Linear Programming $\Rightarrow$ (Lone's' part. matching prob)

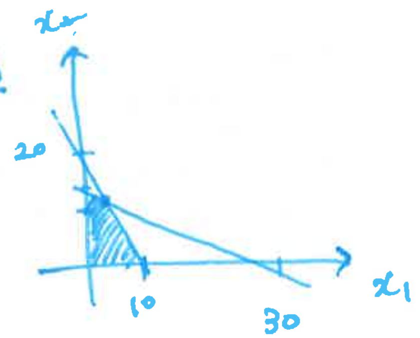
primal $V(A,b,c) = \max_{x} \; c'x \quad s.t \quad \underset{m\times n}{A}\underset{n\times 1}{x} \le \underset{m\times 1}{b}, \quad x \ge 0.$

$x, c \in \mathbb{R}^n, \quad A = \mathbb{R}^{m\times n} \quad b \in \mathbb{R}^m$

Q1. $\max_{x_1,x_2} \; (1 \quad 1)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$s.t \quad \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \le \begin{pmatrix} 10 \\ 15 \end{pmatrix}$

Graphically



$x_1 + \frac{1}{2}x_2 = 10$

$x_1 + 2x_2 = 30$

$\frac{3}{2}x_2 = 20 \Rightarrow x_2 = \frac{40}{3}$

$x_1 = 30 - \frac{80}{3}$

$= \frac{10}{3}.$

Lagrangian $\mathcal{L} = x_1 + x_2 + \lambda_1(10 - x_1 - 0.5x_2) + \lambda_2(\overset{15}{\cancel{10}} - 0.5x_1 - x_2).$

$\partial\mathcal{L}/\partial x_1 = 1 - \lambda_1 - 0.5\lambda_2 = 0$

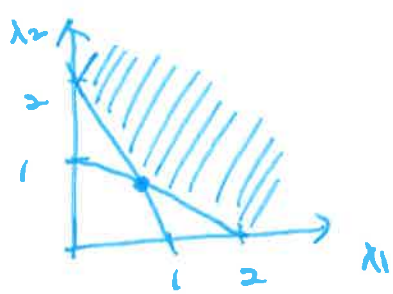
$\partial\mathcal{L}/\partial x_2 = 1 - 0.5\lambda_1 - \lambda_2 = 0$

$\lambda_1 = \lambda_2 = \frac{1}{1.5} = \frac{2}{3}$ & $V = \frac{50}{3}$

dual $W(A,b,c) = \min_{\lambda} \; b'\lambda \quad s.t \quad A'\lambda \ge c, \quad \lambda \ge 0$

Q2. $\min_{\lambda_1,\lambda_2} \; (10 \quad 15)\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} \quad s.t \quad \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} \ge \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

phically



$10\lambda_1 + 15\lambda_2 = W$

$\lambda_2 = -\frac{2}{3}\lambda_1 + \frac{1}{15}.W$

$\boxed{\lambda_1 = \lambda_2 = \frac{2}{3}, \quad W = 25.\frac{2}{3} = \frac{50}{3}}$

Not a coincidence at all !!

<u>Proposition</u>

$V(A, b, c) = W(A, b, c)$

x^* is a shadow value of a dual prob.

λ^* 〃 a primal prob

Intuitively,

$\min \quad b'\lambda \quad s.t \quad \underbrace{A'\lambda}\geq c$

my willingness to pay for a unit of input

total expenditure on the current level of input b.

expenditure of making (1, 1) of product given my w.t.p λ

it should be greater than the price of output

<u>Fixed Point Theorem</u> $\Rightarrow$ (Bill's part. existence of N.E
Rasmus' part. combined w/ contraction MT)

- Brouwer's $A \subset \mathbb{R}^n$ nonempty, cpt, convex

  $f: A \to A$ continuous ftn (onto)

  Then $f(\,)$ has a fixed point; that is, there is an $x \in A$ s.t $x = f(x)$

- Kakutani's

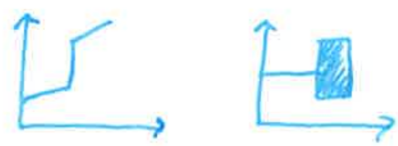
<u>Def</u> A function $f: X \to Y$ is <u>continuous</u> at a point x

if $\forall \boxed{f(x) \in V}$ s.t $x \in V$, $\exists$ an open set U s.t

$\forall x' \in U$, $\boxed{f(x') \in V}$ (ε, δ)

<u>Def</u> correspondence $\Phi: X \to P(Y)$

<u>ex</u>



Now, $\Phi(x)$ is a set! Not a point!

$f(x) \in V$ $<$ $\Phi(x) \subset V$ / $\Phi(x) \cap V \neq \emptyset$

<u>Def</u> A correspondence $\Phi: X \to P(Y)$ is <u>u.h.c</u>

if $\forall V$ s.t $\Phi(x) \subset V$, $\exists$ an open set U s.t

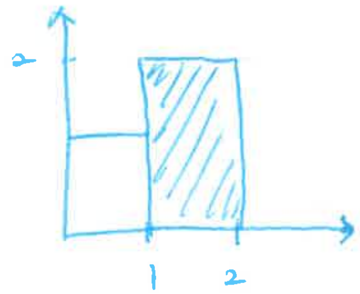
$\forall x' \in U$, $\Phi(x') \subset V$

<u>Def</u> 〃 l.h.c

$\forall V$ s.t $\Phi(x) \cap V \neq \emptyset$

, $\Phi(x') \cap V \neq \emptyset$

Q4. $X = [0, 2]$



- u.h.c at 1

$\Phi(1) = [0, 2]$

$[0, 2] \subset V$

For any $\underbrace{\text{open ball}}_{\equiv U}$ around 1,

$x' \in U \Rightarrow \Phi(U) \subset [0,2] \subset V$

- Not l.h.c

If $V = [1.5, 2]$

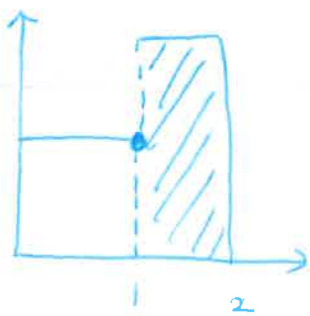
then $\Phi(1) \cap [1.5, 2] \neq \phi$

but any open ball around 1,

$\Phi(x') \cap [1.5, 2] = \phi$

if $x' < 1$

Q5 $X = [0, 2]$



- l.h.c at 1

any V s.t $\Phi(1) \cap V \neq \phi$
$= 1$

$\therefore 1 \in V$.

then for any x'

in the any open ball

around 1,

$\Phi(x') \cap V \Rightarrow$ at least has $\underline{1}$

- u.h.c

$V \ni 1$ is the condition now

if $x' > 1$ then $\forall$

$V \subsetneq [0, 2]$

$\Phi(x') \not\subset V$

- Kakutani's Fixed point thm