Economics 703 : Final Exam

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Please be very explicit in your answers. Carefully state the appropriate definitions and theorems and argue how they apply. Also, make sure that every step in your argument follows logically and directly from the previous step.

Each question is worth 33 points, with one point given away for free.

1. Let $D = \{x \in \mathbb{R} | \ 0 \leq x \leq 1\}$, and let $f : D \to \mathbb{R}$ be given by the rule $f(x) = -x^2 - x$. Prove or disprove each of the following statements.

   (a) f is a concave function on D

   (b) f is a quasiconvex function on D

   (c) f is a pseudoconvex function on D

2. Solve the following problem in $\mathbb{R}^3$. Find the box of given surface area that maximizes volume, by solving the first order conditions associated with the Lagrangean.

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ and $g : \mathbb{R}^2 \to \mathbb{R}$ be functions given by the respective rules

$$f(x, y) = -7x^3 + 4x^2 y - 3xy^2 + 9y^3$$

   and

$$g(x, y) = x^3 - y^2.$$

   Solve the problem of maximizing f subject to the constraint $g \geq 0$.