Group31 ETC5242 Assignment 1

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This report conducts statistic analysis of bank customers, based on a database including data of 200 customers' annual gross income and education level.

Task 1

First part would be some distribution plots and descriptive statistics of the data, and fitted distribution model.

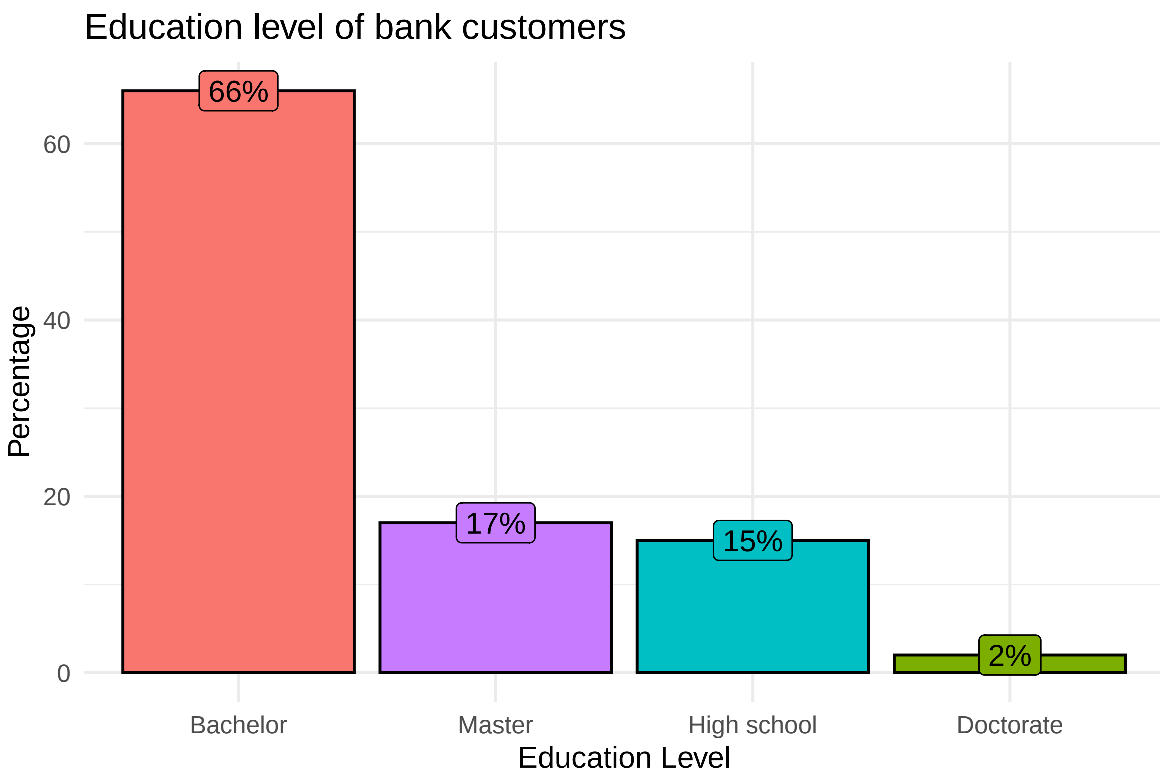
Load data and libraries we needed:

```
# Load necessary libraries
library(tidyverse)
library(kableExtra)
library(fitdistrplus)
library(ggplot2)
library(MASS)
library(boot)

# Load the dataset
bank_data <- read_csv("data/banksurvey.csv")
```

The first plot is about the education level of bank customers.

```
# Descriptive bar plot
bank_data %>%
  group_by(education) %>%
  summarise(perc = n()/nrow(bank_data) * 100) %>%
  as.data.frame() %>%
  ggplot(aes(x = fct_reorder(education, -perc),
             y = perc,
```

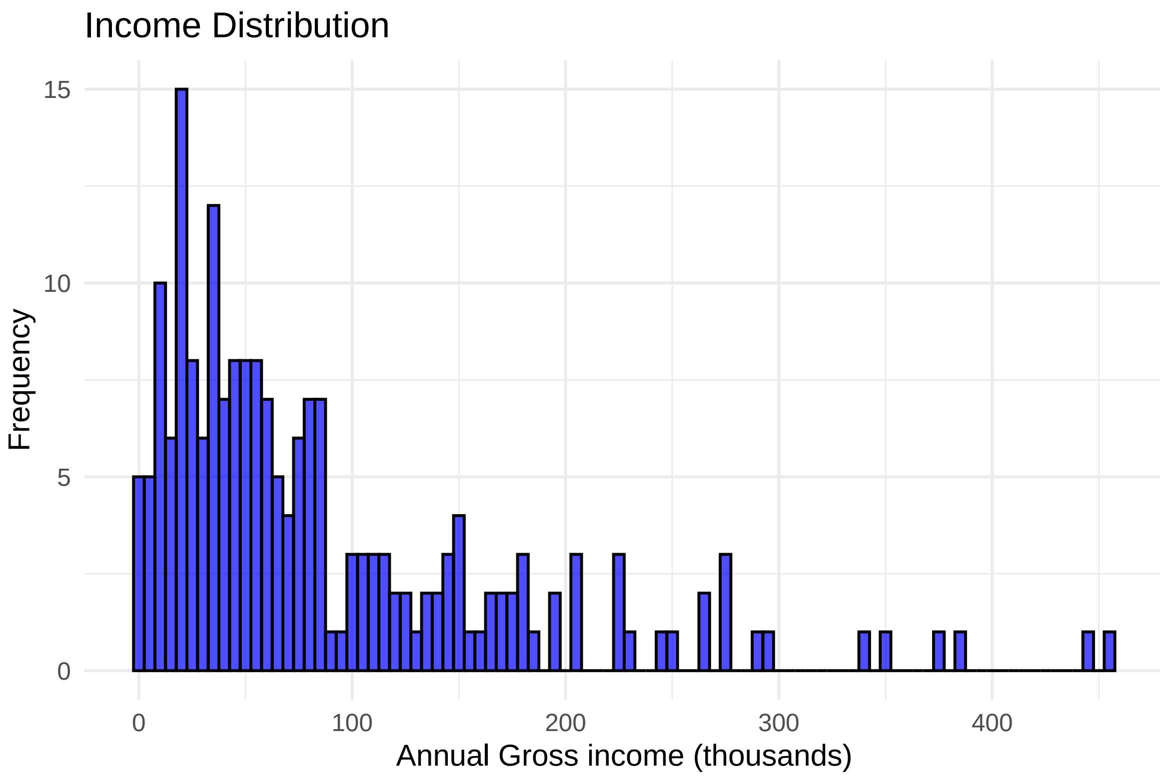
```
             fill = education)) +
  geom_col(color = "black") +
  theme_minimal() +
  labs(x = "Education Level",
       y = "Percentage",
       title = "Education level of bank customers") +
  geom_label(aes(label = paste0(perc, "%"),
                 vjust = 0.5)) +
  guides(fill = "none")
```



The bar plot shows that most of the surveyed bank customers achieved a highest education level of bachelor's degree (66%), followed by master's and high school with similar percentage, and 2% hold a doctorate degree.

The second part is visualizing the distribution of customer incomes with a histogram, which gives us insight into how incomes are spread across our customer sample.

```r
# Histogram of incomes
ggplot(bank_data, aes(x = income)) +
  geom_histogram(binwidth = 5, fill = "blue", color = "black", alpha = 0.7) +
  labs(title = "Income Distribution", x = "Annual Gross income (thousands)", y = "Frequency")
  theme_minimal()
```



The histogram reveals a right-skewed distribution, with most customers having incomes clustered in the lower range and a tail extending toward higher income levels.

We calculate the summary statistics to get more detailed insights into the distribution of incomes.

```r
# Descriptive statistics
summary_stats <- bank_data |>
  summarize(
    Mean = round(mean(income), 1),
    Median = median(income),
    SD = round(sd(income), 1),
```

```r
    Min = min(income),
    Max = max(income)
  )

kable(summary_stats,
      caption = "Statistics of annual gross income of bank customers (thousands)")
```

Table 1: Statistics of annual gross income of bank customers (thousands)

Mean	Median	SD	Min	Max
89.2	58.7	87.6	0.1	453.9

The average income is $89,200, but the median income of $58,700 provides a more accurate representation of the typical customer due to the skewed distribution. The standard deviation of $87,600 reflects a significant variation in incomes.

The next part will be exploring fitting models, including normal, exponential and gamma distribution.

```r
# Fit normal distribution
fit_norm <- fitdist(bank_data$income, "norm")
# Fit exponential distribution
fit_exp <- fitdist(bank_data$income, "exp")
# Fit gamma distribution
fit_gamma <- fitdist(bank_data$income, "gamma")

# Normal distribution summary
tibble(Parameter = c("Mean", "Standard Deviation"),
       Estimate = fit_norm$estimate,
       'Standard Error' = fit_norm$sd) %>%
  kable(caption = "Maximum likelihood estimates of fitted normal distribution model")
```

Table 2: Maximum likelihood estimates of fitted normal distribution model

Parameter	Estimate	Standard Error
Mean	89.2250	6.176770
Standard Deviation	87.3526	4.367631

```r
# Exponential distribution summary
tibble(Parameter = "Rate",
       Estimate = fit_exp$estimate,
       'Standard Error' = fit_exp$sd) %>%
  kable(caption = "Maximum likelihood estimates of fitted exponential distribution model")
```

Table 3: Maximum likelihood estimates of fitted exponential distribution model

Parameter	Estimate	Standard Error
Rate	0.0112076	0.0007861

```r
# Gamma distribution summary
tibble(Parameter = c("Shape", "Rate"),
       Estimate = fit_gamma$estimate,
       'Standard Error' = fit_gamma$sd) %>%
  kable(caption = "Maximum likelihood estimates of fitted gamma distribution model")
```

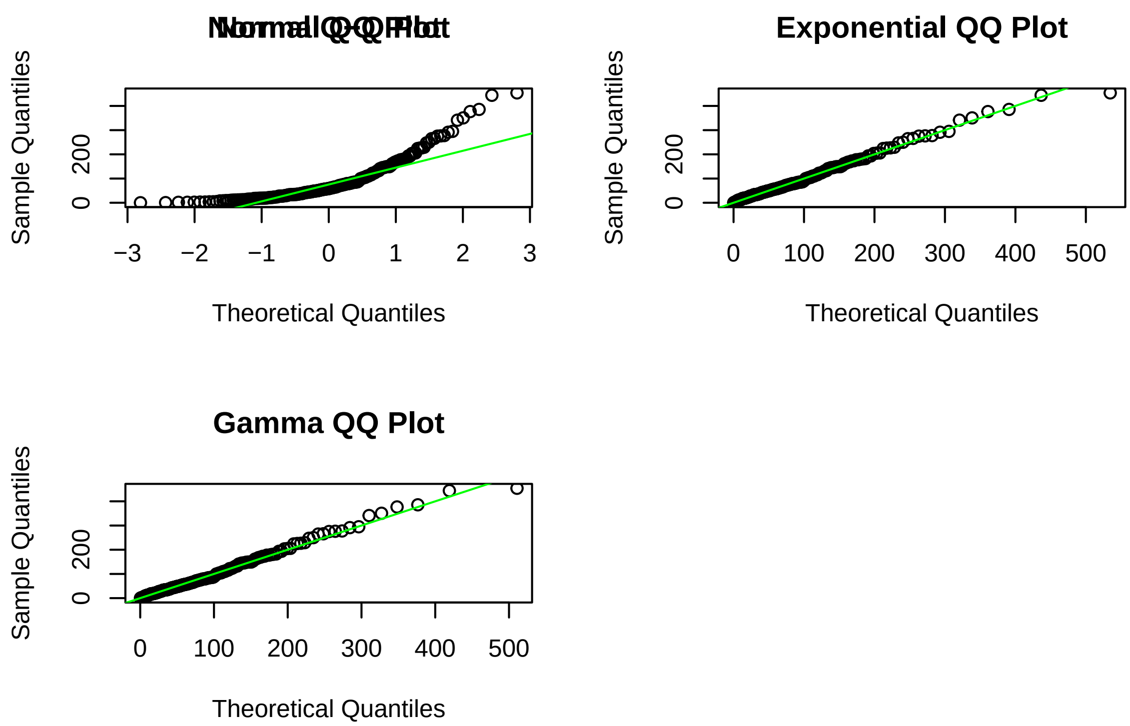
Table 4: Maximum likelihood estimates of fitted gamma distribution model

Parameter	Estimate	Standard Error
Shape	1.083319	0.0953152
Rate	0.012132	0.0013355

Above tables showed the maximum likelihood estimates of the 3 fitted models.

```r
# QQ plots for the normal, exponential, and gamma distributions
par(mfrow = c(2, 2)) # Set up plot window for multiple plots

# QQ plot for normal distribution
qqnorm(bank_data$income)
qqline(bank_data$income,col = "green")
title("Normal QQ Plot")

# QQ plot for exponential distribution
qqplot(qexp(ppoints(length(bank_data$income)),
            rate = fit_exp$estimate),
       sort(bank_data$income),
       xlab = "Theoretical Quantiles",
       ylab = "Sample Quantiles")
```

```r
abline(0, 1,col = "green")
title("Exponential QQ Plot")

# QQ plot for gamma distribution
qqplot(qgamma(ppoints(length(bank_data$income)),
              shape = fit_gamma$estimate[1],
              rate = fit_gamma$estimate[2]),
       sort(bank_data$income),
       xlab = "Theoretical Quantiles",
       ylab = "Sample Quantiles")
abline(0, 1,col = "green")
title("Gamma QQ Plot")
```



Seems like both exponential and gamma distribution models fit the data better than normal distribution. In particular, exponential distribution is slightly better than gamma.

Task 2

In this section, it will focus on estimating the 80th percentile of annual income of customers. In addition, 95% confidence intervals based on each of the estimators will be calculated.

```
# Estimate the 80th percentile using estimators based on fitted model
q80_normal <- qnorm(0.80, mean = fit_norm$estimate[1], sd = fit_norm$estimate[2])
q80_exponential <- qexp(0.80, rate=fit_exp$estimate)
q80_gamma <- qgamma(0.80, shape=fit_gamma$estimate[1], rate=fit_gamma$estimate[2])

# Create a tibble based on these results
results <- tibble(
  Distribution = c("Normal", "Exponential", "Gamma"),
  `80th Percentile Estimate` = c(q80_normal, q80_exponential, q80_gamma))


kable(results, caption = "80th Percentile Estimates based on Different Models")
```

Table 5: 80th Percentile Estimates based on Different Models

Distribution	80th Percentile Estimate
Normal	162.7428
Exponential	143.6021
Gamma	142.7391

The parameter of interest is the 80th percentile of the annual income distribution. It indicates the income level at which 80% of the customers earn less than this amount.

Although we can estimate the 80th percentile of annual income based on a particular model, it is also possible to use a statistic called 'sample quantile', which is an estimator that does not assume any specific model.

```
# Using sample quantile to estimate
q80_sample <- quantile(bank_data$income, probs = 0.80)

new_results <- tibble(
  Distribution = c("Normal", "Exponential", "Gamma","Sample"),
  `80th Percentile Estimate` = c(q80_normal, q80_exponential, q80_gamma,q80_sample))


kable(new_results, caption = "80th Percentile Estimates based on Different Models")
```

Table 6: 80th Percentile Estimates based on Different Models

Distribution	80th Percentile Estimate
Normal	162.7428
Exponential	143.6021
Gamma	142.7391
Sample	146.5600

```
# Calculate the 95% CI for the estimates

# Normal distribution 95% CI
se_normal <- fit_norm$estimate[2] / sqrt(length(bank_data$income))
ci_normal <- q80_normal + c(-1.96, 1.96) * se_normal

# Exponential distribution 95% CI
se_exp <- sqrt(1 / (length(bank_data$income) * fit_exp$estimate^2))
ci_exp <- q80_exponential + c(-1.96, 1.96) * se_exp

# Gamma distribution 95% CI
se_gamma <- sqrt(fit_gamma$estimate[1] / (fit_gamma$estimate[2]^2 * length(bank_data$income)
ci_gamma <- q80_gamma + c(-1.96, 1.96) * se_gamma

ci_results <- tibble(
  Distribution = c("Normal", "Exponential", "Gamma"),
  `80th Percentile Estimate` = c(q80_normal, q80_exponential, q80_gamma),
  `Lower Bound (95% CI)` = c(ci_normal[1], ci_exp[1], ci_gamma[1]),
  `Upper Bound (95% CI)` = c(ci_normal[2], ci_exp[2], ci_gamma[2])
)

kable(ci_results, caption = "80th Percentile Estimates and 95% Confidence Intervals for Diff
```

Table 7: 80th Percentile Estimates and 95% Confidence Intervals for Different Distributions

Distribution	80th Percentile Estimate	Lower Bound (95% CI)	Upper Bound (95% CI)
Normal	162.7428	150.6364	174.8493
Exponential	143.6021	131.2361	155.9681
Gamma	142.7391	130.8490	154.6293

Task 3

```
set.seed(2420)

true_mean <- 100
lambda <- 1 / true_mean
n_sample <- 100
n_bootstrap <- 1000

# Generate simulated data following an exponential distribution
original_data <- rexp(n_sample, rate = lambda)

# Define a function to calculate the 80th percentile
estimate_quantiles <- function(bank_data, indices) {
  sample_data <- bank_data[indices]  # Bootstrap

   # Fit normal, exponential, and gamma distributions
  normal_fit <- fitdist(sample_data, "norm")
  exp_fit <- fitdist(sample_data, "exp")
  gamma_fit <- fitdist(sample_data, "gamma")

  # Return to 80th percentile
  return(
  c(
    qnorm(0.80, mean = normal_fit$estimate[1], sd = normal_fit$estimate[2]),
    qexp(0.80, rate = exp_fit$estimate),
    qgamma(0.80, shape = gamma_fit$estimate[1], rate = gamma_fit$estimate[2])
  ))
}

# Bootstrap sampling
bootstrap_results <- boot(data = original_data, statistic = estimate_quantiles, R = n_bootst

# Extract results
normal_estimates <- bootstrap_results$t[, 1]
exp_estimates <- bootstrap_results$t[, 2]
gamma_estimates <- bootstrap_results$t[, 3]

# Calculate the bias and standard deviation
true_80th_percentile <- qexp(0.80, rate = lambda)
bias_normal <- mean(normal_estimates) - true_80th_percentile
```

```r
bias_exp <- mean(exp_estimates) - true_80th_percentile
bias_gamma <- mean(gamma_estimates) - true_80th_percentile

sd_normal <- sd(normal_estimates)
sd_exp <- sd(exp_estimates)
sd_gamma <- sd(gamma_estimates)

# Print results
cat("Bias and SD for Normal Estimate:\n Bias:", bias_normal, " SD:", sd_normal, "\n")
```

```
Bias and SD for Normal Estimate:
 Bias: 4.532751  SD: 13.78083
```

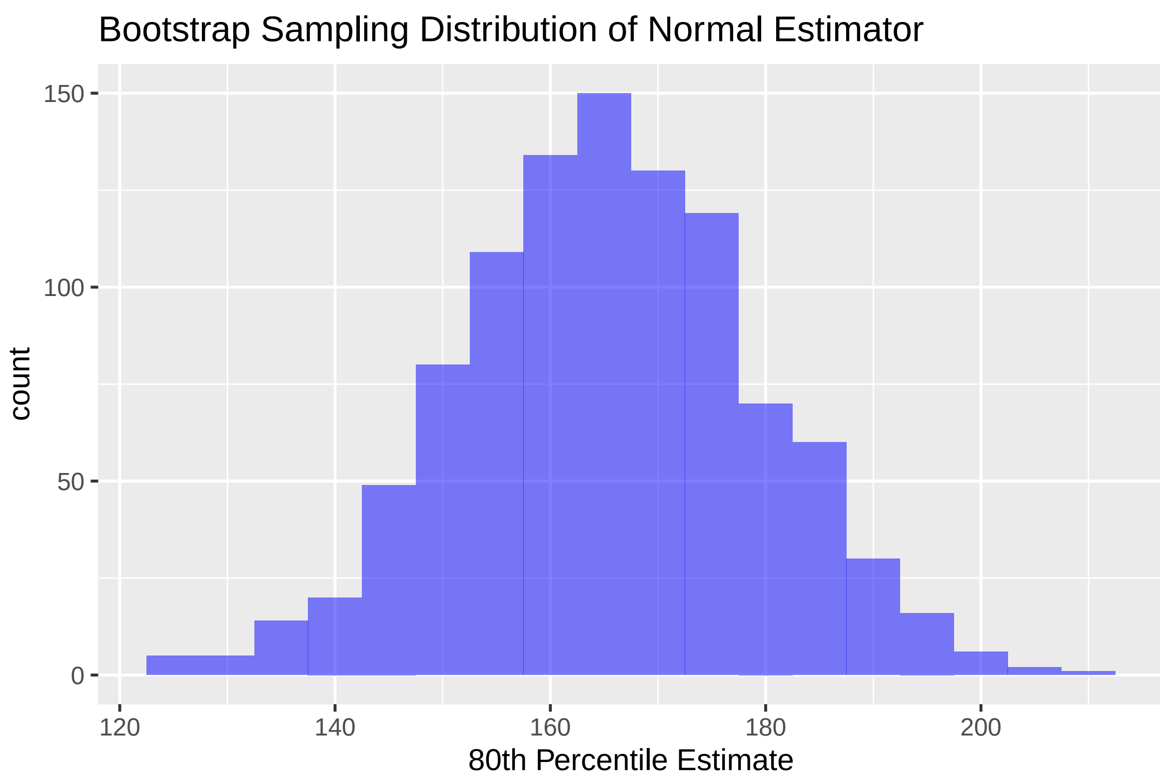
```r
cat("Bias and SD for Exponential Estimate:\n Bias:", bias_exp, " SD:", sd_exp, "\n")
```

```
Bias and SD for Exponential Estimate:
 Bias: -7.061224  SD: 13.69364
```

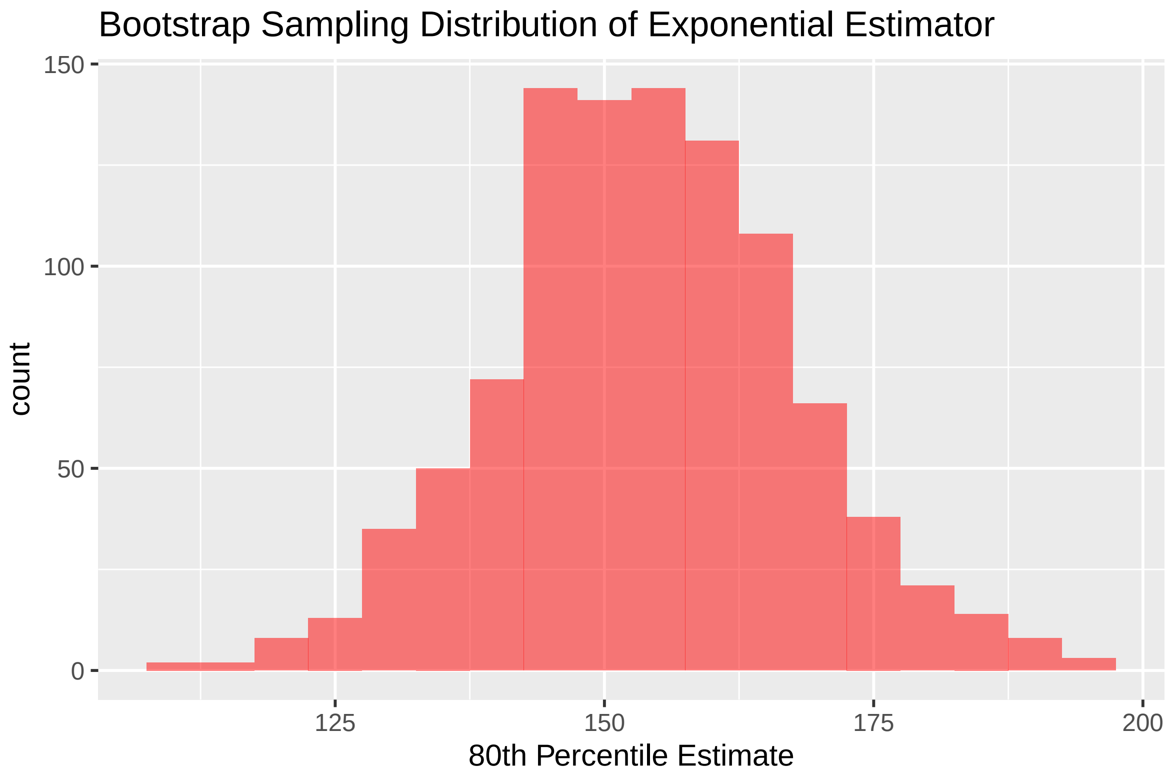
```r
cat("Bias and SD for Gamma Estimate:\n Bias:", bias_gamma, " SD:", sd_gamma, "\n")
```

```
Bias and SD for Gamma Estimate:
 Bias: -10.33077  SD: 13.59149
```

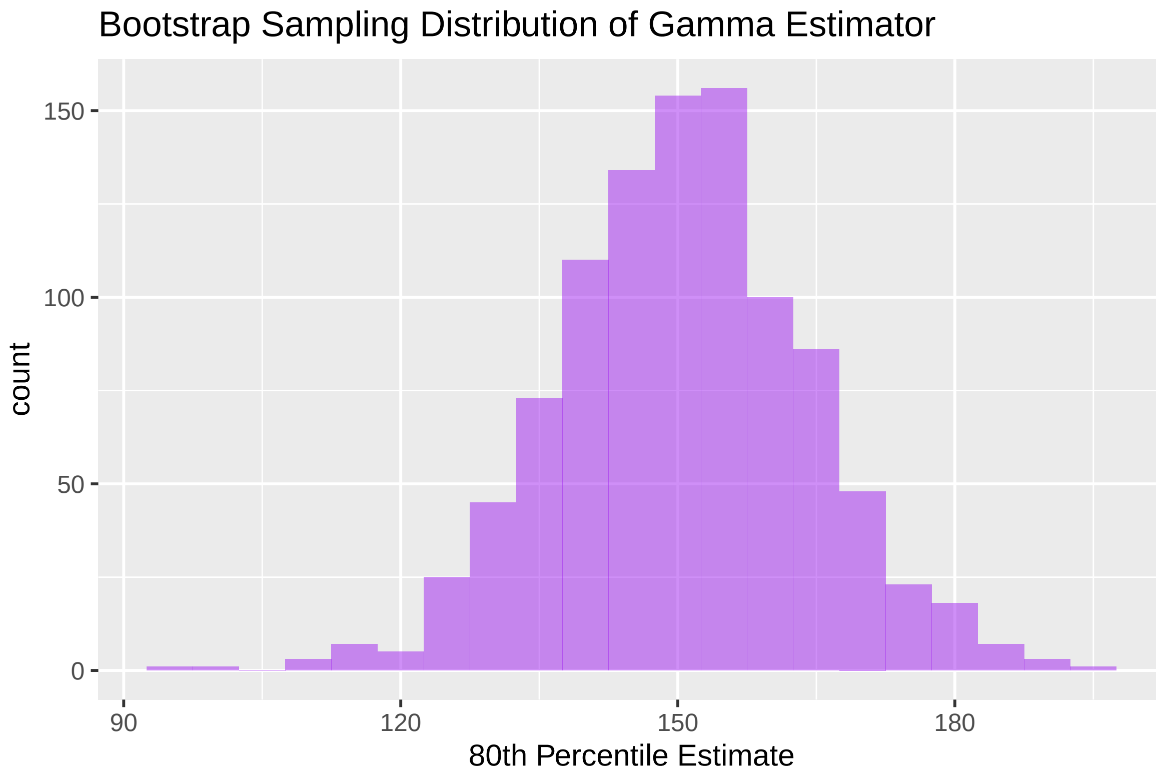
```r
# Visualize the sampling distribution of the normal estimator
ggplot(data.frame(normal_estimates), aes(x = normal_estimates)) +
  geom_histogram(binwidth = 5, fill = "blue", alpha = 0.5) +
  labs(title = "Bootstrap Sampling Distribution of Normal Estimator", x = "80th Percentile Es
```




```
# Visualize the sampling distribution of the exponential estimator
ggplot(data.frame(exp_estimates), aes(x = exp_estimates)) +
  geom_histogram(binwidth = 5, fill = "red", alpha = 0.5) +
  labs(title = "Bootstrap Sampling Distribution of Exponential Estimator", x = "80th Percenti
```




```
#Visualize the sampling distribution of the gamma estimator
ggplot(data.frame(gamma_estimates), aes(x = gamma_estimates)) +
  geom_histogram(binwidth = 5, fill = "purple", alpha = 0.5) +
  labs(title = "Bootstrap Sampling Distribution of Gamma Estimator", x = "80th Percentile Es
```



Task 4

This section will be focusing on the relationship between customer education level and income.

```
# Recode education levels into two groups: "University Degree" vs "No University Degree"
bank_data <- bank_data |>
  mutate(education_group = ifelse(education %in% c("Bachelor", "Master", "Doctorate"),
                                  "University Degree", "No University Degree"))

# Group summary of income by education group
income_education_summary <- bank_data |>
  group_by(education_group) |>
  summarise(mean_income = round(mean(income), 1),
            sd_income = round(sd(income), 1),
            n = n())
```

```r
kable(income_education_summary,
      caption = "Income comparison of university degree holders")
```

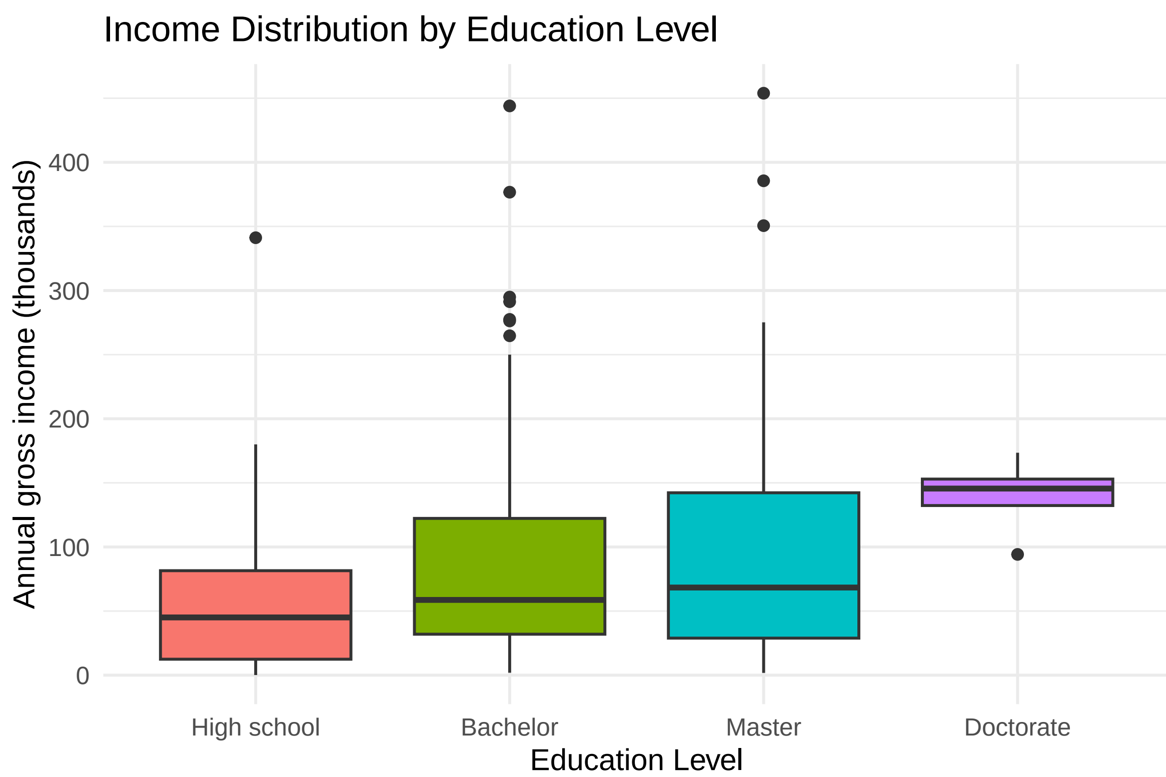
Table 8: Income comparison of university degree holders

education_group	mean_income	sd_income	n
No University Degree	65.1	72.8	30
University Degree	93.5	89.4	170

University degree holders have a higher mean income and standard deviation, with a difference of $28,400 in mean compared to non-univeristy degree holders.

Visualizing Income Distribution by Education Level

```r
# Order education levels
bank_data$education <- factor(bank_data$education,
                         levels = c("High school",
                                    "Bachelor",
                                    "Master",
                                    "Doctorate"))

# Boxplot comparing income across all education levels
ggplot(bank_data, aes(x = education, y = income, fill = education)) +
  geom_boxplot() +
  theme_minimal() +
  ggtitle('Income Distribution by Education Level') +
  xlab('Education Level') +
  ylab('Annual gross income (thousands)') +
  guides(fill = "none")
```



The boxplot showed that the median of annual gross income increases as the education level gets higher. While there is a huge gap between median income of master and doctorate degree holders, the range of doctorate's income is much narrower, and the maximum point is lower than the other 3 education groups as well.

Confidence Interval for the Difference in Mean Incomes

To compare the difference in income of customer who hold a university degree or not, t-test will be conducted to calculate the confidence interval for the difference in mean incomes.

```
# Subset data into two groups
university_graduates <- bank_data |> filter(education_group == "University Degree")
non_graduates <- bank_data |> filter(education_group == "No University Degree")

# Calculate the mean difference and confidence interval
mean_diff <- mean(university_graduates$income) - mean(non_graduates$income)

# Perform a t-test to get the confidence interval
```

```r
t_test_result <- t.test(university_graduates$income, non_graduates$income, var.equal = FALSE)

# Print the results
kable(round(mean_diff, 1),
      caption = "Difference in mean income (thousands)",
      col.names = "")
```

Table 9: Difference in mean income (thousands)

28.4

```r
kable(round(t_test_result$conf.int, 1),
      caption = "95% confidence interval of difference in mean income (thousands)",
      col.names = "")
```

Table 10: 95% confidence interval of difference in mean income (thousands)

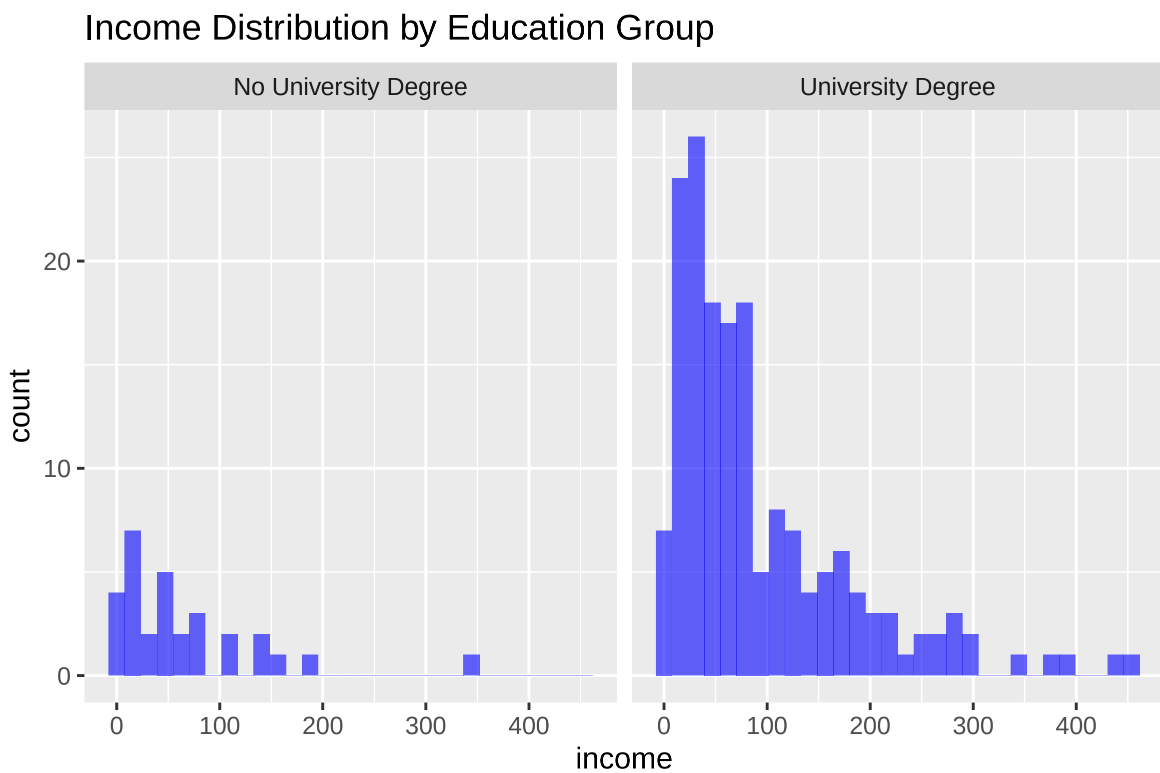
-1.7
58.5

The difference in mean of annual gross income of customers who hold a university degree or not is $28,400, with a 95% confidence interval of -$1,700 and $58,500.

Central Limit Theorem

This part discusses whether it is appropriate to use the Central Limit Theorem when calculating the above confidence interval.

```r
# Histograms to check normality
ggplot(bank_data, aes(x = income)) +
  geom_histogram(bins = 30, fill = 'blue', alpha = 0.6) +
  facet_wrap(~education_group) +
  ggtitle('Income Distribution by Education Group')
```




```
# Check the sample sizes of each group
table(bank_data$education_group) %>%
  kable(col.names = c("Education level", "Count"),
        caption = "Sample size of education groups")
```

Table 11: Sample size of education groups

Education level	Count
No University Degree	30
University Degree	170

To use t-test to calculate confidence interval, the sample needs to be normally distributed. But according to the above plots, both groups are not normally distributed. In this case, Central Limit Theorem is needed, which states that the sample mean gets closer to normal distribution as the sample size increases, even if the data are not under normal distribution. Since both groups have a sample size of at least 30, Central Limit Theorem is applicable and t-test could still be used to calculate confidence interval.

Causation of Education and Income

This section of analysis about education level and income seem to show that the average income would be higher as education level increases. However, these are not evidences of a causal effect. As shown in the t-test results that the confidence interval showed a large positive difference, this could imply that income has a positive relationship with education level, but it could not be summarized with causation that providing greater education would lead to higher incomes.

There could be other underlying factors that are not revealed in this analysis, such as people with higher income have adequate financial support for education, or people in higher level positions are likely to be required by company to complete certain level of education. A positive relationship does not imply causation.

Task 5

This section uses Bayesian inference approach compares the difference in 80th quantile of bank customers annual income of those who holds an university degree or not. Assume an exponential distribution for the annual income, so conjugate prior Gamma(,), and posterior | x1,x2,…,xn Gamma(˜ = + n, ˜ = +nx). Next is simulate 10,000 values from it and calculate the difference in 80th quantile of different groups, lastly calculate 95% credible interval for it.

```r
# Prior
n0 <- nrow(non_graduates)
n1 <- nrow(university_graduates)
alpha <- 1
beta <- 1

# Posterior
alpha_tilda0 <- alpha + n0
alpha_tilda1 <- alpha + n1
beta_tilda0 <- beta + sum(non_graduates$income)
beta_tilda1 <- beta + sum(university_graduates$income)

tibble(Parameter = c("N_0", "N_1","Alpha","Beta",
                     "Alpha_tilda_0","Alpha_tilda_1",
                     "Beta_tilda_0","Beta_tilda_1"),
       Value = c(n0,n1,alpha,beta,alpha_tilda0,alpha_tilda1,
                 beta_tilda0,beta_tilda1),
       Description = c("Number of those who doesn't have a university degree",
                       "Number of those who has a university degree",
```

```
                          "Shape of Gamma distribution",
                          "Rate of Gamma distribution",
                          "Updated shape of posterior(Gamma distribution) for non-uni",
                          "Updated shape of posterior(Gamma distribution) for uni",
                          "Updated rate of posterior(Gamma distribution) for non-uni",
                          "Updated rate of posterior(Gamma distribution) for uni")) %>%
  kable(caption = "Parameter matrix")
```

Table 12: Parameter matrix

Parameter	Value	Description
N_0	30.0	Number of those who doesn't have a university degree
N_1	170.0	Number of those who has a university degree
Alpha	1.0	Shape of Gamma distribution
Beta	1.0	Rate of Gamma distribution
Alpha_tilda_0	31.0	Updated shape of posterior(Gamma distribution) for non-uni
Alpha_tilda_1	171.0	Updated shape of posterior(Gamma distribution) for uni
Beta_tilda_0	1952.8	Updated rate of posterior(Gamma distribution) for non-uni
Beta_tilda_1	15894.2	Updated rate of posterior(Gamma distribution) for uni

So, the posterior for non-uni is X ~ Gamma(31,1952.8), and the posterior for uni X~Gamma(171,15894.2). The below code simulates 10,000 examples from the posterior distribution of the difference between the 80th percentile incomes between the two groups.

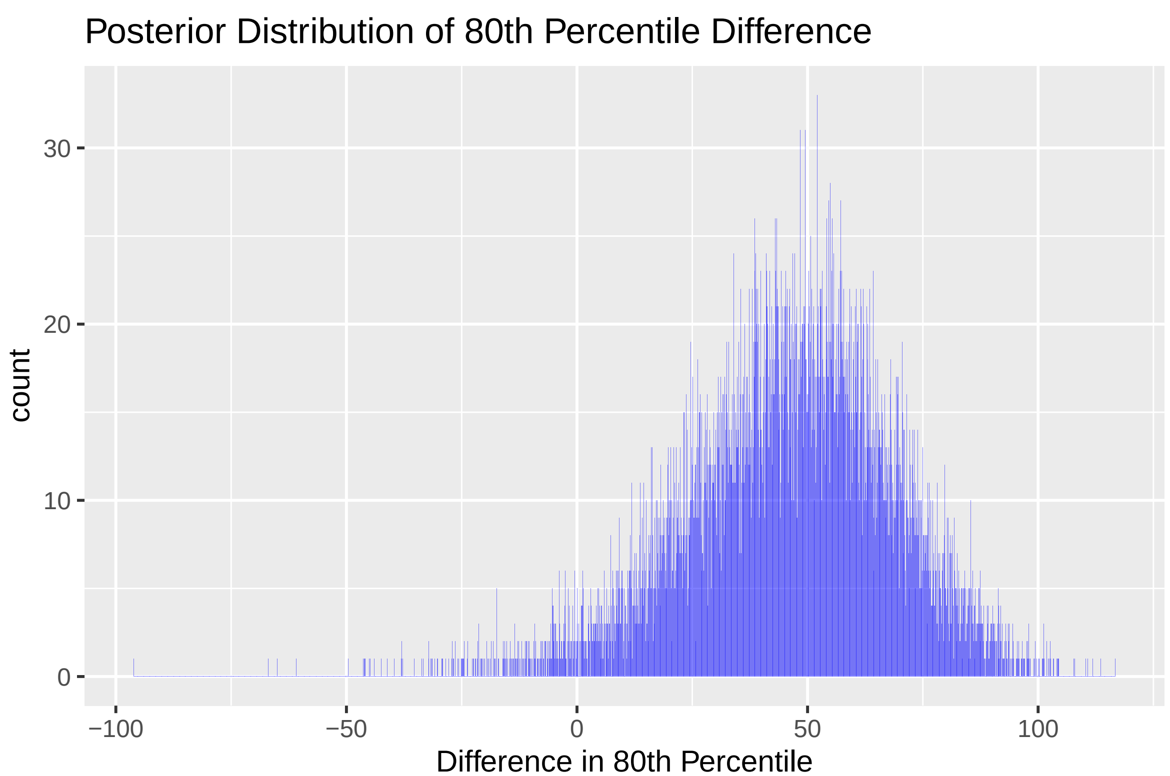
```
# Simulate 10,000 examples
set.seed(2420)
lambda_0_samples <- rgamma(10000, shape = alpha_tilda0, rate = beta_tilda0)
lambda_1_samples <- rgamma(10000, shape = alpha_tilda1, rate = beta_tilda1)


p80_0 <- log(5) / lambda_0_samples
p80_1 <- log(5) / lambda_1_samples


p80_diff <- p80_1 - p80_0


credible_interval <- quantile(p80_diff, c(0.025, 0.975))
```

```
tibble('Credible interval' = credible_interval) %>%
kable(caption = "95% credible interval of difference between 80th percentile incomes")
```

Table 13: 95% credible interval of difference between 80th percentile incomes

Credible interval
-2.782999
86.071030

```
# Plot results
ggplot(data.frame(p80_diff), aes(x = p80_diff)) +
  geom_histogram(binwidth = 0.1, fill = "blue", alpha = 0.5) +
  labs(title = "Posterior Distribution of 80th Percentile Difference", x = "Difference in 80
```



The 95% credible interval for the difference between the 80th percentile incomes between the two groups is -2.78 and 86.07. The interval is mostly positive, which indicates that bank customers with university degree is more likely to earn a higher income than those who don't under 80th percentile.