

Microeconomic Theory (Econ 713)
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Problem Set 3

Due in (=before) class April 25, 2019

Based on Exercise MWG 13.D.1 Consider a job market in which there are two types of workers, θ_L and θ_H , with $0 < \theta_L < \theta_H$. The fraction of workers who are of type θ_H is $p \in (0, 1)$. Suppose that jobs may differ in the *task level* required of workers. The utility of a type θ worker who receives wage w and faces task level $t \geq 0$ is,

$$u(w, t|\theta) = w - c(t, \theta)$$

where $c_t(t, \theta) > 0$ for all $t > 0$ and θ . A type θ worker produces $\theta(1 + \mu t)$ units of output when her task level is t where $\mu > 0$. Assume the reservation utility of each type is \bar{u}_i , for $i \in \{L, H\}$. Identify the perfect Bayesian separating equilibrium of the game where the firm announces a set of contracts (i.e a pair (w, t)) and then workers of each type choose whether to accept a contract and, if so, which one. What other conditions do we need to have a separating equilibrium?

Question 2 (MWG 14.C.7; from J. Tirole)

Assume that there are two types of consumers for a firm's product, θ_H and θ_L . The proportion of type θ_L consumers is λ . A type θ 's utility when consuming amount x of the good and paying a total of T for it is $u(x, T) = \theta v(x) - T$, where

$$v(x) = \frac{1 - (1 - x)^2}{2}.$$

The firm is the sole producer of the good, and its cost of production per unit is $c > 0$. Assume $\theta_H > \theta_L > c$

- A) Consider a nondiscriminating monopolist that may only charge a single price. Derive their optimal pricing policy, and show that they serve both classes of consumers if either θ_L or λ is sufficiently large.
- B) Consider a monopolist who can distinguish the two types but can only charge a simple price p_i to each θ_i . Derive their optimal prices.
- C) Compute the fully optimal nonlinear tariff. How do the quantities purchased by the two types compare with the levels in A) to C)?

Question 3 — Prelim 2012: A company wishes to hire a manager whose effort will secure either success or failure. The manager's effort level e is restricted to $[0, 1]$ and equals the chance of success. Success is worth 1 to the company.

The company is risk neutral over output and wages. An exogenous constraint implies that wages cannot be negative. The manager's utility over wages is $u(w) = w(2 - w)$, while their effort cost is $c(e) = e^2$. Altogether, the manager enjoys expected utility $U(w, e) = u(w) - c(e)$ given wage w and effort e . The manager's outside option is zero.

A) Assume verifiably observable effort, so that the company may contractually specify both the wage and the effort. Solve for the optimal contract, describe the wages, effort, and payoffs of the company and manager.

B) Assume unobservable effort. Solve for the optimal contract (Hint: what form should such a contract assume?)

C) Compare the effort levels, contractual riskiness, and efficiency of the contracts with observable and unobservable effort. Relate these facts.

Question 4: Deterministic and Stochastic Returns (Prelim 2012)

A firm can earn profits from two different activities undertaken by the worker. The firm's return π_1 from activity 1 is a *deterministic* function of the worker's effort on this activity. If the worker exerts high effort e_h on activity 1, then $\pi_1 = Y$; if they exert low effort e_l , then $\pi_1 = Z$, with $Z < Y$. The firm's return π_2 from activity 2 is the following *stochastic* function of the worker's effort: If the worker exerts high effort e_h in activity 2, then $\pi_2 = X$ and $\pi_2 = 0$ with equal chances 0.5. If the worker puts in low effort e_l , then $\pi_2 = 0$ always.

The worker can exert high effort on at most one activity. Exerting high effort is costly for the worker. Specifically, the worker's utility as a function of the contractually specified wage w is \sqrt{w} if they put low effort into both activities and $\sqrt{w} - g$ if they exert high effort on one activity, where g is the utility effort cost. The worker's utility if they do not work for the firm at all is 0. Assume that $0.5X > Y - Z > g^2$. Thus, the expected marginal return of the high effort in activity 2 exceeds the marginal return of high effort in activity 1. The firm is risk neutral.

A) Assume that the firm observes the worker's effort level in each activity. Characterize the contract (wages and effort levels) that the firm will offer.

B) Suppose that the firm observes both returns π_1 and π_2 but *not* the worker's effort choice. Find the profit-maximizing contracts for inducing each combination of efforts in both activities. Under what conditions on X, Y and Z is the effort combination chosen by the firm different from that in part A)?

C) For each combination of efforts in parts A) and B), provide economic intuition for why the wage schedules are the same/different. Verify whether the monotone likelihood ratio property holds and use this in your explanation. Interpret the risk sharing properties of the optimal contracts.