# Econ 711 Midterm

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(sorry for the messy work!!!)

1) $u(x) = \min\{x_1, x_2\}^\alpha (x_3 + x_4)^{1-\alpha}$

$p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 x_4 = w$

a) Since prices are strictly positive, we can ignore non-negativity constraints. ~~Our lagrangian is:~~

The consumer can maximize their utility by choosing $x_1 = x_2$ since $\max(\min(x_1, x_2))$ occurs at $x_1 = x_2$. The consumer will choose the cheapest good between $x_3$ and $x_4$ and consume only that good. Let $x_a = x_1 = x_2$, and let $x_b$ be the cheaper good between $x_3$ and $x_4$. We can rewrite out utility function as:

$$u(x) = x_a^\alpha x_b^{1-\alpha} \quad \text{with BC}$$

$$2p_a x_a + p_b x_b = w.$$

So our lagrangian is:

$$\mathcal{L} = x_a^\alpha x_b^{1-\alpha} + \lambda(w - 2p_a x_a - p_b x_b)$$

Taking our FOC, we see:

$$d\mathcal{L}/dx_a : \alpha x_a^{\alpha-1} x_b^{1-\alpha} - 2\lambda p_a = 0$$

$$\alpha x_a^{\alpha-1} x_b^{1-\alpha} = 2\lambda p_a$$

$$x_a = \left(\frac{2\lambda p_a}{\alpha x_b^{1-\alpha}}\right)^{\frac{1}{\alpha-1}}$$

$$d\mathcal{L}/dx_b : (1-\alpha) x_a^\alpha x_b^{-\alpha} - \lambda p_b = 0$$

$$(1-\alpha) x_a^\alpha x_b^{-\alpha} = \lambda p_b,$$

$$x_b = \left(\frac{\lambda p_b}{(1-\alpha) x_a^\alpha}\right)^{\frac{1}{-\alpha}}$$

So ~~our Marshallian~~ demand is:

$$\cancel{X_a = \frac{2\lambda p_a}{\alpha}}$$

$$\alpha x_a^{\alpha-1}\left(\frac{w-2p_a x_a}{p_b}\right)^{1-\alpha} = 2\lambda p_a$$

$$\left(x_a^{\alpha-1}\left(\frac{w-2p_a x_a}{p_b}\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha}} = \left(\frac{2\lambda p_a}{\alpha}\right)^{\frac{1}{1-\alpha}}$$

$$x_a^{\frac{\alpha-1}{1-\alpha}}(w-2p_a x_a) = p_b\left(\frac{2\lambda p_a}{\alpha}\right)^{\frac{1}{1-\alpha}}$$

$$\frac{w-2p_a x_a}{x_a} = p_b\left(\frac{2\lambda p_a}{\alpha}\right)^{\frac{1}{1-\alpha}}$$

$$\frac{w}{x_a} - 2p_a = p_b\left(\frac{2\lambda p_a}{\alpha}\right)^{\frac{1}{1-\alpha}}$$

$$\frac{w}{x_a} = p_b\left(\frac{2\lambda p_a}{\alpha}\right)^{\frac{1}{1-\alpha}} + 2p_a$$

$$\boxed{x_a = w\left[p_b\left(\frac{2\lambda p_a}{\alpha}\right)^{\frac{1}{1-\alpha}} + 2p_a\right]^{-1}}$$

$$(1-\alpha)x_a^{\alpha} x_b^{-\alpha} = \lambda p_b$$

$$(1-\alpha)\left(\frac{w-p_b x_b}{2p_a}\right)^{\alpha} x_b^{-\alpha} = \lambda p_b$$

$$\left(\frac{w-p_b x_b}{2p_a}\right) x_b^{-1} = \left(\frac{\lambda p_b}{1-\alpha}\right)^{\frac{1}{\alpha}}$$

$$\frac{w}{x_b} - p_b = 2p_a\left(\frac{\lambda p_b}{1-\alpha}\right)^{\frac{1}{\alpha}}$$

$$\frac{w}{x_b} = 2p_a\left(\frac{\lambda p_b}{1-\alpha}\right)^{\frac{1}{\alpha}} + p_b$$

$$\boxed{x_b = w\left[2p_a\left(\frac{\lambda p_b}{1-\alpha}\right)^{\frac{1}{\alpha}} + p_b\right]^{-1}}$$

So our Marshallian demand is

$$x(p,w) = \begin{cases} x_1 = x_2 = x_a \\ x_3 = \begin{cases} x_b & \text{if } p_3 < p_4 \\ 0 & \text{if } p_3 > p_4 \\ 0 \le c \le x_b & \text{if } p_3 = p_4 \end{cases} \\ x_4 = \begin{cases} x_b & \text{if } p_4 < p_3 \\ 0 & \text{if } x_4 > p_3 \\ 1-c & \text{if } p_3 = p_4 \end{cases} \end{cases}$$

Our indirect utility function is:

$$v(p,w) = \left[ w \left[ p_b \left( \frac{2\lambda p_a}{\alpha} \right)^{\frac{1}{1-\alpha}} + 2p_a \right]^{-1} \right]^{\alpha} \left[ w \left[ 2p_a \left( \frac{\lambda p_b}{1-\alpha} \right)^{\frac{1}{\alpha}} + p_b \right]^{-1} \right]^{1-\alpha}$$

Good 1 is normal (inc. in $w$).

1b) $e(p,u) = \min \{ p_a x_a + p_b x_b \}$ s.t $u(x) \geq u$.

$h_1(p,u) = \operatorname{argmin} \; p \cdot x$ s.t $u(x) \geq u$.

$$= \operatorname{argmin} \left\{ p_a \left[ w \left[ p_b \left( \frac{2\lambda p_a}{\alpha} \right)^{\frac{1}{1-\alpha}} + 2p_a \right]^{-1} \right] + p_b \left[ w \left[ 2p_a \left( \frac{\lambda p_a}{1-\alpha} \right)^{\frac{1}{\alpha}} + p_b \right]^{-1} \right] \right.$$

s.t $v(p,w) \geq u$.

Good 2 is a complement for good 1.
Goods 3 & 4 are substitutes for good 1.

1c) If $p_3 > p_4$, the consumer will be a net seller of good 3. Demand for good 1 is decreasing in $p_3$ (increasing in $-p_3$), so consumption of good 1 will increase.

1d) If $p_3 < p_4$, the consumer will be a net ~~buyer~~ seller of $x_3$ when $x_a^{\alpha} > x_3^{1-\alpha}$ and a net buyer when $x_a^{\alpha} < x_3^{1-\alpha}$.

2) a)

| | at $p_1$: | at $p_2$: | at $p_3$: |
|---|---|---|---|
| $y_1$ | * $10-3-4=3$ | $20-3-4=13$ | $10-3-8=-1$ |
| $y_2$ | $15-6-8=1$ | * $30-6-8=16$ | $15-6-16=-7$ |
| $y_3$ | $8-5-1=2$ | $16-5-1=10$ | * $8-5-2=1$ |

The data is consistent with a profit maximizing firm since $p_i \cdot y_i > p_i \cdot y_j$ for all $i,j$. The smallest production set that rationalizes the data is $Y = \{y_1, y_2, y_3\}$.

b) The data is convex if for all $y, y' \in Y$, $y \neq y'$, $t \in (0,1)$ $ty + (1-t)y' \in int(Y)$. This production set is convex.

c) ~~Supermodular if~~

The production plan is not supermodular because $f(z_1, z_2)$ does not have increasing differences in $z_1$. When $z_1$ increased from 3 to 5, output decreased from 10 to 8. The plan would be supermodular if $y_3 = (8, -2, -1)$.

3) a) - Preferences $\succsim_{mnx}$ are complete b/c for any $L, L'$, either $L \succsim_{mnx} L'$ or $L' \succsim_{mnx} L$. (either $L_* \geq L'_*$ or $L'_* \geq L_*$)

- $\succsim_{mnx}$ are transitive because for any $L, L', L''$, if $L \succsim_{max} L'$, then $L_* \geq L'_*$. If $L' \succsim_{max} L''$, then $L'_* \geq L''_*$. So $L_* \geq L''_*$, so $L \succsim_{mnx} L''$.

- preferences $\succsim_{mnx}$ are not continuous. for any sequence $\{L_n\} \to \bar{L}_*$ and $\{L'_n\} \to \bar{L}'_*$, $L \succsim_{mnx} L' \nrightarrow \bar{L} \succsim \bar{L}'$. The min of the convergence may not hold in the inequality

- The preferences $\succsim_{mnx}$ are not independent.

$\succsim_{mnx}$ can't be represented by the expected utility function b/c $U(L)$ does not evaluate the lottery <u>purely</u> based on the worst case outcome with a positive probability. For example,

$U(L_1) = (0.999)(1000) + (0.001)(-100$[redacted]$) >$

$U(L_2) = 0.9(1000) + (0.1)(-10)$

But $\succsim$ would indicate that $L_2 \succsim_{mnx} L_1$.

b) $U(L) = \sum_{i=1}^{k} p_i(1-e^{-cL_i}) = 1 + \sum_{i=1}^{k} -p_i e^{-cL_i}$

$U(L') = \sum_{i=1}^{k} p'_i(1-e^{-cL'_i}) = 1 + \sum_{i=1}^{k} -p'_i e^{-cL'_i}$

If $L \succsim_{mnx} L'$, then as $c \to \infty$, $U(L') < U(L)$.

c) For $c$ sufficiently large, $L$ is preferred to $L'$. As $c \to \infty$, CARA utility approaches 1