Problem set 5

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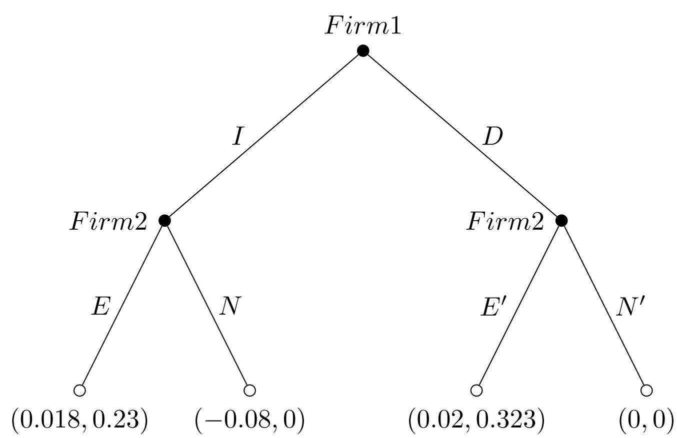
1. (a) The sequential game of this interaction between incumbent and entrant is shown below:



To compute the payoffs of this game first we find the best response of Firm 2 $p_2(p_1)$ and then plug it into the profits of Firm 1 to find its choice of price. The best response of firm 2 is:

$$max_{p_2} \quad \pi_2 = (1 - 2p_2 + p_1)p_2$$
$$p_2 = \frac{1 + p_1}{4}$$

Given this, firm 1's best response and payoffs to both players in each final node are:

- (I,E)

$$max_{p_1} \quad \pi_1 = \left(1 - 2p_1 + \frac{1 + p_1}{4}\right) p_1 - 0.205$$
$$p_1 = \frac{5}{14}, \quad p_2 = \frac{19}{56}$$
$$\pi_1 = 0.018, \quad \pi_2 = 0.23$$

- (D,E)

$$max_{p_1} \quad \pi_1 = \left(1 - 2p_1 + \frac{1+p_1}{4}\right)(p_1 - 0.5)$$
$$p_1 = \frac{17}{28}, \quad p_2 = \frac{45}{112}$$
$$\pi_1 = 0.0201, \quad \pi_2 = 0.323$$

- (I,N)

$$max_{p_1} \quad \pi_1 = (1 - 2p_1)\, p_1 - 0.205$$
$$p_1 = \frac{1}{4}, \quad p_2 = 0$$
$$\pi_1 = -0.08, \quad \pi_2 = 0$$

- (D,N)

$$max_{p_1} \quad \pi_1 = (1 - 2p_1)\,(p_1 - 0.5)$$
$$p_1 = \frac{1}{2}, \quad p_2 = 0$$
$$\pi_1 = 0, \quad \pi_2 = 0$$

By backward induction, there is a unique subgame perfect equilibrium $\{D, (EE')\}$. Therefore, Firm 1 does not invest.

(b) When the entrant does not observe the incumbent's investment choice, this becomes a simultaneous-move game. The solution concept is then Nash equilibrium and not subgame perfection. To determine whether a strategy profile is an equilibrium we need to check that there are no profitable one-shot deviations.

In this game a strategy for player 1 is

$$s_1 \in \{(I, p) | I = \mathbf{1}\{Invest > 0\}, p\{0, 1\} \to \mathcal{R}_+]\}$$

And for player 2:

$$s_2 \in p : \mathcal{R}_+ \to \mathcal{R}_+$$

(c) The best response of firm 2 is:

$$max_{p_2} \quad \pi_2 = (1 - 2p_2 + p_1)p_2$$
$$p_2 = \frac{1+p_1}{4}$$

Given this, firm 1's best response and payoffs to both players in each final node are:

- (I,E)

$$
\begin{aligned}
&max_{p_1} \quad \pi_1 = (1 - 2p_1 + p_2)\, p_1 - 0.205 \\
&p_1 = \frac{1 + p_2}{4} \\
&p_1 = p_2 = \frac{1}{3} \\
&\pi_1^{eq} = 0.0172, \quad \pi_2 = 2/9
\end{aligned}
$$

Now consider a one-shot deviation for firm 1 into not investing:

$$
\begin{aligned}
&max_{p_1} \quad \pi_1 = (1 - 2p_1 + 1/3)\,(p_1 - 0.5) \\
&p_1 = 7/12 \\
&\pi_1^{dev} = 1/72, \quad \pi_2 = 11/36
\end{aligned}
$$

Since $\pi_1^{eq} > \pi_1^{dev}$ then firm 1 investing is an equilibrium.

- (D,E)

$$
\begin{aligned}
&max_{p_1} \quad \pi_1 = (1 - 2p_1 + p_2)\,(p_1 - 0.5) \\
&p_1 = \frac{2 + p_2}{4} \\
&p_1 = \frac{3}{5}, \quad p_2 = \frac{2}{5} \\
&\pi_1^{eq} = 1/50, \quad \pi_2 = 8/25
\end{aligned}
$$

Now consider a one-shot deviation from firm 1 into investing:

$$
\begin{aligned}
&max_{p_1} \quad \pi_1 = (1 - 2p_1 + 2/5)\, p_1 - 0.205 \\
&p_1 = 7/20 \\
&\pi_1^{dev} = 0.04, \quad \pi_2 = 0.22
\end{aligned}
$$

Since $\pi_1^{dev} > \pi_1^{eq}$ then there exists a profitable deviation in which firm 1 invests.

- (I,N)

$$
\begin{aligned}
&max_{p_1} \quad \pi_1 = (1 - 2p_1)\, p_1 - 0.205 \\
&p_1 = \frac{1}{4}, \quad p_2 = 0 \\
&\pi_1 = -0.08, \quad \pi_2 = 0
\end{aligned}
$$

- (D,N)

$$
\begin{aligned}
&max_{p_1} \quad \pi_1 = (1-2p_1)(p_1-0.5) \\
&p_1 = \frac{1}{2}, \quad p_2 = 0 \\
&\pi_1 = 0, \quad \pi_2 = 0
\end{aligned}
$$

2. (a) The problem for firm i is:

$$max_{p_i, A_i} \quad (a-bp_i)\left(\frac{A_i}{\sum A_j}\right)(p_i - c) - A_i$$

So FOC with respect to price:

$$(1-2bp_i)\left(\frac{A_i}{\sum A_j}\right) = 0 \iff p_i = \frac{a+cb}{2b}$$

Which is just the monopoly price.

The FOC with respect to advertising expenditure is:

$$\frac{(a-cb)^2}{4b}\left(\frac{\sum_{j\neq i} A_j}{(\sum A_j)^2}\right) = 1$$

Under symmetric Nash equilibrium $A_i = A_j = A$, and we can solve for A from the previous expression:

$$
\begin{aligned}
\frac{(a-cb)^2}{4b}\left(\frac{(N-1)A}{N^2A^2}\right) &= 1 \\
A = \left(\frac{(a-cb)^2}{4b}\right)\frac{N-1}{N^2}
\end{aligned}
$$

(b) With free entry $\pi_i = 0$ which means:

$$
\begin{aligned}
\pi_i = \frac{(a-cb)^2}{4bN} - \left(\frac{(a-cb)^2}{4b}\right)\frac{N-1}{N^2} - F = 0 \\
\frac{(a-cb)^2}{4bN^2} = F \\
N = \frac{a-cb}{2\sqrt{bF}}
\end{aligned}
$$

(c) **Accomodation**: By backward induction, first Firm 2's best response is given by:

$$max_{p_2, A_2} \quad (a-bp_2)\left(\frac{A_2}{A_1+A_2}\right)(p_2 - c) - A_2$$

So FOC with respect to price yields:

$$p_2 = \frac{a + cb}{2b}$$

And FOC with respect to advertising expenditure is:

$$A_2 = \frac{(a - cb)}{2}\sqrt{\frac{A_1}{b}} - A_1$$

Plugging in this best response into firm 1's optimization problem we get:

$$max_{p_1, A_1} \quad (a - bp_1))(p_1 - c)\frac{2\sqrt{A_1 b}}{a - cb} - A_1$$

So the FOC with respect to price yields:

$$p_1 = \frac{a + cb}{2b}$$

and with respect to A_1 yields:

$$A_1^A = \frac{(a - cb)^2}{16b}$$

which in turn yields advertising expenditure for firm 2 of:

$$A_2 = \frac{(a - cb)^2}{16b}$$

And firm 1's payoff if it accommodates is:

$$\pi_1^A = \frac{(a - cb)^2}{16b}$$

(d) Under **deterrence** we want to find the advertising expenditure of firm 1 that makes firm 2 indifferent between entering and staying out given that firm 2 will best respond. This is:

$$\pi_2(A_1, A_2(A_1)) = 0$$

$$\frac{(a - cb)^2}{4b}\left(1 - \frac{4\sqrt{A_1 b}}{a - cb} + \frac{4bA_1}{(a - cb)^2}\right) = 0$$

$$A_1^D = \frac{(a - cb)^2}{4b}$$

So profits for firm 1 under deterrence are

$$\pi_1^D = 0$$

Therefore since $\pi_1^A > \pi_1^D$, firm 1 will never deter entry.