Econ 703 Homework 4

Fall 2008, University of Wisconsin-Madison

Prof. Raymond Deneckere

Due on Oct. 2, Thu. (in the class)

---

1. Prove the following two statements:

$$
\begin{aligned}
(a) \quad & (\cap_{i\in I} X_i) \cup (\cap_{j\in J} Y_j) = \cap_{i\in I, j\in J}(X_i \cup Y_j); \\
(b) \quad & (\cup_{i\in I} X_i) \cap (\cup_{j\in J} Y_j) = \cup_{i\in I, j\in J}(X_i \cap Y_j),
\end{aligned}
$$

where I and J are arbitrary indexsets.

2. Consider the metric space $(\mathbb{R}^n, \|\cdot\|_2)$, where $\|\cdot\|_2$ is the l_2-norm. Under what conditions is $\|x+y\|_2 = \|x\|_2 + \|y\|_2$? Prove your statement.

3. Consider the two metric spaces $(\mathbb{R}^n, d_2)$ and $(\mathbb{R}^n, d_\infty)$, where d_2 and d_∞ are the metrics derived from the l_2 and the l_∞ norm, respectively. Prove that an open ball in $(\mathbb{R}^n, d_2)$ is an open set in $(\mathbb{R}^n, d_\infty)$, and conversely that an open ball in $(\mathbb{R}^n, d_\infty)$ is an open set in $(\mathbb{R}^n, d_2)$. Use this result to prove that the collection of open subsets of $(\mathbb{R}^n, d_2)$ is the same as the collection of open subsets of $(\mathbb{R}^n, d_\infty)$.

4. Prove that every open set in $(\mathbb{R}^n, d_2)$ is infinite. (Hint: Use the result from problem 3)

5. Is every point of every open subset E of $(\mathbb{R}^n, d_2)$ a limit point of E? What if E is closed?