Econ 703 Fall 2007
Homework 4

Due Tuesday, October 16.

1. Sundaram, #52, p. 72. (Unless finished last week.)
2. Let $B \subset \mathbb{R}^2$ be as defined as follows: $B = \{(x, y) \in \mathbb{R}^2 : y = \sin \frac{1}{x},\ x > 0\} \cup \{(0, 0)\}$. Is B closed? Open? Bounded? Compact?
3. Is every point of every open set $E \subset \mathbb{R}^2$ a limit point of E? What if $E \subset \mathbb{R}^2$ is closed?
4. Let X, Y, and Z be metric spaces (with norms defined on each) and let $f : X \times Y \to Z$. We say that f is *continuous in each variable separately* if, for each x_0 in X, the function $h : Y \to Z$ defined by $h(y) = f(x_0, y)$ is continuous and if for each y_0 in Y the function $g(x) = f(x, y_0)$ is continuous. Prove that if f is continuous, then f is continuous in each variable separately.

   (Remark: whenever considering product spaces, we use the product metric to define open sets).
5. Let X and Y be a metric spaces. Show that f is continuous if and only if the graph of f, $G(f) = \{(x, f(x)) : x \in X\}$,is a closed subset of $X \times Y$ (using the product metric).

   (HINT: If $G(f)$ is closed, and V is a ball around $f(x_0)$, find a tube about $x_0 \times (Y \backslash V)$ not intersecting $G(f)$).