University of Wisconsin

Microeconomics Prelim Exam

Monday, July 29, 2019: 9AM - 2PM

- There are four parts to the exam. All four parts have equal weight.
- Answer all questions. No questions are optional.
- Hand in 12 pages, written on only one side.
- Write your answers for different parts on different pages. So do not write your answers for questions in different parts on the same page.
- Please place a completed label on the top right corner of each page you hand in. On it, write your assigned number, and the part of the exam you are answering (I,II,III,IV). Do not write your name anywhere on your answer sheets!
- Show your work, briefly justifying your claims. Some solutions might be faster done by drawing a suitable diagram.
- You cannot use notes, books, calculators, electronic devices, or consultation with anyone else except the proctor.
- Please return any unused portions of yellow tablets and question sheets.
- There are five pages on this exam, including this one. Be sure you have all of them.
- Best wishes!

Part I

1. Consider the following observations of an individual's choices at three price and wealth levels:

observation	prices	wealth	demand
A	$p^A = (6, 2)$	120	$x^A = (15, 15)$
B	$p^B = (7, 2)$	120	$x^B = (12, 18)$
C	$p^C = (6, 3)$	120	$x^C = (10, 20)$

   Show that the data is consistent with rational choice given locally non-satiated preferences. Give the most complete preference ordering you can over the bundles (x^A, x^B, x^C).

2. The "data" above was generated by the demand function

$$x(p, w) \quad = \quad \left(\frac{w - 30p_2}{p_1 - p_2}, \quad \frac{30p_1 - w}{p_1 - p_2} \right) \tag{1}$$

   defined for prices $p_1 > p_2$ and wealth $w \in [30p_2, 30p_1]$.

   (a) Verify that $x(p, w)$ has a symmetric Slutsky matrix.

   (b) Is good 1 normal or inferior? Is good 2 normal or inferior? Is either good a Giffen good?

3. Now suppose $p_1 > p_2$ and $w \in [30p_2, 30p_1]$, and consider the maximization problem

$$\max x_1 \qquad \text{subject to} \qquad p_1 x_1 + p_2 x_2 \leq w \qquad \text{and} \qquad x_1 + x_2 \geq 30 \tag{2}$$

   (We omit the non-negativity constraints on x_1 and x_2 because they turn out not to bind; to keep things simple, you may ignore them.)

   (a) Show the constraint $p_1 x_1 + p_2 x_2 \leq w$ holds with equality at any solution to (2).

   (b) Show the constraint $x_1 + x_2 \geq 30$ holds with equality at any solution to (2).

   (c) Show that the solution to (2) is the demand function (1).

   (d) Knowing that (1) is the solution to (2), give a story for what the goods could be, and why demand would take the form (1). Describe a utility function (not necessarily continuous) which would give this demand.

Solutions:

1. *To be consistent with rational choice, the data must satisfy GARP, or have no cycles in the revealed-preference relations. The data yields the following revealed preference relations:*

   - *Since* $p^A \cdot x^B = 108 < 120 = w^A$, *at observation A, the individual chose* x^A *when they could have afforded* x^B, *so* $x^A \succ x^B$
   - *Since* $p^A \cdot x^C = 100 < 120 = w^A$, *so* $x^A \succ x^C$
   - *Since* $p^B \cdot x^A = 135 > 120$, x^B *is not revealed-preferred to* x^C
   - *Since* $p^B \cdot x^C = 110 < 120$, $x^B \succ x^C$
   - *Since* $p^C \cdot x^A = 135 > 120$, x^C *is not revealed-preferred to* x^A
   - *Since* $p^C \cdot x^B = 126 > 120$, x^C *is not revealed-preferred to* x^B

   So we have a complete preference ordering $x^A \succ x^B \succ x^C$ *and no cycles; thus, the data is consistent with rational choice.*

2. (a) *Symmetry of the Slutsky matrix requires* $\frac{\partial x_1}{\partial p_2} + \frac{\partial x_1}{\partial w} x_2 = \frac{\partial x_2}{\partial p_1} + \frac{\partial x_2}{\partial w} x_1$; *differentiating,*

$$\frac{\partial x_1}{\partial p_2} + \frac{\partial x_1}{\partial w} x_2 = \frac{-30}{p_1 - p_2} + \frac{w - 30p_2}{(p_1 - p_2)^2} + \frac{1}{p_1 - p_2}\frac{30p_1 - w}{p_1 - p_2} = \frac{-30}{p_1 - p_2} + \frac{30p_1 - 30p_2}{(p_1 - p_2)^2} = 0$$

$$\frac{\partial x_2}{\partial p_1} + \frac{\partial x_2}{\partial w} x_1 = \frac{30}{p_1 - p_2} - \frac{30p_1 - w}{(p_1 - p_2)^2} + \frac{-1}{p_1 - p_2}\frac{w - 30p_2}{p_1 - p_2} = \frac{30}{p_1 - p_2} - \frac{30p_1 - 30p_2}{(p_1 - p_2)^2} = 0$$

   so the two are equal because they're both 0.

   (b) x_1 *is increasing in* w, *so good 1 is normal.*

   x_2 *is decreasing in* w, *so good* 2 *is inferior.*

   Good 1 *can't be a Giffen good, since it's normal. Good* 2, *however, is a Giffen good, as* x_2 *is increasing in* p_2.

3. (a) *At any* (x_1, x_2) *where* $p_1x_1 + p_2x_2 < w$, *we could increase* x_1 *a bit, which would increase the value of the maximand without violating the second constraint. Thus, if* $p_1x_1 + p_2x_2 < w$, *we can't be at a solution; so at a solution,* $p_1x_1 + p_2x_2 = w$.

   (b) *If* $x_1 + x_2 > 30$, *then we could increase* x_1 *a little bit, and decrease* x_2 *a little bit, such that we're spending the same amount of money as before and we're still not violating the second constraint; this would increase the maximand without violating either constraint. For example, if* $x_1 + x_2 - \varepsilon \geq 30$, *we could decrease* x_2 *by* ε, *and increase* x_1 *by* $\frac{p_2}{p_1}\varepsilon$; *this would leave* $p_1x_1 + p_2x_2$ *unchanged, and still satisfy the constraint* $x_1 + x_2 \geq 30$, *while increasing* x_1. *So again, if* $x_1 + x_2 > 30$, *we can't be at a solution; so at any solution,* $x_1 + x_2 = 30$.

(c) *Knowing both constraints must hold, the optimum must satisfy*

$$p_1x_1 + p_2(30 - x_1) = w \longrightarrow (p_1 - p_2)x_1 = w - 30p_2 \longrightarrow x_1 = \frac{w - 30p_2}{p_1 - p_2}$$

and

$$x_2 = 30 - x_1 = 30 - \frac{w - 30p_2}{p_1 - p_2} = \frac{30(p_1 - p_2) - (w - 30p_2)}{p_1 - p_2} = \frac{30p_1 - w}{p_1 - p_2}$$

so the solution matches (1).

(Or, knowing that $p_1x_1 + p_2x_2 = w$ at the optimum, we could rewrite the problem as

$$\begin{aligned} & \max x_1 \quad \textit{subject to} \quad p_1x_1 + p_2x_2 = w \quad \textit{and} \quad x_1 + x_2 \geq 30 \\ = & \max x_1 \quad \textit{subject to} \quad x_2 = \tfrac{1}{p_2}(w - p_1x_1) \quad \textit{and} \quad x_1 + x_2 \geq 30 \\ = & \max x_1 \quad \textit{subject to} \quad x_1 + \tfrac{1}{p_2}(w - p_1x_1) \geq 30 \end{aligned}$$

The constraint $x_1 + \frac{w - p_1x_1}{p_2} \geq 30$ simplifies to $x_1 \leq \frac{w-30p_2}{p_1-p_2}$, so we're just maximizing x_1 subject to $x_1 \leq \frac{w-30p_2}{p_1-p_2}$, which of course is solved by setting $x_1 = \frac{w-30p_2}{p_1-p_2}$. x_2 then follows from plugging x_1 into either constraint.)

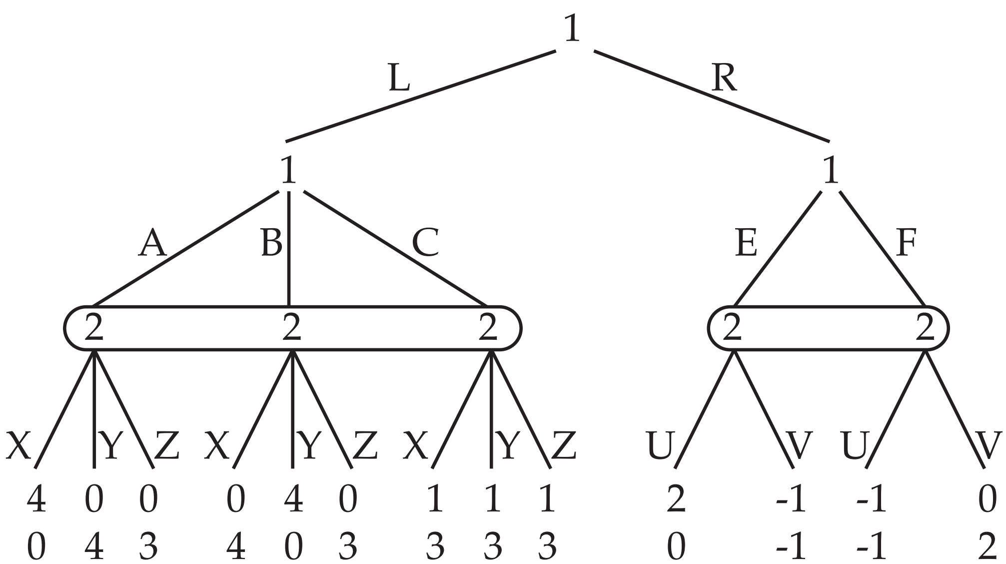
(d) *The obvious story is about food: say, good 1 is steak and good 2 is ramen. You need 30 dinners a month to survive, so $x_1 + x_2 \geq 30$ is a hard constraint; but you prefer steak to ramen, so you want to maximize the number of steak dinners you eat, subject to having something to eat every night. A utility function of the form*

$$u(x_1, x_2) = \begin{cases} x_1 & \textit{if} \quad x_1 + x_2 \geq 30 \\ x_1 - K & \textit{if} \quad x_1 + x_2 < 30 \end{cases}$$

would work, with K sufficiently large — basically, you get very low utility if $x_1 + x_2 < 30$, and otherwise your utility is simply increasing in x_1.

Part II

Consider the extensive form game Γ below.



1. *Find all subgame perfect equilibria of Γ.*
2. *Could it be beneficial to apply sequential equilibrium rather than subgame perfect equilibrium in Γ, in the sense that the set of equilibrium predictions becomes smaller? Your answer should be based on general principles, not on computation.*
3. *Are any subgame perfect equilibria of Γ ruled out by forward induction? Explain.*

Solution:

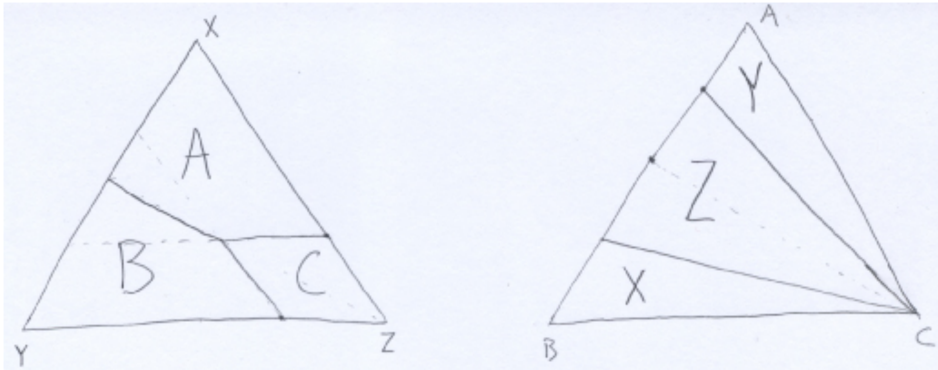
1. *We first find the Nash equilibria of the proper subgames.*

   We first compute player 1's best response correspondence.

$$\begin{aligned} A \geq B &\iff x \geq y \\ A \geq C &\iff x \geq \tfrac{1}{4} \\ B \geq C &\iff y \geq \tfrac{1}{4} \end{aligned}$$

   We next compute player 2's best response correspondence.

$$\begin{aligned} X \geq Y &\iff b \geq a \\ X \geq Z &\iff b \geq 3a \\ Y \geq Z &\iff a \geq 3b \end{aligned}$$



We now consider supports for player 1's equilibrium strategy:

$\{A\}$. *Implies that 2 plays X, but then A is not a best response for 1.* $\updownarrow$

$\{B\}$. *Implies that 2 plays Y, but then B is not a best response for 1.* $\updownarrow$

$\{A, B\}$. *For this to be a best response for 1 we need* $\sigma_2(X) = \sigma_2(Y) \geq \frac{1}{4}$, *but this is only a best response for 2 if 1 plays C.* $\updownarrow$

$\{C\}$. *All strategies are best responses for 2. C is optimal for 1 if* $\sigma_2(X) \leq \frac{1}{4}$ *and* $\sigma_2(Y) \leq \frac{1}{4}$. *These are equilibria.*

$\{A, C\}$. *This is optimal for 1 if* $\sigma_2(X) = \frac{1}{4}$ *and* $\sigma_2(Z) \geq \frac{1}{2}$. *For such a strategy to be optimal for 2, we need* $\sigma_1(B) = 3\sigma_1(A)$, *so the support if* σ_1 *cannot be* $\{A, C\}$. $\updownarrow$

$\{B, C\}$. *This is optimal for 1 if* $\sigma_2(Y) = \frac{1}{4}$ *and* $\sigma_2(Z) \geq \frac{1}{2}$. *For such a strategy to be optimal for 2, we need* $\sigma_1(A) = 3\sigma_1(B)$, *so the support if* σ_1 *cannot be* $\{B, C\}$. $\updownarrow$

$\{A, B, C\}$. *This is optimal for 1 if* $\sigma_2 = \frac{1}{4}X + \frac{1}{4}Y + \frac{1}{2}Z$. *That is only optimal for 2 if 1 plays C.* $\updownarrow$

Thus in the left subgame, the Nash equilibria are the profiles of the form (C, σ_2), *where* $\sigma_2(X) \leq \frac{1}{4}$ *and* $\sigma_2(Y) \leq \frac{1}{4}$. *These equilibria all yield payoffs of* $(1, 3)$.

In the right subgame, the Nash equilibria are (E, U), (F, V), *and* $(\frac{3}{4}E + \frac{1}{4}F, \frac{1}{4}U + \frac{3}{4}V)$, *which yield payoffs of* $(2, 0)$, $(0, 2)$, *and* $(-\frac{1}{4}, -\frac{1}{4})$.

At the initial node, player 1 plays R if (E, U) *is played in the right subgame, and L if one of the other two Nash equilibria are played in the right subgame. This describes the three types of subgame perfect equilibria of* Γ.

2. *The sequential equilibria and the subgame perfect equilibria are the same because the game has stagewise perfect information. In particular, in a sequential equilibrium, player 2's beliefs in both subgames will be determined by player 1's choices in those subgames.*

3. *Forward induction says that when player 1 chooses R, player 2 should realize that player 1 has given up a certain payoff of 1 to play the right subgame. Thus player 2 should expect player 1 to play E and should play U in response. Thus only the equilibrium with payoffs* $(2, 0)$ *survives forward induction.*

Part III

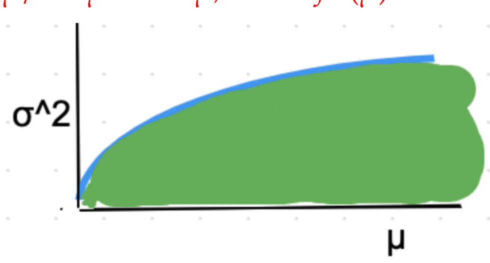
Ben's (Bernoulli) utility for money m is $U(m) = m - bm^2$, where $b > 0$. Throughout, we assume money wealth is bounded by $0 < m < 1/(2b)$.

1. Ben is willing to accept a gamble. If his wealth falls, will he still accept the gamble?
2. Ben faces random wealth risk ε, so that his final wealth is $m+\varepsilon$. Assume $E[\varepsilon] = \mu$ and $\sigma^2 = E[(\varepsilon-\mu)^2]$. Does Ben have convex preferences over (μ, σ^2) when $0 < \mu < 1/(2b)$? Illustrate your argument.
3. Ben is willing to accept wealth risk (μ, σ^2). If b falls, is he still willing to accept it?
4. Assume Ann's utility of leisure ℓ and money m is $U(\ell, m) = a\ell + m - bm^2$, where $0 < a < 1$. Ann lives the American dream, and works for herself. If she works h hours, she earns corporate profits $h + \varepsilon$, where ε is random, with mean 0 and variance σ^2. The government taxes a fraction $t \in (0, 1)$ of profits. Knowing this rate, Ann decides how many hours to work to maximize her expected utility, subject to the constraint $h + \ell = 1$. Does more luck (higher σ^2) lead Ann to work more or less?
5. Show that expected tax revenues from Ann is a hump-shaped function of the tax rate t, namely, where Ann pays a fraction t of her profits in taxes.

   PS Make sure your plot fits on the back of Arthur Laffer's napkin.
6. Assume that a social planner wants higher tax revenue and greater welfare of Ann. Is the socially optimal tax higher or lower than the tax with maximum tax revenue?

Solutions:

1. *Ben is risk averse. This (extremely famous) utility function (from finance) is not CARA or CRRA, but one can easily see that his Arrow-Pratt risk aversion coefficient $-U''/U' = 2b/(1-2bm)$ rises in m. So yes.*
2. *Yes. Ben's expected money utility is $m + E[\varepsilon] - bE[m+\varepsilon]^2 = m + \mu - b(m^2 + 2m\mu + \mu^2 + \sigma^2)$. In (μ, σ^2) space, the upper contour set is high μ and low σ^2 — namely, $\sigma^2 \leq f(\mu) \equiv \mu/b - \mu^2 - 2m\mu$, where $f''(\mu) = -2 < 0$ implies convex preferences.*



3. *Yes. Since $f(\mu) \equiv \mu/b - \mu^2 - 2m\mu$, if b falls, then the gamble is then strictly acceptable. Or, the Arrow-Pratt risk aversion coefficient rises in b.*

4. *Let* $s = 1 - t$. *Since* $\mu = 0$, *Ann's expected utility is* $a(1-h) + E[sh - bs^2h^2 - s^2\sigma^2]$. *The FOC for* h *is* $a = s - 2bs^2h$ *for an interior solution. But if* $s \le a$, *then the solution is* $h = 0$. *Assume* $s > a$. *Then Ann's hours* $h(s) = (s-a)/(2bs^2)$ *are constant in* σ^2.

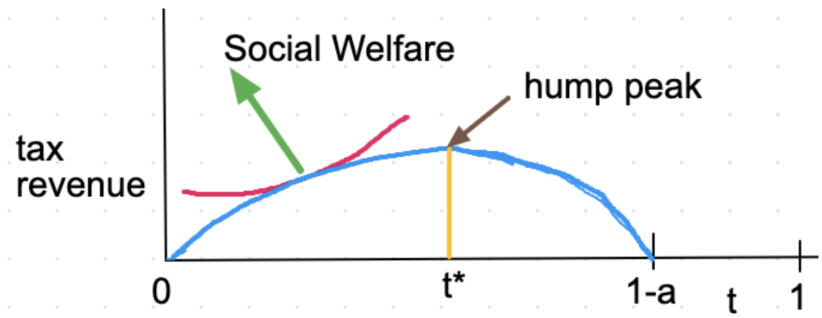
5. *Expected tax revenue is* $th(s)$, *or*

$$(1-s)h(s) = \frac{(1-s)(s-a)}{2bs^2}$$

This plot zeros at $t = 1 - s = 0$ $t = 1 - s = 1 - a$. *Hump-shape for* $s \in (a, 1)$ *follows since*

$$[\log(1-s) + \log(s-a) - \log s^2]'' = \left(\frac{1}{1-s} + \frac{1}{s-a} - \frac{2}{s}\right)' = \left(\frac{-a}{(1-s)(s-a)}\right)' + 2/s^2 > 0$$

6. *The socially optimal tax maximizes in the northeast direction below, since Ann's welfare falls in* t, *but tax revenue rises moving up. This maximizer is obviously below the tax yielding the maximum tax revenue.*



Part IV

Consider the labor market for sales representatives. Good sales reps produce output θ_H while bad reps produce output θ_L, where $\theta_H > \theta_L > 0$. Output is each representative's private information; i.e., companies interested in hiring a representative do not observe his productivity. Assume that $\Pr(\theta = \theta_H) = 1/2$. Suppose that prior to applying for jobs, the representatives can accumulate ratings $r \in \mathbb{R}_+$ (their choice variable). Since the accumulation of ratings requires effort, a representative's payoff is given by

$$u(w, r; \theta) = w - r/\theta\,,$$

where w is the wage. Assume that the labor market is competitive: $w = \mathbb{E}(\theta \mid r)$.

1. Characterize all separating equilibria, assuming pessimistic off-equilibrium beliefs.

   Solution: In a separating equilibrium, θ_L type chooses $r = 0$ and θ_H type chooses $r = \bar{r}$, where $\bar{r}$ can be any r value between r' and r''.

   Here, r' solves the condition $\theta_L = \theta_H - \frac{r'}{\theta_L}$ and r'' solves $\theta_L = \theta_H - \frac{r'}{\theta_H}$. The posterior of the companies is $(r) = \Pr(\theta = \theta_H) = 1$ only if $r = \bar{r}$ and 0 otherwise.

   The wage offer is $w = \theta_H$ if $r = \bar{r}$ and θ_L otherwise.

2. Characterize all pooling equilibria, assuming pessimistic off-equilibrium beliefs.

   Solution: In any pooling equilibrium, both types chooses $r = r^*$, where r^* can be any r value between 0 and r'. Here, r' solves $\theta_L = \mathbb{E}[\theta] - \frac{r'}{\theta_L}$.

   The posterior of the companies is $(r) = \Pr(\theta = \theta_H) = 1/2$ if $r = r^*$ and 0 otherwise.

   The wage offer is $w = \mathbb{E}[\theta]$ if $r = r^*$ and θ_L otherwise.

3. Suppose that an unemployment benefit becomes available: a representative receives a payment of $\beta \in (\theta_L, \theta_H)$ if he chooses to remain unemployed. How will this unemployment benefit change the payoffs of the two types of the representatives in the pooling and separating equilibria?

   Solution: The separating equilibria: As $\beta > \theta_L$, in a separating equilibrium, the low type will choose to remain unemployed. Now, the condition that ensures no mimicking becomes $\beta = \theta_H - \frac{r^{**}}{\theta_L}$. As $\beta > \theta_L$, $r^{**} < r'$. Thus, the high type needs to spend less on ratings and therefore, earns a higher payoff in equilibrium.

   The pooling equilibria: Note that, if $\beta > \mathbb{E}\theta$, in any pooling equilibrium both types choose to remain unemployed. If $\beta \leq \mathbb{E}[\theta]$, then any r between 0 and r'' can be supported by a pooling PBE where r'' is defined as follows:

$$\beta = \mathbb{E}[\theta] - \frac{r''}{\theta_L}.$$