

Homework #11

1. Sundaram, #3, p. 198.
2. Let $C \subset \mathbb{R}^n$ be a convex set. Show that $X = \{x \in \mathbb{R}^p : x = A\rho, \rho \in C\}$, where A is a given $p \times n$ real matrix, is a convex set in \mathbb{R}^p .
3. Show that the real-valued function

$$f(x) = \sum_{j=1}^n x_j \ln x_j - \sum_{j=1}^n x_j \ln \left(\sum_{k=1}^n x_k \right)$$

is convex on the set $C(\alpha) = \{x \in \mathbb{R}^n : x > 0, x_1 + \dots + x_n = \alpha\}$, where $\alpha > 0$.

4. Sundaram, #7, p. 199.
5. Show that a real-valued, positively homogeneous function on \mathbb{R}^n is convex if and only if the inequality

$$f(x + y) \leq f(x) + f(y)$$

holds for every pair of x and y in \mathbb{R}^n