Answer Key to Homework #1

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1. Prove the following statement: If $x \in \phi$ then x is a blue banana. (Hint: Use a contrapositive proof).

   We must prove that if x is not a blue banana, then $x \notin \phi$. Since for any x it is true that $x \notin \phi$, this must also hold if x is not a blue banana. Hence the contrapositive of the proposition holds.

2. Consider an exchange economy, in which the utility functions and endowments are a continuous function of a vector of parameters $\rho \in \mathbb{R}^k$. Let $E(\varrho)$ denote the set of competitive equilibrium prices of this exchange economy. Let $p \in E(\rho)$, and interpret the following statement:

   For every $\varepsilon > 0$ there exists $\delta > 0$ such that for all ρ' satisfying $||\rho - \rho'|| < \delta$ there exists $p' \in E(\rho')$ such that $||p - p'|| < \varepsilon$.

   Find the negation of this statement.

   The statement says that nearby economies have nearby equilibria. The negation of the statement is that there exists $\varepsilon > 0$ such that for all $\delta > 0$, there exists ρ' satisfying $||\rho - \rho'|| < \delta$ such that for all $p' \in E(\rho')$ we have $||p - p'|| \geqslant \varepsilon$.

3. Write the contrapositive and converse of the following statement: "If $x < 0$, then $x^2 - x > 0$", and determine which (if any) of the three statements is true.

   The contrapositive of the statement is that $x^2 - x \leqslant 0$ implies that $x \geqslant 0$. The converse is that $x^2 - x > 0$ implies $x < 0$. The original statement is true, as can be seen by the following argument. We have $x^2 \geq 0 > x$, so $x^2 - x > 0$. The contrapositive is true, as we have $x \geq x^2 \geq 0$, so $x \geq 0$. Finally, the converse statement is false, as can be seen by taking $x = 2$.

4. Let $f : \mathbb{R} \to \mathbb{R}$ be given by the rule $f(x) = x^3 - x$. By restricting the domain and range of f appropriately, obtain from f a bijective function g. Draw the graphs of g and g^{-1}(there are several possible choices for g).

   Let $g : [0, \frac{1}{\sqrt{2}}] \to [0, -\frac{1}{2\sqrt{2}}]$. Then g is strictly decreasing on its domain, and hence invertible.

5. Sundaram, #5, p. 67.

   Let x_n in $\mathbb{R}$ be given by

$$x_n = \begin{cases} n, & \text{if } n \text{ is even} \\ \frac{1}{n}, & \text{if } n \text{ is odd} \end{cases}$$

   Then $\{x_n\}$ has a convergent subsequence given by $\{x_{2n-1}\}$ and $x_{2n-1} \to 0$. However, the sequence $\{x_n\}$ does not converge, because it contains the divergent subsequence $\{x_{2n}\}$.