Some of the most important results (e.g. Cauchy's theorem) are so surprising at first sight that nothing short of a proof can make them credible - Sir Harold Jeffreys

1 Review Topics

Metric spaces, convergence in metric spaces

2 Exercises

2.1 Confirm that each of the following is a metric function on the given metric space

- $d(x, y) = \left|\frac{1}{x} - \frac{1}{y}\right|$, on $(0, \infty)$.

- The "post-office" metric: $d(x, y) = \|x\|_2 + \|y\|_2$ for $x \neq y$, $d(x, x) = 0$, on $\mathbb{R}^n$.

2.2 Prove the triangle inequality for $|\cdot|$, i.e. $|x - y| \leq |x - z| + |z - y|$.

2.3 Prove that every monotone increasing sequence in $\mathbb{R}$ that is bounded above converges to a limit in $\mathbb{R}$

2.4 Prove that the sequence $a_n = 1 + \frac{(-1)^n}{n}$ converges.

2.5 Let a_n be a sequence of positive real numbers, such that $\frac{a_{n+1}}{a_n}$ converges to $a < 1$. Prove that a_n converges.

2.6 Prove that if $a_n \to a$, $a_n > 0$ for all n, then $\sqrt{a_n} \to \sqrt{a}$.