University of Wisconsin

Microeconomics Prelim Exam

Tuesday, June 4, 2019: 9AM - 2PM

- There are four parts to the exam. All four parts have equal weight.
- Answer all questions. No questions are optional.
- Hand in 12 pages, written on only one side.
- Write your answers for different parts on different pages. So do not write your answers for questions in different parts on the same page.
- Please place a completed label on the top right corner of each page you hand in. On it, write your assigned number, and the part of the exam you are answering (I,II,III,IV). Do not write your name anywhere on your answer sheets!
- Show your work, briefly justifying your claims. Some solutions might be faster done by drawing a suitable diagram.
- You cannot use notes, books, calculators, electronic devices, or consultation with anyone else except the proctor.
- Please return any unused portions of yellow tablets and question sheets.
- There are five pages on this exam, including this one. Be sure you have all of them.
- Best wishes!

Part I

SoftDog is a small company on Madison's east side which turns recycled bicycle parts into robotic dogs. SoftDog's output of robotic dogs depends on the amount of inputs used — scrap metal from old bikes, silicon (for custom microchips), and labor — and the quality of management, since better management can "make more out of less." Specifically, output is

$$y = Mf(z_1, z_2, z_3)$$

where z_1 is old bike parts, z_2 is silicon, z_3 is labor, and M is management quality. Assume f is strictly increasing in all its arguments, and SoftDog is a price taker in both input and output markets. Let p be the price of robotic dogs, $g(M)$ the price of a management team of quality M, and (w_1, w_2, w_3) input prices, so profit is

$$pMf(z_1, z_2, z_3) - g(M) - w_1 z_1 - w_2 z_2 - w_3 z_3$$

1. Show that if g is linear, then SoftDog's profits must be either zero or infinite.

2. Suppose at each set of prices $(p, w_1, w_2, w_3) \gg 0$, the firm's problem has a unique solution. If f is supermodular, find the effect of an increase in the price of old bike parts (w_1) on SoftDog's use of each input, management quality, and output of robotic dogs.

3. Consider the consumer market for robotic dogs. Would you expect robotic dogs to be a gross complement or a gross substitute for real-live cats? Explain. Given your answer, what type of change in the price of cats would lead to an increase in the demand for robotic dogs?

Now consider the "pre-management" level of output $X = f(z_1, z_2, z_3)$ as an intermediate good, with production cost

$$c(X) = \min \{w_1 z_1 + w_2 z_2 + w_3 z_3\} \qquad \text{subject to} \qquad f(z_1, z_2, z_3) \geq X$$

You can then think of SoftDogs' problem as first choosing X and M to maximize

$$pMX - g(M) - c(X)$$

and then choosing (z_1, z_2, z_3) to minimize the cost of producing X.

4. Show that even if f is not supermodular, an increase in the price p leads to increases in both SoftDogs' output and the quality level of its management.

5. Let $f(z_1, z_2, z_3) = \sqrt{h(z_1, z_2, z_3)}$, where h is homogeneous of degree 1. Let $z^* = (z_1^*, z_2^*, z_3^*)$ be the cheapest way to produce one unit of the intermediate good at prices (w_1, w_2, w_3).

   Show that $X^2 z^*$ is the cheapest way to produce a given quantity X of the intermediate good; and therefore SoftDog's use of every input z_i increases in p.

Part II

1. There are two firms, each of which owns half of an entertainment complex. Firm $i \in \{1,2\}$ chooses the quality $x_i \in [0,1]$ of the offerings in its half of the complex. The firms' quality choices are complementary: if the firms' quality choices are x_1 and x_2, each firm earns a revenue of $12\,x_1x_2$. Firm i can secure a quality of $x_i = 0$ at zero cost; quality $x_i \in (0,1]$ costs the firm $3(x_i)^2 + 2x_i + \frac{1}{12}$.

   (a) What are the firms' payoff functions in this game?

   (b) Determine firm i's best response correspondence in this game.

   (c) Find all Nash equilibria of this game.

2. Let G^3 be a three-period repeated game with no discounting ($\delta = 1$) and the following normal form game G:

$$
\begin{array}{cc|c|c|c|c|}
 & & \multicolumn{4}{c}{2} \\
 & & A & B & C & D \\
\hline
 & A & 6,6 & 0,0 & 0,0 & 0,11 \\
\hline
 & B & 0,0 & 5,5 & 0,0 & 0,6 \\
\hline
1 & C & 0,0 & 0,0 & 3,3 & 0,0 \\
\hline
 & D & 11,0 & 6,0 & 0,0 & 1,1 \\
\hline
\end{array}
$$

   (a) What is player 1's set of pure strategies in G^3?

   (b) Construct a pure strategy subgame perfect equilibrium of G^3 with action profile (A,A) played in the initial period. Verify that it is a subgame perfect equilibrium.

Part III

1. Pandora is a risk-averse expected utility maximizer. She prefers playing a gamble: "win \$10 with chance p_1, lose \$10 with chance p_2, and otherwise win \$0" to "win \$2 for sure". What is the tightest upper bound on p_2?

2. A firm's marginal revenue of $n = 1, 2, 3, 4, \ldots$ inputs is respectively $5, 2, 4, 2, 0, 0, 0, \ldots$. Plot the firm's demand curve for inputs, namely, its most profitable demands.

3. Assume potential worker $k = 1, 2, \ldots$ has opportunity cost $2k$ of working for a firm. All workers are equally productive: potential firm $k = 1, 2, \ldots$ has potential revenues $15 - Bk$ if it hires a single worker.

   (a) Characterize all competitive equilibria for $B = 1$ and $B = 3$.

   (b) As part of "Drive for Fair Wages" campaign, a higher minimum wage is pushed. Could it backfire and reduce total wages to workers? Specifically, for $B = 1$ and $B = 3$, does there exist any minimum wage that can raise workers total wages? If so, what is the highest minimum wage that raises total wages to workers?

   Hint: Assume that zero surplus trades are agreed upon.

Part IV

A company hires a trader to generate return that can be either high or low: $r \in \{r_H, r_L\}$. The level of return depends on the trader's effort e: The trader may either work ($e = 1$) or browse the Web ($e = 0$). When $e = 1$, the trader incurs a cost of c; browsing costs 0. Let $\Pr(r = r_H \mid e = 1) \equiv p_1$ and $\Pr(r = r_H \mid e = 0) \equiv p_0$, where $p_1 > p_0$. The trader's effort level is privately known, i.e., it is not observable to the company. The company offers a contract that specifies a wage of w_H if the return is r_H and a wage of w_L if the return is r_L. The company and the trader are both risk neutral. Assume that the reservation utility of the trader is 0. [*Hint:* You should be able to answer all questions without setting up a Lagrangean.]

1. Assume that the company wants to implement the high effort level (i.e., $e = 1$). Write down the optimization problem for the company. Show that the relevant constraint(s) must either bind or can be treated as binding without loss of generality.

2. Based on your answer to part (1), solve for the wages that characterize the optimal contract. What is the trader's payoff in the optimal contract? (The wages paid to the trader in the optimal contract are not restricted to be nonnegative.) Interpret the risk sharing and incentive properties of the optimal contract.

3. Suppose that the wages specified in the contract offered by the company to the trader must respect a minimal wage constraint: the company must pay at least m regardless of the return:
$$w_H \geq m \text{ and } w_L \geq m.$$
   (a) Write down the new optimization problem for the company, assuming that the company intends to implement the high effort level.

   (b) Assume that $-p_0 c/(p_1 - p_0) < m < 0$. Determine which constraints must bind. Note that $m < 0$; again, there is no restriction that wages must be nonnegative.

   (c) Based on your answer to part (b), solve for the optimal wages.

   (d) Compute the trader's payoff in the optimal contract. Compare the worker's payoff to the payoff from part 2 and provide a brief explanation in terms of the constraints from part (a).

4. How would you implement low effort level (i.e., $e = 0$)?