Monetary Policy

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- two sources of inefficiency:
  - markups
  - sticky prices
- eliminate markups with a subsidy τ.

Baseline model is summarized with the two equations:

1. NKIS: output gap

$$\sigma \mathbb{E}_t \Delta x_{t+1} = i_t - \mathbb{E}_t \pi_{t+1} - r_t^n, \tag{1}$$

2. NKPC: inflation

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1}. \tag{2}$$

prices are sticky but Fed adjusts interest rate s.t prices don't move

Divine coincidence if the flexible-price equilibrium of the model is efficient, then there is no inflation-output trade-off and the optimal policy replicates the first-best allocation. In particular, the planner can always fully offset demand shocks. More surprisingly, in contrast to the IS-LM analysis, the NK model shows that neither temporary nor permanent productivity shocks generate a policy trade-off.

don't necessarily have flexible prices but can achieve that allocation

first best allocation: no frictions, flexible prices, no inflation, no output gap

Dixit Stiglitz pricing holds with a production subsidy

The Fed can only target one friction at a time!

Commitment becomes important when the divine coincidence does not hold and there is a meaningful trade-off between inflation and output. In particular, introducing markup shocks implies that the flexible-price allocation is no longer efficient and the monetary policy can potentially implement a better equilibrium. Let x_t denote the output deviation from the first-best allocation, while u_t is the markup shock that creates a wedge between the first-best and flexible-price allocation. The planner's problem is

LS relation

sticky prices: inflation comes from some firms changing P and others being left behind

$$\min_{\{x_t,\pi_t,i_t\}} \frac{1}{2}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[x_t^2 + \alpha\pi_t^2\right]$$

Ramsey problem

→ primal approach- remove the prices, only look at allocations. In this case we can drop the 1st constraint (i_t) $i_t \geq 0$

$$\text{s.t. } \sigma \mathbb{E}_t \Delta x_{t+1} = i_t - \mathbb{E}_t \pi_{t+1} - r_t^n,$$

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + \underbrace{u_t}_{\text{markup shock}},$$

Labor Supply: $c_t = a_t$

Note $x_t = c_t - a_t$

↑ if $\neq 0$, violates LS condition

where $\alpha \equiv \frac{\theta}{(\sigma+\phi)\kappa}$. A few things to notice:

— The objective function is derived using the second-order approximations, which involve tedious algebra. The important contribution of the NK literature, however, is that it is possible to obtain the welfare function from microfoundations. The steady state is assumed to be efficient due to production subsidy.

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— We follow the *primal approach*: solve for the optimal allocation rather than the optimal value of policy instruments. In particular, the NKIS can be dropped as a side equation that determines i_t. $\mathcal{L} = \frac{1}{2}\sum\left[\beta^t(x_t^2 + \alpha\pi_t^2) + \lambda_t(\pi_t - \kappa x_t - \beta\pi_{t+1} - u_t)\right]$

Given the certainty equivalence, take the FOCs ignoring the expectation operator:

$$\beta^t x_t = \kappa\lambda_t,$$

$$\beta^t\alpha\pi_t = \beta\lambda_{t-1} - \lambda_t \text{ if } t \geq 1, \quad \text{and} \quad \beta^t\alpha\pi_t = -\lambda_t \text{ if } t = 0.$$

Combing the equations, the optimal policy rule is

$\beta^t\alpha\pi_t = \beta(\beta^{t-1}x_{t-1}) - \beta^t x_t$

$\alpha\pi_t = x_{t-1} - x_t$

$$\alpha\kappa\pi_t + \Delta x_t = 0 \tag{3}$$

for periods $t \geq 1$ and

$\pi_t = p_t - p_{t-1}$

$$\alpha\kappa\pi_0 + x_0 = 0 \tag{4}$$

for the initial period. Define $\hat{p}_t = p_t - p_{-1}$ as the deviation of the price level from the initial level. It follows the optimal rule implements

NOTE!
$x_{-1} = 0$
$p_{-1} \neq 0$
necessarily

LR:
$x_t \to 0$
$\hat{p}_t \to 0$
$p_t \to p_{-1}$

$$\alpha\kappa\hat{p}_t + x_t = 0 \tag{5}$$

in any period t. Thus, as long as the shocks are stationary, both the output gap and prices converge to the initial steady-state level in the long run. This policy is much closer to price targeting rather than inflation targeting.

One can solve explicitly for prices and output gap along the transition path. Substitute the optimal rule into the NKPC and rewrite it as follows:

$$\begin{bmatrix} -\beta & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} E\hat{p}_{t+1} \\ \hat{p}_t \end{bmatrix} = \begin{bmatrix} -1-\beta-\alpha\kappa^2 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \hat{p}_t \\ \hat{p}_{t-1} \end{bmatrix}$$

First order system

2nd order diff eq'n:

$$-\beta\mathbb{E}_t\hat{p}_{t+1} + [1 + \beta + \alpha\kappa^2]\hat{p}_t - \hat{p}_{t-1} = u_t.$$

The roots of the equation on the left-hand side are equal

$$\lambda_{1,2} = \frac{1}{-2\beta}\left((1 + \beta + \alpha\kappa^2) \pm \sqrt{(1 + \beta + \alpha\kappa^2)^2 - 4\beta}\right),$$

- one forward looking var
- one backward looking var

→ $\lambda_1 > 1$, $\lambda_2 < 1$

where $\lambda_1 > 1$ and $\lambda_2 < 1$. The dynamic equation can then be written as

$$-\beta(1 - \lambda_1 L)(1 - \lambda_2 L)L^{-1}\hat{p}_t = u_t,$$

lag operator

where did this come from?

see next page

Where $\lambda_1 \lambda_2 = \frac{1}{\beta}$ came from:

$$\lambda_{1,2} = \frac{1}{2\beta}\left((1+\beta+\alpha\kappa^2) \pm \sqrt{(1+\beta+\alpha\kappa^2)^2 - 4\beta}\right),$$

Note Dima has this equation wrong in notes, corrected here

$$\lambda_1 \lambda_2 = \frac{1}{4\beta^2}\left((1+\beta+\alpha\kappa^2) + \sqrt{(1+\beta+\alpha\kappa^2)^2 - 4\beta}\right)\left((1+\beta+\alpha\kappa^2) - \sqrt{(1+\beta+\alpha\kappa^2)^2 - 4\beta}\right)$$

$$\lambda_1 \lambda_2 = \frac{1}{4\beta^2}\left[(1+\beta+\alpha\kappa^2)^2 - (1+\beta+\alpha\kappa^2)^2 + 4\beta\right]$$

$$\lambda_1 \lambda_2 = \frac{1}{4\beta^2} \cdot 4\beta$$

$$\lambda_1 \lambda_2 = \frac{1}{\beta}$$

Where the lag eq'n came from:

$$-\beta(1-\lambda_1 L)(1-\lambda_2 L)L^{-1}\hat{p}_t = u_t,$$

$\lambda_1 \lambda_2 = \frac{1}{\beta}$

$\rightarrow \lambda_2^{-1} = \lambda_1 \beta$

$$(\beta\lambda_1 L - \beta)(1-\lambda_2 L)L^{-1} p_t = u_t$$

$$(\beta\lambda_1 L - \beta - \beta\lambda_1\lambda_2 L^2 + \beta\lambda_2 L)L^{-1} p_t = u_t$$

$$(\beta\lambda_1 - \beta/L - \beta\lambda_1\lambda_2 L + \beta\lambda_2) p_t = u_t$$

$$\beta\lambda_1 p_t - \beta/L\, p_t - \beta\lambda_1\lambda_2 L p_t + \beta\lambda_2 p_t = u_t$$

$$\beta\lambda_1 p_t - \beta E_t p_{t+1} - \beta\lambda_1\lambda_2 p_{t-1} + \beta\lambda_2 p_t = u_t$$

$$-\beta E_t p_{t+1} + \beta(\lambda_1+\lambda_2) p_t - \beta\lambda_1\lambda_2 p_{t-1} = u_t$$

Note,

$$-\beta\mathbb{E}_t\hat{p}_{t+1} + [1+\beta+\alpha\kappa^2]\hat{p}_t - \hat{p}_{t-1} = u_t.$$

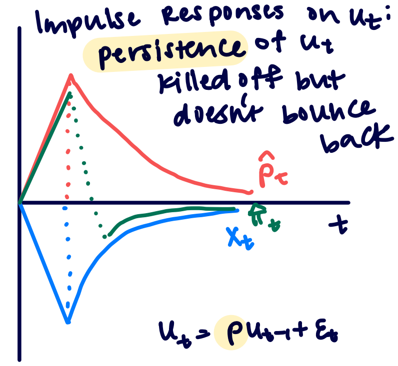
$\beta(\lambda_1+\lambda_2) = 1+\beta+\alpha\kappa^2$ bc $\frac{1}{2\beta}(1+\beta+\alpha\kappa^2 + \sqrt{\cdots}) + \frac{1}{2\beta}(1+\beta+\alpha\kappa^2 - \sqrt{\cdots})$

$\beta\lambda_1\lambda_2 = 1$ bc $\lambda_1\lambda_2 = 1/\beta$ as shown above

Where does this come from?
see previous page

where L is the lag operator. Given $\lambda_1\lambda_2 = 1/\beta$, we get

$$(1-\beta\lambda_2 L^{-1})(1-\lambda_2 L)\hat{p}_t = \lambda_2 u_t.$$



Assume that u_t follows AR(1) process with parameter ρ to obtain $\to L^{-1} = \rho$

$$\hat{p}_t = \lambda_2\hat{p}_{t-1} + \overbrace{\frac{\lambda_2}{1-\rho\beta\lambda_2}}^{\lambda_2 \sum_{j=0}^{\infty}(\rho\beta\lambda_2)^j E_t u_{t+j}} u_t, \qquad x_t = \lambda_2 x_{t-1} - \frac{\alpha\kappa\lambda_2}{1-\rho\beta\lambda_2}u_t.$$

This expression shows that the optimal policy smooths output gap in time: a one-time shock ($\rho = 0$) generates a jump ($\pi_0 > 0, x_0 < 0$) followed by a gradual reversal ($\pi_t < 0$, $x_0 < 0,\ t > 0$). Intuitively, from the NKPC, to avoid higher inflation and lower output in period $t = 0$, the planner promises a negative inflation in the future.

Timeless perspective was proposed by Woodford (1999) and is one way to address the difference between policy rules (3)-(4). Keeping the assumption of full commitment, the basic idea of the timeless perspective is to consider a planner who solves the problem before observing any shocks and before private agents form their expectations, i.e. in period $t = -1$, and chooses a state-contingent response to future shocks. The optimal rule for period $t = -1$ is irrelevant because of the absence of shocks, while the rule becomes $\alpha\kappa\pi_t + \Delta x_t = 0$ for all periods $t \geq 0$. The equilibrium dynamics is then the same as above except that now we can impose $p_{-1} = 0$ and hence, $\hat{p}_t$ is replaced with p_t.

Discretionary policy is different from the policy under commitment in the presence of markup shocks. Indeed, the monetary rule in the initial period (4) is different from the rule (3) in other periods generating a time inconsistency. If the planner can reoptimize its decisions every period, the optimal discretionary policy implements (4) in every period:

$$\alpha\kappa\pi_t + x_t = 0. \tag{6}$$

at t, $\min \frac{1}{2} E[x_t^2 + \alpha\pi_t^2]$ s.t. NKPC holds — no sum

Substituting the optimal rule into the NKPC, we get

$= \frac{\beta}{1+\alpha\kappa} E\pi_{t+2} + \frac{1}{1+\alpha\kappa}u_{t+1}$ $\qquad \frac{1}{(1+\alpha\kappa)}\frac{u_{t+j}}{1+\frac{\beta\rho}{1+\alpha\kappa}} = \frac{u_{t+j}}{1+\alpha\kappa+\beta\rho}$

$$\pi_t = \underline{\frac{\beta}{1+\alpha\kappa}}\mathbb{E}_t\pi_{t+1} + \frac{1}{1+\alpha\kappa}u_t = \cdots = \frac{1}{1+\alpha\kappa}\mathbb{E}_t\sum_{j=0}^{\infty}\left(\frac{\beta}{1+\alpha\kappa}\right)^j u_{t+j} = \frac{1}{1+\alpha\kappa-\beta\underline{\rho}}u_t.$$

acts as discount factor $\qquad$ ρ comes from $E_t u_{t+j} = \rho^j u_t$

It follows that both inflation and output gap follow the same stochastic process as the markup shocks. In particular, the optimal discretionary response to an idiosyncratic shock ($\rho = 0$) is a one-time inflation and output gap and no deviations in the future periods. By definition, the welfare is lower in this case than under commitment.

Inflationary bias was first described by Kydland and Prescott (JPE'1977) and eventually brought them the Nobel Prize. To see it within our model, consider a permanent markup shock $u_t = u$. For simplicity, adopt the timeless perspective with the optimal rule (3), which implies $\pi_t = 0$ and $x_t = -u$, i.e. the planner tolerates the inefficiently low value of output. Adao, Correia and Teles (RES'2003) describe sufficient conditions for this result in a fully non-linear model. In contrast, the optimal discretionary policy (4) implies $\pi_t \approx \frac{1}{\alpha\kappa}u$, $x_t \approx -u$. Thus, the planner does not close the output gap, but introduces a positive inflation.

Forward guidance is the monetary policy that changes future interest rates to affect current output and inflation. Such policies clearly require much commitment and were considered largely redundant until the ZLB restricted the use of the short-term rates in Japan in 1990s and in several developed countries after 2008. To see how it works, consider the NK model with perfect foresight and suppose the monetary policy sets $i_t - \mathbb{E}_t\pi_{t+1} - r_t^n$ at zero for all periods except for period T when it is equal $-\sigma$. From the NKIS (1), it follows

$$x_0 = \cdots = x_T = x_{T+1} - 1 = x_{T+2} - 1 = \ldots.$$

Given that the economy converges to the steady state in the long-run, we have $x_\infty = 0$ and $x_0 = 1$ independently from the value of T. Thus, the forward guidance is as effective as the change in short-term rates in the model. Empirical evidence does not confirm this prediction giving rise to the "forward guidance puzzle".

No commitment → no first best allocation. But
Inflation targeting w/o commitment – $E\,\pi_{t+1} = 0$
→ welfare losses only affected by output gap, determined by markup shock
– rearrange NKPC w/ all πs $= 0$

NKIS – linearized Euler
NKPC – linearized price setting (firms)