

Problem Set #3 (2<sup>nd</sup> Half)  
(Due Tuesday, December 1 before midnight)

Economics 709

Fall 2020

The numbered exercises listed below are from Hansen, *Econometrics*.

1. 3.24 - 3.25
2. 7.2 - 7.4
3. 7.8
4. 7.9 (a)
5. 7.10
6. 7.13 - 7.15
7. 7.17
8. 7.19
9. Suppose  $y_i = 1 + x_i\gamma + \varepsilon_i$ , where  $y_i, x_i, \varepsilon_i$  are scalar.  
Define  $w_i = \begin{pmatrix} 1 \\ x_i \end{pmatrix}$  and  $\beta = \begin{pmatrix} 1 \\ \gamma \end{pmatrix}$ .  
Assume that  $x_i$  has a discrete distribution:

$$\Pr(x_i = 1) = \Pr\left(x_i = \frac{4}{3}\right) = \Pr\left(x_i = \frac{5}{3}\right) = \Pr(x_i = 2) = \frac{1}{4}$$

We will use the following assumptions:

- (A0)  $(y_i, x_i)$  i.i.d.
- (A1)  $E(\varepsilon_i | w_i) = 0$
- (A1')  $E(w_i \varepsilon_i) = 0$
- (A2)  $Var(\varepsilon_i | w_i) = \sigma^2$

Assume that you will observe data  $(y_1, x_1), \dots, (y_n, x_n)$  (a sample of size  $n$ ). Below, state any additional assumptions *needed* to obtain your answers.

Consider the following OLS estimator from regressing  $y_i$  on  $w_i$  using only observations where  $x_i = 1$  or  $x_i = 2$ :

$$\hat{\beta} = \left[ \frac{1}{n} \sum_{i=1}^n w_i w_i' \mathbf{1}\{x_i \in \{1, 2\}\} \right]^{-1} \frac{1}{n} \sum_{i=1}^n w_i y_i \mathbf{1}\{x_i \in \{1, 2\}\}$$

where  $\mathbf{1}\{A\}$  is an indicator function for the event  $A$ .

- (a) Under (A0) and (A1), does  $\hat{\beta} \xrightarrow{p} \beta$ ?
- (b) Under (A0) and (A1'), does  $\hat{\beta} \xrightarrow{p} \beta$ ?
- (c) Under (A0), (A1), and (A2), what is the asymptotic distribution of  $\sqrt{n}(\hat{\beta} - \beta)$ ? Simplify as much as possible.
- (d) Consider the following OLS estimator from regressing  $y_i$  on  $w_i$  using only observations where  $x_i = \frac{4}{3}$  or  $x_i = \frac{5}{3}$ :

$$\hat{\hat{\beta}} = \left[ \frac{1}{n} \sum_{i=1}^n w_i w_i' \mathbf{1} \left\{ x_i \in \left\{ \frac{4}{3}, \frac{5}{3} \right\} \right\} \right]^{-1} \frac{1}{n} \sum_{i=1}^n w_i y_i \mathbf{1} \left\{ x_i \in \left\{ \frac{4}{3}, \frac{5}{3} \right\} \right\}$$

Let  $\hat{\beta}_2$  and  $\hat{\hat{\beta}}_2$  denote the second elements of the vectors  $\hat{\beta}$  and  $\hat{\hat{\beta}}$ . Note that  $\hat{\beta}_2$  and  $\hat{\hat{\beta}}_2$  are estimators for  $\gamma$ .

Under (A0), (A1), and (A2), which estimator for  $\gamma$  do you prefer  $\hat{\beta}_2$  or  $\hat{\hat{\beta}}_2$ ? Explain.

- (e) Consider the OLS estimator,  $\hat{\alpha}$ , from regressing  $y_i$  on  $x_i$  (no constant term) using only the observations where  $x_i = 1$  or  $x_i = 2$ . Under (A0), (A1), and (A2), what is the probability limit of  $\hat{\alpha}$ ?
- (f) Let  $\alpha$  denote your answer to part (e). Under (A0), (A1), and (A2), what is the asymptotic distribution of  $\sqrt{n}(\hat{\alpha} - \alpha)$ ?

Here's the empirical problem that will be due with the next problem set ...

7.29 (a) - (d): Use the subsample of the CPS that you used for problems 3.24 and 3.25 (instead of the subsample requested in the problem)