# Econ 711 PS 1

Sarah Bass *

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## Question 1

### Part A

Consider the original production vector $y = \langle y_1, y_2, y_3 \rangle$ and price vector $p = \langle p_1, p_2, p_3 \rangle$ where goods one and two are inputs and good three is an output. Consider a scenario where $p_3$ falls and $p_1$ and $p_2$ stay the same. For the sake of contradiction, assume the firm's output $y_3$ can increase. Let the vector $p'$ defined as $p' = \langle p_1, p_2, p_3' \rangle$ represent the increase in the price of $p_3$, and let $y'$ defined as $y' = \langle y_1', y_2', y_3' \rangle$ represent the resulting change in the production vector. By the definition of these vectors, we know that:

$(p' - p) = \langle 0, 0, \Delta p_3 \rangle$, where $\Delta p_3$ is negative, and
$(y' - y) = \langle \Delta y_1, \Delta y_2, \Delta y_3 \rangle$, where $\Delta y_3$ is positive

Therefore, by the law of supply, we see:

$$\begin{aligned}(p' - p) \cdot (y' - y) &= 0\Delta y_1 + 0\Delta y_2 + \Delta p_3 \Delta y_3 \\ &= 0 + 0 + \Delta p_3 \Delta y_3 \\ &< 0, \text{ which violates the law of supply.}\end{aligned}$$

Thus, if $p_3$ falls and $p_1$ and $p_2$ stay the same, the firm's output $y_3$ cannot go up.

### Part B

Consider the production set $Y = \{y_a, y_b\}$ with $y_a = \langle -3, -1, 10 \rangle$ and $y_b = \langle -1, -5, 11 \rangle$ and price vector $p = \langle 1, 1, 1 \rangle$ where goods one and two are inputs and good three is an output. At this original price level, the profit at $y_a$ is $\pi(p) = p \cdot y_a = 6$ and the profit at $y_b$ is $\pi(p) = p \cdot y_b = 5$. So the firm will choose the production vector $y_a$ to maximize profits.

*I have discussed this problem set with Emily Case, Michael Nattinger, Alex Von Hafften, and Danny Edgel.

Consider a scenario where $p_1$ rises and $p_2$ and $p_3$ stay the same. Let the new price vector be $p' = \langle 2, 1, 1 \rangle$. At the new price level, the profit at $y_a$ is $\pi(p) = p' \cdot y_a = 3$ and the profit at $y_b$ is $\pi(p) = p' \cdot y_b = 4$. So the firm will choose the production vector $y_b$ to maximize profits.

Thus, a firm's output can go up if $p_1$ rises and $p_2$ and $p_3$ stay the same.

## Part C

Consider the production set from part B, $Y = \{y_a, y_b\}$ with $y_a = \langle -3, -1, 10 \rangle$ and $y_b = \langle -1, -5, 11 \rangle$ and price vector $p = \langle 1, 1, 1 \rangle$ where goods one and two are inputs and good three is an output. At described in part B, at this original price level, the firm will choose the production vector $y_a$ to maximize profits.

Consider a scenario where $p_1$ and $p_2$ both rise and $p_3$ stay the same. Let the new price vector be $p' = \langle 2, 1.1, 1 \rangle$. At the new price level, the profit at $y_a$ is $\pi(p) = p' \cdot y_a = 2.9$ and the profit at $y_b$ is $\pi(p) = p' \cdot y_b = 3.5$. So the firm will choose the price vector $y_b$ to maximize profits.

Thus, a firm's output can go up if $p_1$ and $p_2$ rise and $p_3$ stay the same.

Now consider a scenario where $p_1$ and $p_2$ both increase by 10% and $p_3$ remains the same. Note that the optimal production correspondence $Y^*(p)$ for a given price level depend on the relative magnitude of the prices (e.g. $Y^*(p_1, p_2, p_3) = Y^*(2p_1, 2p_2, 2p_3))$. Therefore increasing $p_1$ and $p_2$ by 10% and holding $p_3$ constant can be written as $y(1.1p_1, 1.1p_2, p_3) = y(p_1, p_2, \frac{1}{1.1}p_3)$. Effectively, this is the same as decreasing $p_3$, which as described in part A, cannot lead to increased output for $y_3$.

# Question 2

## Dataset 1

Dataset 1 is not profit maximizing. At the price level $p = \langle 5, 5 \rangle$, the firm has chosen the production vector $y(p) = \langle -50, 60 \rangle$. This gives us a profit of:

$$\begin{aligned}
\pi &= p \cdot y(p) \\
&= \langle 5, 5 \rangle \cdot \langle -50, 60 \rangle \\
&= 5(-50) + 5(60) \\
&= -250 + 300 \\
&= 50
\end{aligned}$$

However, given the data provided, we know other production vectors in the production set, and we can see that these production vectors are more profitable than $y(p)$. For example, using the production vector $y = \langle -20, 40\rangle$[1]:

$$\begin{aligned}\pi(p) &= p \cdot y \\ &= \langle 5, 5\rangle \cdot \langle -20, 40\rangle \\ &= 5(-20) + 5(40) \\ &= -100 + 200 \\ &= 100\end{aligned}$$

Thus the firm portrayed by Dataset 1 has not optimized their production set based on the prices listed, so it is not profit maximizing.

## Dataset 2

1. The smallest production set that can rationalize the data is the set listed: $Y = \{\langle -20, 40\rangle\langle -40, 70\rangle\langle -70, 90\rangle\}$
2. Figure 1 (below) shows the smallest convex production set with free disposal and the shutdown property that can rationalize the data. The set is created to be convex by connecting the production vectors listed in Dataset 2. The set fulfills the shutdown property because there is an option to produce at the vector $\langle 0, 0\rangle$. Finally, the set has free disposal since it includes production vectors that are below the border, so the producer has the option to produce less output than is feasible with the technology and inputs they have.
3. Figure 2 shows the largest production set that can rationalize the data. This set is created by drawing lines that are perpendicular to the price vectors and that intersect the tip of the production vectors. The production set also includes the area below these lines, including below the x-axis and across the y-axis.

[1] Note, the production vector $y = \langle -70, 90\rangle$ also produces higher profit than $y(p)$.

Figure 1:

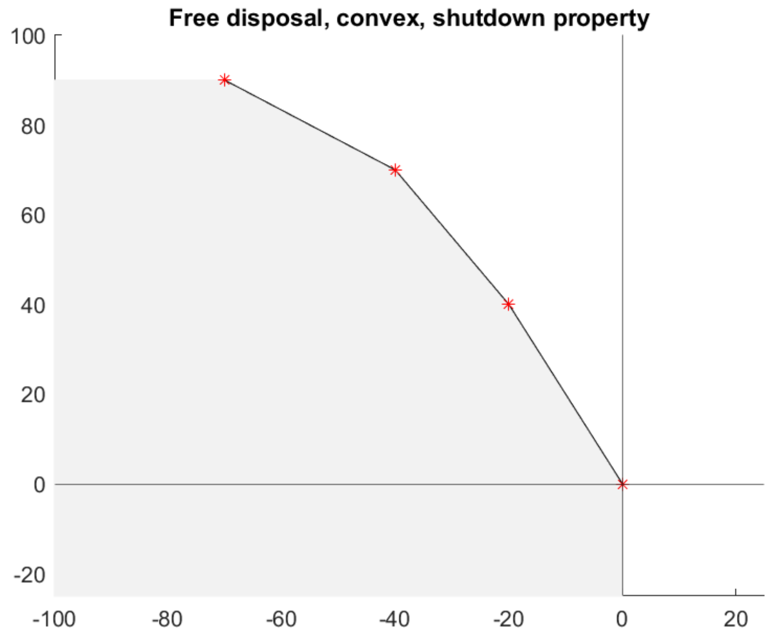
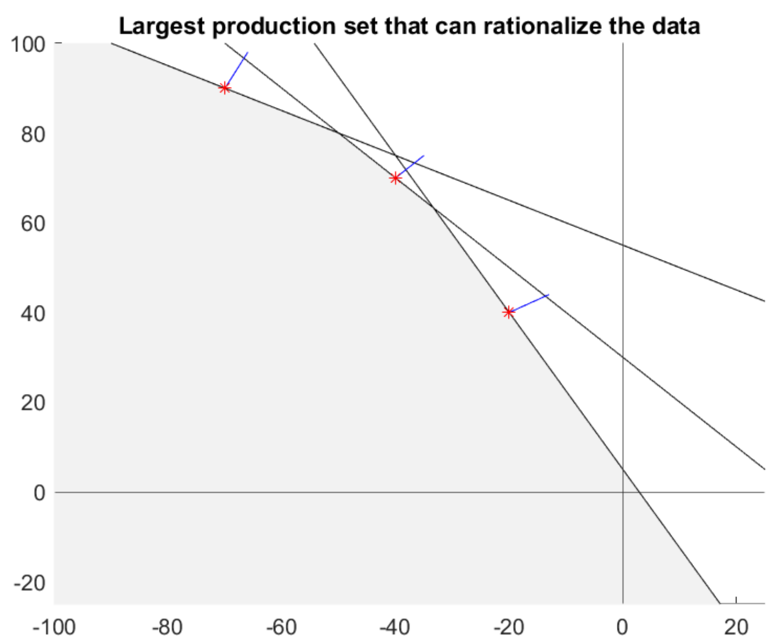



Figure 2:



## Question 3

Because each firm in the industry is profit-maximizing and price-taking, each firm satisfies the Weak Axiom. Thus for each firm, the production vector $y_n$ chosen satisfies $y_n \in Y_n^*(p) = argmax_{y_n \in Y_n} p \cdot y_n$, and $p \cdot y_n \geq p \cdot y_n'$ for all $y_n' \in Y_n$. Define $y = y_1 + ... + y_n$ as the aggregate production across the industry. We can see that:

$$\begin{aligned}
\pi(p) &= p \cdot y \\
&= p(y_1 + ... + y_n) \\
&= py_1 + ... + py_n \\
&\geq py_1' + ... + py_n' \\
&= p(y_1' + ... + y_n') \\
&= p \cdot y'
\end{aligned}$$

Thus, the Weak Axiom holds for aggregate data, and industry production can be rationalized as if it were the choice of a single profit-maiximizing firm.