Econ 712 Final Exam - Sarah Bass 12/16/20

1)

A) The social planner's problem is:

$$\max \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}$$

such that $c_t + K_{t+1} - (1-\delta) K_t = F(A_t K_t, N_t)$

Note, $Y_t = F(A_t K_t, N_t) = f(A_t K_t)$ since $N_t = 1$

So, $c_t = f(A_t K_t) - K_{t+1} + (1-\delta) K_t$

The Bellman equation is:

$$V(K, A) = \max_{K'} \left(\frac{(f(AK) - K' + (1-\delta)K)^{1-\gamma}}{1-\gamma} + \beta V(K', A') \right)$$

Taking FOCs and applying the envelope condition:

$$(f(AK) - K' + (1-\delta)K)^{-\gamma} = \beta V'(K', A')$$

$$V'(K, A) = f'(AK) + 1 - \delta)(f(AK) - K' + (1-\delta)K)^{-\gamma}$$

So the laws of motion governing K and C are:

$$C^{-\gamma} = \beta C'^{-\gamma}(f'(A'K') + 1 - \delta) \quad \text{our Euler eq'n}$$

$$K' = (1-\delta)K + f(AK) - C$$

B) We can normalize $c_t = C_t / A_t$, $k_t = K_t / A_t$

(New c, k are lowercase, sorry if its not clear from my handwriting!)

Then our transformed laws of motion are:

$$(Ac)^{-\gamma} = \beta (A'c')^{-\gamma}(f'(k') + 1 - \delta)$$

$$A'k' = (1-\delta)(Ak) + f(k) - Ac$$

Note, $A' = (1+g)A$ So

$$c^{-\gamma} = \beta (1+g)^{-\gamma} c'^{-\gamma} (f'(k') + 1 - \delta)$$

$$(1+g) k' = (1-\delta) k + f(k)/A - c$$

At the steady state:

$$\bar{c} = (1+g)\bar{k} - (1-\delta)\bar{k} - f(\bar{k})/A$$

$$\bar{k} = (f')^{-1}\left(\frac{1}{\beta(1+g)^{-\gamma}} - 1 + \delta \right)$$

1c) Suppose there is a 1 time increase in g to g'. On impact, the system would jump to the new saddle path with the same level of capital and an increase in consumption. Over time, the system would travel along the saddle path to a new steady state with higher levels of consumption and capital

2 A) We can conjecture that prices are a function of dividends and preference shocks, $p_t = p(s_t, B_t)$

So our constraint becomes:

$$c_t + p(s_t, B_t)a_{t+1} = (p(s_t, B_t) + s_t)a_t$$

The Bellman equation is:

$$V(a, s, B) = \max_{a'} \left(u((p(s,B)+s)a - p(s,B)a', B) + \beta E[V(a', s', B' | s, B] \right.$$

Taking FOCs and applying the envelope condition we have

$$p(s,B)\,u_1((p(s,B)+s)a - p(s,B)a', B) = \beta E[V'(a', s', B' | s, B)]$$

$$V'(a, s, B) = (p(s,B)+s)\,u_1((p(s,B)+s)a - p(s,B)a', B)$$

Our Euler equation is:

$$u_1(c, B) = \beta \iint u_1(c', B') \left[\frac{p(s,B)+s'}{p(s,B)} \right] Q(s, ds')\, F(B, s, dB')$$

We also know that under a competitive equilibrium,

$$c = s, \quad a' = a = 1$$

2 B) A competitive equilibrium is a set of allocations $\{c_t\}_{t=0}^{\infty}$ and prices $\{p_t, s_t\}_{t=0}^{\infty}$ such that agents optimize and markets clear.

A recursive competitive equilibrium is an initial distribution $\{a_0\}$, pricing kernel $Q(s, ds')$, transition function $F(B, s, dB')$, value function $V(a, s, B)$, with decision rules for c and a s.t.

1) Agents solve the Bellman
2) Markets clear.

2C) The equilibrium pricing function is:

$$p(s,B) = \beta \iint \frac{u'(s',B)}{u'(s,B)} \left(p(s',B')+s'\right) Q(s,ds') F(B,s,dB')$$

This differs from the standard case where B is constant because both the stochastic discount factor and future prices can change as a result of a preference shock. Consequently, there is higher volatility in the prices and more uncertainty under this system.

2D) Let $u(c_t, B_t) = B_t \log(c_t)$. Note $u_1(c,B) = B/c$. So our equilibrium pricing function becomes:

$$p(s,B) = \beta \iint \frac{B'}{c'}\left(\frac{c}{B}\right)\left(p(s',B')+s'\right) Q(s,ds') F(B,s,dB')$$

$$= \beta \iint \frac{B's}{s'B} \left(p(s',B')+s'\right) Q(s,ds') F(B,s,dB')$$

Asset 1 positively correlated with B'

Asset 2 positively correlated with s'

Asset 2 would have a higher price because of the s' in the $(p(s',B') + \underline{\underline{s'}})$ term.

3A) True - Dynamic programming just rewrites recursive problems using a Bellman equation.

3B) True - RCE and A-D equilibrium will provide initial distributions, pricing kernels, and decision rules s.t. agents optimize & markets clear.

3C) False - fluctuations in income may affect firms and governments as well as agents.

3D) True - in equilibrium heterogeneity across agents are merged across the distribution of agents. In aggregate, everyone is just one large mass which is unaffected by heterogeneity of individual agents.