# Sets:

1. $A \subset B \Leftrightarrow$ if $x \in A$ then $x \in B$
   $\Leftrightarrow$ if $x \notin B$ then $x \notin A$ (this statement is also often used in the proof)

   $A \subsetneq B \Leftrightarrow A \subset B$ and $A \neq B$

   $A=B \Leftrightarrow A \subset B$ and $B \subset A$

   $A \neq B \Leftrightarrow$ there exists $x \in A$ and $x \notin B$, or there exists $x \in B$ and $x \notin A$

2. $A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$
   $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$
   $A \cap B^c = A - B = \{ x \mid x \in A \text{ or } x \notin B \}$

   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   $(B \cap C)^c = B^c \cup C^c$
   $(B \cup C)^c = B^c \cap C^c$

3. $\mathscr{A}$ : collection of sets, only require the elements are sets. the element of $\mathscr{A}$ can be any set, not necessary the subset of A.
   P(A): power set of A
   power set must be related to some set A. It is a collection of all subsets of A.

   eg. $\mathscr{A} = \{ \{a\}, \{b\}, \{c\} \}$
   $A = \{ a, b, c \}$
   then which of the following statement are true?

   $\{a, b\} \in P(\mathscr{A})$ (Ans: False)
   $\{a, b\} \subset P(\mathscr{A})$ (Ans: False)
   $\{a, b\} \in P(A)$ (Ans: true)
   $\{a, b\} \subset P(A)$ (Ans: False)
   $\{ \{a\}, \{b\} \} \in P(\mathscr{A})$ (Ans: true)
   $\{ \{a\}, \{b\} \} \subset P(\mathscr{A})$ (Ans: False)
   $\{ \{a\}, \{b\} \} \in P(A)$ (Ans: False)
   $\{ \{a\}, \{b\} \} \subset P(A)$ (Ans: true)

   Therefore, a element of P(B) should be subset of B, and a subset of P(B) is a set containing subsets of B.

4. Notations about sets collections:

$\bigcup_{A\in \mathscr{A}} A = \{x \mid x\in A$ for some (at least one) $A \in \mathscr{A}\}$

$\bigcap_{A\in \mathscr{A}} A = \{x \mid x\in A$ for all (every) $A \in \mathscr{A}\}$

e.g $\mathscr{A} = \{ \{a\}, \{b\}, \{c\} \}$

then $\bigcup_{A\in \mathscr{A}} A = \{a\}\cup\{b\}\cup\{c\} = \{a,b,c\}$

$\bigcap_{A\in \mathscr{A}} A = \{a\}\cap\{b\}\cap\{c\} = \phi$

Suppose $E_a$, $A$ are sets, then

$\bigcup_{a\in A} E_a = \{x \mid x\in E_a$ for some (at least one) $a \in A\}$

$\bigcap_{a\in A} E_a = \{x \mid x\in E_a$ for all (every) $a \in A\}$

e.g. $A = \{1,3,5\}$ $E_1=\{1,2\}$ $E_2=\{4,5\}$ $E_3=\{2\}$ $E_4=\{3,4\}$ $E_5=\{2,3\}$

then $\bigcup_{a\in A} E_a = E_1\cup E_3\cup E_5=\{1,2,3\}$

$\bigcap_{a\in A} E_a = E_1\cap E_3\cap E_5=\{2\}$

$$\bigcup_{m=1}^{n} E_m = E_1 \cup E_2 \cup E_3 \ldots \cup E_m$$

$$\bigcap_{m=1}^{n} E_m = E_1 \cap E_2 \cap E_3 \ldots \cap E_m$$

5. $\phi \subset$ any set

$\phi \in$ any powerset.