Econ 711 Game Theory Final Exam

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1) a) $U_i = \begin{cases} -[x^3+1] & \text{for route A} \\ -[(1-x)^3+1] & \text{for route B} \end{cases}$ where x is the proportion of drivers on route A.

The game is supermodular.

b) If route A takes less time than route B, drivers on route B would be better off choosing route A. Similarly, if route B is faster than route A, drivers on route A would be better off choosing route B. So an equilibrium occurs where the time for both routes is equal

$$x^3+1 = (1-x)^3+1$$
$$x^3 = (1-x)^3$$
$$x = 1-x$$
$$x = \tfrac{1}{2}$$

The equilibrium occurs where half of the drivers choose route A and half choose route B. The equilibrium commute time $= \frac{1}{2}^3+1 = 1.125$ hours.

c & d) $S \to A \to T = x^3+1$

$S \to B \to T = (1-x)^3+1$

$S \to A \to B \to T = x^3 + x^3$

The equilibrium commute time will increase to 2 hours. All drivers who choose to go from $S \to B$ will be better off choosing to go from $S \to A$. Since the entire unit mass of drivers will go to A, $x^3 = 1$. All drivers who go $A \to T$ would be better off going $A \to B \to T$. Since the entire unit mass of drivers will go $A \to B \to T$, $x^3=1$. No driver who drives on the narrow road will be better off on the wide road. So $x^3+x^3 = 1+1 = 2$ hours

2) a) We can rewrite our table in terms of expected utility:

	L	M	R
T	$(1, \frac{1}{2})$	$(1, \frac{3}{8})$	$(1, \frac{3}{8})$
B	$(2, 2)$	$(0, \frac{3}{2})$	$(0, \frac{3}{2})$

For player 2, L strictly dominates M and R
For player 1, if player 2 is choosing L, they will choose B.
So the BNE is at (B, L).

b) We can rewrite our table s.t player 2 will choose R for state ω_1 and M for state ω_2

	L	M $\vert \omega_2$ R $\vert \omega_1$
T	$(1, \frac{1}{2})$	$(1, 3/4)$
B	$(2, 2)$	$(0, 3)$

Now choosing M or R strictly dominates choosing L for player 2.
Given that player 1 knows that player 2 will choose R for state ω_1 and M for state ω_2, player 1 will always be better off choosing T.
So the BNE is at (T, R) for ω_1 and (T, M) for ω_2.

c) Player 2 doesn't gain from being informed. Because they know their strictly dominant strategy, they choose this strategy. However, choosing M or R causes player 1 to choose T, which ultimately leads to player 2 having lower utility than when they didn't know the state ($3/4 < 2$). It seems that player 2 has learned the hard way that ignorance is bliss!

3) If both players play c: $\sum_{t=0}^{\infty} \delta^t$

If 1 deviates:

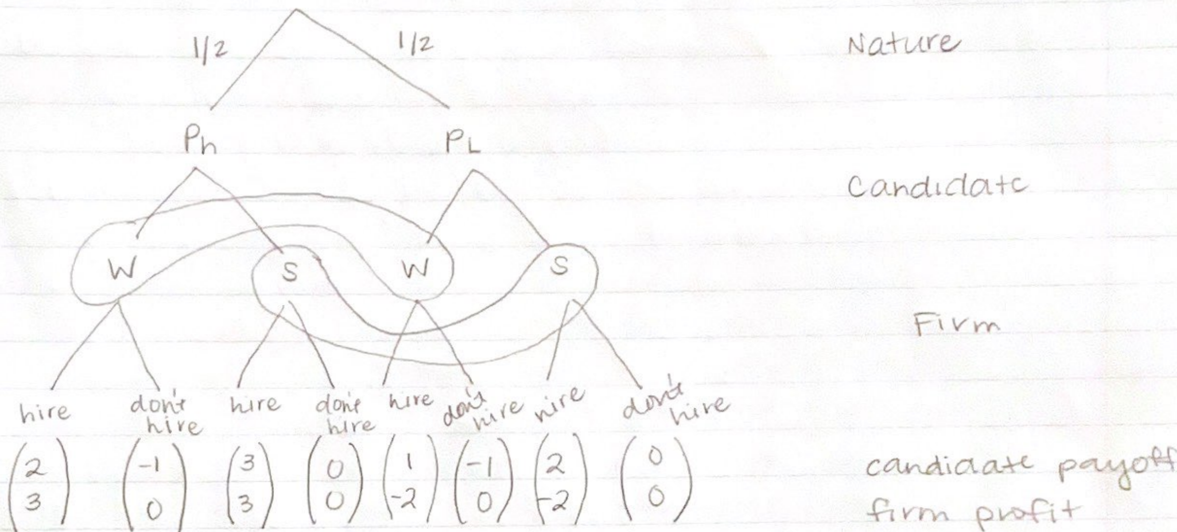
$$2 + \delta(-1) + \delta^2(2) + \delta^3(-1) \ldots$$

$U = \sum_{t=0}^{\infty} (2\delta^{2t} + (-1)\delta^{2t+1})$ for deviant

$U = \sum_{t=0}^{\infty} ((-1)\delta^{2t} + 2\delta^{2t+1})$ for rule follower

If both deviate: 0

4) a)



$P(P_h) + P(P_L) = 1 \rightarrow P(P_L) = 1 - P(P_h)$

b) Firms:

$E[\text{hire} \mid W] = 3P(P_h) + (-2)(1 - P(P_h))$

$E[\text{don't hire} \mid W] = 0$

$\Rightarrow$ hire since $3P(P_h) - 2(1 - P(P_h)) > 0$

$3(\frac{1}{2}) - 2(\frac{1}{2}) > 0$

$1.5 - 1 > 0$

$0.5 > 0$

$E[\text{hire} \mid S] = 3P(P_h) - 2(1 - P(P_h)) = 0.5$

$E[\text{don't hire} \mid S] = 0$

$\Rightarrow$ hire since $0.5 > 0$

Candidates:

$E[W \mid P_h] = 2$

$E[S \mid P_h] = 3$

$E[W \mid P_L] = 1$

$E[S \mid P_L] = 2$

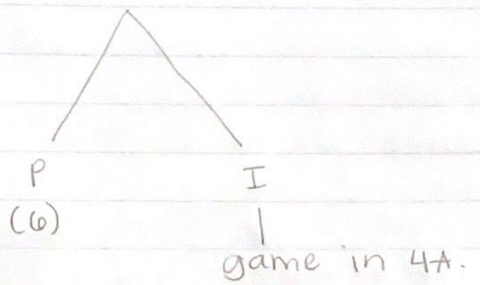
Since the firm will always hire the candidate, the candidate will always dress like a slob. The equilibria are:

(S, hire | P_L)

(S, hire | P_h)

No equilibria are eliminated by the Intuitive Criterion.

4c)



The firm will always promote from within. Since low productivity workers are fired immediately, the firm knows that anyone promoted from within the company would always be a high productivity worker. Further, the payoff of promoting from within is higher, even for high productivity workers. Regardless of the firm's beliefs while interviewing, the weak sequential equilibrium is to always hire from within.

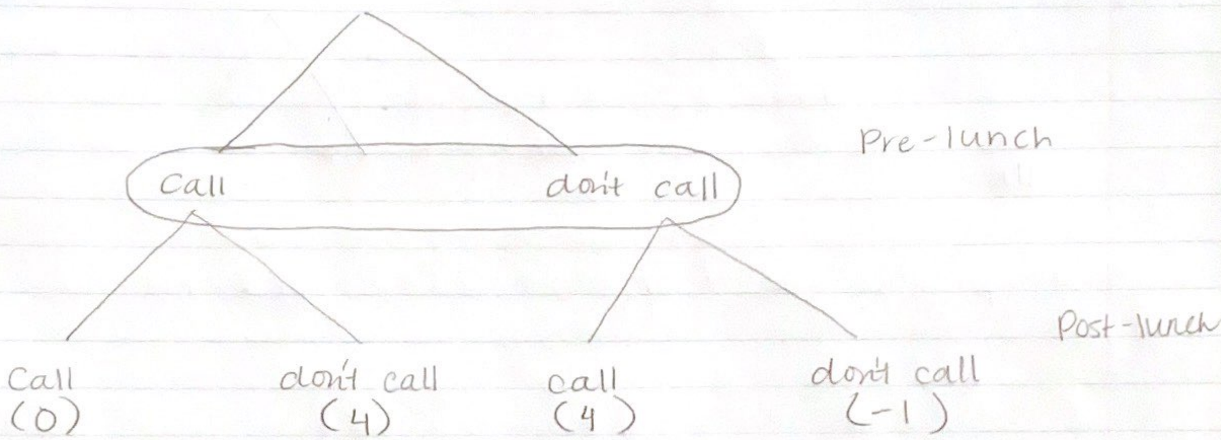
4d) The sequential equilibrium is still to hire from within. However, if firms were to interview, they would follow the beliefs outlined in 4B.

5a) The nash equilibrium is
- c is strictly dominated for player 1
- z is strictly dominated for player 2

A → Y → B → X → A ...
(A, Y), (B, Y), (B, X), (A, X), (A, Y) is the Nash equilibrium

b)
*

6a)



Cody, I am worried about your memory loss!

b) I have assumed the probability of calling is the same before and after lunch. We'll call if

$$0P(C) + 4P(D) > 4P(C) + (-1)P(D)$$

$$0P(C) + 4(1-P(C)) > 4P(C) + (-1)(1-P(C))$$

$$4 - 4P(C) > 4P(C) - 1 + P(C)$$

$$5 > 9P(C)$$

$$\boxed{P(C) = 5/9}$$

c) - If cody thinks there's more than a 5/9 prob he called before lunch, he won't call
- If cody thinks there's less than a 5/9 prob he called before lunch, he'll call.
- If cody thinks there's a 5/9 probability he called before lunch, he will mix between calling again and not calling after lunch with P(call) = 5/9