Problem Set 6

Due on Canvas Wednesday September 30, 11pm Central Time

(1) Alice lives on the farm "Happy Cow"[1], which is located in the forest 5 kilometres from the main road. Bob lives in the house, located on the Main road and 13 kilometres from Alice's farm (see Figure 1). Bob wants to visit his friend Alice. He walks with the speed 5km/hour on the road and 3km/hour in the forest. What is the smallest time Bob needs to reach "Happy Cow" from home?

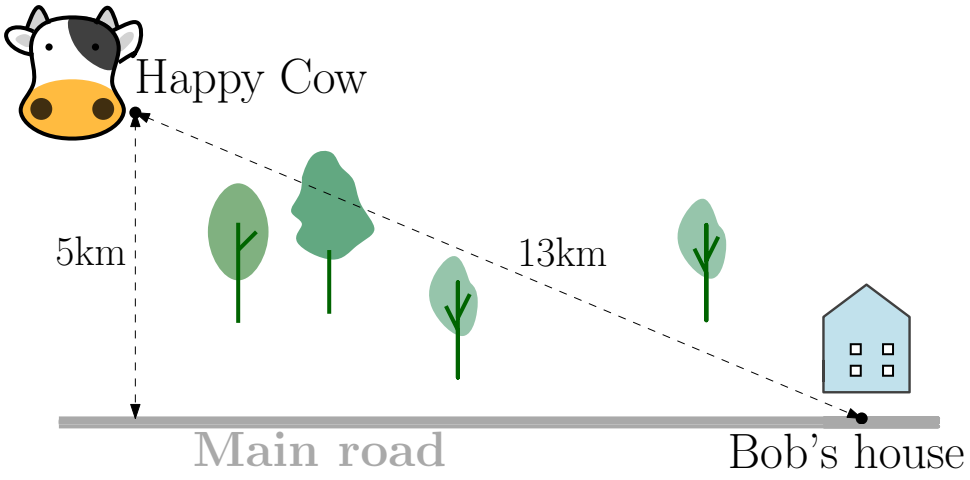

FIGURE 1. Illustration to Problem 1.

(2) Suppose that a function $f : \mathbb{R} \to \mathbb{R}$ is differentiable on $B_\varepsilon(x_0)$ for some $x_0 \in \mathbb{R}$, $\varepsilon > 0$. Suppose also that $f'(x) < 0$ for any $x \in B_\varepsilon(x_0) \setminus \{x_0\}$. Can the point x_0 be a local maximum or minimum of f?

(3) Let $w = f(x, y, z) = xy^2z$, with

$$x = r + 2s + t,\ y = 2r + 3s + t,\ z = 3r + s + t.$$

Use the chain rule to calculate $\partial w/\partial r$, $\partial w/\partial s$, $\partial w/\partial t$.

(4) Let $f : X \to \mathbb{R}^n$ be a continuously differentiable function on the open set $X \subset \mathbb{R}^n$. Show that f is locally Lipschitz on X (use Euclidean distance).

(See the end of Lecture 4 for the definition of locally Lipschitz.)

[1] On, Wisconsin!

(5) Let $f(x,y) = x^5 - x^2 + x - y^3 - 2y + 2$ and let $x(y)$ satisfy $x(1) = 1$ and $f(x(y), y) = 0$. Calculate $\frac{\partial x(y)}{\partial y}\Big|_{y=1}$.

(6) Find all local minima/maxima of $f(x,y) = 2x^4 + y^2 - xy + 1$. Does it have a global maximum/minimum?