Problem set 3 Sarah Bass

1) using the Pareto efficiency rule, we know that a review session is efficient if:

$$1)\ U^i(I=1, m-g_i) \geq u^i(I=0, m) \qquad m\text{-private good.}$$
$$2)\ \sum_i g_i \geq M$$

Taking FOCs of the utility function w.r.t I, we have:

$$\partial u / \partial I = a$$

So: $a + m - g_i \geq m$

$$a - g_i \geq 0$$
$$a \geq g_i$$

Summing across all students:

$$Na \geq \sum_i g_i \geq M$$

where Na is the social benefit of a review session and M is the social cost. The review session is efficient if $N > \frac{M}{a}$.

2) $p_H = H - X_H$ $\qquad H > L$

$p_L = L - X_L$ $\qquad b$- cost of operation per ticket

$\qquad \beta$- parking cost

Our consumer surplus, producer surplus, and Lagrangian are:

$$CS(X_L, X_H) = X_L^2/2 + X_H^2/2$$
$$PS(X_L, X_H) = (H - b - X_H)X_H + (L - b - X_L)X_L - \beta\bar{X}$$
$$\mathcal{L} = X_L^2/2 + X_H^2/2 + (H - b - X_H)X_H + (L - b - X_L)X_L - \beta\bar{X} + \lambda_H(\bar{X} - X_H) + \lambda_L(\bar{X} - X_L)$$

Taking FOCs w.r.t $X_H, X_L, \bar{X}$:

$[X_H]$: $X_H + H - b - 2X_H = \lambda_H$

$[X_L]$: $X_L + L - b - 2X_L = \lambda_L$

$[\bar{X}]$: $\beta = \lambda_H + \lambda_L$ $\qquad$ Note $\lambda_H, \lambda_L \geq 0$, $\bar{X} \geq X_H, X_L$,

If $\lambda_L = 0$, $\lambda_H = \beta = H - b - x_H$. Since $\lambda_H \neq 0$ and $\lambda_H(\bar{x} - x_H) = 0$,
$x_H^* = \bar{x} = H - b - \beta$.

Since $\lambda_L = 0$, $\lambda_L = 0 = L - b - x_L \rightarrow x_L^* = L - b$

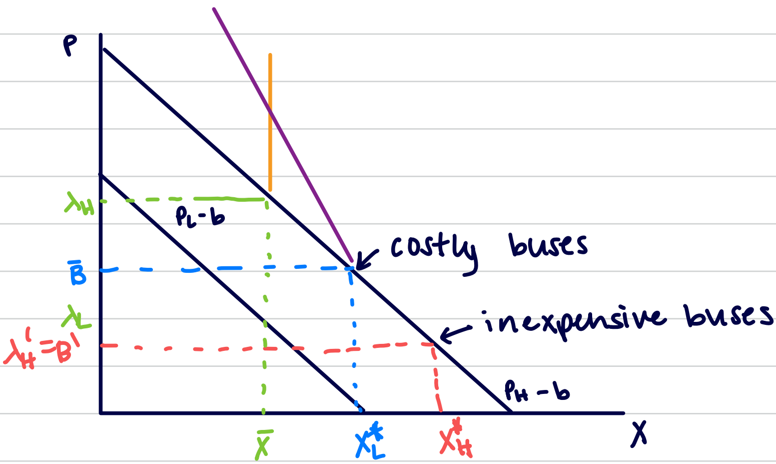
$x_L^* < \bar{x} \rightarrow L - b < H - b - \beta \longrightarrow \beta < H - L$

So if $\beta < H - L$, the efficient pricing is:

$$p_H^* = H - x_H^* = b + \beta$$
$$p_L^* = L - x_L^* = b$$

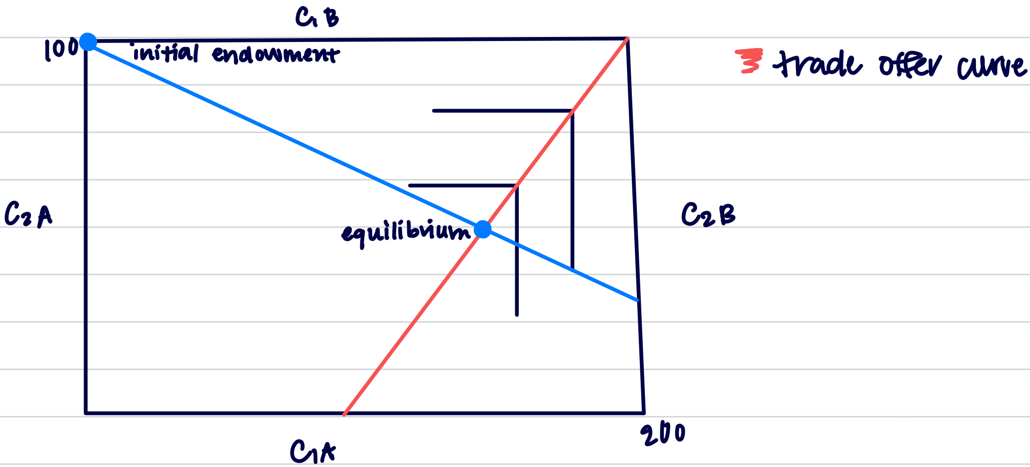
If $\lambda_L, \lambda_H > 0$, $x_L = \bar{x} = x_H$ since $\lambda_L(\bar{x} - x_L) = 0$ and $\lambda_H(\bar{x} - x_H) = 0$

$$\beta = H - b - x_H + L - b - x_L$$
$$\beta + 2b = (H - x_H) + (L - x_L)$$
$$\beta + 2b = p_H^* + p_L^* \quad \text{if } \beta \geq H - L$$



3) $U^A = c_1 c_2$ $\qquad$ $0\,c_1,\ 100\,c_2$

$U^B = \min\{c_1, c_2\}$ $\qquad$ $200\,c_1,\ 0\,c_2$



$c_1^1 + c_1^2 = 200$

$c_2^1 + c_2^2 = 100$

$p_1 c_1^1 + p_2 c_2^1 = 100 p_2$

$\mathcal{L}_1 = c_1^1 c_2^1 - \lambda(p_1 c_1^1 + p_2 c_2^1 - 100 p_2)$

FOC: $[c_1^1]: c_2^1 = \lambda p_1 \qquad \lambda = c_2^1 / p_1$

$[c_2^1]: c_1^1 = \lambda p_2$

$c_1^1 = \dfrac{c_2^1 p_2}{p_1}$

$p_1 c_1^1 + p_2 c_2^1 = 100 p_2$

$p_1 \left[\dfrac{c_2^1 p_2}{p_1}\right] + p_2 c_2^1 = 100 p_2$

$2 c_2^1 = 100$

$c_2^1 = 50$

$c_1^1 = \dfrac{50 p_2}{p_1}$

$p_1 c_1^2 + p_2 c_2^2 = 200 p_1$

$c_1^2 = c_2^2$

$p_1 c_1^2 + p_2 c_1^2 = 200 p_1$

$c_1^2 = \dfrac{200}{1 + p_2/p_1}$

$c_1^1 + c_1^2 = 200$

$\dfrac{50 p_2}{p_1} + \dfrac{200}{1 + p_2/p_1} = 200$

$\dfrac{p_2}{4 p_1} + \dfrac{1}{1 + p_2/p_1} = 1$

$\rightarrow p_2 / p_1 = 3$

Using $p_2/p_1 = 3$,

$c_1^1 = 50\, p_2/p_1 = 150$

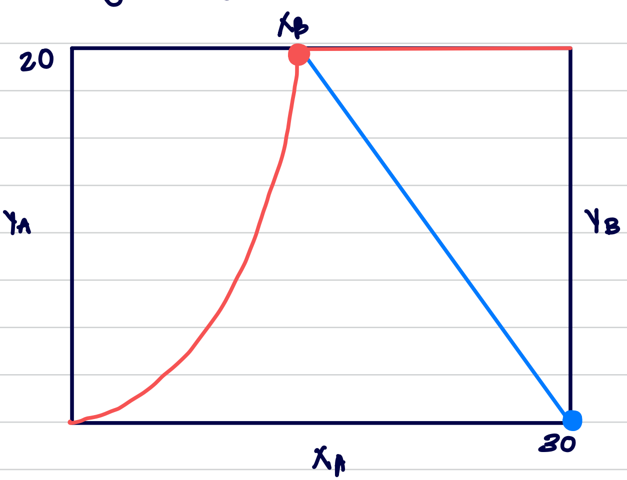
$c_2^1 = 50$

$c_1^2 = 200/1 + p_2/p_1 = 50$

$c_2^2 = 50$

4) $u^A = xy$ $\quad$ $30x,\ 0y$

$u^B = y + 20\log x$ $\quad$ $0x,\ 20y$



$MRS_A = MRS_B$ $\quad$ contract curve

$$\frac{Y_A}{X_A} = \frac{20}{X_B}$$

$$Y_A = \frac{20X_A}{X_B}, \quad X_A + X_B = 30$$

$$Y_A = \frac{20X_A}{30 - X_A}$$

$$\mathcal{L}^A = x_A y_A - \lambda[p_x x_A + p_y y_A - 30p_x]$$

FOC: $[x_A]: y_A = \lambda p_x$

$[y_A]: x_A = \lambda p_y \qquad \lambda = x_A/p_y$

$$y_A = \frac{x_A p_x}{p_y}$$

$$p_x x_A + p_y\left[\frac{x_A p_x}{p_y}\right] = 30p_x$$

$$2x_A = 30$$

$$x_A = 15$$

$$y_A = \frac{15p_x}{p_y}$$

$$\mathcal{L}^B = y_B + 20\log x_B - \lambda[p_x x_B + p_y y_B - 20p_y]$$

FOC: $[x_B]: \frac{20}{x_B} = \lambda p_x$

$[y_B]: 1 = \lambda p_y$

$$\frac{20}{x_B} = \frac{p_x}{p_y}$$

$$\frac{x_B}{20} = \frac{p_y}{p_x}$$

$$x_B = \frac{20p_y}{p_x}$$

$$p_x\left[\frac{20p_y}{p_x}\right] + p_y y_B = 20p_y$$

$$20p_y + p_y y_B = 20p_y$$

$$y_B = 0$$

$Y_A = 20$ $\qquad$ $X_A = 15$

$Y_B = 0$ $\qquad$ $X_B = 15$

$Y_A = \frac{15 P_x}{P_y} = 30 P_x$ $\qquad$ $15 = X_B = \frac{20 P_y}{P_x} = \frac{20(\frac{1}{2})}{P_x}$

$\frac{15}{P_y} = 30$ $\qquad$ $P_x = 2/3$

$P_y = 15/30 = 1/2$

$P_x =$