Nothing takes place in the world whose meaning is not that of some maximum or minimum - Leonhard Euler

1 Review Topics

Continuity, uniform continuity, Lipschitz continuity

2 Exercises

2.1 Classify the following functions as continuous, uniformly continuous, or not continuous

- $\sin \frac{1}{x}$, on $(0, \infty)$.

- $\frac{1}{x}$, on $\mathbb{R}$.

- $\frac{1}{x}$ on $[a, \infty)$, for $a > 0$.

2.2 For $f(x) = x^2$, what is the pre-image of $(-2, -1)$. How about $(1, 2)$?

2.3 Consider the functions x^2 and $\sqrt{x}$ as functions on $(0, \infty)$. Thus, each function is the inverse of the other. Provide an intuitive explanation for why $\sqrt{x}$ is uniformly continuous and x^2 is not.

2.4 **Let $f(x) = \frac{\sin x}{x}$. Is there an alteration of this function such that $\tilde{f}(x) = f(x)$ at all continuity points of x, and $\tilde{f}$ is continuous?**

2.5 **Let $f : X \to \mathbb{R}$ be a continuous function, and let $A = \{x \in X \mid f(x) < a\}$, for some $a \in \mathbb{R}$. Show that A is an open set.**

2.6 **Prove x^2 is Lipschitz on any bounded interval in $\mathbb{R}$.**

2.7 **Let f on $A \subset \mathbb{R}$ have bounded derivative. Prove f is Lipschitz.**