \lambda = 0 \end{aligned}$$

Then:

$$\begin{aligned} &\frac{1}{l} = \frac{1}{a + c} = \frac{1}{a + g} \\ &\Rightarrow l = a + c = a + g \end{aligned}$$

Substituting this into the constraint, we can see that:

$$\begin{aligned} l + c + g &= 1 \\ (a + c) + c + g &= 1 \\ a + c + c + c &= 1 \\ a + 3c &= 1 \\ \Rightarrow c &= \frac{1 - a}{3} \\ \Rightarrow g &= \frac{1 - a}{3} \\ \Rightarrow l &= \frac{2a + 1}{3} \end{aligned}$$

Thus the solution to the Planner's Problem is $(l, c, g) = (\frac{2a+1}{3}, \frac{1-a}{3}, \frac{1-a}{3})$.

Part 2

Under the Ramsey Equilibrium, the government will act first, so they will anticipate the household's actions and respond accordingly. The maximization problem is as follows:

$$\begin{aligned} &\max(\ln(l) + \ln(\alpha + c) + \ln(\alpha + g)) \\ &\text{and } c = (1-\tau)(1-l) \end{aligned}$$

By substituting the constraint into the objective function, we have:

$$\max(\ln(l) + \ln(\alpha + (1-\tau)(1-l)) + \ln(\alpha + g))$$

Differentiating with respect to labor, we see:

$$\begin{aligned} \frac{1}{l} - \frac{1-\tau}{\alpha + (1-\tau)(1-l)} = 0 \\ \frac{1}{l} &= \frac{1-\tau}{\alpha + (1-\tau)(1-l)} \\ l &= \frac{\alpha}{2(1-\tau)} + \frac{1}{2} \end{aligned}$$

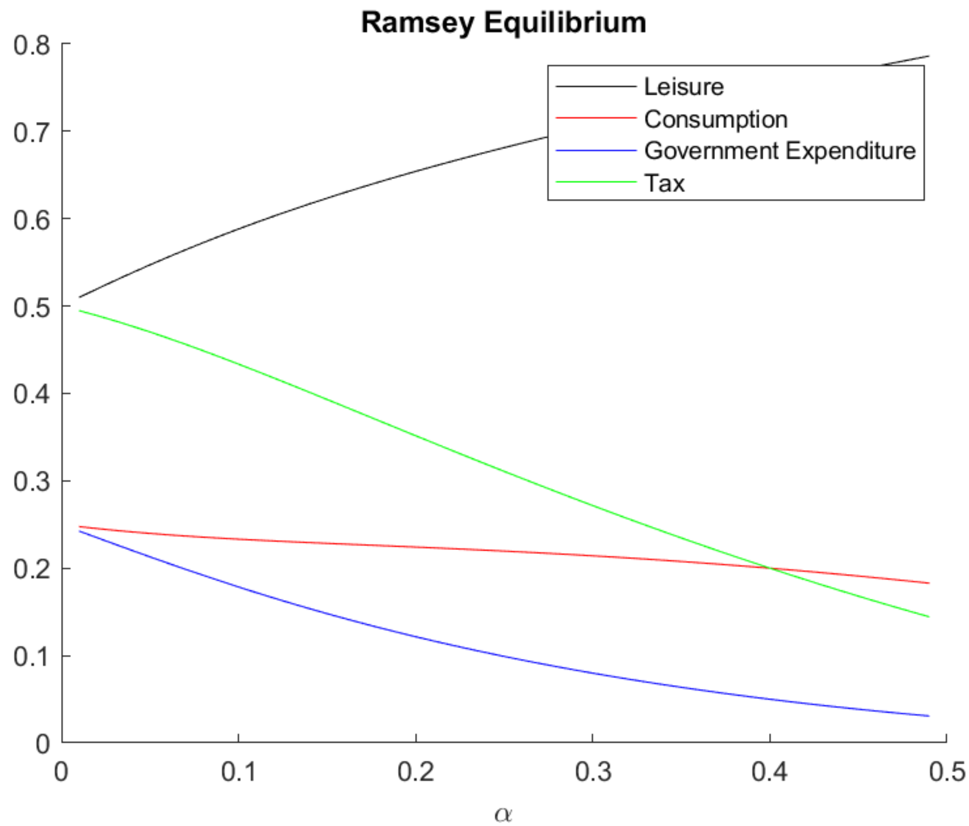
Next we can substitute this value of l into our constraint:

$$\begin{aligned} c &= (1-\tau)(1 - \frac{\alpha}{2(1-\tau)} - \frac{1}{2}) \\ &= (1-\tau)(\frac{1}{2} - \frac{\alpha}{2(1-\tau)}) \\ &= \frac{1-\tau-\alpha}{2} \end{aligned}$$

We can substitute these values of c and l, along with our constraint for g into our objective function to solve for τ.

$$\max(\ln(\frac{\alpha}{2(1-\tau)} + \frac{1}{2}) + \ln(\alpha + \frac{1-\tau-\alpha}{2}) + \ln(\alpha + \tau(1-l)))$$

Solving the for τ has no solution analytically, so I have completed this problem in the attached Matlab code (per Katya's recommendation). The graph below shows the values for l, c, g, τ depending on



the value of α.

Part 3

Under the Nash Equilibrium, the households will act first, so they will anticipate the government's strategy and act accordingly. The maximization problem is as follows:

$$\max(\ln(l) + \ln(\alpha + c) + \ln(\alpha + g))$$
$$\text{such that } g = \tau(1-l)$$
$$\text{and } c = (1-\tau)(1-l)$$

Using the constraints, we can rewrite our maximization problem as follows:

$$\max(\ln(l) + \ln(\alpha + (1-\tau)(1-l)) + \ln(\alpha + \tau(1-l)))$$

Taking the derivative with respect to τ, we find the following first order condition:

$$\begin{aligned}
-\frac{1-l}{\alpha+(1-\tau)(1-l)}+\frac{1-l}{\alpha+\tau(1-l)}&=0\\
\Rightarrow \frac{1-l}{\alpha+(1-\tau)(1-l)}&=\frac{1-l}{\alpha+\tau(1-l)}\\
\Rightarrow \alpha+(1-\tau)(1-l)&=\alpha+\tau(1-l)\\
\Rightarrow 1-\tau&=\tau\\
\Rightarrow \tau&=0.5
\end{aligned}$$

Next we can substitute this value of τ into our maximization problem:

$$\max(\ln(l)+\ln(\alpha+(1-0.5)(1-l))+\ln(\alpha+0.5(1-l)))$$

Taking the derivative with respect to l, we find:

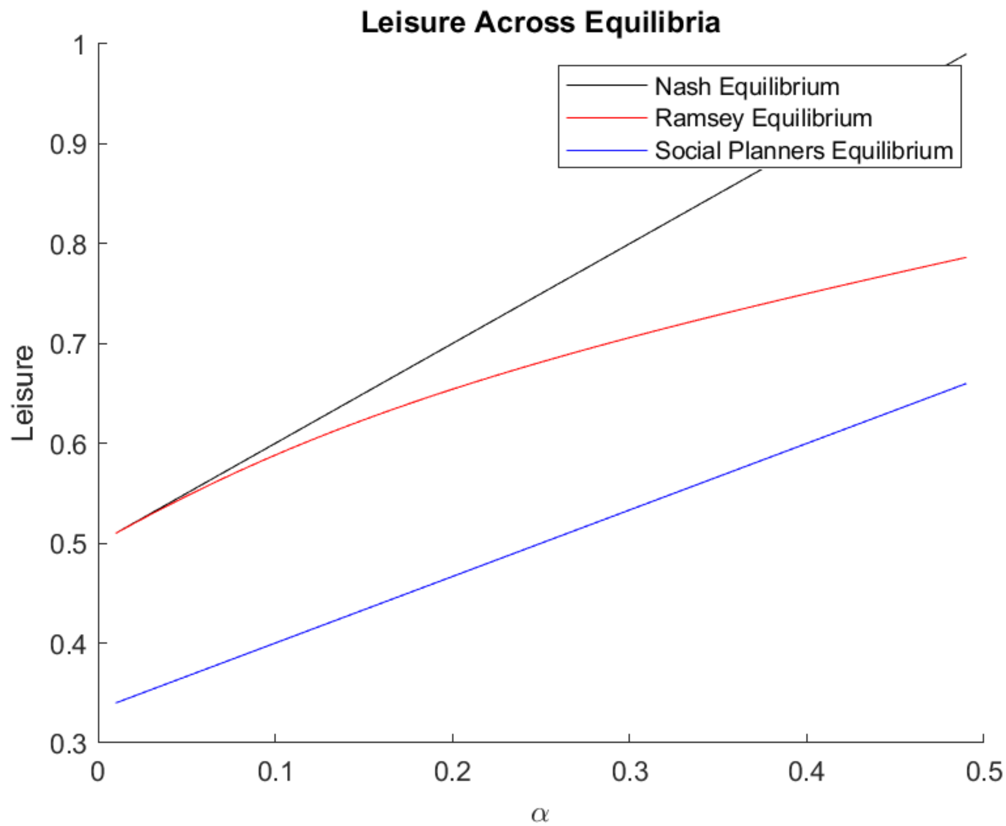
$$\begin{aligned}
\frac{1}{l}-\frac{0.5}{\alpha+0.5(1-l)}&=0\\
\Rightarrow \frac{1}{l}&=\frac{0.5}{\alpha+0.5(1-l)}\\
\Rightarrow 0.5l&=\alpha+0.5(1-l)\\
\Rightarrow l&=\alpha+0.5
\end{aligned}$$

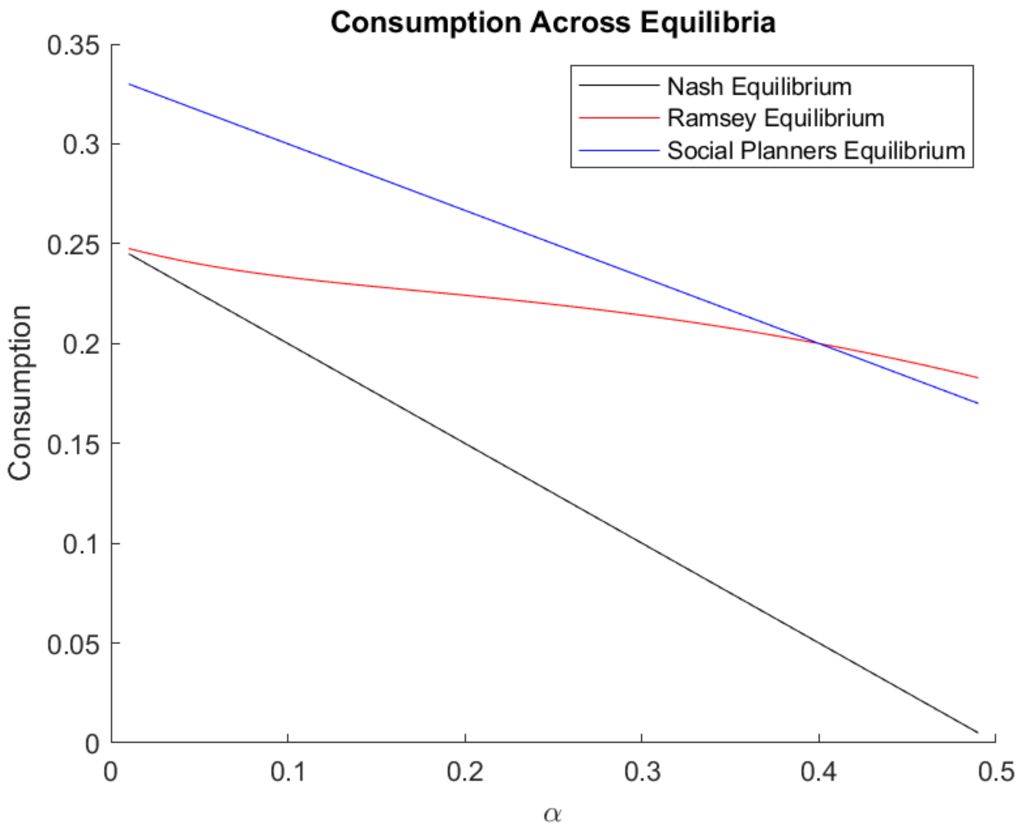
Next we can use these values of τ and l in our constraints to solve for c and g:

$$\begin{aligned}
c&=(1-0.5)(1-\alpha-0.5)\\
&=0.25-0.5\alpha\\
g&=0.5(1-\alpha-0.5)\\
&=0.25-0.5\alpha
\end{aligned}$$

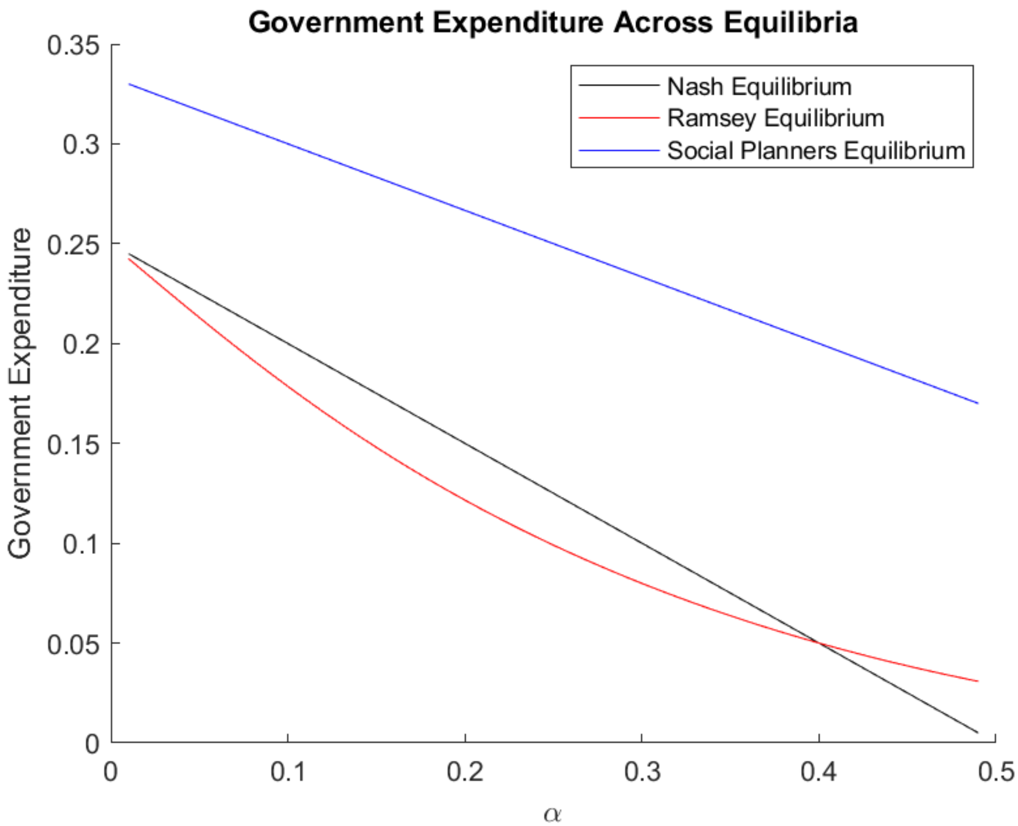
Part 4

The Social Planner's Equilibrium dominates the Ramsey and Nash Equilibria, and the Ramsey Equilibrium dominates the Nash Equilibrium. The differences in the equilibria are reflected in the following graphs:

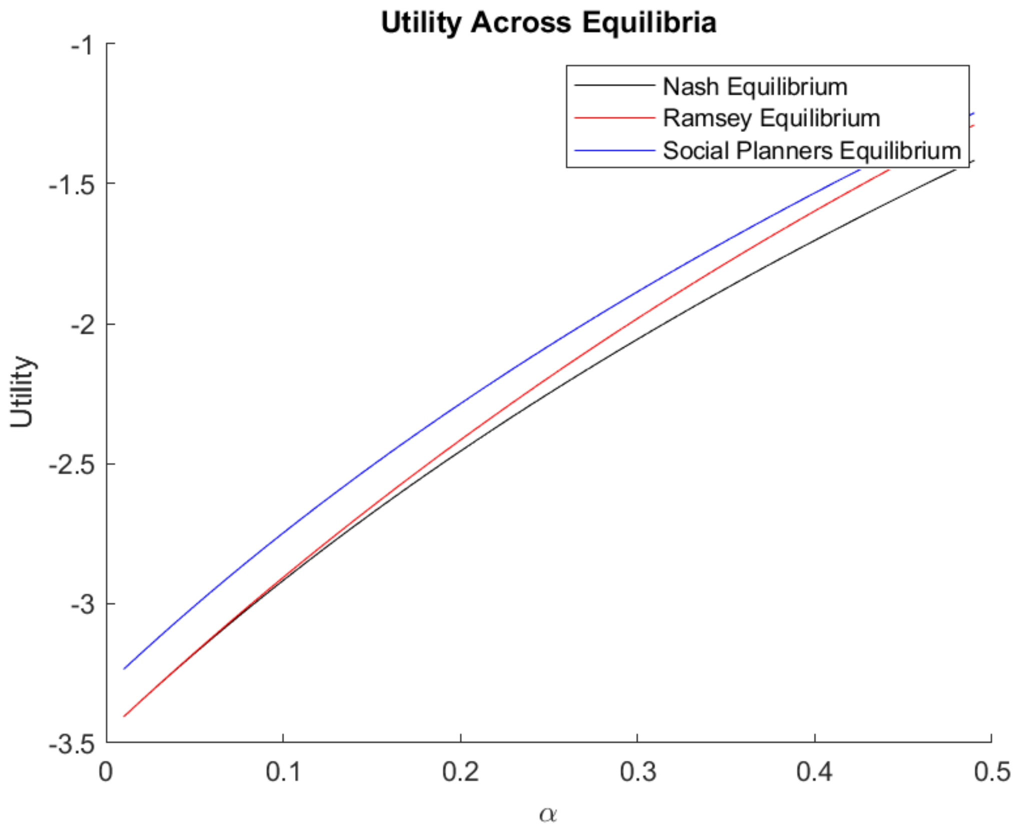

Leisure Across Equilibria
Nash Equilibrium
Ramsey Equilibrium
Social Planners Equilibrium
Leisure
α
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
0.1
0.2
0.3
0.4
0.5



Consumption Across Equilibria
Nash Equilibrium
Ramsey Equilibrium
Social Planners Equilibrium
Consumption
α
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.1
0.2
0.3
0.4
0.5



Government Expenditure Across Equilibria
Nash Equilibrium
Ramsey Equilibrium
Social Planners Equilibrium
Government Expenditure
α
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.1
0.2
0.3
0.4
0.5



Part 5

The Ramsey Equilibrium can be sustained if the value of β corresponds to higher utility over the infinite periods of the economy. Thus, the following inequality must hold:

$$u(l_R|\tau = \tau_R) + \sum_{i=1}^{\infty} \beta^i u(l_R|\tau = \tau_R) > u(l_R|\tau = \tau_N) + \sum_{i=1}^{\infty} \beta^i u(l_N|\tau = \tau_N)$$

I have solved for the minimum value of β given α in Matlab, which is illustrated in the plot below.