Whatever the progress of human knowledge, there will always be room for ignorance, hence for chance and probability - Émile Borel

1 Review Topics

Compactness, extreme value theorem (again!)

2 Exercises

2.1 Which of the following sets are compact?

- $\mathbb{Q} \cap [0,\, 1]$, in $\mathbb{R}$.

- $\left\{x \in \mathbb{Q} : 2 < x^2 < 3\right\}$, in $\mathbb{Q}$.

- $f^{-1}\left([2,\, 3]\right)$, where $f\left(x\right) = x^2$ on $\mathbb{R}$.

2.2 Prove that any closed set of a compact space is compact.

2.3 Prove that if K is a compact set, then $f\left(K\right)$ is compact whenever f is continuous.

2.4 Let K be a compact subset of $\mathbb{R}$. Prove that $\inf K$ and $\sup K$ exist and are in K.

2.5 Prove that $\sin x - \cos y$ attains a maximum on $[0,\, 2\pi] \times [0,\, 2\pi]$.

2.6 Prove that if a space $(X,\, d)$ is compact, then it is complete