

University of Wisconsin-Madison  
Department of Economics

Econ 703  
Fall 2003

Prof. R. Deneckere

**Homework**  
**(due on Oct. 10, 2003)**

1. A point  $x$  is an interior point of set  $A$  if there exists a neighbourhood  $N$  of  $x$  such that  $N \subset A$ . Let  $\mathring{A}$  be the interior of the set  $A$ , i.e. the collection of all of its interior points. Prove the following :
  - (1)  $\mathring{A}$  is an open set;
  - (2)  $A$  is open iff  $A = \mathring{A}$
  - (3) If  $B \subset A$ , and  $B$  is open, then  $B \subset \mathring{A}$ .
2. Let  $K$  be the union of the set  $\{0\}$  and the set  $\{1/n, n \in \mathbb{Z}_{++}\}$ . Prove that  $K$  is compact directly from the definition (i.e., without using the Heine\_Borel Theorem).
3. Sundaram, #26, p. 68.
4. Sundaram, #52, p. 72.
5. Consider the set of all rational numbers,  $\mathbb{Q}$ , and make it into a metric space by defining  $d(p,q) = |p-q|$  for all  $p,q \in \mathbb{Q}$ . Let  $E$  be the set of all  $p \in \mathbb{Q}$  such that  $2 < p^2 < 3$ . Show that  $E$  is closed and bounded in  $\mathbb{Q}$ , but that  $E$  is not compact. Conclude that  $\mathbb{Q}$  is not a compact space. Is  $E$  open in  $\mathbb{Q}$ ?  
HINT : Be very careful here. The notions closed, open, compact are all with reference to the metric space  $(\mathbb{Q},d)$ , not the metric space  $(\mathbb{R},d)$ !