UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MACROECONOMICS THEORY Preliminary Exam

August 1, 2016

9:00 am - 2:00 pm

INSTRUCTIONS

- **Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:**

  (1) your assigned number
  (2) the number of the question you are answering
  (3) the position of the page in the sequence of pages used to answer the questions

Example:

MACRO THEORY 6/13/16

ASSIGNED # ____________

Qu # __1__ (Page __2__ of __6__):

- **Do not answer more than one question on the same page !**
  When you start a new question, start a new page.
- **DO NOT write your name anywhere on your answer sheets!**
  After the examination, the question sheets and answer sheets will be collected.
- **Please DO NOT WRITE on the question sheets.**
- **The number of points for each question is provided on the exam.**
- **Answer all questions.**
- **Do not continue to write answers onto the back of the page – write on one side only.**
- **Answers will be penalized for extraneous material; be concise.**
- **You are not allowed to use notes, books, calculators, or colleagues.**
- **Do NOT use colored pens or pencils.**
- **There are five pages in the exam, including this instruction page—please make sure you have all of them.**

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.

- All scratch paper, unused tablet paper, and exams are to be turned in after the exam. Your proctor will give you directions, listen to your proctor.

- Good luck!

1. OG **[total 50 points]:** Consider the following overlapping generations problem. Each period $t = 1, 2, 3, ...$ a unit measure of two period lived agents is born. There is no population growth. A generation t household has preferences over their per-capita consumption of nonstorable goods and labor supply given by

$$U(c_t^t, c_{t+1}^t, \ell_t^t) = \ln\left(c_t^t\right) - \frac{\gamma}{2}\left(\ell_t^t\right)^2 + \ln(c_{t+1}^t)$$

with constraints on consumption $c_t^t \geq 0, c_{t+1}^t \geq 0$ and labor supply $\ell_t^t \in [0, 1]$ when young only. The initial old have preferences $U(c_1^0) = \ln\left(c_1^0\right)$. The production technology is given by $y_t = A \cdot L_t$ where L_t is per capita labor input. Assume $\gamma > 2$ and $A > 0$.

Planner's problem

(a). **[10 points]** State and solve the planner's problem with objective given by $\ln(c_1^0) + \sum_{t=1}^{\infty} U(c_t^t, c_{t+1}^t, \ell_t^t)$ (where we appeal to the overtaking criterion for finite horizon economies).

Decentralized's problem

The unit measure of initial old are endowed with $\overline{M}_1$ units of fiat money. The money supply changes over time at constant rate $z > -1$. That is, $\overline{M}_{t+1} = (1+z)\overline{M}_t$ for $t \geq 1$. The change in money supply is lump sum transferred (or taxed) each period to old agents in direct proportion to the amount of money that they choose when young. In other words, if a young agent chooses $M_{t+1}^t \geq 0$, they will receive $(1+z)M_{t+1}^t$ units of money when old. Let p_t be the price of consumption goods denominated in terms of money at time t.

Competitive firms hire workers at wage w_t in terms of money (called the nominal wage) and use the production technology to produce goods that they sell at price p_t to maximize profits π_t which are distributed to the old.

(b). **[5 points]** Write down the optimization problems of the initial old and of generation t for $t \geq 1$ given prices, wages, and profits. State their budget constraints sequentially (i.e. in period t and $t+1$).

(c). **[2.5 points]** Denoting L_t the quantity of labor demanded by the firm, write down the firm's optimization problem.

(d). **[2.5 points]** Define a competitive equilibrium stating clearly the market clearing conditions (you need not re-write optimization problems stated above).

(e). **[10 points]** First solve the firm's optimization problem. Derive nominal (w_t) and real (w_t/p_t) wages. Show that equilibrium profits are zero. Assuming $\pi_t = 0, \forall t \geq 1$ solve the initial old and generation t's problems given goods prices (p_t, w_t) for $t \geq 1$. That is, solve for $(c_t^t, c_{t+1}^t, \ell_t^t, M_{t+1}^t)$ given (p_t, w_t, p_{t+1}).

(f). **[12.5 points]** Solve for allocations and prices in an active monetary equilibrium. Does the level ($\overline{M}_t$) of the money supply $\overline{M}_t$ affect the real allocation of resources in this economy (if not, this is called the neutrality of money)? Does the growth rate (z) of the money supply affect the real allocation of resources in this economy (if not, this is called the superneutrality of money)? Why or why not?

(g). **[7.5 points]** Does the decentralized equilibrium implement the planner's solution? If so, why? If not, why not and how can the government use monetary policy to implement the planner's solution?

2. Asset Pricing **[total 25 points]:** Consider a representative agent, stochastic growth model in which output y_t, is equal to $y_t = z_t k_t$ where k_t is capital at the beginning of the period and z_t is a random shock. This shock is i.i.d. over time and can take n possible values with π_i being the probability of $z_t = z_i$, $i = 1, ..., n$. The depreciation rate is $\delta = 1$ so that capital depreciates fully within the period. The social planner chooses sequences of consumption and capital in order to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma} - 1}{1 - \gamma}$$

with $\beta \in (0, 1)$. Given this environment, answer the following questions:

(a). [**5 points**] Express the maximization problem as a dynamic program explicitly identifying the state and policy variables.

(b). [**5 points**] Find the necessary conditions for an optimum, that is, the envelope conditions. Interpret these conditions.

(c). [**8 points**] Derive a closed form solution for the policy function describing optimal consumption. (8 points)

(d). [**7 points**] Suppose the following asset, called equity, is introduced in this economy. If the agent buys the asset at period t he is entitled to the consumption stream c_{t+j} for $j \geq 1$. The price of the asset at period t is q_t. Compute this price. [Note: the price should be described using the primitives of the problem].

3. Growth with Externality [**total 25 points**]: Consider an economy with infinitely many agents each with preferences given by the utility function

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{\gamma}}{\gamma} \qquad 0 < \beta < 1 \qquad \gamma < 1.$$

Each agent has one unit of time available in each period and can produce consumption goods according to

$$c_t = \alpha h_t^{\theta} \overline{h}_t^{1-\theta} u_t,$$

where $0 < \theta < 1$, $\alpha > 0$, u_t is the time spent in consumption goods production by the agent and $\overline{h}_t$ denotes the average human capital across all agents in the economy. The externality is such that each agent's productivity increases with the amount of human capital that other agents have. An agent's human capital evolves according to

$$h_{t+1} = \delta h_t (1 - u_t),$$

where $\delta > 0$. Assume that each agent has the same initial stock of human capital.

(a). [**5 points**] Set up the social planner's problem as a dynamic program. Note that the social planner internalizes the externality and treats $\overline{h}_t = h_t$.

(b). [**10 points**] Solve for the optimal growth rates of human capital and consumption, and solve for u_t.

(c). [**10 points**] Set up the representative agent's dynamic programming problem and solve for the competitive equilibrium. Also, solve for a balanced growth path of the economy.

4. Incomplete Markets **(50 points total)** Consider a two period endowment economy with a continuum of individuals of two types. The total population size is normalized to 1 and each type has measure $\frac{1}{2}$. Endowments are i.i.d. over time, common across agent types, and take on the values 0 or 1 in each period with equal probability. The aggregate endowment is constant at 1 in each period, and the relevant state of nature $s_t \in \{0,1\}, \quad t = 1,2$ is an i.i.d. random variable indicating which type receives the high endowment in period t. All agents have common preferences, defined over their own consumption c_t^i $i = 1,2$ at the two dates:

$$U(c_1^i, c_2^i) = -E\left[(c_1^i - 1)^2 + \beta(c_2^i - 1)^2\right].$$

Assume that $0 \leq c_t^i \leq 1$, as it will be in equilibrium below, so that utility is increasing.

(a). **[15 points]** Suppose that agents can trade in a complete set of state-contingent securities (in zero net supply) at date 0.

(i). State explicitly what the relevant securities are, and solve fully for the equilibrium allocation of consumption for the two agent types.

(ii). Show how the equilibrium allocation is implemented – that is, what are the equilibrium holdings of the different securities by each type?

(iii).What is the equilibrium rate of return r on a risk-free bond?

(b). **[15 points]** Now consider an incomplete markets economy, in which agents can only trade in a risk free bond in zero net supply. Let a_t^i be the holdings of this bond at the start of period t for each type i. Each agent is endowed with $a_1^i = 0$ initial holdings, so the holdings at date 2 are:

$$a_2^i = (1+r)(e_1^i - c_1^i)$$

There is no reason to carry bonds after period 2 so:

$$c_2^i = a_2^i + e_2^i.$$

(i). Taking the interest rate r as given, solve for each agent's optimal (state-contingent) consumption and bond holdings $\{c_t^i, a_t^i\}$.

(ii). Discuss and interpret how the interest rates and the allocations compare to the complete markets case.

(c). **[10 points]** Suppose that agents have access to a full set of state-contingent securities, but they cannot commit to deliver on them. After endowments are at each date agents have the option to default on their promises and revert to autarky for the remainder of time. Find the efficient allocation. Interpret your result and explain whether it would extend to longer horizon economies.

(d). **[10 points]** Now consider efficient allocations with private information. At date 1, after observing his endowment realization, an agent reports either 0 or 1 to the planner, receiving a transfer b_1^0 or b_1^1. In the second period, the transfer may depend on the agent's reports at both date 1 and 2. In any incentive compatible allocation, how does the second period transfer depend on the second period report? Interpret and explain how private information limits risk sharing.

Short Questions

5. **[10 points]** In the model of Burnside and Eichenbaum (1996), non-technology shocks (e.g., government spending shocks) affect conventional measures of technology (i.e., the Solow residual). Explain in 2-3 sentences the mechanism for this result.

6. **[10 points]** Consider the variant of the Bernanke, Gertler, and Gilchrist (1999) model that we studied in class. Suppose there are two economies, the first with a higher auditing cost parameter $\mu_1 > \mu_2$. In which of the two economies is the (steady-state) bankruptcy rate higher? Explain, as clearly as you can, the intuition for this result.

7. **[10 points]** Consider the problem of a household which invests in housing and also a second asset. Use B to denote the household's beginning of period non-housing wealth, and H as the beginning of period size of the house. In any period in which the consumer chooses a housing stock $H' \neq H$, a fixed cost of size $f \cdot H$ is incurred. Housing is the numeraire good. Period utility is given by H'. Between the current period and the following period, non-housing wealth has a stochastic gross return of R', i.i.d. across periods. Households discount with discount factor β.

a. Write a Bellman equation that describes the household's lifetime utility maximization problem. Use $V(B, H)$ to refer to the value function.

b. Re-formulate the value function into one that is a function of a single variable, $h \equiv \frac{H}{B}$. Hint: You might also want to use the transformation $h' \equiv \frac{H'}{B}$.

8. **[10 points]** Regarding the Kiyotaki and Moore (1997) model: Starting from the steady-state allocation, what would happen to aggregate productivity (measured as the number of units of fruit produced per unit of land) if some land was reallocated from the gatherers to the farmers?

9. **[10 points]** Let Δc_t, Δh_t, Δi_t, and Δy_t denote the quarterly growth rates of consumption, hours, investment, and aggregate output. How do the volatilities of these variables compare to one another, as observed in the U.S. data? Rank these four variables from least to most volatile. How do the volatilities of these variables compare to one another, as predicted by a baseline RBC model with only factor neutral productivity shocks?