I have had my results for a long time: but I do not yet know how I am to arrive at them - Carl Friedrich Gauss

1 Review Topics

$\mathbb{R}^n$, $\mathbb{R}^{n\times m}$

2 Exercises

2.1 Find the coordinate vector for the given vector in the given basis

- $$v = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, C = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

- $$v = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \; C = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$$

2.2 Find the set of solution vectors:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} x = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

What is an interpretation of x in terms of bases?

2.3 **Show that similarity is an equivalence relation; that is: A is similar to A, if A is similar to B, then B is similar to A, and if A is similar to B and B is similar to C, then A is similar to C.**

2.4 **Consider the basis for polynomials of degree 2 $\{x^2, x, 1\}$, and the corresponding basis $\{x, 1\}$ for linear functions. What is the matrix representation of differentiation as a map $D : \mathcal{P}^2 \to \mathcal{P}^1$? What if we view D as mapping $\mathcal{P}^2 \to \mathcal{P}^2$?**

2.5 **Prove that two similar operators must have the same rank**

2.6 **Define for a matrix A the new matrix A^k as $A \cdots A$ k times (as an operator this is function composition k times. Prove that if A is similar to B, then A^k is similar to B^k for any k.**