Dixit-Stiglitz Model

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Primitives of the static model:

1. preferences: $U = C = \left(\int C_i^{\frac{\theta-1}{\theta}} \mathrm{d}i\right)^{\frac{\theta}{\theta-1}}$, love of variety, CES aggregated, i firms
2. technology: $Y_i = AL_i$, all firms have the same productivity
3. endowment: $L = \int L_i = 1$. households inelastically supply labor

In contrast to the growth and RBC model, assume that only one firm can produce each variety and hence, is a monopoly in the market of that product. At the same time, the firm takes GE prices as given. Individual firms do not affect aggregate prices

Households choose consumption of each product:

- i type of good
- only one time period
- continuum of goods → integrate

Inner shell - i's on everything (D-S)
- given consumption, how much do I buy of each type

Outer shell - no i's (N-K)
- solves aggregate

$$\max_{\{C_i\}} \left(\int C_i^{\frac{\theta-1}{\theta}} \mathrm{d}i\right)^{\frac{\theta}{\theta-1}}$$

$$\text{s.t.} \int P_i C_i \mathrm{d}i = \overset{W \cdot L,\ L=1}{W} + \int \Pi_i \mathrm{d}i \equiv E,$$

where E is the total income of a representative consumer. It is more convenient, however, to solve the dual problem of minimizing expenditures:

$$\min_{\{C_i\}} \int P_i C_i \mathrm{d}i$$

$$\text{s.t.} \left(\int C_i^{\frac{\theta-1}{\theta}} \mathrm{d}i\right)^{\frac{\theta}{\theta-1}} = C.$$

Denote the Lagrange multiplier with P and take the FOC wrt C_i:

$$\frac{\theta}{\theta-1} - \frac{\theta-1}{\theta-1} = \frac{1}{\theta-1}$$

$$\mathcal{L} = \int P_i C_i \mathrm{d}i - P\left[\left(\int C_i^{\frac{\theta-1}{\theta}} \mathrm{d}i\right)^{\frac{\theta}{\theta-1}} - C\right]$$

$$P_i = P \underbrace{\left(\int C_i^{\frac{\theta-1}{\theta}} \mathrm{d}i\right)^{\frac{1}{\theta-1}}}_{\left(C^{\frac{\theta-1}{\theta}}\right)^{\frac{1}{\theta-1}} = C^{\frac{1}{\theta}}} C_i^{-\frac{1}{\theta}}.$$

$$\frac{\theta-1}{\theta} - \frac{\theta}{\theta} = \frac{-1}{\theta}$$

$$\left(\int \left(\left(\frac{P_i}{P}\right)^{-\theta} C\right)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}} = C$$

$$\left(\int \left(\left(\frac{P_i}{P}\right)^{-\theta}\right)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}} = C$$

This implies that demand for product i is equal

$-\theta\left(\frac{\theta-1}{\theta}\right) = 1-\theta$

$$C_i = \left(\frac{P_i}{P}\right)^{-\theta} C. \tag{1}$$

$$\left(\int P_i^{1-\theta} P^{\theta-1} di\right)^{\frac{\theta}{\theta-1}} = 1$$

Substitute into the constraint to solve for the Lagrange multiplier:

$$P^{\theta}\left(\int P_i^{1-\theta} di\right)^{\theta/\theta-1} = 1$$

$$\left(\int C_i^{\frac{\theta-1}{\theta}} \mathrm{d}i\right)^{\frac{\theta}{\theta-1}} = C \qquad P = \left(\int P_i^{1-\theta} \mathrm{d}i\right)^{\frac{1}{1-\theta}}.$$

Remember P is Lagrange multiplier

Note that $\int P_i C_i \mathrm{d}i = PC$, so it makes sense to call it the aggregate (ideal) price index, i.e. the price of one unit of consumption bundle.

Firms maximize profits subject to household demand (1), production technology and taking decisions of other firms as given:

$$\max_{P_i} P_i\left(\frac{P_i}{P}\right)^{-\theta} C - \frac{W}{A}\left(\frac{P_i}{P}\right)^{-\theta} C$$

Aggregate terms don't affect maximization

max profit
s.t production tech
& household demand

$$\max_{C_i, P_i} P_i C_i - W L_i$$

$$\text{s.t. } C_i = \left(\frac{P_i}{P}\right)^{-\theta} C, \quad \leftarrow \text{take as given from HH problem}$$

$$C_i = A L_i. \qquad L_i = \frac{C_i}{A}$$

Substitute constraints in the objective function and take the FOC: w.r.t P_i

see steps on next page!

$$C_i - \theta\left(P_i - \frac{W}{A}\right)\left(\frac{P_i}{P}\right)^{-\theta} \frac{C}{P_i} = 0,$$

which can be solved for the optimal price:

- mark up constant, $⫫ A_i$
  - monopolistic competition [vs. oligopoly]
  - CES demand [vs. Kimball demand]
- homothetic demand $P_i ⫫ C$
  - $C\uparrow$, $c_i\uparrow$

$$P_i = \frac{\theta}{\theta-1}\frac{W}{A}. \tag{2}$$

Given symmetry across firms, we obtain $P = \frac{\theta}{\theta-1}\frac{W}{A}$. Note that the equilibrium conditions only describe the real wage and relative prices of products, while nominal prices and wages are undetermined. Finally, the market clearing condition

$$\int \frac{C_i}{A} \mathrm{d}i = \frac{C}{A} = 1 \qquad C=A \tag{3}$$

then pins down the output $C_i = C = 1$, so that the aggregate welfare is equal $U = C = 1$.

$$\max_{P_i, C_i} \; P_i C_i - W L_i$$

$$\max_{P_i} \; P_i \left(\frac{P_i}{P}\right)^{-\theta} C - \frac{W}{A}\left(\frac{P_i}{P}\right)^{-\theta} C$$

$$\max_{P_i} \; P_i \left(\frac{P^{\theta}}{P_i^{\theta}}\right) C - \frac{W}{A}\left(\frac{P^{\theta}}{P_i^{\theta}}\right) C$$

Agg terms that appear in both terms don't affect maximization – pull out C, P

$$\max_{P_i} \; \frac{1}{P_i^{\theta-1}} - \frac{W}{A}\frac{1}{P_i^{\theta}}$$

FOCs: $$-\frac{(\theta-1)}{P_i^{\theta-2}} + \frac{W}{A}\frac{\theta}{P_i^{\theta-1}}$$

$$\frac{(\theta-1)}{P_i^{\theta-2}} = \frac{W}{A}\frac{\theta}{P_i^{\theta-1}}$$

$$\frac{P_i^{\theta-1}}{P_i^{\theta-2}} = \frac{\theta}{\theta-1}\frac{W}{A}$$

$$P_i = \frac{\theta}{\theta-1}\frac{W}{A}$$

SPP is to allocate labor across firms in an optimal way:

$$\max_{\{C_i\}} \left(\int C_i^{\frac{\theta-1}{\theta}} \mathrm{d}i \right)^{\frac{\theta}{\theta-1}}$$

$$\text{s.t.} \int \frac{C_i}{A} \mathrm{d}i = 1.$$

The FOC implies $C_i = C = 1$. Therefore, the monopolistically competitive equilibrium coincides with the first-best allocation and there is no room for government interventions.