UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MACROECONOMICS THEORY Preliminary Exam

June 9, 2017

9:00 am - 2:00 pm

INSTRUCTIONS

- **Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:**

  (1) your assigned number
  (2) the number of the question you are answering
  (3) the position of the page in the sequence of pages used to answer the questions

Example:	
MACRO THEORY	6/09/17
ASSIGNED # ____________	
Qu # __1__ (Page __2__ of __8__):	

- **Do not answer more than one question on the same page !**
  When you start a new question, start a new page.
- **DO NOT write your name anywhere on your answer sheets!**
  After the examination, the question sheets and answer sheets will be collected.
- **Please DO NOT WRITE on the question sheets.**
- **The number of points for each question is provided on the exam.**
- **Answer all questions.**
- **Do not continue to write answers onto the back of the page – write on one side only.**
- **Answers will be penalized for extraneous material; be concise.**
- **You are not allowed to use notes, books, calculators, or colleagues.**
- **Do NOT use colored pens or pencils.**
- **There are eight pages in the exam, including this instruction page—please make sure you have all of them.**

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.

- All scratch paper, unused tablet paper, and exams are to be turned in after the exam. Your proctor will give you directions, listen to your proctor.

- Good luck!

Question 1. This question has three parts:

Consider an overlapping generations model with two-period-lived agents. Each generation has a unit measure of agents. Each agent is indexed by $\alpha \in [0,1]$ uniformly distributed. Agent α in generation t has an endowment profile $\{\alpha y, (1-\alpha)y\}$. That is, their income is αy in youth and $(1-\alpha)y$ in old age. Thus in each generation there are agents who are more heavily endowed in their youth (those with α above $\frac{1}{2}$) and agents who are more heavily endowed in their old age (those with α less than $\frac{1}{2}$). The standard OG model typically assumes a degenerate distribution on $\alpha = 1$. We assume that all initial old agents have an endowment of $\frac{y}{2}$ and that there is no population growth. All agents have identical preferences given by $\ln\left(c^t_{\alpha,t}\right) + \ln\left(c^t_{\alpha,t+1}\right)$ where $c^i_{\alpha,j}$ denotes consumption at time $j = 1,2,3,...$ by an agent with endowment profile α born at time i with initial old preferences simply given by $\ln\left(c^0_{\frac{1}{2},1}\right)$.

A. Planner's Problem (10 points) Suppose there is a social planner who weights all agents equally across generations and chooses consumption allocations.

i. (5 points) State the planner's problem.

ii. (5 points) Solve for the optimal allocation.

B. Competitive Equilibrium (17.5 points) Suppose there is a competitive bond market with no ability to default. That is, if agent α of generation t borrows or lends $B_{\alpha,t}$ in period t they must pay back or are paid back $(1+r_t)B_{\alpha,t}$ at time $t+1$ where r_t is the real interest rate on bonds issued in period t. Specifically, $B_{\alpha,t} > 0$ corresponds to lending and $B_{\alpha,t} < 0$ corresponds to borrowing.

i. (7.5 points) State the conditions for a competitive bond market equilibrium (i.e. the optimization problem facing any agent α of generation t and market clearing). Characterize how $B_{\alpha,t}$ depends on r_t.

ii. (7.5 points) Solve for the equilibrium interest rate.

iii. (2.5 points) Does the competitive equilibrium implement the planner's solution?

C. Competitive Equilibrium with Costly Participation (22.5 points) Now assume that an individual who chooses to borrow or lend using bonds must incur a small fixed transactions cost $\tau > 0$. You are to state and characterize certain properties of a competitive bond market equilibrium with transactions costs.

i. (15 points) To start, state the value functions for an agent born in period t with earnings α if they participate (i.e. incur τ to borrow or lend) denoted $V_{P,t}(\alpha)$ and if they do not participate denoted $V_{N,t}(\alpha)$. For each agent α, the agent chooses $V_t(\alpha) = \max\{V_{P,t}(\alpha), V_{N,t}(\alpha)\}$. Then you should characterize when $V_{P,t}(\alpha)$ exceeds or is exceeded by $V_{N,t}(\alpha)$ at $\alpha \in \left\{0, \frac{1}{2}, 1\right\}$ when the interest rate is evaluated at the value you found in part **B**. At that interest rate, what does it imply about cutoff values of α such that agents borrow, lend, or do not participate?

ii. (7.5 points) Without explicitly solving for interest rates, provide an argument why or why not real interest rates may be different from their value in part **B**. Specifically, does the interest rate which you found in part **B** constitute an equilibrium in part **C**? If not, is the interest rate higher or lower than what you found in part **B**? Are there multiple equilibria?

Question 2. This question has three parts:

A. Incomplete Markets: (15 points) Consider the following problem of a retiree. She faces a constant probability of death p every period. She begins retirement with assets a_0. In every period, she decides how much to consume and save. She receives social security income s in every period. This is constant. Further, assume that annuity markets are missing. In other words, complete insurance against the possibility of being alive are not available. Also, the individual cannot borrow against future income. If the individual dies with positive assets, these assets are redistributed to everyone in society equally. There are a continuum of such individuals. To make things specific, assume that the momentary utility function is given by $U(c)$. Further, think of a standard neoclassical firm with a constant-returns-to scale production function $F(K, L)$. As usual, the solution to the firm's first-order conditions give the interest and wage rates.

i. (2.5 points) Formulate the individual's dynamic programming problem.

ii. (2.5 points) Write down the first-order and envelope conditions.

iii. (10 points) In General Equilibrium, If annuity markets were indeed present, what do you think would happen to the capital stock? The interest rate? Imagine there were a tax on capital income with the proceeds rebated to everyone in a lumpsum fashion. Would such a tax transfer scheme improve welfare? Outline your argument.

B. Altruism: (20 points) One topic that has been subject to considerable debate is whether or not individuals have a bequest motive. That is, is the life cycle model a good description of reality, or is the altruistic model a better description. To be concrete, and to simplify matters greatly, consider a very simple model where parents and children live for two periods, one as a child in which they make no decisions and one as an adult. An adult parent has assets a_p and assume that the child will possess assets a_k when she grows up. Parent cares about utility $u(c_p)$ from own consumption and also derives utility from kid consumption $u(c_k)$. Parent's consumption equal assets less bequests b. And child consumption equals child assets plus the value of bequests. Let the degree of altruism be α.

i. (5 points) Write down the decision programming problem facing the parent. Write down the FOC for bequests.

ii. (15 points) The objective here is to come up with a clean testable implication from this model with bequests that will help test whether individuals indeed have a bequest motive. The optimal decision rule for bequests depend on parent and child states of the world. Consider a parent who optimally chooses to leave positive bequests to her daughter. The objective here is to calculate the Transfer Derivative and then come up with a prediction. Calculate the the difference between the derivative of bequests with respect to parent assets and the derivative of bequests with respect to child assets? Do you think the transfer derivative holds in the real world data? How would you test this prediction?

C. Growth (15 points): Consider an economy where a single homogeneous good is traded. the good can be used for consumption or for investment. The economy is populated by a large number of identical agents. Life-time utility of a representative agent is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln C_t + \gamma(1 - N_t)], \quad 0 < \beta < 1$$

where N_t is the time spent working. Output is produced according to $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, 0 < \alpha < 1$. A_t is a stochastic productivity shock. Assume that

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_t, \varepsilon_t \sim iid(0, \sigma_{\varepsilon}^2).$$

Further assume that ε_t is observed before any period-t decisions are undertaken. Physical capital depreciates at rate δ.

i. (2.5 points)What is the Bellman's equation that corresponds to the above problem. What are the states and controls?

ii. (2.5 points) Derive the Euler equation and the FOC for the fraction of time spent.

iii. (5 points) One of the 'stylized facts' over the last hundred years in the United States is that while GDP per capita has been growing at a constant rate, fraction of life-time spent working has been decreasing. Now assume that $\delta = 1$. Is there any hope for this model to match the above 'fact'? In answering this question you could alter the manner in which productivity evolves. Explain and account for it clearly.

iv. (5 points) Now, assume that $\gamma = 0$ and $A_t = A$ for all t. If $\delta < 1$, can the value function take the form $V(K) = E + F \log(K)$ for constants E and F? Why?

Question 3. This question has two parts:

A. (15 points) Consider an endowment economy where a representative agent has recursive preferences of the Epstein-Zin type. That is, the utility V_t of a consumption stream $\{c_s\}_{s=t}^{\infty}$ is evaluated recursively:

$$V_t = \left((1-\beta)c_t^{1-\rho} + \beta \left(E_t V_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right)^{\frac{1}{1-\rho}},$$

where $\rho > 0$ and $\alpha > 0$. Notice that this is a combination of a CES aggregate (with parameter ρ) of current utility of consumption and a risk-adjustment (with parameter α) of future utility.

(i) **(5 points)** Show that when $\alpha = \rho$ these preferences collapse to standard expected utility with a power utility function.

(ii) **(5 points)** Epstein-Zin preferences allow us to disentangle risk aversion and intertemporal substitution. How are these properties characterized here?

(iii) **(5 points)** Find an expression for the intertemporal marginal rate of substitution (stochastic discount factor), which we can define here as:

$$S_t = \frac{\frac{\partial V_t}{\partial V_{t+1}} \frac{\partial V_{t+1}}{\partial C_{t+1}}}{\frac{\partial V_t}{\partial C_t}}.$$

B. (35 points) Now suppose that the endowment process (fruit from the Lucas tree) has i.i.d. growth rates, that is:

$$c_{t+1}/c_t = g + \sigma_c \varepsilon_{t+1}$$

where $g > 0$ and $\sigma_c > 0$ are constants and $\varepsilon_t \sim N(0,1)$.

(i) **(10 points)** Conjecture a Markov pricing function, then write down the Bellman equation for the representative agent and find his optimality conditions.

(ii) **(10 points)** Define a recursive competitive equilibrium, being specific about the objects which make it up.

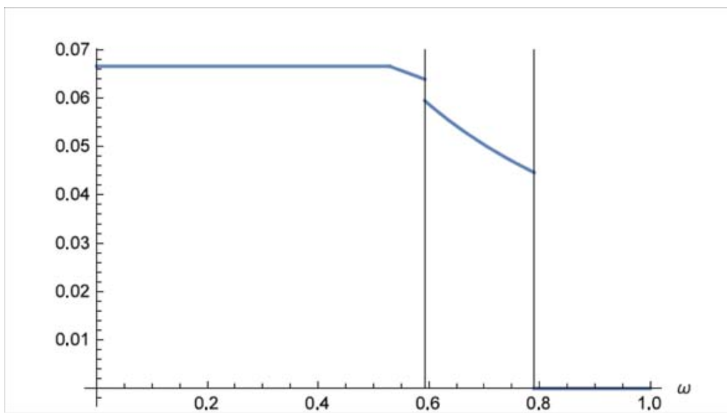
(iii) **(5 points)** Show that the value function can be written $V(c_t) = vc_t$ for some constant v, and find an expression for $\log S_t$.

(iv) **(5 points)** Find an expression for the risk-free rate. How does this differ from the standard CRRA case?

(v) **(5 points)** Find an expression for the return on the Lucas tree. How does this differ from the standard CRRA case?

Question 4. This is a question with four parts:

A. (14 points) The following plot is taken from our discussion of Bernanke and Gertler (1989). The plot was constructed by assuming that each successful investment project produces $\kappa_2 = 0.1$ units of capital; unsuccessful projects produce $\kappa_1 = 0$. The probability of a successful project is $\pi_2 = \frac{2}{3}$ (so $\pi_1 = \frac{1}{3}$); this explains why the capital produced by the most efficient entrepreneurs equals $\frac{2}{3} \cdot 0.1 \approx 0.067$. The horizontal axis gives entrepreneur type, $\omega \in [0, 1]$. The vertical axis plots the expected units of capital which are produced by each entrepreneur of type ω (for a given parameterization, productivity level, etc...).



i. There are two vertical lines (one just below ω =0 .6, and a second just below ω=0.8). Call the locations of the vertical lines $\underline{\omega}$, $\bar{\omega}$. Why do the expected units of capital output begin to decline towards the right portion of the $[0, \underline{\omega}]$ interval? In this interval, how does the probability of auditing vary with ω?

ii. Why does the expected units of capital output decrease in ω within the $[\underline{\omega}, \bar{\omega}]$ interval? In this interval, how does the probability of auditing vary with ω?

iii. Draw a sketch: What would the above plot look like in a parameterization in which lenders can audit without cost?

B. (12 points) Consider the following neoclassical growth model: Equation 1 describes the preferences of the representative consumer. Equations 2 and 3 describe the relationship between capital and investment and the production function and good-market-clearing condition. Equation 4 describes the evolution of productivity. Assume labor supply, H_t, is fixed across time at $\bar{H}$.

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\theta}}{1-\theta}, \theta \neq 1 \tag{1}$$

$$K_{t+1} = (1-\delta) K_t + I_t, \delta \in (0,1) \tag{2}$$

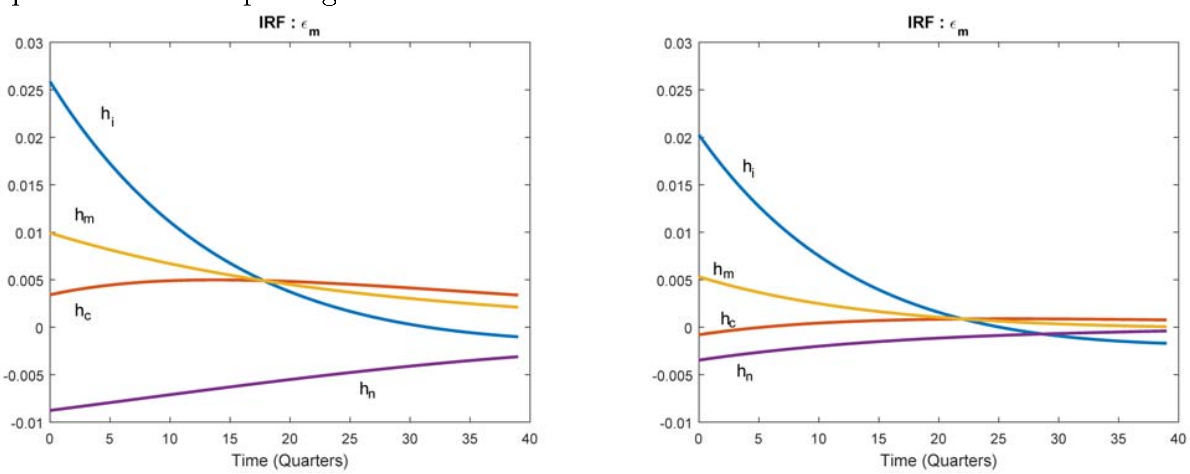
$$C_t + I_t = Y_t = A_t \cdot \left(\alpha^{\frac{1}{\sigma}} (K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} \bar{H}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad \alpha, \sigma \in (0,1) \tag{3}$$

$$\log A_t = \log A_{t-1} + \gamma, \gamma > 0 \tag{4}$$

i. Does this model have a balanced growth path with positive investment? Sketch a proof of your assertion.

ii. Does this model have a balanced growth path with positive investment if instead $\sigma \to 1$? Compute the growth rate of output.

C. (12 points) The following two figures are taken from our discussion of Benhabib, Rogerson, and Wright (1991, Homework in Macroeconomics: Household Production and Aggregate Fluctuations). Each plot gives the impulse responses resulting from a shock to the productivity of the market produced good, depicting the effect of an increase in productivity on i) hours worked in the market sector, h_m, ii) hours worked in the market sector in order to increase next-period capital, h_i, iii) hours worked in the market sector to produce consumption, h_c, iv) and hours worked in home production, h_n. The two plots differ only in terms of the assumed elasticity of substitution in consumers' preferences between home-produced consumption goods and market-produced consumption goods.



i. Does the right panel correspond to the parameterization with the high elasticity of substitution? Or the low elasticity of substitution?

ii. In the parameterization in which h_c declines on impact (as in the right panel), what is the intuition for the negative co-movement between h_c and h_i?

iii. Building on the answer to part **ii**, why does the parameterization of the left panel imply positive co-movement between h_c and h_i?

D. (12 points) In a paragraph, summarize the contribution of Chari, Kehoe, and McGrattan (2007). Included in this paragraph, answer the following: How does the paper's accounting procedure indicate that financial frictions like the ones modeled in Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) do not provide the primary explanations for understanding the early 1980s or 2007-09 recessions?