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Econ 761 HW 2

1. a) We are given $P = a_0 - a_1 Q + \nu \Rightarrow Q = \frac{1}{a_1}(a_0 + \nu - P)$

Elasticity of demand is $\varepsilon = -\frac{P}{Q}\frac{\partial Q}{\partial P} = \frac{P}{Q}\left(\frac{1}{a_1}\right)$

$\Rightarrow \varepsilon = \frac{1}{a_1 Q}(a_0 - a_1 Q + \nu) = \frac{a_0}{a_1 Q} + \frac{\nu}{a_1 Q} - 1$

$\frac{\partial \varepsilon}{\partial Q} = -\frac{a_0}{a_1 Q^2} - \frac{\nu}{a_1 Q^2} = -\frac{a_0 + \nu}{a_1 Q^2}$

For $a_1 > 0$, the denominator $a_1 Q^2 > 0$

Since $a_0, \nu \geq 0$, the numerator $-(a_0 + \nu) \leq 0$

$\Rightarrow \boxed{\frac{\partial \varepsilon}{\partial Q} \leq 0}$, so as quantity Q increases, elasticity of demand $\frac{\partial \varepsilon}{\partial Q}$ decreases

$\frac{\partial \varepsilon}{\partial \nu} = \frac{1}{a_1 Q} \geq 0$ for $a_1 > 0 \Rightarrow \boxed{\frac{\partial \varepsilon}{\partial \nu} \geq 0}$

$\Rightarrow$ as ν increases, $\frac{\partial \varepsilon}{\partial \nu}$ increases which makes sense since as maximum willingness to pay rises, there is higher demand elasticity

b) Since $b_1 = 0 \Rightarrow C = F + (b_0 + \eta) Q$

Then the problem for firm i is $\max_{q_i} P(Q) q_i - c(q_i)$

$\Rightarrow \max_{q_i} \left[a_0 - a_1(q_i + Q_{-i}) + \nu\right] q_i - \left[F + (b_0 + \eta) q_i\right]$

$\Rightarrow \max_{q_i} \left[a_0 - a_1(q_i + Q_{-i}) + \nu - b_0 - \eta\right] q_i - F$

Taking FOC wrt q_i, $a_0 - 2a_1 q_i - a_1 Q_{-i} + \nu - b_0 - \eta = 0$

Since firms are symmetric, let $q_i = q \;\forall i = 1, \dots, N \Rightarrow Q = Nq$

$\Rightarrow a_0 - a_1(N+1) q + \nu - b_0 - \eta = 0 \Rightarrow q^* = \frac{a_0 + \nu - b_0 - \eta}{a_1 (N+1)}$

$\Rightarrow$ total quantity is $Q^* = Nq^* = N\left(\frac{a_0 + \nu - b_0 - \eta}{a_1 (N+1)}\right)$

Price is $P^* = a_0 - a_1 Q^* + \nu = a_0 - N\left(\frac{a_0 + \nu - b_0 - \eta}{N+1}\right) + \nu = \frac{a_0 + \nu + N(b_0 + \eta)}{N+1}$

Profits for each firm are $\pi = P^* q^* - c(q^*)$

$$\Rightarrow \pi = \left[\frac{a_0+\nu+N(b_0+\eta)}{N+1}\right]\left[\frac{a_0+\nu-b_0-\eta}{a_1(N+1)}\right] - \left[F+(b_0+\eta)\left[\frac{a_0+\nu-b_0-\eta}{a_1(N+1)}\right]\right]$$

$$\pi = \frac{1}{a_1(N+1)^2}\left[a_0+\nu+N(b_0+\eta)\right]\left[a_0+\nu-b_0-\eta\right] - \frac{1}{a_1(N+1)^2}\left[(b_0+\eta)(N+1)(a_0+\nu-b_0-\eta)\right] - F$$

$$\pi = \frac{1}{a_1}\left[\frac{a_0+\nu-b_0-\eta}{N+1}\right]^2 - F$$

$$\Rightarrow \text{Cournot equilibrium is } \begin{cases} q^* = \dfrac{a_0+\nu-b_0-\eta}{a_1(N+1)} \Rightarrow Q^* = N\left(\dfrac{a_0+\nu-b_0-\eta}{a_1(N+1)}\right) \\ P^* = \dfrac{a_0+\nu+N(b_0+\eta)}{N+1} \\ \pi = \dfrac{1}{a_1}\left[\dfrac{a_0+\nu-b_0-\eta}{N+1}\right]^2 - F \quad \text{for each firm} \end{cases}$$

c) Firms enter until profits are zero $\Rightarrow 0 = \frac{1}{a_1}\left[\frac{a_0+\nu-b_0-\eta}{N+1}\right]^2 - F$

$$\Rightarrow F(N+1)^2 = \frac{1}{a_1}(a_0+\nu-b_0-\eta)^2 \Rightarrow \boxed{N = \frac{1}{\sqrt{Fa_1}}(a_0+\nu-b_0-\eta) - 1}$$

d) Lerner index: $L_I = \frac{P-MC}{P} = \dfrac{\frac{a_0+\nu+N(b_0+\eta)}{N+1} - (b_0+\eta)}{\frac{a_0+\nu+N(b_0+\eta)}{N+1}}$

$$\Rightarrow L_I = \frac{a_0+\nu-b_0-\eta}{a_0+\nu+N(b_0+\eta)}$$

Using $N = \frac{a_0+\nu-b_0-\eta}{\sqrt{Fa_1}} - 1$ from (c), $L_I = \dfrac{a_0+\nu-b_0-\eta}{a_0+\nu+\frac{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}{\sqrt{Fa_1}}(b_0+\eta)}$

$$\Rightarrow L_I = \frac{\sqrt{Fa_1}(a_0+\nu-b_0-\eta)}{a_0\sqrt{Fa_1}+\nu\sqrt{Fa_1}+(a_0+\nu-b_0-\eta-\sqrt{Fa_1})(b_0+\eta)}$$

$$\Rightarrow L_I = \frac{\sqrt{Fa_1}(a_0+\nu-b_0-\eta)}{\sqrt{Fa_1}(a_0+\nu-b_0-\eta)+(a_0+\nu-b_0-\eta)(b_0+\eta)}$$

$$\Rightarrow L_I = \frac{\sqrt{Fa_1}(a_0+\nu-b_0-\eta)}{(\sqrt{Fa_1}+b_0+\eta)(a_0+\nu-b_0-\eta)} = \boxed{\frac{\sqrt{Fa_1}}{\sqrt{Fa_1}+b_0+\eta} = L_I}$$

Herfindahl index : $H = \frac{1}{N}$ since firms are symmetric

$$\Rightarrow H = \frac{1}{\frac{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}{\sqrt{Fa_1}}} = \boxed{\frac{\sqrt{Fa_1}}{a_0+\nu-b_0-\eta-\sqrt{Fa_1}} = H}$$

demand elasticity : $\varepsilon = \frac{a_0+\nu-a_1Q}{a_1Q}$

using $Q = N\left(\frac{a_0+\nu-b_0-\eta}{a_1(N+1)}\right)$, $\varepsilon = \frac{a_0+\nu-N\left(\frac{a_0+\nu-b_0-\eta}{N+1}\right)}{N\left(\frac{a_0+\nu-b_0-\eta}{N+1}\right)}$

using $N = \frac{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}{\sqrt{Fa_1}}$, $\frac{N}{N+1} = \frac{\frac{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}{\sqrt{Fa_1}}}{\frac{a_0+\nu-b_0-\eta}{\sqrt{Fa_1}}} = \frac{a_0+\nu-b_0-\eta-\sqrt{Fa_1}}{a_0+\nu-b_0-\eta}$

$$\Rightarrow \varepsilon = \frac{a_0+\nu-(a_0+\nu-b_0-\eta-\sqrt{Fa_1})}{a_0+\nu-b_0-\eta-\sqrt{Fa_1}} = \boxed{\frac{b_0+\eta+\sqrt{Fa_1}}{a_0+\nu-b_0-\eta-\sqrt{Fa_1}} = \varepsilon}$$

e) $$\frac{\partial\varepsilon}{\partial F} = \frac{(a_0+\nu-b_0-\eta-\sqrt{Fa_1})\frac{\sqrt{a_1}}{2\sqrt{F}} + (b_0+\eta+\sqrt{Fa_1})\frac{\sqrt{a_1}}{2\sqrt{F}}}{(a_0+\nu-b_0-\eta-\sqrt{Fa_1})^2}$$

$$\Rightarrow \frac{\partial\varepsilon}{\partial F} = \frac{\sqrt{a_1}(a_0+\nu)}{2\sqrt{F}(a_0+\nu-b_0-\eta-\sqrt{Fa_1})^2} \geq 0 \Rightarrow \boxed{\frac{\partial\varepsilon}{\partial F} \geq 0}$$

As entry costs rise, potentially fewer firms enter the market leading to an increase in demand elasticity.

$$\frac{\partial\varepsilon}{\partial\nu} = -\frac{b_0+\eta+\sqrt{Fa_1}}{(a_0+\nu-b_0-\eta-\sqrt{Fa_1})^2} \leq 0 \Rightarrow \boxed{\frac{\partial\varepsilon}{\partial\nu} \leq 0}$$

Hence with endogenous number of firms entering, as willingness to pay increases, number of firms entering increases which drives the demand elasticity down.

$$\frac{\partial \varepsilon}{\partial \eta} = \frac{(a_0+\nu-b_0-\eta-\sqrt{Fa_1}) + (b_0+\eta+\sqrt{Fa_1})}{(a_0+\nu-b_0-\sqrt{Fa_1})^2}$$

$$\Rightarrow \frac{\partial \varepsilon}{\partial \eta} = \frac{a_0+\nu}{(a_0+\nu-b_0-\sqrt{Fa_1})^2} \geq 0 \Rightarrow \boxed{\frac{\partial \varepsilon}{\partial \eta} \geq 0}$$

As marginal cost increases, firms produce less and so the demand elasticity increases.

$$\ln(L_I) = \tfrac{1}{2}\ln(Fa_1) - \ln(\sqrt{Fa_1}+b_0+\eta)$$
$$\ln(H) = \tfrac{1}{2}\ln(Fa_1) - \ln(a_0+\nu-b_0-\eta-\sqrt{Fa_1})$$

$$\left.\begin{aligned} \frac{\partial \ln(L_I)}{\partial F} &= \frac{1}{2F} - \frac{\sqrt{a_1}}{(\sqrt{Fa_1}+b_0+\eta)2\sqrt{F}} \\ \frac{\partial \ln(H)}{\partial F} &= \frac{1}{2F} + \frac{\sqrt{a_1}}{(a_0+\nu-b_0-\eta-\sqrt{Fa_1})2\sqrt{F}} \end{aligned}\right\} \Rightarrow \frac{\partial \ln(L_I)}{\partial F} \neq \frac{\partial \ln(H)}{\partial F}$$

$$\left.\begin{aligned} \frac{\partial \ln(L_I)}{\partial \nu} &= 0 \\ \frac{\partial \ln(H)}{\partial \nu} &= -\frac{1}{a_0+\nu-b_0-\eta-\sqrt{Fa_1}} \end{aligned}\right\} \Rightarrow \frac{\partial \ln(L_I)}{\partial \nu} \neq \frac{\partial \ln(H)}{\partial \nu}$$

$$\left.\begin{aligned} \frac{\partial \ln(L_I)}{\partial \eta} &= -\frac{1}{\sqrt{Fa_1}+b_0+\eta} \\ \frac{\partial \ln(H)}{\partial \eta} &= \frac{1}{a_0+\nu-b_0-\eta-\sqrt{Fa_1}} \end{aligned}\right\} \Rightarrow \frac{\partial \ln(L_I)}{\partial \eta} \neq \frac{\partial \ln(H)}{\partial \eta}$$

$\Rightarrow$ $\ln(L_I)$ and $\ln(H)$ do not change at the same rate due to exogenous changes in F, ν, η

f) Firms now have $\max\limits_Q P(Q)Q - C(Q)$

$$\Rightarrow \max_Q (a_0 - a_1 Q + \nu)Q - [(b_0+\eta)Q + F]$$

FOC is $a_0 - 2a_1 Q + \nu - b_0 - \eta = 0 \Rightarrow Q^* = \dfrac{a_0+\nu-b_0-\eta}{2a_1}$

Price is $P^* = a_0 - a_1 Q^* + \nu = a_0 - \frac{a_0+\nu-b_0-\eta}{2} + \nu = \frac{1}{2}(a_0+\nu+b_0+\eta)$

$\Rightarrow$ profits are $\pi = P^* \frac{Q^*}{N} - C(\frac{Q^*}{N})$

$= \frac{1}{2}(a_0+\nu+b_0+\eta) \frac{a_0+\nu-b_0-\eta}{2Na_1} - \left[(b_0+\eta)\frac{a_0+\nu-b_0-\eta}{2Na_1} + F\right]$

$\Rightarrow \pi = \frac{1}{4Na_1}[a_0+\nu-b_0-\eta]^2 - F$ for each firm

Firms enter until zero profits $\Rightarrow \frac{1}{4Na_1}[a_0+\nu-b_0-\eta]^2 - F = 0$

$\Rightarrow 4NFa_1 = (a_0+\nu-b_0-\eta)^2 \Rightarrow N = \frac{1}{4Fa_1}(a_0+\nu-b_0-\eta)^2$

Lerner index : $L_I = \frac{P-MC}{P} = \frac{\frac{1}{2}(a_0+\nu+b_0+\eta)-(b_0+\eta)}{\frac{1}{2}(a_0+\nu+b_0+\eta)}$

$\Rightarrow \boxed{L_I = \frac{a_0+\nu-b_0-\eta}{a_0+\nu+b_0+\eta}}$

Herfindahl index : $H = \frac{1}{N} = \boxed{\frac{4Fa_1}{(a_0+\nu-b_0-\eta)^2} = H}$

demand elasticity : $-\frac{P}{Q}\frac{dQ}{dP} = \frac{P}{Q}\left(\frac{1}{a_1}\right) = \boxed{\frac{a_0+\nu+b_0+\eta}{a_0+\nu-b_0-\eta} = \varepsilon}$

9) (a) $\ln P = c_0 - c_1 \ln Q + \xi \Rightarrow P = \exp(c_0 - c_1 \ln Q + \xi)$

$c_1 \ln Q = c_0 - \ln P + \xi \Rightarrow Q = \exp\left[\frac{1}{c_1}(c_0 - \ln P + \xi)\right]$

$\Rightarrow$ elasticity of demand $\varepsilon = -\frac{P}{Q}\frac{dQ}{dP} = \frac{P}{Q}\left[\exp\left[\frac{1}{c_1}(c_0-\ln P+\xi)\right]\frac{1}{P}\right]$

$\Rightarrow \varepsilon = \frac{1}{Q}\exp\left[\frac{1}{c_1}(c_0 - \ln P + \xi)\right]$

$\frac{\partial \varepsilon}{\partial Q} = -\frac{1}{Q^2}\exp\left[\frac{1}{c_1}(c_0-\ln P+\xi)\right] \Rightarrow \boxed{\frac{\partial \varepsilon}{\partial Q} \leq 0}$

$\frac{\partial \varepsilon}{\partial \xi} = \frac{1}{Q}\exp\left[\frac{1}{c_1}(c_0-\ln P+\xi)\right] \Rightarrow \boxed{\frac{\partial \varepsilon}{\partial \xi} \geq 0}$

$\Rightarrow$ we have the same results as part (a) previously

(b) Problem for firm i is $\max_{q_i} P(Q) q_i - c(q_i)$

$$\Rightarrow \max_{q_i} \exp(c_0 - c_1 \ln(q_i + Q_{-i}) + \xi) q_i - [F + (b_0 + \eta) q_i]$$

FOC wrt q_i: $\exp(c_0 - c_1 \ln(q_i + Q_{-i}) + \xi) - (q_i)\exp(c_0 - c_1 \ln(q_i + Q_{-i}) + \xi)\frac{c_1}{q_i + Q_{-i}}$
$- b_0 - \eta = 0$

Since firms are symmetric, $q_i = q$ for all $i = 1, \dots, N$ $\Rightarrow Q = Nq$

$$\Rightarrow \exp(c_0 - c_1 \ln(Nq) + \xi) - \frac{q c_1}{Nq}\exp(c_0 - c_1 \ln(Nq) + \xi) - b_0 - \eta = 0$$
$$\Rightarrow \frac{N - c_1}{N}\exp(c_0 - c_1 \ln(Nq) + \xi) - b_0 - \eta = 0$$

$$\Rightarrow \exp(c_0 - c_1 \ln(Nq) + \xi) = \frac{N}{N - c_1}(b_0 + \eta)$$
$$\Rightarrow c_0 - c_1 \ln(Nq) + \xi = \ln\left[\frac{N}{N - c_1}(b_0 + \eta)\right]$$

$$\Rightarrow c_1 \ln(Nq) = c_0 + \xi - \ln\left[\frac{N}{N - c_1}(b_0 + \eta)\right]$$

$$\Rightarrow q^* = \frac{1}{N}\exp\left[\frac{1}{c_1}\left(c_0 + \xi - \ln\left[\frac{N}{N - c_1}(b_0 + \eta)\right]\right)\right]$$

Hence $Q^* = Nq^* = \exp\left[\frac{1}{c_1}\left(c_0 + \xi - \ln\left[\frac{N}{N - c_1}(b_0 + \eta)\right]\right)\right]$

Price is $P^* = \exp(c_0 - c_1 \ln Q^* + \xi)$ with Q^* defined above

$$\Rightarrow P^* = \exp\left(c_0 - c_1 \ln\left[\exp\left[\frac{1}{c_1}\left(c_0 + \xi - \ln\left[\frac{N}{N - c_1}(b_0 + \eta)\right]\right)\right]\right] + \xi\right)$$
$$= \exp\left(c_0 - \left[c_0 + \xi - \ln\left[\frac{N}{N - c_1}(b_0 + \eta)\right]\right] + \xi\right)$$

$$\Rightarrow P^* = \frac{N}{N - c_1}(b_0 + \eta)$$

Profits are $\pi = P^* q^* - c(q^*)$ for each firm

$$\Rightarrow \pi = \frac{N}{N - c_1}(b_0 + \eta)\frac{1}{N}\exp\left[\frac{1}{c_1}\left(c_0 + \xi - \ln\left[\frac{N}{N - c_1}(b_0 + \eta)\right]\right)\right]$$
$$- \left[F + (b_0 + \eta)\frac{1}{N}\exp\left[\frac{1}{c_1}\left(c_0 + \xi - \ln\left[\frac{N}{N - c_1}(b_0 + \eta)\right]\right)\right]\right]$$

Because we will not use the profits anymore, I will not simplify it further.

(d) Lerner index: $L_I = \frac{P-MC}{P} = \frac{\frac{N}{N-c_1}(b_0+\eta) - (b_0+\eta)}{\frac{N}{N-c_1}(b_0+\eta)}$

$$\Rightarrow L_I = \frac{\frac{N}{N-c_1} - 1}{\frac{N}{N-c_1}} = \frac{\frac{N-(N-c_1)}{N-c_1}}{\frac{N}{N-c_1}} = \boxed{\frac{c_1}{N} = L_I}$$

Herfindahl index: $\boxed{H = \frac{1}{N}}$

demand elasticity: $\varepsilon = \frac{1}{Q} \exp\left[\frac{1}{c_1}(c_0 - \ln P + \xi)\right]$

$$\Rightarrow \varepsilon = \frac{\frac{1}{c_1}\exp\left[\frac{1}{c_1}\left(c_0 - \ln\left[\frac{N}{N-c_1}(b_0+\eta)\right] + \xi\right)\right]}{\exp\left[\frac{1}{c_1}\left(c_0 + \xi - \ln\left[\frac{N}{N-c_1}(b_0+\eta)\right]\right)\right]} = \frac{1}{c_1} \Rightarrow \boxed{\varepsilon = \frac{1}{c_1}}$$

(e) We take N as fixed and exogenous, so the Lerner index L_I, Herfindahl index H, and demand elasticity all do not vary with F, ν, η.

For ε, $\frac{\partial \varepsilon}{\partial F} = \frac{\partial \varepsilon}{\partial \nu} = \frac{\partial \varepsilon}{\partial \eta} = 0$.

Similarly, $\begin{cases} \frac{\partial \ln(L_I)}{\partial F} = \frac{\partial \ln(H)}{\partial F} = 0 \\ \frac{\partial \ln(L_I)}{\partial \nu} = \frac{\partial \ln(H)}{\partial \nu} = 0 \\ \frac{\partial \ln(L_I)}{\partial \eta} = \frac{\partial \ln(H)}{\partial \eta} = 0 \end{cases}$

$\Rightarrow$ $\ln(L_I)$ and $\ln(H)$ do change at the same rate (0) in response to changes in F, ν, η.

2. a) see attached code

b) see attached code

c) The results of the three regressions are presented below.

	Collusion Possible N=500	no Collusion N=500	Pooled Sample N=1000
Constant	0.387 (0.081)	-0.163 (0.001)	0.077 (0.056)
ln (Herfindahl)	0.619 (0.049)	0.962 (0.001)	0.764 (0.034)
F-stat for test $H_0: \ln(\text{Herfindahl})=1$	61.59 $\Rightarrow$ reject	4261.89 $\Rightarrow$ reject	47.77 $\Rightarrow$ reject

When collusion is possible, a one percent increase in Herfindahl index increases the Lerner index by 0.619 percent. This is much higher when there is no collusion (0.962) and in between these two for the pooled sample (0.764).

For this demand function, the elasticity of demand is $\frac{1}{c_1} = \frac{1}{0.9} = 1.111$. Due to the high demand elasticity, monopolies can charge higher prizes leading to a lower correlation between H and L_I in the sample with collusion.

It makes sense that the pooled sample yields a coefficient between either of the individual samples, because it contains them both. Hence, in all three samples there is a positive correlation between H and L_I.

The f-statistic in each test of ln(Herfindahl)=1 is large and leads to a rejection of the null. However, the reason it is rejected in the sample with no collusion is because of a very small standard error. By inspection, for the no collusion sample, there is very near a 1-1 correlation between H and L_I.

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d) Now we use linear demand, still taking N as fixed.

For nonlinear demand in (c) we had $L_I = \frac{c_1}{N}$, $H = \frac{1}{N}$, $\varepsilon = \frac{1}{c_1}$.

In this case, we have $L_I = \frac{a_0 + \nu - b_0 - \eta}{a_0 + \nu + N(b_0 + \eta)}$, $H = \frac{1}{N}$, $\varepsilon = \frac{a_0 + \nu + N(b_0 + \eta)}{a_0 + \nu - b_0 - \eta}$.

The results from the structure-conduct-performance paradigm regressions as well as the hypothesis tests are below.

	collusion possible N=500	no collusion N=500	Pooled sample N=1000
constant	-0.473 (0.041)	-0.658 (0.005)	-0.580 (0.027)
ln(Herfindahl)	0.294 (0.024)	0.480 (0.003)	0.376 (0.016)
F-stat for test H_0: ln(Herfindahl)=1	893.73 ⇒ reject	28309.22 ⇒ reject	1481.95 ⇒ reject

There is still a higher correlation between H and L_I for the sample with no collusion. However, with linear demand, the magnitude of the relationship in each sample is lower than the respective coefficient with nonlinear demand. In all three samples with linear demand, we reject the null hypothesis that ln(Herfindahl) = 1, as can be seen in the table above.

In the case of linear demand, demand elasticity depends on quantity whereas in part (c), elasticity is a constant. With collusion, the equilibrium quantity is lower than when collusion isn't possible, so demand elasticity is higher in the former, leading to potentially lower markups. These factors explain these results and why they differ from part (c).

e) From part (d), if we know demand is linear and we suspect collusion in markets 1-250, we can run a structure-conduct-performance paradigm regression and the coefficient on ln(Herfindahl) should be quite low, around 0.3.

However, this may not necessarily help us generally. If we do not know the functional form of demand, the regression could result in a higher coefficient (higher than the no collusion sample with linear demand), which could mislead us in concluding collusion. Hence, the functional form of demand is important in the subsequent regression and analysis for us to be certain whether or not some markets are colluding.

3. a,b) The results of the regressions are below.

	$\nu \sim U[-1,1]$ $\eta = 0$	$\nu = 0$ $\eta \sim U[-1,1]$
constant	-0.851 (0.0005)	-2.116 (0.011)
ln(Herfindahl)	-0.146 (0.0004)	-1.359 (0.011)

c) When $\nu = 0$, maximum willingness to pay is a constant across cities, and the Lerner index drops by 1.359 % for a 1% increase in the Herfindahl index. For $\eta = 0$, marginal costs across cities are the same and the effect of Herfindahl index on Lerner index is much closer to zero.

With more firms, higher competition opposes higher markups and demand is more elastic when $\nu \sim U[-1,1]$ than $\eta \sim U[-1,1]$, leading to coefficient on ln(Herfindahl) that is closer to 0.

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Notes:
      1.  Unicode is supported; see help unicode_advice.
      2.  Maximum number of variables is set to 5000; see help set_maxvar.
      3.  New update available; type -update all-

. do "C:\z_toshiba\course work\phd\econ 761\hw\hw2\hw2.do"

. // code for questions 2 and 3 of hw2
. 
. // clear workspace
. clear

. 
. // 2a (setup)
. 
. // for each city, number of firms uniformly distributed on {1,2,...,10}
. set obs 1000
number of observations (_N) was 0, now 1,000

. gen unif = runiform()

. gen num_firms = int(unif*10+1)

. 
. // in 500 cities, firms collude perfectly when num_firms <= 8
. gen collude = 0

. replace collude = 1 if _n <= 500 & num_firms <= 8
(400 real changes made)

. 
. // 2b (construct L_i, H, e)
. 
. // demand function parameter initialization
. gen c0 = 1

. gen c1 = 0.9
```

```
23 . gen xi = 0

24 . 
25 . // cost function parameter initialization
26 . gen F = 1

27 . gen b0 = 1

28 . gen b1 = 0

29 . gen eta = 0

30 . 
31 . // construct Lerner index, Herfindahl index, demand elasticity
32 . gen lerner_cournot = c1/num_firms

33 . gen lerner_monopoly = c1

34 . gen lerner = collude*lerner_monopoly + (1-collude)*lerner_cournot

35 . gen observed_lerner = ln(lerner) + 0.1*(unif - 0.5)

36 . gen herfindahl = 1/num_firms

37 . gen elasticity = 1/c1

38 . 
39 . // 2c (regressions and tests)
40 . 
41 . // structure-conduct-performance paradigm regressions
42 . gen ln_herfindahl = ln(herfindahl)

43 . regress observed_lerner ln_herfindahl if _n <= 500  // collusion is possible

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(1, 498)       =    163.08
       Model |  95.6248438         1  95.6248438   Prob > F        =    0.0000
    Residual |  292.006505       498  .586358443   R-squared       =    0.2467
-------------+----------------------------------   Adj R-squared   =    0.2452
       Total |  387.631349       499   .77681633   Root MSE        =    .76574

------------------------------------------------------------------------------
observed_le~r |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
--------------+---------------------------------------------------------------
ln_herfindahl |   .6193705   .0485006    12.77   0.000     .5240795    .7146614
        _cons |   .3871213   .0812891     4.76   0.000     .2274094    .5468331
------------------------------------------------------------------------------

44 . test ln_herfindahl = 1

 ( 1)  ln_herfindahl = 1

       F(  1,   498) =   61.59
            Prob > F =    0.0000
```

```
45 . regress observed_lerner ln_herfindahl if _n > 500  // no collusion

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(1, 498)       >  99999.00
       Model |  236.102625         1  236.102625   Prob > F        =    0.0000
    Residual |  .043826911       498  .000088006   R-squared       =    0.9998
-------------+----------------------------------   Adj R-squared   =    0.9998
       Total |  236.146452       499  .473239383   Root MSE        =    .00938

------------------------------------------------------------------------------
observed_le~r |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ln_herfindahl |   .9616705   .0005871  1637.93   0.000      .960517    .9628241
        _cons |  -.1633087   .0009573  -170.59   0.000    -.1651896   -.1614278
------------------------------------------------------------------------------

46 . test ln_herfindahl = 1

    ( 1)  ln_herfindahl = 1

          F(  1,   498) = 4261.89
               Prob > F =    0.0000

47 . regress observed_lerner ln_herfindahl  // pooled sample

      Source |       SS           df       MS      Number of obs   =     1,000
-------------+----------------------------------   F(1, 998)       =    500.45
       Model |  294.916379         1  294.916379   Prob > F        =    0.0000
    Residual |   588.12952       998  .589308137   R-squared       =    0.3340
-------------+----------------------------------   Adj R-squared   =    0.3333
       Total |  883.045899       999  .883929829   Root MSE        =    .76766

------------------------------------------------------------------------------
observed_le~r |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ln_herfindahl |   .7639598   .0341501    22.37   0.000     .6969455    .8309741
        _cons |   .0769168    .056465     1.36   0.173    -.0338869    .1877205
------------------------------------------------------------------------------

48 . test ln_herfindahl = 1

    ( 1)  ln_herfindahl = 1

          F(  1,   998) =   47.77
               Prob > F =    0.0000

49 .
50 . // 2d (repeat 2b and 2c for linear demand)
51 .
52 . // demand function parameter initialization
53 . gen a0 = 3

54 . gen a1 = 1
```

```
55 . gen nu = 0

56 .
57 . // construct Lerner index, Herfindahl index, demand elasticity
58 . gen lerner_cournot2 = (a0+nu-b0-eta)/(a0+nu+num_firms*(b0+eta))

59 . gen lerner_monopoly2 = (a0+nu-b0-eta)/(a0+nu+b0+eta)

60 . gen lerner2 = collude*lerner_monopoly2 + (1-collude)*lerner_cournot2

61 . gen observed_lerner2 = ln(lerner2) + 0.1*(unif - 0.5)

62 . gen herfindahl2 = 1/num_firms

63 . gen elasticity2 = (a0+nu+b0+eta)/(a0+nu-b0-eta)

64 .
65 . // structure-conduct-performance paradigm regressions
66 . gen ln_herfindahl2 = ln(herfindahl2)

67 . regress observed_lerner2 ln_herfindahl2 if _n <= 500  // collusion is possible

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(1, 498)       =    146.01
       Model |  21.5861585         1  21.5861585   Prob > F        =    0.0000
    Residual |  73.6262089       498  .147843793   R-squared       =    0.2267
-------------+----------------------------------   Adj R-squared   =    0.2252
       Total |  95.2123673       499  .190806347   Root MSE        =     .3845

------------------------------------------------------------------------------
observed_ler~2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------+----------------------------------------------------------------
ln_herfindahl2 |   .2942746   .0243538    12.08   0.000     .2464258    .3421235
         _cons |  -.4726272    .040818   -11.58   0.000     -.552824   -.3924304
------------------------------------------------------------------------------

68 . test ln_herfindahl2 = 1

 ( 1)  ln_herfindahl2 = 1

       F(  1,   498) =  839.73
            Prob > F =    0.0000

69 . regress observed_lerner2 ln_herfindahl2 if _n > 500  // no collusion

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(1, 498)       =  24124.38
       Model |  58.8244536         1  58.8244536   Prob > F        =    0.0000
    Residual |  1.21431447       498  .002438382   R-squared       =    0.9798
-------------+----------------------------------   Adj R-squared   =    0.9797
       Total |  60.0387681       499  .120318173   Root MSE        =    .04938

------------------------------------------------------------------------------
observed_ler~2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------+----------------------------------------------------------------
ln_herfindahl2 |   .4800151   .0030905   155.32   0.000     .4739431    .4860871
         _cons |  -.6577959   .0050391  -130.54   0.000    -.6676963   -.6478954
------------------------------------------------------------------------------
```

```
70 . test ln_herfindahl2 = 1

 ( 1)  ln_herfindahl2 = 1

       F(  1,   498) =28309.22
            Prob > F =    0.0000

71 . regress observed_lerner2 ln_herfindahl2  // pooled sample

      Source |       SS           df       MS      Number of obs   =     1,000
-------------+----------------------------------   F(1, 998)       =    537.11
       Model |  71.3593422         1  71.3593422   Prob > F        =    0.0000
    Residual |  132.591543       998  .132857258   R-squared       =    0.3499
-------------+----------------------------------   Adj R-squared   =    0.3492
       Total |  203.950885       999   .20415504   Root MSE        =     .3645

------------------------------------------------------------------------------
observed_ler~2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------+----------------------------------------------------------------
ln_herfindahl2 |   .3757911   .0162149    23.18   0.000     .3439719    .4076103
         _cons |  -.5796315   .0268102   -21.62   0.000    -.6322424   -.5270206
------------------------------------------------------------------------------

72 . test ln_herfindahl2 = 1

 ( 1)  ln_herfindahl2 = 1

       F(  1,   998) = 1481.95
            Prob > F =    0.0000

73 .
74 . // 3a (setup, regressions, construct num_firms, L_i, H)
75 .
76 . // clear workspace
77 . clear

78 .
79 . // 1000 cities
80 . set obs 1000
   number of observations (_N) was 0, now 1,000

81 . gen unif = runiform()

82 .
83 . // demand function parameter initialization
84 . gen a0 = 5

85 . gen a1 = 1

86 . gen nu = 2*(unif - 0.5)

87 .
88 . // cost function parameter initialization
```

```
. gen F = 1

. gen b0 = 1

. gen b1 = 0

. gen eta = 0

.
. // firms enter until profits are zero
. gen num_firms = (a0+nu-b0-eta-sqrt(F*a1))/(sqrt(F*a1))

.
. // construct Lerner index, Herfindahl index, demand elasticity
. gen lerner = (a0+nu-b0-eta)/(a0+nu+num_firms*(b0+eta))

. gen observed_lerner = ln(lerner) + 0.1*(unif - 0.5)

. gen herfindahl = 1/num_firms

. gen elasticity = (a0+nu+b0+eta)/(a0+nu-b0-eta)

.
. // structure-conduct-performance paradigm regression
. gen ln_herfindahl = ln(herfindahl)

. regress observed_lerner ln_herfindahl

      Source |       SS           df       MS      Number of obs   =     1,000
-------------+----------------------------------   F(1, 998)       >  99999.00
       Model |  .81922049         1  .81922049     Prob > F        =    0.0000
    Residual |  .006599608       998  6.6128e-06   R-squared       =    0.9920
-------------+----------------------------------   Adj R-squared   =    0.9920
       Total |  .825820098       999  .000826647   Root MSE        =    .00257

------------------------------------------------------------------------------
observed_le~r |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ln_herfindahl |  -.1458873   .0004145  -351.97   0.000    -.1467007    -.145074
        _cons |  -.8506153   .0004606 -1846.81   0.000    -.8515191   -.8497114
------------------------------------------------------------------------------

.
. // 3b (repeat 3a for new eta and nu)
.
. // parameter initialization
. drop nu

. drop eta

. gen nu = 0

. gen eta = 2*(unif - 0.5)
```

```
114 .
115 . // firms enter until profits are zero
116 . gen num_firms2 = (a0+nu-b0-eta-sqrt(F*a1))/(sqrt(F*a1))

117 .
118 . // construct Lerner index, Herfindahl index, demand elasticity
119 . gen lerner2 = (a0+nu-b0-eta)/(a0+nu+num_firms2*(b0+eta))

120 . gen observed_lerner2 = ln(lerner2) + 0.1*(unif - 0.5)

121 . gen herfindahl2 = 1/num_firms2

122 . gen elasticity2 = (a0+nu+b0+eta)/(a0+nu-b0-eta)

123 .
124 . // structure-conduct-performance paradigm regression
125 . gen ln_herfindahl2 = ln(herfindahl2)

126 . regress observed_lerner2 ln_herfindahl2

      Source |       SS           df       MS      Number of obs   =     1,000
-------------+----------------------------------   F(1, 998)       =  16414.98
       Model |  72.1224304         1  72.1224304   Prob > F        =    0.0000
    Residual |  4.38490729       998  .004393695   R-squared       =    0.9427
-------------+----------------------------------   Adj R-squared   =    0.9426
       Total |  76.5073377       999  .076583922   Root MSE        =    .06628

------------------------------------------------------------------------------
observed_ler~2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------+--------------------------------------------------------------
ln_herfindahl2 |  -1.359395   .0106102  -128.12   0.000    -1.380216   -1.338575
         _cons |  -2.116006   .0114953  -184.08   0.000    -2.138564   -2.093448
------------------------------------------------------------------------------

127 .
    end of do-file

128 .
```