Econ 712 Midterm
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$w<1$ endowment
k investment in productive tech, $\to Rk$
n labor, $(1-n)$ leisure, $y=n$ in $t=2$
$u(c_1, c_2, n) = \ln(c_1+c_2) + \ln(1-n)$
g exogenous, $g > Rw$
$\frac{(1+g-Rw)}{2} \in (0,1)$

1) Planner's Problem

1) $\max_{c_1, c_2, n} \ln(c_1+c_2) + \ln(1-n)$

such that $c_1 + k = w$ in $t=1$
and $c_2 = Rk + y$ in $t=2$
$y = n$
$c_1, c_2 \geq 0$

2) $\mathcal{L} = \ln(c_1+c_2) + \ln(1-n) + \lambda_1(w - c_1 - k) + \lambda_2(Rk + y - c_2)$

$d\mathcal{L}/dc_1 = \frac{1}{c_1+c_2} - \lambda_1 = 0$ $\qquad c_1 + c_2 = \frac{1}{\lambda_1}$

$d\mathcal{L}/dc_2 = \frac{1}{c_1+c_2} - \lambda_2 = 0$ $\qquad c_1 + c_2 = \frac{1}{\lambda_2}$

$d\mathcal{L}/dn = \frac{-1}{1-n} + \lambda_2 = 0$ $\qquad 1-n = \frac{1}{\lambda_2}$

$n = 1 - \frac{1}{\lambda_2}$
$n = 1 - c_1 - c_2$

$\boxed{c_1 + c_2 = 1-n}$

$k = w - c_1$
$c_2 = R(w - c_1) + n$
$c_2 = Rw - Rc_1 + n$

$\tau \in [0, 1-\omega]$
$\delta \in [0,1]$ $\quad k \rightarrow (R(1-\delta)k)$

3) Gov budget constraint:

$$\tau n + \delta R k = g$$

4) Household Problem:

$$\max_{c_1, c_2, n} \ln(c_1 + c_2) + \ln(1-n)$$
$$\text{s.t.} \quad c_1 + k = w$$
$$\text{and} \quad c_2 = (1-\delta)Rk + (1-\tau)n$$

5) Substituting our constraints into the objective function, we have:

$$\max \ln(w - k + (1-\delta)Rk + (1-\tau)n) + \ln(1-n)$$

Taking the FOC, we see:

w.r.t k: $$\frac{-1 + (1-\delta)R}{w - k + (1-\delta)Rk + (1-\tau)n} = 0$$

w.r.t n: $$\frac{(1-\tau)}{w - k + (1-\delta)Rk + (1-\tau)n} - \frac{1}{1-n} = 0$$

$$\frac{-1 + (1-\delta)R}{w - k + (1-\delta)Rk + (1-\tau)n} = \frac{(1-\tau)}{w - k + (1-\delta)Rk + (1-\tau)n} - \frac{1}{1-n}$$

$$\frac{1}{1-n} = \frac{(1-\tau) + 1 - (1-\delta)R}{w - k + (1-\delta)Rk + (1-\tau)n}$$

$$w - k + (1-\delta)Rk + (1-\tau)n = (1-n)((1-\tau) + 1 - (1-\delta)R)$$

$$((1-\delta)R - 1)k = (1-n)((1-\tau) + 1 - (1-\delta)R) - w - (1-\tau)n$$

$$k = \frac{(1-n)((1-\tau) + 1 - (1-\delta)R) - w - (1-\tau)n}{(1-\delta)R - 1}$$

$$k = \frac{(1-\tau) + 1 - (1-\delta)R - (1-\tau)n - n + (1-\delta)Rn - w - (1-\tau)n}{(1-\delta)R - 1}$$

$$k = -1 + \frac{(1-\tau) - 2(1-\tau)n - n + (1-\delta)Rn - w}{(1-\delta)R - 1}$$

6) max $g = \tau n + \delta R k$.

Using the decision rules from 5, the gov problem is:

max $g = \tau n^r(\tau, \delta) + \delta R k^r(\tau, \delta)$

7) The gov maximizes where $d/d\tau = d/d\delta = 0$.
At the points τ^* and δ^* such that $d/d\tau^* = d/d\delta^* = 0$, increasing or decreasing τ or δ would result in less tax revenue.

8) $\max \ \ln(c_1+c_2) + \ln(1+n)$ ← corrected to $-$!

$$c_1 + k = w$$
$$c_2 = (1-\delta)Rk + (1-\tau)n$$

Substituting our constraints:

~~max ln(w-k+c2-(1-δ)Rk~~

$$\max \ \ln\left(w-k+(1-\delta)Rk+(1-\tau)n\right) + \ln(1+n)$$

Using the FOC for n:

$$\frac{(1-\tau)}{w-k+(1-\delta)Rk+(1-\tau)n} - \frac{1}{1-n} = 0$$

$$1-n = \frac{w-k+(1-\delta)Rk}{1-\tau} + \frac{(1-\tau)n}{1-\tau}$$

$$1-n = \frac{w-k+(1-\delta)Rk}{1-\tau} + n$$

$$-2n = \frac{w-k+(1-\delta)Rk}{1-\tau} - 1$$

$$\boxed{n = \frac{1}{2} - \frac{w-k+(1-\delta)Rk}{2(1-\tau)}}$$

~~8~~ 9a) $\max \ g = \tau n^{*}(\tau,\delta,k) + \delta Rk$

$$\max \ \tau\left[\frac{1}{2} - \frac{w-k+(1-\delta)Rk}{2(1-\tau)}\right] + \delta Rk$$

The government maximizes where $d/d\tau = d/d\delta = 0$. At the points τ^* and δ^* such that $d/d\tau^* = d/d\delta^* = 0$, increasing or decreasing τ will result in lower tax earnings revenue ~~[illegible]~~, and increasing or decreasing δ will result in lower storage returns tax revenue. For both τ^* and δ, τ^* and δ^* are the maximum pts of their respective Laffer curves.

10) Given $\tau^n(k)$ and $\delta^n(k)$, the household chooses k s.t.

$$\max \ \ln(w - k + (1-\delta^n(k))Rk + (1-\tau^n(k))n) + \ln(1-n)$$

FOC: $$\frac{(-1 + (1-\delta^n(k))(-\delta^{n\prime}(k))}{w - k + (1-\delta^n(k))Rk + (1-\tau^n(k))n} = 0$$

$$\Rightarrow (-1 + (1-\delta^n(k))(-\delta^{n\prime}(k)) = 0$$

$$\Rightarrow (-\delta^n(k))(-\delta^{n\prime}(k)) = 0$$

$$\boxed{\text{either } \delta^n(k) = 0 \text{ or } \delta^{n\prime}(k) = 0}$$

11) The Ramsey equilibrium can not be implemented in a finite economy without commitment. In the last period, the government will choose τ and δ to maximize g to try to maximize utility, but households will predict this move. As a result, households will choose not to work and not to invest in the productive storage technology. In the second to last period, the gov will predict this future choice, so they will again choose τ^n and δ^n over τ^r and δ^r. This pattern will continue back to the first time period, so a Ramsey eq. is not possible without commitment.