University of Wisconsin-Madison
Department of Economics

Econ 703 Prof. R. Deneckere
Fall 2003

Homework #6
(due Nov. 11)

1. Let f,g : $[0,1] \rightarrow R$ be continuous functions, and suppose that $f(x) > g(x)$ for all $x \in [0,1]$. Prove or disprove the following statement : There exists Ä > 0 such that $f(x) \geq g(x) +$ Ä for all $x \in [0,1]$. What if instead f and g were only left continuous?

2. (Brouwer fixed point theorem) Let $I = [0,1]$, and that suppose that $f : I \rightarrow I$ is continuous. Prove that there exists $x \in I$ such that $f(x) = x$.

3. Let f be a continuous real-valued function on R, of which it is known that $f'(x)$ exists for all $x \neq 0$ and that $f'(x) \rightarrow 3$ as $x \rightarrow 0$. Does it follow that $f'(0)$ exists? Either prove or disprove your statement.

4. Suppose $f'(x)$ exists, $g'(x)$ exists, $g'(x) \neq 0$, and $f(x)=g(x)=0$. Prove that
$$\lim_{t \to x} f(t)/g(t) = f'(t) / g'(t).$$

5. Let f: $R \rightarrow R$ be difined by $f(x)=x^2 \sin(1/x)$ for $x \neq 0$, and $f(x)=0$ for $x=0$. Show that $f'(x)$exists at all points $x \in R$, but that $f'(x)$ is not continuous at $x=0$.