PROBLEM SET 3

1. Suppose there are three coffee houses along Main Street. The street is one mile. One hundred residents are uniformly distributed along this stretch. Each resident is willing to buy one cup of coffee per day. A cup of coffee differs only in its location and price and not in any other way. Each customer derives a utility of v = $3.00 from the cup of coffee. A consumer's (round-trip) cost of travel is quadratic in the distance from home to any of the coffee houses.

Starbucks Coffee House is located at either end of the one mile stretch and Esquire Coffee House is located halfway between the two end points of the street. The prices of coffee at Starbucks' two locations are p_0 and p_1 respectively. Esquire's price of coffee is denoted by q. Marginal costs of a cup of coffee are zero.

(a) Determine the location of the two marginal consumers – the one who is indifferent between purchasing from Esquires and Starbucks located at the left end point and the one who is indifferent between purchasing from Esquires and the Starbucks located the right end point.
(b) Derive the best reply functions to the pricing game in which the coffee houses choose prices simultaneously. Assume that Starbucks can set different prices at its two locations.
(c) Determine the equilibrium prices and market shares.
(d) Suppose Starbucks and Esquires swap houses so that the Starbucks houses are located at one of the endpoints and the halfway point and Esquires is located at the other endpoint. Derive the equilibrium prices and market shares and explain why it differs (if at all) from (c).
(e) Suppose instead of swapping houses, Starbucks sells one of its coffee houses to Seattle Best Coffee. Derive the equilibrium prices and market shares. Explain why it differs (if at all) from (c).

2. A county consists, for all intents and purposes, of two towns, Right and Left. They are connected by a straight road of length one mile. The population is uniformly distributed on this one-mile stretch. Everyone is an imbiber. Jim Beam and Jack Daniels own the only two liquor licenses. Each drinker will go regularly to whichever bar is closest to him. Unfortunately for Jim and Jack, Mr. Nag has convinced the powers that be that if these two gentlemen were unregulated, they would conspire to raise prices. Hence the price of a drink in each bar is equal and fixed at $p by the county Pickled Brain Commission. Thus, the only variable that Jack and Jim have control over is their location. Each drinker's cost of travel is quadratic in the distance from home to the bar. Assume that the utility obtained from a drink is sufficiently high that market is covered for every pair of locations.

(a) Formulate this location game by defining strategies and payoffs. Let a denote the location of Jim Beam and 1-b denote the location of Jack Daniels. Be sure to derive demands for locations (a, 1-b).
(b) Determine the Nash equilibrium locations of the two establishments and explain the intuition behind your answer.

(c) Are the Nash equilibrium locations socially optimal (i.e., do they minimize total travel costs)? Why or why not? If not, set up the social planner's problem and derive the optimal locations.

3. Consider a model of vertical differentiation in which a customer of type θ obtains the net benefit $U = s(\theta - p(s))$ from an item of quality s. Here θ is uniformly distributed in the population between zero and one. Marginal costs of providing a good of quality s is $C(s) = cs$ where $c \leq 1/2$.

(a) Find the optimal prices which a monopolist should charge when it offers two goods with qualities $s = 1$ and $s = 2$.
(b) Find the optimal price which a monopolist should charge when it offers only one good with quality $s = 1$.
(c) Determine the conditions on c under which these two solutions are optimal.

Now suppose there are two firms: firm 1 offers low quality product with $s = 1$ and firm 2 offers high quality product with $s = 2$. The firms compete in prices.

(d) Determine the relevant range for p_2 and derive the demand for good 1 on that range.
(e) Derive firm 1's best reply as a function of c and plot it in price space. (Hint: it consists of four segments.)
(f) Derive firm 2's best reply and compute the Nash equilibrium prices as a function of c. Can the firms coexist?

4.* Bonus question.

Consider an industry with two retail stores located along a line segment of unit length. Goods are supplied by the stores at zero variable costs, but at positive fixed costs. The fixed costs depend on the distance each store is located from the endpoints. In particular, the first firm's fixed costs are given by

$$F_1 = cd_1$$

where c is a positive constant and d_1 is the distance that firm 1 is located from one endpoint (say, the west).

The second firm's fixed costs are proportional to its distance from the other endpoint (the east), and so can be written as:

$$F_2 = c(1-d_2),$$

where d_2 is the distance from the west endpoint.

The price of the industry's product is exogenously fixed at \$1, where $c < 1$. There is one customer with inelastic demand for one unit of the commodity of the closest store. Suppose that the customer pays \$t per unit distance traveled for a round trip to any store. (Hence if a store is a distance x away, the customer must incur travel costs of tx). If the suppliers are equally close, the consumer randomizes between the stores.

(a) Derive the (pure) strategy Nash equilibrium location choices when the stores know where the customer is located, and they locate simultaneously. Characterize the conditions under which the equilibrium exists. If a pure strategy equilibrium does not exist, find the mixed strategy equilibrium.)

(b) Suppose instead that the location of the customer is unknown when the stores choose their locations, and that each store believes the customer's location is a random draw from the uniform distribution on the line segment. Assuming each firm is risk neutral, find the Nash equilibrium in pure strategies.