Econ 713A Problem Set 1

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Question 1

If the men propose first, M1 will propose to W1, M2 will propose to W2, and M3 will propose to W3. Since all of the women are better off accepting the proposals than remaining single, they will all match in one round.

If the women propose first, W1 will propose to M3, W2 will propose to M1, and W3 will propose to M2. Since all of the men are better off accepting the proposals than remaining single, they will all match in one round. Thus the Gale-Shapley algorithm yields different outcomes depending on whether men or women propose first.

If the utilities for woman 1 change to (10,5,15) and the utilities for woman 3 change to (12,16,20), we have the following new payoff matrix:

	W1	W2	W3
M1	(10,10)	(8,3)	(6,12)
M2	(4,5)	(5,2)	(3,16)
M3	(6,15)	(7,1)	(8,20)

Using the new payoff matrix, if the men propose first, M1 will propose to W1, M2 will propose to W2, and M3 will propose to W3. Since all of the women are better off accepting the proposals than remaining single, they will all match in one round. This is the same as with the original payoff matrix.

However, if the women propose first, W1 will propose to M3, W2 will propose to M1, and W3 will propose to M3. M1 will accept the proposal from W2 because there is no better offer, and M3 will accept the proposal from W3 since it has the higher payoff. After getting rejected by M3, W1 will propose to M1, who is already engaged to W2. Since the payoff from being engaged to W1 is higher for M1 than the payoff from being engaged to W2, M1 will leave W2 for W1. Now a broken hearted W2 will propose to the only remaining option, M2, who will accept their offer. Thus the Gale-Shapley algorithm yields the same outcome regardless of whether men or women propose first.

*I have discussed this problem set with Emily Case, Michael Nattinger, Alex Von Hafften, Katherine Kwok, and Danny Edgel.

Question 2

Part A

When men move first, M1 proposes to W2, M2 proposes to W2, and M3 proposes to W3. W3 accepts M3's offer, and W2 accepts M2's offer. Then M1 will proppose to W1, who will accept.

When the women move first, W1 proposes to M2, W2 proposes to M3, and W3 proposes to M3. M2 accepts W1's offer, and M3 accepts W3's offer. W2 then proposes to M2, who leaves W1 for W2. W1 then proposes to M1, who accepts.

Since the same matching occurs when either men or women move first, the male-optimal matching is the same as the male-pessimal matching, so this is a unique stable match.

Part B

Once side transfers are possible, we have the following new payoff matrix with the efficient matching indicated in red:

	M1	M2	M3
W1	3	7	5
W2	4	6	5
W3	4	4	8

This is the efficient matching because the sum of these payoffs ($7 + 4 + 8 = 19$) is the maximum possible when selecting only one entry in each row and column.

Part C

By free entry and free exit, we know the following inequalities exist:

$$
\begin{aligned}
w_2 + v_2 &\geq 6 \\
w_2 + v_1 &\leq 4 \\
\Rightarrow v_2 - v_1 &\geq 2 \\
\Rightarrow v_2 &\geq 2 + v_1
\end{aligned}
$$

Since v_1 is bounded below by 0, we know that $v_2 \geq 2$.

Question 3

Part A

Type x matches with type y if

$$
\begin{aligned}
0 &\leq y + axy \\
\Rightarrow 0 &\leq 1 + ax \\
\Rightarrow x &\leq \frac{1}{-a}
\end{aligned}
$$

Similarly, type y matches with type x if

$$\begin{aligned}
&0 \leq x + axy \\
\Rightarrow\ &0 \leq 1 + ay \\
\Rightarrow\ &y \leq \frac{1}{-a}
\end{aligned}$$

So a match will occur if both $x \leq \frac{1}{-a}$ and $y \leq \frac{1}{-a}$.

Part B

We can solve for the wages of each type that centralize the market using our profit function:

$$\begin{aligned}
\pi = y + axy + x + axy - v(x) - w(y) &= 0 \\
\frac{\partial \pi}{\partial x} = 1 + 2ay - v'(x) &= 0 \\
\Rightarrow 1 + 2ax - v'(x) &= 0 \\
\Rightarrow v'(x) &= 1 + 2ax \\
\Rightarrow v(x) &= x + ax^2 + c_x \\
\frac{\partial \pi}{\partial y} = 1 + 2ax - w'(y) &= 0 \\
\Rightarrow 1 + 2ay - w'(y) &= 0 \\
\Rightarrow w'(y) &= 1 + 2ay \\
\Rightarrow w(y) &= y + ay^2 + c_y
\end{aligned}$$

Next we can solve for the constants as follows:

$$\begin{aligned}
\pi = y + axy + x + axy - (x + ax^2 + c_x) - (y + ay^2 + c_y) &= 0 \\
\Rightarrow 2axy - ay^2 - ax^2 - c_x - c_y &= 0 \\
\Rightarrow -a(x-y)^2 &= c_x + c_y
\end{aligned}$$

Note that since $-1 < a < 0$, $-a(x-y)^2 < 0$. Since c_x and c_y must be non-negative (due to non-negative utility), we can conclude that $c_x = c_y = 0$. Thus our wage functions are:

$$\begin{aligned}
v(x) &= x + ax^2 \\
w(y) &= y + ay^2
\end{aligned}$$

Question 4

Students 2, 4, 6,...,30 are lenders with available returns 3.04, 3.08, 3.12,...,3.60. Students 1, 3, 5,...,29 are borrowers with project returns 3.01, 3.03, 3.05,...,3.29. A borrower borrows if she can get an interest rate below her project's return, and a lender lends if she can get an interest rate above her available return. The lenders with the lowest available returns will lend to the borrowers with the highest available returns, so long as the interest rate is higher than all lender returns and lower than all borrower returns. So students 2, 4, 6, 8, and 10 (with available returns 3.04, 3.08, 3.12, 3.16, and 3.20, respectively) will lend to students 21, 23, 25, 27, and 29 (with available returns 3.21, 3.23, 3.25, 3.27, and 3.29, respectively), and the market-clearing interest rate is anything between 3.2% and 3.21%.