University of Wisconsin

Microeconomics Prelim Exam

Friday, July 30, 2018: 9AM - 2PM

- There are four parts to the exam. All four parts have equal weight.
- Answer all questions. No questions are optional.
- Hand in **at most 16 pages**, written on only one side.
- Write your answers for different parts on different pages. So do not write your answers for questions in different parts on the same page.
- Please place a completed label on the top right corner of each page you hand in. On it, write your assigned number, and the part of the exam you are answering (I,II,III,IV). Do not write your name anywhere on your answer sheets!
- Show your work, briefly justifying your claims. At the same time, aim for brevity and clarity.
- You cannot use notes, books, calculators, electronic devices, or consultation with anyone else except the proctor.
- Please return any unused portions of yellow tablets and question sheets.
- There are six pages on this exam, including this one. Be sure you have all of them.
- Best wishes!

Part I

We consider consumption decisions in the presence of an addictive good 2, say cigarettes, and a composite good 1 capturing "all other goods", written $x = (x_1, x_2)$. Normalize the price of good 1 to $p_1 = 1$. Assume henceforth enough wealth so that consuming only cigarettes is not optimal. Consider first a "standard" consumer model with preferences represented by the utility function

$$u(x) \quad = \quad x_1 + k\sqrt{x_2} - x_2$$

where k is a constant. You can think of $k\sqrt{x_2}$ as the consumption utility of smoking cigarettes, and $-x_2$ as the disutility coming from the long-term health consequences. Assume (even if you personally disagree) that $k > 0$.

1. Are preferences monotone? Are they locally non-satiated? Explain.

2. Given prices $p = (1, p_2)$ and wealth w, calculate Marshallian demand $x(p, w)$ and indirect utility $v(p, w)$. Is v increasing or decreasing in p_2?

Addictive behavior violates many of the implications of standard consumer theory. For example, addicts frequently view their own consumption choices as mistakes, believing later that they would have been better off (even at the time) if they had consumed less of the addictive good. One way to model this is to imagine that people sometimes make decisions based on the "wrong" preferences: Suppose you have two different emotional states, a "hot" state and a "cold" state. In the cold state, you choose rationally based on your true utility

$$u_C(x) \quad = \quad x_1 + 10\sqrt{x_2} - x_2$$

which is also used to evaluate your welfare. But external factors sometimes push you into the hot state, in which you over-value the joy of smoking relative to your true preferences, and consume as if your utility function was

$$u_H(x) \quad = \quad x_1 + 10\alpha\sqrt{x_2} - x_2$$

where $\alpha > 1$ measures of how severe the addiction is.

3. Consider the problem facing an empirical researcher studying consumer behavior. Suppose I observe you choosing consumption at different wealth and price levels, sometimes in your hot state and sometimes in your cold state, and I can only observe your consumption choices and the prices you were facing, not which state you were in. Will the observations satisfy GARP? Why or why not?

4. Now suppose there are many smokers with preferences identical to yours — enough so that at each point in time, we can assume a constant fraction of the consumers are in the hot state. If I observe aggregate consumption choices at different price levels, will the observations satisfy GARP? Why or why not?

There are things you can do when you're in your cold state to make it harder for you to smoke later — you can throw out your cigarettes, ask your friends to not lend you a cigarette if you ask for one, and so on. We can think of these "commitment devices" as ways you can increase the price p_2 you face when you choose your consumption.

5. Suppose that you consume as if your preferences were u_H, but evaluate your utility using u_C. Calculate your indirect utility function $v_H(p, w)$, the "actual" indirect utility you get from the consumption you choose in your hot state.

6. If you know you'll be making your consumption choice in the hot state, under what conditions can you benefit from a commitment device that raises p_2?

7. If $\alpha = 3$, how high would you optimally set p_2 if you knew that you'd be choosing your consumption with equal 50% probability in the hot and cold states?

Finally, consider another addictive product like coffee. Let's now assume that u_C and u_H do not have the functional forms specified above but are simply general differentiable utility functions over coffee (good 2) and all other goods (good 1). Suppose that coffee is not actually bad for you: your true utility function u_C is increasing in both goods; your hot-state utility function u_H simply leads you to consume more of good 2 (and so less of good 1) than your true preferences. Assume that neither good is a Giffen good under either your hot or cold preferences.

8. Show that if goods 1 and 2 are gross complements according to your hot-state preferences, then you cannot benefit from a commitment device that raises the price p_2.

Part II

Consider the following two-player, alternating-move game: There is a pile containing $a \geq 0$ acorns and a pile containing $b \geq 0$ blueberries. During a move, a player must take one or two items of the same kind (i.e., a player can't take an acorn and a blueberry). The player who takes the last item loses, and his opponent wins. Player 1 goes first.

1. What is a pure strategy for player 1 in this game?

   Hint: This question can be answered without any notation.

2. For any $a, b \geq 0$, describe the set of subgame perfect equilibrium outcomes.

3. For any $a, b \geq 0$, describe the set of pure-strategy subgame perfect equilibria. Justify your answer carefully.

   Suggestions: Denote by (α, β) the numbers of items in each pile at an arbitrary moment during play, and refer to the player whose turn it is as the active player.

Now change the rules of the game as follows: when it is a player's turn to move, he must take some positive number of items of the same kind. (Thus if there are currently $\alpha \geq 1$ acorns and $\beta \geq 1$ blueberries, he must take between 1 and α acorns (inclusive), or between 1 and β blueberries (inclusive).) All other rules are as above.

4. For any $a, b \geq 0$, describe the set of subgame perfect equilibrium outcomes.

5. For any $a, b \geq 0$, describe the set of pure-strategy subgame perfect equilibria. Justify your answer carefully.

Part III

1. There are two equally likely states of the world $s = \delta, \rho$. There is an election between Mr. Elephant $\mathcal{E}$ and Ms. Donkey $\mathcal{D}$. Suppose that if $\mathcal{E}$ wins, Avery earns 0 in state δ and 1 in state ρ. If $\mathcal{D}$ wins, Avery earns 0 in state ρ and a payoff $\pi > 0$ in state δ.

   Avery maximizes his expected money payoff. Avery initially prefers $\mathcal{D}$. Then Avery sees the flip of a coin tossed by a prophet who knows the state. The coin is fair if $s = \delta$, but comes up heads 2/3 of the time in state $s = \rho$ wins. In fact, heads comes up and Avery then prefers $\mathcal{E}$.

   Using the information, provide the tightest possible upper and lower bounds on π.

2. Two allied nations A and B have a mutual defense pact. Nation A has a military budget of 100 resource units, while nation B has a military budget of 200 resource units. Each nation divides its military budget between a domestic military budget and a common defense fund. If nation $N \in \{A, B\}$ spend D_N resource units on the domestic military budget and C_N on the common defense fund, then A gets net benefit: $U_A = D_A + 10\log(C_A + C_B)$, while B gets net benefit $U_B = D_B + 5\log(C_A + C_B)$.

   The nations play a simultaneous move Nash equilibrium in choosing (C_A, C_B). Is it unique? Is it efficient?

Part IV

Consider a game between a buyer (b) and a seller (s). The seller can make a costly investment x to improve the quality of his product. The payoff function of the seller is $p - cq - x$ and the payoff function of the buyer is $u(q, x) - p$, where p is price and and c is the marginal cost of production. We assume $u_x > 0$, $u_q > 0$, $u_{qx} > 0$, $u_{qq} < 0$, and $u_{xx} < 0$. Information about the payoffs is complete. Quality is observable but is not contractible; prices and quantities are contractible.

The game is as follows. First, the buyer and the seller write a reservation contract (p_R, q_R). Then, the seller chooses x, which then becomes a sunk cost. Next, the buyer and the seller bargain: with probability 0.5, the buyer makes a take-it-or-leave-it offer (p, q) to the seller, while with probability 0.5, the seller makes a take-it-or-leave-it offer (p, q) to the buyer. After the offer is made, the other party either accepts the offer or chooses the reservation contract.

1. Find the first-best values of q and x.
2. Suppose there is no reservation contract (i.e., assume that the reservation contract is $(p_R, q_R) = (0, 0)$). Find the equilibrium values of q and x.
3. Suppose that the reservation contract is (p_R, q_R). Characterize the equilibrium quality in the contract. Can the price that satisfies the ex ante participation constraint for the buyer be chosen as equilibrium price (explain)? Does the reservation contract affect equilibrium?
4. Suppose now that the payoff functions are $u(q) - p$ for the buyer and $p - c(x)q - x$ for the seller; assume $c'(x) < 0$. Is there a reservation contract (p_R, q_R) that achieves the first-best outcome (for the new payoffs)?