Practice Problems 12

Implicit Fuction Theorem

1. * Show that there is a vector $p \in R^2$ such that for given $(x_0, y_0) = (\sqrt{2}, \sqrt{2})$, $p \cdot (x_0, y_0) \leq p \cdot (x, y)$ for all $(x, y) \in \{(x, y)|xy \geq 2\}$. Can you derive p?

2. Show that there is a vector $p \in R^2$ such that for given $(x_0, y_0) = (\sqrt{2}, \sqrt{2})$, $p \cdot (x_0, y_0) \geq p \cdot (x, y)$ for all $(x, y) \in \{(x, y)|x^2 + y^2 \leq 4, x, y \geq 0\}$. Can you derive p?

3. * Prove that the expression $x^2 - xy^3 + y^5 = 17$ is an implicit function of y in terms of x in a neighborhood of $(x, y) = (5, 2)$. Then Estimate the y value which corresponds to $x = 4.8$.

4. Define $f : \mathbb{R}^3 \to \mathbb{R}$ by
$$f(x, y, z) = y^2x + e^y + z.$$
Show that there exists a differentiable function $g(x, z)$, such that $g(1, -1) = 0$ and
$$f(x, g(x, z), z) = 0$$
Specify the domain of g. Compute $Dg(1, -1)$.

Brouwer's Fixed Point Theorem

5. * Show that there is a Nash Equilibrium in the battle of sexes where the payoff is given as in the table below.

		Tony	
		Action	Comedy
Maria	Action	2, 5	−1, −1
	Comedy	1, 1	5, 2

Jensen's Inequality

6. * Homework Q28