Economics 703 : Final Exam

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Fall 2005

Please be very explicit in your answers. Carefully state the appropriate definitions and theorems and argue how they apply. Also, make sure that every step in your argument follows logically and directly from the previous step.

Each question is worth 33 points, with one point given away for free.

1. Let $f : \mathbb{R}^3 \to \mathbb{R}$ and $g : \mathbb{R}^3 \to \mathbb{R}$ respectively be given by the rules $f(x, y, z) = x + y + z$ and $g(x, y, z) = (1/x) + (1/y) + (1/z)$. Find the maximum of f subject to the constraint $g(x, y, z) = 1$.

2. Let T be some positive integer. Solve the following maximization problem:

$$\text{Maximize } \sum_{t=0}^{T} \left(\frac{1}{2}\right)^t \sqrt{x_t}$$
$$\text{subject to } \sum_{t=0}^{T} x_t \leq 1$$
$$x_t \geq 0$$

   What are the values of the Lagrange multipliers at the optimum?

3. Let T be some positive integer. Consider the following problem:

$$\text{Maximize } \sum_{t=0}^{T} \beta^t u(c_t)$$
$$\text{subject to } c_1 + x_1 \leq x$$
$$c_t + x_t \leq f(x_{t-1}),\ t = 1, ..., T$$
$$c_t,\ x_t \geq 0,\ t = 0, ..., T,$$

   where $x \in \mathbb{R}_+$, $u : \mathbb{R}_+ \to \mathbb{R}$ and $f : \mathbb{R}_+ \to \mathbb{R}_+$ are non-decreasing functions. Derive the Kuhn-Tucker first-order conditions for this problem, and provide an economic interpretation of them. Explain under what circumstances these conditions are necessary and sufficient.