25.1 $P(\text{purchase} \mid X=x) = 1 - P(\text{no purchase} \mid X=x)$

$$\Phi(x'\beta_1) = 1 - \Phi(x'\beta_2) = \Phi(-x'\beta_2)$$

$$\rightarrow \beta_1 = -\beta_2$$

25.3 (25.1): $e = \begin{cases} 1 - P(x) & \text{with prob } P(x) \\ P(x) & \text{with prob } 1-P(x) \end{cases}$

$$Y = P(x) + e$$

If $Y=1$ with prob $P(x)$:

$$\rightarrow 1 = P(x) + e$$

$$\rightarrow e = 1 - P(x)$$

with prob $1-P(x)$:

$$1 = (1-P(x)) + e$$

$$\rightarrow e = P(x)$$

25.2): $\text{var}(e \mid X) = P(x)(1-P(x))$

$$\text{ar}(e \mid x) = E[e^2 \mid X] - E[e \mid x]^2$$

$$= E[e^2 \mid X] - 0$$

$$= (1-P(x))^2 P(x) + P(x)^2 (1-P(x))$$

$$= P(x)(1-P(x))$$

25.9 The conditional PMF is:

$$\pi(y \mid x) = \Lambda(z'\beta) = (1 + e^{-z'\beta})^{-1} \qquad z = \begin{cases} x & \text{if } Y=1 \\ -x & \text{if } Y=0 \end{cases}$$

Then the log likelihood function is:

$$L(\beta) = -\sum_{i=1}^{n} \log(1 + e^{-z_i'\beta})$$

FOC: $$\sum_{i=1}^{n} \frac{e^{-z_i'\beta} \cdot z_i}{1 + e^{-z_i'\beta}} = 0$$

25.12. $E[Y|X] = P(Y=1|X) = \Phi(X'\beta)$

$$\beta^* = \operatorname{argmin} \sum_{i=1}^{n} (Y_i - \Phi(X_i'\beta))^2$$

25.14. a. $P(Y>0) = P(Y^*>0)$

$$= P(m(X)+e>0)$$

$$= P(e<-m(X))$$

$$= 1-\Phi_{\sigma^2_X}(-m(X))$$

Where $\Phi_{\sigma^2_X}$ is the CDF of $N(0, \sigma^2(X))$

b. No, assume $m(X)$ and $\sigma^2(X)$ were both uniquely identified. Then $m'(X) = c\,m(X)$ and $\sigma^{2\prime}(X) = c^2\sigma^2(X)$ would have the same response probability, a contradiction.

c. Let $\sigma^2(X)=1$. Then $m^*(X) = 1-\Phi(-m(X))$.

d. No, $\sigma^2(X)=1$ implies homoskedasticity.

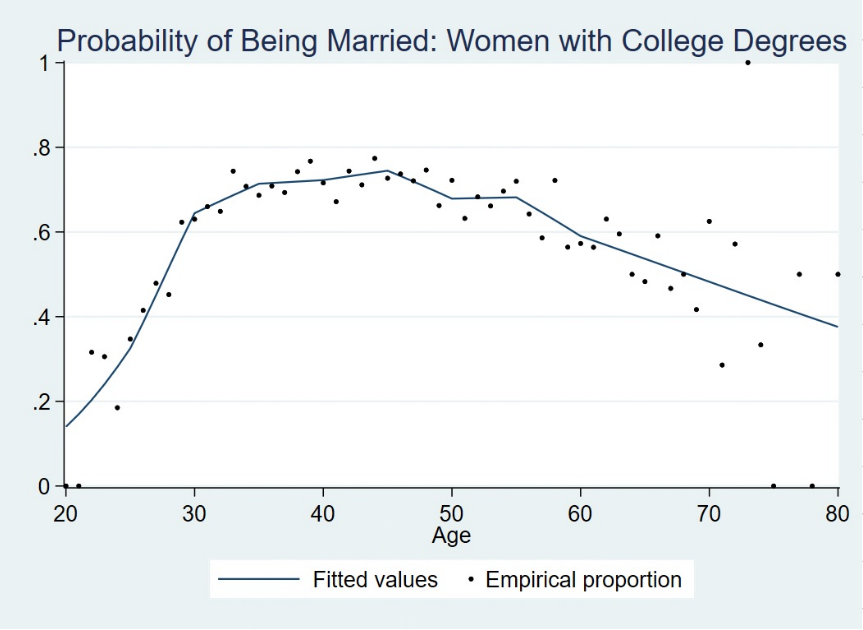
25.15

```
Probit regression                               Number of obs     =     29,140
                                                LR chi2(4)        =      71.31
                                                Prob > chi2       =     0.0000
Log likelihood = -3105.3454                     Pseudo R2         =     0.0114
```

y	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	.0079255	.0014313	5.54	0.000	.0051202	.0107308
education	-.025504	.0063144	-4.04	0.000	-.03788	-.013128
hisp	-.297746	.0565305	-5.27	0.000	-.4085437	-.1869483
black	-.0540834	.0599733	-0.90	0.367	-.171629	.0634621
_cons	-1.953831	.1085302	-18.00	0.000	-2.166546	-1.741115

As we can see from the regression results, age, education, and being hispanic have significant relationships with union membership.

25.17



I chose a probit model with linear splines at 25, 30, 35, 40, 45, 50, 55, and 60. I included splines because different stages of life may lead to different dynamics between age and marriage.

Comparing to Figure 25.1, the probability of being married for educated women appears to be lower than for educated men. Also, for men the probability of being married appears to be increasing monotonically, but there is a significant decrease in the probability of being married for women starting around age 45.

26.1 $$P_j(x) = \frac{\exp(x'\beta_j)}{\sum_{\ell=1}^{J} \exp(x'\beta_\ell)}$$

Note $\exp(x'\beta) > 0 \quad \forall \beta \rightarrow P_j(x) > 0$

Further, $$\frac{\exp(x'\beta_j)}{\sum_{\ell=1}^{J} \exp(x'\beta_\ell)} < \frac{\exp(x'\beta_j)}{\exp(x'\beta_j)} = 1.$$

Thus $0 < P_j(x) < 1$.

$$\sum_{j=1}^{J} P_j(x) = \sum_{j=1}^{J} \frac{\exp(x'\beta_j)}{\sum_{\ell=1}^{J} \exp(x'\beta_\ell)} = \frac{\sum_{j=1}^{J} \exp(x'\beta_j)}{\sum_{\ell=1}^{J} \exp(x'\beta_\ell)} = 1.$$

26.3

$$\frac{\partial P_j(x)}{\partial x} = \frac{\partial}{\partial x} \frac{\exp(x'\beta_j)}{\sum_{\ell=1}^{J} \exp(x'\beta_\ell)}$$

$$= \frac{\exp(x'\beta_j)}{\sum_{\ell=1}^{J} \exp(x'\beta_\ell)} \cdot \beta_j - \frac{\exp(x'\beta_j)}{\sum_{\ell=1}^{J} \exp(x'\beta_\ell)} \frac{\sum_{j=1}^{J} \exp(x'\beta_j)\beta_j}{\sum_{\ell=1}^{J} \exp(x'\beta)}$$

$$= P_j(x)\left(\beta_j - \sum_{j=1}^{J} P_j(x)\beta_j \right)$$

26.7.

$$AME_{jj} = E[\delta_{jj}(w, x)]$$
$$= E[\gamma P_j(w, x)(1 - P_j(w, x))]$$

$$\rightarrow \widehat{AME}_{jj} = \frac{1}{n} \sum_{i=1}^{n} \hat{\gamma} \hat{P}_j(w_i, x_i)(1 - \hat{P}_j(w_i, x_i))$$

26.8 $$P_j(w, x|\theta) = \frac{\dfrac{\exp(w'\beta_j + x_j'\gamma)}{\sum_{i=1}^{J} \exp(w'\beta_i + x_j'\gamma)}}{\dfrac{\exp(w'\beta_\ell + x_\ell'\gamma)}{\sum_{i=1}^{J} \exp(w'\beta_i + x_i'\gamma)}}$$

$$= \frac{\exp(w'\beta_j + x_j'\gamma)}{\exp(w'\beta_\ell + x_\ell'\gamma)}$$