Problem Set 3 — Sarah Bass

1) Using the Pareto efficiency rule, we know that a review session is efficient if:

1) $U^i(I=1, m-g_i) \geq u^i(I=0, m)$ — m = private good.

2) $\sum_i g_i \geq M$

Taking FOCs of the utility function w.r.t I, we have:

$\partial u / \partial I = a$

So: $a + w - g_i \geq m$

$a - g_i \geq 0$

$a \geq g_i$

Summing across all students:

$Na \geq \sum_i g_i \geq M$

where Na is the social benefit of a review session and M is the social cost. The review session is efficient if $N > \frac{M}{a}$.

2) $p_H = H - X_H$ $\quad H > L$

$p_L = L - X_L$ $\quad b$ - cost of operation per ticket

β - parking cost

Our consumer surplus, producer surplus, and Lagrangian are:

$CS(X_L, X_H) = X_L^2/2 + X_H^2/2$

$PS(X_L, X_H) = (H - b - X_H)X_H + (L - b - X_L)X_L - \beta \bar{X}$

$\mathcal{L} = X_L^2/2 + X_H^2/2 + (H - b - X_H)X_H + (L - b - X_L)X_L - \beta \bar{X} + \lambda_H(\bar{X} - X_H) + \lambda_L(\bar{X} - X_L)$

Taking FOCs w.r.t $X_H, X_L, \bar{X}$:

$[X_H]$: $X_H + H - b - 2X_H = \lambda_H$

$[X_L]$: $X_L + L - b - 2X_L = \lambda_L$

$[\bar{X}]$: $\beta = \lambda_H + \lambda_L$ $\quad$ Note $\lambda_H, \lambda_L \geq 0$, $\bar{X} \geq X_H, X_L$,

If $\lambda_L = 0$, $\lambda_H = \beta = H - b - X_H$. Since $\lambda_H \neq 0$ and $\lambda_H(\bar{x} - X_H) = 0$,
$X_H^* = \bar{X} = H - b - \beta$.

Since $\lambda_L = 0$, $\lambda_L = 0 = L - b - X_L \rightarrow X_L^* = L - b$

$X_L^* < \bar{X} \rightarrow L - b < H - b - \beta \longrightarrow \beta < H - L$

So if $\beta < H - L$, the efficient pricing is:

$p_H^* = H - X_H^* = b + \beta$

$p_L^* = L - X_L^* = b$

If $\lambda_L, \lambda_H > 0$, $X_L = \bar{X} = X_H$ since $\lambda_L(\bar{X} - X_L) = 0$ and $\lambda_H(\bar{X} - X_H) = 0$

$\beta = H - b - X_H + L - b - X_L$

$\beta + 2b = (H - X_H) + (L - X_L)$

$\beta + 2b = p_H^* + p_L^*$ if $\beta \geq H - L$