Econ 711 Problem Set 1

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Question 1

Let $t_i \in S_i$ be strictly dominated by $x_i \in \Delta S_i$ such that $u(x_i) > u(t_i)$. Then for any $y_i \in \Delta S_i$ and $p \in (0,1)$, consider the mixed strategies $\sigma_i = pt_i + (1-p)y_i$ and $z_i = px_i + (1-p)y_i$. Then $u(x_i) > u(t_i) \Rightarrow u(z_i) > u(\sigma_i)$. Thus any mixed strategy σ_i with t_i in its support is strictly dominated.

Question 2

Part A

$N = \{1, 2\}$. For each player, $S_i = \{2, ..., 500\}$, and for each player,

$$u(s_1, s_2) = \begin{cases} (s_1 + 2, s_1 - 2) & \text{if } s_1 < s_2 \\ (s_1, s_2) & \text{if } s_1 = s_2 \\ (s_2 - 2, s_2 + 2) & \text{if } s_1 > s_2 \end{cases}$$

Part B

Suppose that player 1 thinks player 2 will choose some $s_2' \in \{2, ..., \bar{s_2}\}$. Then

$$u_1(s_1, s_2') = \begin{cases} s_2' - 2 & \text{if } s_1 > s_2' \\ s_2' & \text{if } s_1 = s_2 \\ s_2' + 1 & \text{if } s_1 = s_2' - 1 \\ s_2' & \text{if } s_1 = s_2' - 2 \\ s_1' + 2 < s_2' & \text{if } s_1 < s_2' - 2 \end{cases}$$

So player 1's best choice is to play $s_1' = s_2' - 1 < \bar{s_2}$.

Part C

Consider the maximum choice (500, 500). At this selection, the players will each yield a payoff of \$500. Knowing this, player 1's best response is to choose 499, so that they receive a payoff of \$501 and player 2 receives a payoff of \$497. However, then player 2's best response is to choose 498 so

*I have discussed this problem set with Emily Case, Michael Nattinger, Alex Von Hafften, and Danny Edgel.

that they get a payoff of \$500 and player 1 gets a payoff of \$496. This pattern iterates backwards until the players choose (2,2), where each player gets a payoff of \$2.

Question 3

Part A

No pure strategies are strictly dominated by other pure strategies. For player 1, since we can't make a non-pure mixed strategy that doesn't include both T and B in the support, it must be the case that neither T nor B is dominated.

For player 2, we clearly can't make a non-pure mixed strategy of L and C that dominates R, and we clearly can't make a non-pure mixed strategy of R and C that dominates L. In order to make a non-pure mixed strategy of L and R that dominates C, the following criteria must be met for some $p \in (0,1)$:

$$
\begin{aligned}
& u_2(T,C) < pu_2(T,L) + (1-p)u_2(T,R) \\
& \quad \Rightarrow 6 < 4p + 7(1-p) \\
& \quad \Rightarrow p < \frac{1}{3} \\
& \quad \text{and} \\
& u_2(B,C) < pu_2(B,L) + (1-p)u_2(B,R) \\
& \quad \Rightarrow 5 < 9p + (1-p) \\
& \quad \Rightarrow p > \frac{1}{2}
\end{aligned}
$$

Clearly it is not possible for $\frac{1}{2} < p < \frac{1}{3}$, so there cannot be any mixed strategy of L and R that dominates C.

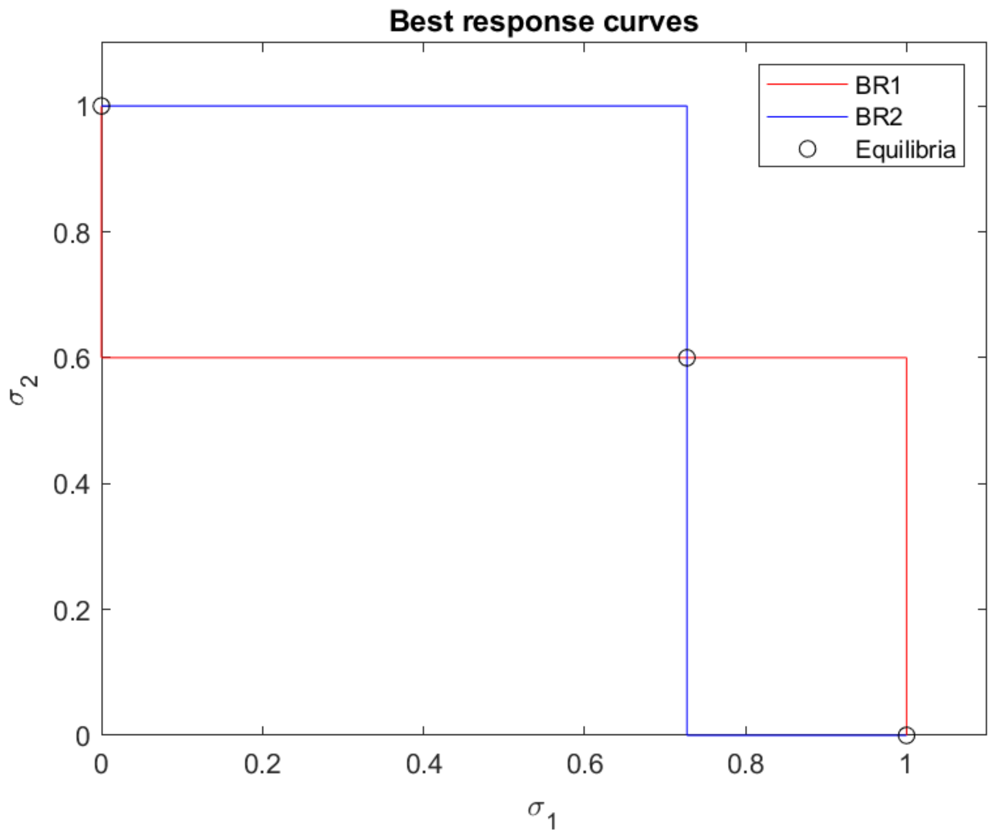
Part B

Let us first consider the pure strategies:

- If player 1 thinks player 2 will choose L, player 1 will choose B. If player 1 chooses B, player 2 will choose L. So (B,L) is a Nash equilibrium.
- If player 1 thinks player 2 will choose C, player 1 will choose B. If player 1 chooses B, player 2 will choose L. So C is not part of a closed rationalizable cycle.
- If player 1 thinks player 2 will choose R, player 1 will choose T. If player 1 chooses T, player 2 will choose R. So (T,R) is a Nash equilibrium.

Now let us consider mixed strategies. Let $\sigma_T \in (0,1)$ be the probability that player 1 chooses T and let $\sigma_L \in (0,1)$ be the probability that player 2 chooses L. Since C is not part of a closed rationalizable cycle, we know that $1-\sigma_T$ is the probability that player 1 chooses B and $1-\sigma_L$ is the probability that player 2 chooses R. Then a Nash equilibrium occurs when

$$
\begin{aligned}
4\sigma_T + 9(1-\sigma_T) = 7\sigma_T + (1-\sigma_T) \Rightarrow \sigma_T = \frac{8}{11} \\
8(1-\sigma_L) = 2\sigma_L + 5(1-\sigma_L) \Rightarrow \sigma_L = \frac{3}{5}
\end{aligned}
$$



Question 4

If player 1 thinks player 2 will choose a, player 1 chooses A, then player 2 chooses b, then player 1 chooses B, then player 2 chooses c, then player 1 chooses C, then player 2 chooses d, then player 1 chooses D, then player 2 chooses a. The loop (A, a), (A, b), (B, b), (B, c), (C, c), (C, d), (D, d), and (D, a), and (A, a) is rationalizable.

If player 1 thinks player 2 will choose e, player will will choose E. Then player 2 will choose e. So (E,e) is rationalizable.

Question 5

Part A

$N = \{1, 2\}$. For each player, $S_i = \{$ seek approval, do not seek approval$\}$. For each player,

$$u(s_1, s_2) = \begin{cases} (R - c, 0) & \text{if firm 1 seeks approval and firm 2 does not seek approval} \\ (r - c, r - c) & \text{if both firms seek approval} \\ (0, R - c) & \text{if firm 2 seeks approval and firm 1 does not seek approval} \\ (0, 0) & \text{if neither firm seeks approval} \end{cases}$$

Part B

In order for seeking approval to be a strictly dominant strategy, the payoff from seeking approval must be higher than the payoff from not seeking approval regardless of what the other firm does. Considering the lowest return possible from seeking approval, we can see that $r - c > 0 \Rightarrow r > c$.

In order for not seeking approval to be a strictly dominant strategy, the payoff from not seeking approval must be higher than the payoff from seeking approval regardless of what the other firm does. Considering the highest return possible from seeking approval, we can see that $R - c < 0 \Rightarrow R < c$.