Econ 703 Fall 2007
Homework 8

Due Tuesday, November 20

1. (Skip this question if already completed for homework 7.)

   Define $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(x, y) = 2x^3 - 3x^2 + 2y^3 + 3y^2$.

   (a) Find the four points in $\mathbb{R}^2$ at which the gradient of f is zero. Show that f has exactly one local maximum and one local minimum.

   (b) Let S be the set of all $(x, y) \in \mathbb{R}^2$ at which $f(x, y) = 0$. Find those points of S that have no neighborhoods in which the equation $f(x, y)$ can be solved for y in terms of x (or for x in terms of y). Describe S as precisely as you can.

2. Show that $F(x, y) = (e^y \cos x, e^y \sin x)$ is locally one-to-one and onto, but not globally one-to-one.

3. Give an example of a function $f : \mathbb{R}^2 \to \mathbb{R}^2$ satisfying $\frac{\partial^2 f_i}{\partial x_j} > 0$ for all $i, j \in \{1, 2\}$ such that $f\,(x)$ is not globally invertible.

4. Show that the system of equations

$$\begin{aligned} 3x + y - z + u^2 &= 0 \\ x - y + 2z + u &= 0 \\ 2x + 2y - 3z + 2u &= 0 \end{aligned}$$

   can be solved for x,y,u in terms of z; for x,z,u in terms of y; for y,z,u in terms of x; but not for x,y,z in terms of u.

5. Let $f : \mathbb{R} \to \mathbb{R}$ be a C^1 function, and let

$$\begin{aligned} u &= f(x) \\ v &= -y + xf(x). \end{aligned}$$

   Under what conditions is this transformation invertible near (x_0, y_0)? What is the inverse?