Section 1.

0.1 Logical Operators

And, or statements, if/then statements, negation.

0.2 Methods of Proof

Direct, contrapositive, contradiction, induction.

0.3 Set Operations

$\cap$, $\cup$, $\subset$, ^{c}, $\backslash$.

Section 2.

0.4 Metric Spaces

Distance/metric functions: nonnegativity and identification, symmetry, and the triangle inequality.

0.5 Convergence in Metric Spaces

$a_n \to a \Leftrightarrow \forall \epsilon > 0, \, \exists N \, s.t. \, \forall n \geq N, \, d(a_n, \, a) < \epsilon$.

Section 3.

0.6 Open & Closed Sets

A is open if $\forall x \in A, \, \exists \epsilon > 0 \, s.t. \, B_\epsilon(x) \subset A$. A is closed if A^c is open.

0.7 Limits of Functions

For $f : X \to Y$, $\lim_{x \to x_0} f(x) = y$ iff $\forall x > 0, \, \exists \delta \, s.t.$ if $d_X(x, \, x_0) < \delta$, then $d_Y(f(x), \, y) < \epsilon$.

Section 4.

0.8 Continuity

$f(x)$ is continuous at x_0 if $\lim_{x \to x_0} f(x) = f(x_0)$.

0.9 Uniform Continuity

For all $\epsilon > 0$, there exists $\delta > 0$ such that for all x, y such that $d_X(x, \, y) < \delta$, then $d_Y(x, \, y) < \epsilon$.

0.10 Lipschitz

For all x, y, $\exists B > 0 \, s.t. \, d_Y(f(x), \, f(y)) \leq B d_X(x, \, y)$.

Section 5.

0.11 Supremum and Infimum

$\sup A = \min \{B : x \leq B, \forall x \in A\}$, $\inf A = \min \{B : x \leq B, \forall x \in A\}$.

0.12 Extreme Value Theorem

Continuous functions on closed intervals achieve their maximum and minimum.

0.13 Intermediate Value Theorem

Continuous functions on closed intervals achieve any value between the value at the endpoints on the interval.

0.14 Monotone Functions

Left continuous/right continuous.

Section 6.

0.15 Complete Metric Spaces

Cauchy sequences: $\forall \epsilon > 0, \exists N\, s.t. \forall n, m \geq N, d(x_n, x_m) < \epsilon$. Complete spaces: all Cauchy sequences converge.

0.16 Contraction Mapping Theorem

$T : X \to Y$ has a fixed point if T is a contraction: $d(T(x), T(y)) \leq Cd(x, y)$, $C\,[0, 1)$.

Section 7.

0.17 Compactness

Compact: closed and totally bounded, every open cover has a finite subcover, every sequence has a convergent subsequence.

0.18 Extreme Value Theorem (again)

Instead of closed intervals, we can consider compact sets.

Section 8.

0.19 Vector Spaces

Closed under linear compositions, identity elements, null elements, well-behaved operations.

0.20 Linear Transformations

$T(\alpha v + \beta w) = \alpha T v + \beta T w$.

0.21 Isomorphisms

X is isomorphic to Y iff $\exists\, T : X \to Y$, T 1-to-1 and onto.

Section 9.

0.22 $\mathbb{R}^n$

Any n-dimensional metric space is isomorphic to $\mathbb{R}^n$.

0.23 $\mathbb{R}^{n\times m}$

Isomorphic to spaces of linear transformations from m to n dimensional spaces.