Econ 711 Problem Set 6

Sarah Bass *

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Question 1

Part A

$$\begin{aligned}5(12)+5(4)+5(4)&=100\\7(9)+4(3)+5(5)&=100\\2(27)+4(9)+5(5)&=100\\7(15)+4(5)+5(5)&=150\end{aligned}$$

Since $p \cdot x = w$ for each bundle, these data are consistent with Walras Law.

Part B

Table 1. Price of each possible bundle

	x_1	x_2	x_3	x_4
p_1	100	85	230	125
p_2	120	100	275	150
p_3	44	35	100	55
p_4	120	100	275	150

Note that at the p_1, the consumer can afford x_1 and x_2, and chooses x_1. So $x_1 \succ x_2$. At the p_3, the consumer can afford all of the consumption bundles, and chooses x_3. So $x_3 \succ x_1, x_2, x_4$. At the p_4, the consumer can afford x_1, x_2, and x_4, and chooses x_4. So $x_4 \succ x_1, x_2$. So we can conclude that $x_3 \succ x_4 \succ x_1 \succ x_2$. Since this satisfies GARP, the data is rationalizable.

Question 2

Part A

Using Roy's Identity, $x^i(p, w_i) = -\frac{\partial v^i}{\partial p} / \frac{\partial v^i}{\partial w_i} = -\frac{a_i + b'(p) w_i}{b(p)}$.

*I have discussed this problem set with Emily Case, Michael Nattinger, Alex Von Hafften, and Danny Edgel.

Part B

Using Roy's Identity,

$$\begin{aligned}
X(p,W) &= -\frac{\partial v^i}{\partial p} / \frac{\partial v^i}{\partial W} \\
&= -\frac{\sum_{i=1}^n a_i + b'(p)W}{b(p)} \\
&= -\frac{\sum_{i=1}^n (a_i + b'(p)w_i)}{b(p)} \\
&= \sum_{i=1}^n -\frac{(a_i + b'(p)w_i)}{b(p)}
\end{aligned}$$

Question 3

Part A

Let preferences be homothetic and represented by a utility function $u(x)$. Let $x, y \in B(p,w)$ and let $x \in x(p,w) = \arg\max_{x \in B(p,w)} u(x)$. Let $x(p,tw) = \arg\max_{x \in B(p,tw)} u(x)$. Since $p \cdot x(p,w) \leq w$, $p \cdot tx(p,w) \leq tw$, which implies that $tx(p,w) \in B(p,tw)$.

Let $ty \in B(p,tw)$. Then,

$$\begin{aligned}
y \in B(p,w) &\rightarrow x(p,w) \succ y \\
&\rightarrow tx(p,w) \succ ty \\
&\rightarrow tx(p,w) = x(p,tw)
\end{aligned}$$

The argument would go the opposite way as well by parallel logic. Thus Marshallian Demand is homogeneous of degree 1 in wealth.

Part B

Let $u(x) = \alpha$ where $x \sim (\alpha, ..., \alpha)$. If preferences are continuous and monotone, $u(x)$ represents the preference relation.

$$\begin{aligned}
& x \sim (\alpha, ..., \alpha) \\
\Rightarrow\ & tx \sim t(\alpha, ..., \alpha) \\
\Rightarrow\ & tx \sim (t\alpha, ..., t\alpha) \\
\Rightarrow\ & u(tx) = t\alpha
\end{aligned}$$

Thus our utility function is homogeneous of degree 1.

Part C

Given A and B,

$$
\begin{aligned}
v(p,w) &= u(x(p,w)) \\
&= wu(x(p,1)) \\
&= wb(p) \text{ where } b(p) = u(x(p,1))
\end{aligned}
$$

Question 4

Part A

Since preferences are LNS, the budget constraint must hold with equality. Note that $x_1+\sum_{i=2}^n p_i x_i = w \Rightarrow x_1 = w - \sum_{i=2}^n p_i x_i$. Then,

$$
\begin{aligned}
X(p,w) &= \underset{X\in B(p,w)}{\arg\max}\ u(x) \\
&= \underset{X\in B(p,w)}{\arg\max}\ x_1 + U(x_2,....,x_k) \\
&= \arg\max_X w - \sum_{i=2}^n p_i x_i + U(x_2,....,x_k) \\
&= \arg\max_X w - \sum_{i=2}^n p_i x_i + U(x_2,....,x_k) \\
&= \arg\max_X - \sum_{i=2}^n p_i x_i + U(x_2,....,x_k) \\
&= X_{2,...,k}(p)
\end{aligned}
$$

Part B

$$
\begin{aligned}
v(p,w) &= u(X(p,w)) \\
&= u\left(\left(\left(w - \sum_{i=2}^n p_i x_i\right) X_{2,...,k}^T(p)\right)^T\right) \\
&= w - \sum_{i=2}^n p_i x_i + U(X_{2,...,k} \\
&= w - g(X_{2,...,k}) + U(X_{2,...,k} \\
&= w + \tilde{v}(X_{2,...,k})
\end{aligned}
$$

Part C

$$\begin{aligned}
e(p,u) &= \min_{u(x)\leq u} p \cdot X \\
&= \min_{u(x)\leq u} x_1 + \sum_{i=2}^{n} p_i x_i
\end{aligned}$$

Note that our utility constraint must hold with equality. So $u = x_1 + U(x_2, ..., x_k) \Rightarrow x_1 = u - U(x_2, ..., x_k)$. So,

$$\begin{aligned}
e(p,u) &= \min_{u(x)\leq u} x_1 + \sum_{i=2}^{n} p_i x_i \\
&= \min_{X} u - U(x_2, ..., x_k) + \sum_{i=2}^{n} p_i x_i \\
&= u - \min_{X} -U(x_2, ..., x_k) + \sum_{i=2}^{n} p_i x_i \\
&= u - f(p)
\end{aligned}$$

Part D

$$\begin{aligned}
h(p,u) &= \arg\min_{u(x)\leq u} p \cdot x \\
&= \arg\min_{u(x)\leq u} x_1 + \sum_{i=2}^{n} p_i x_i \\
&= \arg\min_{X} u - U(x_2, ..., x_k) + \sum_{i=2}^{n} p_i x_i \\
&= \arg\min_{X} -U(x_2, ..., x_k) + \sum_{i=2}^{n} p_i x_i \\
&= h(p)
\end{aligned}$$

Part E

Compensating variation is:

$$\int_{p_i^1}^{p_i^0} h_i(p,u^0)dpi = \int_{p_i^1}^{p_i^0} h_i(p)dpi = \int_{p_i^1}^{p_i^0} h_i(p,u^1)dpi$$

which is equivalent variation. Consumer surplus is:

$$\int_{p_i^1}^{p_i^0} x_i(p,w)dpi = \int_{p_i^1}^{p_i^0} x_i(p)dpi = \int_{p_i^1}^{p_i^0} h_i(p)dpi$$

Since Hicksian and Marshallian demand for good i are functions only of price, they must be equal at each price.