Econ 714A Problem Set 3

Sarah Bass *

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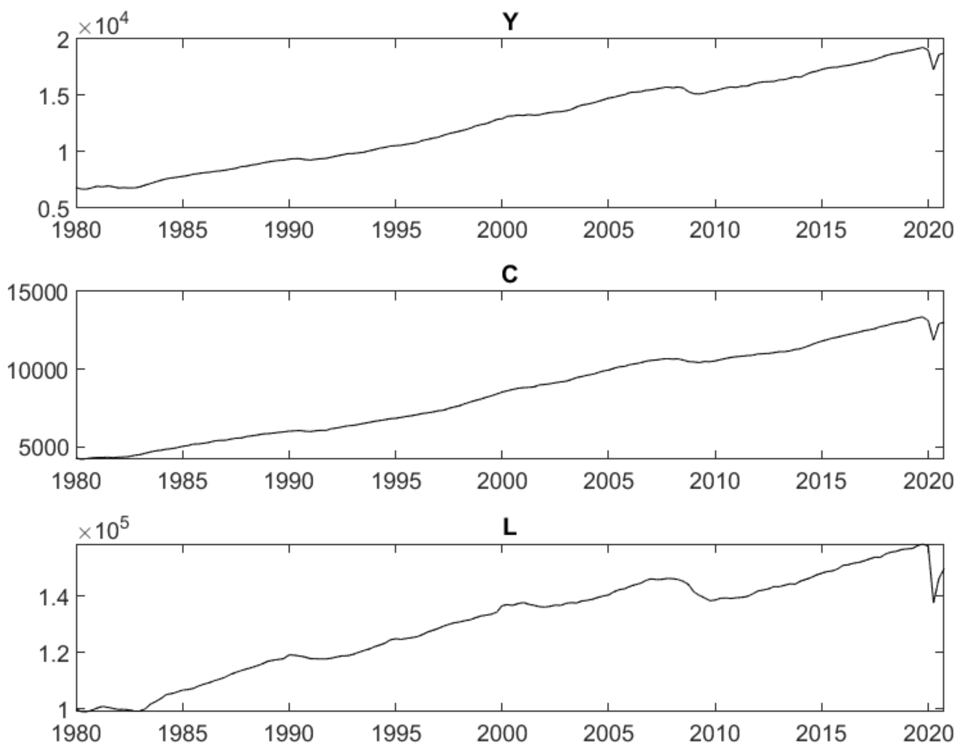
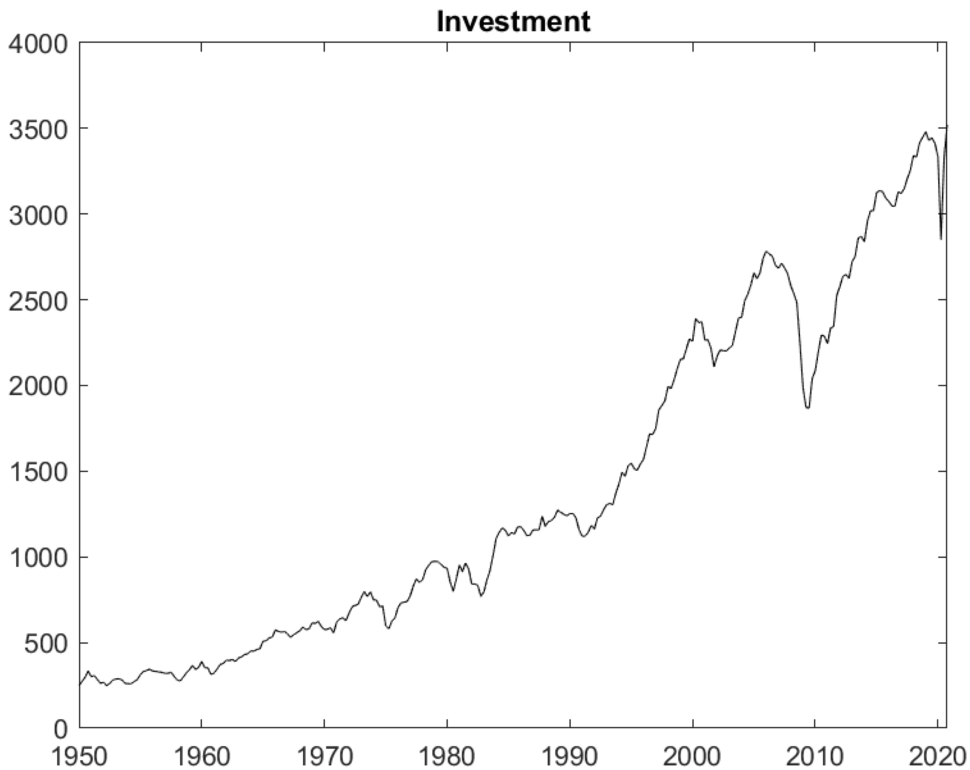
Question 1

I have pulled the data from the FRED website, and the graphs below show the time trends for the key variables.

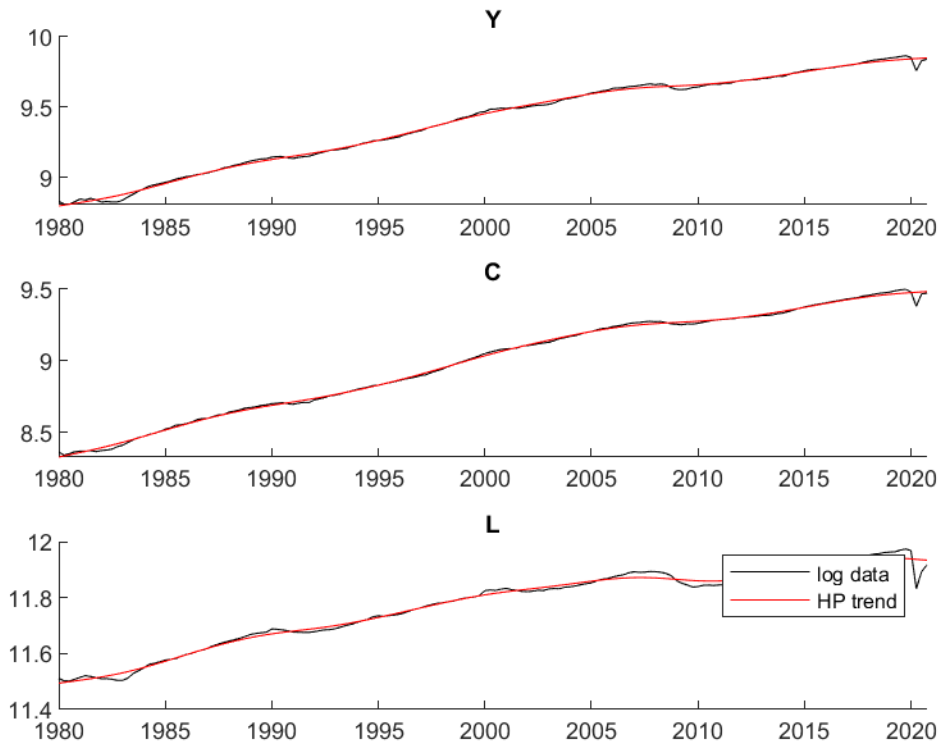


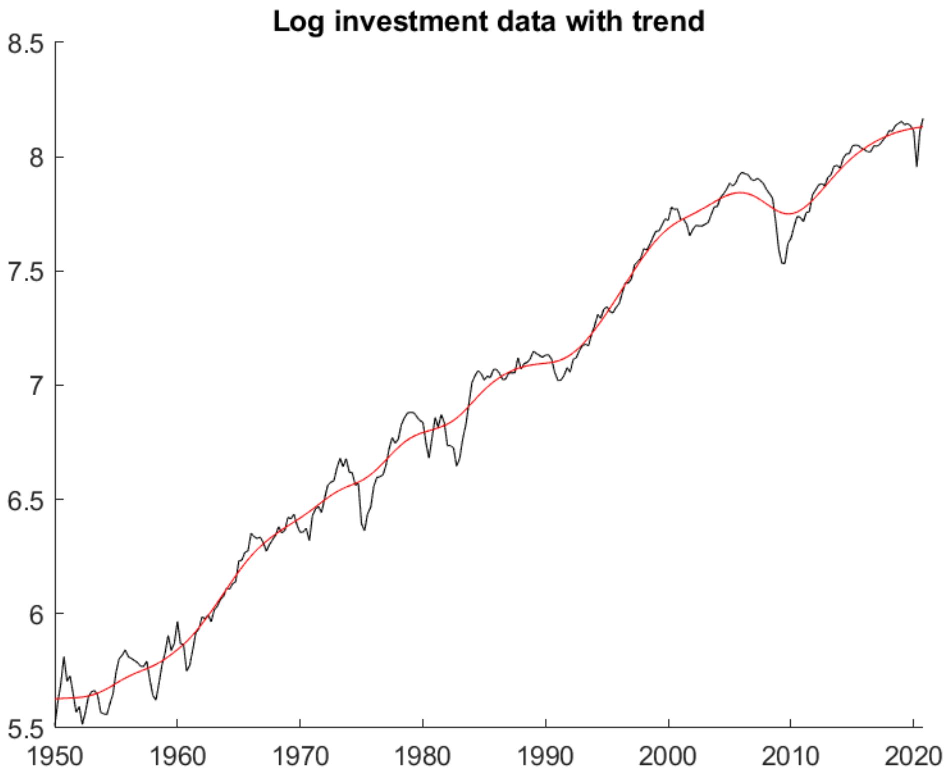
*I have discussed this problem set with Emily Case, Michael Nattinger, Alex Von Hafften, Katherine Kwok, and Danny Edgel.



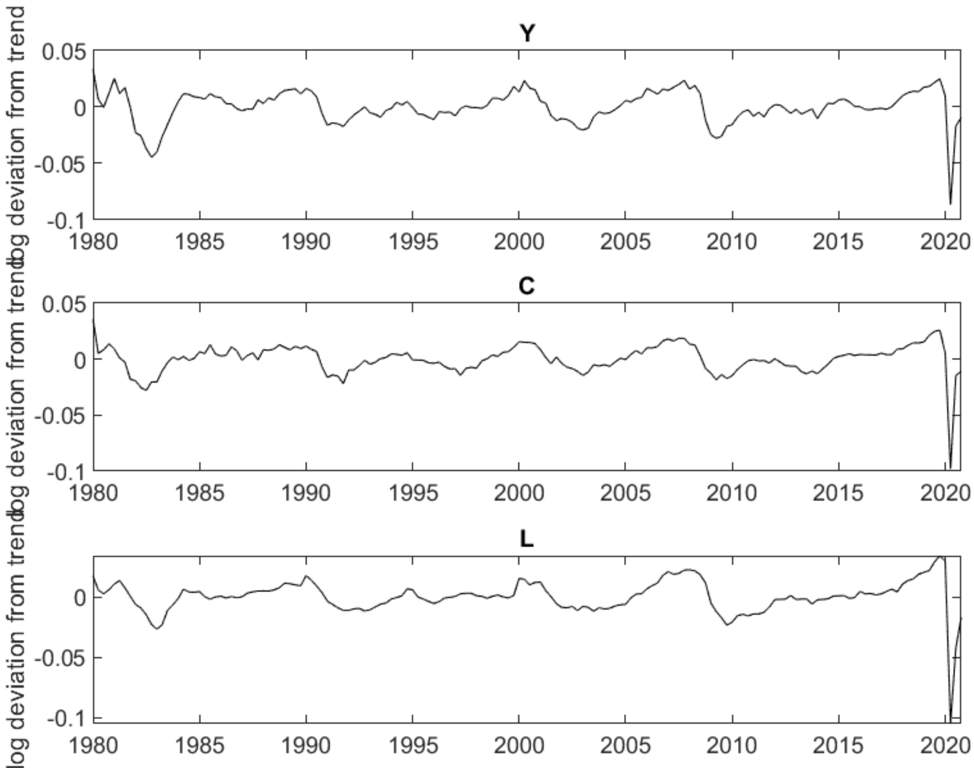
Question 2

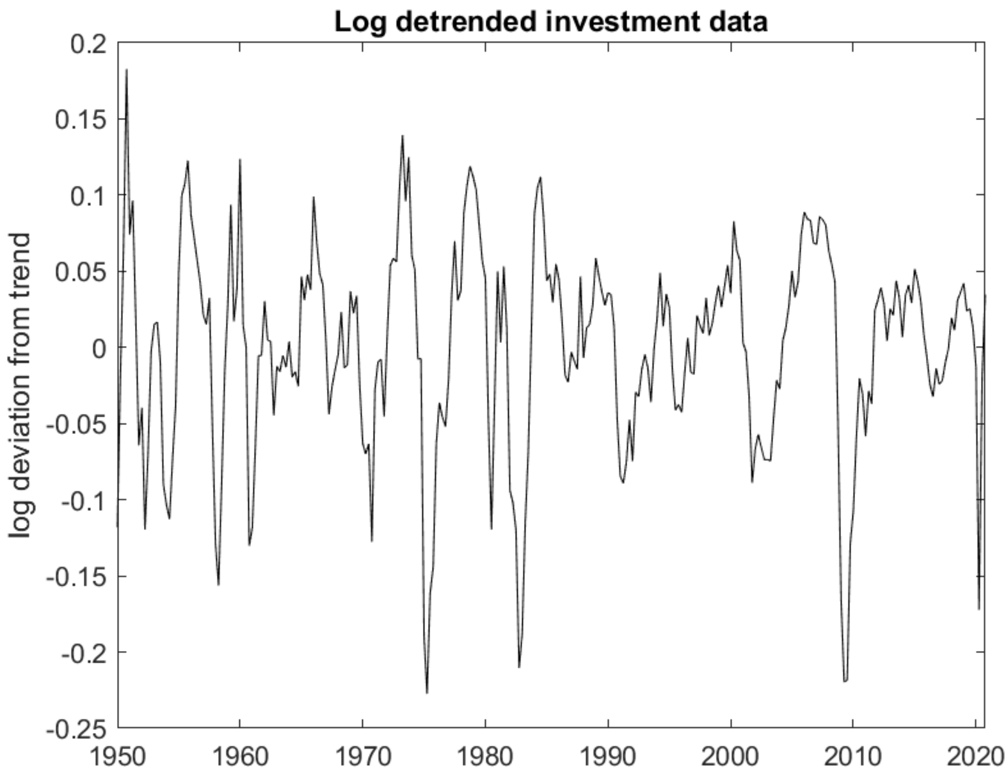
We next log the data, and extract an HP filter (smoothing parameter 1600) from the logged data. The filtered data is shown in the graphs below.


Y
10
9.5
9
1980
1985
1990
1995
2000
2005
2010
2015
2020
C
9.5
9
8.5
1980
1985
1990
1995
2000
2005
2010
2015
2020
L
12
11.8
11.6
11.4
1980
1985
1990
1995
2000
2005
2010
2015
2020
log data
HP trend



Next, we've detrended the data. This is shown in the graphs below.


Y
log deviation from trend
0.05
0
-0.05
-0.1
1980
1985
1990
1995
2000
2005
2010
2015
2020
C
log deviation from trend
0.05
0
-0.05
-0.1
L
log deviation from trend
0
-0.05
-0.1



Question 3

The log linearized capital law of motion is:

$$
\begin{aligned}
K_{t+1} &= (1-\delta)K_t + I_t \\
\bar{I} &= \delta \bar{K} \\
\bar{K}(1+k_{t+1}) &= \bar{K}(1-\delta)(1+k_t) + \bar{I}(1+i_t) \\
\bar{K}k_{t+1} &= \bar{K}(1-\delta)k_t + \bar{I}i_t \\
k_{t+1} &= (1-\delta)k_t + \delta i_t
\end{aligned}
$$

Next, we can iteratively decompose the law of motion:

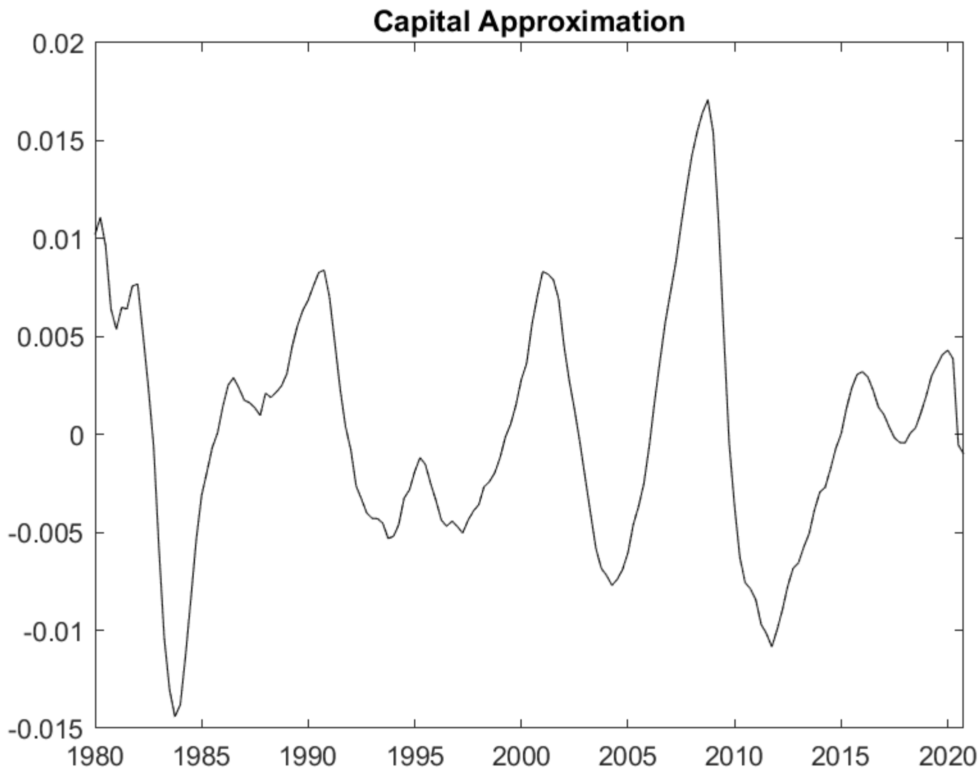
$$\begin{aligned}
k_{t+1} &= (1-\delta)k_t + \delta i_t \\
&= \delta i_t + (1-\delta)((1-\delta)k_{t-1} + \delta i_{t-1}) \\
&= \delta(i_t + (1-\delta)i_{t-1}) + (1-\delta)^2((1-\delta)k_{t-2} + \delta i_{t-2}) \\
&= \delta \sum_{j=0}^{J} (1-\delta)^j i_{t-j} + (1-\delta)^{J+1} k_{t-J}
\end{aligned}$$

We have 30 years of quarterly investment data before our capital series starts (120 observations). Although we do not know our capital deviation from trend in 1950, this term has very little impact on our capital deviation from trend in 1980.

$$k_{1980Q1} = \delta \sum_{j=0}^{119} (1-\delta)^j i_{1979Q4-j} + (1-\delta)^{120} k_{1950Q1}$$

Note that the k_{1950Q1} term above is multiplied by $(1-\delta)^{120}$. Given $\delta = 0.025$, $(1-\delta)^{120} = 0.048$. As a result, our capital level in the first quarter of 1950 accounts for less than 5 percent of our capital level at the beginning of 1980, and even less as we get later in the sample. Since cross-terms in the log linearization are approximately zero, this term is on the same order of magnitude and can also be approximated to zero.

I implemented the perpetual inventory method to estimate the capital stock over 1980-2020. This is shown in the graphs below.



Question 4

Our equilibrium consists of the following equations:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$
$$Y_t = C_t + I_t + G_t \tag{2}$$
$$L_t^{\phi} C_t^{\sigma} = (1-\tau_{L,t}) A_t (1-\alpha) K_t^{\alpha} L_t^{-\alpha} \tag{3}$$
$$C_t^{-\sigma}(1+\tau_{I,t}) = \beta E_t[C_{t+1}^{-\sigma}[A_{t+1}\alpha K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + (1-\delta)(1+\tau_{I,t+1})]] \tag{4}$$

First, we will linearize the production function (1):

$$y_t = a_t + \alpha k_t + (1-\alpha) l_t. \tag{5}$$

Next, our market clearing condition (2):

$$y_t = \frac{\bar{C}}{\bar{Y}} c_t + \frac{\bar{I}}{\bar{Y}} i_t + \frac{\bar{G}}{\bar{Y}} g_t \tag{6}$$

Then, our labor supply equation (3). Since $\bar{\tau_I}, \bar{\tau_L}$ equal 0, $\tau_{I,t}, \tau_{L,t}$ are linearized and not log linearized:

$$
\begin{aligned}
\bar{L}^{\phi}\bar{C}^{\sigma}(1+\sigma c_t+\phi l_t) &= \bar{X}\bar{A}(1-\alpha)\bar{K}^{\alpha}\bar{L}^{-\alpha}(1+x_t+a_t+\alpha k_t-\alpha l_t) \\
\sigma c_t+\phi l_t &= x_t+a_t+\alpha k_t-\alpha l_t \\
\bar{X}(1+x_t) &= 1-\hat{\tau}_{L,t} \\
x_t &= \hat{\tau}_{L,t}
\end{aligned}
$$

$$
\phi l_t+\sigma c_t = -\hat{\tau}_{L,t}+\alpha k_t-\alpha l_t \tag{7}
$$

Finally, we linearize our Euler equation:

$$
\begin{aligned}
\bar{C}(1-\sigma c_t)\bar{Y}(1+y_t) &= \beta E_t[\bar{C}(1-\sigma c_{t+1})\bar{X}(1+x_{t+1})] \\
\sigma(E_t[c_{t+1}]-c_t)+y_t &= E_t[x_{t+1}], \\
Y_t &= 1+\tau_{I,t} \\
\bar{Y}(1+y_t) &= 1+\hat{\tau}_{I,t} \\
y_t &= \hat{\tau}_{I,t} \\
\bar{X}(1+x_t) &= \bar{A}(1+a_t)\alpha\bar{K}^{\alpha-1}(1+(\alpha-1)k_t)\bar{L}_t^{1-\alpha}(1+(1-\alpha)l_t)+(1-\delta)\bar{Y}(1+y_t) \\
\bar{X}x_t &= \bar{A}\alpha\bar{K}^{\alpha-1}\bar{L}^{1-\alpha}(a_t+(1-\alpha)(l_t-k_t))+(1-\delta)\bar{Y}y_t \\
x_t &= (\bar{A}\alpha\bar{K}^{\alpha-1}\bar{L}^{1-\alpha}(a_t+(1-\alpha)(l_t-k_t))+(1-\delta)y_t)/\bar{X} \\
\sigma(E_t[c_{t+1}]-c_t)+\frac{\bar{\tau}_I}{1+\bar{\tau}_I}\hat{\tau}_{I,t} &= E_t[(\bar{A}\alpha\bar{K}^{\alpha-1}\bar{L}^{1-\alpha}(a_{t+1}+(1-\alpha)(l_{t+1}-k_{t+1}))+(1-\delta)y_{t+1})/\bar{X}]
\end{aligned}
$$

$$
\begin{aligned}
\bar{X} &= \beta^{-1} \\
\sigma(E_t[c_{t+1}]-c_t)+\hat{\tau}_{I,t} &= \beta E_t[(\bar{A}\alpha\bar{K}^{\alpha-1}\bar{L}^{1-\alpha}(a_{t+1}+(1-\alpha)(l_{t+1}-k_{t+1}))+(1-\delta)y_{t+1})]
\end{aligned}
$$

$$
\sigma(E_t[c_{t+1}]-c_t)+\hat{\tau}_{I,t} = \beta E_t\left[\alpha\bar{A}\bar{K}^{\alpha-1}\bar{L}^{1-\alpha}(a_{t+1}+(1-\alpha)(-k_{t+1}+l_{t+1}))+(1-\delta)\hat{\tau}_{I,t+1}\right] \tag{8}
$$

Using the data for y_t, k_t, l_t, i_t we can back out $a_t, g_t, \hat{\tau}_{L,t}$. First, we will solve the following system of equations for steady state values:

$$
\begin{aligned}
\bar{Y} &= \bar{A}\bar{K}^{\alpha}\bar{L}^{1-\alpha} \\
\bar{Y} &= \bar{C}+\bar{I}+\bar{G} \\
\bar{L}^{\phi}\bar{C}^{\sigma} &= (1-\bar{\tau}_L)\bar{A}(1-\alpha)\bar{K}^{\alpha}\bar{L}^{-\alpha} \\
(1+\bar{\tau}_I) &= \beta[\bar{A}\alpha\bar{K}^{\alpha-1}\bar{L}^{1-\alpha}+(1-\delta)(1+\bar{\tau}_I)]
\end{aligned}
$$

I have solved this system in Matlab using the symbolic toolbox and estimated the following shock persistencies:

	ρ
a	0.72622
g	0.85589
τ_L	0.57029
τ_I	0.69941

Questions 5,6,7, and 8

Now that we have steady state values, we can rewrite the question as a fixed point problem and proceed with the Blanchard-Kahn method. First we solve for labor in terms of consumption, capital, and $\hat{\tau}$ via equation (7):

$$\begin{aligned}
\phi l_t + \sigma c_t &= -\hat{\tau}_{L,t} + \alpha k_t - \alpha l_t \\
\Rightarrow l_t &= \frac{-\sigma}{\phi+\alpha} c_t + \frac{\alpha}{\phi+\alpha} k_t + \frac{-\bar{\tau}_L}{(\alpha+\phi)(1-\bar{\tau}_L)} \hat{\tau}_{L,t}
\end{aligned}$$

We next solve for investment in terms of consumption, capital, and labor using (5) and (6) :

$$\begin{aligned}
y_t &= a_t + \alpha k_t + (1-\alpha) l_t \\
y_t &= \frac{\bar{C}}{\bar{Y}} c_t + \frac{\bar{I}}{\bar{Y}} i_t + \frac{\bar{G}}{\bar{Y}} g_t \\
\Rightarrow i_t &= (\bar{Y}/\bar{I})(a_t + \alpha k_t + (1-\alpha) l_t - (\bar{C}/\bar{Y}) c_t - (\bar{G}/\bar{Y}) g_t)
\end{aligned}$$

Using these, we can rewrite our law of motion of capital:

$$k_{t+1} = (1-\delta) k_t + \delta (\bar{Y}/\bar{I})(a_t + \alpha k_t + (1-\alpha) l_t - (\bar{C}/\bar{Y}) c_t - (\bar{G}/\bar{Y}) g_t)$$

Next we can solve analytically for $E_t c_{t+1}$ using the Euler equation. I have solved for this using Matlab.

With that solution, we can write our log linearized law of motion:

$$E_t X_{t+1} = E_t \begin{pmatrix} k_{t+1} \\ c_{t+1} \end{pmatrix} = A X_t + B Z_t = A \begin{pmatrix} k_t \\ c_t \end{pmatrix} + B \begin{pmatrix} a_t \\ g_t \\ \hat{\tau}_{L,t} \\ \hat{\tau}_{I,t} \end{pmatrix}$$

Decomposing A:

$$\begin{aligned}
E_t X_{t+1} &= Q \Lambda Q^{-1} X_t + B Z_t \\
E_t Y_{t+1} &= E_t Q^{-1} X_{t+1} = \Lambda Y_t + C Z_t = \Lambda Q^{-1} X_t + Q^{-1} B Z_t
\end{aligned}$$

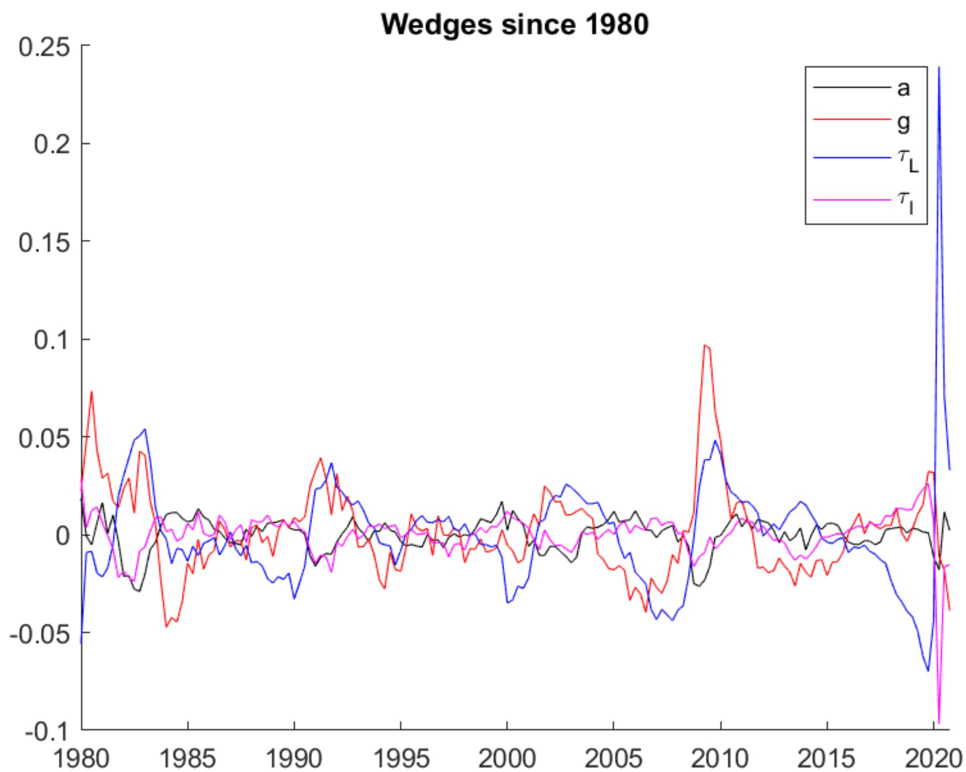
One of the eigenvalues is greater than 1. Without loss of generality, we will call it λ_1. We iterate Y_1 forward:

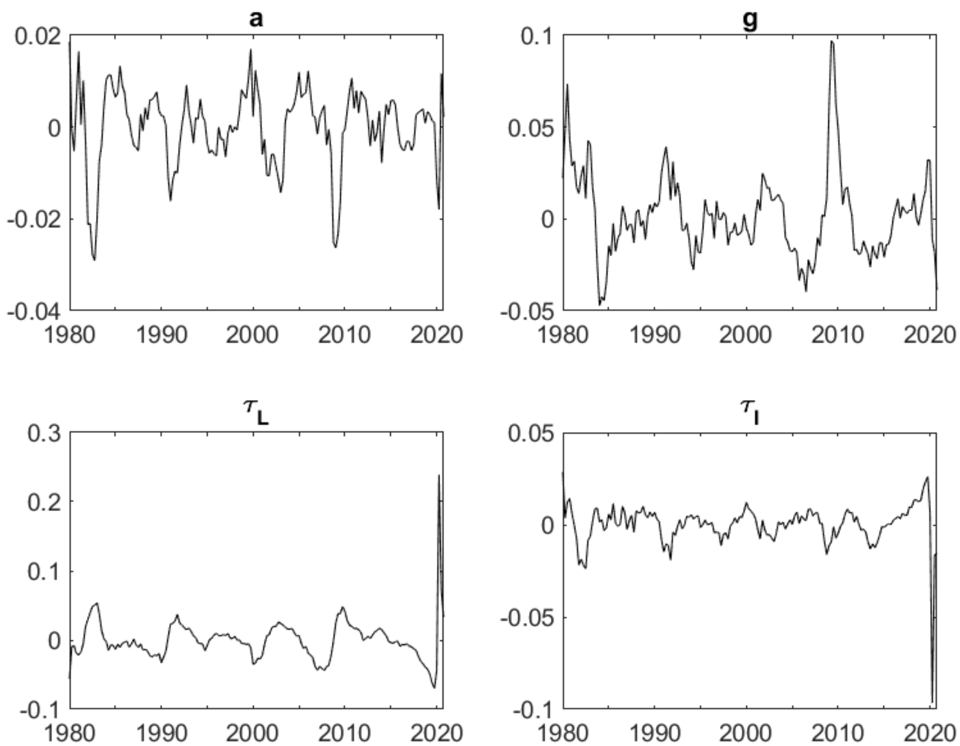
$$\begin{aligned}
Y_{1,t} &= -\lambda_1^{-1} C_1 Z_t + \lambda_1^{-1} E_t Y_{1,t+1} \\
&= -\lambda_1^{-1} C_1 \sum_{j=0}^{\infty} \lambda_1^{-j} E_t Z_{t+j} + \lim_{j\to\infty} \lambda_1^{-j} y^1_{t+j} \\
&= -\lambda_1^{-1} C_1 \sum_{j=0}^{\infty} \lambda_1^{-j} \rho^j Z_t \\
&= -\lambda_1^{-1} C_1 (I_4 - \lambda_1^{-1} \rho)^{-1} Z_t \\
&= \Theta Z_t.
\end{aligned}$$

where ρ is a 4×4 matrix with the shock persistences on the main diagonal and zeroes elsewhere. Let $Q^{-1} = \begin{pmatrix} q_{1,1} & q_{1,2} \\ q_{2,1} & q_{2,2} \end{pmatrix}$. Then we can derive a formula for $\hat{\tau}_{I,t}$:

$$\begin{aligned} q_{1,1}k_t + q_{1,2}c_t &= \Theta_1 a_t + \Theta_2 g_t + \Theta_3 \hat{\tau}_{L,t} + \Theta_4 \hat{\tau}_{I,t} \\ \Rightarrow \hat{\tau}_{I,t} &= \frac{q_{1,1}k_t + q_{1,2}c_t - (\Theta_1 a_t + \Theta_2 g_t + \Theta_3 \hat{\tau}_{L,t})}{\Theta_4}. \end{aligned}$$

With this formula, I can now calculate all of the wedges. The wedges are shown in the graphs below.

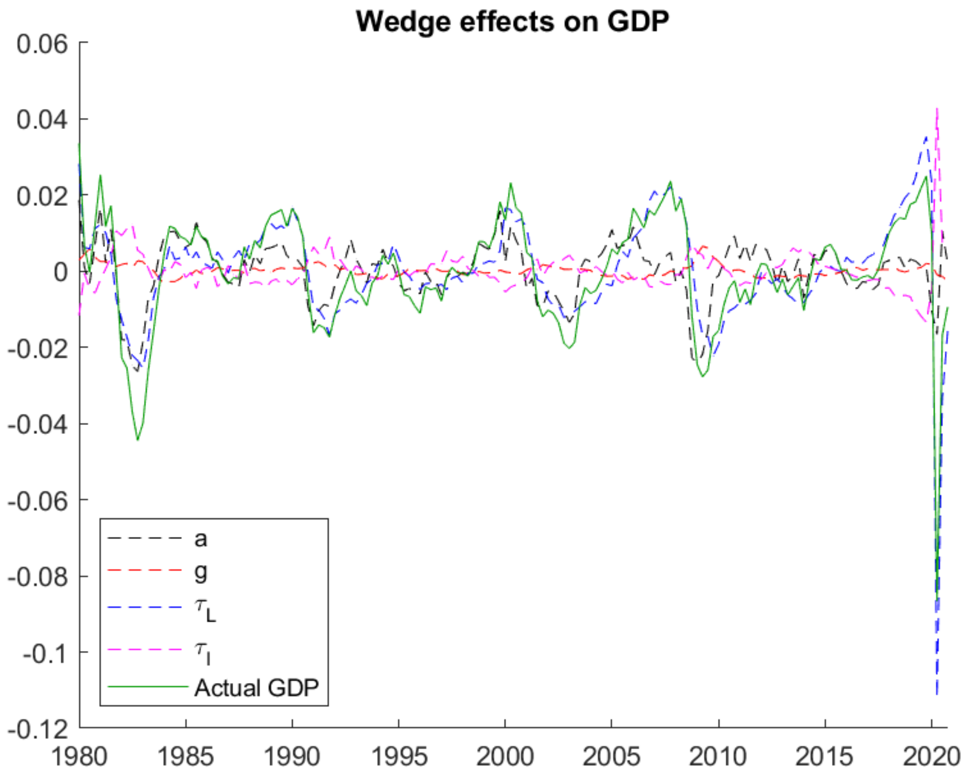




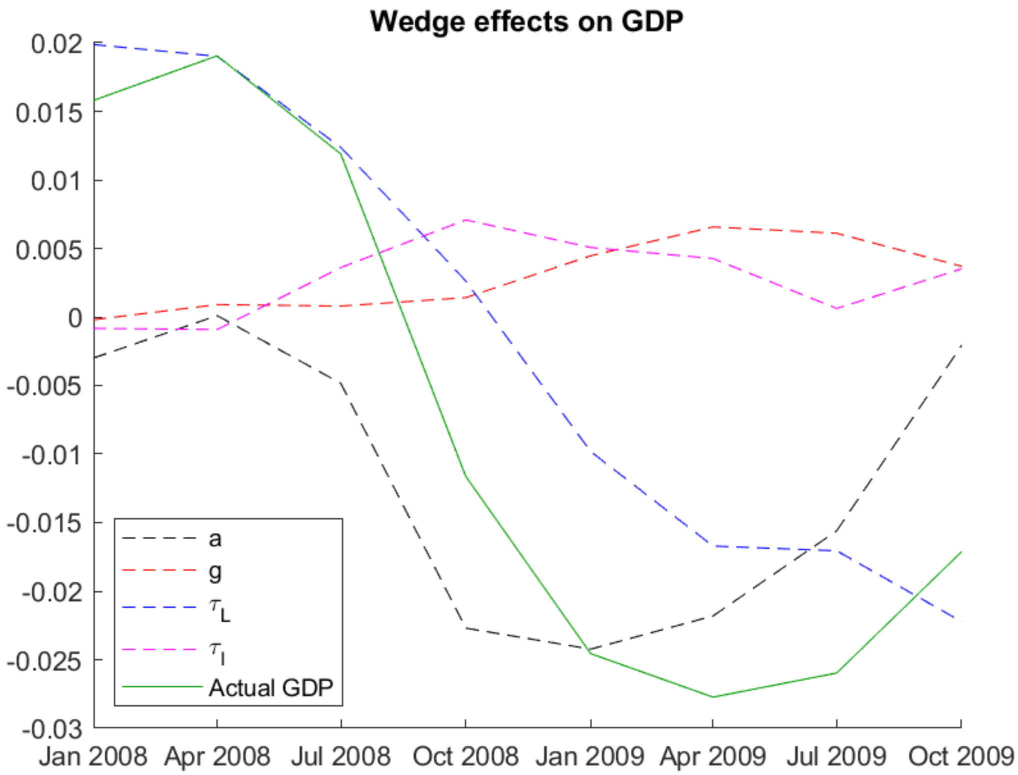
Next We can solve for each wedge separately and calculate the GDP using these wedges. We can calculate these using formulas we derived earlier:

$$q_{1,1}k_t + q_{1,2}c_t = \Theta_1 a_t + \Theta_2 g_t + \Theta_3 \hat{\tau}_{L,t} + \Theta_4 \hat{\tau}_{I,t}$$
$$\Rightarrow c_t = \frac{-q_{1,1}k_t + \Theta Z_t}{q_{1,2}}$$

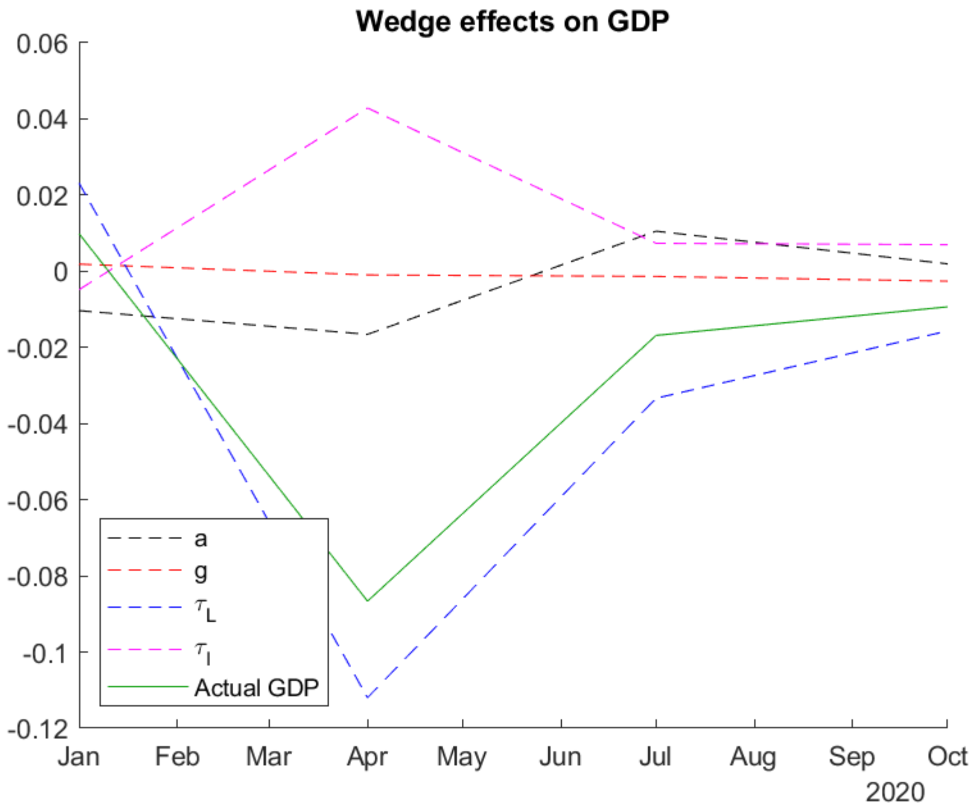
The figure below shows the counterfactual GDP calculated using only one of the wedge shocks at a time. As we can see, the a and τ_L track closely with GDP until the early 2000s. After this point, τ_L continues to track very closely with GDP, but a diverges some. Also note that there appears to be an inverse relationship between τ_I and g with GDP - periods of contracted GDP growth also tend to see more investment activity and more government spending.



The next figure shows the counterfactual GDP calculated using only one wedge shock at a time, focusing on the time period surround the Great Recession of 2009. The model matches intuition that labor and productivity shocks would be large drivers of the economy during the Great Recession, given that there was mass unemployment and reductions in productivity.



The next figure shows the counterfactual GDP calculated using only one wedge shock at a time, focusing on the time period surround the Great Lockdown of 2020. The model matches intuition that a labor shock would be a large driver of the economy during the Covid pandemic, given that Covid caused mass unemployment that is not accounted for elsewhere in the model.



Wedge effects on GDP
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
2020
a
g
τ_L
τ_I
Actual GDP