Econ 711 Problem Set 5

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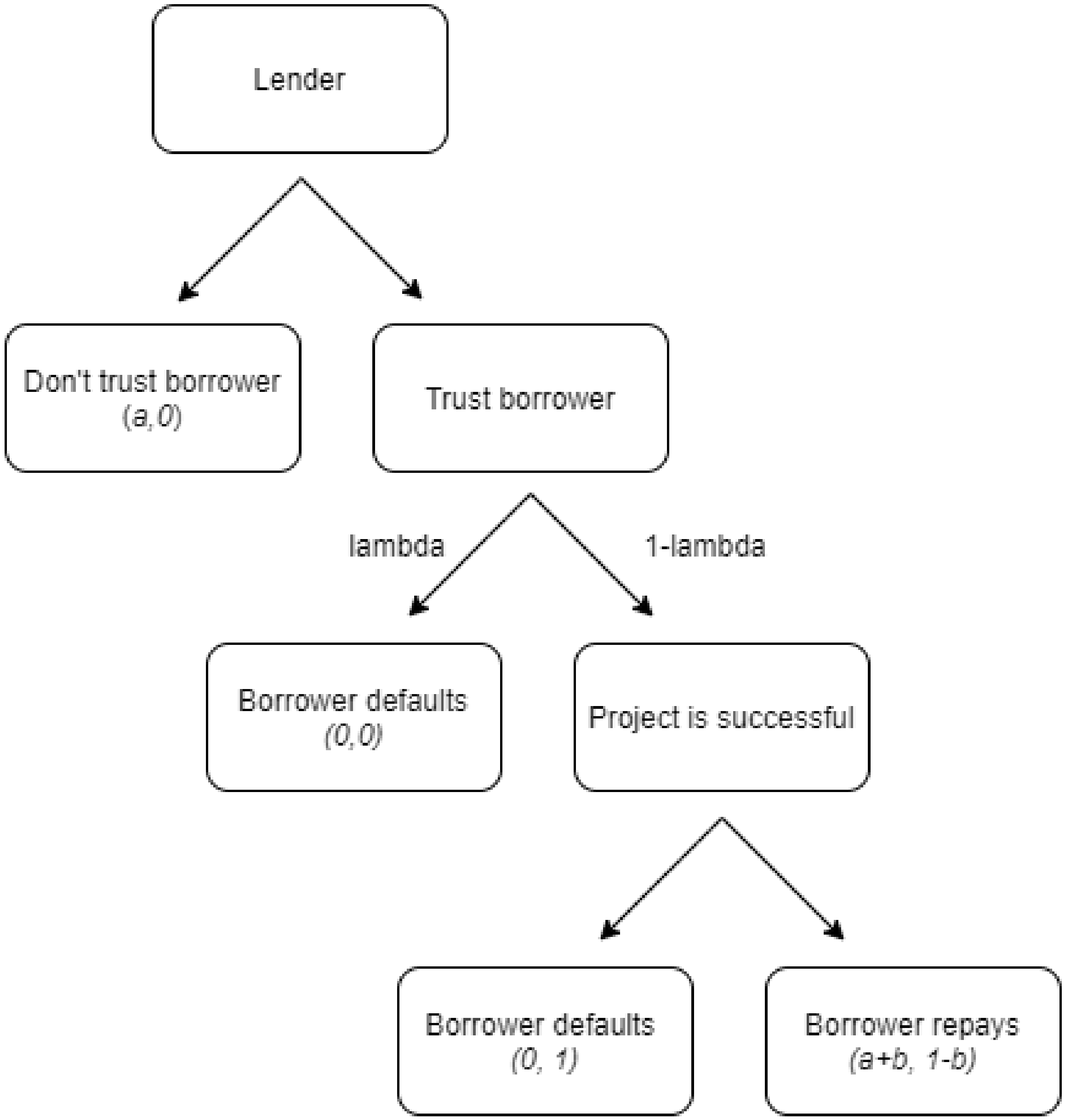
Question 1

After Rowena divides the cake into two pieces, Colin will choose the larger slice of cake, so Rowena will be stuck with the smaller slice of cake. Knowing this, Rowena will make the smaller slice of cake as large as possible, and consequently the larger slice of cake as small as possible, in order to maximize her utility. The largest the small slice can be without becoming the larger slice (which would then be chosen by Colin) is by splitting the cake exactly in half.

*I have discussed this problem set with Emily Case, Michael Nattinger, Alex Von Hafften, and Danny Edgel.

Question 2



We can deduce the SPE using backwards induction. If the project occurs and does not fail, the lender has no reason to repay the investment since they are strictly better off by defaulting on their loan. So when the investor looks at the game tree, there is an expected utility of 0 from investing, and an expected utility of a from not investing. So the subgame perfect equilibrium is for the investor to not invest.

Question 3

(a)

For $n = 1$, when the coin is flipped the person who wins the coin toss can either accept the offer or reject the offer. Since accepting the offer is weakly better than rejecting the offer (which has no payoff), the person who wins the coin toss should always accept the offer. Knowing this, the person dividing the dollar will offer 0 to the other person and 1 for themself. Since there is a 50% chance of winning the coin toss, there is a 50% chance of winning the full dollar, so the expected utility is \$0.50.

(b)

For $n = 2$, if the offer is declined in the first round, then the expected payoff is 0.5δ. So in the first round, the person who wins the coin toss will accept offers that are weakly greater than 0.5δ. Knowing this, the person dividing the dollar will offer 0.5δ, leaving $1 - 0.5\delta$. So the expected utility is $0.5(1 - 0.5\delta) + 0.5(0.5\delta) = 0.5$

(c)

Similar to (b), the expected utility of rejecting and continuing to the next round is 0.5 for any $n \geq 3$. So the person who divides the dollar should offer offer 0.5δ, leaving $1 - 0.5\delta$ for themselves. The player who wins the coin toss should accept because this offer will always leave them weakly better off than rejecting the offer.

Question 4

We can use backward induction by to determine the consumers' choice of coffee shop given x_1, x_2, p_1, and p_2, the pricing decision by the coffee shops given x_1 and x_2, and the location decision by the coffee shops.

First consider the case where both coffee shops pick the same location. If $x_i = x_j$ and $p_i < p_j$, then coffee shop i serves all of the costumers and coffee shop j serves no one. If $x_1 = x_2$ and $p_1 = p_2$, then each coffee shop serves coffee to half of the customers because everyone along the $[0, 1]$ interval is indifferent between going to either shop and flips a coin. In this case, each coffee shop has the incentive to lower prices to serve the entire unit mass until $p_1 = p_2 = 0$.

Next, consider the case when coffee shops choose different locations. Without loss of generality, assume that $x_1 < x_2$. So consumers are indifferent between coffee shop 1 and 2 if:

$$
\begin{aligned}
c(w-x_1)^2+p_1 &= c(w-x_2)^2+p_2 \\
\Rightarrow c(w^2-2x_1w+x_1^2)+p_1 &= c(w^2-2x_2w+x_2^2)+p_2 \\
\Rightarrow cw^2-2cx_1w+cx_1^2+p_1 &= cw^2-2cx_2w+cx_2^2+p_2 \\
\Rightarrow 2cw(x_2-2cx_1) &= c(x_2^2-x_1^2)+p_2-p_1 \\
\Rightarrow \bar{w} &= \frac{c(x_2^2-x_1^2)+p_2-p_1}{2c(x_2-x_1)}
\end{aligned}
$$

Coffee shop 1 serves the consumers below the cutoff $w \in [0, \bar{w})$, and coffee shop 2 serves the consumers above the cutoff $w \in [\bar{w}, 1]$. Since consumers are uniformly distributed, the quantity of consumers served by coffee shop 1 is $\bar{w}$ and the quantity of consumers served by coffee shop 2 is $1-\bar{w}$.

Next, consider the pricing decision by by each coffee shop given x_1 and x_2. The total revenue for coffee shop 1 is:

$$
p_1\bar{w} = \frac{p_1c(x_2^2-x_1^2)+p_1p_2-p_1^2}{2c(x_2-x_1)}
$$

Taking first order conditions with respect to p_1, we see:

$$
\begin{aligned}
0 &= \frac{c(x_2^2-x_1^2)+p_2-2p_1}{2c(x_2-x_1)} \\
\Rightarrow p_1^*(x_1,x_2,p_2) &= \frac{c(x_2^2-x_1^2)+p_2}{2}
\end{aligned}
$$

So the total revenue for coffee shop 2 is:

$$
\begin{aligned}
p_2(1-\bar{w}) &= p_2(1-\frac{c(x_2^2-x_1^2)+p_2-p_1}{2c(x_2-x_1)}) \\
&= p_2 - \frac{p_2c(x_2^2-x_1^2)+p_2^2-p_1p_2}{2c(x_2-x_1)}
\end{aligned}
$$

Taking first order conditions with respect to p_2, we see:

$$
\begin{aligned}
0 &= 1-\frac{c(x_2^2-x_1^2)+2p_2-p_1}{2c(x_2-x_1)} \\
\Rightarrow 2c(x_2-x_1) &= c(x_2^2-x_1^2)+2p_2-p_1 \\
\Rightarrow p_2^*(x_2,x_1,p_1) &= \frac{2c(x_2-x_1)-c(x_2^2-x_1^2)+p_1}{2}
\end{aligned}
$$

If both coffee shops anticipate that their competitor is best responding, we can see:

$$
\begin{aligned}
p_1^*(x_1,x_2) &= p_1^*(x_1,x_2,p_2^*(x_2,x_1,p_1^*)) \\
\Rightarrow p_1^* &= \frac{c(x_2^2-x_1^2)+\frac{2c(x_2-x_1)-c(x_2^2-x_1^2)+p_1^*}{2}}{2} \\
\Rightarrow \frac{3p_1^*}{4} &= \frac{c(x_2^2-x_1^2)}{4}+\frac{c(x_2-x_1)}{2} \\
\Rightarrow p_1^*(x_1,x_2) &= \frac{2c(x_2-x_1)}{3}+\frac{c(x_2^2-x_1^2)}{3}
\end{aligned}
$$

$$
\begin{aligned}
p_2^*(x_1,x_2) &= p_2^*(x_1,x_2,p_1^*) \\
&= \frac{2c(x_2-x_1)-c(x_2^2-x_1^2)+\frac{c(x_2^2-x_1^2)}{3}+\frac{2c(x_2-x_1)}{3}}{2} \\
&= \frac{4c(x_2-x_1)}{3}-\frac{c(x_2^2-x_1^2)}{3}
\end{aligned}
$$

Simplifying $\bar{w}$, we can see:

$$
\begin{aligned}
\bar{w} &= \frac{c(x_2^2-x_1^2)+\frac{4c(x_2-x_1)}{3}-\frac{c(x_2^2-x_1^2)}{3}-(\frac{2c(x_2-x_1)}{3}+\frac{c(x_2^2-x_1^2)}{3})}{2c(x_2-x_1)} \\
&= \frac{c(x_2^2-x_1^2)+\frac{2c(x_2-x_1)}{3}-\frac{2c(x_2^2-x_1^2)}{3}}{2c(x_2-x_1)} \\
&= \frac{2c(x_2-x_1)+c(x_2^2-x_1^2)}{6c(x_2-x_1)}
\end{aligned}
$$

$$
1-\bar{w} = 1-\frac{2c(x_2-x_1)+c(x_2^2-x_1^2)}{6c(x_2-x_1)} = \frac{c(x_2^2-x_1^2)-4c(x_2-x_1)}{6c(x_2-x_1)}
$$

Thus, the total revenue of coffee shop 1 given x_1 and x_2 is:

$$
\begin{aligned}
p_1^*\bar{w} &= \left(\frac{2c(x_2-x_1)}{3}+\frac{c(x_2^2-x_1^2)}{3}\right)\frac{2c(x_2-x_1)+c(x_2^2-x_1^2)}{6c(x_2-x_1)} \\
&= \frac{1}{18}c(x_1+x_2+2)^2(x_2-x_1)
\end{aligned}
$$

The total revenue for coffee shop 2 given x_1 and x_2 is:

$$
\begin{aligned}
p_2^*(1-\bar{w}) &= (\frac{4c(x_2-x_1)}{3}-\frac{c(x_2^2-x_1^2)}{3})\frac{c(x_2^2-x_1^2)-4c(x_2-x_1)}{6c(x_2-x_1)} \\
&= \frac{1}{18}c(x_1+x_2-4)^2(x_1-x_2)
\end{aligned}
$$

Next, consider the location decision. Since neither coffee shop knows where they will be the higher or lower coffee shop when choosing their location, we can set away from the assumption that $x_1 < x_2$. For both of the total revenue functions, since the maximum is outside the unit interval, the coffee shops will choose a corner solution. Thus, the subgame perfect equilibrium is one coffee shop at zero and the other coffee shop at 1 with both coffee shops charging c:

$$p_1^*(0,1) = \frac{2c(1-0)}{3} + \frac{c(1^2-0^2)}{3} = c$$
$$p_2^*(0,1) = \frac{4c(1-0)}{3} - \frac{c(1^2-0^2)}{3} = c$$