

University of Wisconsin-Madison
Department of Economics

Econ 703
Fall 2004

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Homework #6

1. Let $f, g : [0, 1] \rightarrow \mathbb{R}$ be continuous functions, and suppose that $f(x) > g(x)$ for all $x \in [0, 1]$. Prove or disprove the following statement : There exists $\delta > 0$ such that $f(x) \geq g(x) + \delta$ for all $x \in [0, 1]$. What if instead f and g were only left continuous?
2. (Brouwer fixed point theorem) Let $I = [0, 1]$, and that suppose that $f : I \rightarrow I$ is continuous. Prove that there exists $x \in I$ such that $f(x) = x$.
3. Let f be a continuous real-valued function on \mathbb{R} , of which it is known that $f'(x)$ exists for all $x \neq 0$ and that $f'(x) \rightarrow 3$ as $x \rightarrow 0$. Does it follow that $f'(0)$ exists? Either prove or disprove your statement.
4. Suppose $f'(x)$ exists, $g'(x)$ exists, $g'(x) \neq 0$, and $f(x) = g(x) = 0$. Prove that
$$\lim_{t \rightarrow x} f(t)/g(t) = f'(t) / g'(t).$$
5. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 \sin(1/x)$ for $x \neq 0$, and $f(x) = 0$ for $x = 0$. Show that $f'(x)$ exists at all points $x \in \mathbb{R}$, but that $f'(x)$ is not continuous at $x = 0$.