# Practice Problems 3

Office Hours: Tuesdays, Thursdays from 3:30 to 4:30 at SS 6439.
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If you need help, reach out: your classmates, the TA, textbooks, or the Professor.

**NEGATIONS**

1. Negate the following:
   (a) * Exists $x \in \mathbb{R}$ such that $\log x = 30$
   (b) $\forall\ a \in \mathbb{Q},\ \sqrt{a} \in \mathbb{Q}$
   (c) * If you're Madisonian, then you were born in Wisconsin.
   (d) A person can be happy while not loving spicy food.
   (e) * $\forall \epsilon \in \mathbb{R}$ such that $\epsilon > 0,\ \exists N \in \mathbb{N}$ such that $\forall\ n \in \mathbb{N}$, satisfying $n \geq N,\ 1/n < \epsilon$.
   (f) Between any rational numbers, there exists another rational.

**SEQUENCES AND LIMITS**

2. * Show that the every convergent sequence is a Cauchy sequence.
3. * Show that $logx$, $x = 1, 2, 3...$ does not have any convergent subsequence.
4. * Define $a_n = \sum_{i=1}^{n}(-1)^n \frac{1}{n}$. Show that $\{a_n\}$ is Cauchy to argue it converges somewhere.
5. * Show that if $\{x_k\} \subset \mathbb{R}$ converges to $x \in \mathbb{R}$, so does every subsequence.
6. * Show that $\{x_k\} \subset \mathbb{R}$ converges to $x \in \mathbb{R}$ iff every subsequence of it has a subsequence that converges to $x$.
7. * Suppose $\{p_n\}$ and $\{q_n\}$ are Cauchy sequences in a metric space $X$. Show that the sequence $\{d(p_n, q_n)\}$ converges.
8. Prove or disprove the following:
   (a) $y_k = \frac{1}{k}$ is a subsequence of $x_k = \frac{1}{\sqrt{k}}$.
   (b) $x_k = \frac{1}{\sqrt{k}}$ is a subsequence of $y_k = \frac{1}{k}$.

**COMPLETENESS**

9. * Prove or disprove the following:
   (a) $\mathbb{Q}$ is a complete space.
   (b) Any subset of $\mathbb{R}$ is a complete space.