

Problem Set 2

Sarah Bass

1) cost: $k^2 + q + q^2$

$$\pi = pq - (k^2 + q + q^2) \quad \text{Take } p \text{ as given}$$

$$\text{FOC: } p - 1 - 2q = 0$$

$$q^* = \frac{p-1}{2}$$

The minimum production occurs if only $k=1$ produces.

$$\pi(1) = p \left(\frac{p-1}{2} \right) - 1 - \left(\frac{p-1}{2} \right) - \left(\frac{p-1}{2} \right)^2 = 0$$

$$p^2 - p - 2 - p + 1 - \left(\frac{p^2 - 2p + 1}{2} \right) = 0$$

$$p^2 - 2p + 1 - 4 = 0$$

$$p^2 - 2p - 3 = 0$$

$$(p-3)(p+1) = 0$$

We can't have a negative price, so $p=3$ is the minimum price at which there is non-zero production.

2a) Given γ , a firm will choose q that maximizes profits and will enter the market if $\pi \geq 0$.

$$\pi = (1-\gamma) \cdot 2\theta x - x^2 - 1 \quad \text{choose } x \text{ to max } \pi$$

$$\text{FOC: } 2\theta(1-\gamma) = 2x$$

$$\rightarrow x = \theta(1-\gamma)$$

$$\pi = 2\theta^2(1-\gamma)^2 - \theta^2(1-\gamma)^2 - 1 \geq 0$$

$$\theta^2(1-\gamma)^2 \geq 1$$

$$\theta(1-\gamma) \geq 1$$

$$\theta \geq \frac{1}{1-\gamma}$$

$$\text{Let } F = 1 - M(\theta). \text{ Then } f = -M'(\theta) = \beta \theta^{-\beta-1}$$

$$Q_2 = \int_{\frac{1}{1-\gamma}}^{\infty} 2(1-\gamma)\theta^2 \cdot (\beta \theta^{-\beta-1}) d\theta$$

$$= \int_{\frac{1}{1-\gamma}}^{\infty} 2\beta(1-\gamma)\theta^{1-\beta} d\theta$$

$$= \frac{2\beta(1-\gamma)}{2-\beta} \theta^{2-\beta} \Big|_{\frac{1}{1-\gamma}}^{\infty}$$

$$= 0 - \frac{2\beta(1-\gamma)}{\beta-2} \left(\frac{1}{1-\gamma}\right)^{2-\beta}$$

$$= \frac{2\beta(1-\gamma)^{\beta-1}}{\beta-2}$$

Note $\beta > 2$

2b) The cutoff to produce is $\theta = 1/(1-\tau)$, so if τ increases, fewer developers will produce. Further the production function is $q = 2\theta x$, where $x = \theta(1-\tau)$, so production developers that continue to produce when τ increases will produce a smaller quantity.

$$2c) R = \tau Q_3 = \tau \frac{2B(1-\tau)^{B-1}}{B-2}$$

$$\text{FOC: } \frac{2B(1-\tau)^{B-1}}{B-2} = \frac{2(B-1)\tau B(1-\tau)^{B-2}}{B-2}$$

$$1-\tau = \tau(B-1)$$

$$\tau^* = \frac{1}{B}$$

2d) When B rises, the distribution $m(\theta)$ changes so that fewer firms produce, and firms are less productive. As a result Q_3 will decrease.

$$3) \begin{array}{ll} P_1 = a_1 - b_1 Q & \text{Fruit Bowl} \\ P_2 = a_2 - b_2 Q & \text{Triple chocolate chunk} \end{array}$$

$a_1 > a_2$ b/c FB has higher price intercept than TCC

$P_1 = P_2$ at interior point

then $b_1 > b_2$

$$\pi_i = P_i Q_i - MC Q_i \quad \text{where } MC \text{ is constant}$$

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3) $\pi_1 = P_1 Q_1 - MC Q_1$ where MC is constant

$$= (a_1 - b_1 Q_1) Q_1 - MC Q_1$$

$$= a_1 Q_1 - b_1 Q_1^2 - MC Q_1$$

Foc: $a_1 - 2b_1 Q_1 - MC = 0$

$$Q_1^* = \frac{a_1 - MC}{2b_1}$$

By symmetry, $Q_2^* = \frac{a_2 - MC}{b_2}$

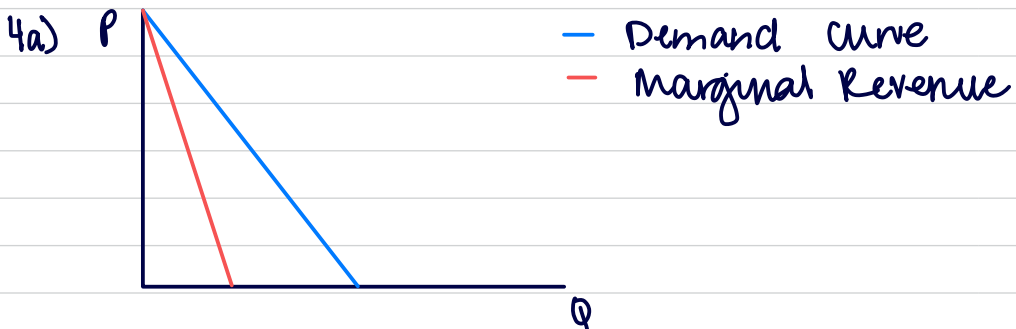
$$P_1 = a_1 - b_1 Q_1$$

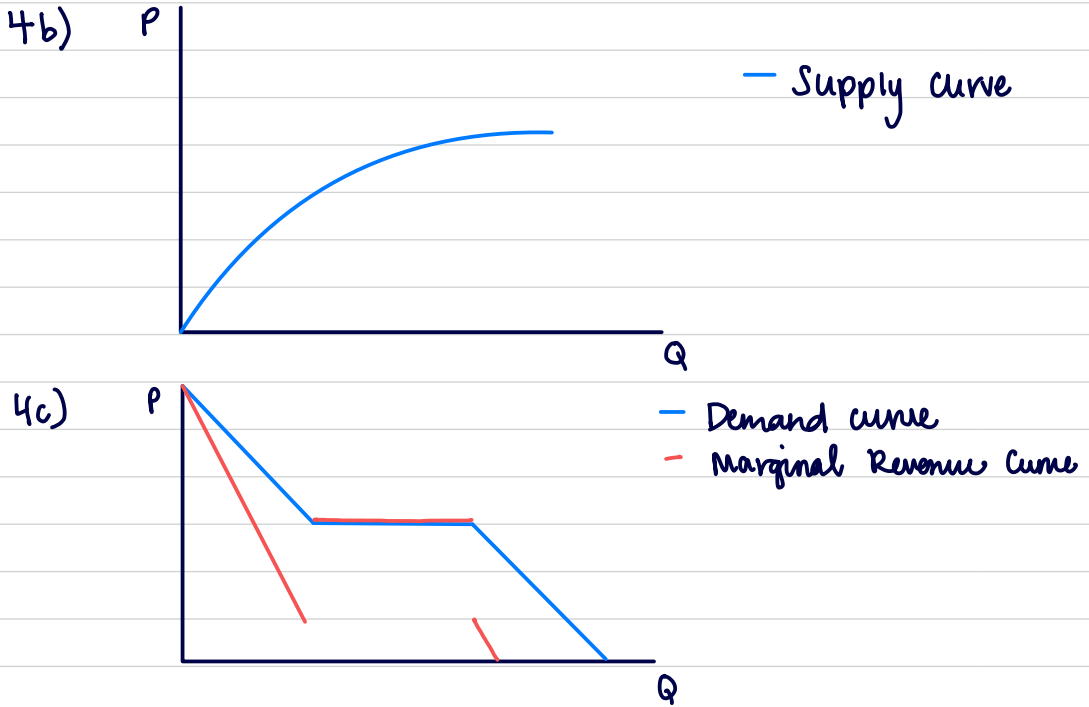
$$= a_1 - b_1 \left(\frac{a_1 - MC}{2b_1} \right)$$

$$= \frac{a_1}{2} - \frac{MC}{2}$$

$$P_2 = \frac{a_2}{2} - \frac{MC}{2}$$

Since $a_1 > a_2$, $P_1 > P_2$.





5) $P_1 = 3 - Q_1$ shops in person

$P_2 = 5 - Q_2$ shops online

For Group 1, monopolists maximize:

$$\pi = p \cdot Q - 1 \cdot Q = 3Q - Q^2 - Q = 2Q - Q^2$$

$$\text{FOC: } 2 - 2Q = 0$$

$$Q = 1$$

$$P = 3 - 1 = 2$$

For Group 2, monopolists maximize:

$$\pi = 5Q - Q^2 - Q = 4Q - Q^2$$

$$\text{FOC: } 4 - 2Q = 0$$

$$Q = 2$$

$$p = 5 - 2 = 3$$

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Now let $MC = Q$, $Q = Q_1 + Q_2$. Monopolists maximise:

$$\begin{aligned}\Pi &= P_1 Q_1 + P_2 Q_2 - MC Q_1 - MC Q_2 \\ &= (3 - Q_1) Q_1 + (5 - Q_2) Q_2 - \frac{1}{2} (Q_1 + Q_2)^2\end{aligned}$$

$$FOC: 3 - 2Q_1 - Q_1 - Q_2 = 0$$

$$3 - 3Q_1 - Q_2 = 0$$

$$3Q_1 + Q_2 = 3$$

$$5 - 2Q_2 - Q_2 - Q_1 = 0$$

$$3Q_2 + Q_1 = 5$$

$$Q_2 = 3 - 3Q_1$$

$$3(3 - 3Q_1) + Q_1 = 5$$

$$9 - 9Q_1 + Q_1 = 5$$

$$8Q_1 = 4$$

$$Q_1 = \frac{1}{2} \quad Q_2 = \frac{3}{2}$$

$$P_1 = \frac{5}{2} \quad P_2 = \frac{7}{2}$$