# Problem set 3

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1. (a) The marginal consumer between Starbucks-0 and Esquire, $\hat{x}_0$ is such that:

$$u(\hat{x}_0) = 3 - p_0 - t\hat{x}_0^2 = 3 - q - t(1/2 - \hat{x}_0)^2 = u(1/2 - \hat{x}_0)$$
$$p_0 + t\hat{x}_0^2 = t/4 - t\hat{x}_0 + t\hat{x}_0^2 + q$$

$$\hat{x}_0 = \frac{1}{4} + \frac{q}{t} - \frac{p_0}{t}$$

The marginal consumer between Starbucks-1 and Esquire, $\hat{x}_1$ is such that:

$$u(\hat{x}_1) = 3 - q - t(\hat{x}_1^2 - 1/2)^2 = 3 - p_1 - t(1 - \hat{x}_1)^2 = u(1 - \hat{x}_1)$$
$$q + t\hat{x}_1^2 - t\hat{x}_1 + t/4 = p_1 + t - 2t\hat{x}_1 + t\hat{x}_1^2$$

$$\hat{x}_1 = \frac{3}{4} + \frac{p_1}{t} - \frac{q}{t}$$

(b) The demand for Starbucks-0 is:

$$Q_0 = \frac{1}{4} + \frac{q}{t} - \frac{p_0}{t}$$

The demand for Esquire is:

$$Q_e = \hat{x}_1 - \hat{x}_0 = \frac{1}{2} + \frac{p_1 + p_0}{t} - \frac{2q}{t}$$

The demand for Starbucks-1 is:

$$Q_e = 1 - \hat{x}_1 = \frac{1}{4} + \frac{q}{t} - \frac{p_1}{t}$$

Therefore, Starbucks solves the problem of

$$\max_{p_0,p_1} \left(\frac{1}{4}+\frac{q}{t}-\frac{p_0}{t}\right)p_0+\left(\frac{1}{4}+\frac{q}{t}-\frac{p_1}{t}\right)p_1$$

$$\text{FOC wrt } p_0\text{:}\quad p_0=\frac{t}{8}+\frac{q}{2}$$

$$\text{FOC wrt } p_1\text{:}\quad p_1=\frac{t}{8}+\frac{q}{2}$$

Esquire, on the other hand solves:

$$\max_{q} \left(\frac{1}{2}+\frac{p_1+p_0}{t}-\frac{2q}{t}\right)q$$

$$\text{FOC:}\quad q=\frac{t}{8}+\frac{p_0+p_1}{4}$$

(c) Solving for the equilibrium:

$$\left(\begin{array}{ccccc} 1 & 0 & -1/2 & | & t/8 \\ 0 & 1 & -1/2 & | & t/8 \\ 1/4 & 1/4 & -1 & | & -t/8 \end{array}\right)$$

Reducing this matrix yields:

$$p_0=p_1=q=t/4$$

(d) In this case, the demand for Esquire is:

$$Q_e=\frac{1}{4}+\frac{p_0}{t}-\frac{q}{t}$$

The demand for Starbucks-0 is:

$$Q_0=\hat{x}_1-\hat{x}_0=\frac{1}{2}+\frac{p_1+q}{t}-\frac{2p_0}{t}$$

And the demand for Starbucks-1 is:

$$Q_1=1-\hat{x}_1=\frac{1}{4}+\frac{p_0}{t}-\frac{p_1}{t}$$

So, Esquire solves the following problem

$$\max_{q} \left(\frac{1}{4}+\frac{p_0}{t}-\frac{q}{t}\right)q$$

$$\text{FOC:}\quad q=\frac{t}{8}+\frac{p_0}{2}$$

And Starbucks solves the following problem:

$$\max_{p_0,p_1} \left(\frac{1}{2} + \frac{p_1+q}{t} - \frac{2p_0}{t}\right) p_0 + \left(\frac{1}{4} + \frac{p_0}{t} - \frac{p_1}{t}\right) p_1$$

FOC wrt $p_0$: $p_0 = \frac{t}{8} + \frac{p_1}{2} + \frac{q}{4}$

FOC wrt $p_1$: $p_1 = \frac{t}{8} + p_0$

Solving for the equilibrium:

$$\left(\begin{array}{ccccc} -1/2 & 0 & 1 & | & t/8 \\ 1 & -1/2 & -1/4 & | & t/8 \\ -1 & 1 & 0 & | & t/8 \end{array}\right)$$

Reducing this matrix yields:

$$p_0 = 7t/12, \quad p_1 = 17t/24, \quad q = 5t/12$$

(e) The reason why these two frameworks yield different equilibria in prices is because in the first case, Starbucks' stores are not competing with each other, so the only strategic substitute is Esquire for both of them. This allows Starbucks to charge the same price in both stores. While in the second case, Starbucks' stores compete against each other, so the strategic substitute is not only esquire but the other store as well. In this case, because the store located in the middle faces competition from Esquire and Starbucks itself, it has to set a price that is lower than the price set by the Starbucks' store located at the end of the unit line. Also because competition for Esquire is lower when it locates at the end it can charge a higher price compared to the situation where it is located in the middle.

2. (a) A consumer is indifferent between buying s=1 and s=2 if:

$$2(\theta_2 - p_2) = \theta_2 - p_1 \rightarrow \ \theta_2 = 2p_2 - p_1$$

A consumer is indifferent between buying s=1 and the outside option if:

$$\theta_1 - p_1 = 0 \rightarrow \ \theta_1 = p_1$$

Therefore, demand for s=2 is:

$$Q_2 = Prob(\theta \geq \theta_2) = 1 - 2p_2 + p_1$$

Demand for s=1 is:

$$Q_1 = Prob(\theta_1 \leq \theta \leq \theta_2) = 2(p_2 - p_1)$$

The monopolist producing both qualities solves the following problem:

$$\max_{p_1,p_2} 2(p_2 - p_1)(p_1 - c) + (1 - 2p_2 + p_1)(p_2 - 2c)$$

$$\text{FOC wrt } p_1\text{:} \quad p_1 = \frac{3p_2}{4}$$

$$\text{FOC wrt } p_2\text{:} \quad p_2 = \frac{3p_1 + 2c + 1}{4}$$

So the solution to this system of linear equations yields:

$$p_1 = \frac{3}{7}(2c+1), \quad p_2 = \frac{4}{7}(2c+1)$$

And equilibrium profits are:

$$\pi_2 = \frac{4}{7}c^2 - \frac{8}{7}c + \frac{2}{7}$$

(b) If the monopolist only offers low quality, then it solves the problem:

$$\max_{p_1} (1 - p_1)(p_1 - c)$$

$$\text{FOC:} \quad p_1 = \frac{1+c}{2}$$

And the equilibrium profits are:

$$\pi_1 = \frac{(1-c)^2}{4}$$

(c) This suggests that offering both products is optimal whenever $\pi_2 \geq \pi_1$ or:

$$\frac{4}{7}c^2 - \frac{8}{7}c + \frac{2}{7} \geq \frac{(1-c)^2}{4} \iff c \geq \frac{1+2\sqrt{2}}{3}$$

(d) Now, suppose we have two firms competing, and demand for each product is weakly positive. From this condition we can derive the relevant range for $p_2$. So we have:

$$Q_1 = 2(p_2 - p_1) \geq 0 \iff p_2 \geq p_1$$

$$Q_2 = 1 - 2p_2 + p_1 \geq 0 \iff p_2 \leq (1 + p_1)/2$$

$$p_1 \leq p_2 \leq (1 + p_1)/2$$

Given the relevant range for $p_2$, demand for firm 1 is:

$$Q_1 = \begin{cases} 1 - p_1 & \text{if} \quad p_2 > (1+p_1)/2 \\ 2(p_2 - p_1) & \text{if} \quad p_1 \le p_2 \le (1+p_1)/2 \\ 0 & \text{if} \quad p_2 < p_1 \end{cases}$$

What this is telling us, is that when $p_2$ exceeds $(1+p_1)/2$, firm 1 is a monopolist in the market, while when $p_2 < p_1$ every consumer will strictly prefer to buy the high quality good.

(e) The best response function for firm 1 is[1]:

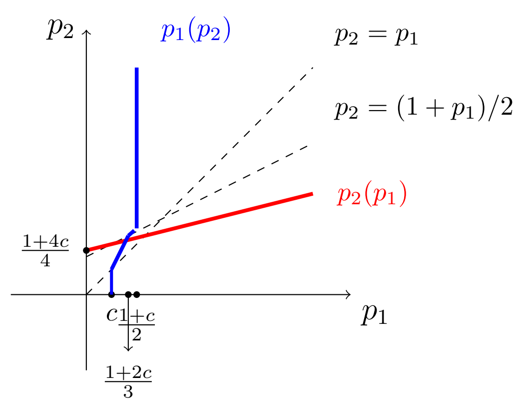
$$p_1(p_2) = \begin{cases} (1+c)/2 & \text{if} \quad p_2 > (1+p_1)/2 \\ (p_2 + c)/2 & \text{if} \quad p_1 \le p_2 \le (1+p_1)/2 \\ 0 & \text{if} \quad p_2 < p_1 \end{cases}$$

(f) Firm 2 is solving the problem of:

$$\max_{p_2} (1 - 2p_2 + p_1)(p_2 - 2c)$$

$$\text{FOC:} \quad p_2 = \frac{1 + 4c + p_1}{4}$$

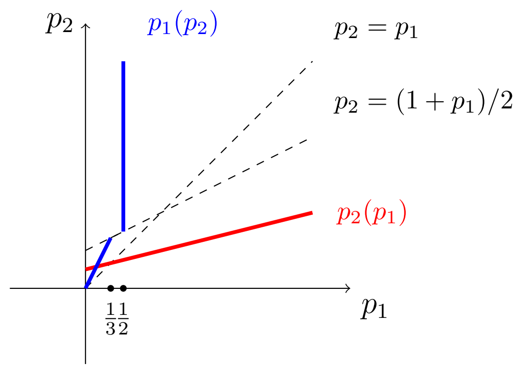
The graph of the best response functions is shown below:



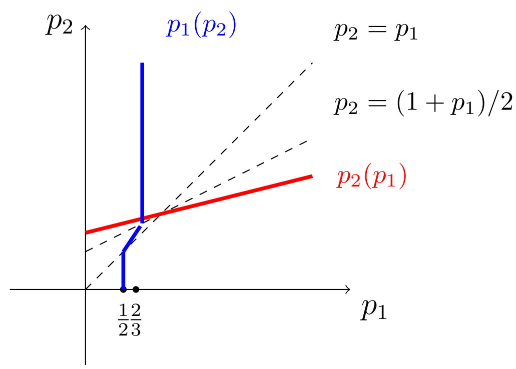

Now let's consider the extreme cases where $c = 0$ and $c = 1/2$

[1] In the case $p_2 \le (1+p_1)/2$, $p_1 = arg\max 2(p_2 - p_1)(p_1 - c)$

$c = 0$




$c = 1/2$




So the firms can coexist depending on the value of $c$. For $c$ low enough, the Nash equilibrium in prices is:

$$p_1 = \frac{1}{7} + \frac{8}{7}c, \quad p_2 = \frac{2}{7} + \frac{9}{7}c$$

While for c close to 1/2 we have a monopoly of firm 1.