Econ 712 Problem Set 5

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Question 2.1

To solve for the workers labor supply, we want to solve the following maximization problem:

$$V_j(k) = \max_{k',l}\{u^W(c,l) + V_{j+1}(k')\}$$
$$\text{s.t. } c = (1-\tau)we_jl + (1+r)k - k'$$
$$u^W(c,l) = \frac{(c^\gamma(1-l)^{1-\gamma})^{1-\sigma}}{1-\sigma}$$

Taking the first order conditions, we can see that:

$$\frac{\partial V}{\partial l} = 0$$
$$\Rightarrow (1-\sigma)\frac{(c^\gamma(1-l)^{1-\gamma})^{-\sigma}}{1-\sigma}((\gamma c^{\gamma-1}\frac{\partial c}{\partial l}(1-l)^{1-\gamma} + c^\gamma(1-\gamma)(1-l)^{-\gamma}) = 0$$
$$\Rightarrow (c^\gamma(1-l)^{1-\gamma})^{-\sigma}((\gamma c^{\gamma-1}\frac{\partial c}{\partial l}(1-l)^{1-\gamma} + c^\gamma(1-\gamma)(1-l)^{-\gamma}) = 0$$
$$\Rightarrow (c^\gamma(1-l)^{1-\gamma})^{-\sigma}((\gamma c^{\gamma-1}((1-\tau)we_j)(1-l)^{1-\gamma} + c^\gamma(1-\gamma)(1-l)^{-\gamma}) = 0$$
$$\Rightarrow \gamma\Big(\frac{1-l}{c}\Big)^{1-\gamma}(1-\tau)we_j = \Big(\frac{1-l}{c}\Big)^{\gamma}(1-\gamma)$$
$$\Rightarrow \frac{\gamma}{1-\gamma}(1-l)(1-\tau)we_j = c = (1-\tau)we_jl + (1+r)k - k'$$
$$\Rightarrow \frac{\gamma}{1-\gamma}(1-\tau)we_j = \Big(\frac{\gamma}{1-\gamma}+1\Big)(1-\tau)we_jl + (1+r)k - k'$$
$$\Rightarrow \frac{\gamma}{1-\gamma}(1-\tau)we_j - [(1+r)k - k'] = \Big(\frac{1}{1-\gamma}\Big)(1-\tau)we_jl$$
$$\Rightarrow \frac{\frac{\gamma}{1-\gamma}(1-\tau)we_j - [(1+r)k-k']}{\Big(\frac{1}{1-\gamma}\Big)(1-\tau)we_j} = l$$
$$\Rightarrow \frac{\gamma(1-\tau)we_j - (1-\gamma)[(1+r)k-k']}{(1-\tau)we_j} = l$$

Question 2.2

See the Matlab script uploaded with this problem set.

*I have discussed this problem set with Emily Case, Michael Nattinger, Alex Von Hafften, and Danny Edgel.

Question 2.3

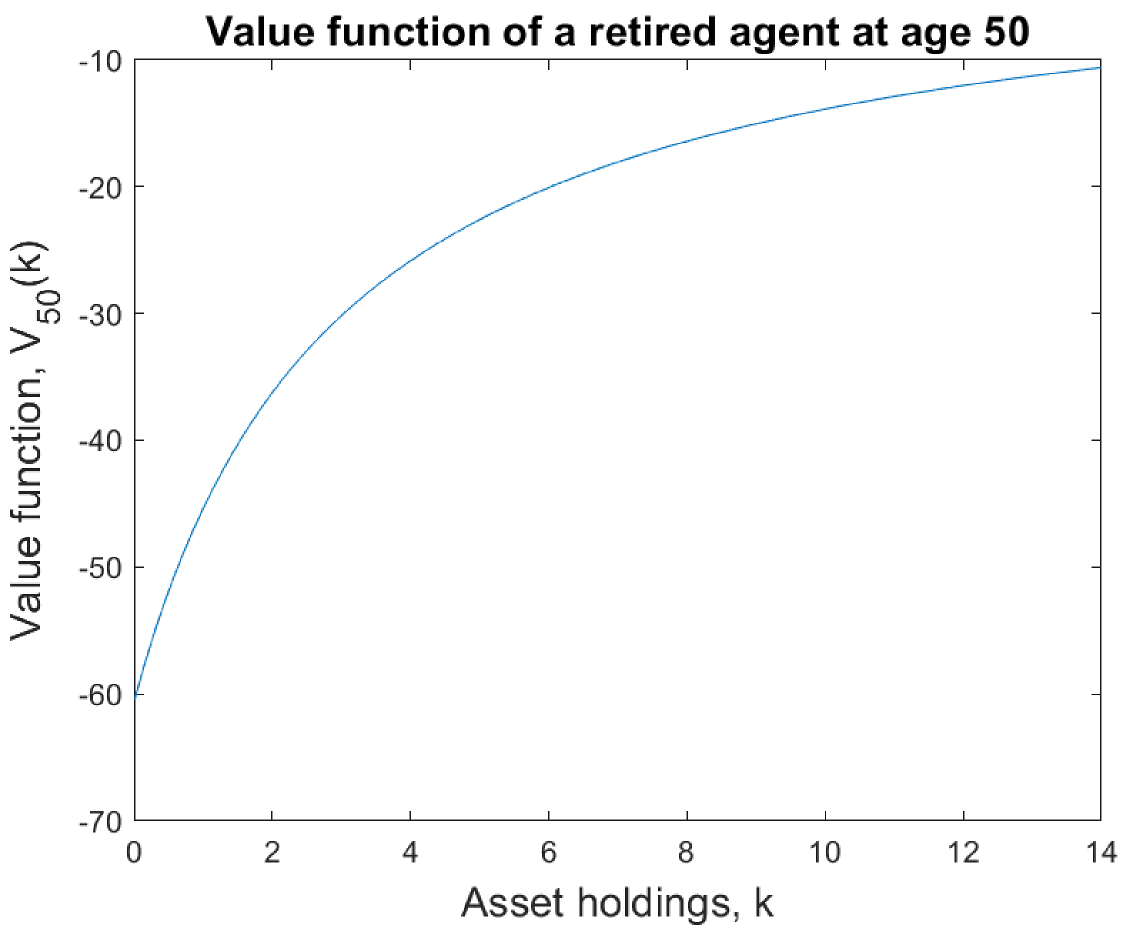
In my code, I added a loop for the different values of tau, setting the initial capital demand and labor demand guesses based on the tau values. Then while the labor demand and capital demand isn't converged and we haven't hit the maximum iteration number, we run the procedure until this condition is met. We take a guess for the demand for capital and labor for firms, then start backing out what the implied value for other parameters (r, w, b, etc.) are based on the capital and labor guesses. From this, we can find the last period consumption and the last period utility.

Next, we complete backwards induction. Going one generation at a time, we calculate the amount saved for tomorrow based on the assets today. We'll determine the value of holding the asset for tomorrow, and what is the best option for tomorrow. Then using the combination of assets today and tomorrow, we can calculate the consumption and utility. This happens for both retirees and workers.

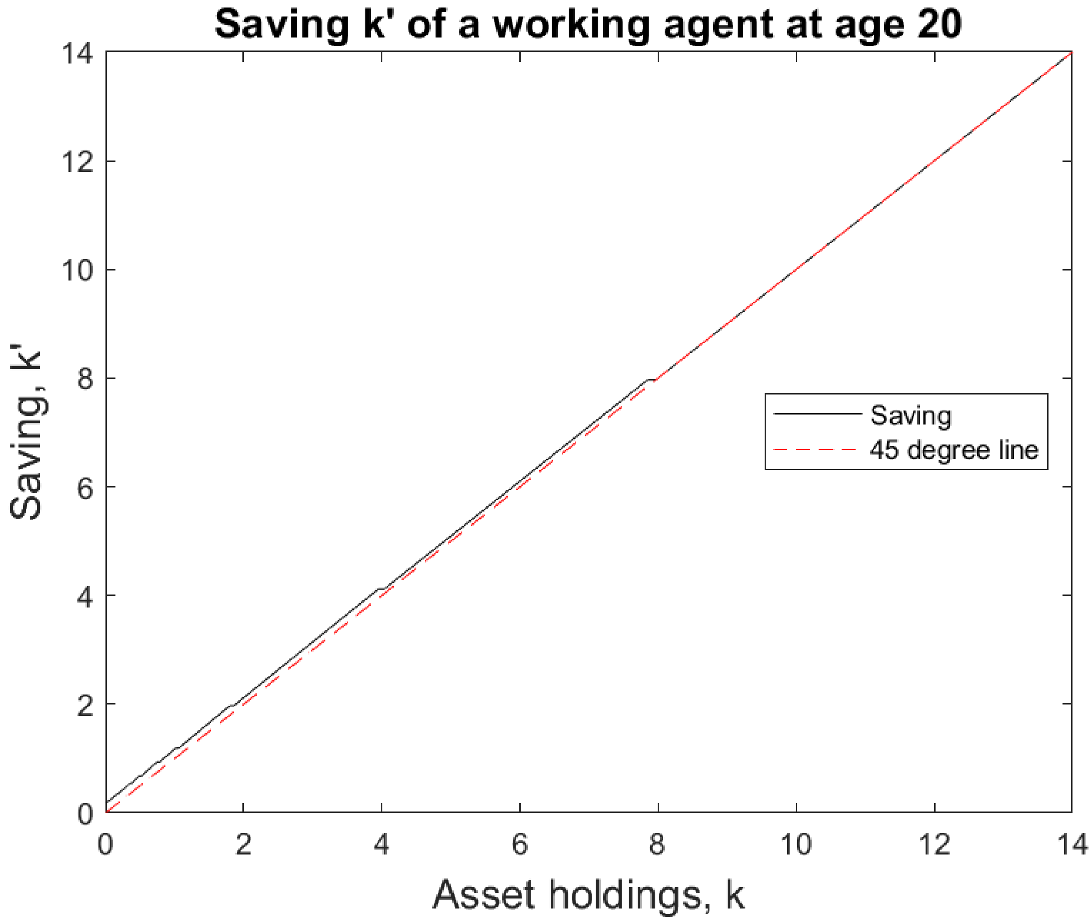
Next, we solve for the indirect utility function using the optimal value to save calculated in the previous loop. Then we calculate the labor supplied by individuals, and do this until the aggregate labor supply converges to the labor demanded so that the labor market can clear.

Question 2.4

The value function for a retired agent is increasing and concave.



Savings are increasing in k, but net savings are decreasing in k.



Question 2.5

Table 1. Results of the policy experiment: Benchmark model

	Social Security	No Social Security
Capital	2.9777	3.9114
Labor	0.351	0.36866
Wage	1.3819	1.4978
Interest	0.031622	0.019398
Pension benefit	0.21869	0
Newborn welfare	-54.6134	-52.7642
Aggregate welfare	-35.0692	-36.0338

Part 1

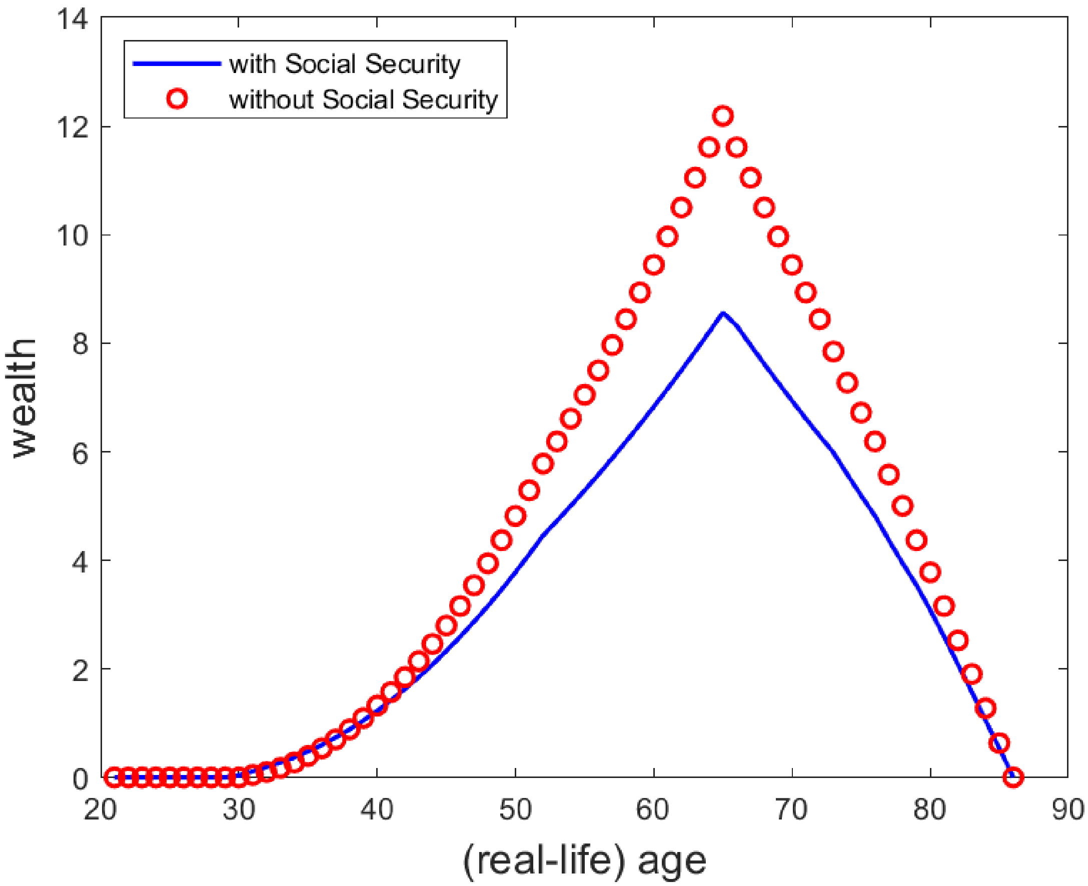
The model is dynamically efficient since the interest rate is greater than the population growth rate.

Part 2

Capital increases since the agents without social security must save for retirement. Wages without social security are slightly higher since agents do not need to pay taxes for social security. Labor also increases slightly for agents without social security.

Part 3

Agents without social security are encourage to save more for retirement throughout their lives, so their wealth profile is higher for their entire lifetime.



Part 4

A newborn generation will prefer to start in a steady state without social security since the newborn welfare is higher without social security.

Part 5

Since aggregate welfare is higher with social security, most people would not vote for social security to be removed.