9/3/04

- vacuously true
- power set
- Is the contrapositive equivalent to the original statement?
- Negation and quantifiers.

1. vacuously true

If P, then Q

if P is always false, then Q is always right.

e.g. if pig can fly, then your father is your son

2. Power set

Def: set of all subsets

Note: in mathematics, "set" = "class" = "collection"

e.g. $A = \{1, 2\}$, $\mathcal{A} = \{\{1\}, \{2\}\}$

$P(A) = \{\{1\}, \{2\}, \{1, 2\}, \phi\}$

$P(\mathcal{A}) = \{\{\{1\}\}, \{\{2\}\}, \{\{1\}, \{2\}\}, \phi\}$

Is the following true or false?

$\{2\} \in P(\mathcal{A})$	(X)	$\{\{2\}\} \subset P(\mathcal{A})$	(X)
$\{2\} \subset P(\mathcal{A})$	(X)	$\{\{2\}\} \in P(\mathcal{A})$	(√)
$\{\{2\}\} \subset P(A)$	(√)	$\{\{\{2\}\}\} \subset P(\mathcal{A})$	(√)
$\{2\} \in P(A)$	(√)	$\{\{2\}\} \in P(A)$	(X)
$\{2\} \subset P(A)$	(X)		
$\phi \in P(A)$	(√)	$\phi \subset P(A)$	(√)

3. Let's check the following statement:

If $x<0$, then $x^2-x>0$

Contrapositive: If $x^2-x\leq 0$, then x could not be less than 0, that is, $x\geq 0$.

Someone said "$x\geq 0$, x could be 2, but $2^2-2>0$, so the contrapositive is not equivalent to original statement".

why is this wrong?

Actually, this person is checking the negation of the original statement.

The contrapositive is just that "if $x\in[0,1]$, then $x\in[0,\infty)$"

Corollary: Converse $\Leftrightarrow$ Negation

Note1: Contrapositive $\Leftrightarrow$ original statement" is is logic basis of "Contradiction".

a deeper question is why is contradiction right?

Actually, this is related to the dichotomy /daiˈkɔtəmi/ of mathematical logic, that is, the world is divided into something and its negation, no interim.

Note2: In mathematics, there are three popular proof methods: Contradiction, Deduction & Induction.

A further example "If you don't study math well, you won't be a good economist" $\Leftrightarrow$ "If you want to be a good economist, you must study math well", that is, math is a necessary condition of being a good economist

4. ① If all the quantifiers in a given proposition are of the same type, the order of the quantifiers is immaterial.

e.g. $\forall x \in \mathbb{R}, y \in \mathbb{R}, \quad (x+y)^2 = x^2 + 2xy + y^2$

$\Leftrightarrow \forall y \in \mathbb{R}, x \in \mathbb{R}, \quad (x+y)^2 = x^2 + 2xy + y^2$

② The order of quantifiers becomes significant if quantifiers of different types are involved

e.g. $\forall x > 0, \ \exists y > 0, \ \text{s.t } y^2 = x \quad (\checkmark)$

$\not\Leftrightarrow \exists y > 0, \ \forall x > 0, \ \text{s.t } y^2 = x \quad (\times)$

Negation: $\exists x > 0, \ \forall y > 0, \quad y^2 \neq x \quad (\times)$

$\forall y > 0, \ \exists x > 0, \quad y^2 \neq x \quad (\checkmark)$