Problem Set 2

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1) cost: $k^2 + q + q^2$

$\pi = pq - (k^2 + q + q^2)$ Take p as given

FOC: $p - 1 - 2q = 0$

$$q^* = \frac{p-1}{2}$$

The minimum production occurs if only $k=1$ produces.

$$\pi(1) = p\left(\frac{p-1}{2}\right) - 1 - \left(\frac{p-1}{2}\right) - \left(\frac{p-1}{2}\right)^2 = 0$$

$$p^2 - p - 2 - p + 1 - \left(\frac{p^2 - 2p + 1}{2}\right) = 0$$

$$p^2 - 2p + 1 - 4 = 0$$

$$p^2 - 2p - 3 = 0$$

$$(p-3)(p+1) = 0$$

We can't have a negative price, so $p=3$ is the minimum price at which there is non-zero production.

2a) Given τ, a firm will choose q that maximizes profits and will enter the market if $\pi \geq 0$.

$$\pi = (1-\tau) \cdot 2\theta x - x^2 - 1 \quad \text{choose } x \text{ to max } \pi$$

$$\text{FOC: } 2\theta(1-\tau) = 2x$$

$$\rightarrow x = \theta(1-\tau)$$

$$\pi = 2\theta^2(1-\tau)^2 - \theta^2(1-\tau)^2 - 1 \geq 0$$

$$\theta^2(1-\tau)^2 \geq 1$$

$$\theta(1-\tau) \geq 1$$

$$\theta \geq \frac{1}{1-\tau}$$

Let $F = 1 - M(\theta)$. Then $f = -M'(\theta) = \beta\theta^{-\beta-1}$

$$Q_s = \int_{\frac{1}{1-\tau}}^{\infty} 2(1-\tau)\theta^2 \cdot (\beta\theta^{-\beta-1})\, d\theta$$

$$= \int_{\frac{1}{1-\tau}}^{\infty} 2\beta(1-\tau)\theta^{1-\beta}\, d\theta$$

$$= \frac{2\beta(1-\tau)}{2-\beta}\theta^{2-\beta}\Big|_{\frac{1}{1-\tau}}^{\infty} \qquad \text{Note } \beta > 2$$

$$= 0 - \frac{2\beta(1-\tau)}{\beta-2}\left(\frac{1}{1-\tau}\right)^{2-\beta}$$

$$= \frac{2\beta(1-\tau)^{\beta-1}}{\beta-2}$$

2b) The cutoff to produce is $\theta = 1/1-\tau$, so if τ increases, fewer developers will produce. Further the production function is $q = 2\theta x$, where $x = \theta(1-\tau)$, so production developers that continue to produce when τ increases will produce a smaller quantity.

2c) $R = \tau Q_s = \tau \frac{2\beta(1-\tau)^{\beta-1}}{\beta-2}$

FOC: $\frac{2\beta(1-\tau)^{\beta-1}}{\beta-2} = \frac{2(\beta-1)\tau\beta(1-\tau)^{\beta-2}}{\beta-2}$

$1-\tau = \tau(\beta-1)$

$\tau^* = \frac{1}{\beta}$

2d) When β rises, the distibution $m(\theta)$ changes so that fewer firms produce, and firms are less productive. As a result Q_s will decrease.

3) $P_1 = a_1 - b_1 Q$ Fruit Bowl

$P_2 = a_2 - b_2 Q$ Triple chocolate chunk

$a_1 > a_2$ b/c FB has higher price intercept than TCC

$P_1 = P_2$ at interior point

then $b_1 > b_2$

$\Pi_1 = P_1 \cdot Q_1 - MC\,Q_1$ where MC is constant

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3) $\pi_1 = P_1 \cdot Q_1 - MC\,Q_1$ where MC is constant

$$= (a_1 - b_1 Q_1) Q_1 - MC\,Q_1$$

$$= a_1 Q_1 - b_1 Q_1^2 - MC\,Q_1$$

FOC: $a_1 - 2b_1 Q_1 - MC = 0$

$$Q_1^* = \frac{a_1 - MC}{2b_1}$$

By symmetry, $Q_2^* = \dfrac{a_2 - MC}{b_2}$

$$P_1 = a_1 - b_1 Q_1$$

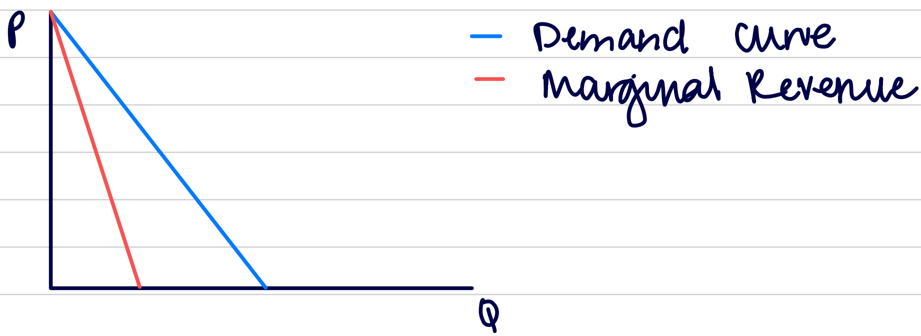
$$= a_1 - b_1 \left(\frac{a_1 - MC}{2b_1} \right)$$

$$= \frac{a_1}{2} - \frac{MC}{2}$$

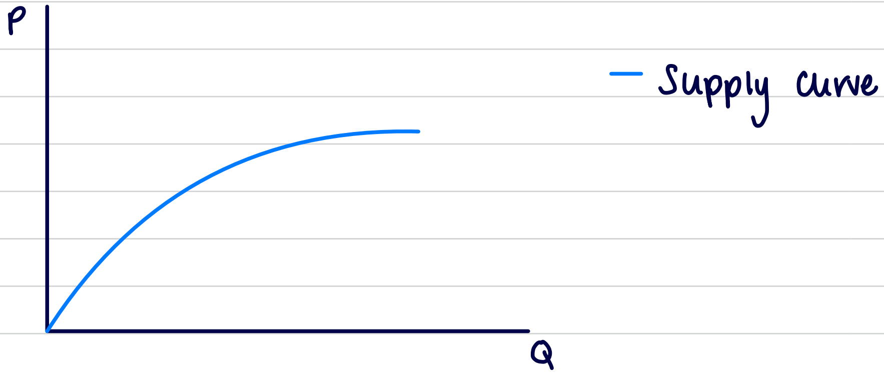
$$P_2 = \frac{a_2}{2} - \frac{MC}{2}$$

Since $a_1 > a_2$, $P_1 > P_2$.

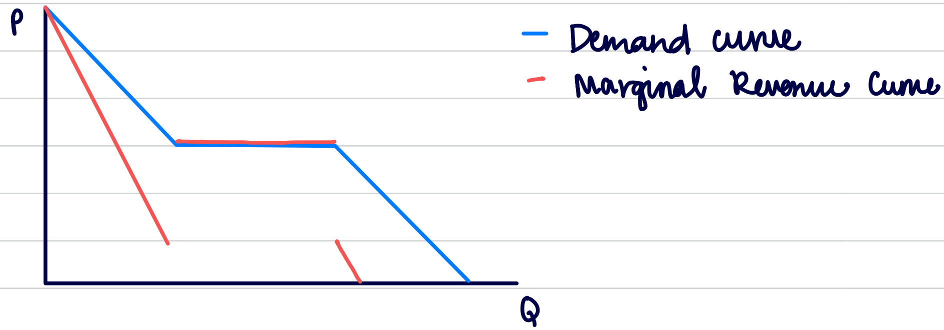
4a)



4b)



4c)



5) $P_1 = 3 - Q_1$ shops in person

$P_2 = 5 - Q_2$ shops online

For Group 1, monopolists maximize:

$\pi = p \cdot Q - 1 \cdot Q = 3Q - Q^2 - Q = 2Q - Q^2$

FOC: $2 - 2Q = 0$

$Q = 1$

$P = 3 - 1 = 2$

For Group 2, monopolists maximize:

$\pi = 5Q - Q^2 - Q = 4Q - Q^2$

FOC: $4 - 2Q = 0$

$Q = 2$

$p = 5 - 2 = 3$

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Now let $MC = Q$, $Q = Q_1 + Q_2$. Monopolists maximize:

$$\pi = P_1 Q_1 + P_2 Q_2 - MC\,Q_1 - MC\,Q_2$$
$$= (3 - Q_1)Q_1 + (5 - Q_2)Q_2 - (Q_1 + Q_2)^2$$
$$= 3Q_1 - Q_1^2 + 5Q_2 - Q_2^2 - Q_1^2 - 2Q_1Q_2 - Q_2^2$$

FOC:
$$3 - 2Q_1 - 2Q_1 - 2Q_2 = 0$$
$$3 - 4Q_1 - 2Q_2 = 0$$
$$4Q_1 + 2Q_2 = 3$$
$$5 - 2Q_2 - 2Q_2 - 2Q_1 = 0$$
$$4Q_2 + 2Q_1 = 5$$

$$Q_1 = 1/6, \quad Q_2 = 7/6$$
$$P_1 = 17/6, \quad P_2 = 23/6$$