University of Wisconsin-Madison
Department of Economics

Econ 703 Prof. R. Deneckere
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**Final Exam**

1. Let $f : \mathbb{R} \to \mathbb{R}^2$ be of class $C^3$ in a neighborhood of a point **a** in its domain. Suppose the gradient of f is **0** at **a**, and that not all second order derivatives of f are 0 at **a**. Show that one can then determine from the Taylor polynomial (of degree 2) of f at **a** whether f has a local maximum, or a local minimum, or neither, at the point **a**.

2. Let $f : \mathbb{R} \to \mathbb{R}^2$ and $g : \mathbb{R} \to \mathbb{R}^2$ be given by $f(x,y) = x^2 + y^2$ and $g(x,y) = (x-1)^3 - y^2$. Find the minimum of $f(x,y)$ subject to $g(x,y) \geq 0$.

3. Can the intersection of the surfaces $e^y - xyz = e$ and $x^2 + y^2 - z = 2$ be written in terms of differentiable functions $x^*(z)$ and $y^*(z)$ near $(0,1,-1)$? Defend your answer.

4. Consider the nonlinear program $\min f(x) = \sum_{j=1}^{n} \frac{c_j}{x_j}$, subject to $\sum_{j=1}^{n} a_j x_j = b$ and $x \geq 0$, where $a_j$, $b_j$, and $c_j$ are strictly positive constants. Provide an economic interpretation of this problem, and solve it.