Homework #4

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Fall 2016

1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = 2x^3 - 3x^2 + 2y^3 + 3y^2$.

   (a) Find the four points in $\mathbb{R}^2$ at which the gradient of f is equal to zero. Show that f has exactly one local maximum and one local minimum.

   (b) Let S be the set of all $(x, y) \in \mathbb{R}^2$ at which $f(x, y) = 0$. Describe$S$ as precisely as you can. Find those points of S that have no neighborhoods in which the equation $f(x, y) = 0$ can be solved for y in terms of x, or for x in terms of y.

2. Let $f : E \subset \mathbb{R}^n \to \mathbb{R}$ be of class C^1, and suppose that E is open. Let $x \in E$ be such that f does not have a local maximum at x. Find the direction of greatest increase in f. (HINT: Compute the directional derivative of f in the directiom of the vector u, where $\| u \| = 1$).

3. Suppose that $f'(x)$ exists, $g'(x)$ exists, $g'(x) \neq 0$, and $f(x) = g(x) = 0$. Prove that

$$\lim_{t \to x} \frac{f(t)}{g(t)} = \frac{f'(x)}{g'(x)}$$

4. Sundaram, #4, (a)-(b), p. 110