FALL 2018: PROBLEM SET 5

1. There are two firms, i = 1,2, with demand functions $D_i = 1 - 2P_i + P_j$. Firm 1 is the incumbent and its marginal cost is ½. Firm 2 is the entrant and its marginal cost is zero. By investing I = .205, the incumbent can buy a new technology which reduces its marginal cost to zero. The timing is as follows. The incumbent chooses whether to invest first and the entrant observes the incumbent's investment decision. In the second stage, both firms compete in prices.

(a) Show that, in a subgame perfect equilibrium, the incumbent does not invest.

Now suppose the investment decision made by the incumbent is not observable to the entrant.

(b) Define strategies and payoffs for this game.
(c) Show that the incumbent investing in the new technology is an equilibrium.

2. Consider an industry in which each of N firms produces at the same, constant average cost c. Each consumer buys from only one firm. Each firm's share of consumers (market share) is determined solely by the advertising expenditures of the firms as follows:

$$s_i = A_i/(\Sigma A_j)$$

where s_i and A_i are firm i's market share and advertising expenditure respectively. Note that the shares are independent of prices.

Advertising only affects brand selection; after a consumer has selected a brand, the number of q units he purchases is a function of only that brand's price p. Each individual's demand function is q = a – bp.

(a) Characterize a static, symmetric Nash equilibrium in which firms simultaneously choose prices and advertising expenditures, for each of the two specifications.
(b) Determine the number of firms under free entry, when each firm incurs a fixed cost F, for each of the two specifications.

Now consider a single market in which there is an incumbent firm and a potential entrant and there are no fixed costs. There are two periods: in period 1, the incumbent chooses its level of advertising; in period two, the entrant chooses its level of advertising and both firms choose their prices.

(c) If the incumbent chooses to deter entry, how much will it spend on advertising?
(d) If it chooses to accommodate, how much will it spend on advertising? Which of deterrence and accommodation will it in fact choose?

Bonus Question:

3. Consider a market for a homogenous good. Let $R(p) = pq(p)$ denote the revenues obtained from selling $q(p)$ units at price p. Let p^M denote the price that maximizes $R(p)$ and assume that R is increasing for $p < p^M$. Two firms can enter this market by paying a fixed cost c. Assume that $c < R(p^M)$. Production costs are zero for both firms.

The timing is as follows. Each firm decides whether to enter or not and if it enters, it chooses a price. In making these decisions, it does not observe the entry or prices of the other firm. In other words, the game is a simultaneous move game in which each firm chooses whether to enter and, if it enters, a price. After the prices are set, the market opens and consumers purchase from the firm with the lowest price. If the two firms charge the same price, each gets ½ of demand at that price.

(a) Explain why there is no equilibrium in which firms play pure entry strategies (i.e., enter with probability 1 or 0).
(b) Suppose that each firm randomizes over entry and let α_i denote the entry probability of firm i. Explain why there is no pure strategy equilibrium in prices.
(c) Let G_i denote the distribution of firm i's prices conditional on entry. Define the expected profit to firm j from entering and charging price p.

You may assume that $G_1 = G_2 = G$ (i.e., the equilibrium is symmetric) and that G is continuously increasing on the interior of its support.

(d) Explain why the upper bound of the support of G is p^M. Use this result to show that each firm enters with probability $1 - c/R(p^M)$.
(e) Determine the equilibrium price distribution.
(e) Find an expression determining the lower bound of the support of G.
(f) What happens to the distribution of prices as $c \to 0$?