

Problem Set #2 (2<sup>nd</sup> Half)  
(Due Sunday, November 22 before midnight)

Economics 709

Fall 2020

You may work in groups. However, please note that you must write up your own solutions in your own words.

The exercises listed below are from Hansen, *Econometrics*.

1. 3.2
2. 3.5 - 3.7
3. 3.11 - 3.13
4. 3.16
5. 3.21 - 3.23
6. 3.24 - 3.25 (due with the next problem set)
7. Given the  $n \times 1$  vector  $y$  and the  $n \times k$  matrix  $X$ . Assume:  $\text{rank}(X) = k$ ;  $E(y|X) = X\beta$ ; and  $\text{Var}(y|X) = \sigma^2 I$ .  
Partition  $X$ :  $X = [X_1 \ X_2]$  where  $X_1$  is  $n \times k_1$ ,  $X_2$  is  $n \times k_2$ , and  $k_1 + k_2 = k$ . And similarly partition  $\beta$ :  $\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$ , where  $\beta_1$  is  $k_1 \times 1$  and  $\beta_2$  is  $k_2 \times 1$ .

- (a) Consider the OLS regression of  $y$  on  $X$  that yields the OLS estimator  $\hat{\beta}$ . What is  $E(\hat{\beta}_1|X)$ ? Simplify your answer.
- (b) Let  $\hat{y} = X\hat{\beta}$ . Now, consider the OLS regression of  $\hat{y}$  on  $X_1$  that yields the OLS estimator  $\hat{\hat{\beta}}_1$ . What is  $E(\hat{\hat{\beta}}_1|X)$ ? (Simplify your answer.) Is  $\hat{\hat{\beta}}_1$  an unbiased estimator of  $\beta_1$ ?
- (c) Consider the OLS regression of  $y$  on  $X_1$  that yields the OLS estimator  $\tilde{\beta}_1$ . Let  $\tilde{y} = X_1\tilde{\beta}_1$ .  
Now consider the OLS regression of  $\tilde{y}$  on  $X$  that yields the OLS estimator  $\tilde{\tilde{\beta}}$ . How is  $\tilde{\tilde{\beta}}$  related to  $\tilde{\beta}_1$ ? (Provide a mapping between  $\tilde{\tilde{\beta}}$  and  $\tilde{\beta}_1$  that does not involve  $X$ .)
- (d) What is the  $R^2$  for the OLS regression of  $\tilde{y}$  on  $X$  (from part (c))? Simplify your answer.
- (e) What is  $\text{Var}(\tilde{\tilde{\beta}}|X)$ ? Simplify your answer.