Money-in-the-Utility Model

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In other models, money is indicated with other assets: C, K.

Primitives of the endowment model:

1. preferences: $U = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}-1}{1-\sigma}+\kappa\frac{(M_t/P_t)^{1-\phi}-1}{1-\phi}\right)$,
2. endowment: Y_t is given. exogenous

We can introduce money as:
1) ad hoc
- money in utility
- transaction constraint
- cash in advance const.
2) deep theory of money
- search friction
- financial friction
- memory

Competitive equilibrium boils down to the household problem

$$\max \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}-1}{1-\sigma}+\kappa\frac{(M_t/P_t)^{1-\phi}-1}{1-\phi}\right)$$

$$\text{s.t. } C_tP_t + B_t + M_t = Y_tP_t + \underbrace{(1+i^b_{t-1})}_{\text{banks pay fed}}B_{t-1} + \underbrace{(1+i^m_{t-1})}_{\text{fed pays banks}}M_{t-1}.$$

borrow, ceiling, store money, floor

may not apply

- interbank rate between i^b_{t-1}, i^m_{t-1}

Take the FOC wrt C_t, B_t and M_t respectively:

Gov BC: $(1+i_{t-1})B_{t-1} = B_t + T_t + \Delta M_t$

$$\beta^t C_t^{-\sigma} = \lambda_t P_t, \qquad \lambda_t = \frac{\beta^t C_t^{-\sigma}}{P_t}$$

Market clearing: $C_t = Y_t$

$$\lambda_t = \mathbb{E}_t\lambda_{t+1}(1+i^b_t), \qquad \mathbb{E}_t\lambda_{t+1} = \frac{\lambda_t}{1+i^b_t}$$

$$\kappa\beta^t\frac{M_t^{-\phi}}{P_t^{1-\phi}} - \lambda_t + \mathbb{E}_t\lambda_{t+1}.(1+i^m_t) = 0,$$

where B_t and M_t are the amount of bonds and money held by a representative household, i^b_t and i^m_t are the respective nominal interest rates. Substitute in the Lagrange multiplier from the first equation into the second one to obtain a standard Euler equation:

$$\beta\mathbb{E}_t\underbrace{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}}_{\text{stochastic discount factor}}\frac{P_t}{P_{t+1}}(1+i^b_t) = 1. \tag{1}$$

Substitute the first two FOCs into the third one to obtain the optimal money demand:

$$\left(\frac{M_t}{P_t}\right)^{\phi} = \kappa\frac{1+i^b_t}{i^b_t - i^m_t}C_t^{\sigma}. \tag{2}$$

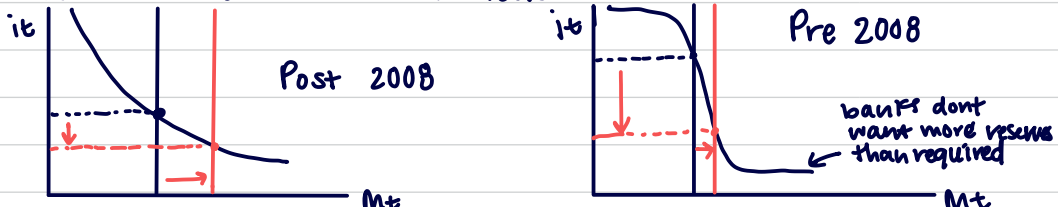
$\frac{M_t}{P_t} = f(y_t, i_t)$ **IS/LM model**

- When the interest rate goes up, demand for money goes down
  - interest rate is the opportunity cost of holding money.
- $M_t \geq 0 \rightarrow i_t > 0$ zero lower bound
  - lower bound of zero because holding money would have a higher return b/c you wouldn't be losing money.
  - now we do see countries with negative interest rates for banks, not for households. households would cause a bank rush. For banks, there are transaction costs, need for financial stability.
  - effective lower bound - interest rates can go negative, but not too negative
- Government instruments:
  - interest rate i_t } monetary tools
  - money supply M_t }
  - taxes T_t } fiscal tools
  - bonds B_t }
  - fiscal tools - fix $\{i_t, M_t\}$ and change $\{T_t, B_t\}$.
    - Ricardian equivalence - lower T today → higher T tomorrow
    - "monetary dominance"
  - monetary tools - fix $\{T_t, B_t\}$ and change $\{i_t, M_t\}$.
    - "fiscal dominance"
  - fiscal theory of price level - changes gov BC (unconventional)

$$\Delta M_t + B_t - (1+i_{t-1})B_{t-1} + \underbrace{P_t \cdot T_t}_{\text{real price level}} = P_t \cdot g_t$$

  - gov chooses $\{M_t, B_t, i_t, T_t \mid g_t\}$.

- 1980 - 2008 monetary policy
  - fed rate i_t - interbank rate



- Supply of reserves

- By changing the supply of reserves slightly, the interest rate changed dramatically before 2008. very erastic demand for money.
  - quantitative easing
- now the HHBC is:

$$C_t P_t + M_t + B_t = P_t Y_t + T_t + (1+i_{t-1})B_{t-1} + (1+i^M_{t-1})M_{t-1}$$

  - earn interest on money reserves

- can shift money demand

- Optimal monetary policy - $M_t/P_t \uparrow$, $i_t = 0$.
  - Friedman rule
  - $K \to 0$ "cashless limit"