Homework #2

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Fall 2016

1. Sundaram, #23, p. 68.

2. Sundaram, #25, p. 68.

3. Let (X, d) be a metric space. Prove the following statement : $A \subset X$ is closed iff for every sequence $\{x_n\} \subset A$, $x_n \to x$ implies $x \in A$.

4. Consider the set of all rational numbers $\mathbb{Q}$, and make it into a metric space by defining $d(p, q) = |p - q|$ for all $p, q \in \mathbb{Q}$. Let E be the set of all $p \in \mathbb{Q}$ such that $2 < p^2 < 3$. Show that E is closed and bounded in $\mathbb{Q}$, but that E is not compact. Conclude that $\mathbb{Q}$ is not a compact space. Is E open in $\mathbb{Q}$?

   HINT: Be very careful here. The notions closed, open and compact are all with reference to the space $\mathbb{Q}$, not the space $\mathbb{R}$.

5. Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be defined by :

$$f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

   (a) Show that f is continuous in each variable separately.

   (b) Compute the function $g(x) = f(x, x)$.

   (c) Show that f is not continuous.