**FALL 2020**

**Problem Set 2: Estimating Markups**

Instructions: You may work in groups. However, you should create, comment, and run your own code after discussions with classmates, and turn in an individual write-up.

1. *Theory of how equilibria change with demand and cost parameters.* Suppose that price is determined by

$$P = a_0 - a_1 Q + \nu \tag{1}$$

and total costs are given by

$$C = F + (b_0 - b_1 Q + \eta)Q, \tag{2}$$

where $F$ is an entry cost that must be borne prior to production. Initially, suppose $b_1 = 0$.

(a) How does the elasticity of demand change with $Q$ and $\nu$?

(b) Solve for the Cournot equilibrium for fixed $F$ and $N$.

(c) If firms enter up to the point where it is no longer profitable, what is the equilibrium number of firms? Do not worry about integer constraints.

(d) Calculate the industry Lerner index $L_I$, Herfindahl index $H$, and demand elasticity as a function of the other parameters.

(e) How does the equilibrium elasticity change with $F$, $\nu$, and $\eta$? Do $\ln(L_I)$ and $\ln(H)$ change at the same rate in response to exogenous changes in $F$, $\nu$, and $\eta$ (that is, is the derivative of $\ln(L_I)$ with respect to the exogenous variables equal to that of $\ln(H)$ with respect to the same exogenous variables)?

(f) Suppose now that firms collude perfectly and split the profits, but firms choose to enter if it is profitable (anticipating collusive conduct). Calculate the equilibrium Lerner index $L_I$, Herfindahl index $H$, and demand elasticity as a function of the other parameters.

(g) Repeat exercises a,b,d, and e for the following demand function:

$$\ln P = c_0 - c_1 \ln Q + \xi \tag{3}$$

Do not do part c. That is, take $N$ as fixed and exogenous when doing the computations.

2. *SCP regression analysis: exogenous market structure, Cournot competition.*

(a) Suppose that there are 1000 different cities in the population, and the number of firms per city is uniformly distributed on {1,2,...,10}. In 500 cities, the antitrust authority is inactive, and firms collude perfectly when the number of firms is less than or equal to 8. Otherwise, firms play Cournot.

(b) For demand function (3), assume that $c_0 = 1$, $c_1 = .9$, and $\xi = 0$; .for the cost function, assume that $F = 1$, $b_0 = 1$, $b_1 = 0, \eta = 0$. Construct a dataset that simulates these 1000 cities, and for each city determine the equilibrium Lerner index, Herfindahl index, and demand elasticity as a function of the other parameters. Suppose there is measurement error on the Lerner index, so what is observed is $\ln(L_I) + \varepsilon$, where $\varepsilon \sim U[-.05, -.05]$. You can do this in Stata. Initial code would read:

```
set obs 1000;
gen unif = uniform();
gen unif2 = uniform();
gen Num = int(unif*10+1);
gen collude = 0;
replace collude = 1 if _n <=500 & Num <=8; /* first 500 markets collude if Number firms <=8 */
gen F = 1;
gen c0 = 1;
gen c1 = .9
gen b0 = 1;
gen b1 = 0;
gen z = 0;
gen h = 0;
gen LernerCN = {insert formula as function of c0, b0, etc.}; /* Eqm Lerner index for Num firms playing Cournot*/
gen LernerM = {insert formula as function of c0, b0, etc.}; /* Eqm Lerner index for Monopoly*/
gen Herf = 1/Num; /* Herfindahl for symmetric firms with fixed N*/
gen Lerner = collude * LernerM + (1-collude)*LernerCN; /* Lerner index depends on conduct */
gen lnLernerObs = ln(Lerner) + .1*(unif2 - .5);
```

In Matlab, the (uncommented) code would read:

```
J = 1000;
unif = rand(J,1);
unif2 = rand(J,1);
Num = ceil(10*unif);
collude = [(Num(1:500)>=8); zeros(500,1)];
F = 1;
c0 = 1;
c1 = .9;
b0 = 1;
b1 = 0;
```

```
z = 0;
h = 0;
LernerCN = {insert}
LernerM = {insert};
Herf = 1./Num;
Lerner = collude.*LernerM + (1-collude).*LernerCN;
lnLernerobs = log(Lerner) + .1*(unif2-.5);
```

(c) Run the structure-conduct-performance paradigm regression: regress $\ln(ObservedLerner)$ on $\ln(observedHerfindahl)$. Do this separately for the 500 cities where collusion is possible, and the 500 cities where it is not, as well as for the pooled sample. Test the hypothesis that the coefficient on $\ln(Herf) = 1$, and interpret your results.

(d) Repeat exercises (b) and (c) for the linear demand (1), with the same cost coefficients and demand coefficients $a_0 = 3$, $a_1 = 1$, $v = 0$. How do your results differ? Why? Explain very carefully and precisely your interpretation, referring back to the theory in question 1.

(f) In part (e), can you learn anything positive from your analysis? For example, what if you suspect collusion in markets $1 - 250$. Could you use this analysis to compare their competitiveness? Can you speculate whether that would be true in more general models?

3. *SCP regression analysis: endogenous market structure, Cournot competition.*

For this question, consider the linear demand equation (1) and the cost function (2) and suppose that all firms compete a la Cournot, and anticipating that they enter until profits are zero (ignoring integer constraints). Assume that $a_0 = 5$, $a_1 = 1$, and $F = 1$, $b_0 = 1$, and $b_1 = 0$. Suppose there are 1000 cities in the population.

(a) Assume that for each city, the demand parameter $v$ is distributed $U[-1, 1]$ and $\eta = 0$. For each city, simulate a value of $v$ and compute the equilibrium number of firms, Lerner index, and Herfindahl index. As above, there is measurement error on the Lerner index so that what is observed is $\ln(L_I)+\varepsilon$, where $\varepsilon \sim U[-.05, .05]$. Run a regression of the observed Lerner index on the Herfindhal Index.

(b) Repeat part (a) but where the cost parameter $\eta$ is distributed $U[-1, 1]$ and $v = 0$.

(c) Interpret your results carefully in light of the theory you derived above. In particular, what are the effects of $v$ and $\eta$ on the equilibrium Lerner Index, Herfindahl Index, and elasticity?