Homework #2

Raymond Deneckere

Fall 2017

1. Sundaram, #9, p.67.

2. Sundaram, #13, p.68.

3. Sundaram, #23, p.68

4. Let (X, d) be a metric space. Prove the following statement : $A \subset X$ is closed iff for every sequence $\{x_n\} \subset A$, $x_n \to x$ implies $x \in A$.

5. Consider the set of all rational numbers $\mathbb{Q}$, and make it into a metric space by defining $d(p, q) = |p - q|$ for all $p, q \in \mathbb{Q}$. Let E be the set of all $p \in \mathbb{Q}$ such that $2 < p^2 < 3$. Show that E is closed and bounded in $\mathbb{Q}$, but that E is not compact. Conclude that $\mathbb{Q}$ is not a compact space. Is E open in $\mathbb{Q}$?

   HINT: Be very careful here. The notions closed, open and compact are all with reference to the space $\mathbb{Q}$, not the space $\mathbb{R}$.