$$\underbrace{(1-\theta)P_{it} + \theta\frac{W_t}{A_t}}_{x_t} = \varphi E_t \underbrace{\left[\frac{W_t}{C_{it}}(\Pi_t^2+\Pi_t) - \beta\left(\frac{W_{t+1}}{C_{t+1}}\right)\Pi_{t+1}\right]}_{z_t}$$

In steady state, inflation is 0, so $\bar{z} = 0$.
So $\bar{x} = \varphi E[\bar{z}] = \varphi E[0] = 0$.

LHS: $x_t = (1-\theta)p_t + \theta\frac{W_t}{A_t}$ $\qquad A_t^{-1} = \bar{A}^{-1}(1-a_t)$

↓ linearize

$$x_t = (1-\theta)\bar{p}(1+p_t) + \theta\frac{\bar{W}}{\bar{A}}(1+w_t-a_t)$$

$$x_t = \underbrace{(1-\theta)\bar{p}} + (1-\theta)\bar{p}\,p_t + \underbrace{\frac{\theta\bar{W}}{\bar{A}}}_{=0} + \frac{\theta\bar{W}}{\bar{A}}w_t - \frac{\theta\bar{W}}{\bar{A}}a_t \qquad \bar{x} = (1-\theta)\bar{P} + \frac{\theta\bar{W}}{\bar{A}} = 0$$

$$x_t = (1-\theta)\bar{P}p_t + \frac{\theta\bar{W}}{\bar{A}}(w_t - a_t)$$

RHS: $z_t = \underbrace{\frac{W_t}{C_{it}}(\Pi_t^2+\Pi_t)}_{H_t} - \beta\underbrace{\left(\frac{W_{t+1}}{C_{t+1}}\right)\Pi_{t+1}}_{Q_t}$ $\qquad \Pi_t = 0$

$z_t = h_t - q_t$ $\qquad$ Linearized

$$H_t = \frac{W_t}{C_{it}}(\Pi_t^2 - \Pi_t)$$

$h_t = \bar{w}(1+w_t)\bar{c}^{-1}(1-c_t)(\Pi_t^2+\Pi_t)$ $\qquad \bar{\Pi} = 0$, linearise

$h_t = \bar{w}\bar{c}^{-1}(1+w_t-c_t)(\Pi_t^2+\Pi_t)$

$h_t = \bar{w}\bar{c}^{-1}\Pi_t$

$Q_t = \beta\left(\frac{W_{t+1}}{C_{t+1}}\right)\Pi_{t+1}$

$q_t = \beta\bar{w}(1+w_t)\bar{c}^{-1}(1-c_t)\Pi_{t+1}$

$q_t = \beta\bar{w}\bar{c}^{-1}(1+w_t-c_t)\Pi_{t+1}$

$q_t = \beta\bar{w}\bar{c}^{-1}\Pi_{t+1}$

$$z_t = \bar{w}\bar{c}^{-1}\Pi_t - \beta\bar{w}\bar{c}^{-1}\Pi_{t+1}$$

$$\rightarrow (1-\theta)\bar{P}p_t + \frac{\theta\bar{W}}{\bar{A}}(w_t - a_t) = \varphi E_t\left[\bar{w}\bar{c}^{-1}\Pi_t - \beta\bar{w}\bar{c}^{-1}\Pi_{t+1}\right]$$

$$W_t = P_t + C_t$$

$$c_t - a_t = x_t$$ — output gap

$$(1-\theta)\bar{P}_{P_t} + \frac{\theta\bar{W}}{\bar{A}}(w_t - a_t) = \varphi E_t\left[\bar{W}\bar{C}^{-1}\pi_t - \beta\bar{W}\bar{C}^{-1}\pi_{t+1}\right]$$

$$(1-\theta)\bar{P}_{P_t} + \frac{\theta\bar{W}}{\bar{A}}(p_t + x_t) = \varphi E_t\left[\bar{W}\bar{C}^{-1}\pi_t - \beta\bar{W}\bar{C}^{-1}\pi_{t+1}\right]$$

$$\underbrace{\left[(1-\theta)\bar{P} + \frac{\theta\bar{W}}{\bar{A}}\right]}_{=0 \text{ in SS}} p_t + \frac{\theta\bar{W}}{\bar{A}}x_t = \varphi E_t\left[\bar{W}\bar{C}^{-1}\pi_t - \beta\bar{W}\bar{C}^{-1}\pi_{t+1}\right]$$

$$\frac{\theta\cancel{\bar{W}}}{\bar{A}}x_t = \varphi E_t\left[\cancel{\bar{W}}\bar{C}^{-1}\left(\pi_t - \beta\pi_{t+1}\right)\right]$$

$$\frac{\theta\bar{C}}{\bar{A}}x_t = \varphi E_t\left[\pi_t - \beta\pi_{t+1}\right]$$

$$\frac{\theta\bar{C}}{\varphi\bar{A}}x_t = \pi_t - \beta E_t[\pi_{t+1}] \qquad \bar{C} = \bar{A}\bar{L}$$

$$\frac{\theta\bar{L}}{\varphi}x_t = \pi_t - \beta E_t[\pi_{t+1}] \qquad \bar{L} = \frac{\theta-1}{\theta}$$

$$\frac{(\theta-1)}{\varphi}x_t + \beta E_t[\pi_{t+1}] = \pi_t$$

$$x_t = c_t - a_t, \quad c_t = y_t \rightarrow x_t = y_t - a_t$$

$$\frac{(\theta-1)}{\varphi}(y_t - a_t) + \beta E_t[\pi_{t+1}] = \pi_t$$ NKPC